Spin effects and thermal field theory

1. Intro.
2. Response theory
   - Fermions: shear-induced polarization and baryonic spin Hall effect.
   - Vector bosons
3. Outlook

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Based on: Shuai Liu, YY, 2006.12421, PRD; 2103.09200, JHEP 21.

Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song, YY, 2103.10403, PRL 21;

Baochi Fu, Longgang Pang, Huichao Song, YY, 2201.12970.

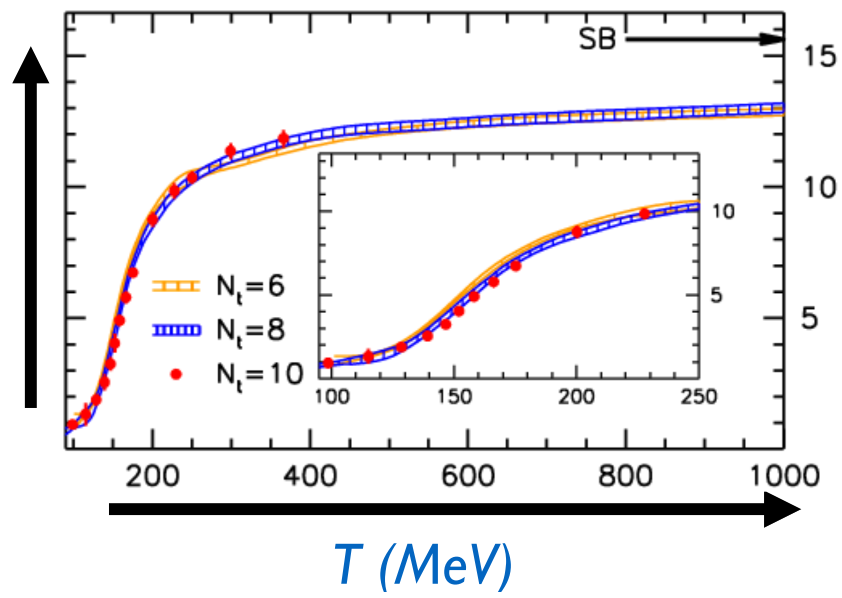
Zonglin Mo, YY, in progress.

Reimei Workshop (Yonsei University and online) , Oct 6-8th, 2022

Motivation

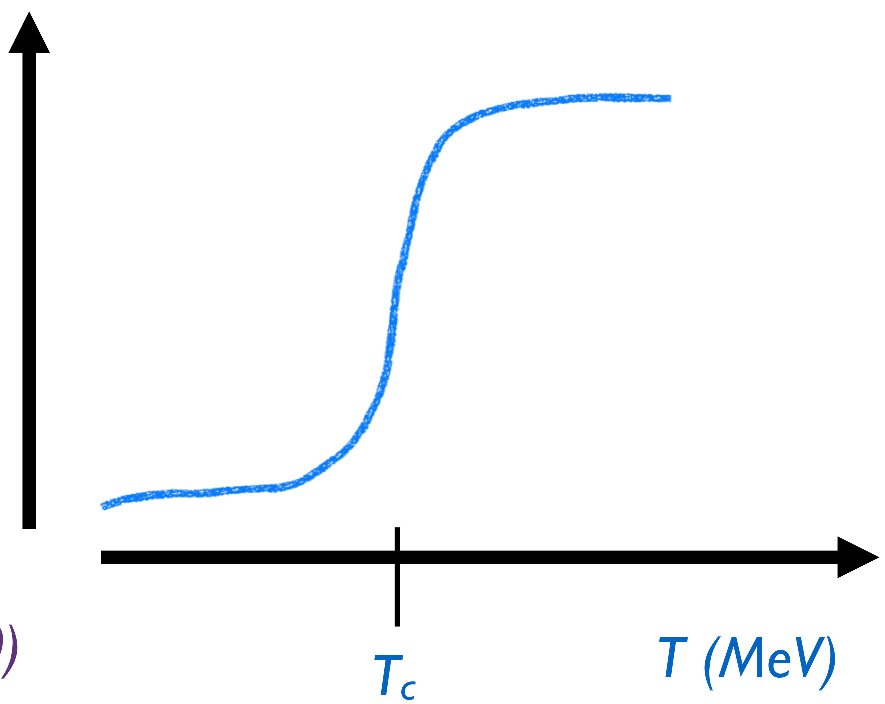
Spin observables probe QCD phase structure.

ε/T^4~# of Degree of freedom



(Budapest-Marseille-Wuppertal Collaboration, JHEP'10)

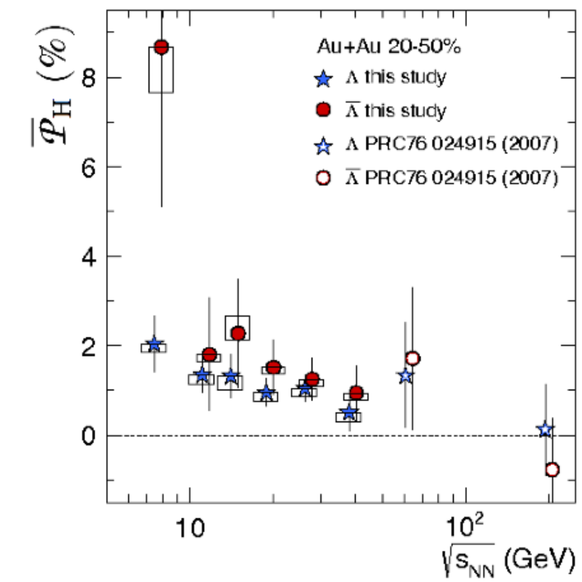
Review: Becattini, Lisa, Annals Phys. 2020



- Density of spin carriers changes dramatically near Tc.
- In-medium properties of spin carriers (e.g. VM) monitor phase transition
- Broad context: spin dynamics of QCD (c.f. proton spin puzzle).

Spin observables in HIC

Review: Becattini, Lisa, Annals Phys. 2020



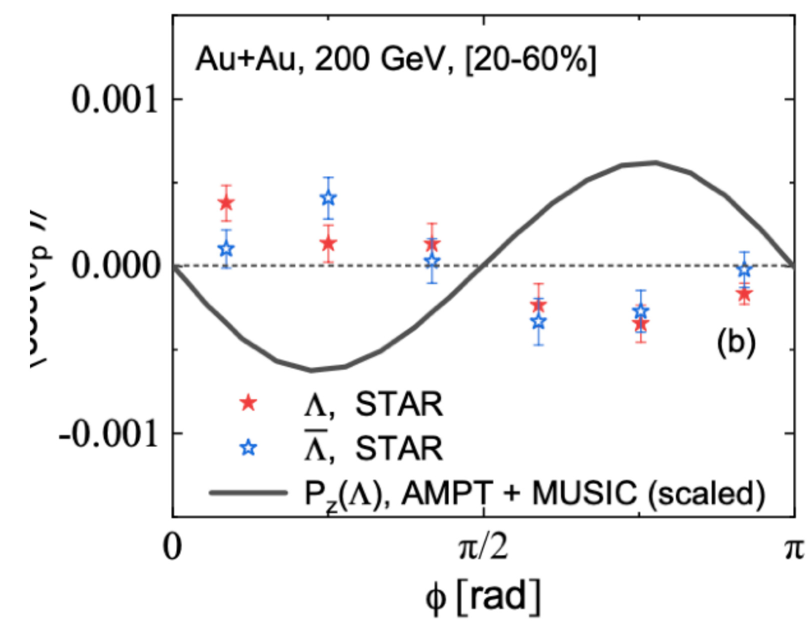
STAR, Nature 17'.

- Informative Λ hyperon polarization data
  - Vorticity effect describes the trends of global (phase-space averaged) Λ.

    Xin-Nian Wang, Zuo-Tang Liang, PRL 05'; Becattini et al, Annals Phys 13'

  - the differential data, i.e. the flow of spin.: intensive study.

    STAR PRL 19'; hydro. simulations by many



Baochi Fu et. al, PRC21'

- Intriguing results on vector meson (kaon, ϕ, J/ψ) spin density matrix.

  e.g. STAR, 2204.02302;ALICE PRL 20', 2204.10171

Mechanism for spin polarization of Fermions

- Rotation polarizes spin: *Landau-Lifshitz volume 5*

$$\Delta\epsilon = -\,\hat{s}\cdot\overrightarrow{\Omega} \rightarrow \hat{s} \,||\, \overrightarrow{\Omega}$$ (similar for B-field)

Xin-Nian Wang, Zuo-Tang Liang, PRL 05'; Becattini et al, 13'

  - extensively studied.
  - induced-polarization is independent of $\hat{p}$.
- A different class of mechanisms: spin polarization that is correlated with momentum (spin-momentum correlation).

Spin Hall effect (SHE)

$$\boxed{\hat{s} \propto \overrightarrow{v} \times \overrightarrow{E}}$$

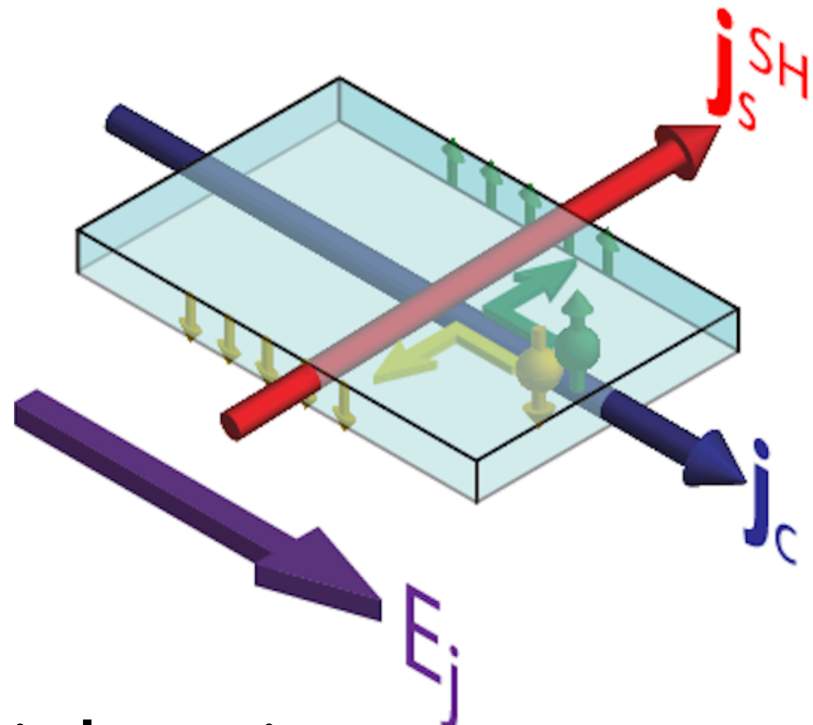


Fig. from Meyer et al, Nature material 17'

- implies non-trivial spin current and spin-momentum correlation.

- is instrumental for characterizing topological phases in 2+1 insulators.

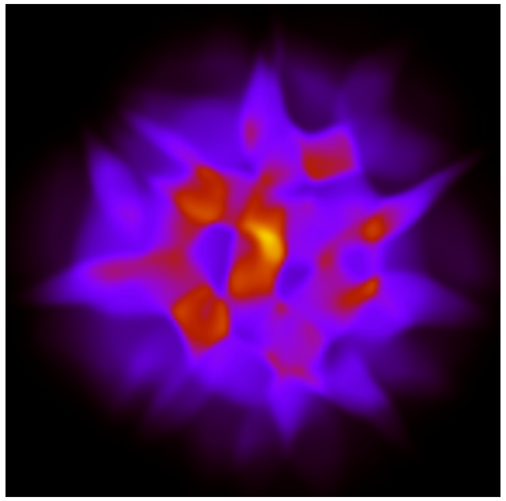
    - SHE conductivity is zero for topological matter class A but non-zero for class AII.

Kane-Mele PRL 05'

- connects to Berry curvature and angular momentum conservation.

Murakami, Nagaosa, Shou-Cheng Zhang, Science 2003'

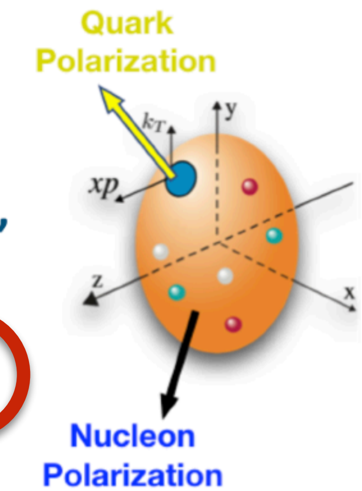
Heavy-ion collisions



- We may generalize SHE by replacing electric field by the effective force $\vec{F}$ such as $T\nabla(\mu/T)$, T-gradient ....
- Spin transport induced by the gradient of hydro. field (e.g. flow and energy/charge density)?
- Towards quantitative description:
  - The popular approach looks different from thermal field theory paradigm.
- Our approach: employing standard and systematic field theory.

Understanding spin-momentum of partons inside a proton is an important QCD frontier

Explore the Flavor-Spin-Motion Correlation



- **Intrinsic & confined parton motion:**
  - **Fundamental information sensitive to how partons are bound together**
  - **Responsible for dynamical contribution to emergent hadron properties, such as spin, mass, ..**
- **Quantum correlation between hadron spin and parton motion:**

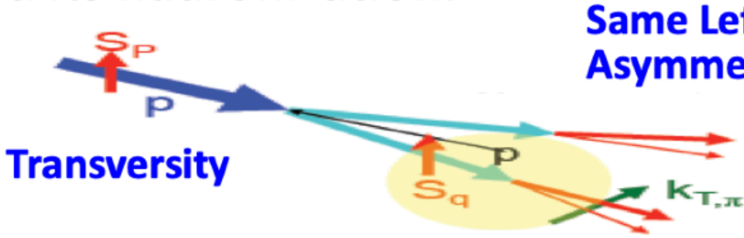
Observed: Left-Right Asymmetry

  - **Sivers effect – Sivers function**

    Hadron spin influences parton's transverse motion

- **Quantum correlation between parton's spin and its hadronization:**

Same Left-Right Asymmetry



  - **Collins effect – Collins function**

    Parton's transverse polarization influences its hadroni

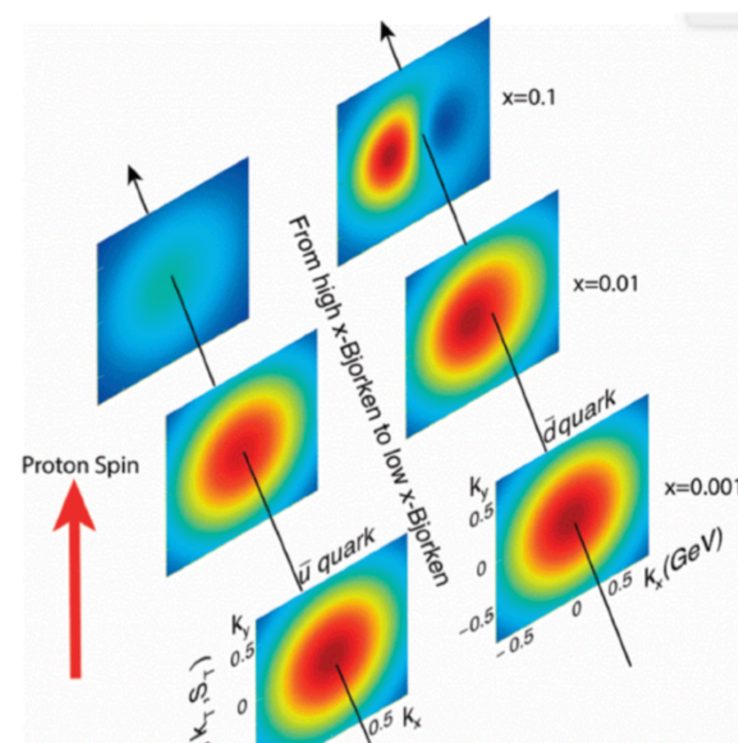


Fig. 2.7 NAS Report

Slides by Jianwei Qiu (JLAb)

Response theory

Field theory

- Response to hydro. gradients:
  - expansion in gradient (assuming non-hydro. modes are expressible in terms of hydro. ones).
  - relating expansion coefficients to correlators $\langle O(x) T^{\mu\nu}(x') \rangle$.
- E.g.: viscous stress-tensor and viscosities.

$$(T^{\mu\nu})_{\text{vis}} \propto \eta \sigma^{\mu\nu} \qquad q^{\mu}_{\text{heat}} \propto \kappa \partial^{\mu}_{\perp} T$$

- Applying similar procedure to spin polarization.

Axial Wigner function

$$\mathscr{A}^\mu(t, \vec{x}, \vec{p}) = \int d^3\vec{y}\, e^{-i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t, \vec{x} - \frac{1}{2}\vec{y})\, \gamma^\mu \gamma^5 \psi(t, \vec{x} + \frac{1}{2}\vec{y}) \rangle$$

- related to the phase space distribution of spin polarization vector.
- Building blocks for the gradient expansion:

$$\theta = \partial_\perp \cdot u\,,$$
$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\lambda} u_\nu \partial^\perp_\alpha u_\lambda\,, \qquad \beta^{-1}\partial^\mu_\perp \beta\,,$$
$$\sigma^{\mu\nu} = \frac{1}{2}(\partial^\mu_\perp u^\nu + \partial^\nu_\perp u^\mu) - \frac{1}{3}\Delta^{\mu\nu}\theta\,.$$

Tensors from gradient (focus on the neutral fluid)

$$p^\mu = \epsilon u^\mu + p^\mu_\perp,$$
$$Q^{\mu\nu} = -\frac{p^\mu_\perp p^\nu_\perp}{p^2_\perp} - \frac{1}{3}\Delta^{\mu\nu}, \ldots$$

Tensors formed by single particle momentum

$$\text{e.g.}: \mathscr{A}^\mu \sim \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho{}_\lambda$$

The derivative expansion

Shuai Liu, YY, 2103.09200, JHEP

- The most general expression consistent with symmetries (for the neutral fluid):

$$u \cdot \mathscr{A} = \tilde{c}_\omega p \cdot \omega \,,$$

$$\mathscr{A}^\mu_\perp = c_\omega \omega^\mu + \boxed{c_T \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log\beta + g_\sigma \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho{}_\lambda + g_\omega Q^{\mu\nu} \omega_\nu}$$

vorticity effects *spin Nernst effect* *shear-induced polarization*

$$\vec{s} \propto \hat{p} \times \nabla \log T$$

Spin-momentum correlation

- Although allowed by symmetry, flow gradient and momentum quadrupole coupling, has been overlook before.

- All above effects might be non-dissipative (associated coefficients are T-even).

Correlators

$$G^{\mu;\alpha\beta} = \int_{\vec{y}} e^{i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t, \vec{x} - \frac{1}{2}\vec{y})\, \gamma^\mu \gamma^5 \psi(t, \vec{x} + \frac{1}{2}\vec{y})\, T^{\alpha\beta}(0,0) \rangle$$

shear,vorticity $\sim G^{\mu;i0}$ $(h_{0i} \sim v_i)$, T-gradient $\sim G^{\mu;00}$ $(h_{00} \sim T)$

- For general fermion mass at one-loop:

$$\mathscr{A}^\mu_\perp = (-n'_{FD}) \left[\omega^\mu + \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log\beta + \frac{-p_\perp^2}{(p\cdot u)} \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho{}_\lambda \right] + 0 \times Q^{\mu\nu}\omega_\nu$$

vorticity effects *spin Nernst effect* *shear-induced polarization*

- Consistent with (collisionless) quantum kinetic theory analysis.

- Relation among different effects?

Collisional effect: ;Shu Lin & Ziyue Wang, 2206.12573;Wagner, Wickgenannt, Speranza, Rischke, 2208.01955

Constraint from Ward identity (WT)

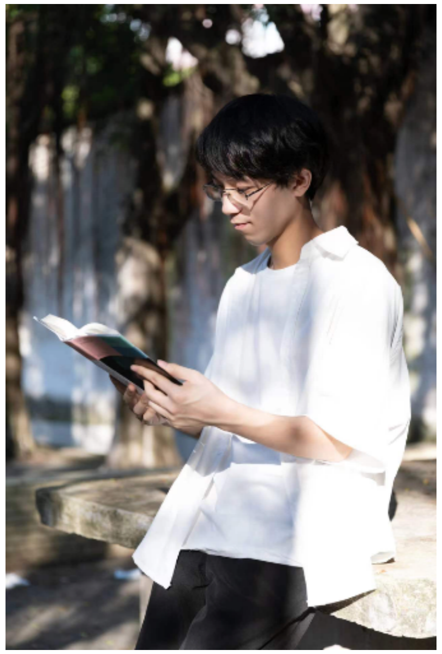
Zonglin Mo, YY, in progress

- WT are non-perturbative relations among correlators from symmetries considerations. Applying to spin effects.

$$G^{i;0j}$$

$$q_0\, G^{i;00} + q_j \left(\#\epsilon^{ijk} q_k + \#\epsilon^{ilm} q_l p_m p^j\right) = \text{contact term}$$

T-gradient effect *vorticity effects* *Shear effect*

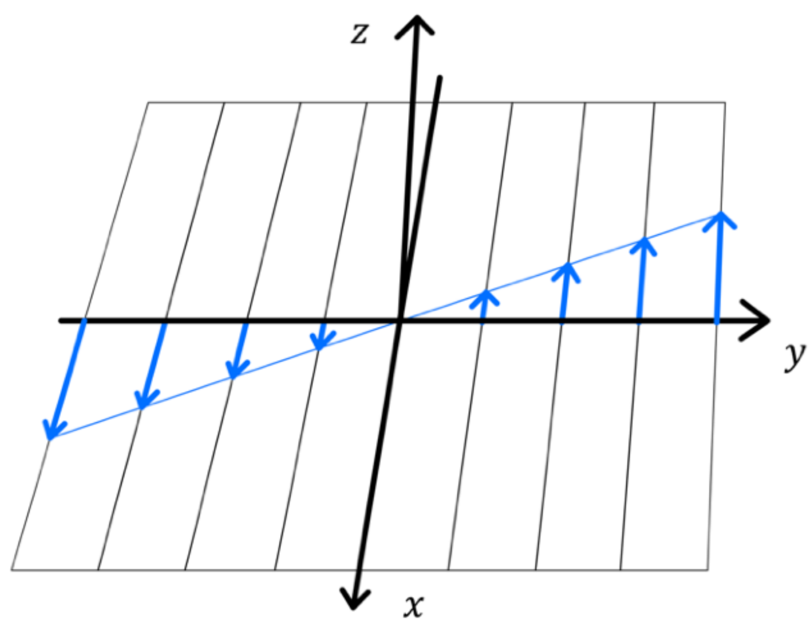


Zonglin Mo, 2nd year graduate from USTC

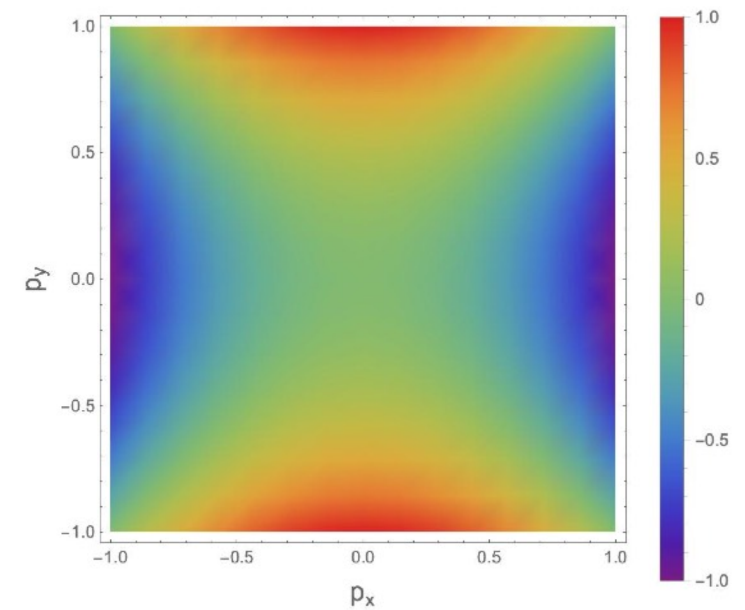
- T-gradient effect may be related to shear-induced polarization. (convention wisdom based on "global equilibrium": T-gradient and vorticity effect are related.)

Shear-induced polarization

Shear-induced polarization: an illustration



A standard shear flow profile:
$\omega^z \neq 0\,, \sigma^{xy} \neq 0$



Spin polarization along z-direction in phase space from SIP.

$$\mathscr{A}^i_{SIP} \propto \epsilon^{ikj} Q_{jl} \sigma^l{}_k\,, \qquad Q_{ij} = \hat{p}_i \hat{p}_j - \frac{1}{3}\delta_{ij}$$

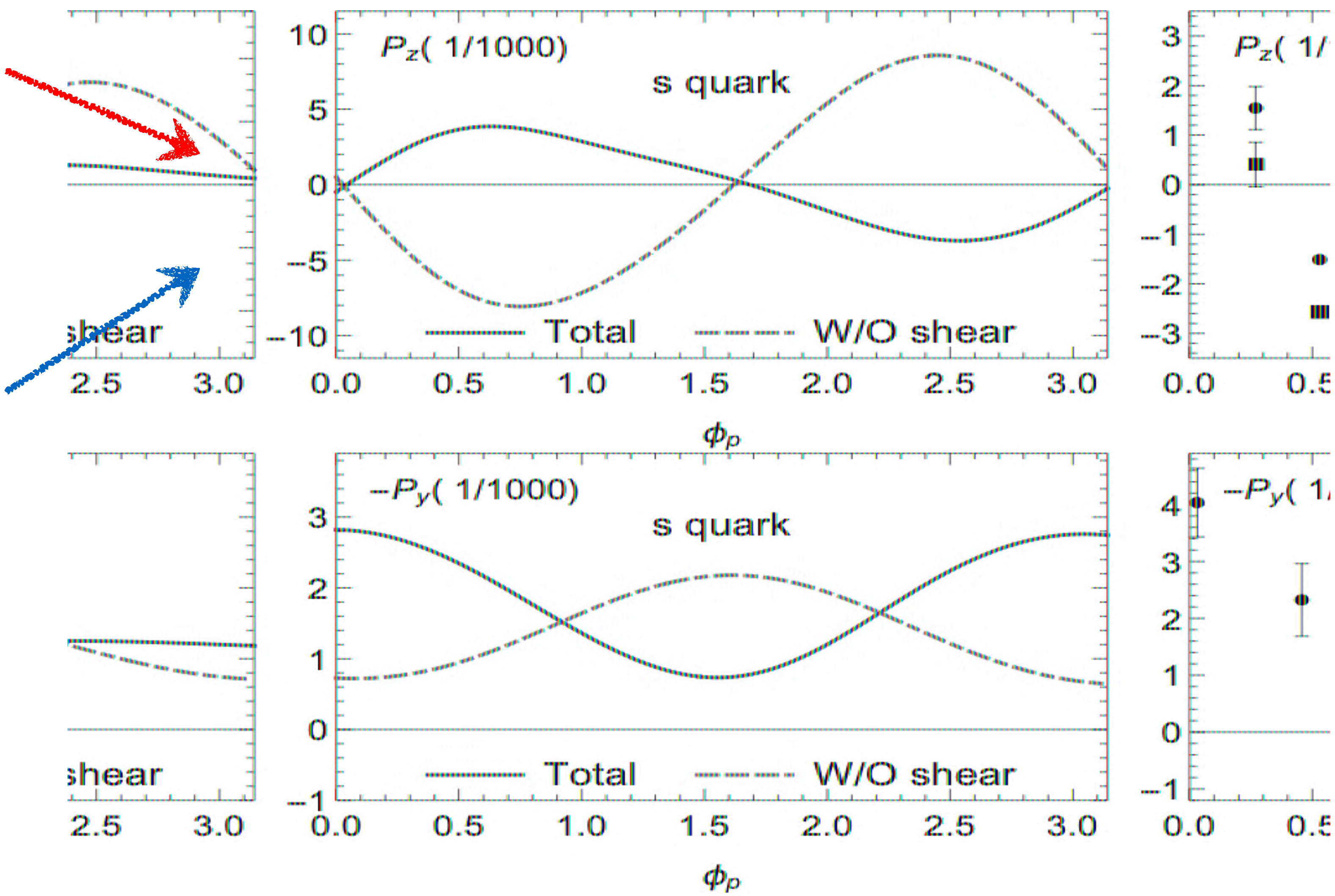
Shear-induced polarization (SIP): imaging anisotropy in a fluid into anisotropy in spin space.

SIP at RHIC

Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song, YY, 2103.10403, PRL 21;

With SIP

Without SIP



vs transverse azimuthal angle ϕ_p

- determines the qualitative feature of differential polarization.

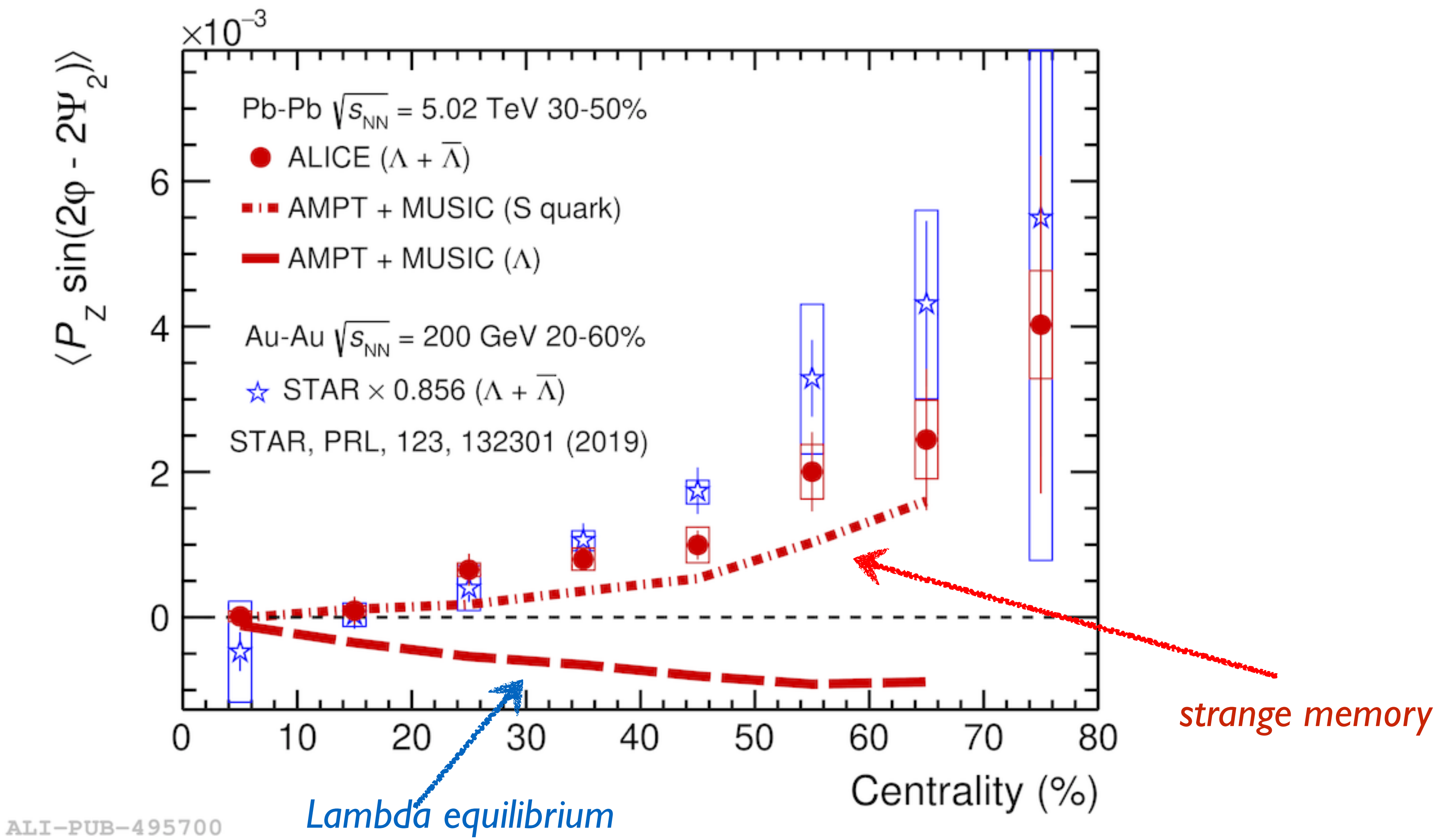
(Main theoretical uncertainties arises from freeze-out prescription for spin and subsequent hadronic evolution.)

see also 2103.14621, PRL 21 by Becattini et al.

SIP at LHC

ALICE collaboration: 2107.11183 PRL ;

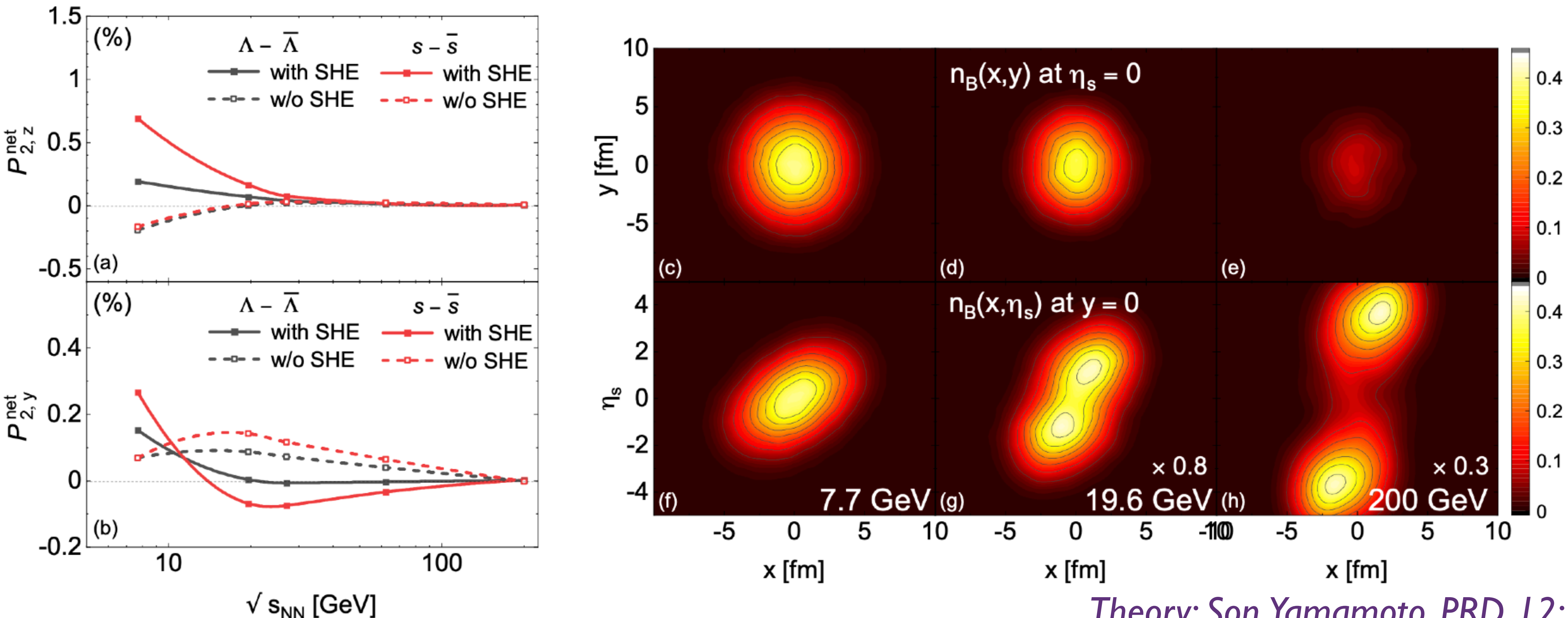
benchmark calculations by Baochi Fu&Huicaho Song



- Tantalizing evidence for SIP; more efforts are needed.

The observation of SIP in QGP might be its first detection among all kinds of fluid!

Baryonic spin Hall effect probes medium created at lower beam energy



Theory: Son, Yamamoto, PRD 12; Di-Lung Yang, Hattori, Yoshimasa, PRD 19' … ; First phenomenological study: Liu-YY, 2006.12421, PRD21; also Shen et al, 2106.08125

$$\hat{s}_{\pm} \propto \pm \hat{p} \times \nabla \mu_B$$

- The proposed observables show the qualitative difference in both sign and collision energy dependence with and without SHE.

- Stay tuned: experimental analysis is under way.

Baochi Fu, Longgang Pang, Huichao Song, YY, 2201.12970.

Vector Mesons

Spin alignment

- measures the imbalance between different spin states and implies anisotropy.

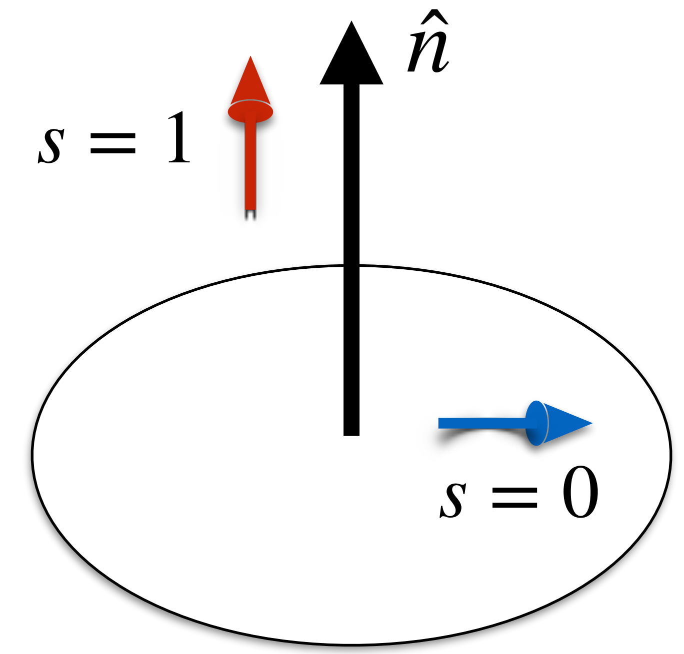
$$\rho_{ss'} = \langle a^{\dagger}_{p,s} a_{p,s'} \rangle \qquad s = 0, \pm$$

Spin density matrix

$$\delta\rho_{00} = \rho_{00}(\hat{n}) - \frac{1}{3} = \frac{1}{3}(2\rho_{00} - \rho_{++} - \rho_{--})$$

imbalance

Spin alignment



- is a component of tensor polarization and hence is conceptually different from polarization.

$$\rho_{ss'} \propto \frac{1}{3}\left[\delta_{ss'} + J^i_{ss'}\mathscr{P}^i + \rho^T_{ss'}\right]$$

c.f spin 1/2 case $\quad \rho_{ss'} \propto \frac{1}{2}\left(\sigma^0_{ss'} + \mathscr{A}^i\sigma^i_{ss'}\right)$

Expectation

- Conventional wisdom $\mathcal{O}(\partial^2)$
  - $\delta\rho_{00} \sim E^2 \sim \mathcal{O}(\partial^2)$ since strange electric field is counted as $\mathcal{O}(\partial)$ when deriving kinetic theory.)

    See Xinli Sheng and Di-Lun Yang's talk.

  - Vorticity effect based on re-combination model (similar for B-field effect etc.)

$$\delta\rho_{00}(\hat{e}_y) = -\frac{1}{9}(\frac{\omega}{T})^2$$

Becattini et al PRC 17', and many others

- However, $\delta\rho_{00}$ is a component of tensor polarization, $\mathcal{O}(\partial)$ from the shear is allowed.

$$(\rho_{ij})_{\mathrm{PRF}} = n_B(1 + n_B)\alpha_{\mathrm{sh}}\,(\sigma_{ij})_{\mathrm{PRF}}$$

Feng Li & Shuai Liu, 2206.11890;Wagner, Wickgenannt, Speranza, 2207.01111

- Shear induced tensor polarizability is T-odd, I.e. dissipative and are sensitive to in-medium dispersion of VM.

$$\alpha_{\mathrm{sh}} = \frac{1}{TE_p}\frac{\mathrm{Re}\Pi}{\Gamma_p} \propto \frac{(\text{momentum exchange})^2}{\text{decay rate}}$$

From field theory calculations

Energy splitting ($\mathcal{O}(\partial^0)$) also induces spin alignment

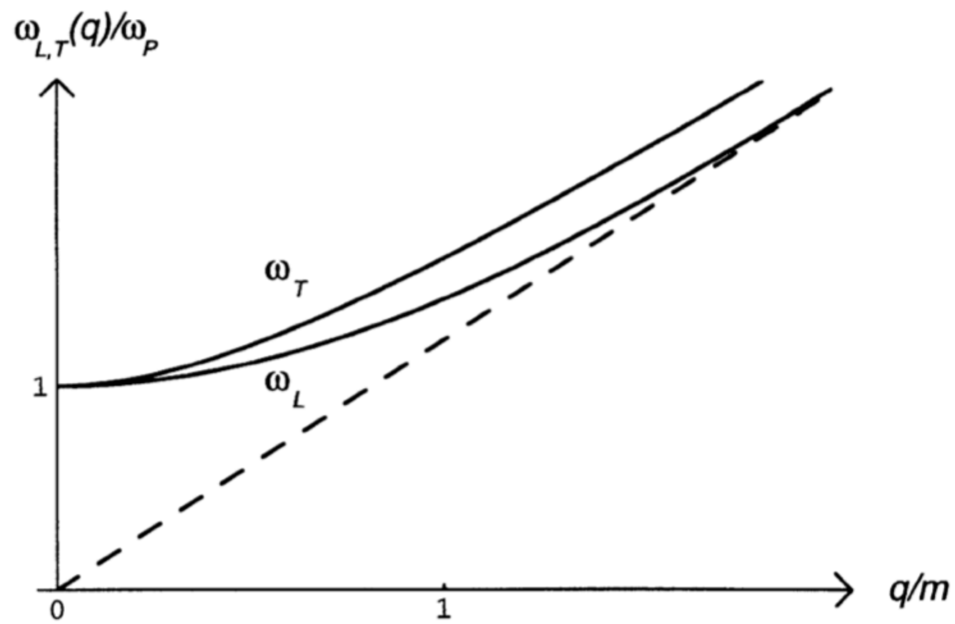
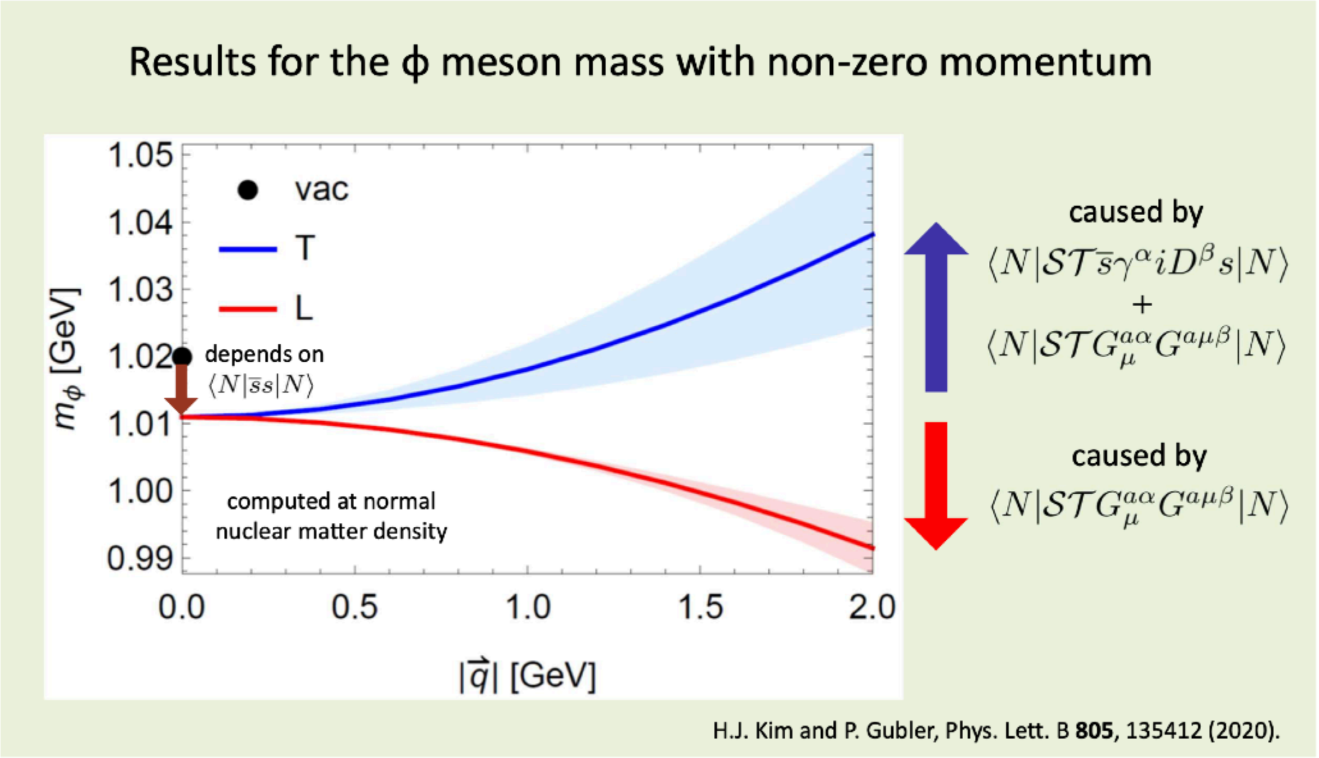


Fig. 6.2 Dispersion laws for transverse and longitudinal photons.

Bellac's textbook



Gubler's talk, see also, Kim-Gubler, PLB 20'.

- Splitting of E_T, E_L induces spin alignment. (benchmark for ρ_{00} in-medium is NOT 1/3)

$$\delta\rho_{00}^{(0)}(\hat{n}) \approx \int_{x,p} \frac{E_L(p) - E_T(p)}{9T} \left[(\hat{n}\cdot\vec{v})^2 - \frac{1}{3} \right]$$

Spin alignment serve as a new probe to in-medium properties.

Summary and outlook

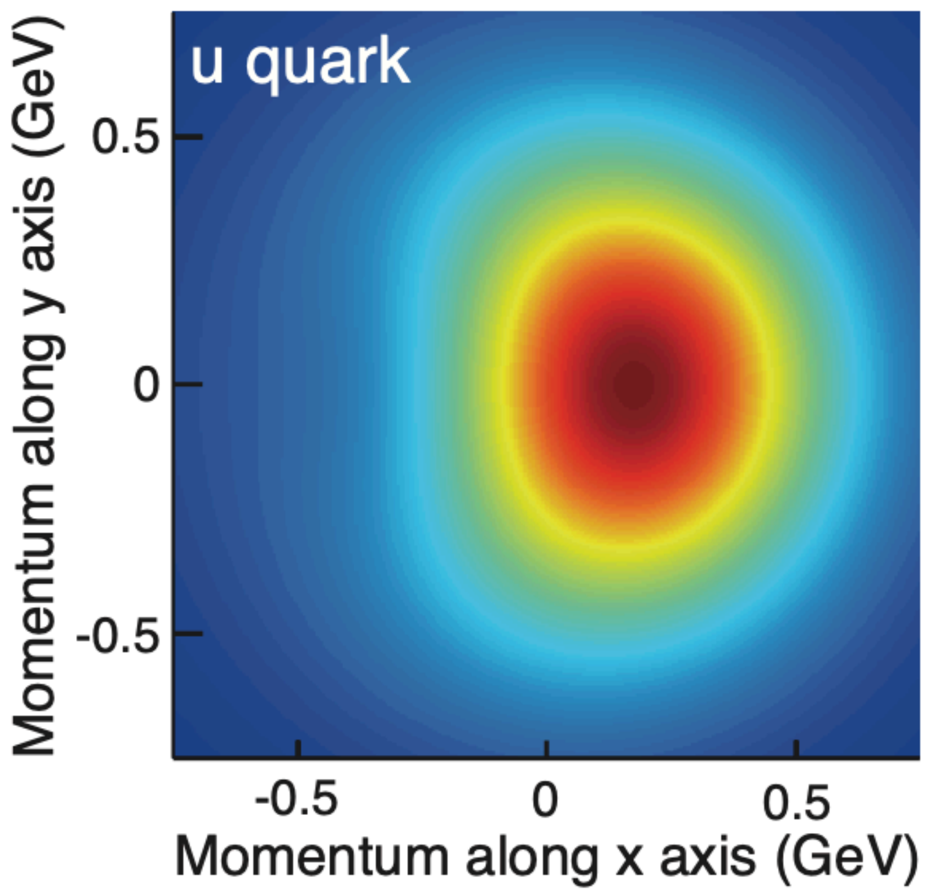
Summary

- Spin observables: new probes into QCD phase structure.
- Status: various “spin puzzles” are demystified if not fully solved.
  - many spin effects can be systematically analyzed within conventional field theory.
- Shear stress are important in generating spin-momentum correlation for fermions and tensor polarizability for vector bosons.
- Discovery potential at high baryon regime (CBM, J-PARC, …).

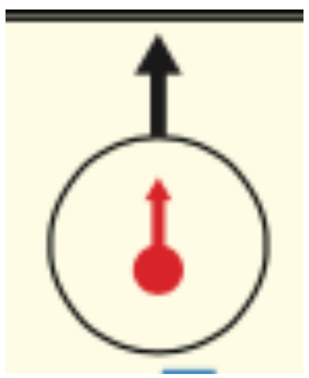
Ultimate goal: spin structure of QCD systems.

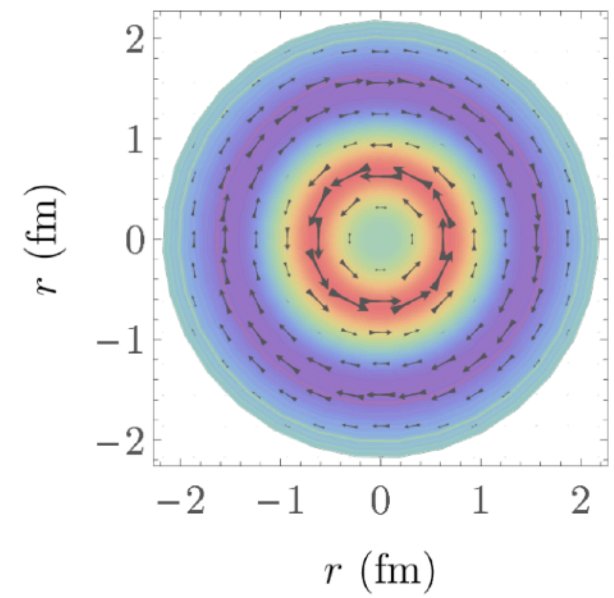
Back-up

Spin transport inside nucleons



transverse momentum dependent quark distribution in a polarized proton (fig from EIC white paper)





lattice calculation of shear force inside proton.

Shanahan-Detmold, PRL 18

- Correlation with shear force?

Ultimate goal: spin structure of QCD systems.

Berry gauge field

- Berry gauge field: average position of an eigenstate.

$$\overrightarrow{a}(\hat{p}) = i\langle u(\overrightarrow{p}, \lambda) \,|\, \partial_{\overrightarrow{p}} \,|\, u(\overrightarrow{p}, \lambda)\rangle > = \langle \hat{x} \rangle_u$$

- Berry curvature: $\overrightarrow{b}(\hat{p}) = \nabla \times \overrightarrow{a}(\hat{p}) \parallel \lambda\hat{s}$
    - carries topological charge;
    - plays an important role in describing quantum transport.