DIS in the dipole picture at one loop with massive quarks

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ILCAC seminar

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HELSINKI INSTITUTE OF PHYSICS

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Outline

Outline of this talk

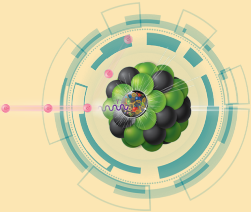
- Eikonal scattering and gluon saturation
- DIS in the dipole picture at NLO: massless quarks

  Balitsky & Chirilli 2010, G. Beuf Phys. Rev. D **94** (2016) no.5, 054016, Phys. Rev. D **96** (2017) no.7, 074033 H. Hänninen, T.L., R. Paatelainen, Annals Phys. **393** (2018), 358-412
- Massive quarks

  Beuf, T.L. Paatelainen, Phys.Rev.Lett. 129 (2022) 7, 072001, Phys.Rev.D 104 (2021) 5, 056032, Phys.Rev.D 106 (2022) 3, 034013

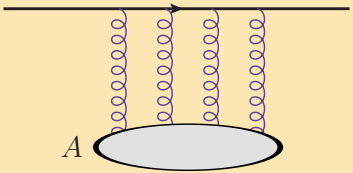
Process of interest

DIS cross section at high energy



High energy collisions as eikonal scattering

Eikonal scattering off target of glue



How to measure small-x glue?

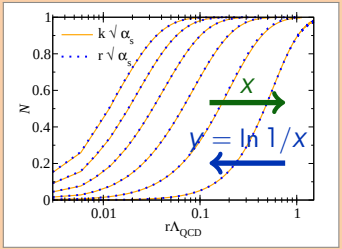
- Dilute probe through target color field
- At high energy interaction is **eikonal**,
  x (2d $\perp$ coordinate) conserved in scattering
  (T-matrix diagonal in $\perp$ coordinate space)

- Amplitude for quark: **Wilson line**

$$\mathbb{P}\exp\left\{-ig\int^{x^+} dy^+ A^-(y^+, x^-, \mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_c)$$

- Amplitude for color dipole

$$\mathcal{N}(r = |\mathbf{x}-\mathbf{y}|) = 1 - \left\langle \frac{1}{N_c}\,\mathrm{tr}\, V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

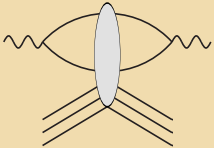


- $r = 0$: color transparency, $r \gg 1/Q_s$: saturation , nonperturbative!

Dipole picture of DIS

Limit of small x, i.e. high γ^*-target energy

Leading order

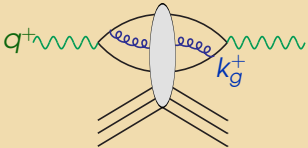


- $\gamma^* \to q\bar{q}$ in vacuum
- $q\bar{q}$ interacts eikonally with target
- σ^{tot} is $2\times$Im-part of amplitude

"Dipole model": Nikolaev, Zakharov 1991

Many fits to HERA data, starting with Golec-Biernat, Wüsthoff 1998

Leading Log: add **soft** gluon



- Soft gluon: large logarithm

$$\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

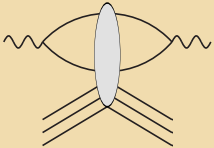
Absorb into renormalization of target:

BK equation Balitsky 1995, Kovchegov 1999

Dipole picture of DIS

Limit of small x, i.e. high γ^*-target energy

Leading order



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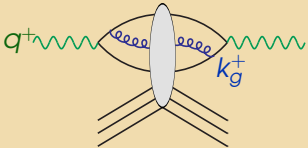

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Absorb into renormalization of target:

BK equation Balitsky 1995, Kovchegov 1999

NLO: the same gluon with full kinematics

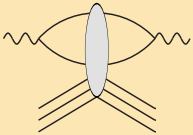
Why light cone perturbation theory

Bjorken, Kogut, Soper: "Quantum Electrodynamics at ∞ Momentum: Scattering from an External Field" Phys.Rev. D (1971)

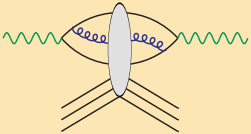
LCPT: ideal computational tool for HE scattering, dipole factorization

Rotational symmetry already broken by physical situation

- LC quantize γ^*
  - Clean perturbative object
  - Evolution $x^+ = -\infty \to 0$ in vacuum
  - Encoded in γ^* **light cone wave function**
  - Less work than covariant P.T.
- Dipole-target amplitude
  - Nonperturbative physics of target
  - Eikonal, $\perp$ coordinate space
  - Naturally includes gluon saturation
  - Satisfies BK evolution equation in $\ln 1/x_{Bj}$

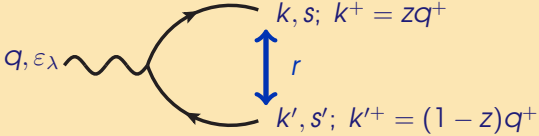


$$\sigma_{\text{tot}}^{\gamma^*+p} = \left|\psi_{T,L}^{\gamma^*\to q\bar{q}}\right|^2 \otimes \sigma_{\text{tot}}^{q\bar{q}+p}$$



$$+ \left|\psi_{T,L}^{\gamma^*\to q\bar{q}g}\right|^2 \otimes \sigma_{\text{tot}}^{q\bar{q}g+p}$$

Example: leading order $\psi^{\gamma^*\to q\bar{q}}$



- Matrix element

$$e\bar{u}_s(k)\not{\varepsilon}_\lambda v_{s'}(k') \quad ; \quad s, s' = \pm\frac{1}{2}; \quad \lambda = 0 = L, \quad \lambda = \pm 1 = T$$

- Energy denominator $(q^- - k^- - k'^-)^{-1}$

$$= -\left(\frac{Q^2}{2q^+} + \frac{\mathbf{k}^2 + m^2}{2zq^+} + \frac{\mathbf{k}^2 + m^2}{2(1-z)q^+}\right) = \frac{-2q^+z(1-z)}{\underbrace{Q^2z(1-z) + m^2}_{\equiv\,\varepsilon^2} + \mathbf{k}^2}$$

Fourier-transform $\mathbf{k} \to \mathbf{r}$, sum over spins; result has Bessel K's that enforce $r \sim 1/Q$:

$$\left|\psi_T^{\gamma^*\to q\bar{q}}\right|^2 = \frac{\alpha_{\text{e.m.}}}{2\pi^2} N_c e_f \left(K_1^2(\varepsilon r)\left[z^2 + (1-z)^2\right] + m_f^2 K_0^2(\varepsilon r)\right)$$

$$\left|\psi_L^{\gamma^*\to q\bar{q}}\right|^2 = N_c e_f \frac{\alpha_{\text{e.m.}}}{2\pi^2} 4Q^2z^2(1-z)^2 K_0^2(\varepsilon r)$$

NLO DIS cross section with massless quarks

DIS at NLO: Fock state expansion

Balitsky & Chirilli 2010, Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

To be specific: want total γ^*-target cross section using optical theorem:

$$\sigma_\lambda^{\gamma^*} = 2\mathrm{Re}\left[(-i)\mathcal{M}^{\mathrm{fwd}}_{\gamma^*_\lambda \to \gamma^*_\lambda}\right],$$

$${}_i\langle\gamma_\lambda(\vec{q}\,', Q^2)|(\hat{\mathcal{S}}_E - \mathbf{1})|\gamma_\lambda(\vec{q}, Q^2)\rangle_i = 2q^+(2\pi)\delta(q'^+ - q^+)i\mathcal{M}^{\mathrm{fwd}}_{\gamma^*_\lambda \to \gamma^*_\lambda}.$$

$\hat{\mathcal{S}}_E$: eikonal scattering $\Longrightarrow$ Wilson line in coordinate space.
At NLO need Fock state decomposition of $|\gamma_\lambda(\vec{q}, Q^2)\rangle_i$ (and ${}_i\langle\gamma_\lambda(\vec{q}\,', Q^2)|$) up to g^2:

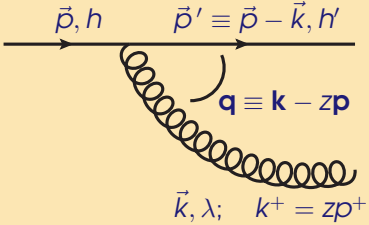
$$|\gamma_\lambda(\vec{q}, Q^2)\rangle_i = \sqrt{Z_{\gamma^*}}\Bigg[|\gamma_\lambda(\vec{q}, Q^2)\rangle + \sum_{q\bar{q}} \Psi^{\gamma^* \to q\bar{q}}|q(\vec{k}_0, h_0)\bar{q}(\vec{k}_1, h_1)\rangle$$
$$+ \sum_{q\bar{q}g} \Psi^{\gamma^* \to q\bar{q}g}|q(\vec{k}_0, h_0)\bar{q}(\vec{k}_1, h_1)g(\vec{k}_2, \sigma)\rangle + \cdots\Bigg]$$

with **Light Cone Wave Functions** $\Psi^{\gamma^* \to q\bar{q}}$ **and** $\Psi^{\gamma^* \to q\bar{q}g}$

DIS at NLO: procedure

Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

1. Evaluate LCPT diagrams
   - $\Psi^{\gamma^* \to q\bar{q}}$ to 1 loop
   - $\Psi^{\gamma^* \to q\bar{q}g}$ at tree level
2. Fourier-transform to transverse coordinate
3. Square to get ${}_i\langle\gamma_\lambda(\vec{q}\,', Q^2)|(\hat{S}_E - \mathbf{1})|\gamma_\lambda(\vec{q}, Q^2)\rangle_i$



- Intermediate ($\ni$ "final") state k^- denominators
- On-shell vertices, most importantly $q\bar{q}g$

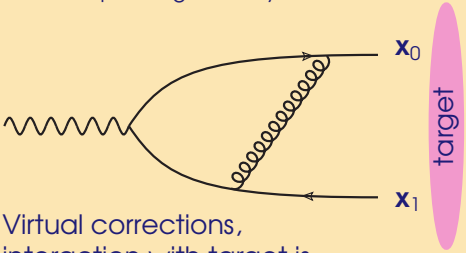
$$\left[\bar{u}_{h'}(p')\not{\varepsilon}^*_\lambda(k)u_h(p)\right] = \frac{-2}{z\sqrt{1-z}}\left[\left(1-\frac{z}{2}\right)\delta_{h',h}\delta^{ij} + \frac{z}{2}ih\delta_{h',h}\varepsilon^{ij}\right]\mathbf{q}^i\varepsilon^{*j}_\lambda,$$

(This is in $d = 4$, generalize for $d < 4$)
Note 2 index structures for massless quarks.

- Regularize: in $2 - 2\varepsilon$-dim $\perp$ + cutoff in k^+

DIS at NLO: real and virtual corrections

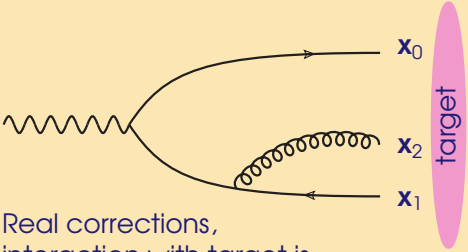
Here example diagams only



Virtual corrections,
interaction with target is

$$\mathcal{N}_{q\bar{q}}(\mathbf{x}_0, \mathbf{x}_1)$$

+ UV divergence in loop



Real corrections,
interaction with target is

$$\mathcal{N}_{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$$

UV (!) divergence in $\mathbf{x}_2$-integral

These UV-divergences cancel because for Wilson lines $\in$ SU(N_c)

$$\mathcal{N}_{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_0) = \mathcal{N}_{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_1) = \mathcal{N}_{q\bar{q}}(\mathbf{x}_0, \mathbf{x}_1)$$

(Why UV, not IR? LC gauge ...)

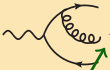
DIS at NLO: subtracting BK equation

B. Ducloué, H. Hänninen, T. L. and Y. Zhu, Phys. Rev. D **96** (2017) no.9, 094017

Evaluate cross section as $\sigma_{L,T}^{\mathrm{NLO}} = \sigma_{L,T}^{\mathrm{LO}} + \sigma_{L,T}^{\mathrm{dip}} + \sigma_{L,T,\mathrm{sub.}}^{qg}$

$$\Rightarrow \quad \sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$

$$-* \Rightarrow \quad \sigma^{\mathrm{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$+* \Rightarrow$

- LL

$k_g^+ \sim z_2$

$$\sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) +* - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) +* \Big].$$

* UV-divergence

DIS at NLO: subtracting BK equation

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Evaluate cross section as $\sigma^{\rm NLO}_{L,T} = \sigma^{\rm LO}_{L,T} + \sigma^{\rm dip}_{L,T} + \sigma^{qg}_{L,T,\rm sub.}$

$$\Rightarrow \quad \sigma^{\rm LO} \sim \int_0^1 {\rm d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\rm LO}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$

$$-* \Rightarrow \quad \sigma^{\rm dip} \sim \alpha_s C_F \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\rm LO}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$

- LL

$k_g^+ \sim z_2$

$$+* \Rightarrow \quad \sigma^{qg}_{\rm sub.} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} {\rm d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) +*$$

$$- \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) +* \Big].$$

* UV-divergence

LL: subtract leading log, already in BK-evolved $\mathcal{N}$ in $\sigma^{\rm LO}$

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$$+* \Rightarrow \quad \sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) + *$$

$$- \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) + * \Big].$$

- LL

$k_g^+ \sim z_2$

* UV-divergence LL: subtract leading log, already in BK-evolved $\mathcal{N}$ in σ^{LO}

- Parametrically $X(z_2) \sim x_{Bj}$, but $X(z_2) \sim 1/z_2$ essential!

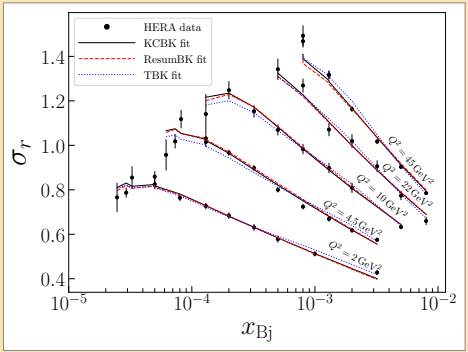
  ("k_T-factorization" with fixed rapidity scale is unstable @ NLO. Analogous probem in $p + A \to h + X$)

Fits to HERA data

G. Beuf, H. Hänninen, T. L. and H. Mäntysaari, (arXiv:2007.01645 (hep-ph)).

Free parameters:

- σ_0: proton area
- Q_{s0}: initial saturation scale
- γ shape of initial condition as function of r
- C^2: scale of α_s as function of r
  (could think of as fitting α_s or Λ_{QCD})



Main conclusions

- Fits are very good, χ^2/N varies 1.03 . . . 2.77
- Different ∼NLO BK-eqs equally good
  (Differences absorbed in initial conditions) .
  Similar to finding of Albacete 2015
  Only see differences at LHeC kinematics
- Generally prefer smallish σ_0

Including quark masses

Heavy quarks, motivation, issues

- Data
  - HERA F_2^c
  - Charm big part of EIC program
- LO F_2^c problematic in existing fits

  Dirty little secret: heavy quarks in rcBK fits do not actually work!

LCPT loops with massive quarks are so much fun!

- Working with fixed helicity states (not Dirac traces=sums) : physics very explicit
- New Lorentz structures $\Longrightarrow$ rotational invariance constraints

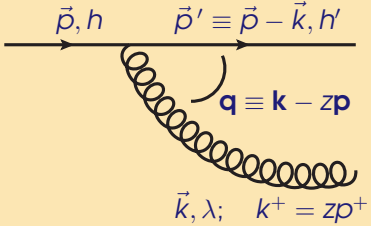
Approach for this talk: start with same regularization as in massless case

- Cutoff in k^+
- $\perp$ dim. reg.

Then see how far we get before trouble!

Beuf, T.L. Paatelainen 2021–2022

Elementary vertex with masses



- h, h': light cone (z-axis) helicities
- $\mathbf{q}$: center-of-mass $\perp$ momentum in splitting
- polarization λ, with $\perp$ polarization vector ε^{*j}_λ

$$\left[\bar{u}_{h'}(p')\not{\varepsilon}^*_\lambda(k)u_h(p)\right] \sim \overbrace{\bar{u}_{h'}\gamma^+ u_h}^{\sim\delta_{h,h'}}\,\delta^{ij}q^i\varepsilon^{*j}_\lambda + \overbrace{\bar{u}_{h'}\gamma^+[\gamma^i,\gamma^j]u_h}^{\sim\delta_{h,h'}}\,q^i\varepsilon^{*j}_\lambda + \overbrace{\bar{u}_{h'}\gamma^+\gamma^j u_h}^{\sim\delta_{h,-h'}}\,m_q\varepsilon^{*j}_\lambda$$

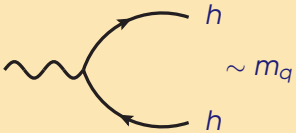
- New 3rd ligh-cone-helicity-flip structure $\sim m_q$ (Loops: also 4th $\bar{u}_{h'}\gamma^+\gamma^i u_h\varepsilon^{*j}_\lambda q^i q^j$)
- Note: $\perp$ momentum in non-flip, but not in flip vertex $\Longrightarrow$ less UV-divergent

Then look at diagrams for $\gamma^* \to Q\bar{Q}$, new UV-divergent and finite contributions in

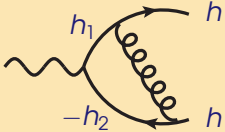
1. "Vertex correction" diagrams: calculation complicated, physics simple
2. "Propagator correction" diagrams: calculation simple, interpretation not!

Vertex corrections to LC helicity flip vertex

Look at LC helicity flip part of LO vertex



Vertex corrections from diagrams like

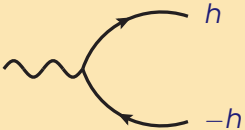


3 vertices $\Longrightarrow$ 2 options

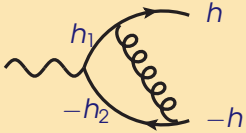
- 1 flip vertex ($h_1 \neq h$, $h_2 \neq h_1$ or $h_2 \neq h$)
  $\Longrightarrow$ log-divergent $\sim m_q \frac{1}{\varepsilon}$ (2 ED's $\sim \mathbf{k}^2$ each, 2 vertices $\mathbf{k}$ each, measure $d^2\mathbf{k}$)
  $\Longrightarrow$ absorb into **vertex mass** counterterm δm_v,
  same as δm_q in conventional perturbation theory
- 3 flip vertices:($h_1 \neq h$, $h_2 \neq h_1$ and $h_2 \neq h$)
  $\Longrightarrow$ finite NLO contribution

Vertex corrections to non-flip vertex

Non-flip part of LO vertex



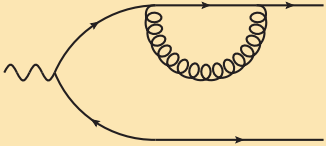
Corrections from



$\Longrightarrow$ again 2 options

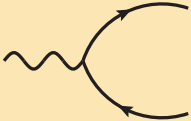
- no flip vertices ($h_1 = h$, $h_2 = h_1$ and $h_2 = h$)
  vertices as in massless theory $\Longrightarrow$ not new contribution
- 2 flip + 1 non-flip ($h_1 = -h$ or $h_2 = -h_1$ or $h_2 = -h$)
  $\Longrightarrow$ again finite NLO contribution
  (2 ED's $\sim \mathbf{k}^2$ each, 1 vertex $\sim \mathbf{k}$, finite integral $\sim \int d^2\mathbf{k} \frac{\mathbf{k}}{((\mathbf{k}-\ldots)^2+\ldots)((\mathbf{k}-\ldots)^2+\ldots)}$)

Quark propagator corrections



can have 0 or 2 flip vertices (1 gives zero by symmetry)

- Loops give m_q-dependent divergence $\sim$ $\left(\int d^2\mathbf{k}\,\frac{m_q^2}{(\mathbf{k}-\ldots)^2+\ldots}\right)$



$$\times\,\frac{m_q^2}{\Delta k_{\rm LO}^-}\,\frac{1}{\varepsilon}$$

- Can absorb into a renormalization of m_q^2 in ED of LO LCWF $(k_q^- = (\mathbf{k}_q^2 + m_q^2)/(2k_q^+))$
- But now the problem, known since 90's e.g. Haridranath, Zhang, also Burkardt in Yukawa th.
- In our regularization: k^+ cutoff, $\perp$ dim. reg.
  this **kinetic mass** counterterm δm_k is **not** same as the vertex correction δm_v
- In fact δm_v is same as in covariant theory, δm_k different

Mass renormalization

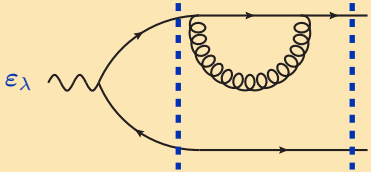
- Mass has 2 conceptually different roles:
  - Kinetic mass: relates energy and momentum
  - Vertex mass: amplitude of helicity flip in gauge boson vertex
- 1 parameter in Lagrangian, but 2 parameters in LCPT Hamiltonian
  — and thus in Hamiltonian LC quantization
- Lorentz-invariance requires they stay the same
- In practical LCPT calculations so far used k^+-cutoff and $\perp$ dim. reg. violates rotational invariance $\Longrightarrow$ $m_v \neq m_k$ at loop level $\Longrightarrow$ "textbook stuff"

There are 3 options to deal with this

1. Smartly combine with instantaneous "normal ordering" diagrams before regularizing & integrating $\Longrightarrow$ can keep $m_k = m_v$ but cannot calculate blindly
   For details see Beuf @ Hard Probes 2018
2. Use some other regularization $\Longrightarrow$ finite parts hard!
3. Regularize as before, but use additional renormalization condition to set separately m_v and m_k $\Longrightarrow$ discuss next

Two mass renormalization conditions

- Pole mass: **on-shell renormalization point**:
  - Timelike virtual $\gamma^* \to q\bar{q}$ with $q^2 = M^2$ (Same diagrams as for spacelike γ^*)
  - On-shell final state $M^2 = (\mathbf{P}^2 + m_q^2)/(z(1-z))$ (i.e. $ED_{\mathrm{LO}} \to 0$)
- One condition:



$\mathbf{P} = (1-z)\mathbf{k}_q - z\mathbf{k}_{\bar{q}}$

is the most divergent at on-shell point $\Longrightarrow$ cancel this $\Longrightarrow$ **kinetic mass**

- Second condition (+ cross checks) from Lorentz-invariance @ on-shell point
  1-loop vertex corrections: 4 scalar coefficients of 4 Dirac structures

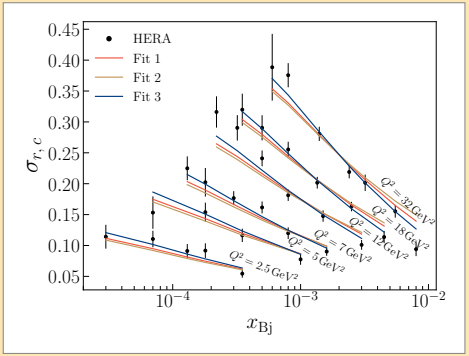
$$\bar{u}(0)\not{\varepsilon}_\lambda(q)v(1) \qquad (\mathbf{P}\cdot\varepsilon_\lambda)\bar{u}(0)\gamma^+ v(1) \qquad \frac{(\mathbf{P}\cdot\varepsilon_\lambda)}{\mathbf{P}^2}\mathbf{P}^j\bar{u}(0)\gamma^+\gamma^j v(1) \qquad \varepsilon_\lambda^j\bar{u}(0)\gamma^+\gamma^j v(1)$$

must reproduce 2 Lorentz-invariant form factors (Dirac & Pauli) $\Longrightarrow$ **vertex mass**

NLO heavy quark cross section in action

H. Hänninen, H. Mäntysaari, R. Paatelainen and J. Penttala, (arXiv:2211.03504 (hep-ph)).

- NLO dipole picture fit to light quark HERA data
- Calculate charm reduced cross section: $\chi^2/N_{\mathrm{dof}} \gtrsim 1$
- LO problem of simultaneous fit of σ_r and σ_r^c **resolved.**



Conclusions

- High energy scattering of dilute probe off strong color fields:
  - Target: classical color field
  - Probe: virtual photon,

    develop in a Fock state expansion in Light Cone Perturbation Theory
- This calculation carried out to one-loop order for massless quarks in 2017
- Successful description of HERA total cross section data
- Calculation for massive quarks, 2022:
  - Many features similar to massless case
  - Sizeable calculation, nontrivial amount of algebra
  - Full practicable solution for mass renormalization, including finite parts
- Future: diffractive DIS, forward proton-nucleus physics, . . .

$\gamma^* \to q\bar{q}$ with massive quarks

New result Beuf, Paatelainen, T.L. 2112.03158, 2204.02486 full LC gauge 1-loop structure of $\gamma^* \to Q\bar{Q}$

$$\widetilde{\psi}^{\gamma^*_T \to q\bar{q}}_{\text{NLO}} = -\frac{ee_f}{2\pi}\left(\frac{\alpha_s C_F}{2\pi}\right)\left\{\left[\left(\frac{k_0^+ - k_1^+}{q^+}\right)\delta^{ij}\bar{u}(0)\gamma^+ v(1) + \frac{1}{2}\bar{u}(0)\gamma^+[\gamma^i,\gamma^j]v(1)\right]\mathcal{F}\left[\mathbf{P}^i\mathcal{V}^T\right] + \bar{u}(0)\gamma^+ v(1)\mathcal{F}\left[\mathbf{P}^j\mathcal{N}^T\right]\right.$$
$$\left. + m\bar{u}(0)\gamma^+\gamma^j v(1)\mathcal{F}\left[\left(\frac{\mathbf{P}^i\mathbf{P}^j}{\mathbf{P}^2} - \frac{\delta^{ij}}{2}\right)\mathcal{S}^T\right] - m\bar{u}(0)\gamma^+\gamma^j v(1)\mathcal{F}\left[\mathcal{V}^T + \mathcal{M}^T - \frac{\mathcal{S}^T}{2}\right]\right\}\varepsilon^j_\lambda.$$

$$\mathcal{F}\left[\mathbf{P}^i\mathcal{V}^T\right] = \frac{i\mathbf{x}^i_{01}}{|\mathbf{x}_{01}|}\left(\frac{\kappa_z}{2\pi|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\left\{\left[\frac{3}{2} + \log\left(\frac{\alpha}{z}\right) + \log\left(\frac{\alpha}{1-z}\right)\right]\left\{\frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})}\Gamma\left(3-\frac{D}{2}\right) + \log\left(\frac{|\mathbf{x}_{01}|^2\mu^2}{4}\right)\right.\right.$$
$$\left.\left. + 2\gamma_E\right\} + \frac{1}{2}\frac{(D_s-4)}{(D-4)}\right\}\kappa_z K_{\frac{D}{2}-1}(|\mathbf{x}_{01}|\kappa_z) + \frac{i\mathbf{x}^i_{01}}{|\mathbf{x}_{01}|}\left\{\left[\frac{5}{2} - \frac{\pi^2}{3} + \log^2\left(\frac{z}{1-z}\right) - \Omega^T_{\mathcal{V}} + L\right]\kappa_z K_1(|\mathbf{x}_{01}|\kappa_z) + I^T_{\mathcal{V}}\right\}$$

$$\mathcal{F}\left[\mathbf{P}^i\mathcal{N}^T\right] = \frac{i\mathbf{x}^i_{01}}{|\mathbf{x}_{01}|}\left\{\Omega^T_{\mathcal{N}}\kappa_z K_1(|\mathbf{x}_{01}|\kappa_z) + I^T_{\mathcal{N}}\right\}$$

$$\mathcal{F}\left[\left(\frac{\mathbf{P}^i\mathbf{P}^j}{\mathbf{P}^2} - \frac{\delta^{ij}}{2}\right)\mathcal{S}^T\right] = \frac{(1-z)}{2}\left[\frac{\mathbf{x}^i_{01}\mathbf{x}^j_{01}}{|\mathbf{x}_{01}|^2} - \frac{\delta^{ij}}{2}\right]\int_0^z \frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty \frac{\mathrm{d}u}{(u+1)^2}|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}$$
$$\times K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) + [z \leftrightarrow 1-z].$$

$$\mathcal{F}\left[\mathcal{V}^T + \mathcal{M}^T - \frac{\mathcal{S}^T}{2}\right] = \left(\frac{\kappa_z}{2\pi|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\left\{\left[\frac{3}{2} + \log\left(\frac{\alpha}{z}\right) + \log\left(\frac{\alpha}{1-z}\right)\right]\left\{\frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})}\Gamma\left(3-\frac{D}{2}\right) + \log\left(\frac{|\mathbf{x}_{01}|^2\mu^2}{4}\right)\right.\right.$$
$$\left.\left. + 2\gamma_E\right\} + \frac{1}{2}\frac{(D_s-4)}{(D-4)}\right\}K_{\frac{D}{2}-2}(|\mathbf{x}_{01}|\kappa_z) + \left\{3 - \frac{\pi^2}{3} + \log^2\left(\frac{z}{1-z}\right) - \Omega^T_{\mathcal{V}} + L\right\}K_0(|\mathbf{x}_{01}|\kappa_z) + I^T_{\mathcal{V}\mathcal{M}\mathcal{S}},$$

$$\Omega^T_{\mathcal{V}} = -\left(1 + \frac{1}{2z}\right)\left[\log(1-z) + \gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right] + \frac{1}{2z}\left[\left(z+\frac{1}{2}\right)(1-\gamma) + \frac{m^2}{Q^2}\right]\log\left(\frac{\kappa_z^2}{m^2}\right) + [z \leftrightarrow 1-z]$$

$$I^T_{\mathcal{V}} = \int_0^1 \frac{\mathrm{d}\xi}{\xi}\left(\frac{2\log(\xi)}{(1-\xi)} - \frac{(1+\xi)}{2}\right)\left\{\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right) - [\xi \to 0]\right\}$$
$$- \int_0^1 \mathrm{d}\xi\left(\frac{\log(\xi)}{(1-\xi)^2} + \frac{z}{(1-z)} + \frac{z}{2}\right)\frac{(1-z)m^2}{\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right)$$
$$- \int_0^z \frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty \frac{\mathrm{d}u}{u(u+1)}\frac{m^2}{\kappa_\chi^2}\left(2\chi + \left(\frac{u}{1+u}\right)^2\frac{1}{z}(z-\chi)(1-2\chi)\right)$$
$$\times\left\{\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) - [u \to 0]\right\}$$
$$- \int_0^z \frac{\mathrm{d}\chi}{(1-\chi)^2}\int_0^\infty \frac{\mathrm{d}u}{(u+1)}(z-\chi)\left[1 - \frac{2u}{1+u}(z-\chi) + \left(\frac{u}{1+u}\right)^2\frac{1}{z}(z-\chi)^2\right]$$
$$\times\frac{m^2}{\sqrt{\kappa_z^2 + u\frac{[z-\chi]}{[1-\chi]}\kappa_\chi^2}}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) + \quad [z \leftrightarrow 1-z].$$

$$\Omega^T_{\mathcal{N}} = \frac{z+1-2z^2}{z}\left[\log(1-z) + \gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right] - \frac{(1-z)}{z}\left[\frac{2z+1}{2} - (1-\gamma) + \frac{m^2}{Q^2}\right]\log\left(\frac{\kappa_z^2}{m^2}\right) - [z \leftrightarrow 1-z] \tag{9}$$

$$I^T_{\mathcal{N}} = \frac{2(1-z)}{z}\int_0^z \mathrm{d}\chi\int_0^\infty \frac{\mathrm{d}u}{(u+1)^2}\left\{\left[(2+u)\kappa_z + u^2\kappa_\chi^2\right]\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)\right. \tag{10}$$
$$\left. + \frac{m^2}{\kappa_\chi^2}\left(\frac{z}{1-z} + \frac{\chi}{1-\chi}[u-2z-2u\chi]\right)\left[\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) - [u \to 0]\right]\right\} - [z \leftrightarrow 1-z].$$

$$I^T_{\mathcal{V}\mathcal{M}\mathcal{S}} = \int_0^1 \frac{\mathrm{d}\xi}{\xi}\left(\frac{2\log(\xi)}{(1-\xi)} - \frac{(1+\xi)}{2}\right)\left\{K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right) - [\xi \to 0]\right\}$$
$$+ \int_0^1 \mathrm{d}\xi\left(-\frac{3(1-z)}{2(1-\xi)} + \frac{(1-z)}{2}\right)K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right)$$
$$+ \int_0^z \frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty \frac{\mathrm{d}u}{(u+1)^2}\left\{-z - \frac{u}{(1+u)}\frac{[z+u\chi]}{z}(\chi-(1-z))\right\}K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)$$
$$+ \int_0^z \mathrm{d}\chi\int_0^\infty \frac{\mathrm{d}u}{(u+1)^3}\left\{\frac{\kappa_z^2}{\kappa_\chi^2}\left[1 + u\frac{\chi(1-\chi)}{z(1-z)}\right] - \frac{m^2}{\kappa_\chi^2}\frac{\chi}{(1-\chi)}\left[2\frac{(1+u)^2}{u} + \frac{u}{z(1-z)}(z-\chi)^2\right]\right\}$$
$$\times\left\{K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) - [u \to 0]\right\} + [z \leftrightarrow 1-z].$$