

# Proton's intrinsic charm remains concealed

**Tim Hobbs**

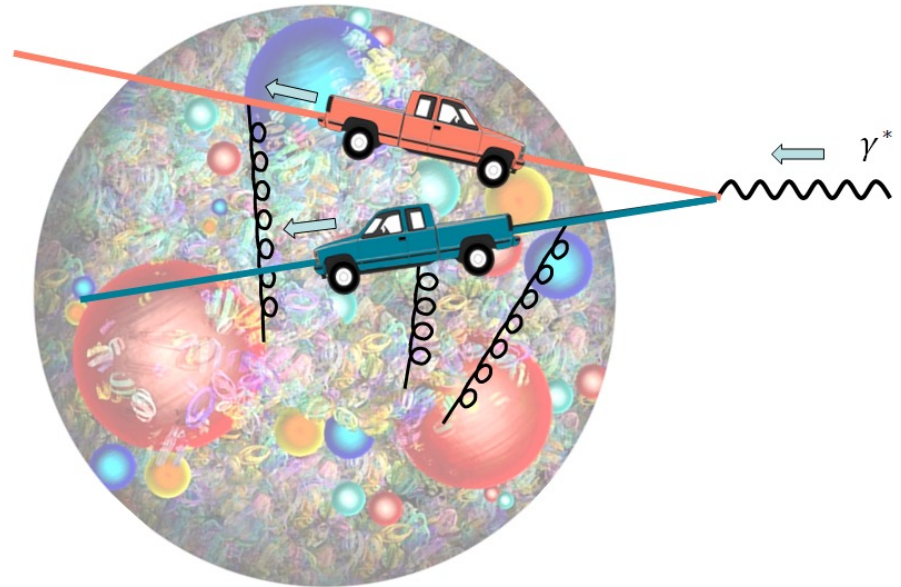
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and members of the  
CTEQ-TEA (Tung Et. Al.) working group



arXiv:2211.01387, 1707.00657  
and 2205.10444

# Contents

## 1. Terminology:

- Nonperturbative, intrinsic (IC), fitted (FC) charm
- What is “nonperturbative” in “nonperturbative charm”?
- Process and scheme (in)dependence of the IC
- SACOT-MPS factorization scheme at NNLO

As a community, we must develop precise terminology for our quantitative (and the public's) understanding

## 2. The IC models of proton structure and CT18 FC NNLO global analysis

## 3. Probability:

- Estimation of PDF uncertainties
- Comparison of CT18 FC and NNPDF4.0 IC studies

# References

## **CTEQ-TEA analyses of fitted charm**

1. T.-J. Hou et al., JHEP 02 (2018) 059; 57 pages, 19 figures: QCD factorization with the NP charm and CT14 IC NNLO pheno analysis
2. M. Guzzi, T. J. Hobbs, K. Xie, et al., arXiv:2211.01387; 10 pages: **new** CT18 FC analysis with the LHC Run-1 and 2 data

## **IC from nonperturbative methods and models:**

1. BHPS: Brodsky, Hoyer, Peterson, Sakai, PLB 93 (1980) 451
2. BHPS3: Bluemlein, PLB 753 (2016) 619
3. Meson-Baryon models (MBM): Hobbs, Londergan, Melnitchouk, PRD 89 (2014) 074008
4. Light-front WF models: Hobbs, Alberg, Miller, PRD 96 (2017) 7, 074023
5. Dyson-Schwinger equations, lattice QCD, ...

**CT18 NNLO analysis and methodology:** T.-J. Hou, J. Gao, T. J. Hobbs, K. Xie, et al., Phys.Rev.D 103 (2021) 1, 014013

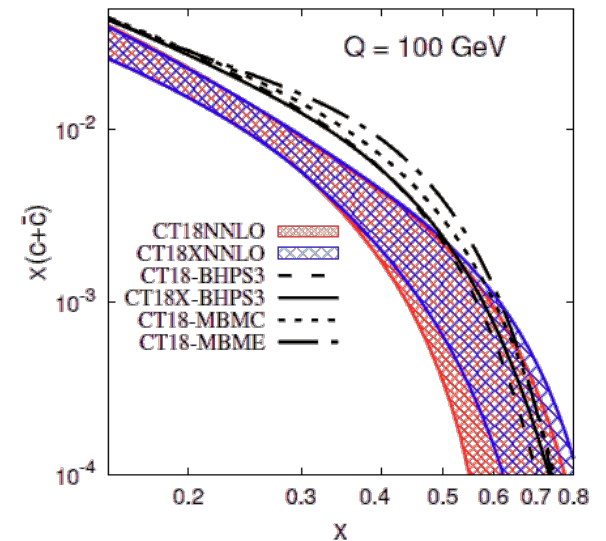
**Strong goodness-of-fit criteria for PDF fits:** K. Kovařík, P. Nadolsky, D. Soper, Rev.Mod.Phys. 92 (2020) 4, 045003

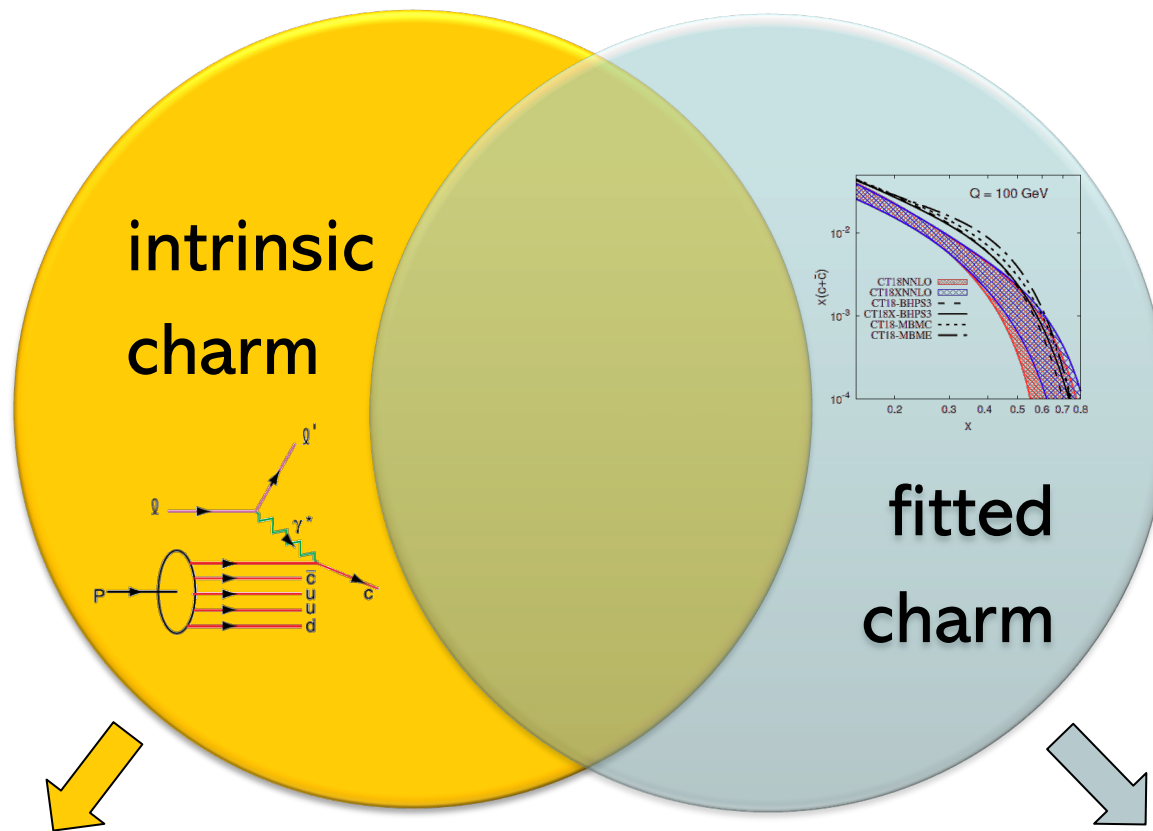
# Part 1. Terminology

# Do global PDF fits constrain intrinsic charm?

“Fitted charm” is a more direct term to describe the charm PDF found in the global QCD fit

Analog: the fitted charm mass





- The concept of nonperturbative methods
- Can refer to a component of the hadronic Fock state or the type of the hard process
- Predicts a typical enhancement of the charm PDF at  $x \gtrsim 0.2$

Connection?

- A charm PDF parametrization at scale  $Q_0 \approx 1 \text{ GeV}$  found by global fits [CT, NNPDF, ...]
- Arises in perturbative QCD expansions over  $\alpha_s$  and operator products
- May absorb process-dependent or unrelated radiative contributions

# PDF fits may include a ‘fitted charm’ PDF

‘Fitted charm’ = ‘higher-twist charm’

+ other (possibly not universal)

higher  $O(\alpha_s)$  / higher power terms

QCD factorization theorem for DIS structure function  $F(x, Q)$  [Collins, 1998]:

All  $\alpha_s$  orders:

$$F(x, Q) = \sum_{a=0}^{N_f} \int_x^1 \frac{d\xi}{\xi} C_a \left( \frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_c}{\mu}; \alpha(\mu) \right) f_{a/p}(\xi, \mu) + \mathcal{O}(\Lambda^2/m_c^2, \Lambda^2/Q^2).$$

The PDF fits implement this formula up to (N)NLO ( $N_{ord} = 1$  or  $2$ ):

PDF fits:

$$F(x, Q) = \sum_{a=0}^{N_f} \int_x^1 \frac{d\xi}{\xi} C_a^{(N_{ord})} \left( \frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_c}{\mu}; \alpha(\mu) \right) f_{a/p}^{(N_{ord})}(\xi, \mu).$$

The leading-power charm PDF component cancels at  $Q \approx m_c$  up to a higher order

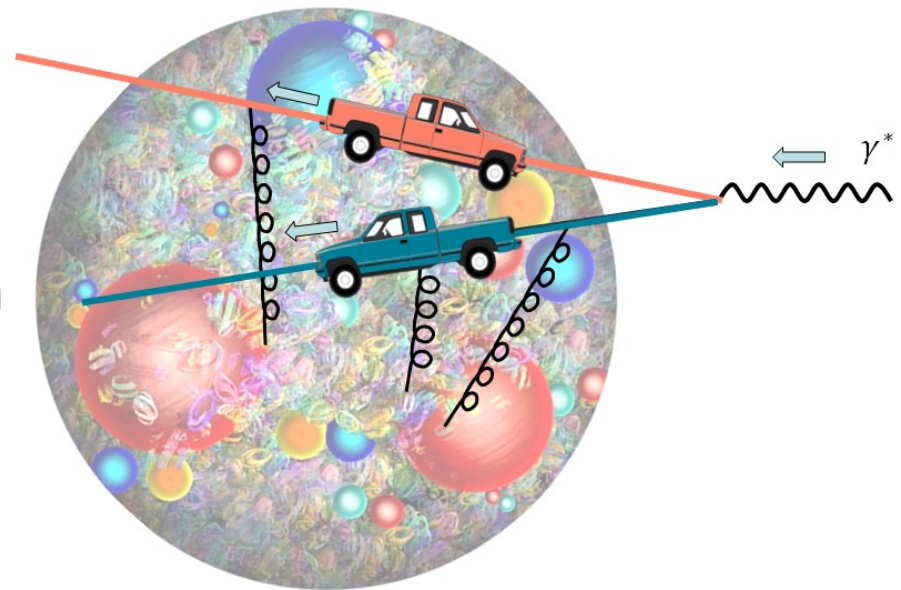
The ‘fitted charm component’ may approximate for missing terms of orders  $\alpha_s^p$  with  $p > N_{ord}$ , or  $\Lambda^2/m_c^2$ , or  $\Lambda^2/Q^2$

# What is nonperturbative in the “nonperturbative charm”?

In perturbative QFT, extrinsic and intrinsic production mechanisms differ in topology of diagrams, not the strength of the coupling

Intrinsic contributions exist in QED, weakly coupled theories

In proton scattering, the intrinsic  $c\bar{c}$  contribution emerges through power-suppressed diagrams with more than 1 gluon connection to the asymptotic Fock state

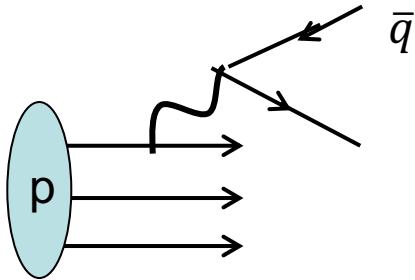




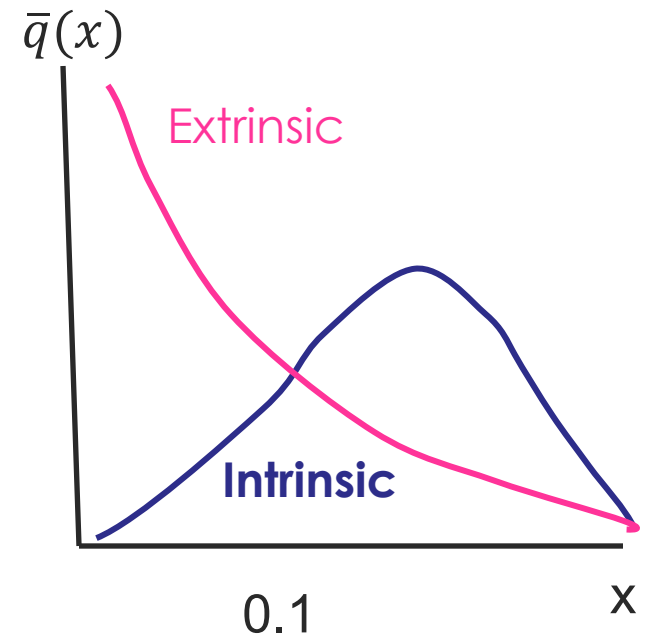
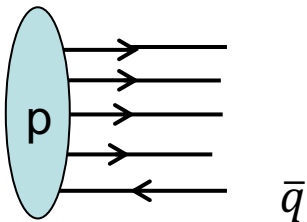
# In nonperturbative models:

## “Extrinsic” sea

[maps onto leading-power sea production from light flavors]



“Intrinsic” sea (excited Fock nonpert. states; beyond the leading-power production)



# In perturbative QFT:

## Intrinsic Chevrolets at the SSC

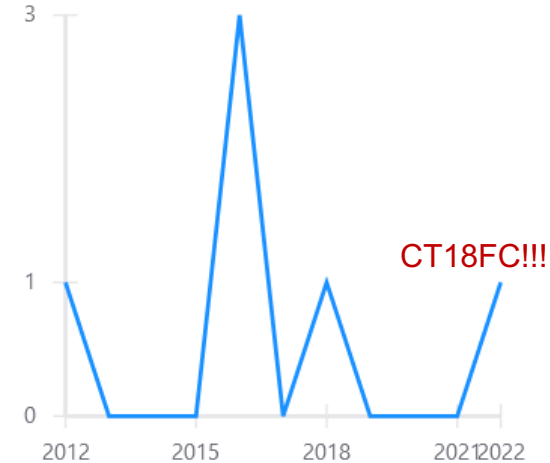
Stanley J. Brodsky (SLAC), John C. Collins (IIT, Chicago and Argonne), Stephen D. Ellis (Washington U., Seattle), John F. Gunion (UC, Davis), Alfred H. Mueller (Columbia U.)  
Aug, 1984

10 pages

Part of DESIGN AND UTILIZATION OF THE SUPERCONDUCTING SUPER COLLIDER. PROCEEDINGS, 1984 SUMMER STUDY, SNOWMASS, USA, JUNE 23 - JULY 13, 1984  
Proceedings of: 1984 DPF Summer Study on the Design and Utilization of the Superconducting Super Collider (SSC) (Snowmass 84), 227  
Report number: DOE/ER/40048-21 P4, SLAC-PUB-15471

pdf links cite claim reference search 6 citations

Citations per year



An intrinsic Chevrolet is attached to the target hadron by  $\geq 2$  gluon propagators

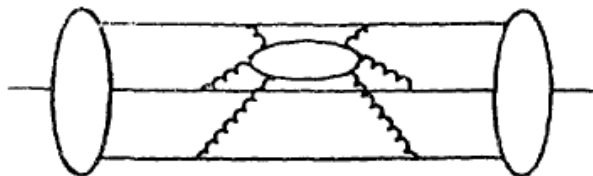


Fig. 1. Example of intrinsic heavy quark contribution to the proton wave function in QCD.

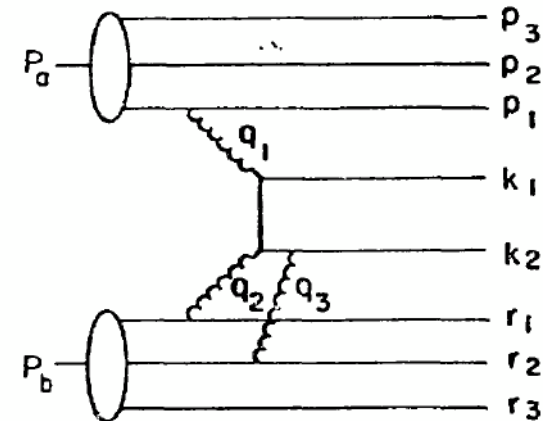


Fig. 3. Typical diagram for heavy quark production with extra gluon corresponding to intrinsic production.

# IC is either process-dependent or scheme-dependent

## A persistent terminological and conceptual ambiguity:

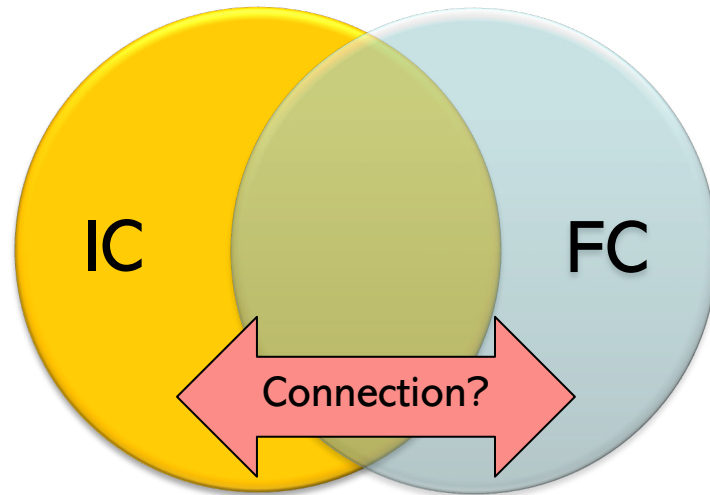
Is IC a type of a QCD observable or a nonperturbative QCD function?

If an observable, it receives process-dependent radiative contributions.

⇒ **Process dependence**

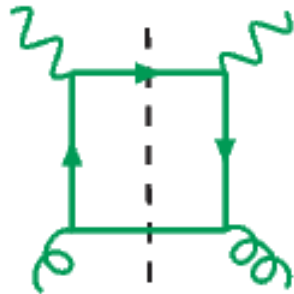
If a nonperturbative function, it can be defined in many ways.

⇒ **Scheme dependence**

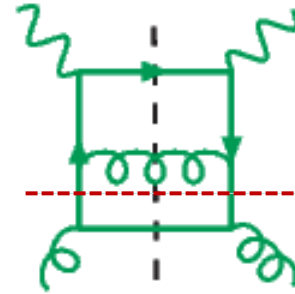


# A twist-4 contribution in HERA DIS charm production ( $\subset$ “intrinsic charm”)

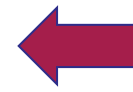
Twist-2  
 $\gamma^* g \rightarrow c\bar{c}$



Order  $\alpha_s(Q)$

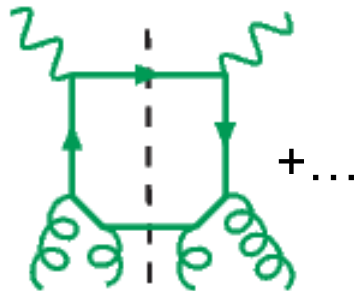


$\alpha_s^2(Q) \cdot \ln(Q^2/m_c^2)$



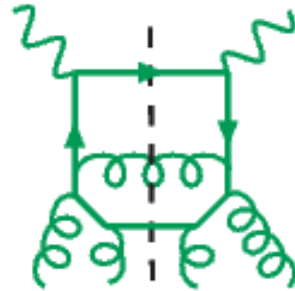
A ladder; must be resummed in  $c(x, Q)$  in the  $N_f = 4$  scheme at  $Q^2 \gg m_c^2$ ; e.g., in the ACOT scheme

Twist-4  
 $\gamma^*(gg) \rightarrow c\bar{c}$



+...

$\alpha_s^2(Q) \cdot (\Lambda^2/Q^2)$   
or  $\alpha_s^2(Q) \cdot (\Lambda^2/m_c^2)$

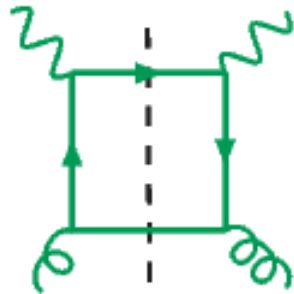


$\alpha_s^3(Q) \cdot (\Lambda^2/m_c^2) \ln(Q^2/m_c^2)$

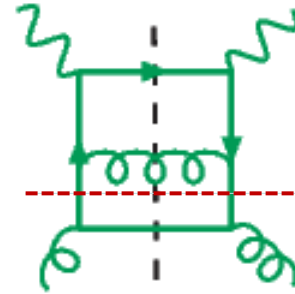
$\Lambda \lesssim 1 \text{ GeV}$

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Twist-2  
 $\gamma^* g \rightarrow c\bar{c}$



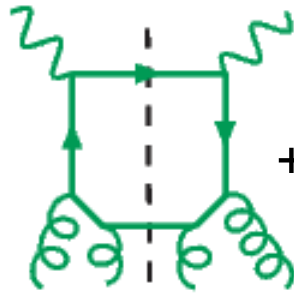
Order  $\alpha_s(Q)$



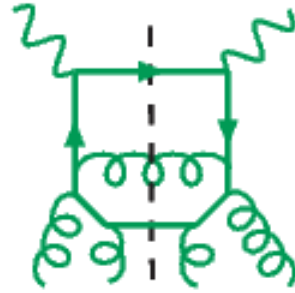
$\alpha_s^2(Q) \cdot \ln(Q^2/m_c^2)$

A ladder; must be resummed in  $c(x, Q)$  in the  $N_f = 4$  scheme at  $Q^2 \gg m_c^2$ ; e.g., in the ACOT scheme

Twist-4  
 $\gamma^*(gg) \rightarrow c\bar{c}$



+...



$\Lambda \lesssim 1 \text{ GeV}$

$\alpha_s^2(Q) \cdot (\Lambda^2/Q^2)$   
or  $\alpha_s^2(Q) \cdot (\Lambda^2/m_c^2)$

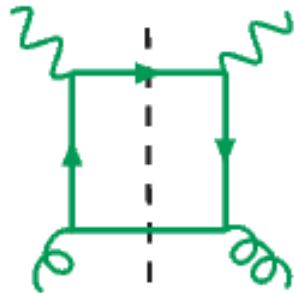
$\alpha_s^3(Q) \cdot (\Lambda^2/m_c^2) \ln(Q^2/m_c^2)$



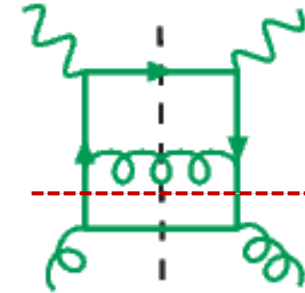
Can be of order 10% of the  
twist-2  $\alpha_s^2$  term

# A twist-4 contribution in HERA DIS charm production ( $\subset$ “intrinsic charm”)

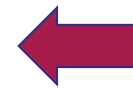
Twist-2  
 $\gamma^* g \rightarrow c\bar{c}$



Order  $\alpha_s(Q)$

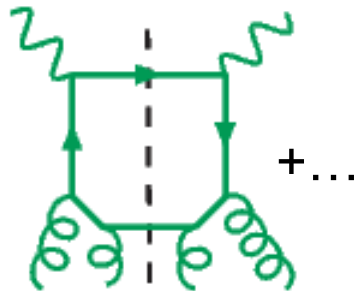


$\alpha_s^2(Q) \cdot \ln(Q^2/m_c^2)$

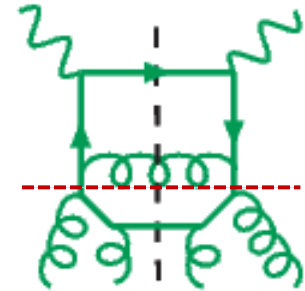


A ladder; must be resummed in  $c(x, Q)$  in the  $N_f = 4$  scheme at  $Q^2 \gg m_c^2$ ; e.g., in the ACOT scheme

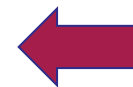
Twist-4  
 $\gamma^*(gg) \rightarrow c\bar{c}$



$\alpha_s^2(Q) \cdot (\Lambda^2/Q^2)$   
or  $\alpha_s^2(Q) \cdot (\Lambda^2/m_c^2)$



$\alpha_s^3(Q) \cdot (\Lambda^2/m_c^2) \ln(Q^2/m_c^2)$



The ladder subgraphs can be resummed as a part of  $c(x, Q)$  in the  $N_f = 4$  scheme at  $Q^2 \gg m_c^2 > \Lambda^2$ ;

contributes to the boundary condition for  $c(x, Q_0)$  at  $Q_0 \approx m_c$ ;

obeys twist-2 DGLAP equations.

$\Lambda \lesssim 1 \text{ GeV}$

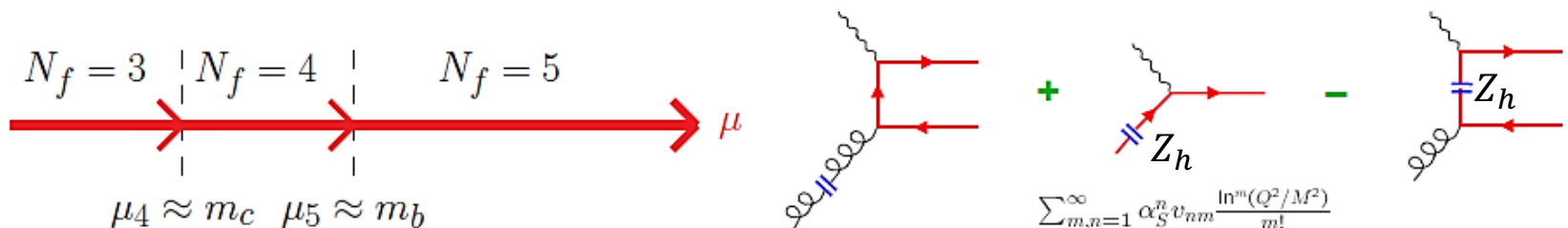


Can be of order  $\sim 10\%$  of the twist-2  $\alpha_s^2$  term

# Factorization scheme dependence of the FC

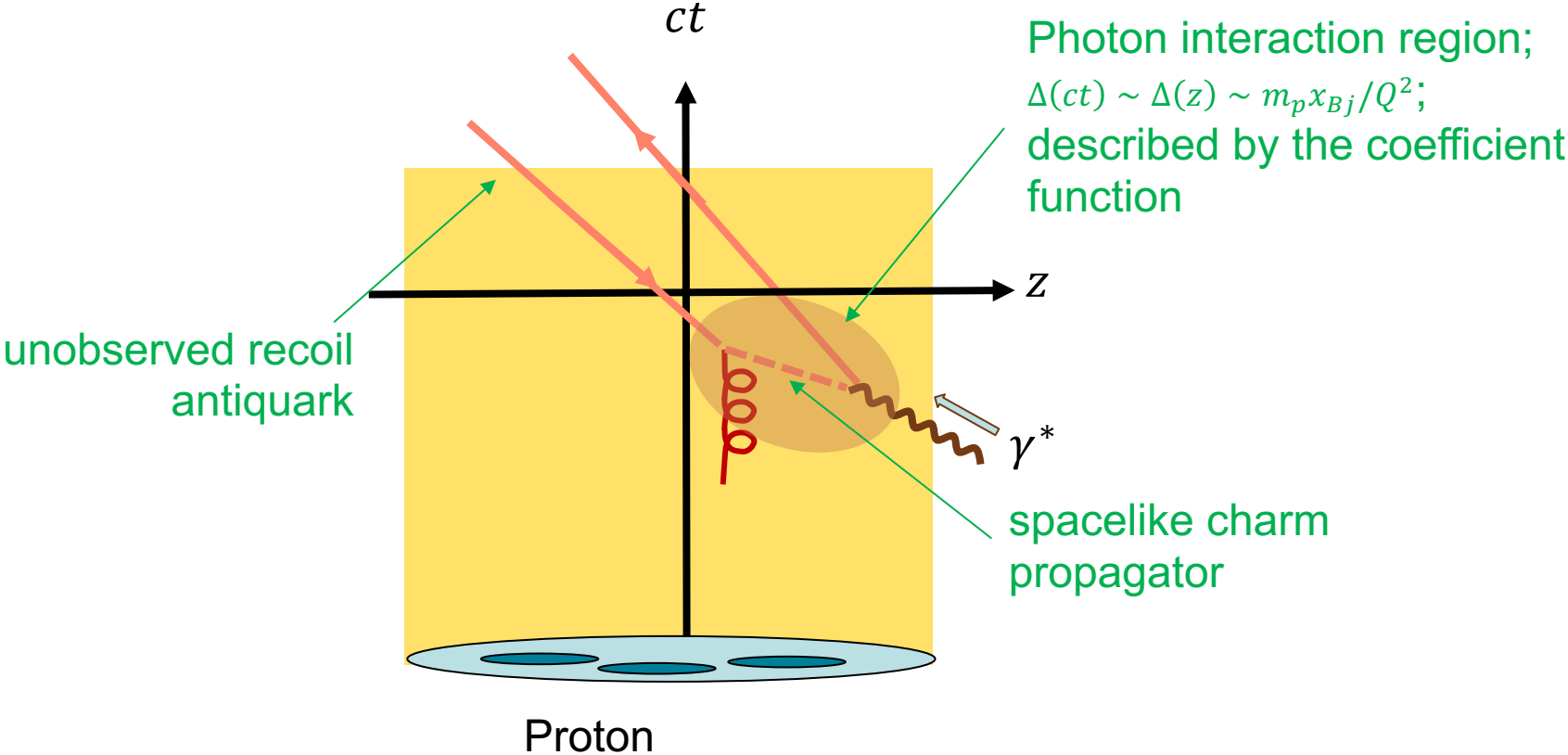
In global PQCD fits, factorization is performed in a **general-mass variable flavor number scheme (GM VFNS)**.

- 1 PI Green functions with charm lines are renormalized in the  $\overline{\text{MS}}$  scheme for  $\mu > \mu_4$  and in the zero-momentum subtraction scheme for  $\mu < \mu_4$ .
- $\mu_4$  is often set equal to  $m_c$  -- either the pole mass or  $\overline{\text{MS}}$  mass.
- Process-specific coefficient functions are factorized from universal PDFs. This factorization is under control only for the leading-power FC.
- **In all factorization schemes**, the flavor-excitation coefficient functions involve a kinematic approximation [realized by operator  $Z_h$ ] that puts the external heavy quark leg on the mass shell and optionally omits irrelevant  $m_c^2/Q^2$  terms.



# DIS in the rest frame of the proton, space-time diagram

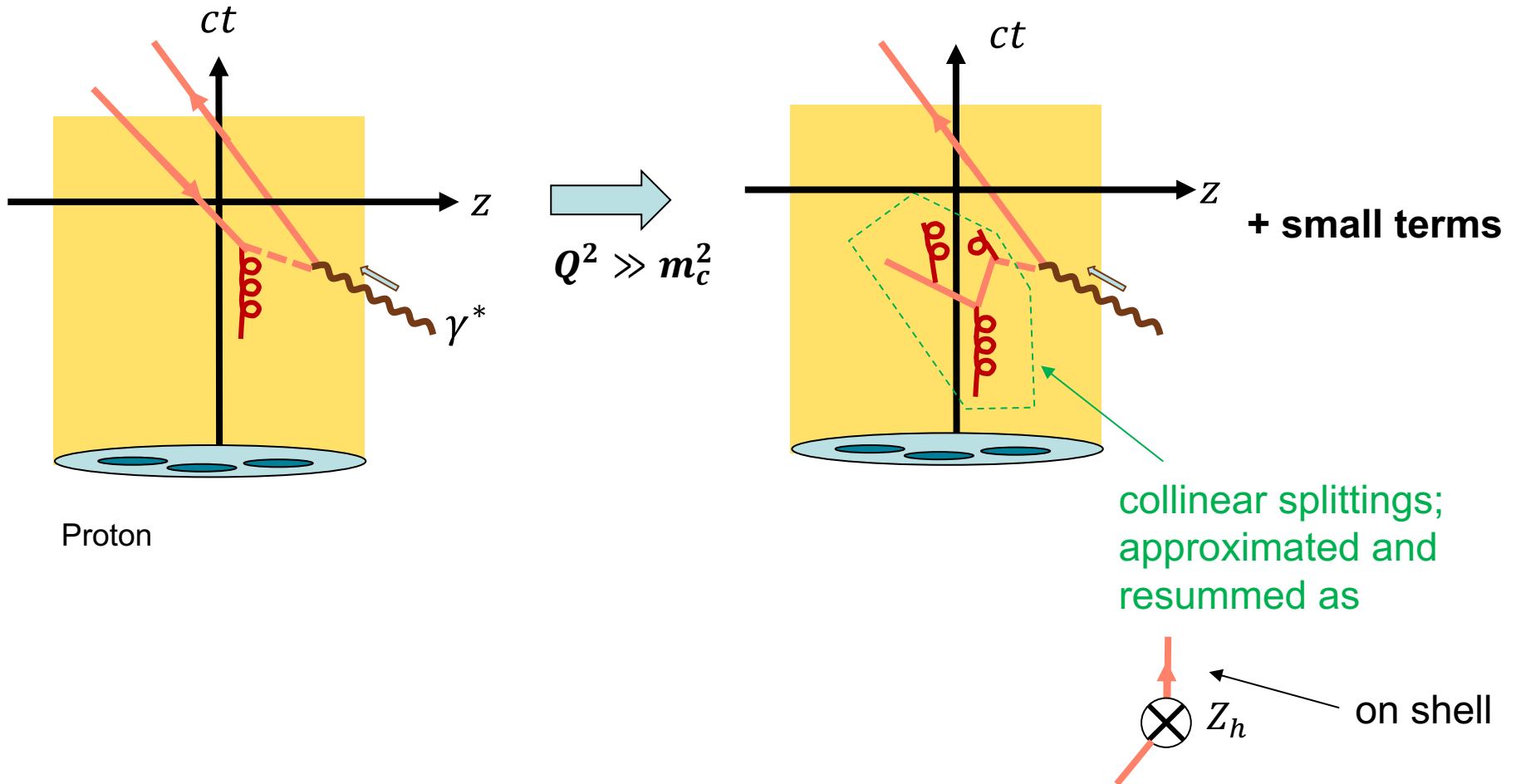
## Extrinsic production





# DIS in the rest frame of the proton, leading kinematic configurations

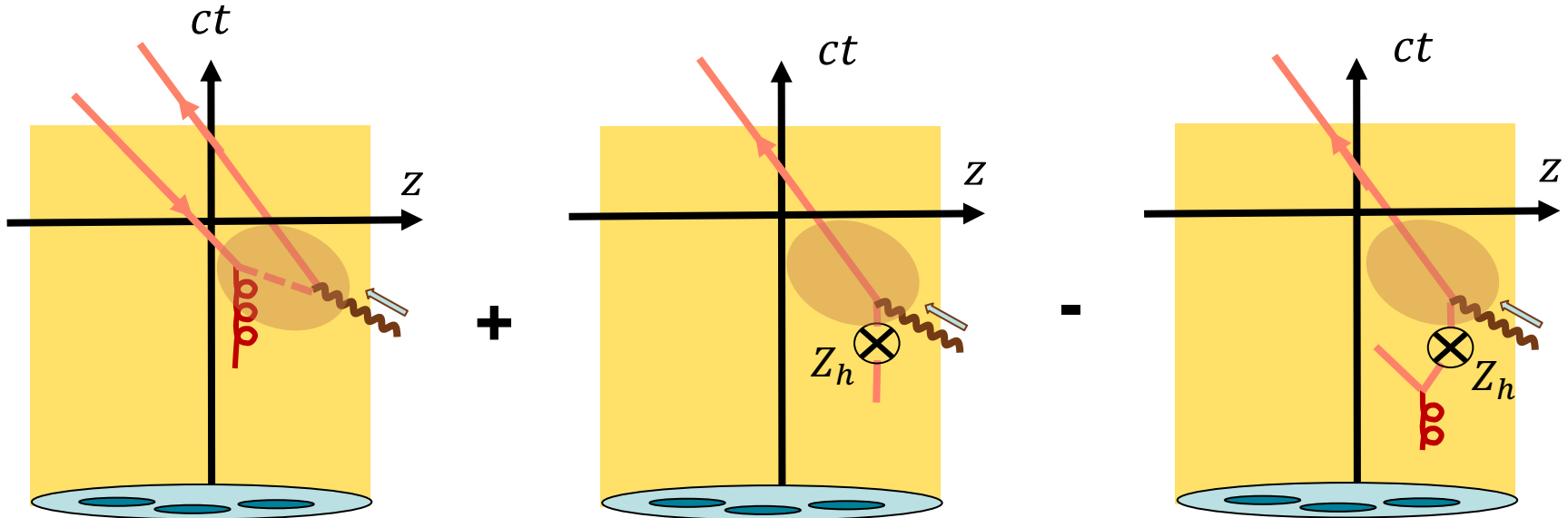
## Extrinsic production



Operator  $Z_h$  applies the kinematic approximation to the coefficient function

# DIS in the rest frame of the proton, leading kinematic configurations

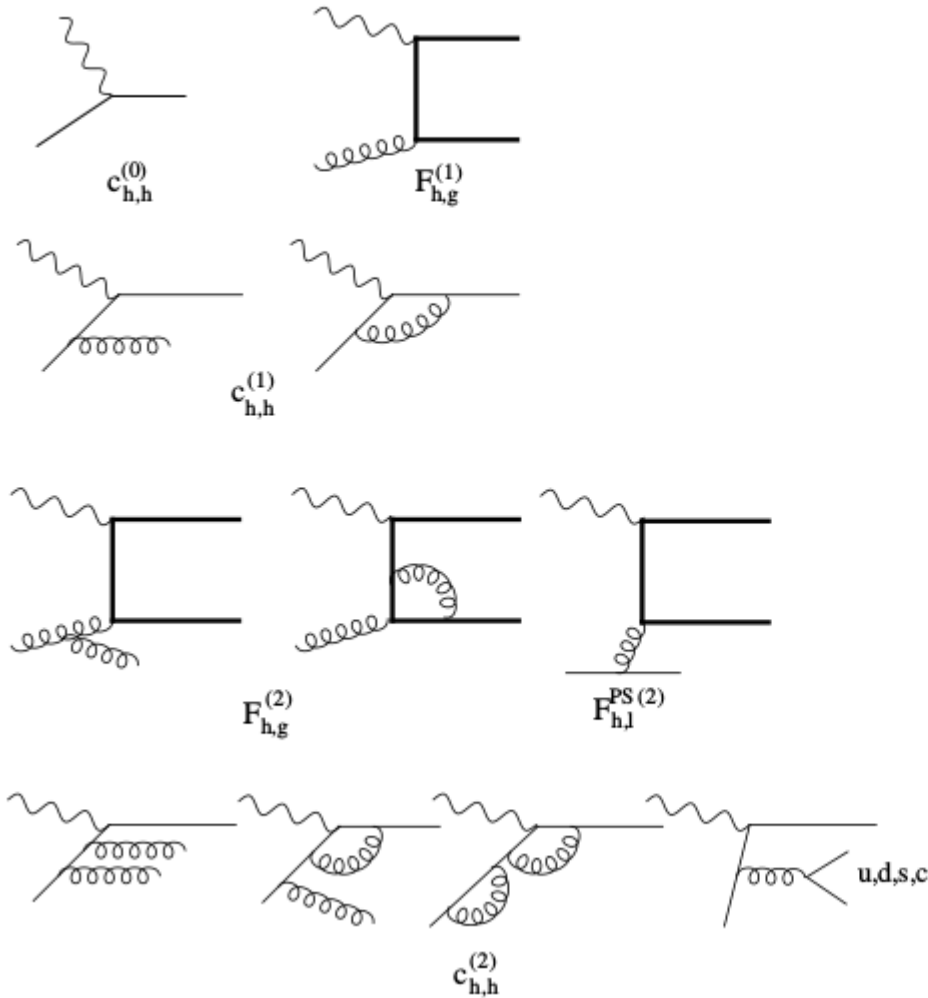
## Extrinsic production, full leading order



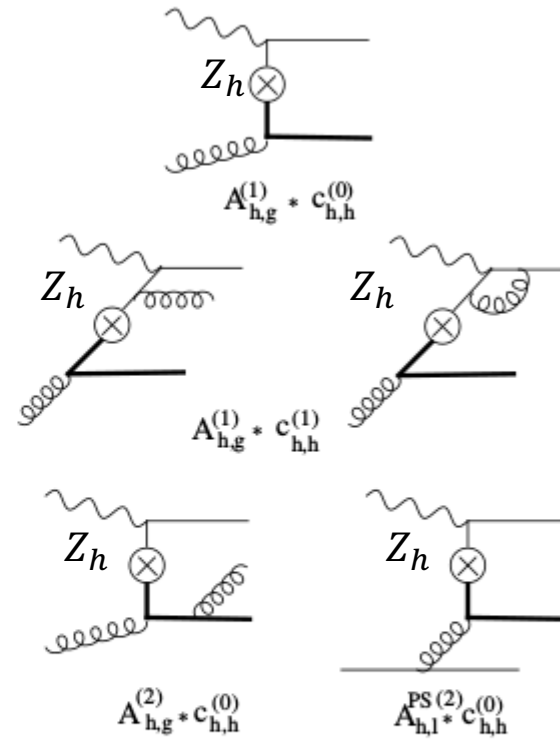
Operator  $Z_h$  applies the kinematic approximation to the coefficient function  $Z_h$  dependence cancels to a higher order. This algorithm is continued to higher orders.

# Twist-2: factorization for DIS in SACOT-MPS scheme up to NNLO

## Structure Functions



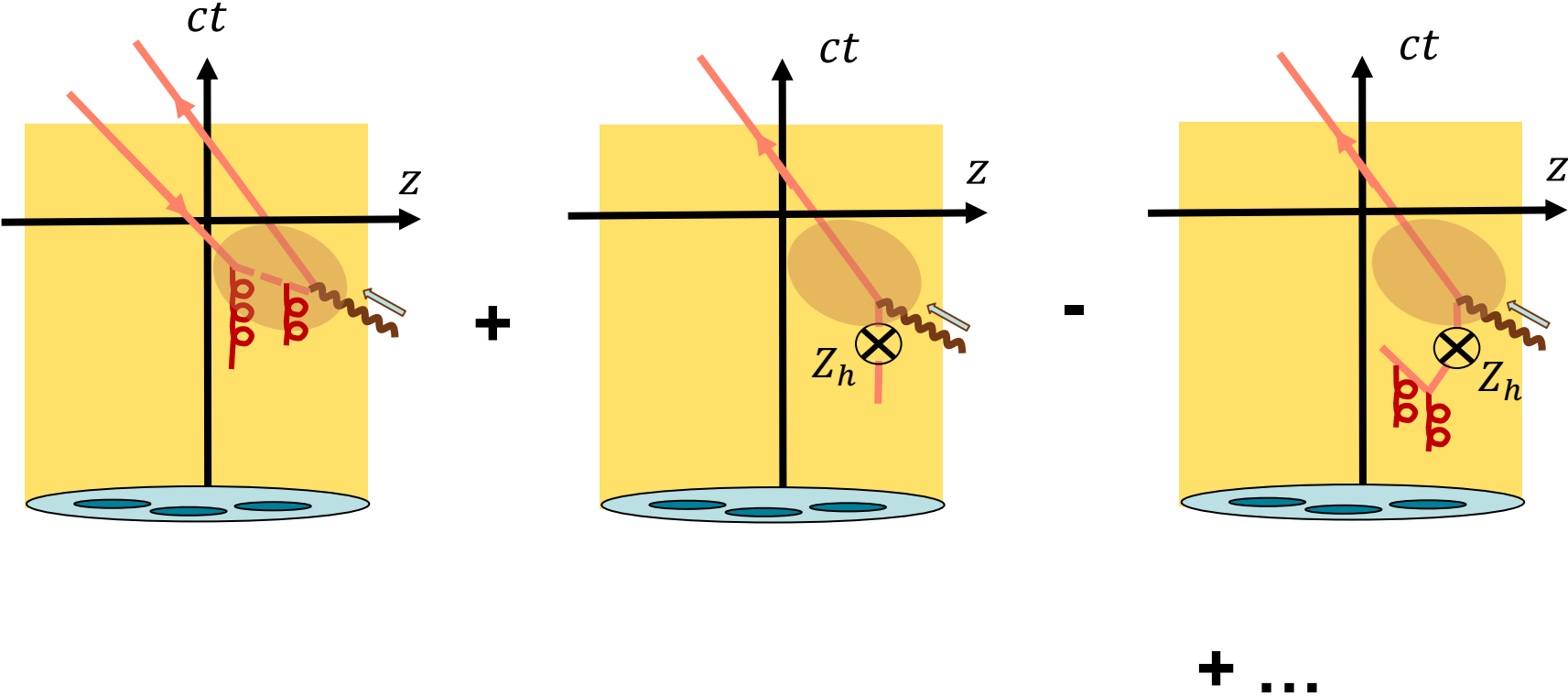
## Subtractions



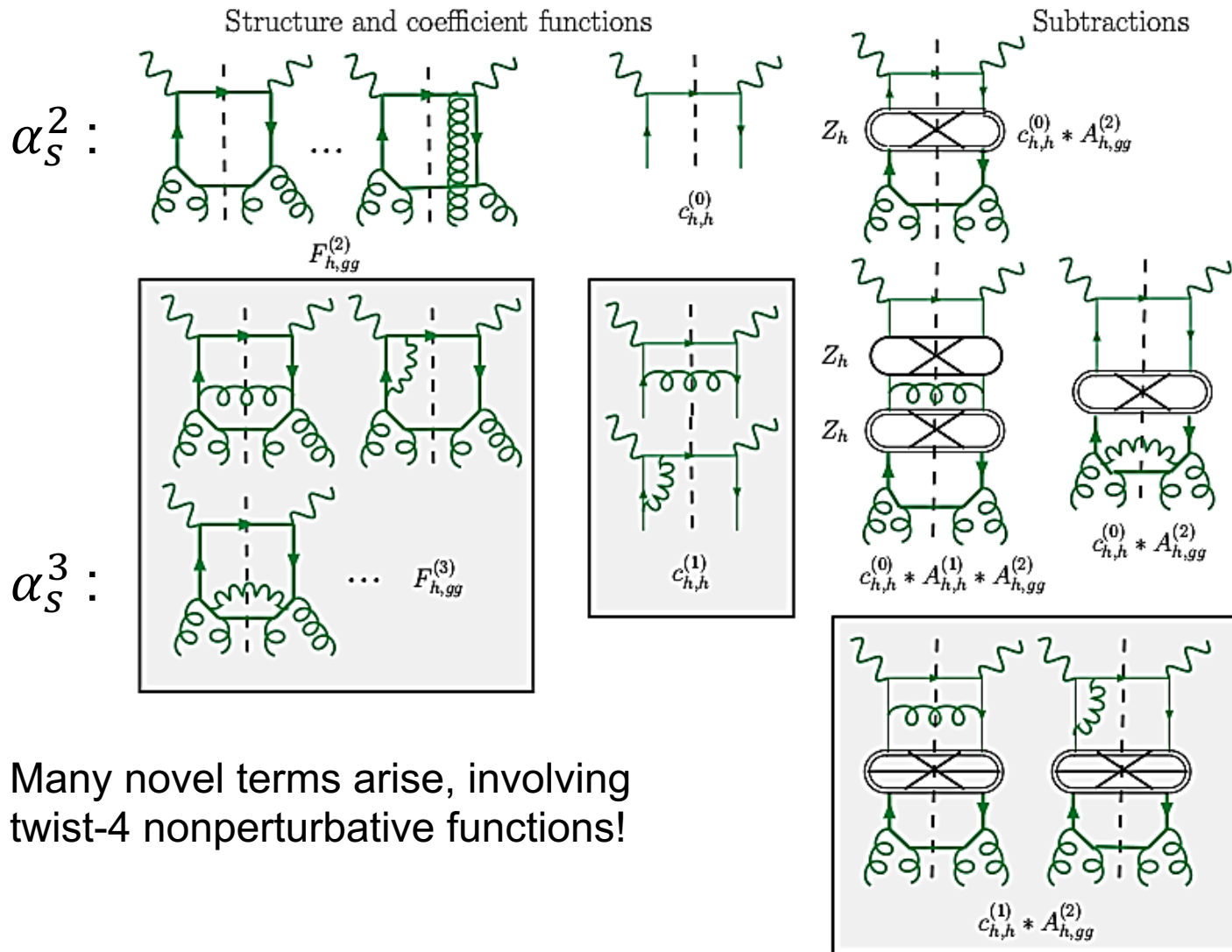
Leading-power radiative contributions to neutral-current DIS charm production in the CTEQ-TEA NNLO analysis

# DIS in the rest frame of the proton, leading kinematic configurations

**Intrinsic production: attach more gluons between the proton and charm**

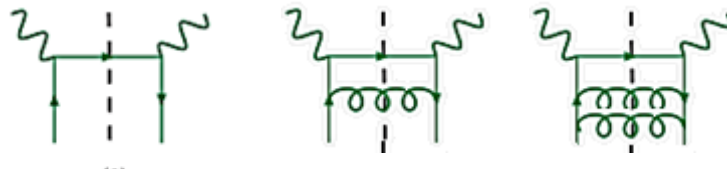


# ACOT-like factorization for twist-4 charm contributions (an example)



# Fitted charm contributions, practical implementation in CT14 and CT18

Keep only  $c_{h,h} \otimes f_h$ :  
Discard  $C_{h,gg}^{(k)} \otimes f_{gg}$ , etc.



In the absence of full computation, we (and other groups) make the simplest approximation:

$$F_{FC}(x, Q_0) = [c_{h,h} \otimes f_{c/p}^{FC}](x, Q_0)$$

$c_{h,h}$  is the **twist-2 charm DIS coefficient function** introduced to factorize the twist-4 ladder terms; defined according to the SACOT-MPS scheme

Start with  $N_f = 3$  at  $\mu_0 = m_c - \epsilon$ , evolve to  $\mu > \mu_0$  by incrementing  $N_f$  to 4 and 5

FC is compatible with any version of the ACOT scheme (cf. arXiv:1707.00657).

Flavor-excitation coefficient functions of these schemes differ by terms of  $O(m_c^2/Q^2)$ . Their overall differences are of  $O(\Lambda^2/Q^2)$ , i.e., within the accuracy of the factorization theorem.

# SACOT

= Simplified Aivazis-Collins-Olness-Tung scheme

ACOT, PRD 50 3102 (1994); Collins, PRD 58 (1998) 094002;  
Kramer, Olness, Soper, PRD (2000) 096007; Tung, Kretzer, Schmidt, J.Phys. G28 (2002) 983

The default heavy-quark scheme of CTEQ-TEA PDFs

Implementation is based upon, and closely follows, the proof of QCD factorization for DIS with massive quarks (*Collins, 1998*)

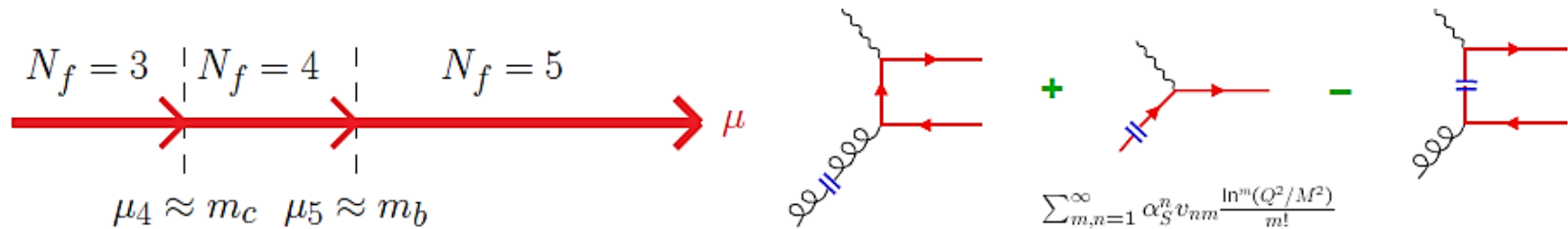
**MPS/ $\chi$  prescription**  $\equiv$  Kinematic matching based on **m**assive **p**hase **s**pace to improve perturbative convergence near HF production threshold

Applied

- to NNLO in NC DIS (Guzzi et al.; arXiv:1108.5112)
- to NLO in heavy-flavor hadroproduction using MCFM (Xie, Campbell, Nadolsky, 2019-2020)

In CC dimuon DIS, the NNLO correction is relatively small (Berger, Gao, arXiv:1710.04258)

# SACOT-MPS scheme: advantages

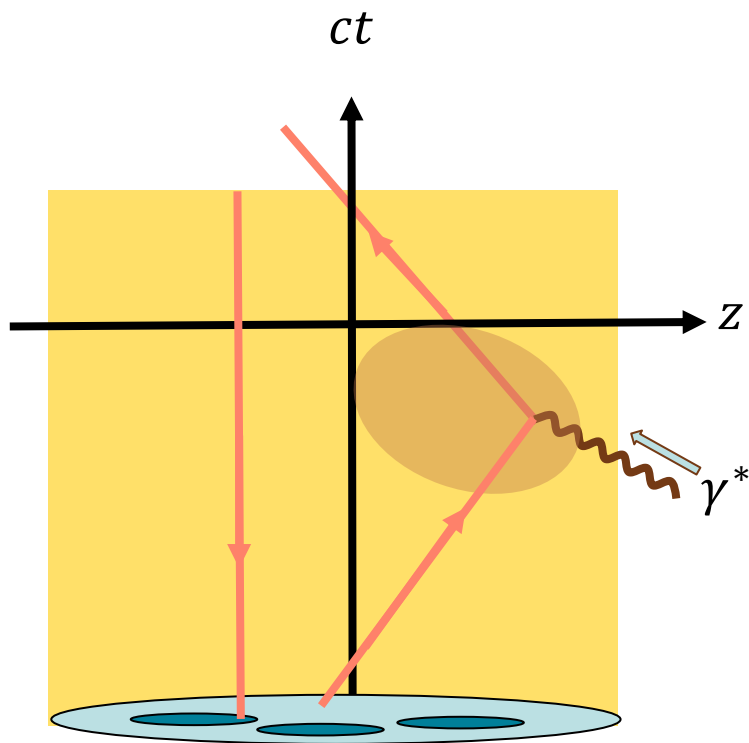


- A general-mass variable-flavor number (GM-VFN) scheme
  - Perturbatively convergent at all factorization scales  $\mu \gtrsim m_Q$
  - reduces to the zero-mass  $\overline{MS}$  scheme at  $\mu^2 \gg m_Q^2$ , without additional renormalization
  - reduces to the fixed-flavor number scheme at  $\mu^2 \sim m_Q^2$
- Relatively simple
  - One value of  $N_f$  (and one PDF set) in each  $\mu$  range
  - Sets  $m_Q = 0$  in  $|M|^2$  with incoming  $c$  or  $b$
  - Straightforward matching based on kinematical rescaling

Other common heavy-quark schemes: FONLL, TR', SACOT- $m_T$ ,...



# What about the pentaquark state?



A long-lived  $|uudc\bar{c}\rangle$  state could strongly mix with the ground  $|uud\rangle$  state. [No evidence exists for such mixed state.]

For this contribution, neither available factorization scheme is kinematically exact:

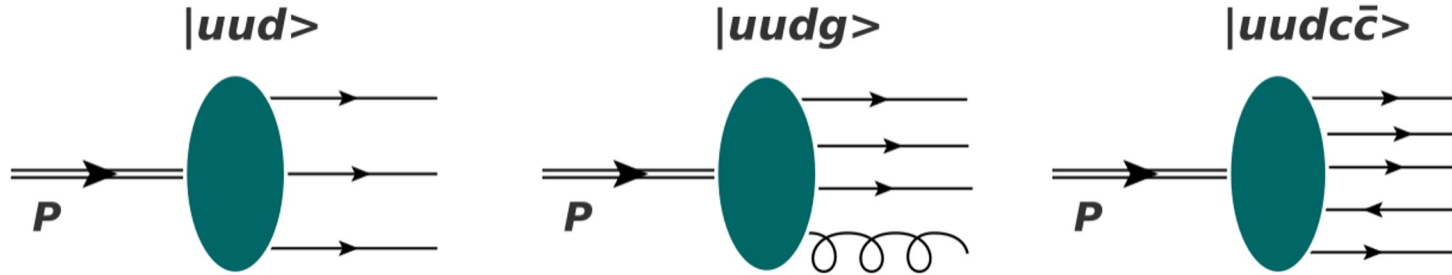
- There is a target-mass correction of order of mass of  $|uudc\bar{c}\rangle$ .
- There is the 2<sup>nd</sup> massive antiquark in the proton remnant. [The SACOT-MPS scheme corrects for it.]

# Part 2. IC models and CT18 FC global analysis

# nonperturbative QCD can generate a low-scale charm PDF

## Fock expansion

Brodsky, Hoyer, Peterson, Sakai (BHPS); Phys. Lett. **B93** (1980) 451.



IC PDF: transition matrix element,  $|\text{proton}\rangle \rightarrow |uudc\bar{c}\rangle$

$$P(p \rightarrow uudc\bar{c}) \sim \left[ M^2 - \sum_{i=1}^5 \frac{k_{\perp i}^2 + m_i^2}{x_i} \right]^{-2}$$

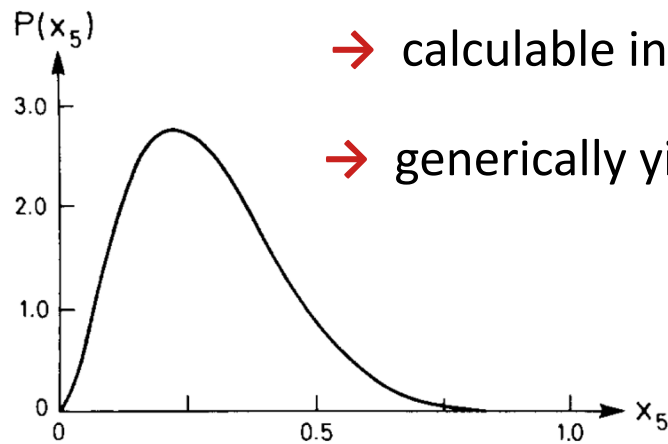
→ calculable in old-fashioned perturbation theory; **scalar** field theory

→ generically yields valence-like shape; governed by charm masses

$$m_c = m_{\bar{c}} \implies c^{\text{BHPS}}(x) = \bar{c}^{\text{BHPS}}(x)$$

alternative but similar representations exist

Blumlein; Phys. Lett. **B753** (2016) 619.



# meson-baryon models (MBMs): 5-quark states from hadronic interactions

- we implement a framework which *conserves spin/parity*

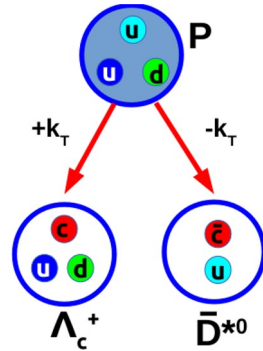
- nonperturbative mechanisms** are needed to break

$$c(x, Q^2 \leq m_c^2) = \bar{c}(x, Q^2 \leq m_c^2) = 0!$$

We build an **EFT** which connects IC to properties of the hadronic spectrum: [TJH, J. T. Londergan and W. Melnitchouk, Phys. Rev. D89, 074008 (2014).]

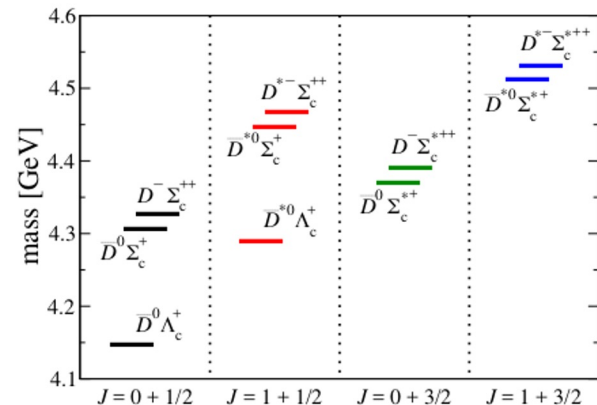
$$|N\rangle = \sqrt{Z_2} |N\rangle_0 + \sum_{M,B} \int dy f_{MB}(\mathbf{y}) |M(y); B(1-y)\rangle$$

$$y = k^+ / P^+ : k \text{ meson, } P \text{ nucleon}$$



$$c(x) = \sum_{B,M} \left[ \int_x^1 \frac{d\bar{y}}{\bar{y}} f_{BM}(\bar{y}) c_B\left(\frac{x}{\bar{y}}\right) \right]$$

- a similar *convolution* procedure may be used for  $\bar{c}(x) \dots$



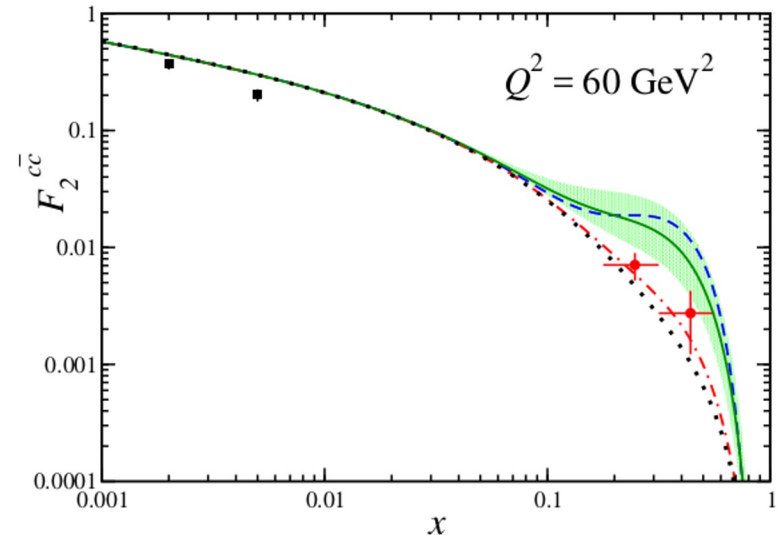
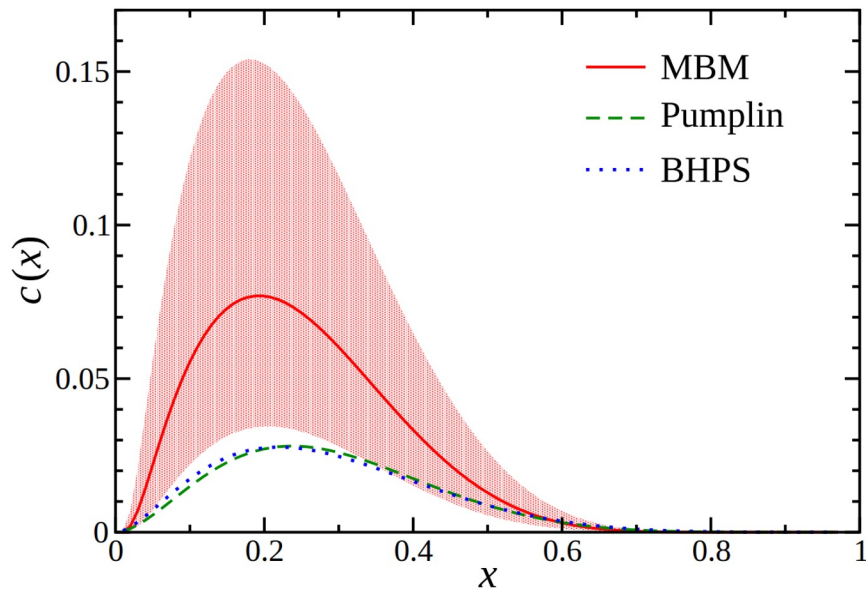
IC (MBM) depends on UV scale parameter,  $\Lambda$ ; predicts high- $x$  excess

- tune **universal** cutoff  $\Lambda = \hat{\Lambda}$  to fit **ISR**  $pp \rightarrow \Lambda_c X$  collider data

**multiplicities, momentum sum:**

$$\langle n \rangle_{MB}^{(\text{charm})} = 2.40\% \begin{matrix} +2.47 \\ -1.36 \end{matrix};$$

$$P_c := \langle x \rangle_{IC} = 1.34\% \begin{matrix} +1.35 \\ -0.75 \end{matrix}$$



$$F_2^{c\bar{c}}(x, Q^2) = \frac{4x}{9} [c(x, Q^2) + \bar{c}(x, Q^2)]$$

→ evolve to **EMC** scale,  $Q^2 = 60 \text{ GeV}^2$

low- $x$  H1/ZEUS data check *massless* **DGLAP** evolution

# IC models and formal QCD

models simulate nucleon wave function; aim to *mimic* nonpert QCD

- bound-state structure driven by constituent-quark masses
- connect to SU(4) flavor-symm breaking (in meson-baryon models [MBMs])

also, light-front models may relate PDFs, form factors, ...

ex.: Hobbs, Alberg, Miller, PRD96 (2017) 7, 074023; Sufian et al, PLB808 (2020) 135633.

$$|\Psi_P^\lambda(P^+, \mathbf{P}_\perp)\rangle = \frac{1}{16\pi^3} \sum_{q=c,\bar{c}} \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}} \psi_{q\lambda_q}^\lambda(x, \mathbf{k}_\perp) |q; xP^+, x\mathbf{P}_\perp + \mathbf{k}_\perp\rangle$$

**BUT:** unclear mapping, IC models → systematically-improvable QCD calc.

- based on *truncated* Fock-state or similar wave function expansions
- no obvious mapping onto factorization theorems
- ambiguity regarding fact. scale,  $\mu$ , in IC models

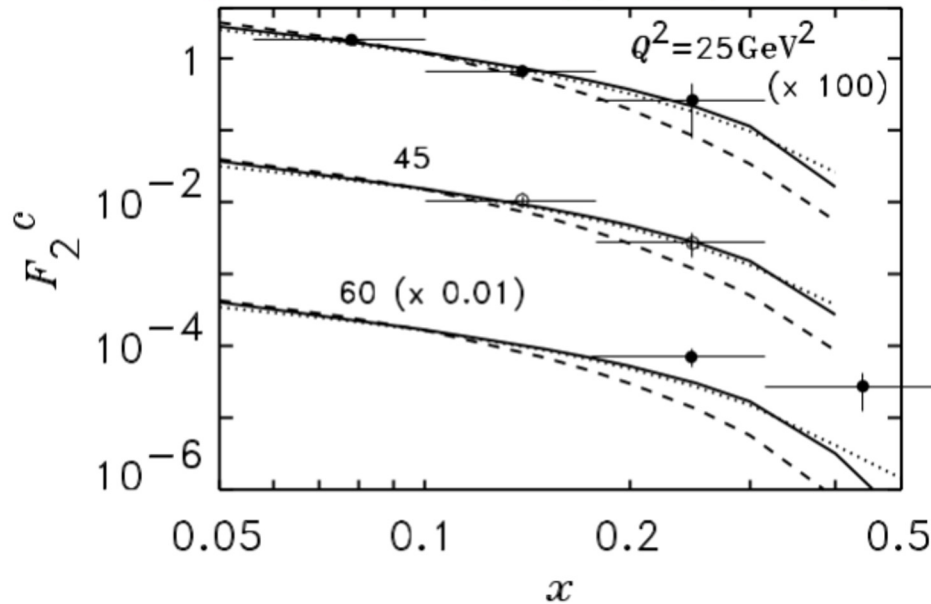
again, intrinsic charm (IC) ≠ fitted charm (FC) extracted in PDF analyses

# few expts with 'smoking gun' sensitivity to FC; but EMC data (?)

historically, charm structure function data,  $F_2^{c\bar{c}}$ , from EMC were suggestive

J. J. Aubert *et al.* (EMC), NPB213 (1983) 31–64.

F. M. Steffens, W. Melnitchouk and A. W. Thomas, Eur. Phys. J. C 11, 673 (1999) [hep-ph/9903441].



- hint of high-x excess in select  $Q^2$  bins
- data were analyzed only at LO
- show anomalous  $Q^2$  dependence
- EMC data fit poorly in CT14 IC study

**we do not include EMC in CT18 FC**

CT14 IC, arXiv: 1707.00657.

Candidate NNLO PDF fits	$\chi^2/N_{\text{pts}}$			
	All Experiments	HERA inc. DIS	HERA $c\bar{c}$ SIDIS	EMC $c\bar{c}$ SIDIS
CT14 + EMC (weight=0), no IC	1.10	1.02	1.26	3.48
CT14 + EMC (weight=10), no IC	1.14	1.06	1.18	2.32
CT14 + EMC in BHPS model	1.11	1.02	1.25	2.94
CT14 + EMC in SEA model	1.12	1.02	1.28	3.46

# FC at LHC: $Z+c$ suggested as sensitive probe

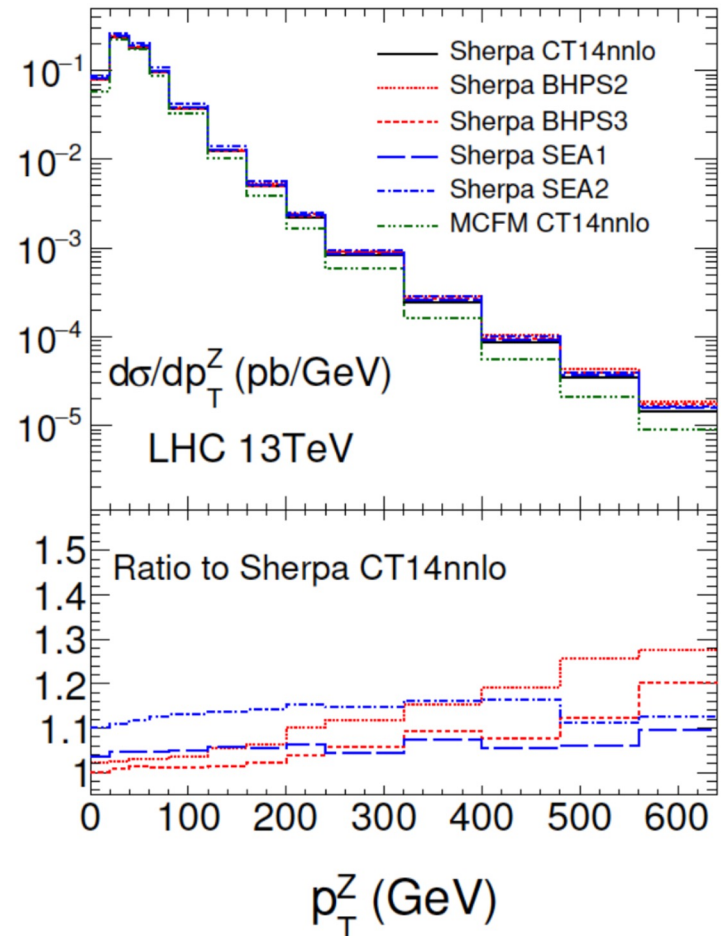
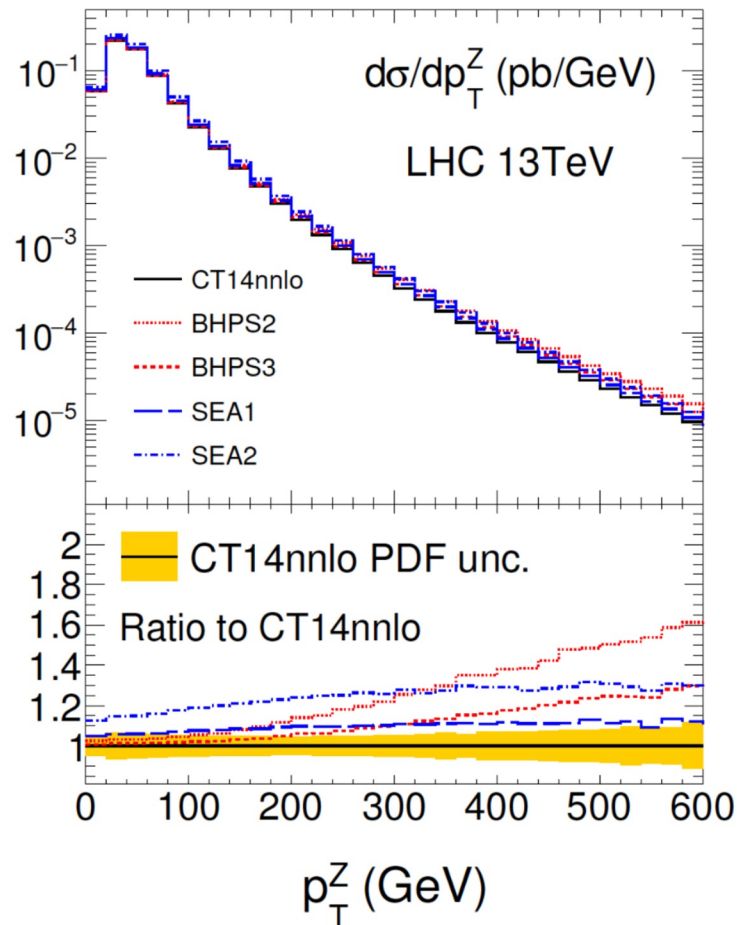
T. Boettcher, P. Ilten, M. Williams, 1512.06666; Bailas, Goncalves, 1512.06007

$p_T$  spectra, rapidity dists nominally sensitive to high- $x$  charm PDF

→ parton-shower effects can dampen high- $p_T$  tails

## $Z+c$ NLO LHC 13 TeV

[Hou et al., arXiv:1707.00657]



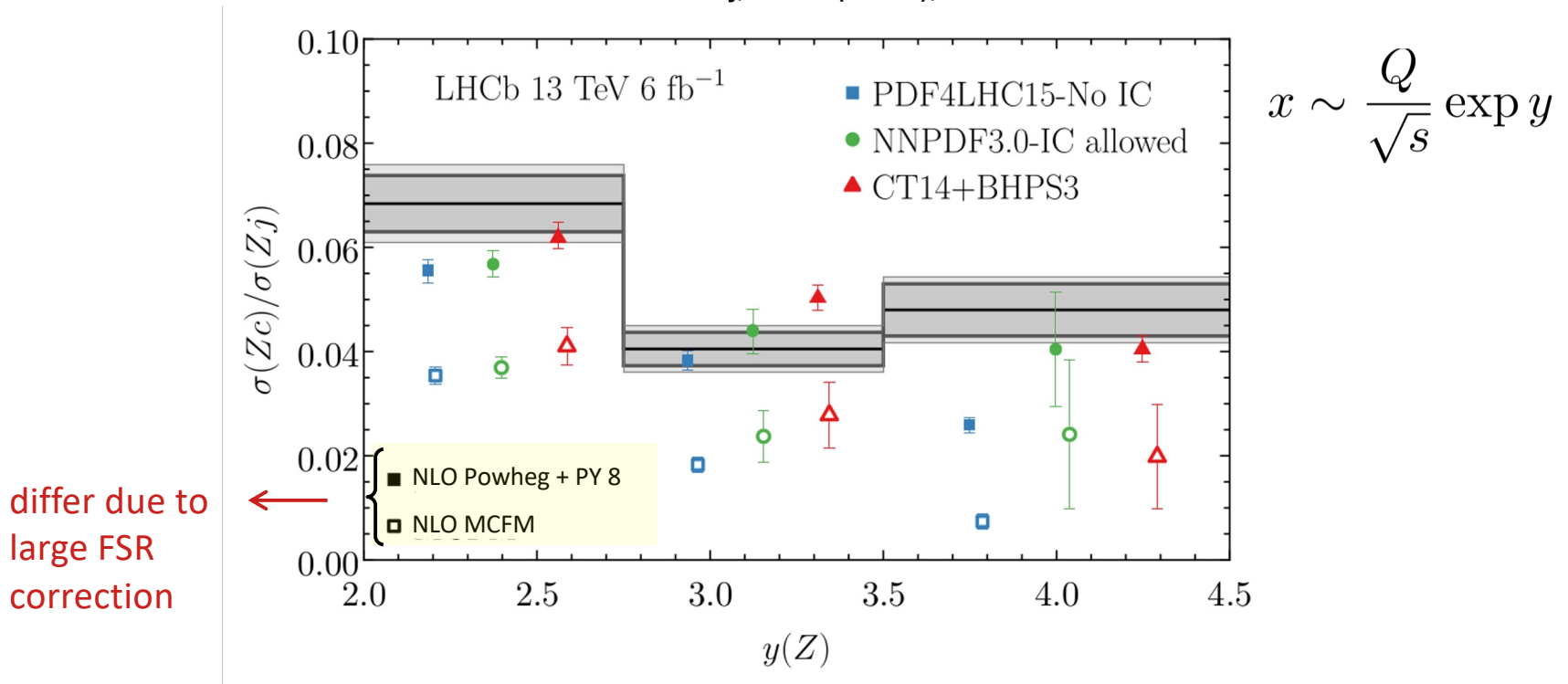


# Z+c theory predictions carry sizable uncertainties

2022 LHCb 13 TeV data: (Z+c) / (Z+jet) ratios; 3 rapidity bins

→ calculated **NLO** cross-section ratio similarly depends on showering, hadronization

R. Aaij, *et al.* (LHCb); arXiv: 2109.08084.



NNLO calculations recently available, but not implemented in PDF fits

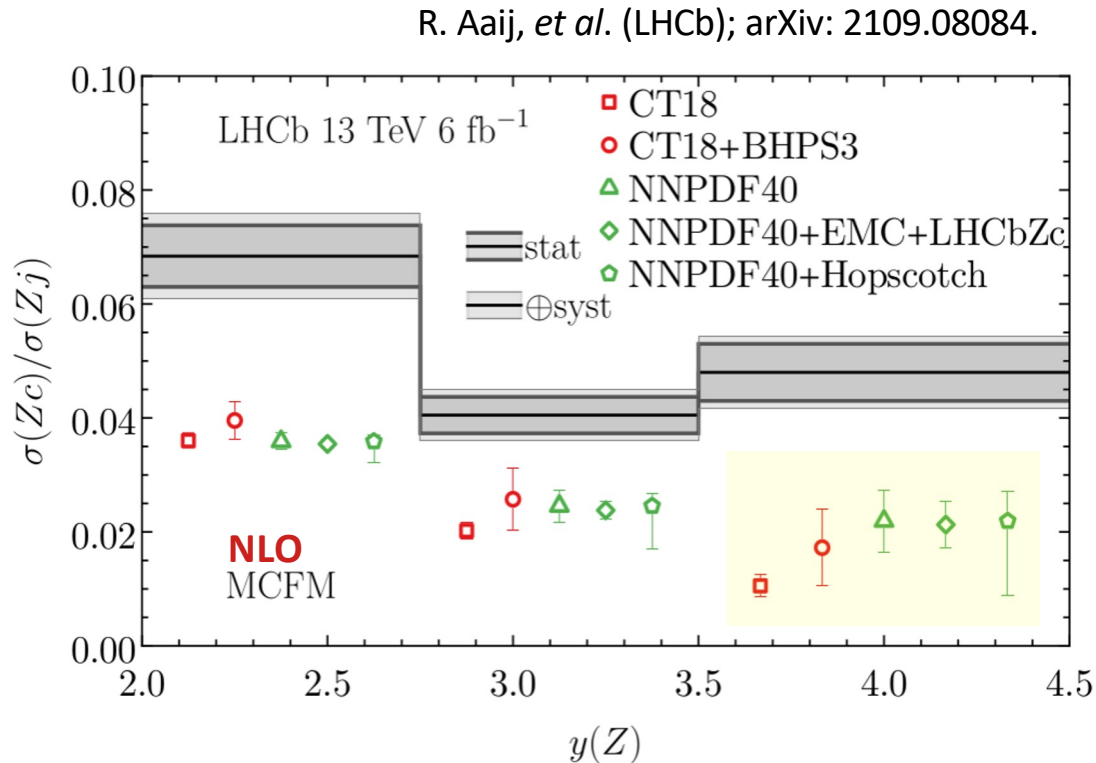
R. Gauld, *et al.*; arXiv: 2005.03016.

M. Czakon, *et al.*; arXiv: 2011.01011.

# theory uncertainties currently larger than PDF variations

assuming MCFM at NLO, can vary underlying PDFs, test inclusion of FC

→ FC slightly enhances ratio; not enough to improve agreement with data



$$x \sim \frac{Q}{\sqrt{s}} \exp y$$

theory accuracy not yet sufficient to leverage expt. precision for PDFs

→ need NNLO theory interface; control over showering, final-state effects

# might other HEP experiments be sensitive to FC?

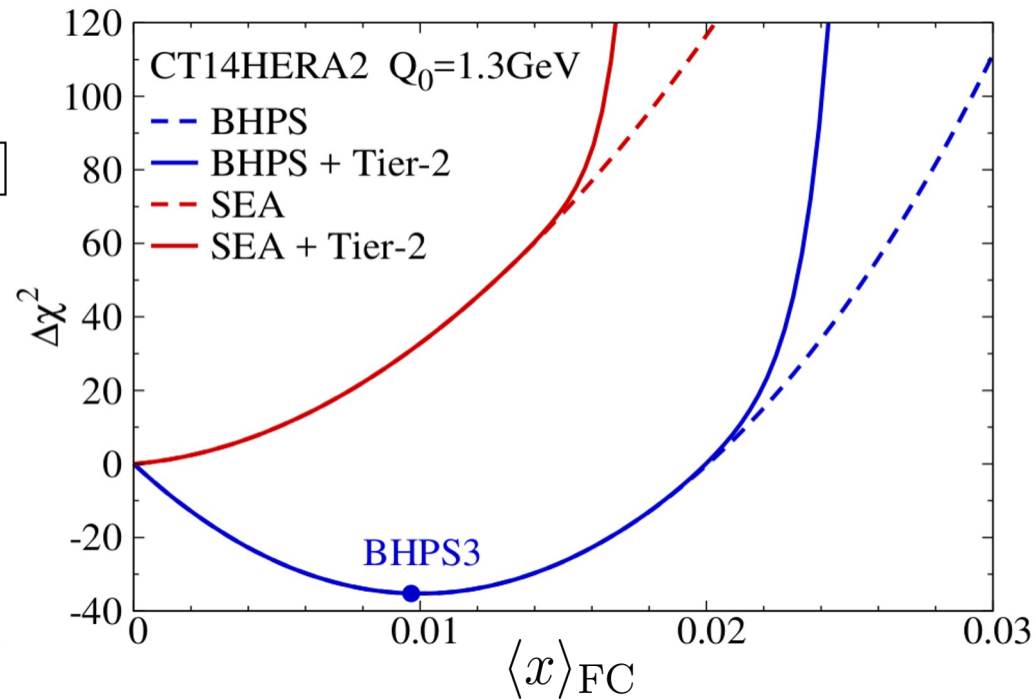
must be assessed using comprehensive global QCD analysis of PDFs

CT performed such an analysis, CT14 IC in [arXiv: 1707.00657](https://arxiv.org/abs/1707.00657)

→ found  $\langle x \rangle_{\text{FC}} < 2\%$ , but with large uncertainty consistent with zero FC

$$\langle x \rangle_{\text{FC}} = \int_0^1 dx x [c(x, Q_0) + \bar{c}(x, Q_0)]$$

included many details on theory and analysis of IC



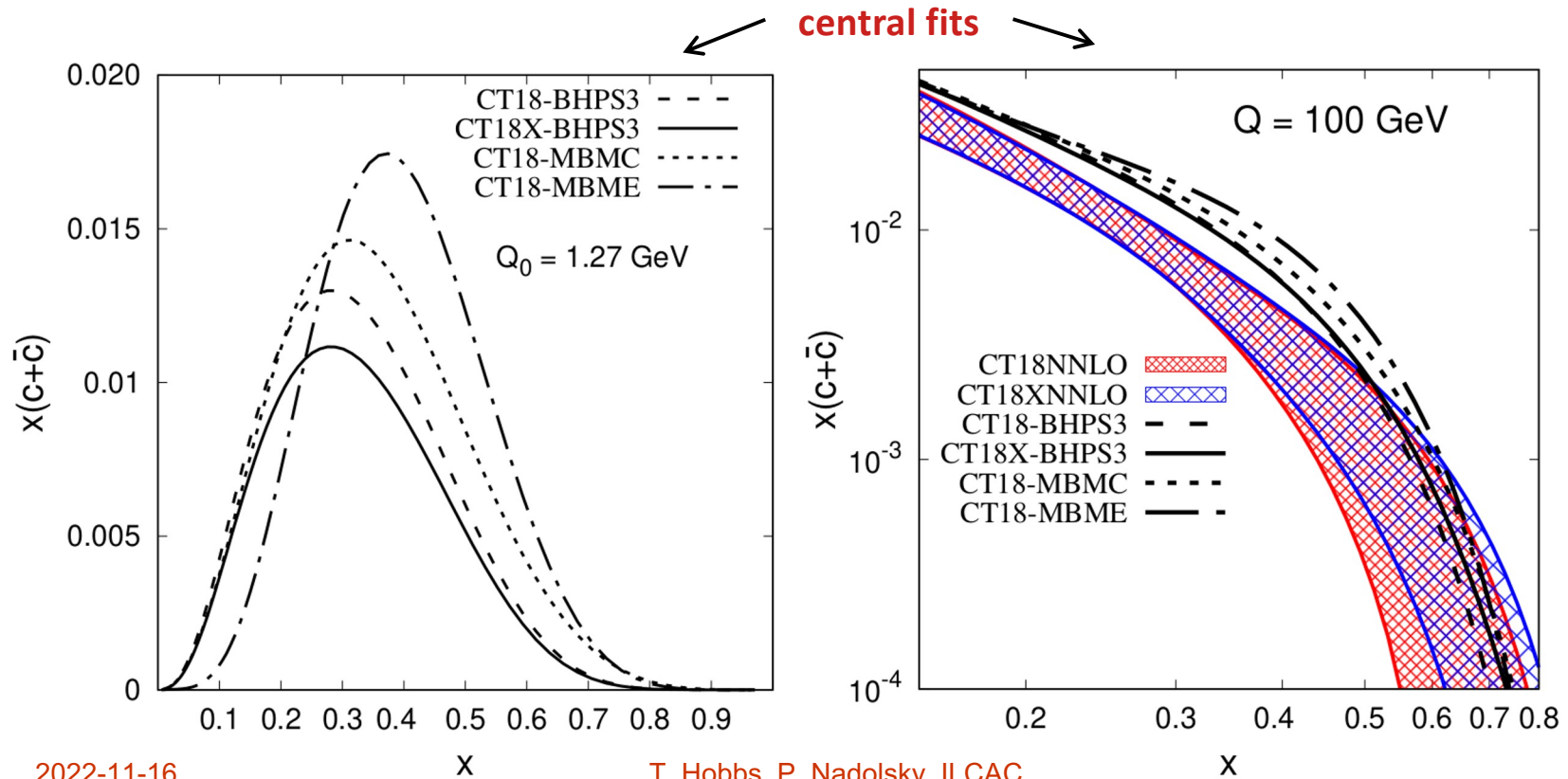
since CT14 IC, many LHC measurements have been released; natural to ask if these possess *collective* sensitivity to FC

# CT18 FC total charm PDFs

FC scenarios traverse range of high- $x$  behaviors from IC models

- fit implementation of BHPS from CT14IC (BHPS3) on CT18 or CT18X (NNLO)
- fit two MBMs: MBMC (confining), MBME (effective mass) on CT18

investigate constraints from newer LHC data in CT18

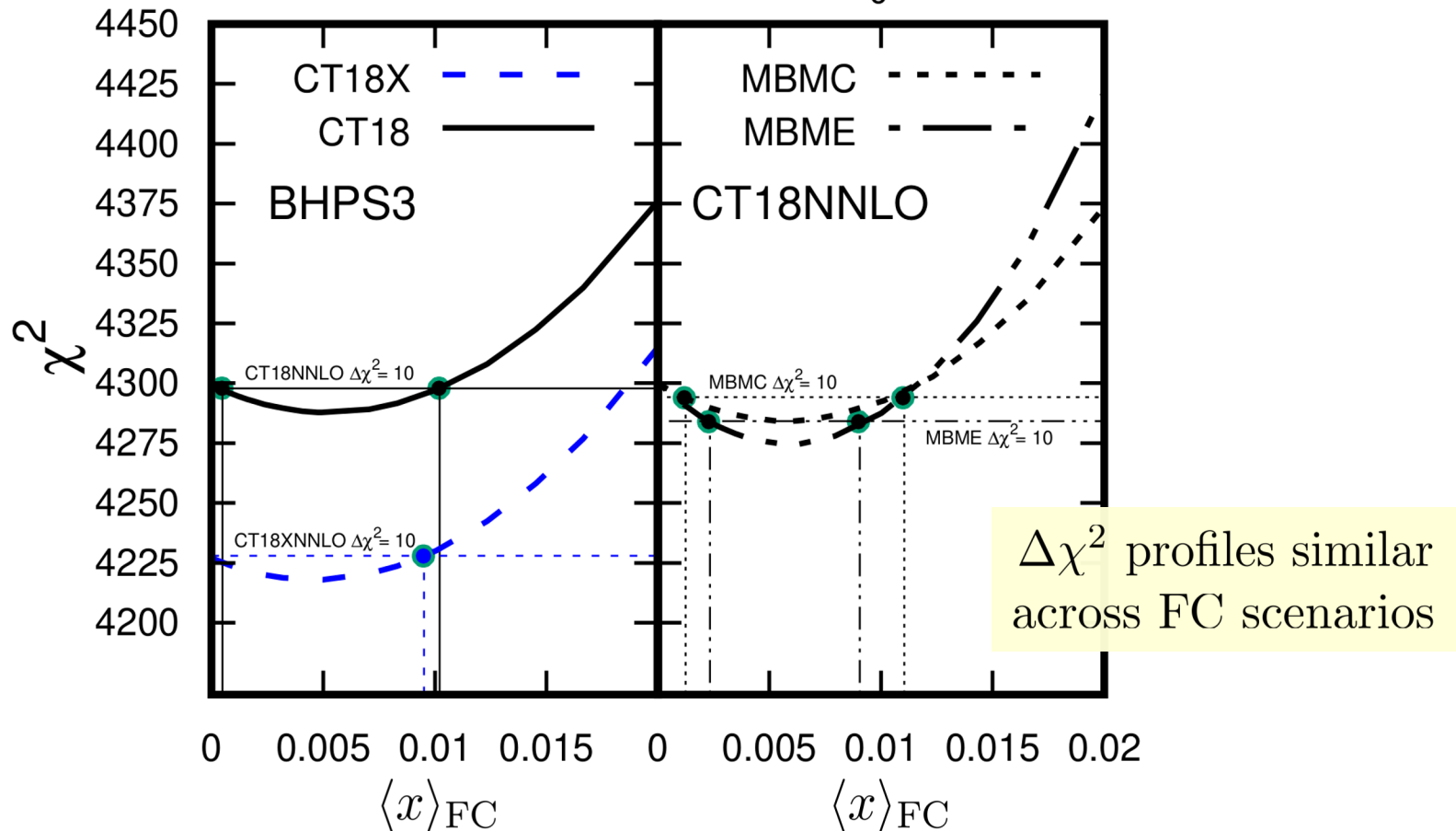


signal for FC in CT18 study, but with shallower  $\Delta\chi^2$  than CT14 IC

FC uncertainty quantified by normalization via  $\langle x \rangle_{\text{FC}}$  for each input IC model

→  $\langle x \rangle_{\text{FC}} \approx 0.5\%$  ( $\Delta\chi^2 \gtrsim -25$ ) vs.  $\langle x \rangle_{\text{FC}} \approx 0.8-1\%$  ( $\Delta\chi^2 \gtrsim -40$ ) **CT14 IC**

CT18 nonperturbative charm fit  $Q_0 = 1.27 \text{ GeV}$



# FC PDF moments as F.o.M.

moments of the FC PDFs often used to characterize magnitude, asymmetry

$$\langle x^n \rangle_{c^\pm} = \int_0^1 dx x^n (c \pm \bar{c})[x, Q]$$

$$\langle x \rangle_{\text{FC}} \equiv \langle x \rangle_{c^+} [Q_0 = 1.27 \text{ GeV}] \quad \dots \text{at NNLO.}$$

$$= 0.0048^{+0.0063}_{-0.0043} \left( {}^{+0.0090}_{-0.0048} \right), \text{ CT18 (BHPS3)}$$

$$= 0.0041^{+0.0049}_{-0.0041} \left( {}^{+0.0091}_{-0.0041} \right), \text{ CT18X (BHPS3)}$$

$$= 0.0057^{+0.0048}_{-0.0045} \left( {}^{+0.0084}_{-0.0057} \right), \text{ CT18 (MBMC)}$$

$$= 0.0061^{+0.0030}_{-0.0038} \left( {}^{+0.0064}_{-0.0061} \right), \text{ CT18 (MBME)}$$

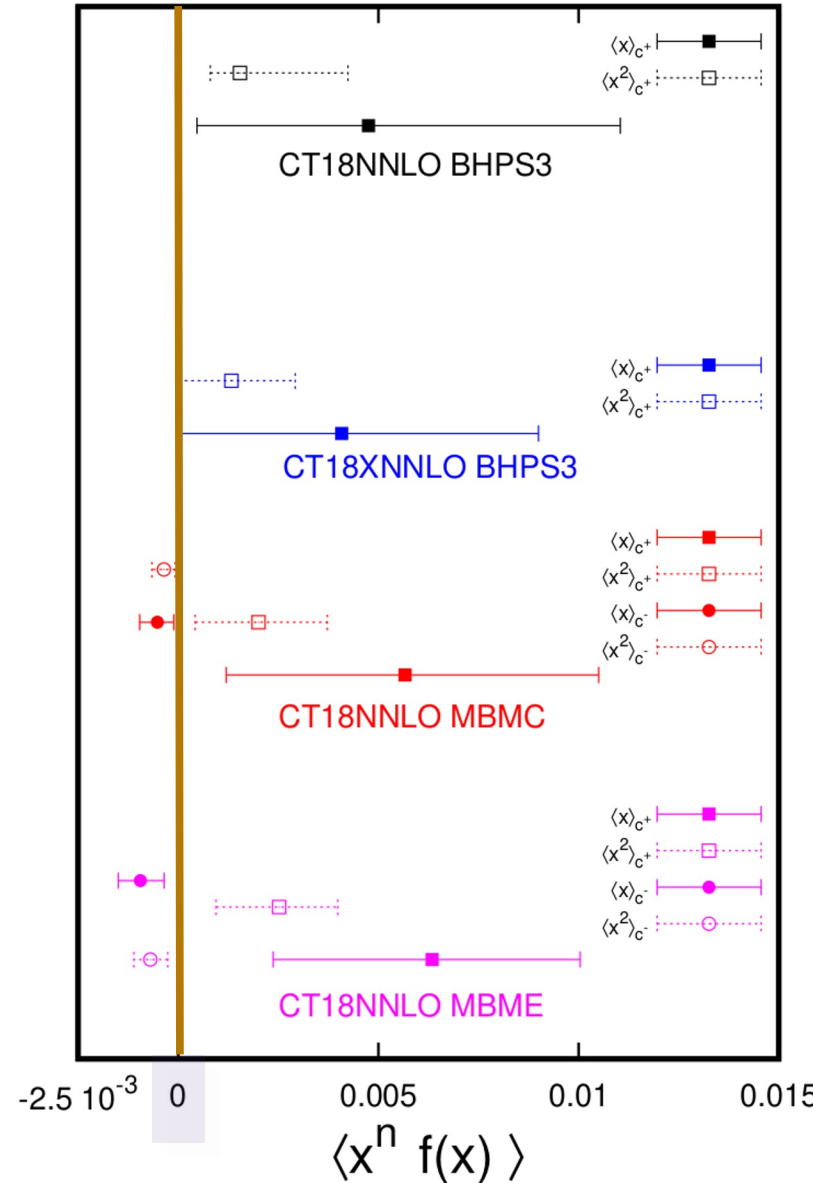
$$\Delta\chi^2 \leq 10$$

$$\Delta\chi^2 \leq 30$$

(restrictive tolerance)

(~CT standard tolerance)

Nonperturbative charm moments  $Q_0 = 1.27 \text{ GeV}$   
Intervals of  $\Delta\chi^2 < 10$



# FC PDF moments as F.o.M.

**even restrictive uncertainties give moments consistent with zero**

→ broaden further for default CT tol.

→ lattice may give  $\langle x \rangle_{c+}$ ,  $\langle x^2 \rangle_{c-}$

$$\langle x \rangle_{\text{FC}} \equiv \langle x \rangle_{c+} [Q_0 = 1.27 \text{ GeV}]$$

$$= 0.0048^{+0.0063}_{-0.0043} \left( {}^{+0.0090}_{-0.0048} \right), \text{ CT18 (BHPS3)}$$

$$= 0.0041^{+0.0049}_{-0.0041} \left( {}^{+0.0091}_{-0.0041} \right), \text{ CT18X (BHPS3)}$$

$$= 0.0057^{+0.0048}_{-0.0045} \left( {}^{+0.0084}_{-0.0057} \right), \text{ CT18 (MBMC)}$$

$$= 0.0061^{+0.0030}_{-0.0038} \left( {}^{+0.0064}_{-0.0061} \right), \text{ CT18 (MBME)}$$

$$\Delta\chi^2 \leq 10$$

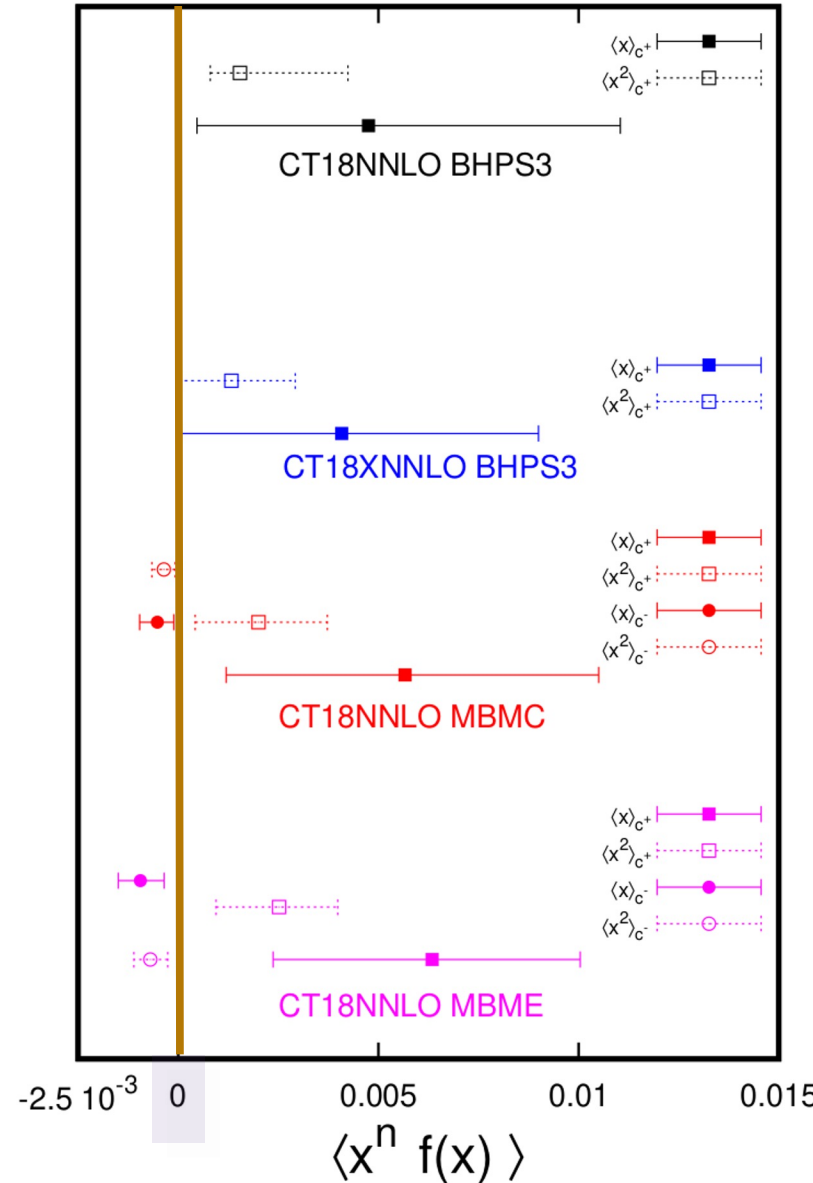
$$\Delta\chi^2 \leq 30$$

(restrictive tolerance)

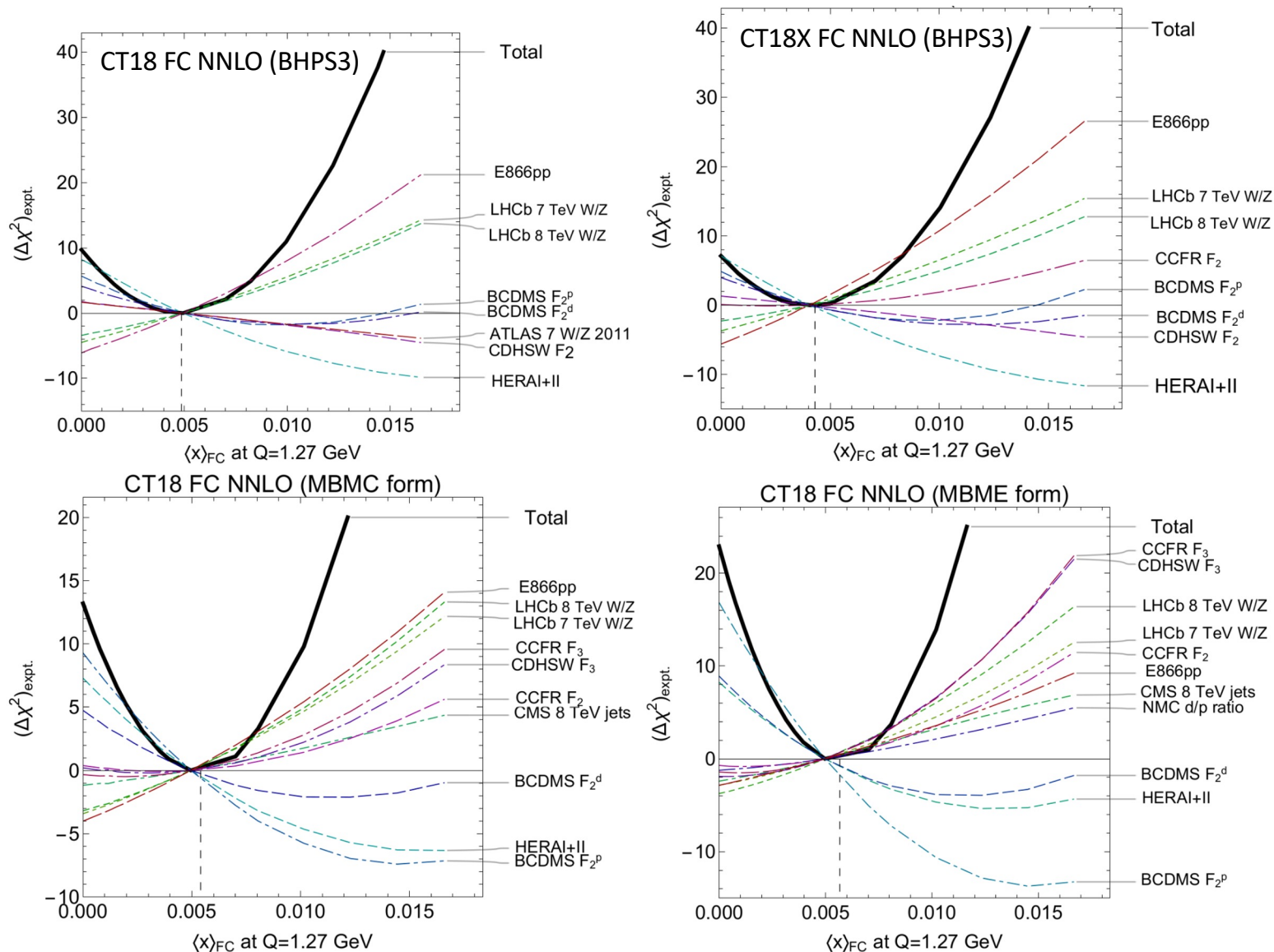
(~CT standard tolerance)

Nonperturbative charm moments  $Q_0 = 1.27 \text{ GeV}$

Intervals of  $\Delta\chi^2 < 10$



data pull opposingly on  $\langle x \rangle_{FC}$ ; depend on FC scenario, enhancing error

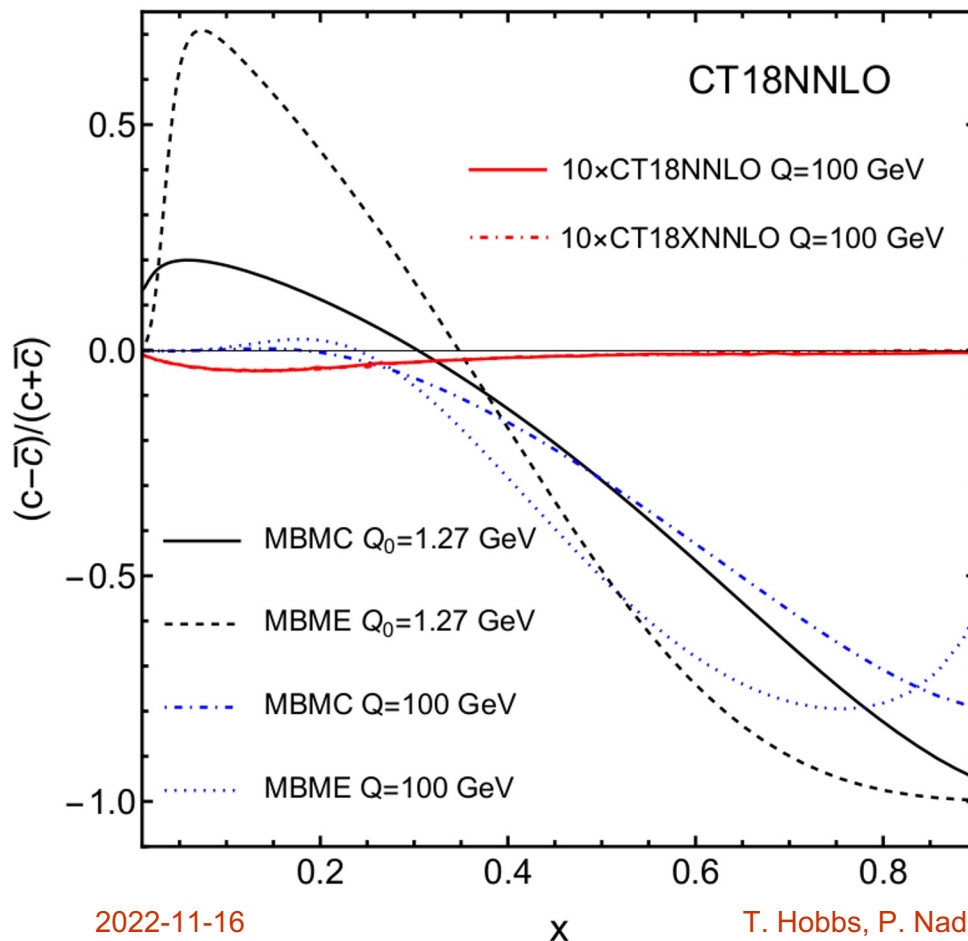




# possible charm-anticharm asymmetries

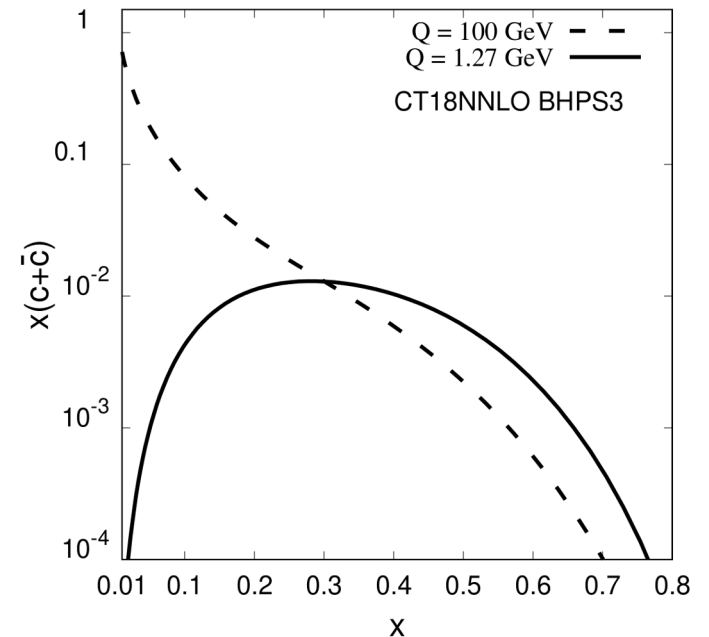
pQCD only very weakly breaks  $c = \bar{c}$  through HO corrections

- large(r) charm asymmetry would signal nonpert dynamics, IC
- MBM breaks  $c = \bar{c}$  through hadronic interactions



consider two MBM models as **examples** (not predictions)

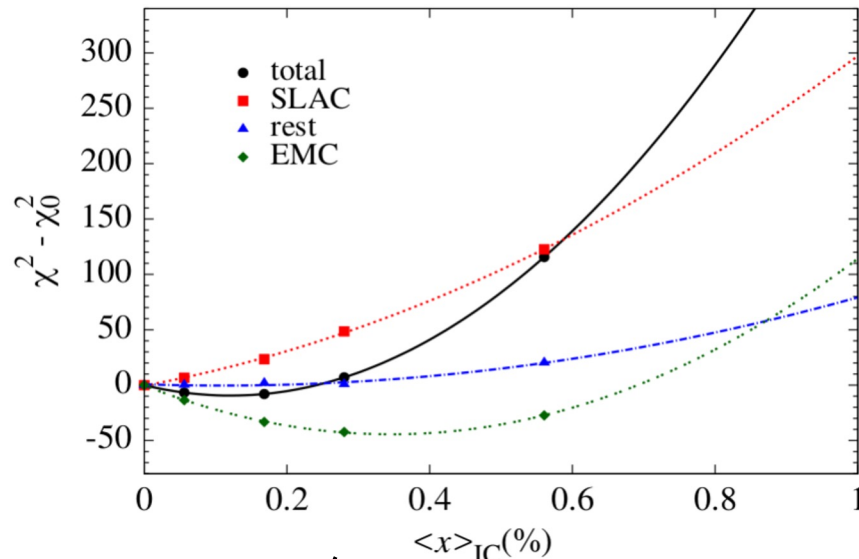
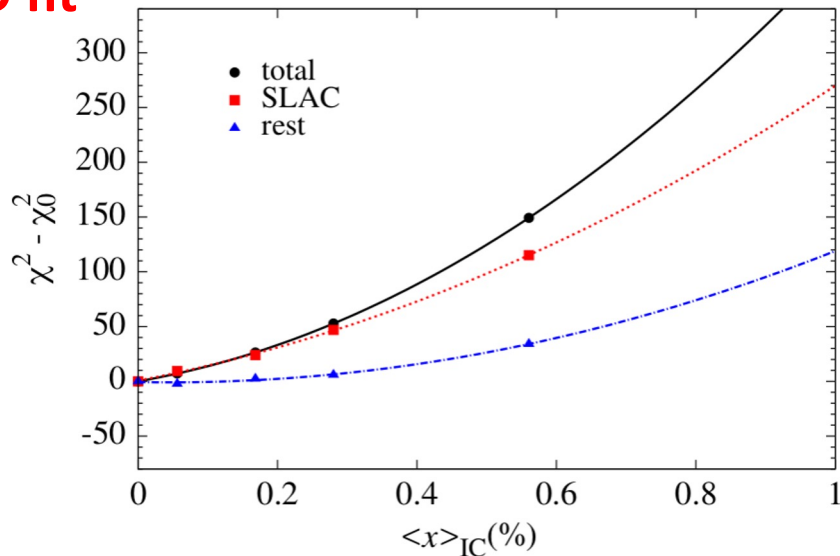
- asym. small but ratio (left) can be bigger; will be hard to extract from data



# re: other studies → first full fit with EMC data; no signal for 'IC'

P. Jimenez-Delgado, TJH, J. T. Londergan and W. Melnitchouk; PRL 114, no. 8, 082002 (2015).

## NLO fit



'SLAC + REST'  $\implies \langle x \rangle_{IC} < 0.1\%$ ; at  $5\sigma$  !

'REST' only  $\implies \langle x \rangle_{IC} < 0.1\%$ ; at  $1\sigma$

EMC alone:  $\langle x \rangle_{IC} = 0.3 - 0.4\%$

+ **SLAC**/'REST':  $\langle x \rangle_{IC} = 0.13 \pm 0.04\%$

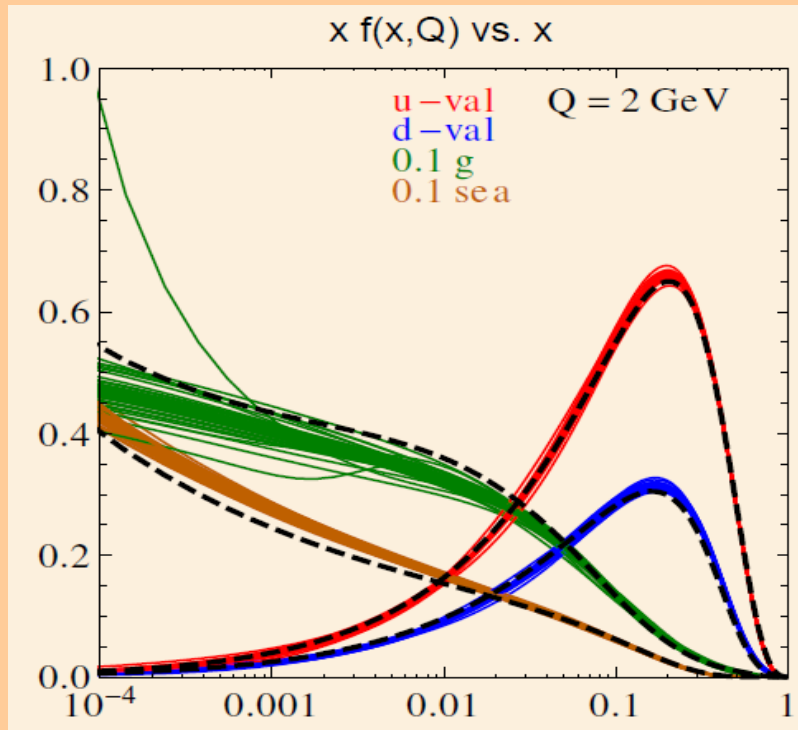
...but  $F_2^{c\bar{c}}$  poorly fit —  $\chi^2 \sim 4.3$  per datum!

included SLAC DIS; used more restrictive tolerance criterion,  $\Delta\chi^2 = 1$

# Part 3. Probability

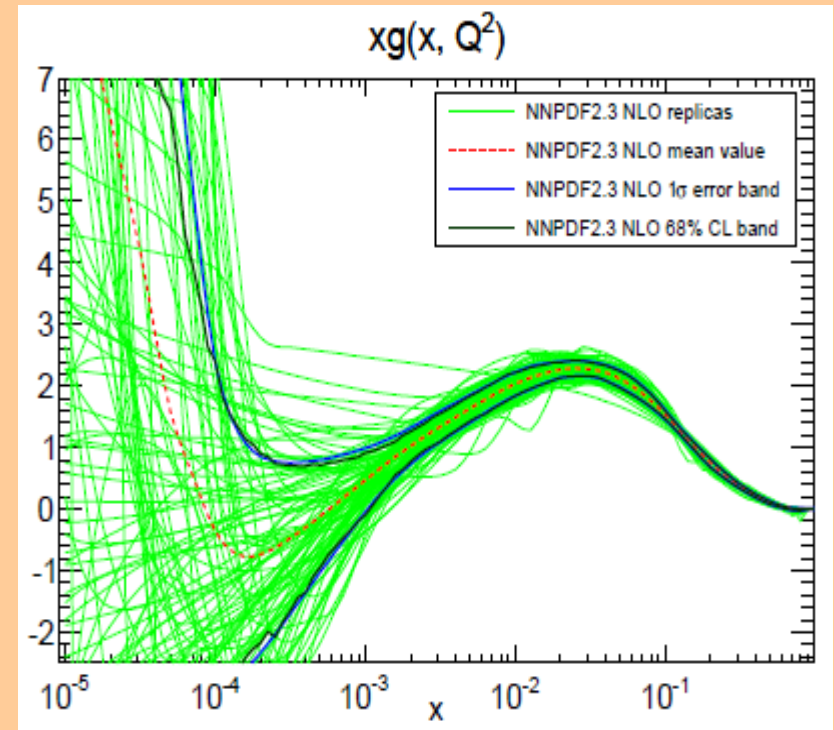
# Two types of modern error PDFs

Analytic parametrizations +  
Hessian PDF eigenvector sets  
(**ABM, CTEQ, HERA, MMHT,...**)



Obtained by analytic minimization of  
the log-likelihood  $\chi^2$

Neural network parameterizations  
+ Monte Carlo PDF replicas  
(**NNPDF**)

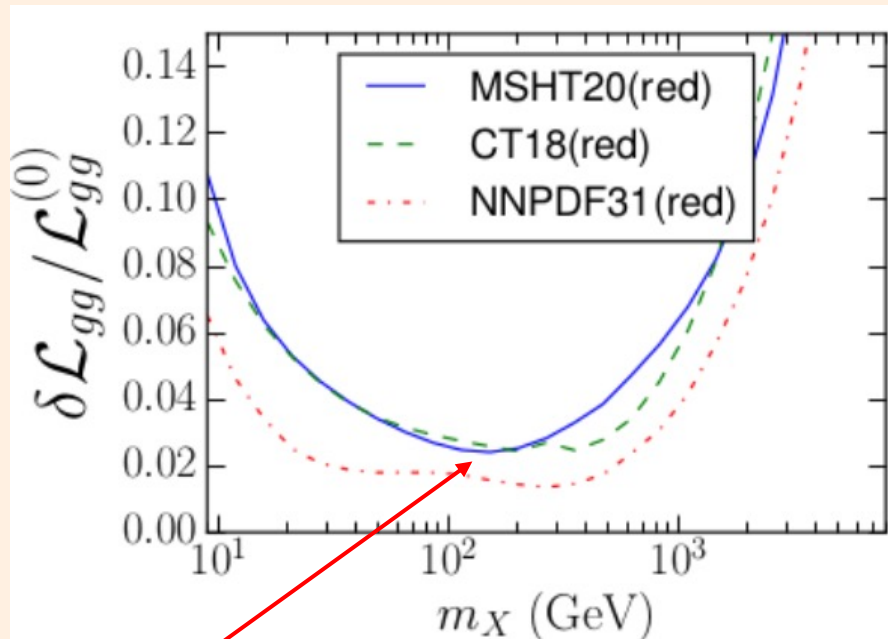


Obtained as NN replicas of an optimized  
architecture trained using an objective function  
with the log-likelihood and prior constraints

# The tolerance puzzle

Relative PDF uncertainties on the  $gg$  luminosity at 14 TeV in three PDF4LHC21 fits to the **identical** reduced global data set

arXiv:2203.05506



×1.5 – 2 difference

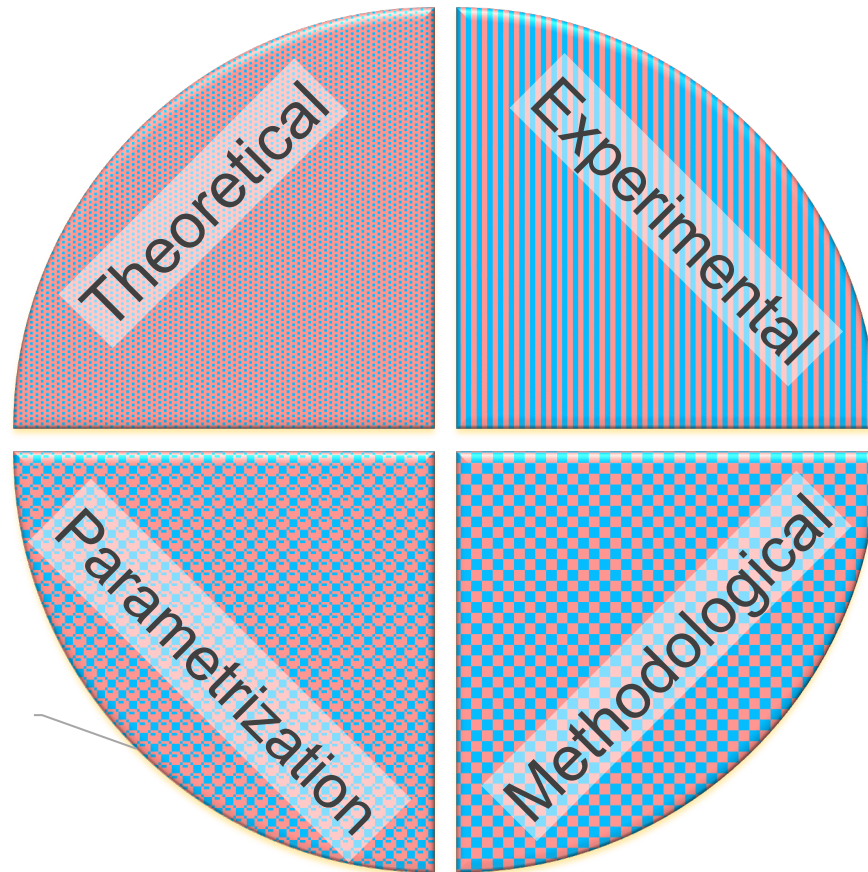
“Tolerance” determines the size of the PDF uncertainty.

While the fitted data sets are identical or similar in several such analyses, the resulting PDF sets may differ because of methodological choices adopted by the PDF fitting groups.

NNPDF3.1' and especially 4.0 (based on the NN's+ MC technique) tend to give smaller uncertainties in data-constrained regions

**Is any error underestimated or overestimated?**

# Components of PDF uncertainty



In each category, one must maximize

**PDF fitting accuracy**  
(accuracy of experimental, theoretical and other inputs)

**PDF sampling accuracy**  
(adequacy of sampling of space of possible solutions)

**NEW AND IMPORTANT**

**Fitting/sampling classification** is borrowed from the statistics of large-scale surveys  
[Xiao-Li Meng, *The Annals of Applied Statistics*, Vol. 12 (2018), p. 685]

Kovarik et al., arXiv: [1905.06957](https://arxiv.org/abs/1905.06957)

# Unrepresentative big surveys significantly overestimated US vaccine uptake

*Nature* v. 600 (2021) 695

<https://doi.org/10.1038/s41586-021-04198-4> Valerie C. Bradley<sup>1,2</sup>, Shiro Kuriwaki<sup>3,2</sup>, Michael Isakov<sup>3</sup>, Dino Sejdinovic<sup>1</sup>, Xiao-Li Meng<sup>4</sup> & Seth Flaxman<sup>2,3</sup>  
Received: 18 June 2021

SCIENCE ADVANCES | RESEARCH ARTICLE

## MATHEMATICS

### Models with higher effective dimensions tend to produce more uncertain estimates

Arnald Puy<sup>1,2,3\*</sup>, Pierfrancesco Beneventano<sup>4</sup>, Simon A. Levin<sup>2</sup>, Samuele Lo Piano<sup>5</sup>, Tommaso Portaluri<sup>6</sup>, Andrea Saltelli<sup>3,7</sup>

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## The Big Data Paradox in Clinical Practice

Pavlos Msaouel

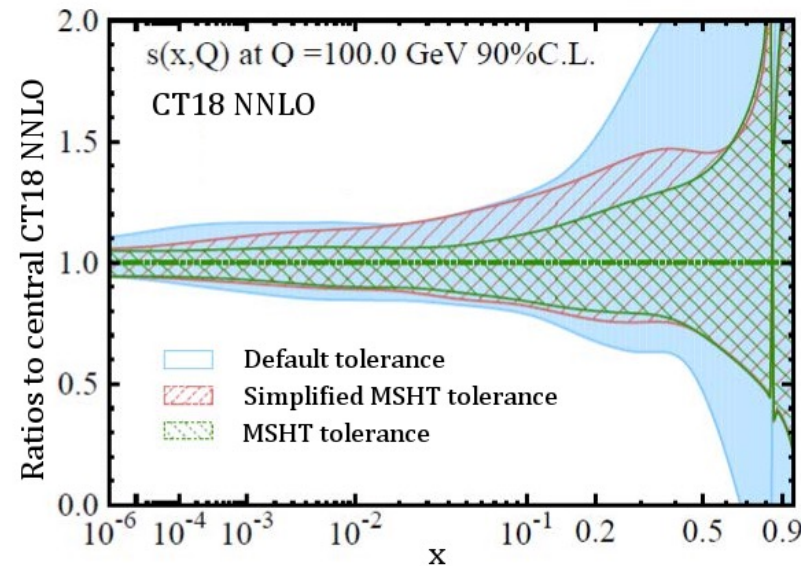
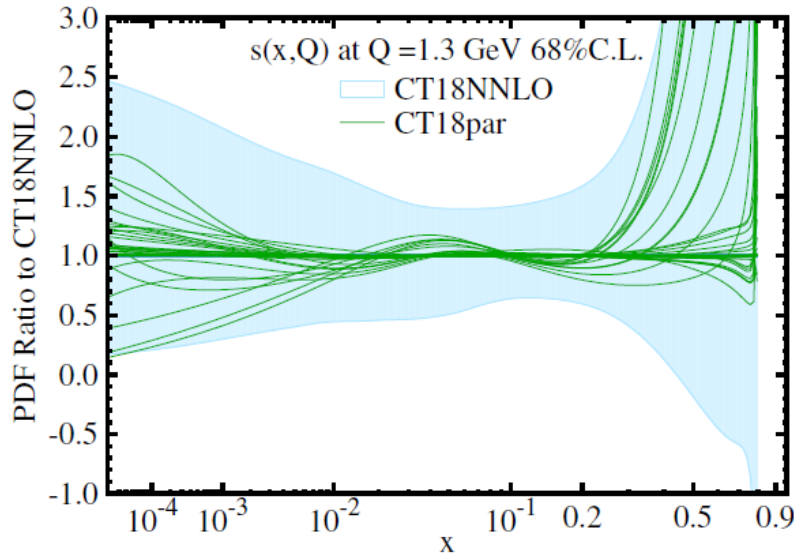
To cite this article: Pavlos Msaouel (2022) The Big Data Paradox in Clinical Practice, Cancer Investigation, 40:7, 567-576, DOI: [10.1080/07357907.2022.2084621](https://doi.org/10.1080/07357907.2022.2084621)

A flurry of recent articles point out statistical difficulties with estimating uncertainties in multiparametric models working with large data sets. Here, subtle **sampling biases can negate** reduction of the uncertainty due to **the law of large numbers**, leading to the situation dubbed **“the big-data paradox”**:

Unrepresentative sampling in a large number of dimensions commonly leads to wrong conclusions with high confidence.

# CT and MSHT tolerances: average $\langle \Delta\chi^2 \rangle \approx 10$ and 30 at 68% C.L.

Tolerance can be understood in the language of sampling over acceptable PDF solutions.



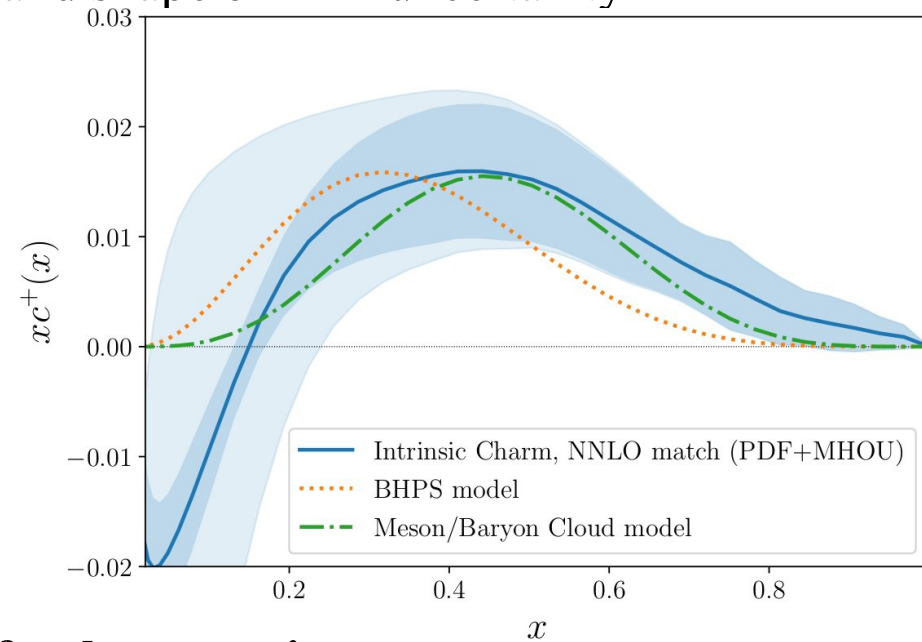
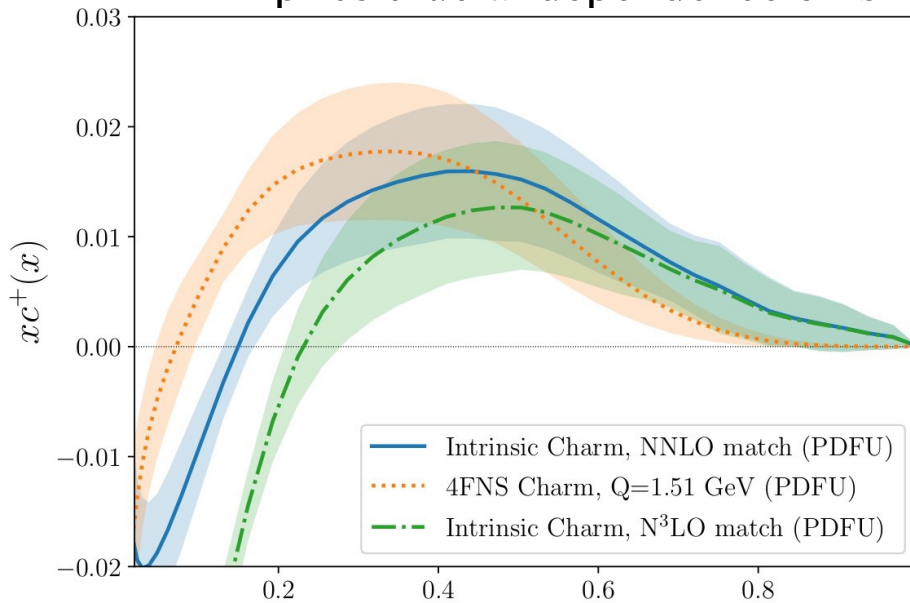
**Upper figure:** A large part of the CT18 PDF uncertainty accounts for the sampling over 250-350 parametrization forms, possible choices of fitted experiments and fitting parameters, definitions of  $\chi^2$

**Lower figure:** this approach sometimes enlarges the uncertainties compared to the other groups. For example, the MSHT tolerance is based on the agreement with each included experiment for a fixed parametrization form.

More restrictive tolerance criteria may introduce a hidden uncertainty due to unrepresentative sampling.



- NNPDF have recently claimed  $3\sigma$  evidence for ‘IC’
  - based on local ( $x$ -dependent) deviation of FC PDF from the ‘no-FC’ scenario
  - implies crucial dependence on size and shape of PDF uncertainty

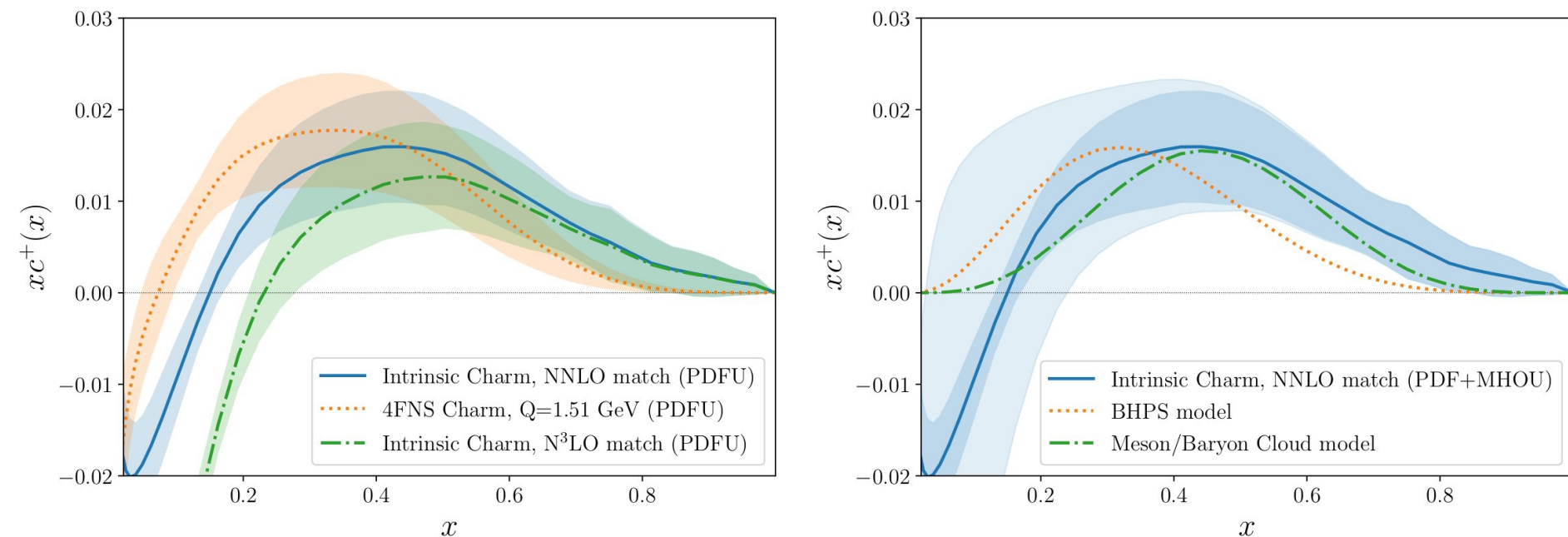


→ **Two classes of uncertainties need further scrutiny:**

1. Missing higher-order uncertainties (MHOU): N3LO in DIS, etc.; N2LO in Z+c production
2. Parametrization sampling uncertainty (PSU): underestimation of PDFU

- large perturbative instability from MHOU in DGLAP affects low- $x$  behavior

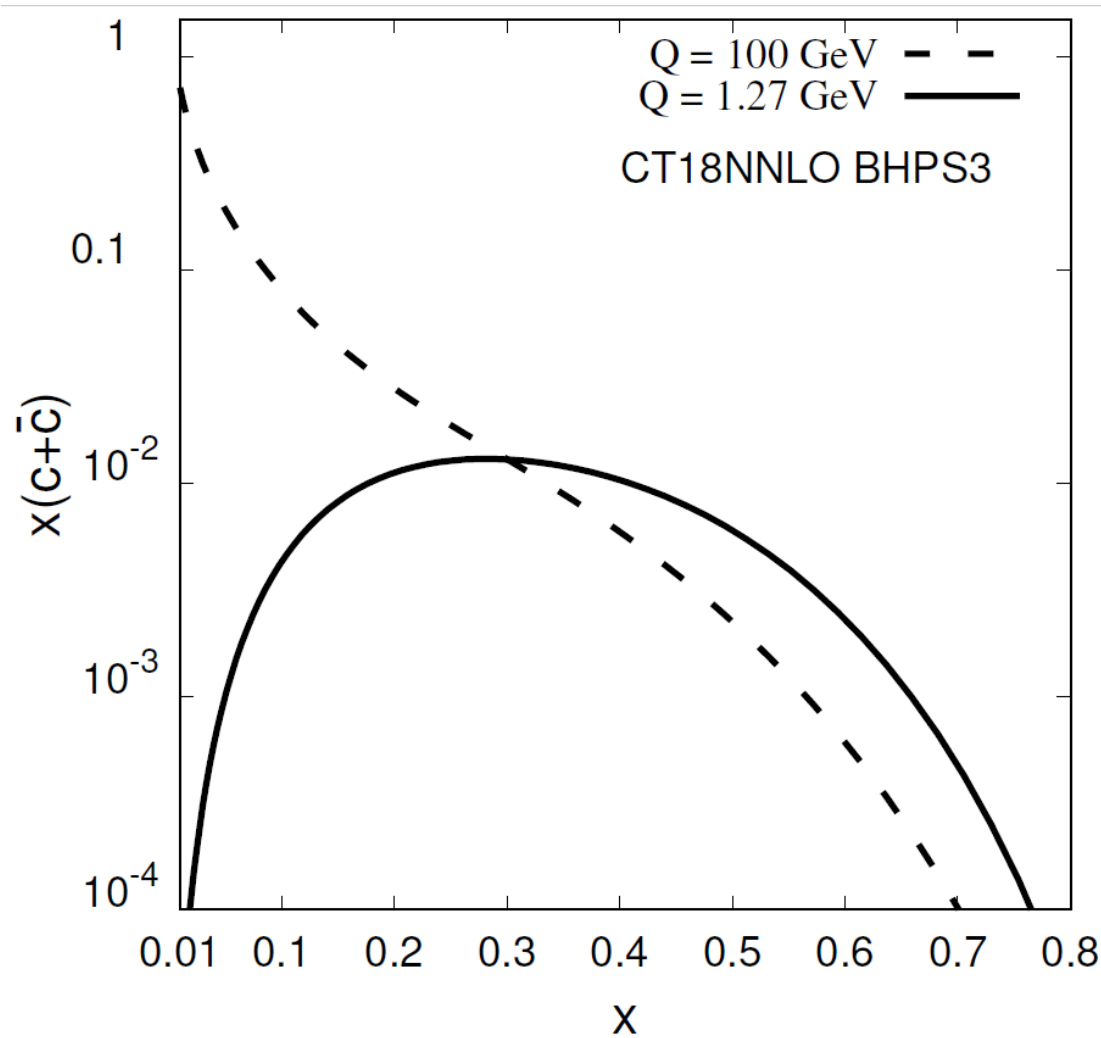
→ matching at fixed NNLO gives negative FC, unlike IC models



→ MHOU excluded to obtain a nominal charm fraction,  $\langle x \rangle_{\text{FC}} = 0.62 \pm 0.28\%$

→ if MHOU is included, consistency with zero:  $\langle x \rangle_{\text{FC}} = 0.62 \pm 0.61\%$

# Backward DGLAP evolution is approximate



Data constrain the PDFs at  $Q > 2 \text{ GeV}$ .

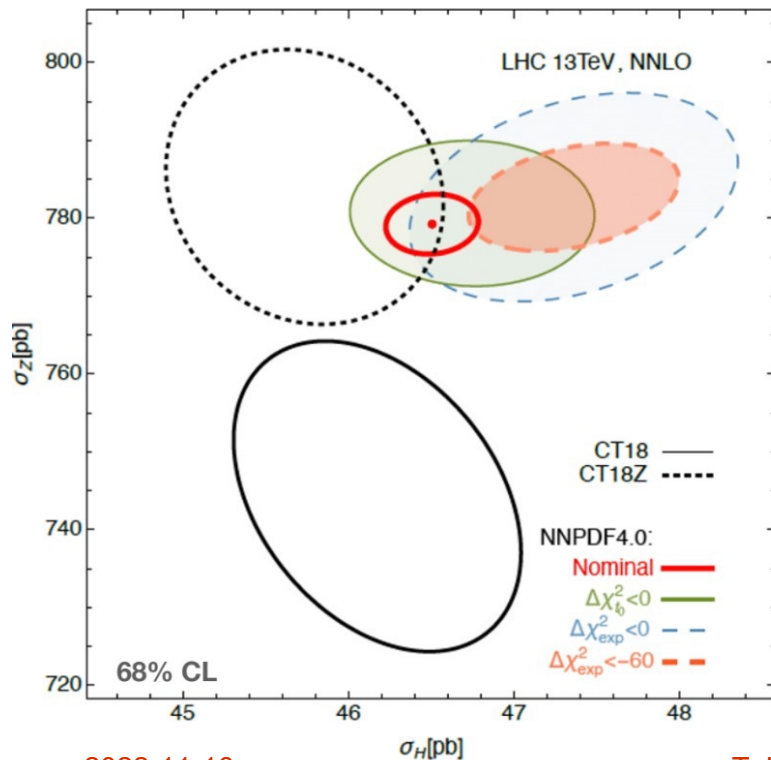
When PDFs are evolved at N2LO down to  $Q \approx 1.3 \text{ GeV}$ , the charm PDF is increased at  $x \gtrsim 0.3$  and decreased at  $x \lesssim 0.3$ .

MHOU in DGLAP evolution can produce the bump-like shape.

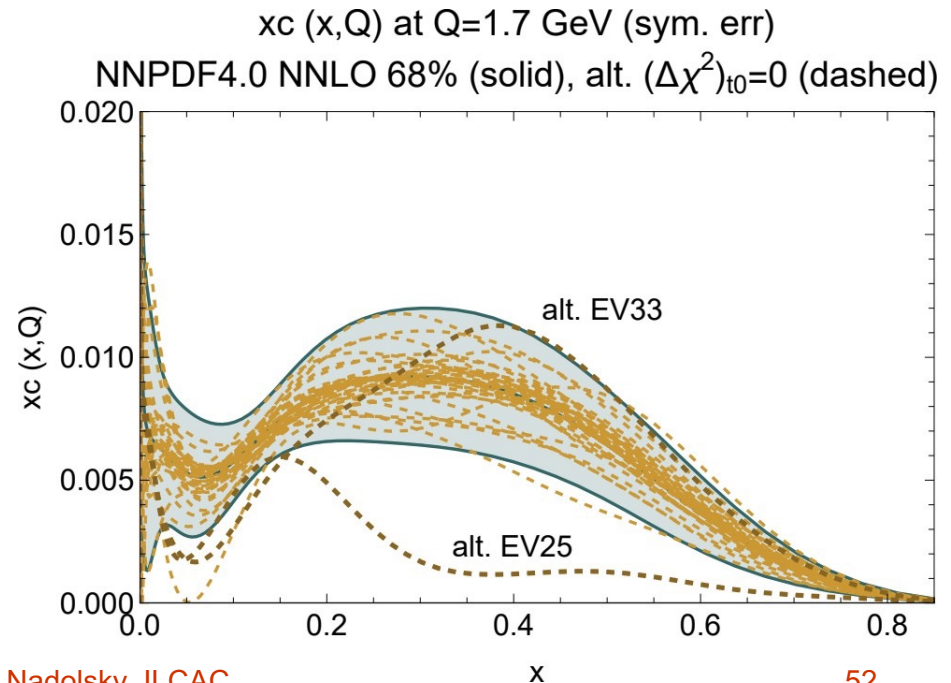
# more representative sampling can enlarge MC uncertainties

Courtoy *et al.*, arXiv: 2205.10444.

- default replica-training in MC studies may omit otherwise acceptable solutions
- more comprehensive sampling with the public NNPDF4.0 code impacts PDF errors of cross sections



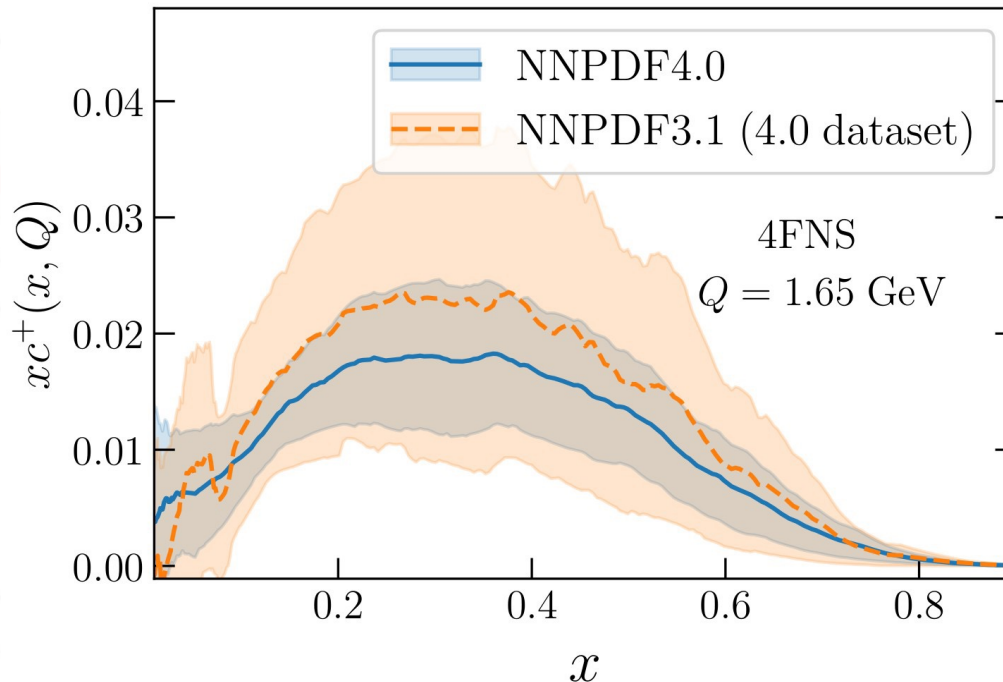
- substantially broadens high- $x$  FC error



# more representative sampling can enlarge MC uncertainties

Courtoy *et al.*, arXiv: 2205.10444.

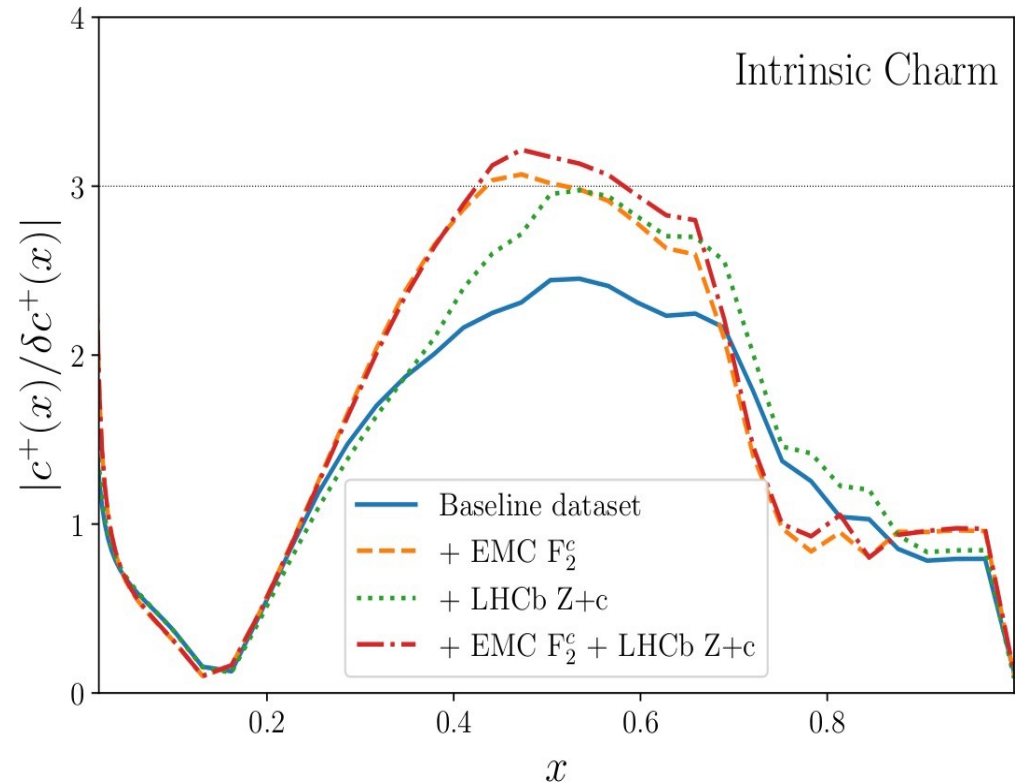
- default replica-training in MC studies may omit otherwise acceptable solutions
- more comprehensive sampling impacts PDF errors of cross sections  
→ alternate fitting methodologies (NNPDF3.1 vs. 4.0) produce significant differences in the PDF uncertainty



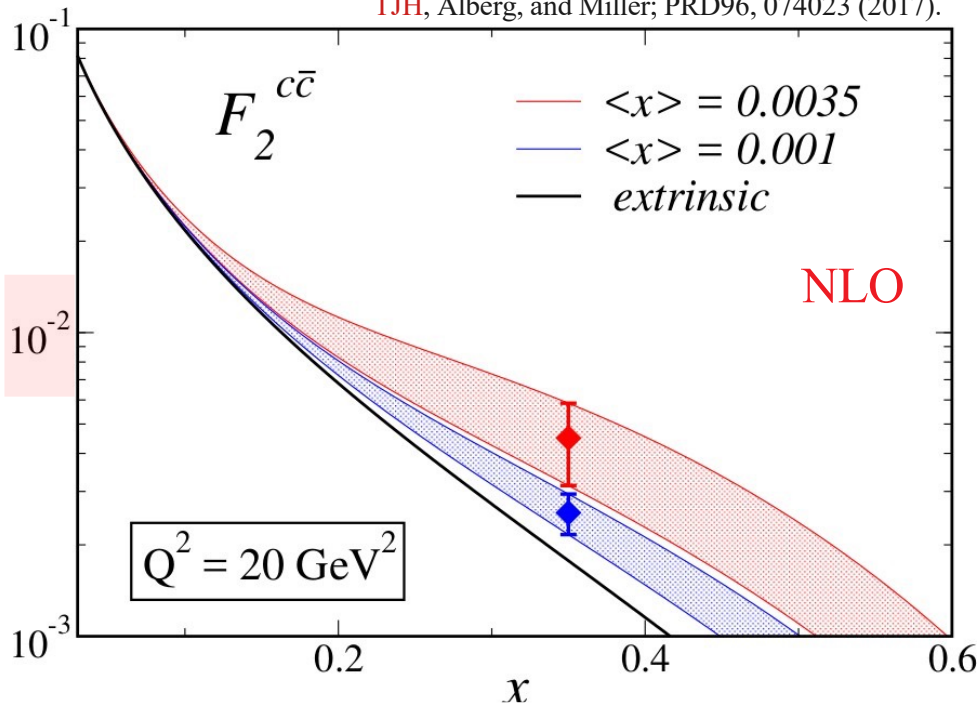
# Revisiting the significance in NNPDF IC

## Important additional uncertainties:

- In the baseline fit due to sampling of MC replicas
- In the NLO LHCb  $Z + c$  analysis due to MHOUs and final-state showering
- In the EMC  $F_2^c$  due to insufficient control of syst. uncertainties and LO analysis



∴ No significant evidence for NNPDF4.0 IC, in compliance with CT18 FC observations



# future data will inform FC

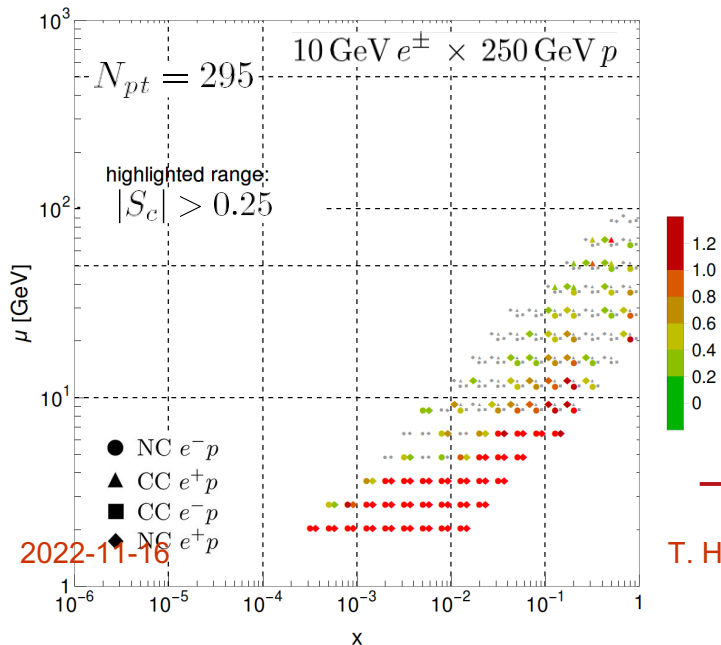
EIC + lattice QCD  
will constrain FC  
scenarios

enhanced FC momentum implied by  
EMC data → small high- $x$  effects in  
structure function; need high precision

essential complementary  
input from LHC; CERN FPF

EIC will measure precisely in the few-  
GeV, high- $x$  region where FC signals  
are to be expected

$|S_f|$  for  $c(x, \mu)$ , CT14 HERA2 NNLO



2022-11-16

T. Hobbs, P. Nadolsky, ILCAC

# conclusions

---

- size, shape of nonpert charm remains **indeterminate**
  - theoretical ambiguities in relation between FC/IC unresolved
  - need more sensitive data; FC currently consistent with zero

concordance with enlarged error estimates:  $\langle x \rangle_{\text{FC}} \sim 0.5\%$ , well below evidence-level

- need more NNLO and better showering calculations (*e.g.*, for  $Z+c$ )
- further progress in quantifying and estimating PDF uncertainties

---

opportunities to improve knowledge of FC:

- - promising experiments at LHC; EIC; CERN FPF
  - lattice data on key charm PDF moments; quasi-PDFs
  - direct benchmarking of FC among PDF fitting groups



# Backup