Anomalous spin polarization from turbulent color fields in heavy ion collisions

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(Reimei workshop, Oct 7th, 2022)

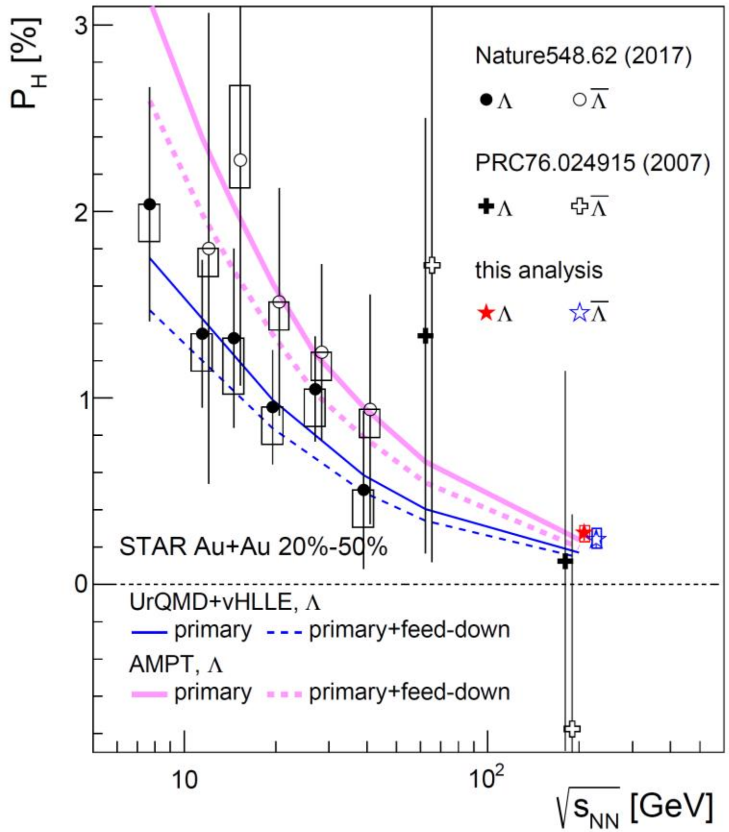
Berndt Müller, DY, PRD 105, L011901 (2022)
DY, JHEP 06, 140 (2022)

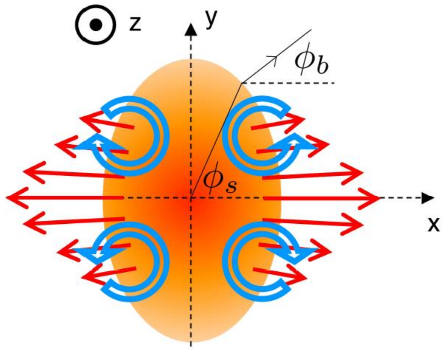
Outline

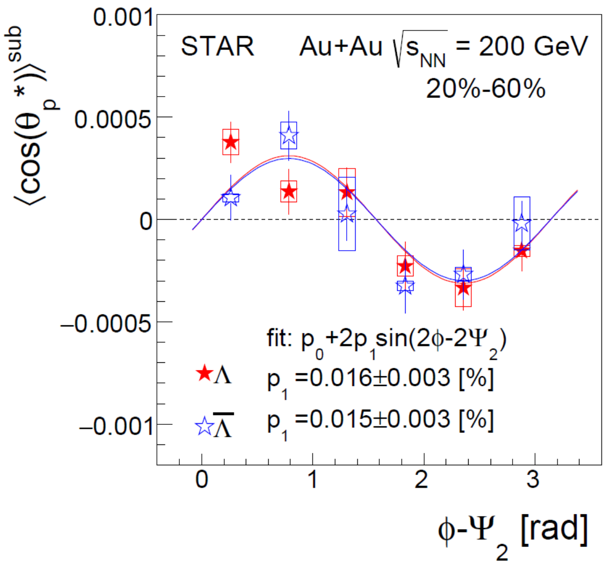
- Motivations : Λ polarization and spin alignment
- Quantum kinetic theory for spin transport
- Generalization to include color degrees of freedom
- Spin polarization from color-field correlators
- Conclusions and outlook

Λ polarization in HIC

- The large AM generated in HIC could induce spin polarization of the QGP via spin-orbit interaction. Z.-T. Liang and X.-N. Wang, PRL. 94, 102301 (2005)
- Global and local polarization of Λ hyperons :







L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

J. Adam et al. (STAR, PRL. 123, 132301 (2019)

Theoretical status

- Spin polarization of a relativistic fermion : $\mathcal{P}^{\mu}(\mathbf{p}) = \frac{\int d\Sigma\cdot p\mathcal{J}_5^{\mu}(p,X)}{2m\int d\Sigma\cdot\mathcal{N}(p,X)}\Big|_{p_0=\epsilon_{\mathbf{p}}}$

- Global equilibrium : $\mathcal{P}^{\mu}(p) = \frac{1}{8m}\epsilon^{\sigma\mu\nu\rho}p_{\sigma}\frac{\int d\Sigma\cdot p\omega_{\nu\rho}f_p^{(0)}(1-f_p^{(0)})}{\int d\Sigma\cdot pf_p^{(0)}}$ $\qquad \omega_{\mu\nu}\equiv\frac{\partial_{[\mu}(u_{\nu]}/T)}{2}$

  F. Becattini, et al., Ann. Phys. 338, 32 (2013)
  R. Fang, et al., PRC 94, 024904 (2016)

  ⇒ ❖ Agrees with global Λ polarization
  ❖ A "spin sign problem" for local polarization

- Near local equilibrium : shear correction, acceleration, chemical-potential gradient, and accompanied dissipative corrections

  Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)
  S. Fang, S. Pu, DY, PRD 106, 016002 (2022)
  S. Y. F. Liu and Y. Yin, PRD 104, 054043 (2021)
  S. Y. F. Liu, Y. Yin, JHEP 07, 188 (2021)
  F. Becattini, M. Buzzegoli, A. Palermo, PLB 820,136519 (2021)

  ⇒ ❖ Potentially resolves the spin sign problem
  ❖ Sensitive to parameters & approx.

  F. Becattini, et al., PRL. 127 (2021) 27, 272302
  B. Fu, et al., PRL 127, 142301 (2021)
  C. Yi, S. Pu, DY, PRC 104 (2021) 6, 064901
  W. Florkowski, et al., PRC105 (2022), 064901

- Could spin polarization of quarks be affected by microscopic properties of the QGP? In particular, what are the roles of gluons and color dof.?

  ("anomalous spin polarization" beyond hydrodynamic gradients)

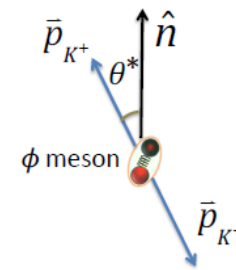
Spin alignment of vector mesons

- Production of the decay daughter w.r.t the quantization axis :

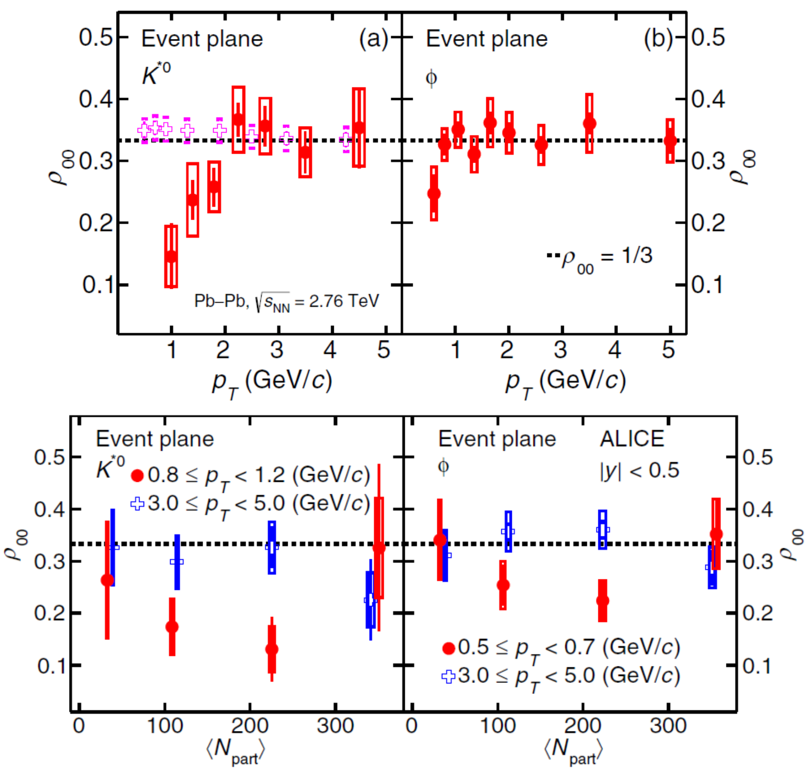
$$\frac{dN}{d\cos\theta^*} \propto [1-\rho_{00}+\cos^2\theta^*(3\rho_{00}-1)]$$

$$\rho_{00}=\frac{1-\mathcal{P}_q^i\mathcal{P}_{\bar{q}}^i}{3+\mathcal{P}_q^i\mathcal{P}_{\bar{q}}^i}$$

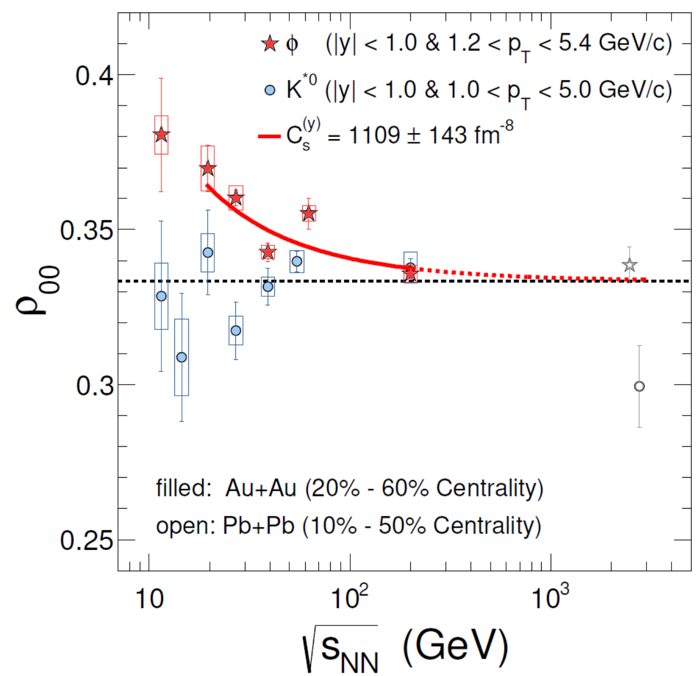
Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)



$\rho_{00} \neq 1/3$: spin polarization



S. Acharya et al. (ALICE), PRL.125, 012301 (2020)



STAR, arXiv:2204.02302 (2022)

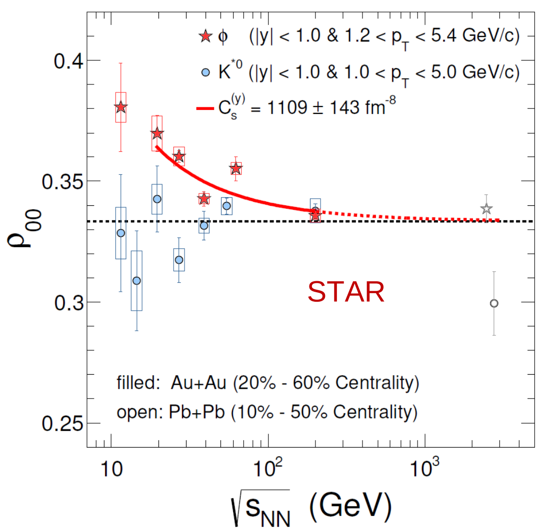
Spin polarization beyond subatomic swirls?

- Spin alignment puzzle : the deviation of ρ_{00} from 1/3 is unexpectedly large

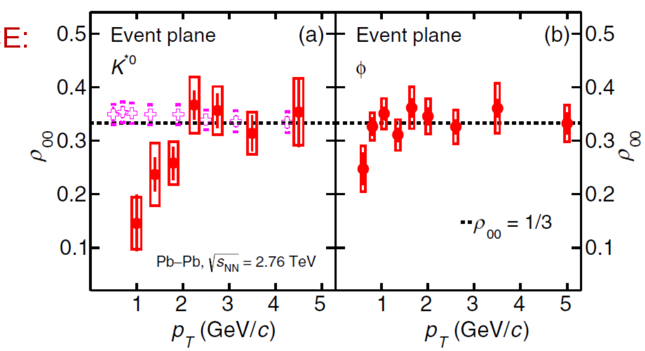
  e.g. $\rho_{00} \approx \frac{1}{3} - \left(\frac{\omega}{T}\right)^2$, $\frac{\omega}{T} \sim 0.1\%$ at LHC energy. (from Λ polarization)

- Different trends in LHC and RHIC :

	ϕ	K^{*0}
ALICE	$\rho_{00} < 1/3$ ($p_T \leq 1$ GeV)	$\rho_{00} < 1/3$
STAR	$\rho_{00} > 1/3$	$\rho_{00} \approx 1/3$



ALICE:



- Other sources for spin polarization (alignment) of quarks & anti-quarks?

  (electromagnetic fields are also weak in late times)

  e.g. hadron mean fields :X. L. Sheng, L. Oliva, Q. Wang, PRD101,096005 (2020)

Relativistic kinetic theory

- How is the spin of a strange quark dynamically polarized when traversing the QGP? (as a probe interacting with light quarks & gluons)
- Kinetic theory : microscopic theory for quasi-particles in phase space

  - Boltzmann (Vlasov) Eq. : $q^\mu \Delta_\mu f(q,X) = q^\mu \mathcal{C}_\mu[f], \qquad \Delta_\mu = \partial_\mu + F_{\nu\mu}\frac{\partial}{\partial q^\nu}.$
  - Charge or energy transport : $J^\mu(X) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{q^\mu}{E_q} f(q,X), \; T^{\mu\nu}(X) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{q^\mu q^\nu}{E_q} f(q,X).$
  - Disadvantage : kinetic theory is subject to weak coupling
  - Advantage : in connection to the underlying QFT
  - Near equilibrium : kinetic theory ⇨ hydrodynamics

- A kinetic equation to delineate the spin evolution in phase space is needed.

  ⇨ Quantum kinetic theory (QKT) for massive fermions

Review : Y. Hidaka S. Pu, Q, Wang, DY, arXiv: 2201.07644

Axial kinetic theory

- QKT can be derived from the Wigner-function approach
- Vector/axial-vector components :

$$S^<(p,X) = \int d^4Y e^{\frac{ip\cdot Y}{\hbar}} \langle \bar{\psi}(X-Y/2)\psi(X+Y/2)\rangle \Rightarrow \left.\begin{aligned} \mathcal{V}^\mu(p,X) &= \frac{1}{4}\mathrm{tr}\left(\gamma^\mu S^<(p,X)\right), \\ \mathcal{A}^\mu(p,X) &= \frac{1}{4}\mathrm{tr}\left(\gamma^\mu\gamma^5 S^<(p,X)\right) \end{aligned}\right\} \mathcal{P}^\mu(\boldsymbol{p}) = \frac{\int d\Sigma\cdot p\mathcal{A}^\mu(\boldsymbol{p},X)}{2m\int d\Sigma\cdot\mathcal{V}(\boldsymbol{p},X)}$$

- Dynamical variables in $\mathcal{V}^\mu/\mathcal{A}^\mu$: $f_{V/A}(p,X)$ & $a^\mu(p,X)$ — spin 4-vector $(\tilde{a}^\mu = a^\mu f_A)$

K. Hattori, Y. Hidaka, DY, PRD100, 096011 (2019)

$$\xrightarrow{m=0} \quad a^\mu = p^\mu,\ f_V = (f_R+f_L)/2,\ f_A = f_R - f_L.$$

- For the $\hbar$ expansion : $f_V \sim \mathcal{O}(\hbar^0),\ \tilde{a}^\mu \sim \mathcal{O}(\hbar).$ $\Rightarrow$ $\mathcal{V}^\mu = 2\pi\delta(p^2-m^2)p^\mu f_V$
- In the particle rest frame : $\mathcal{A}^\mu = 2\pi\left(\delta(p^2-m^2)\tilde{a}^\mu + \hbar\tilde{F}^{\mu\nu}p_\nu\delta'(p^2-m^2)f_V\right)$
- Axial kinetic theory : scalar/axial-vector kinetic eqs. (SKE/AKE)
- SKE : standard Vlasov eq.
- Collisions for AKE : $\square\tilde{a}^\mu = \hat{\mathcal{C}}_1^\mu[\tilde{a}] + \hbar\hat{\mathcal{C}}_2^\mu[f_V]$

DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)
Z. Wang, X. Guo, P. Zhuang, Eur. Phys. J. C 81, 799 (2021)

with quantum corrections — spin relaxation $\propto L^{\mu\nu}\tilde{a}_\nu$ — spin polarization coupled to vector charge $\propto H^{\mu\nu}\partial_\nu f_V$

- Naively : Lorentz force $\propto g\ (g^2)$, collisions $\propto g^4\ (g^4 \ln g)$ S. Li, H.-U. Yee, PRD100, 056022 (2019)

WFs and AKE with color fields

- Incorporation of background color fields into WFs and kinetic theory.
- Color decomposition : $O = \boxed{O^s} I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)

H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B276, 706 (1986).

→ physical observable

e.g. $J_5^\mu = 4\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}_c\,\mathcal{A}^\mu(p,X)$

- SKE, AKE, WFs are decomposed into color-singlet & octet components.

singlet SKE : $\delta(p^2-m^2)\mathcal{K}_{\rm s}[\hat{f}_V]=0,$ octet SKE : $\delta(p^2-m^2)\mathcal{K}^a_{\rm o}[\hat{f}_V]=0,$

$$\mathcal{K}_{\rm s}[O] \equiv p^\mu\Big(\partial_\mu O^{\rm s} + \bar{C}_2 F^a_{\nu\mu}\partial^\nu_p O^a\Big), \quad \mathcal{K}^a_{\rm o}[O] \equiv p^\mu\Big(\partial_\mu O^a - f^{bca}A^b_\mu O^c + F^a_{\nu\mu}\partial^\nu_p O^{\rm s} + \frac{d^{bca}}{2}F^b_{\nu\mu}\partial^\nu_p O^c\Big)$$

$\bar{C}_2 = 1/(2N_c)$

quantum corrections

singlet AKE : $\delta(p^2-m^2)\big(\mathcal{K}_{\rm s}[\hat{a}^\mu] + \boxed{\hbar\mathcal{Q}^\mu_{\rm s}[\hat{f}_V]}\big)=0,$

DY, JHEP 06, 140 (2022)

octet AKE : $\delta(p^2-m^2)\big(\mathcal{K}^a_{\rm o}[\hat{a}^\mu] + \hbar\mathcal{Q}^{a\mu}_{\rm o}[\hat{f}_V]\big) + \hbar\delta'(p^2-m^2)\hat{\Theta}^{a\mu}_{\rm o}[\hat{f}_V]=0.$

e.g. $$\mathcal{Q}^\mu_{\rm s}[O] \equiv -\frac{1}{2}\big[\epsilon^{\mu\nu\rho\sigma}p_\rho\big((\partial_\sigma F^a_{\beta\nu}) - f^{bca}A^b_\sigma F^c_{\beta\nu}\big)\bar{C}_2\partial^\beta_p O^a\big]$$

- AKEs have the anomalous-force terms beyond the Lorentz-force.

Color-field induced polarization

- Perturbatively, we may rewrite f_V^a, $\tilde{a}^{a\mu}$ in terms of f_V^s, $\tilde{a}^{s\mu}$ by solving the color-octet kinetic equations.
- Onshell color-singlet axial WF :

$$\mathcal{A}^{s\mu}(\boldsymbol{p},X)\equiv\int\frac{dp_0}{2\pi}\langle\mathcal{A}^{s\mu}\rangle=\frac{1}{2\epsilon_{\boldsymbol{p}}}\left(\tilde{a}^{s\mu}+\hbar\bar{C}_2\mathcal{A}^{\mu}_{Q}\right)_{p_0=\epsilon_{\boldsymbol{p}}}\Rightarrow\mathcal{P}^{\mu}(\boldsymbol{p})=\frac{\int d\Sigma\cdot p\mathcal{A}^{s\mu}(\boldsymbol{p},X)}{2m\int d\Sigma\cdot p(2\epsilon_{\boldsymbol{p}})^{-1}f^{s}_{V}(\boldsymbol{p},X)}$$

dynamical ($\tilde{a}^{s\mu}$) non-dynamical ($\hbar\bar{C}_2\mathcal{A}^{\mu}_{Q}$)

❖ The dynamical part needs to be solved from the AKE

B. Müller, DY, PRD 105, L011901 (2022)
DY, JHEP 06, 140 (2022)

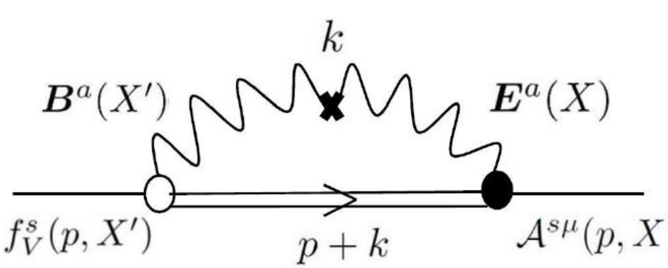
❖ Non-dynamical part : $\mathcal{A}^{\mu}_{Q}=\mathcal{A}^{\mu}_{Q1}+\mathcal{A}^{\mu}_{Q2}$

$$\mathcal{A}^{\mu}_{Q1}=\left[\frac{\partial_{p\kappa}}{2}\int_{k,X'}^{p}p^{\beta}\langle\tilde{F}^{a\mu\kappa}(X)F^{a}_{\alpha\beta}(X')\rangle\partial^{\alpha}_{p}f^{\rm s}_{V}(p,X')\right]_{p_0=\epsilon_{\boldsymbol{p}}},$$

$$\mathcal{A}^{\mu}_{Q2}=\frac{1}{2\epsilon^2_{\boldsymbol{p}}}(p_{\perp\kappa}-\epsilon^2_{\boldsymbol{p}}\partial_{p_{\perp}\kappa})\left[\int_{k,X'}^{p}p^{\beta}\langle\tilde{F}^{a\mu\kappa}(X)F^{a}_{\alpha\beta}(X')\rangle\partial^{\alpha}_{p}f^{\rm s}_{V}(p,X')\right]_{p_0=\epsilon_{\boldsymbol{p}}},$$

$$\langle F^{a}_{\kappa\lambda}(X)F^{a}_{\alpha\rho}(X')\rangle\equiv\langle F^{a}_{\kappa\lambda}(X)U^{ab}(X,X')F^{b}_{\alpha\rho}(X')\rangle,$$

$$\int_{k,X'}^{p}\equiv\int d^4k\int\frac{d^4X'}{(2\pi)^4}e^{ik\cdot(X'-X)}\left(\pi\delta(p\cdot k)+iPV(1/p\cdot k)\right).$$



AKE with the diffusion and source term

- SKE : $0 = p \cdot \partial f_V^s(p, X) - \boxed{\partial_p^\kappa \mathscr{D}_\kappa[f_V^s]}$: anomalous shear viscosity

M. Asakawa, S. A. Bass, B. Muller, PRL. 96, 252301 (2006)

$$\mathscr{D}_\kappa[O] = \bar{C}_2 \int_{k,X'}^{p} p^\lambda p^\rho \langle F^a_{\kappa\lambda}(X) F^a_{\alpha\rho}(X')\rangle \partial_p^\alpha O(p, X')$$

$$\int_{k,X'}^{p} \equiv \int d^4k \int \frac{d^4X'}{(2\pi)^4} e^{ik\cdot(X'-X)} \left(\pi\delta(p\cdot k) + iPV(1/p\cdot k)\right).$$

- AKE: $0 = p \cdot \partial \tilde{a}^{s\mu}(p, X) - \boxed{\partial_p^\kappa \mathscr{D}_\kappa[\tilde{a}^{s\mu}]} + \boxed{\hbar \partial_p^\kappa \left(\mathscr{A}^\mu_\kappa[f_V^s]\right)}$

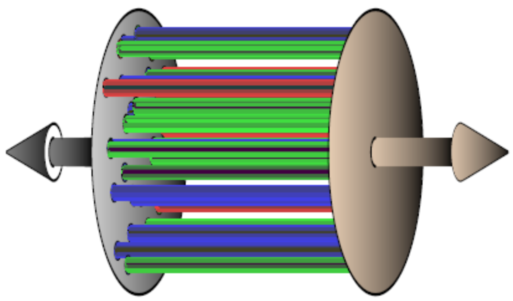
diffusion

source : dynamical spin polarization

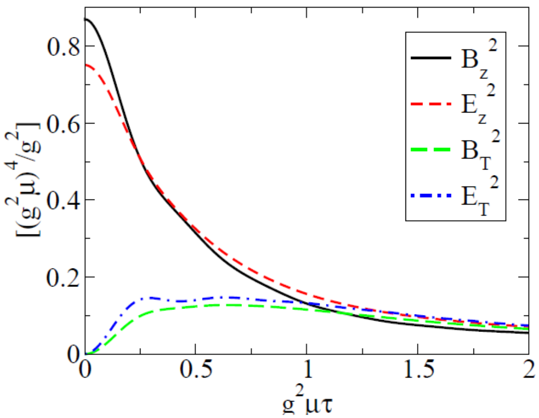
$$\mathscr{A}^\mu_\kappa[O] = \frac{\bar{C}_2}{2}\epsilon^{\mu\nu\rho\sigma} \int_{k,X'}^{p} p^\lambda p_\rho \Big(\partial_{X'\sigma} \langle F^a_{\kappa\lambda}(X) F^a_{\alpha\nu}(X')\rangle + \partial_{X\sigma}\langle F^a_{\kappa\nu}(X) F^a_{\alpha\lambda}(X')\rangle\Big) \partial_p^\alpha O(p, X')$$

Chromo-electromagnetic fields in HIC

- Color flux tubes in the glasma phase : longitudinal chromo-EM fields in early times.

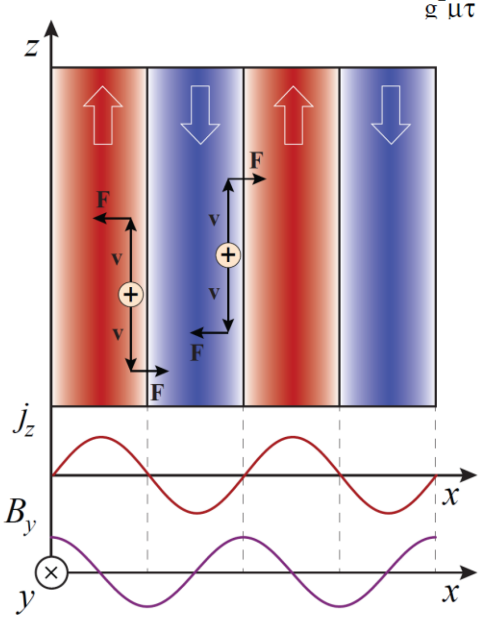


review: F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010



T. Lappi, PLB 643 (2006) 11-16

- Weibel-type plasma instability could generate turbulent color fields in anisotropic QGP.



S. Mrowczynski, PLB 214, 587 (1988), PLB314,118 (1993)

P. Romatschke and M. Strickland, PRD 68, 036004 (2003)

review: S. Mrowczynski, B. Schenke, M. Strickland, Physics Reports 682, 1 (2017)

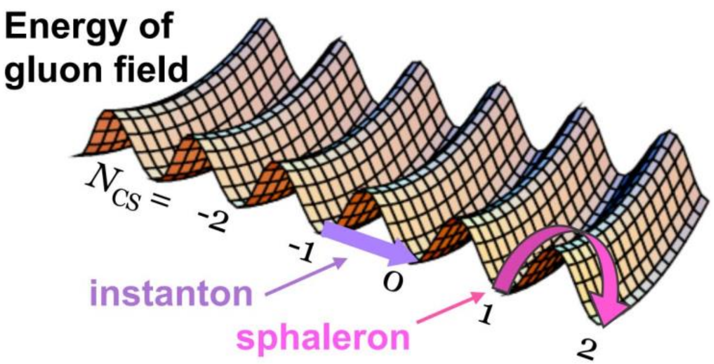
Local parity violation

- No local parity violation :

$$\langle B^a_\mu(X) B^a_\nu(X') \rangle \neq 0, \quad \langle E^a_\mu(X) E^a_\nu(X') \rangle \neq 0, \quad \langle B^a_\mu(X) E^a_\nu(X') \rangle = 0.$$

- Local-parity violation : $\langle B^a_\mu(X) E^a_\nu(X') \rangle \neq 0$

- Local parity violation in QGP : sphaleron transition



$$\partial_\mu J^\mu_5 = -\frac{g^2 N_f}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$$

$$\Rightarrow \partial_t N_5 = -2N_f \partial_t N_{CS} = -2N_f V \Gamma_{\rm sph}$$

F. R. Klinkhammer, N. S. Manton, PRD30, 2212 (1984)

L. McLerran, E. Mottola, M. Shaposhnikov, PRD 43, 2027 (1991)

(Correlators in the QGP phase are unknown)

Spin polarization from parity-odd correlators

- The practical color-field correlators can only be obtained from real-time simulations.

  e.g. non-equilibrium evolution in two-color QCD:
  N. Tanji, N. Mueller, and J. Berges, PRD 93, 074507 (2016)

- Physical assumptions : $\langle F^a_{\kappa\lambda}(X)F^a_{\alpha\rho}(X')\rangle = \langle F^a_{\kappa\lambda}F^a_{\alpha\rho}\rangle e^{-(t-t')^2/\tau_c^2}$ (correlation time)

  $$|\langle B^a_\mu B^a_\nu\rangle| \gg |\langle E^a_\mu B^a_\nu\rangle| \gg |\langle E^a_\mu E^a_\nu\rangle|$$

- f^s_V reaches thermal equilibrium : (assuming $\partial_X f^s_V \ll 1,\ \tilde{a}^{s\mu}(t_0,p)=0$)

$$\tilde{a}^{s\mu}(t,p) = -\frac{\hbar\bar{C}_2(t-t_0)}{2p_0^2}\left(\partial_{p0}f_{\rm eq}(p_0)\right)\left(\langle B^{a\mu}E^{a\nu}\rangle p_\nu - \langle B^a\cdot E^a\rangle p^\mu_\perp\right)$$

$$\mathcal{A}^\mu_Q(\boldsymbol{p},X) \approx \frac{\pi^{1/2}\tau_c}{4\epsilon^3_{\boldsymbol{p}}}\Big[\left(p^\alpha p^\beta\langle E^a_\alpha B^a_\beta\rangle u^\mu - \langle B^{a\mu}E^{a\beta}\rangle\epsilon_{\boldsymbol{p}}(\epsilon^2_{\boldsymbol{p}}\partial_{p\beta} - p_\beta)\right)\Big]\partial_{\epsilon_{\boldsymbol{p}}}f_{\rm eq}(\epsilon_{\boldsymbol{p}})$$

(parity-odd correlators)

❖ strengths and τ_c depend on anisotropy & parity violation

The axial charge currents and Ward identity

- A constant axial charge current (finite τ_c):

$$J_5^\mu = 4N_c \int \frac{d^4p}{(2\pi)^4} \mathrm{sign}(p_0) \mathcal{A}^{s\mu}(p,X) = -\frac{\hbar u^\mu}{8\pi^2}\sqrt{\pi}\tau_c \langle E^a \cdot B^a \rangle \mathcal{I} \qquad \mathcal{I} = 1 \text{ for } m = 0$$

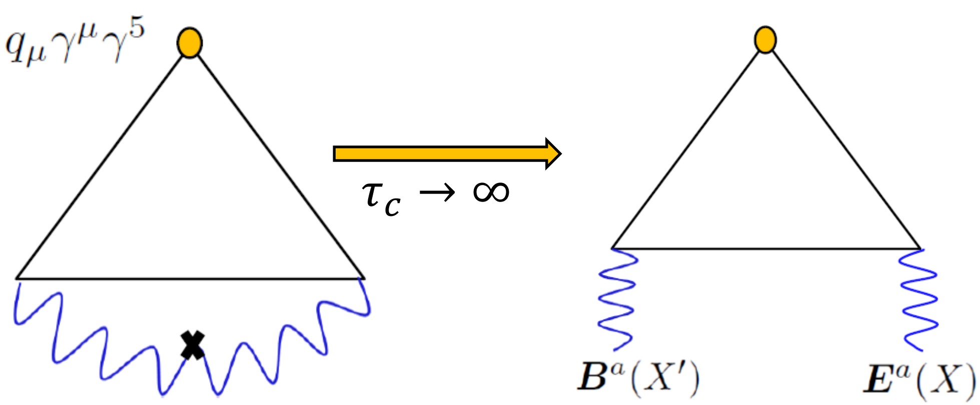
(similar to the steady state in Weyl semimetals $n_5 \sim \tau_R E \cdot B$)

- Vanishing axial-Ward identity : $\partial \cdot J_5 = 0$

- Constant-field limit ($\tau_c \to \infty$) : $\partial_\mu J_5^\mu(X) = -\hbar \frac{\langle B^a \cdot E^a \rangle}{4\pi^2} + 2m\langle \bar{\psi} i \gamma_5 \psi \rangle,$

DY, (2021), JHEP 06, 140 (2022)

$$\langle \bar{\psi} i \gamma_5 \psi \rangle = -\frac{\hbar \langle B^a \cdot E^a \rangle}{8m\pi^2} \int_0^\infty d|\boldsymbol{p}| \left(1 - \frac{|\boldsymbol{p}|}{\epsilon_{\boldsymbol{p}}}\right) \frac{d}{d|\boldsymbol{p}|} \left[f_{\mathrm{eq}}(\epsilon_{\boldsymbol{p}}) - f_{\mathrm{eq}}(-\epsilon_{\boldsymbol{p}}) \right]$$



Potential impact on observables

- Unlike the spin polarization from U(1) background fields, the polarization from color fields is charge-conjugation even, $\mathcal{P}^i_q = \mathcal{P}^i_{\bar{q}}$.

- Parity-odd correlators fluctuate event by event : average spin polarization of a single quark vanishes.

$$\langle \mathcal{P}^i_{\mathrm{q}} \rangle \neq 0, \quad (\langle \mathcal{P}^i_{\mathrm{q}} \rangle)_{\mathrm{ea}} = 0, \quad (\langle \mathcal{P}^i_{\mathrm{q}}(\boldsymbol{p}) \mathcal{P}^i_{\bar{\mathrm{q}}}(\boldsymbol{p}) \rangle)_{\mathrm{ea}} \approx (\langle \mathcal{P}^i_{\mathrm{q}}(\boldsymbol{p}) \rangle \langle \mathcal{P}^i_{\bar{\mathrm{q}}}(\boldsymbol{p}) \rangle)_{\mathrm{ea}}.$$

⇨ spin polarization of Λ should be unaffected

- Non-vanishing contribution to spin alignment : $\rho_{00} = \dfrac{1 - \langle \mathcal{P}^i_q \mathcal{P}^i_{\bar{q}} \rangle}{3 + \langle \mathcal{P}^i_q \mathcal{P}^i_{\bar{q}} \rangle}$ ⇨ $\rho_{00} < 1/3$

(qualitatively consistent with LHC)

- Caveat : spin polarization of quarks is not necessarily along the spin quantization axis.

$$\rho_{00} \approx \frac{1 + \sum_{j=1,2,3} \langle \mathcal{P}^j_q \mathcal{P}^j_{\bar{q}} \rangle - 2 \langle \mathcal{P}^i_q \mathcal{P}^i_{\bar{q}} \rangle}{3 + \sum_{j=1,2,3} \langle \mathcal{P}^j_q \mathcal{P}^j_{\bar{q}} \rangle}$$

⇨ $\rho_{00} = 1/3$ when $\langle \mathcal{P}^j_q \mathcal{P}^j_{\bar{q}} \rangle \neq 0$ is isotropic.

X.-L. Sheng, et al., 2206.05868

Conclusions & outlook

- ✓ From the QKT of massive quarks coupled with background color fields, it is found the spin polarization could be induced by color-field correlators.
- ✓ Based on certain approximations, the contributions from parity-odd correlators of color fields dominate.
- ✓ Such anomalous spin polarization of quarks might lead to $\rho_{00} < 1/3$ for spin alignment of vector mesons yet without affecting the Λ polarization.

- ❑ The information of color fields in the QGP phase is needed.
- ❑ Applying the generic expression for ρ_{00} in the coalescence model.
- ❑ The dynamical source term may be affected by the early-time color fields from the glasma. (under investigation with A. Kumar & B. Müller)
- ❑ For spin alignment, the effect proportional to e.g. $\langle EBEB \rangle \approx \langle EE \rangle \langle BB \rangle$ instead of $\langle EB \rangle \langle EB \rangle$ needs to be studied.

Thank you!

Wigner functions (WFs)

- lesser (greater) propagators :

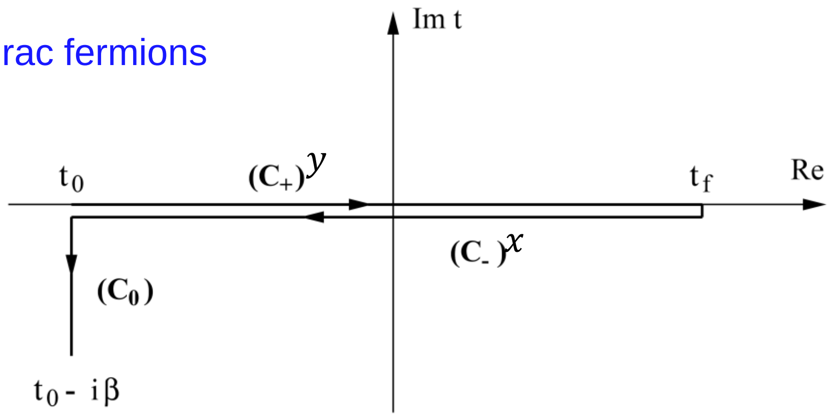
Dirac fermions

$$\tilde{S}^{>}_{\alpha\beta}(x,y) = \langle \psi_\alpha(x) U^\dagger(x,y) \bar{\psi}_\beta(y) \rangle$$

$$\tilde{S}^{<}_{\alpha\beta}(x,y) = \langle \bar{\psi}_\beta(y) \boxed{U(y,x)} \psi_\alpha(x) \rangle$$

gauge link

$$X = \frac{x+y}{2}, \ Y = x - y$$



review : J. Blaizot, E. Iancu, Phys.Rept. 359 (2002) 355-528

Wigner functions : $S^{<(>)}_{\alpha\beta}(q,X) = \int d^4Y e^{\frac{iq\cdot Y}{\hbar}} \tilde{S}^{<(>)}_{\alpha\beta}(x,y)$

- Kadanoff-Baym (KB) equations up to $\mathcal{O}(\hbar)$: ($q \gg \partial$: weak fields)

$$(\not{\Pi} - m)S^< + \gamma^\mu i\frac{\hbar}{2}\nabla_\mu S^< = \frac{i\hbar}{2}\left(\Sigma^< \star S^> - \Sigma^> \star S^<\right)$$

$$\nabla_\mu = \Delta_\mu + \mathcal{O}(\hbar^2),$$
$$\Delta_\mu = \partial_\mu + F_{\nu\mu}\partial/\partial q_\nu$$

(master Eqs. for WFs & KEs)

$$\Pi^\mu = q^\mu + \mathcal{O}(\hbar^2)$$

$$A \star B = AB + \frac{i\hbar}{2}\{A,B\}_{\text{P.B.}} + \mathcal{O}(\hbar^2)$$

Suppression of the diffusion term

- The diffusion term :

$$\partial_p^{\kappa}\mathscr{D}_{\kappa}[\tilde{a}^{s\mu}] \approx \bar{C}_2\epsilon_{\alpha\beta\gamma\kappa}\epsilon_{\nu\rho\lambda\sigma}u^{\alpha}u^{\nu}\partial_p^{\beta}\int_{k,X'}^{p}\langle B^{a\kappa}(X)B^{a\sigma}(X')\rangle p^{\gamma}p^{\lambda}\partial_p^{\rho}\tilde{a}^{s\mu}$$

$$= \frac{\bar{C}_2\sqrt{\pi}\tau_m}{2p_0}\epsilon_{\alpha\beta\gamma\kappa}\epsilon_{\nu\rho\lambda\sigma}u^{\alpha}u^{\nu}\langle B^{a\kappa}B^{a\sigma}\rangle\partial_p^{\beta}\left(p^{\gamma}p^{\lambda}\partial_p^{\rho}\tilde{a}^{s\mu}\right),$$

$$\langle B^{a\kappa}(X)B^{a\sigma}(X')\rangle = \langle B^{a\kappa}B^{a\sigma}\rangle e^{-\delta X_0^2/\tau_m^2}.$$

- The diffusion term plays a role when $\partial_p^{\kappa}\mathscr{D}_{\kappa}[\tilde{a}^{s\mu}] \sim p\cdot\partial\tilde{a}^{s\mu}$

⇨ $(t-t_0)\tau_m|\langle B^{a\kappa}B^{a\sigma}\rangle| \sim p_0^2$ (for $\tilde{a}^{s\mu}(t_0,p)=0$)

- In our setup : $p_0^2 \gg |B^{a\mu}|$

Relativistic angular momentum

- Relativistic angular momentum for fermions : $\mathcal{L} = \bar{\psi}\left(\frac{i\hbar}{2}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu} - m\right)\psi,$ $\overleftrightarrow{\partial}_{\mu} = \overrightarrow{\partial}_{\mu} - \overleftarrow{\partial}_{\mu}.$

  reviews : E. Leader & C. Lorce, Phys.Rept. 541 (2014) 3, 163-248
  K. Fukushima, S. Pu, Lect.Notes Phys. 987 (2021) 381-396

- Noether theorem :

$$\delta\psi \to \frac{\tilde{\omega}_{\mu\nu}}{2}\left(\hat{J}^{\mu\nu} - i\Sigma^{\mu\nu}\right)\psi, \quad \Sigma^{\mu\nu} = \frac{i}{4}[\gamma^{\mu},\gamma^{\nu}], \quad \hat{J}^{\mu\nu} = x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu}. \quad \Rightarrow \quad \partial_{\lambda}M_C^{\lambda\mu\nu} = 0.$$

- Canonical AM tensor :

$$M_C^{\lambda\mu\nu} = M_S^{\lambda\mu\nu} + M_O^{\lambda\mu\nu},$$

$$M_S^{\lambda\mu\nu} = \frac{1}{2}\bar{\psi}\{\gamma^{\lambda},\Sigma^{\mu\nu}\}\psi = -\frac{1}{2}\epsilon^{\lambda\mu\nu\rho}\bar{\psi}\gamma_{\rho}\gamma_5\psi,$$

proportional to the axial-charge current

$$M_O^{\lambda\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^{\lambda}\left(x^{\mu}\overleftrightarrow{\partial}^{\nu} - x^{\nu}\overleftrightarrow{\partial}^{\mu}\right)\psi = x^{\mu}T_C^{\lambda\nu} - x^{\nu}T_C^{\lambda\mu},$$

- Canonical EM tensor : $T_C^{\mu\nu} = T^{\mu\nu} + T_A^{\mu\nu}, \quad T^{\mu\nu} = \frac{i\hbar}{4}\bar{\psi}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi, \quad T_A^{\mu\nu} = \frac{i\hbar}{4}\bar{\psi}\gamma^{[\mu}\overleftrightarrow{D}^{\nu]}\psi.$

- EM & AM conservation :

$$\partial_{\mu}T_C^{\mu\nu} = 0, \quad \partial_{\lambda}M_C^{\lambda\mu\nu} = 0. \quad \Rightarrow \quad -\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_{\lambda}J_{5\rho} + 2T_A^{\mu\nu} = 0$$

spin: $-\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_{\lambda}J_{5\rho}$; orbit: $2T_A^{\mu\nu}$

Spectra for spin polarization

- How to measure the spin polarization in experiments?
- On the theoretical side, how to construct a potential observable in connection to the spin AM?
  - We measure particle spectra in HIC experiments. (unlike CM systems)
  - Generalize the field-theory operators into the phase-space representation : using Wigner functions
  - Pauli-Lubanski (pseudo) vector : $\bar{W}^{\mu}_{C}(\boldsymbol{p},X)=-\frac{1}{2m}\epsilon^{\mu\nu\alpha\beta}p_{\nu}\tilde{M}_{C\alpha\beta}(\boldsymbol{p},X),$

$$\tilde{M}_{C\alpha\beta}(\boldsymbol{p},X)=\bar{n}^{\rho}\tilde{M}_{C\rho\alpha\beta}(\boldsymbol{p},X).$$

$(\bar{n}^{\rho}\parallel d\Sigma^{\rho})$ freeze-out surface

- (modified) Cooper-Frye formula for spin :

$$\mathcal{P}^{\mu}(\boldsymbol{p})=\frac{\int d\Sigma\cdot p\bar{W}^{\mu}_{C}(\boldsymbol{p},X)}{E_{p}\frac{dN}{d^3\boldsymbol{p}}}=\boxed{\frac{\int d\Sigma\cdot p\tilde{J}^{\mu}_{5}(\boldsymbol{p},X)}{2m\int d\Sigma\cdot\mathcal{N}(\boldsymbol{p},X)}}.$$

(By Wigner functions)

F. Becattini, et al., Ann. Phys. 338, 32 (2013)
R. Fang, et al., PRC 94, 024904 (2016)

- In "global" equilibrium : $\mathcal{P}^{\mu}(\boldsymbol{p})=\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_{\nu}\frac{\int d\Sigma\cdot p\omega_{\rho\sigma}f^{(0)}_{p}(1-f^{(0)}_{p})}{\int d\Sigma\cdot pf^{(0)}_{p}},\quad f^{(0)}_{p}=f_{\rm FD},$

Killing cond. : $"\partial_{\mu}\beta_{\nu}+\partial_{\nu}\beta_{\mu}=0"$

$$\beta^{\mu}\equiv u^{\mu}/T$$

$$\boxed{\omega_{\mu\nu}=\frac{1}{2}(\partial_{\mu}\beta_{\nu}-\partial_{\nu}\beta_{\mu})}.\quad\Longrightarrow\quad\omega_{\alpha\beta}=-\epsilon_{\alpha\beta\mu\nu}\omega^{\mu}u^{\nu}$$

thermal vorticity

T=const.