Spin alignment of vector mesons in heavy-ion collisions

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in collaboration with L. Oliva, Q. Wang, Z.-T. Liang, and X.-N. Wang.

@ The second Reimei Workshop "Polarization phenomena and Lorentz symmetry violation in dense matter"

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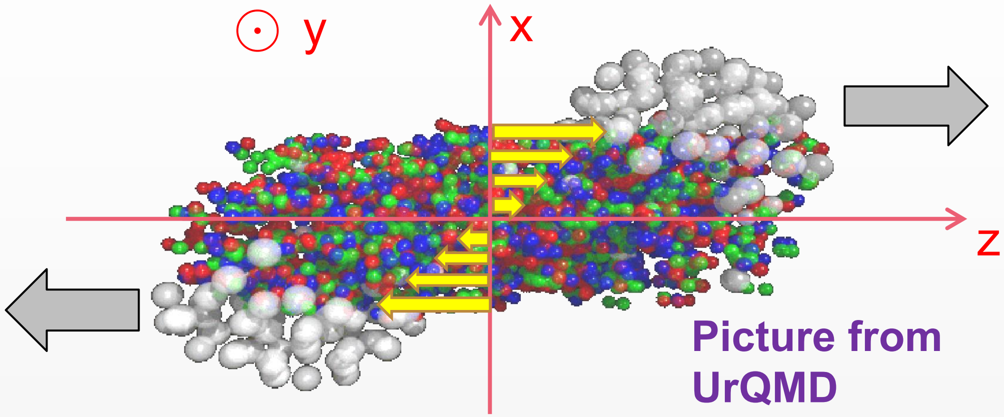
Contents

- Introduction
- Boltzmann equation for vector meson
- Numerical results for ϕ meson's spin alignment
- Mass splitting and spin alignment
- Summary and outlook

Heavy-ion collisions



Picture from UrQMD

Vorticity, magnetic field

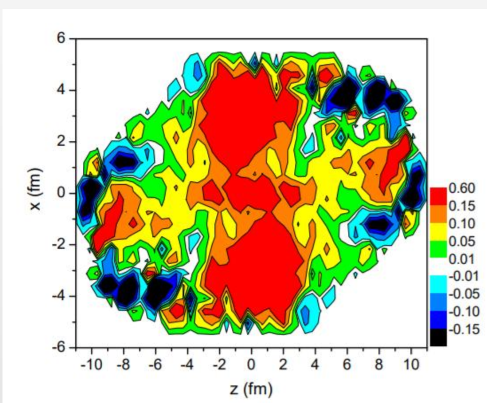
spin-orbit / magnetic coupling

Spin of hadrons

weak / strong p-wave decay

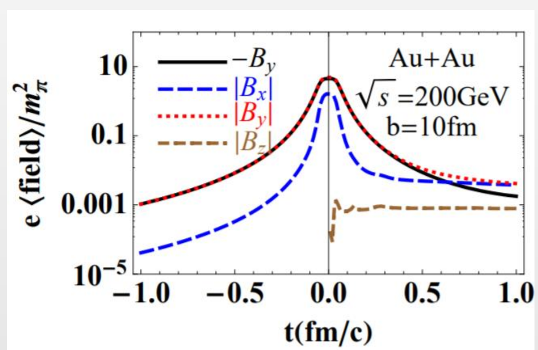
Measurements

Vorticity fields



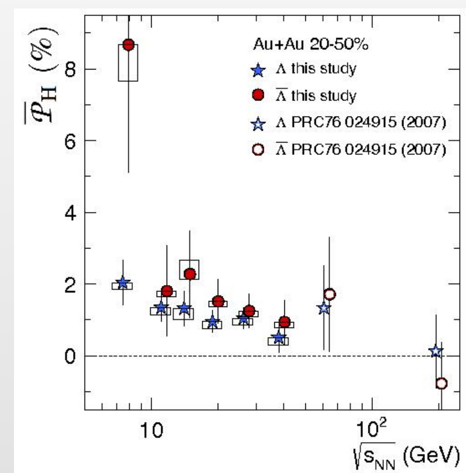
F. Becattini, L. Csernai, D.J. Wang, PRC 88, 034905 (2013); PRC 93, 069901 (2016)

Electromagnetic fields



W.-T. Deng, X.-G. Huang, PRC 85, 044907 (2012).

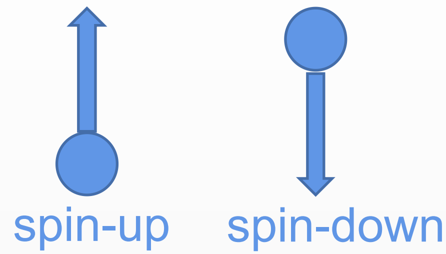
Λ's global polarization



L. Adamczyk, et al. (STAR), Nature 548 (2017) 62.

Polarization

- Spin-1/2 fermions



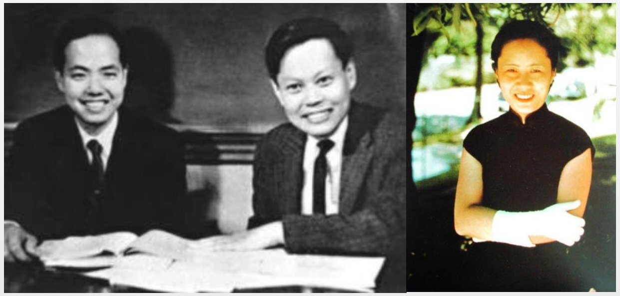
- Density matrix: $\rho = \sum_{r,s=-S,\cdots,S} \rho^{S}_{r,s} \left| r \right\rangle \left\langle s \right|$ $\quad \rho^{S=\frac{1}{2}}_{rs} = \begin{pmatrix} \rho_{+\frac{1}{2},+\frac{1}{2}} & \rho_{+\frac{1}{2},-\frac{1}{2}} \\ \rho_{-\frac{1}{2},+\frac{1}{2}} & \rho_{-\frac{1}{2},-\frac{1}{2}} \end{pmatrix}$

- Polarization for spin-1/2 fermions: $\dfrac{\rho_{+\frac{1}{2},+\frac{1}{2}} - \rho_{-\frac{1}{2},-\frac{1}{2}}}{\rho_{+\frac{1}{2},+\frac{1}{2}} + \rho_{-\frac{1}{2},-\frac{1}{2}}}$

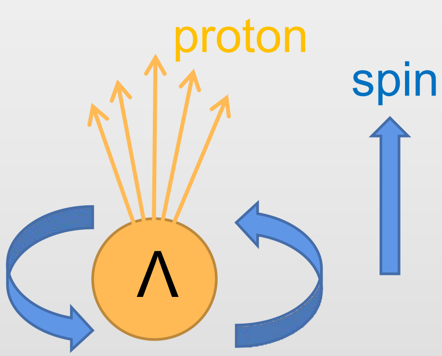
- Example: $\Lambda \rightarrow \mathrm{p} + \pi^{-}$

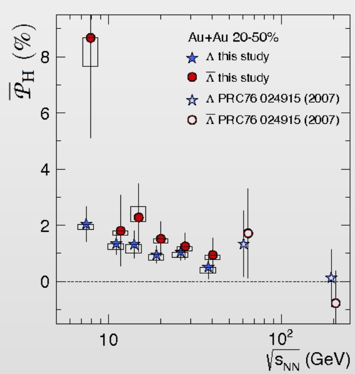
Weak decay

CP violation



T.-D. Lee, C.-N. Yang, C.-S. Wu





L. Adamczyk, et al. (STAR), Nature 548 (2017) 62.

Spin alignment

- Spin alignment for a vector meson ($J^P = 1^-$) is 00-element ρ_{00} of its normalized spin density matrix

  ρ_{00}=1/3 if spin does not have a preferred direction

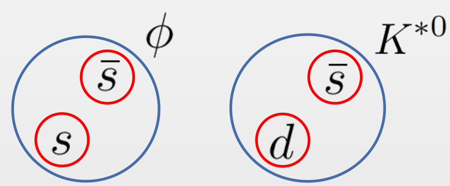
$$\rho_{rs}^{S=1} = \begin{pmatrix} \rho_{+1,+1} & \rho_{+1,0} & \rho_{+1,-1} \\ \rho_{0,+1} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,+1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

- Spin alignment (ρ_{00}) $\neq$ polarization ($\rho_{+1,+1} - \rho_{-1,-1}$)

- Spin alignment can be measured when

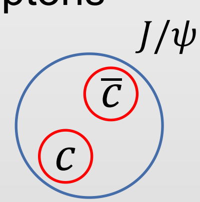
  (1) vector mesons decay to pseudo-scalar mesons

$$\phi \to K^+ + K^-$$
$$K^{*0} \to K^+ + \pi^-$$



  (2) vector mesons decay to dileptons

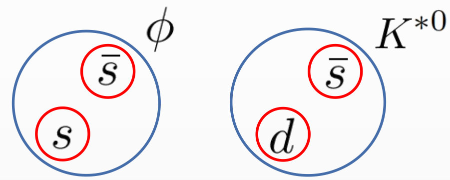
$$J/\psi \to \mu^+ + \mu^-$$



Spin alignment

- Vector mesons decay to pseudo-scalar mesons parity-odd strong decay

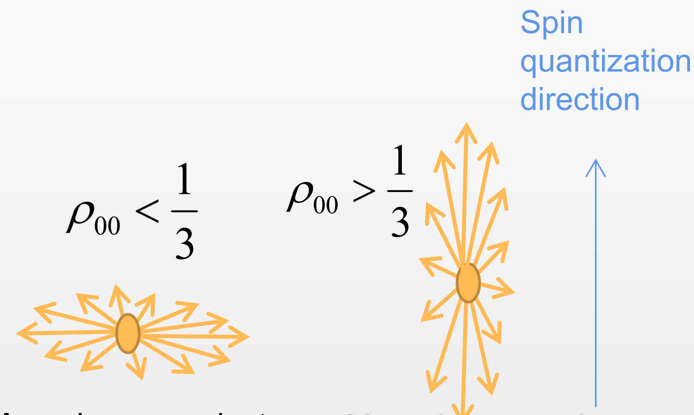
$$\phi \to K^+ + K^-$$
$$K^{*0} \to K^+ + \pi^-$$
$$J^P \quad 1^- \quad 0^- \quad 0^-$$



- Spin angular momentum of vector meson
  → orbital angular momentum of decay products

  For a meson with $S = 1$, $S_z = m$, angular distribution of decay products: $dN/d\Omega \propto |Y_{1,m}(\theta,\phi)|^2$

  Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018).



More decay products in transverse direction

$|S, S_z\rangle = |1, +1\rangle$
$|1, -1\rangle$

Transversely polarized

More decay products in longitudinal direction

$|1,0\rangle$

Longitudinally polarized

- Polar angle distribution

$$\frac{dN}{d\theta} = \frac{3}{4}\left[(1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta\right]$$

K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].

Experimental results

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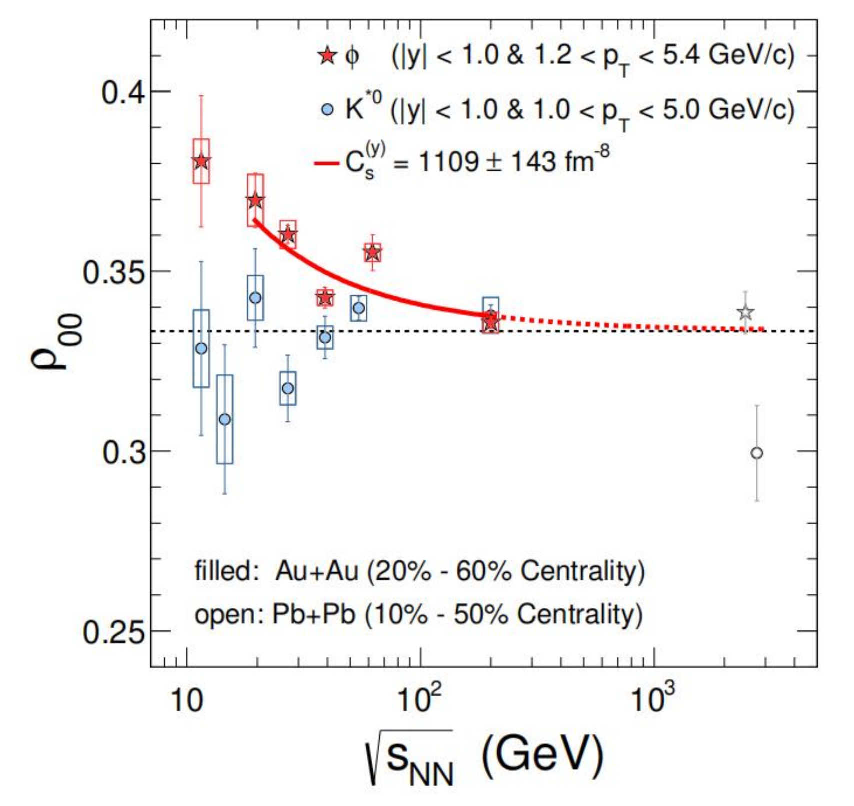
Observation of Global Spin Alignment of ϕ and K^{*0} Vector Mesons in Nuclear Collisions



ϕ meson's ρ_{00} is significantly larger than 1/3 for collision energies of 62 GeV and below (8.4σ !!)

K^{*0} meson's ρ_{00} is consistent with 1/3 within errors.

STAR collaboration, arXiv:2204.02302

Related studies

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- **Spin Alignment of Vector Mesons in Non-central $A+A$ Collisions** PLB 629, 20 (2005).

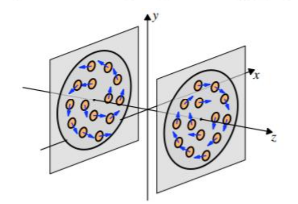
  Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]
  [1]Department of Physics, Shandong University, Jinan, Shandong 250100, China
  uclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 947
  (Dated: November 5, 2018)

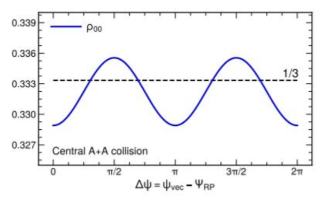
  Quark-antiquark recombination: $\rho_{00}^{V(\mathrm{rec})} = \frac{1-P_q P_{\bar q}}{3+P_q P_{\bar q}}$, Fragmentation: $\rho_{00}^{\rho\,(\mathrm{frag})} = \frac{1+\beta P_q^2}{3-\beta P_q^2}$,

- Contributions from vorticity and magnetic field:
  Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018).

	$\mathscr{P}_\phi$	ρ_{00}^{ϕ}	$\mathscr{P}_\Lambda$	
$\mathscr{P}(\omega)$ or $\rho_{00}^{\phi}(\omega)$	$\frac{2}{3}\beta\omega$	$\frac{1}{3}-\frac{1}{9}(\beta\omega)^2$	$\frac{1}{2}\beta\omega$	
$\mathscr{P}(B)$ or $\rho_{00}^{\phi}(B)$	0	$\boxed{\frac{1}{3}+\frac{4}{9}(\beta\mu_{\mathrm{ms}}B)^2}$	$\beta\mu_{\mathrm{ms}}B$	$\frac{1}{3}\beta B($
$\mathscr{P}(B)$	$\frac{2}{3}\beta\mu_{\mathrm{m}\phi}B$	$\frac{1}{3}-\frac{1}{9}(\beta\mu_{\mathrm{m}\phi}B)^2$	$\beta\mu_{\mathrm{m}\Lambda}B$	

- Local vorticity:
  X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817,136325 (2021).





- Helicity alignment: J.-H. Gao, PRD 104, 076016 (2021).

  Turbulent color fields: B. Mueller, D.-L. Yang, PRD 105, 1 (2022).

  Shear-induced spin alignment: F.Li, S.Liu, arXiv:2206.11890.
  D.Wagner, N.Weickgenannt, E.Speranza, arXiv:2207.0111.

- Strong force fields: XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv: 2206.05868; arXiv: 2205.15689.
  XLS, Q.Wang, X.-N.Wang, PRD 102, 056013 (2020). XLS, L.Oliva, Q.Wang, PRD 101, 096005 (2020).

Contents

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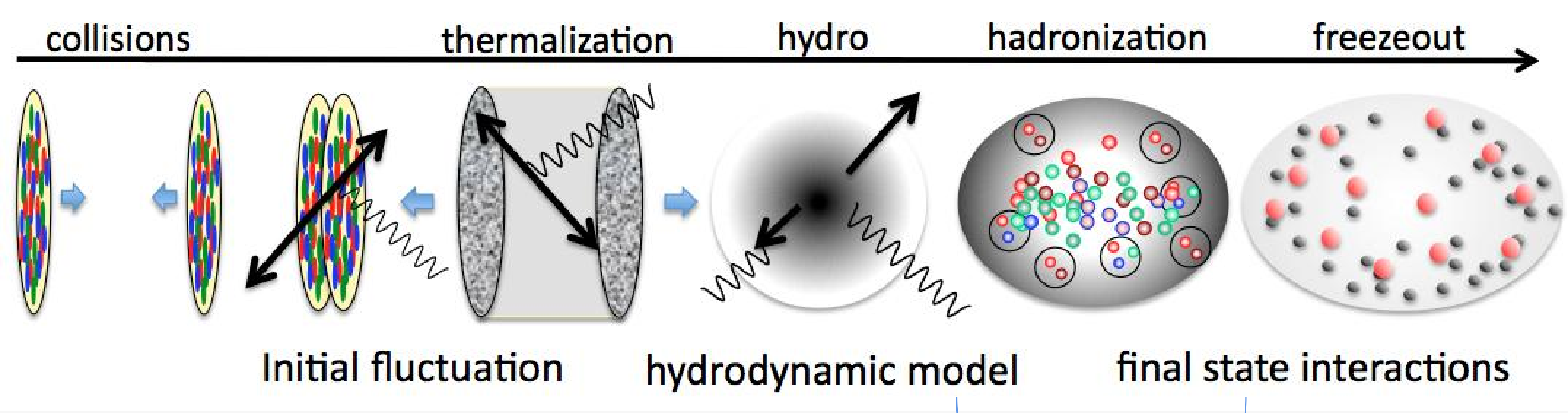
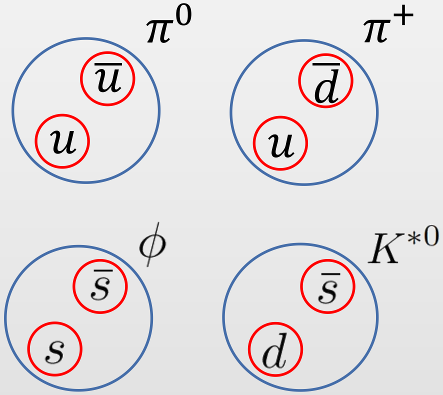
Quark combination



Quarks are confined inside hadrons



Quark combination model

R. C. Hwa and C. B. Yang, Phys. Rev. C 70, 024904 (2004);

R. J. Fries, B. Muller, C. Nonaka, S. A. Bass, Phys. Rev. C 68, 044902 (2003)

V. Greco, C. M. Ko, P. Lévai, L.W. Chen, Phys. Rev. C 68, 034904 (2003);

L. W. Chen, C. M. Ko, Phys. Rev. C 73, 044904 (2006)

include spin DoF

Z.-T. Liang and X.-N. Wang, Phys.Lett.B 629, 20 (2005).

Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys.Rev.C 97, 3 (2018).

Quark combination

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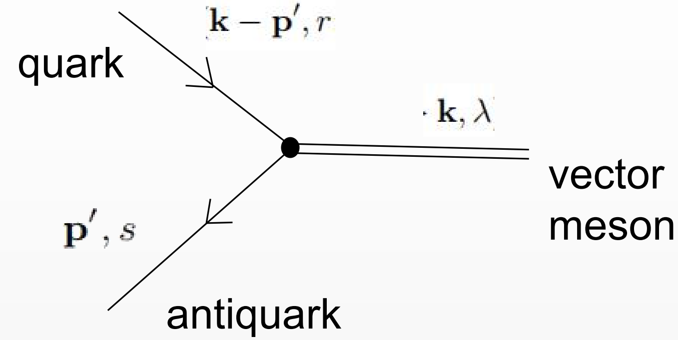
- Boltzmann equation for quark-antiquark combination

$$q+\bar{q}\rightarrow V$$



$$\begin{aligned} k\cdot\partial_x f_\lambda^V(x,\mathbf{k}) \sim & \sum_{r,s=\pm 1/2}\int \frac{d^3\mathbf{p}'}{E_{\mathbf{p}'}^{\bar{q}}E_{\mathbf{k}-\mathbf{p}'}^{q}}\delta\left(E_{\mathbf{k}}^V - E_{\mathbf{k}-\mathbf{p}'}^{q} - E_{\mathbf{p}'}^{\bar{q}}\right) \\ & \times \left|M(\mathbf{k}-\mathbf{p}', r; \mathbf{p}', s \rightarrow \mathbf{k},\lambda)\right|^2 \\ & \times \left\{ \underbrace{f_r^q(\mathbf{k}-\mathbf{p}')f_s^{\bar{q}}(\mathbf{p}')\left[1+f_\lambda^V(\mathbf{k})\right]}_{\text{gain term}} - \underbrace{f_\lambda^V(\mathbf{k})\left[1-f_r^q(\mathbf{k}-\mathbf{p}')\right]\left[1-f_s^{\bar{q}}(\mathbf{p}')\right]}_{\text{loss term (decay process)}} \right\} \end{aligned}$$

→ energy conservation

- $f_\lambda^V(x,\mathbf{k})$ is distribution in phase space for a vector meson with spin λ along spin quantization direction

- General form: Matrix valued spin-dependent distribution

$$\begin{pmatrix} f_1^V & 0 & 0 \\ 0 & f_0^V & 0 \\ 0 & 0 & f_{-1}^V \end{pmatrix} \rightarrow \begin{pmatrix} f_{1,1}^V & f_{1,0}^V & f_{1,-1}^V \\ f_{0,1}^V & f_{00}^V & f_{0,-1}^V \\ f_{-1,1}^V & f_{-1,0}^V & f_{-1,-1}^V \end{pmatrix} \equiv f_{rs}^V(x,\mathbf{k})$$

MVSD

- Matrix valued spin-dependent distribution (MVSD)

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \equiv \int \frac{d^4u}{2(2\pi\hbar)^3}\delta(k\cdot u)e^{-iu\cdot x/\hbar} \times \left\langle a^\dagger_V\left(\lambda_2,\mathbf{k}-\frac{\mathbf{u}}{2}\right)a_V\left(\lambda_1,\mathbf{k}+\frac{\mathbf{u}}{2}\right)\right\rangle$$

3×3 Hermitian matrix in spin space

- MVSD can be parameterized as

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = f_V(x,\mathbf{k})\rho_{\lambda_1\lambda_2}(x,\mathbf{k})$$

spin-independent distributionfunction

normalized density matrix

$$f_V(x,\mathbf{k}) = \sum_{\lambda=0,\pm1} f^V_{\lambda\lambda}(x,\mathbf{k})$$

$$\sum_{\lambda=0,\pm1}\rho_{\lambda\lambda}(x,\mathbf{k}) = 1$$

- Relation with Wigner function

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = 2E^V_{\mathbf{k}}\int\frac{dk^0}{2\pi\hbar}\epsilon^{*\mu}(\lambda_1,\mathbf{k})G^<_{\mu\nu}(x,k)\epsilon^\nu(\lambda_2,\mathbf{k})$$

- Spin vectors for vector meson

$$\epsilon^\mu(\lambda,\mathbf{k}) = \left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda}{m_V}, \boldsymbol{\epsilon}_\lambda + \frac{\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda}{m_V(E^V_{\mathbf{k}}+m_V)}\mathbf{k}\right)$$

If spin alignment is measured along y-direction, we take:

$$\boldsymbol{\epsilon}_0 = (0,1,0)\,,\quad \boldsymbol{\epsilon}_{+1} = -\frac{1}{\sqrt{2}}(i,0,1)\,,\quad \boldsymbol{\epsilon}_{-1} = \frac{1}{\sqrt{2}}(-i,0,1)\,.$$

Kadanoff-Baym equation

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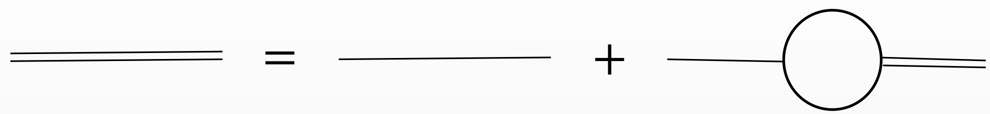
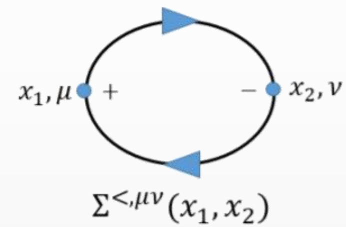
- Dyson-Schwinger equation



Full-propagator = Free-propagator + Self-energy

$$G = G_0 + G_0 \Sigma G$$
$$G = G_0 + G \Sigma G_0$$



One-loop self-energy

- With help of Schwinger-Keldysh (closed-time path) formalism, we derive Kadanoff-Baym equations at leading order in spatial gradient

$$\left[-\left(p^2 - m_V^2 - \frac{\hbar^2}{4}\partial_x^2\right) - i\hbar p\cdot\partial_x\right] G^{<,\mu\nu}(x,p)$$
$$= -\frac{i\hbar}{2}\int \frac{d^4p'}{(2\pi\hbar)^4}\{\mathrm{Tr}\left[\Gamma^\mu S^<(x,p+p')\,\Gamma_\alpha S^>(x,p')\right] G^{>,\alpha\nu}(x,p)$$
$$-\mathrm{Tr}\left[\Gamma^\mu S^>(x,p+p')\,\Gamma_\alpha S^<(x,p')\right] G^{<,\alpha\nu}(x,p)\}$$

$$\left[-\left(p^2 - m_V^2 - \frac{\hbar^2}{4}\partial_x^2\right) + i\hbar p\cdot\partial_x\right] G^{<,\mu\nu}(x,p)$$
$$= -\frac{i\hbar}{2}\int \frac{d^4p'}{(2\pi\hbar)^4}\{G^{<,\mu}{}_\alpha(x,p)\mathrm{Tr}\left[\Gamma^\alpha S^>(x,p+p')\,\Gamma^\nu S^<(x,p')\right]$$
$$-G^{>,\mu}{}_\alpha(x,p)\,\mathrm{Tr}\left[\Gamma^\alpha S^<(x,p+p')\,\Gamma^\nu S^>(x,p')\right]\}$$

Green functions on the closed-time path contour

Mass-shell condition

$$(p^2 - m_V^2) G^{<,\mu\nu} = \ldots$$

Boltzmann equation

$$p\cdot\partial_x G^{<,\mu\nu} = \ldots$$

P. Martin, J. S.Schwinger, PR 115 (1959) 1342.
L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (Benjamin, New York, 1962).
L.V. Keldysh, Zh. Eksp. Teor. Fiz. 47 (1964) 1515.

Boltzmann equation

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- Dyson-Schwinger equation
  ⇒ Kadanoff-Baym equation for Wigner function
  ⇒ Matrix-form Boltzmann equation

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k}) - f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\mathcal{C}_{\mathrm{diss}}(x,\mathbf{k})\right]$$

Dilute gas limit

$f_q \sim f_{\bar{q}} \sim f_V \ll 1$

Coalescence

Dissociation (independent from quark distributions)

- Contribution from coalescence

$$\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k}) = \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^q_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}} - E^{\bar{q}}_{\mathbf{p}'} - E^q_{\mathbf{k}-\mathbf{p}'}\right)$$
$$\times \mathrm{Tr}\left\{\Gamma^\nu (p'\cdot\gamma - m_{\bar{q}})\left[1+\gamma_5\gamma\cdot P^{\bar{q}}(x,\mathbf{p}')\right]\right.$$
$$\left.\times\Gamma^\mu[(k-p')\cdot\gamma + m_q]\left[1+\gamma_5\gamma\cdot P^q(x,\mathbf{k}-\mathbf{p}')\right]\right\}$$
$$\times f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}'),$$

Energy conservation (all particles are on their normal mass shells)

Quark-antiquark-meson vertex

Polarizations of quark/antiquark

unpolarized quark/antiquark distributions

Spin alignment

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- Neglecting space inhomogeneity of $f^V_{\lambda_1\lambda_2}$ and assuming that $f^V_{\lambda_1\lambda_2} = 0$ at a given t_0, we obtain formal solution

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1-\exp[-\mathcal{C}_{\rm diss}(x,\mathbf{k})\Delta t]}{\mathcal{C}_{\rm diss}(x,\mathbf{k})}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\rm coal}(x,\mathbf{k})\right] \qquad \Delta t \equiv t - t_0$$

$$\sim \begin{cases} \Delta t\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\rm coal}(x,\mathbf{k})\right] & \Delta t \ll \frac{1}{\mathcal{C}_{\rm diss}(x,\mathbf{k})} \\ \frac{1}{\mathcal{C}_{\rm diss}(x,\mathbf{k})}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\rm coal}(x,\mathbf{k})\right] & \Delta t \gg \frac{1}{\mathcal{C}_{\rm diss}(x,\mathbf{k})} \end{cases}$$

⇒ reach thermal equilibrium

- Spin alignment only depend on coalescence process

$$\boxed{\rho_{00} \equiv \frac{f^V_{00}}{f^V_{+1,+1}+f^V_{00}+f^V_{-1,-1}} = \frac{\epsilon^*_\mu(0,\mathbf{k})\epsilon_\nu(0,\mathbf{k})\mathcal{C}^{\mu\nu}_{\rm coal}(x,\mathbf{k})}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\rm coal}(x,\mathbf{k})}}$$

Relativistic description of spin combination.

Agree with non-relativistic results

$$\begin{cases} |1,+1\rangle_M = \left|\frac{1}{2},+\frac{1}{2}\right\rangle_q \left|\frac{1}{2},+\frac{1}{2}\right\rangle_{\bar q} \\ |1,0\rangle_M = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2},+\frac{1}{2}\right\rangle_q \left|\frac{1}{2},-\frac{1}{2}\right\rangle_{\bar q} + \left|\frac{1}{2},-\frac{1}{2}\right\rangle_q \left|\frac{1}{2},+\frac{1}{2}\right\rangle_{\bar q}\right) \\ |1,-1\rangle_M = \left|\frac{1}{2},-\frac{1}{2}\right\rangle_q \left|\frac{1}{2},-\frac{1}{2}\right\rangle_{\bar q} \end{cases}$$

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- Summary

Polarization of quarks

- Polarizations of strange quark/antiquark in a thermal equilibrium system

F.Becattini, V.Chandra, L.Del Zanna, E.Grossi, Annals Phys. 338, 32 (2013)

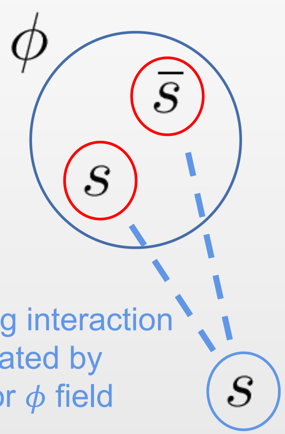
Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys.Rev.C 97, 3 (2018).

XLS, L.Oliva, Q.Wang, PRD 101, 096005 (2020);

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

$$P_s^\mu(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\alpha\beta}p_\nu\left[\omega_{\rho\sigma} + \frac{Q_s}{(u\cdot p)T}F_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T}F^\phi_{\rho\sigma}\right]$$

$$P_{\bar{s}}^\mu(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\alpha\beta}p_\nu\left[\omega_{\rho\sigma} - \frac{Q_s}{(u\cdot p)T}F_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T}F^\phi_{\rho\sigma}\right]$$



strong interaction mediated by vector ϕ field

thermal vorticity field (rotation and acceleration)

classical electromagnetic field

$$\frac{e^2}{4\pi} \sim \frac{1}{137}$$

vector ϕ field coupled to $s/\bar{s}$

$$\frac{g_\phi^2}{4\pi} \sim \mathcal{O}(1) \gg \frac{e^2}{4\pi}$$

- Vector meson field has been used to explain the difference between polarizations of Λ and $\bar{\Lambda}$

L.P.Csernai, J.I.Kapusta, T.Welle, PRC 99, 021901 (2019)

Vector meson field

- At low energies, strong interaction is mediated by mesons, which was proposed by Yukawa in 1935.

H. Yukawa, Proc. Phys. Math. Soc. Jap. 17, 48 (1935)

- Supposing the strong interaction is mediated by a scalar field σ and a vector meson field V^μ, the effective Lagrangian reads

$$V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$\begin{aligned}\mathcal{L}_{\text{eff}}(x) &= \overline{\psi}(x)\left[i\partial\cdot\gamma - (m_0 + g_\sigma\sigma) - g_V\gamma\cdot V\right]\psi(x) \\ &+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}m_V^2 V_\mu V^\mu - \frac{1}{4}V^{\mu\nu}V_{\mu\nu}\end{aligned}$$

Similar to EM field, anti-symmetric tensor, can be decomposed into electric part and magnetic part

- In flavor space, V^μ is a 3×3 matrix,

$$V^\mu = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \phi \end{pmatrix}$$

- The ϕ field can be generated from strangeness current

$$(\partial^2 + m_\phi^2)\phi^\mu(x) - \partial^\mu\partial_\nu\phi^\nu(x) = g_\phi \underbrace{\overline{\psi}_s(x)\gamma^\mu\psi_s(x)}_{\text{strangeness current}}$$

$$m_\phi^2\phi^\mu(x) \gg \partial^2\phi^\mu(x) \Longrightarrow \phi^\mu(x) \simeq \frac{g_\phi}{m_\phi^2}\overline{\psi}_s(x)\gamma^\mu\psi_s(x)$$

Spin alignment

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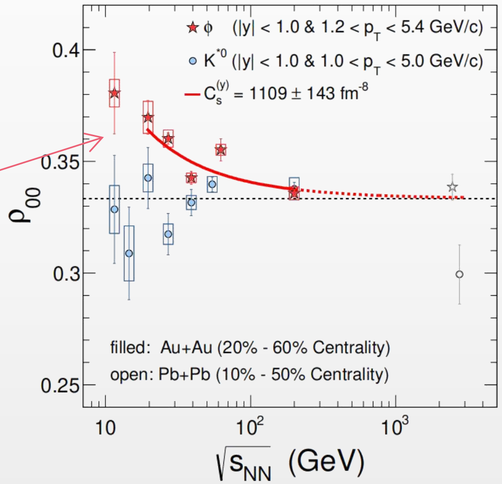
- Spin alignment of the ϕ meson in its rest frame measuring along the direction of ϵ_0

$$\rho_{00}(x,\mathbf{0}) \approx \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\epsilon_0\cdot\boldsymbol{\omega}')^2\right] + C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\epsilon_0\cdot\boldsymbol{\varepsilon}')^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2}C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\epsilon_0\cdot\mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2}C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\epsilon_0\cdot\mathbf{E}'_\phi)^2\right],$$

effective temperature at freeze-out

larger at lower energies ~ $1/T^2$

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)},$$
$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)}.$$



STAR collaboration, arXiv:2204.02302

- Features:

  - Cancellation for mixing terms

  - All fields appear in squares, spin alignment measures anisotropy of fluctuations in meson's rest frame

e.g., contribution from $\mathbf{B}'_\phi$ to spin alignment along *y*-direction $\propto (B'_{\phi,y})^2 - \frac{(B'_{\phi,x})^2 + (B'_{\phi,z})^2}{2}$

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

Numerical set-up

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- Momentum dependence is recovered by taking a Lorentz boost from rest frame to lab frame

$$\mathbf{B}'_\phi = \gamma \mathbf{B}_\phi - \gamma \mathbf{v} \times \mathbf{E}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_\phi}{v^2}\mathbf{v},$$
$$\mathbf{E}'_\phi = \gamma \mathbf{E}_\phi + \gamma \mathbf{v} \times \mathbf{B}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_\phi}{v^2}\mathbf{v},$$
$$\boldsymbol{\omega}' = \gamma \boldsymbol{\omega} - \gamma \mathbf{v} \times \boldsymbol{\varepsilon} + (1-\gamma)\frac{\mathbf{v}\cdot\boldsymbol{\omega}}{v^2}\mathbf{v},$$
$$\boldsymbol{\varepsilon}' = \gamma \boldsymbol{\varepsilon} + \gamma \mathbf{v} \times \boldsymbol{\omega} + (1-\gamma)\frac{\mathbf{v}\cdot\boldsymbol{\varepsilon}}{v^2}\mathbf{v},$$

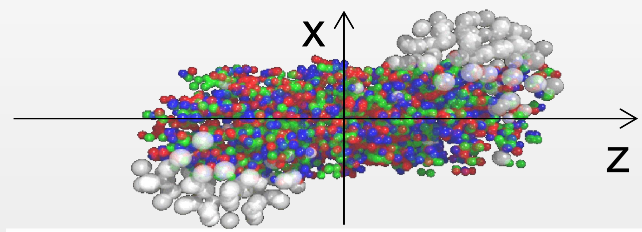
$$\gamma = \frac{E^\phi_{\mathbf{k}}}{m_\phi}, \quad \mathbf{v} = \frac{\mathbf{k}}{E^\phi_{\mathbf{k}}}$$

spin alignment in lab frame $\rho_{00}(\mathbf{k})$

- Set-up for field fluctuations in lab frame

$$\left\langle (\omega_i)^2 \right\rangle = \left\langle (\varepsilon_i)^2 \right\rangle = 0$$
$$\left\langle (g_\phi \mathbf{B}^\phi_{x(y)})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{x(y)})^2 \right\rangle = F^2$$
$$\left\langle (g_\phi \mathbf{B}^\phi_z)^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_z)^2 \right\rangle = r_z F^2 < F^2$$

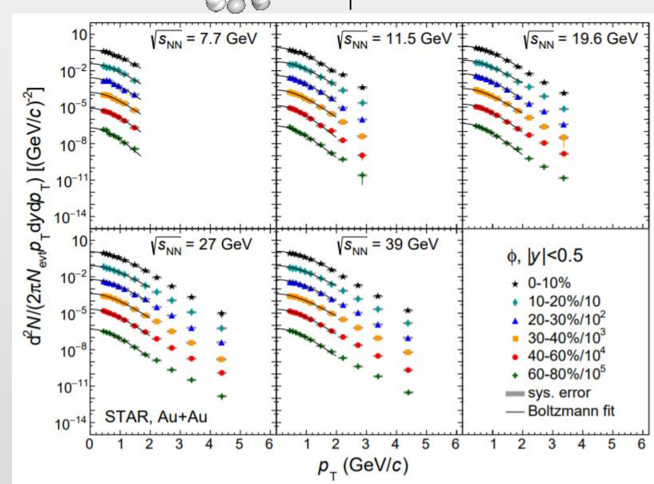
r_z denotes the anisotropy between longitudinal and transverse directions



- Spectra of ϕ meson

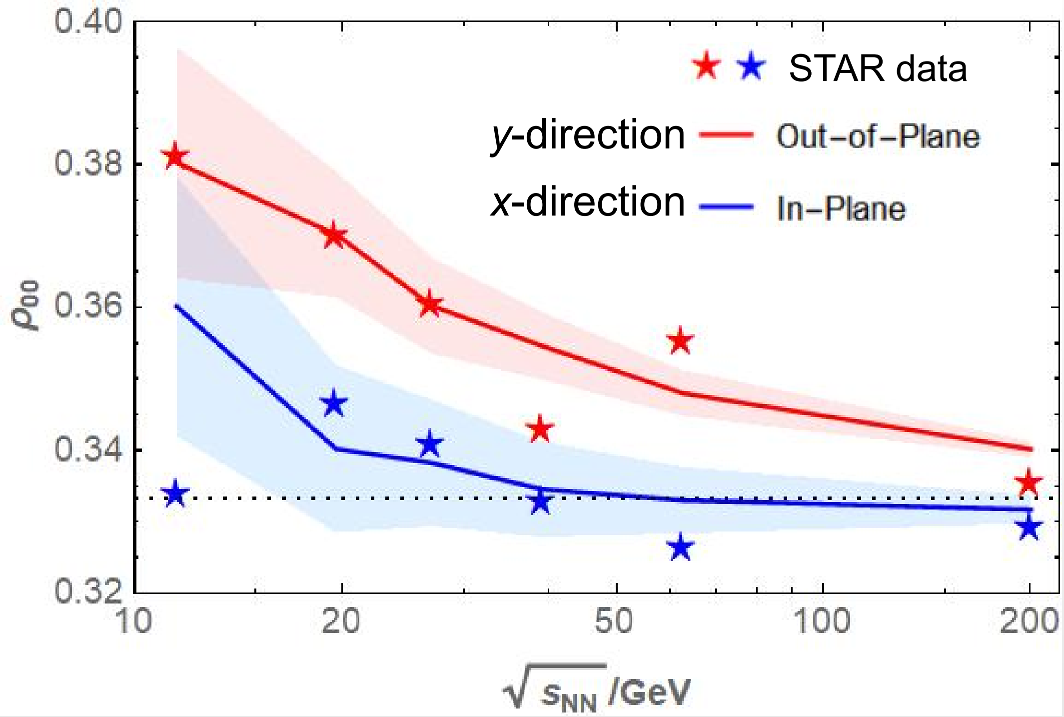
$$\frac{dN}{d^2\mathbf{k}_T dy} = \frac{1}{4\pi}\left[1 + 2v_2(k_T)\cos(2\phi)\right]\frac{dN}{k_T dk_T dy}$$

STAR collaboration, PRL 99,112301 (2007); PRC 79, 064903 (2009); PRC 88,014902 (2013); PRC 102, 034909(2020).



Spin alignment

- Spin alignment as a function of collision energy



$F^2 = 0.45\, m_\pi^4$ if $m_s = 170$ MeV

Solid line: $r_z = 0.79$
Upper bound $r_z = 0.59$
Lower bound $r_z = 0.99$

Reproduce out-of-plane data and in-plane data at the same time!

- Agree with STAR's recent data, arXiv:2204.02302.
- Difference between red line and blue line is attribute to v_2

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689.

Spin alignment

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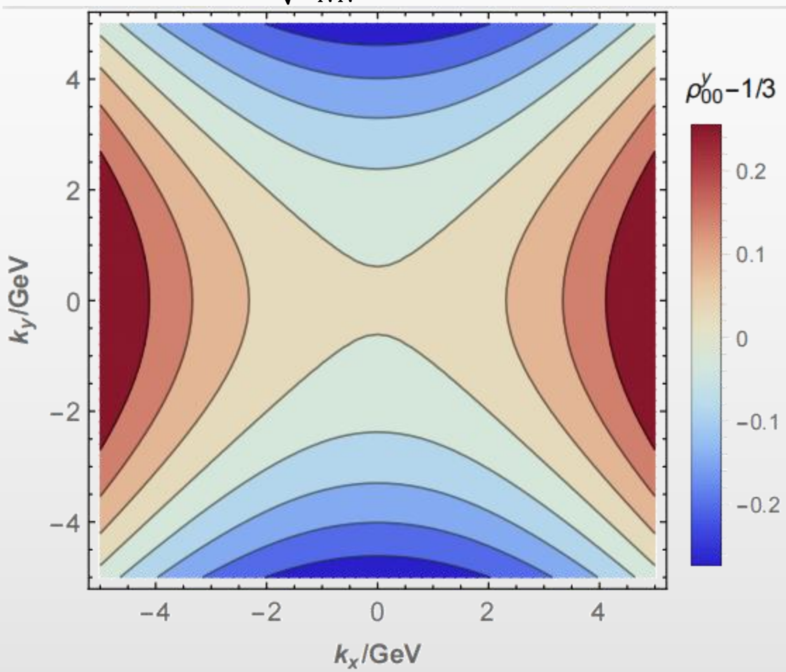
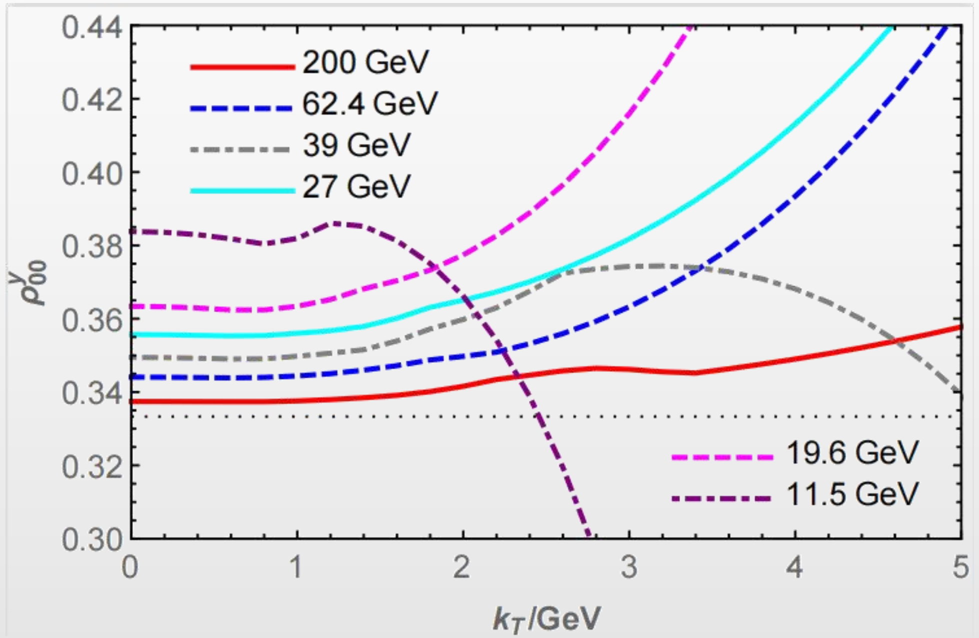
- Spin alignment in *y*-direction as a function of transverse momentum (k_z is integrated out by taking an average over rapidity range |y|<1)

$\sqrt{s_{NN}}$ = 200 GeV

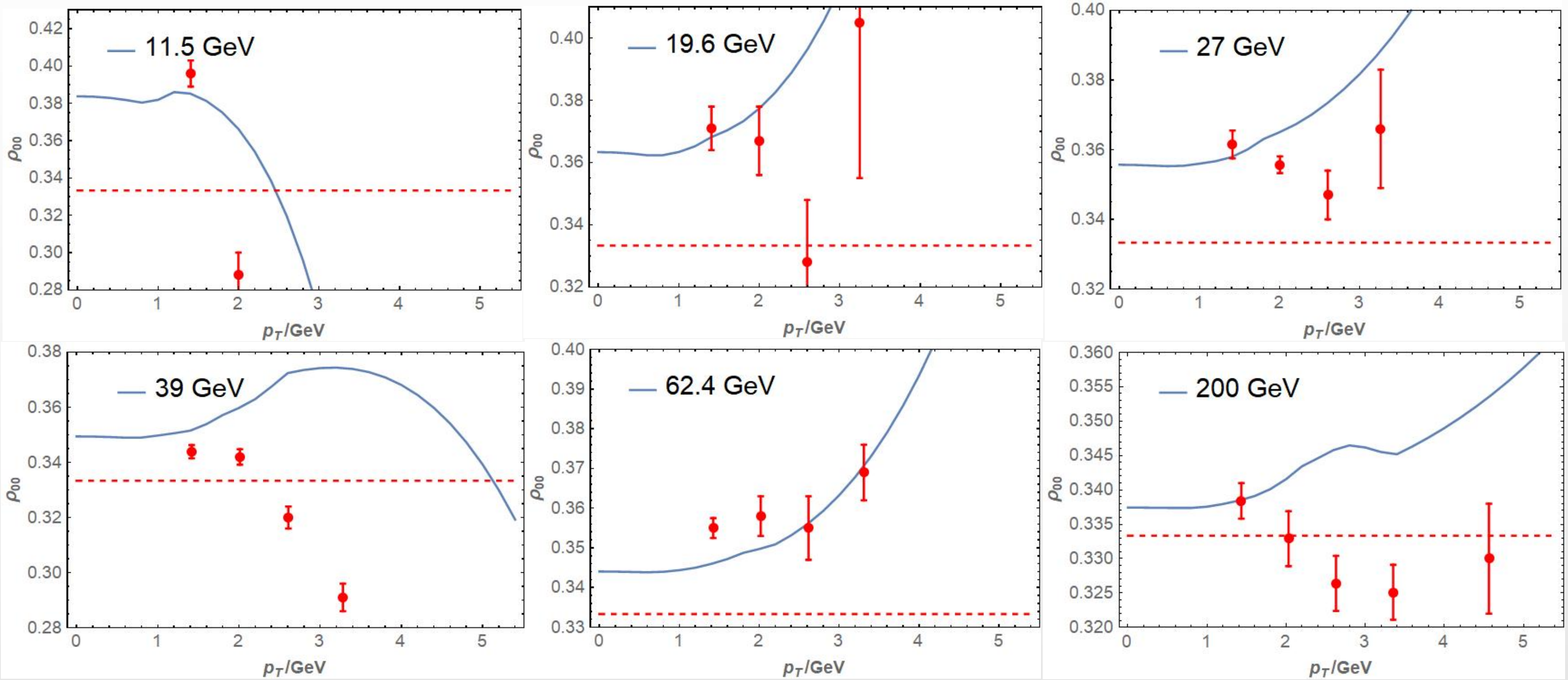




XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689.

Spin alignment

- Spin alignment as a functions of transverse momentum



Red dots and error bars are read from STAR's paper arXiv:2204.02302

Centrality dependence

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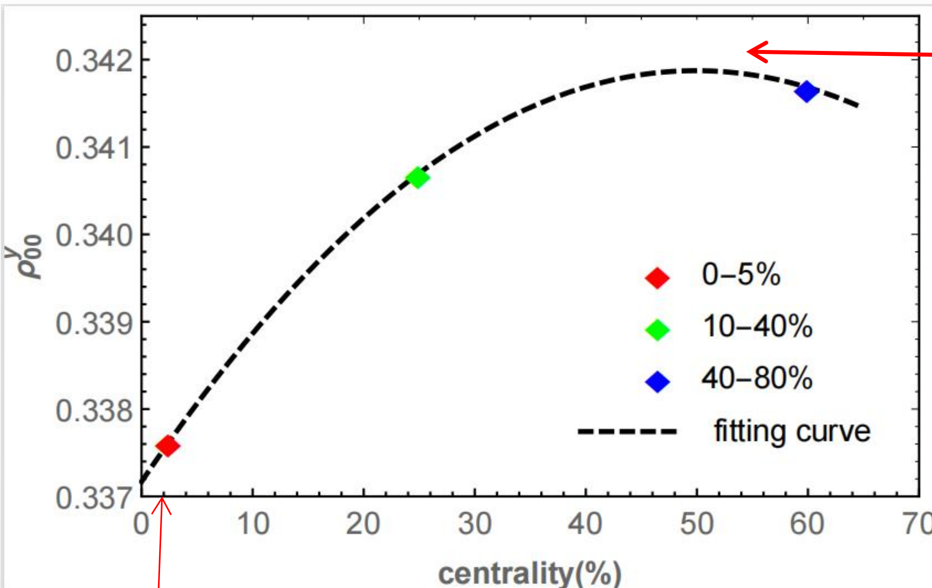
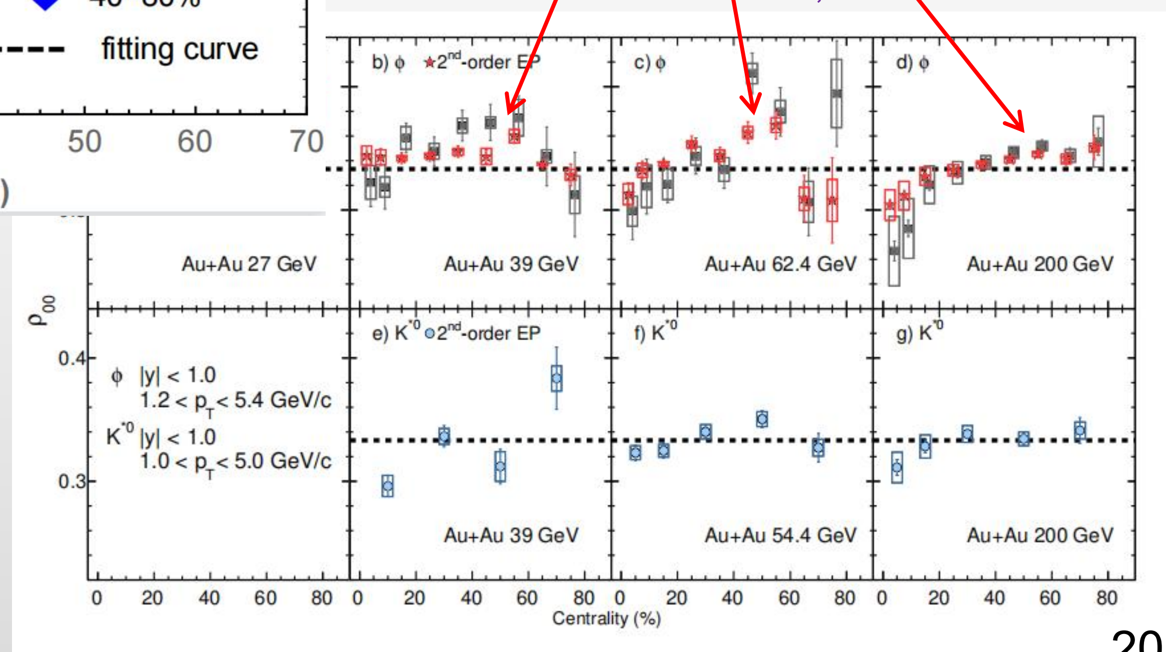
- Spin alignment as a function of centrality at $\sqrt{s_{NN}}$ = 200 GeV



peak at 50% centrality

STAR collaboration, arXiv:2204.02302



>1/3 in most central collisions, may be contributions from local vorticity

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689.

Contents

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- Introduction
- Boltzmann equation for vector meson
- Numerical results for ϕ meson's spin alignment
- Mass splitting and spin alignment
- Summary and outlook

Mass splitting

- Kadanoff-Baym equation for Wigner function

$$\left[-\left(p^2 - m_V^2 - \frac{\hbar^2}{4}\partial_x^2\right) - i\hbar p\cdot\partial_x\right] G^{<,\mu\nu}(x,p)$$ = (contributions from interactions)

Mass-shell condition

$(p^2 - m_V^2)G^{<,\mu\nu} = \ldots$

Boltzmann equation

$p\cdot\partial_x G^{<,\mu\nu} = \ldots$

- Meson's in-medium mass is different from its vacuum mass

  If Lorentz symmetry is broken, vector meson's mass will depend on its spin

  $\Rightarrow$ spin alignment

For ϕ meson's in-medium mass:

talks by P. Gubler, K. Aoki, S. H. Lee, H. Sako

$$M_{\phi,0} = \overline{M}_\phi + \Delta$$

$$M_{\phi,\pm1} = \overline{M}_\phi - \frac{\Delta}{2}$$

$$\overline{M}_\phi \equiv \frac{1}{3}\sum_{\lambda=0,\pm1} M_{\phi,\lambda}$$

$$f_\lambda \sim \frac{1}{1+\exp\left(M_{\phi,\lambda}/T\right)}$$

$$\rho_{00} \equiv \frac{f_0}{f_1+f_0+f_{-1}} \simeq \frac{1}{3} - \frac{\Delta}{3T}\left[1 - \frac{1}{1+\exp\left(\overline{M}_\phi/T\right)}\right] + \mathcal{O}\left[\left(\frac{\Delta}{T}\right)^2\right]$$

SU(3) NJL

- Effective Lagrangian for SU(3) NJL model

Four-quark interactions

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_q + G_S \sum_{a=0}^{8} \left[(\overline{\psi}\lambda_a\psi)^2 + (\overline{\psi}i\gamma_5\lambda_a\psi)^2 \right] - G_V \sum_{a=0}^{8} \left[(\overline{\psi}\gamma_\mu\lambda_a\psi)^2 + (\overline{\psi}i\gamma_\mu\gamma_5\lambda_a\psi)^2 \right]$$
$$-K \left\{ \det_f \left[\overline{\psi}(1+\gamma_5)\psi \right] + \det_f \left[\overline{\psi}\,(1-\gamma_5)\,\psi \right] \right\},$$

Six-quark Kobayashi-Maskawa-'t Hooft interaction

G_S, G_V are coupling constants for scalar and vector interactions, respectively.

- Lagrangian for quarks in a classical EM field

$$\mathcal{L}_q = \sum_{f=u,d,s} \overline{\psi}_f (i\gamma_\mu D_f^\mu - m_f)\psi$$

Covariant derivative

$$D_f^\mu \equiv \partial^\mu + iq_f A^\mu$$

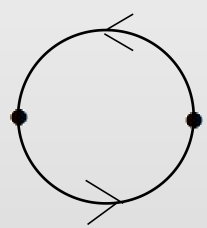
- Vector meson propagator

$$D_\lambda(k) = -4G_V - 4G_V \Sigma_\lambda(k) D_\lambda(k) \quad \Longrightarrow \quad D_\lambda(k) = -\frac{4G_V}{1+4G_V\Sigma_\lambda(k)}$$

- Self-energy at one-loop level

Quark propagators

$$\Sigma_\lambda(k) \equiv \epsilon^\alpha(\lambda,k)\Sigma_{\alpha\beta}(k)\epsilon^{*\beta}(\lambda,k)$$
$$= -iN_c \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\alpha S_f(k+p)\gamma_\beta S_{f'}(p)\right] \epsilon^\alpha(\lambda,k)\epsilon^{*\beta}(\lambda,k)$$



Mass spectra

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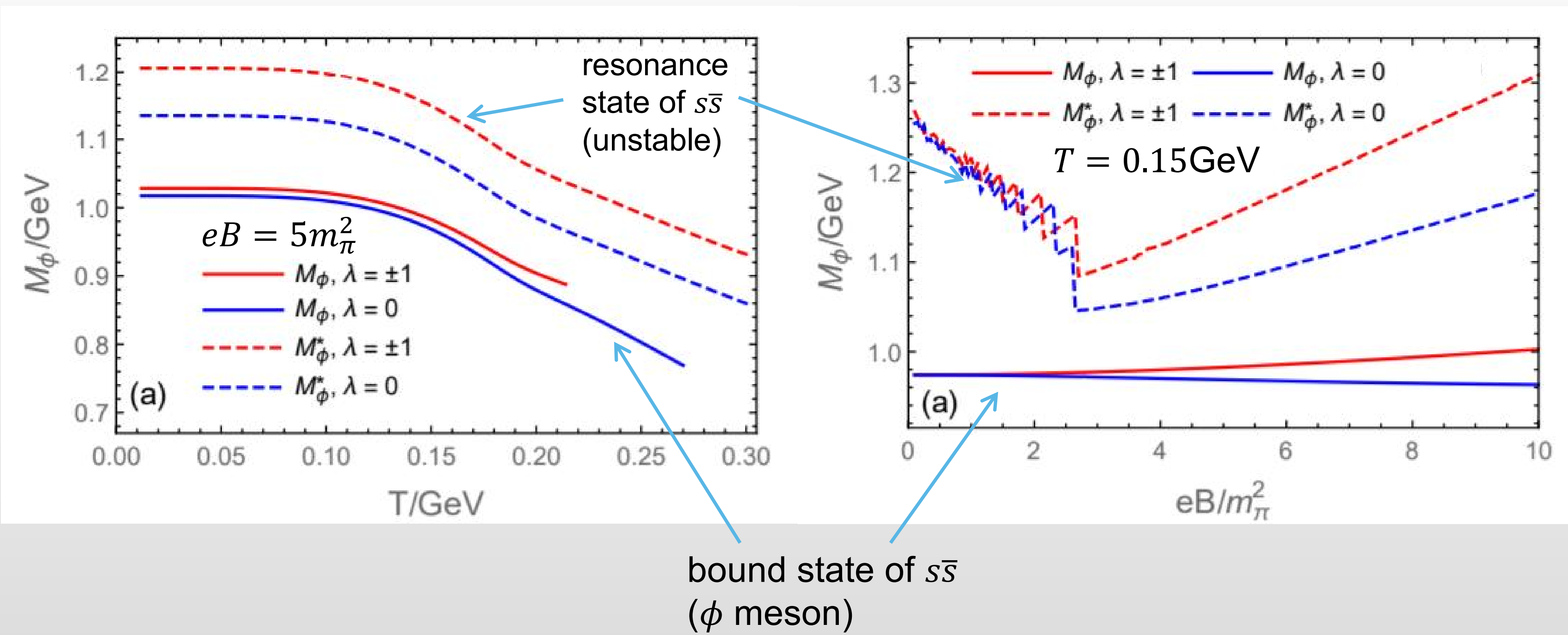
- Dynamical mass of vector ϕ meson in a constant magnetic field

XLS, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv:2209.01872

$\lambda = 0, \pm 1$ is quantum number of spin,
spin quantization direction is direction of magnetic field



Spin alignment

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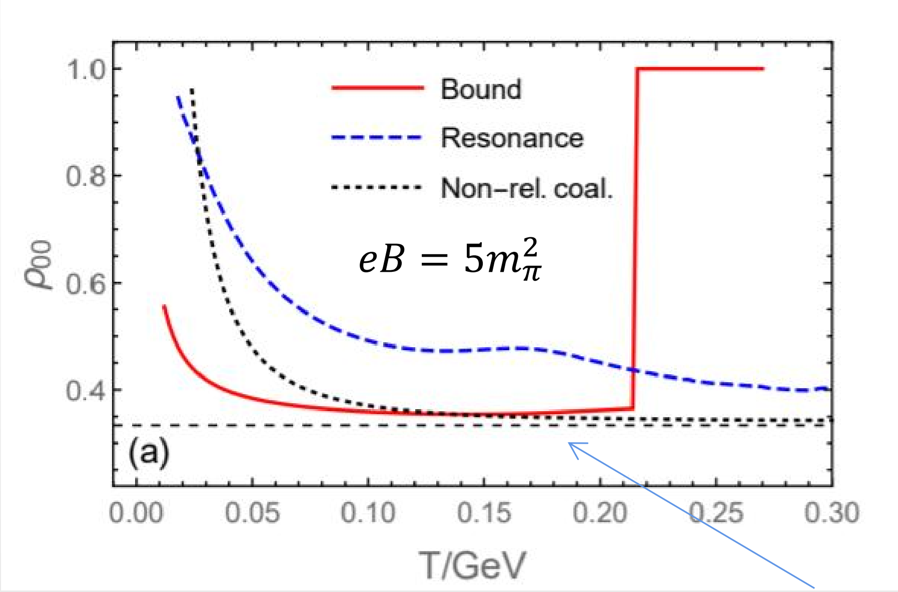
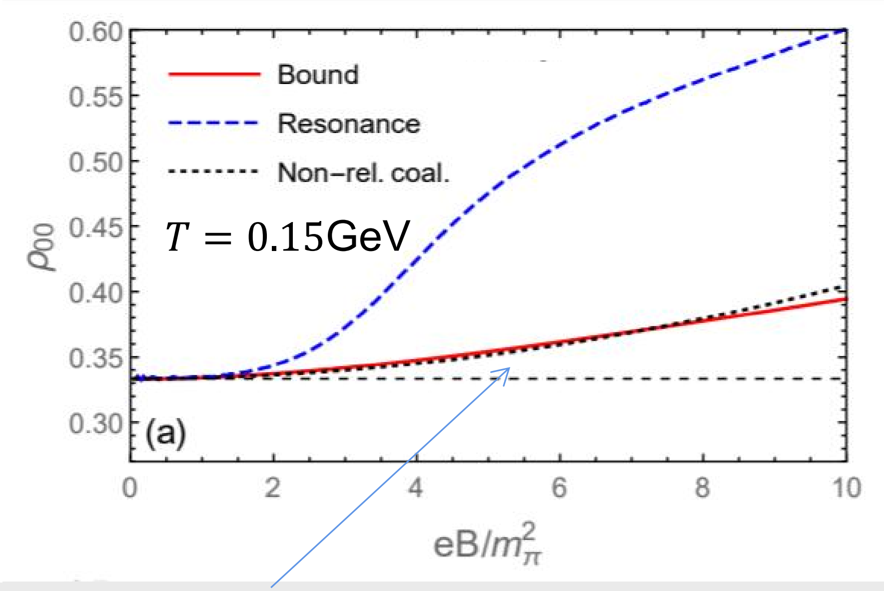
- Spin alignment for ϕ meson in a constant magnetic field

  XLS, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv:2209.01872





spin alignment generated by mass splitting agree with result from non-relativitic coalescence model around T_c

- Spin alignment from non-relativistic coalescence model: $\rho_{00} = \frac{1}{3} + \frac{Q_s^2}{9M_s^2T^2}|eB|^2$

  Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys.Rev.C 97, 3 (2018).

Summary

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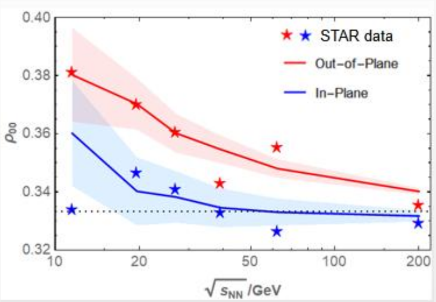
- We derive a relativistic Boltzmann equation for quark-antiquark combination and form vector meson

- Spin alignment measures anisotropy of fluctuations in meson's rest frame

- Using two parameters (fluctuations for transverse and longitudinal components of strong force field), we reproduce most of recent STAR data for ϕ meson spin alignment

$$\left\langle (g_\phi \mathbf{B}^{\phi}_{x(y)})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^{\phi}_{x(y)})^2 \right\rangle = F^2$$

$$\left\langle (g_\phi \mathbf{B}^{\phi}_{z})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^{\phi}_{z})^2 \right\rangle = r_z F^2 < F^2$$



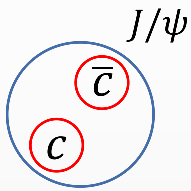
- External fields can result in mass splitting of different spin states, corresponding to different distributions and thus contribute to spin alignment

- Spin alignment calculated using mass-splitting agrees with that from coalescence model. Two equivalent ways? Or coincidence?

Spin alignment of J/ψ

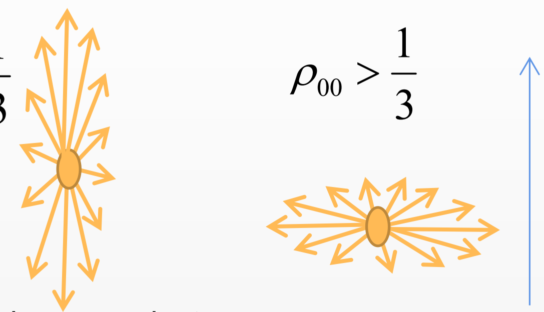
- Vector mesons decay to dileptons

$$J/\psi \rightarrow l^{+} + l^{-}$$



- Spin angular momentum of vector meson → spin angular momentum of dileptons
  Angular distribution of decay products is determined by spin conservation.

$\rho_{00} < \frac{1}{3}$ $\rho_{00} > \frac{1}{3}$



More decay products in transverse direction

$|1,+1\rangle$
$|1,-1\rangle$

Transversely polarized

More decay products in longitudinal direction

$|1,0\rangle$

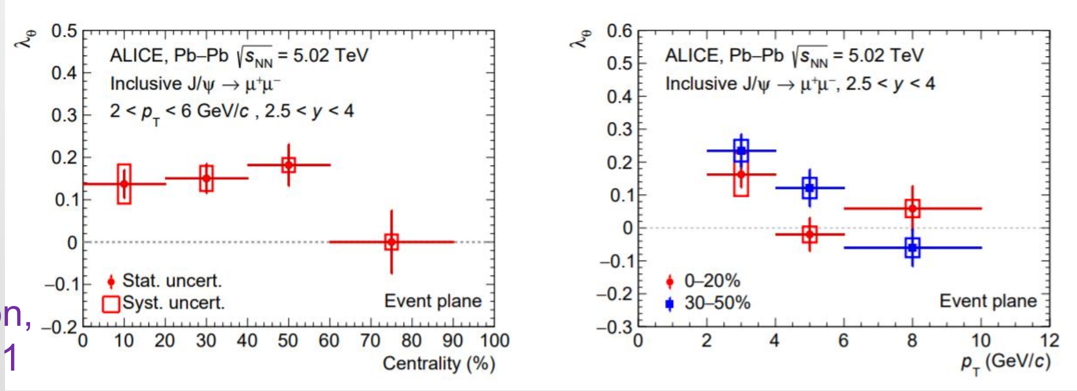
Longitudinally polarized

- Polar angle distribution

$$\frac{dN}{d\theta} = \frac{3}{8}[(1+\rho_{00}) - (3\rho_{00}-1)\cos^2\theta]$$
$$\propto \frac{1}{3+\lambda_\theta}(1+\lambda_\theta \cos^2\theta)$$

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}} \approx -\frac{9}{4}\left(\rho_{00}-\frac{1}{3}\right)$$

$$\lambda_\theta > 0,\ \rho_{00} < \frac{1}{3}$$



ALICE collaboration, e-Print: 2204.10171

Open questions

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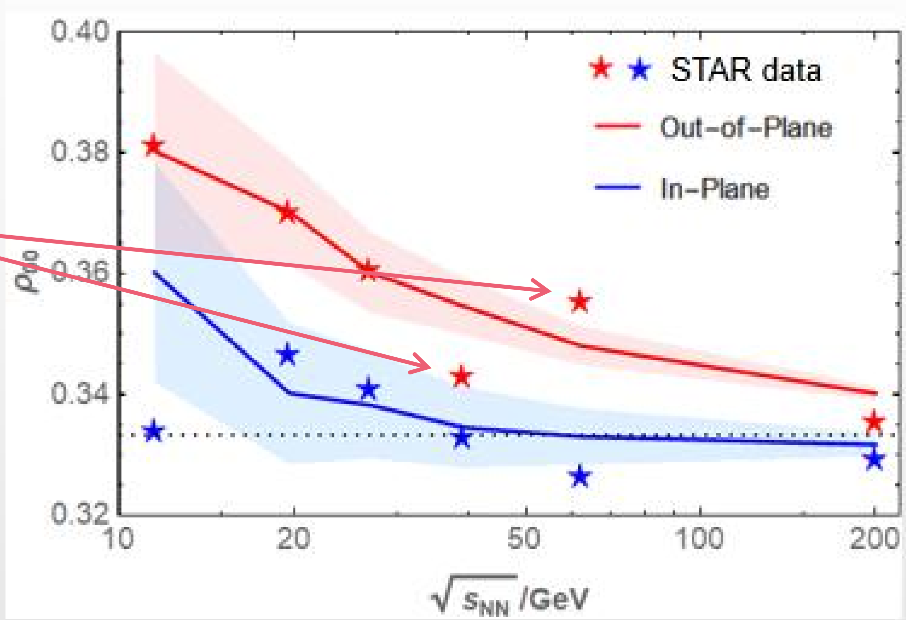
- When measuring along y-direction, why ϕ mesons at RHIC are longitudinally polarized while J/ψ are transversely polarized?

- How to understand spin alignment at collision energies 62.4 GeV and 39 GeV?



Thank you!