Dihadrons and Lambdas at the EIC

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APCTP Workshop on the
Physics of the EIC

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Outline

- SIDIS Dihadron Kinematics and Cross Section
- TMD PDFs and EIC Impact
- Dihadron Fragmentation Functions and Partial Waves
- Lambdas and TMD Fragmentation

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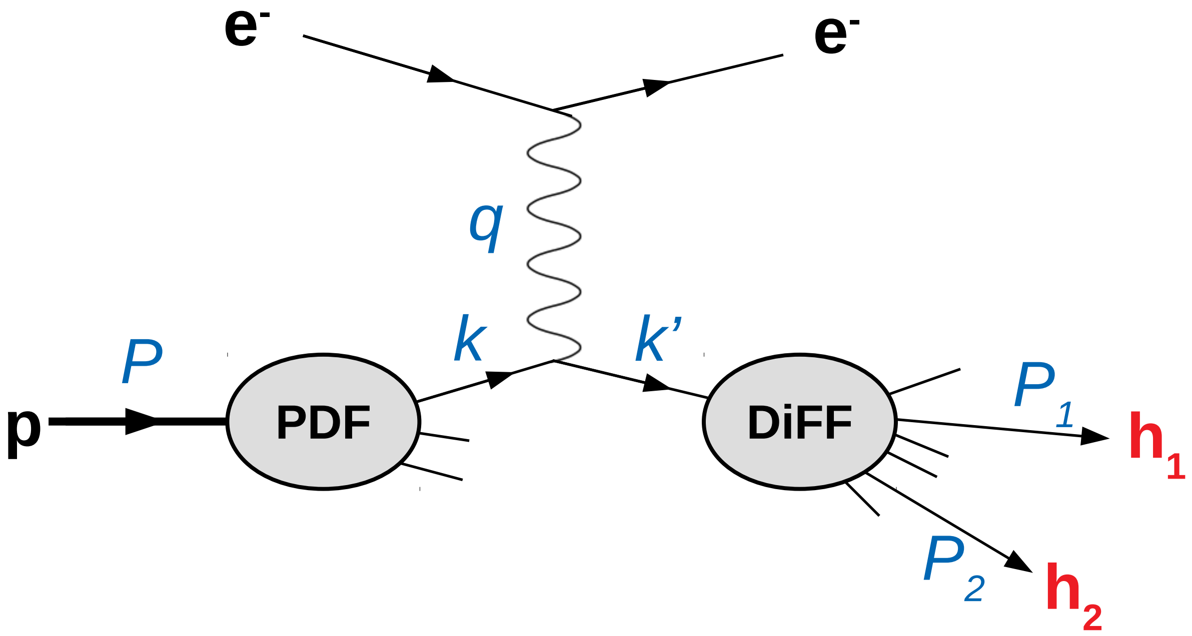
- **SIDIS Dihadron Kinematics and Cross Section**
- TMD PDFs and EIC Impact
- Dihadron Fragmentation Functions and Partial Waves
- Lambdas and TMD Fragmentation

Dihadrons in SIDIS

$$eN \to e + h_1(P_1) + h_2(P_2) + X$$



Dihadron Kinematics

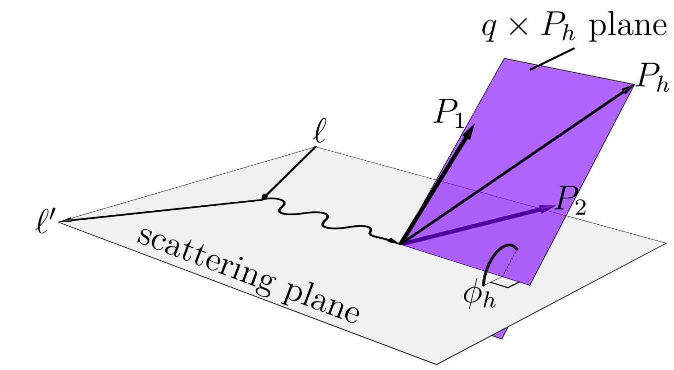
$$eN \to e + h_1(P_1) + h_2(P_2) + X$$

Dihadrons:

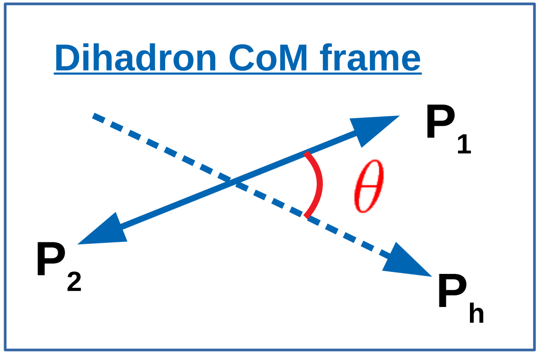
$$\text{momentum:}\;\; P_h = P_1 + P_2$$

$$\text{kinematics:}\;\; M_h,\;\; z,\;\; p_T$$

$$\text{angles:}\;\; \phi_h,\;\; \phi_R,\;\; \phi_S,\;\; \theta$$



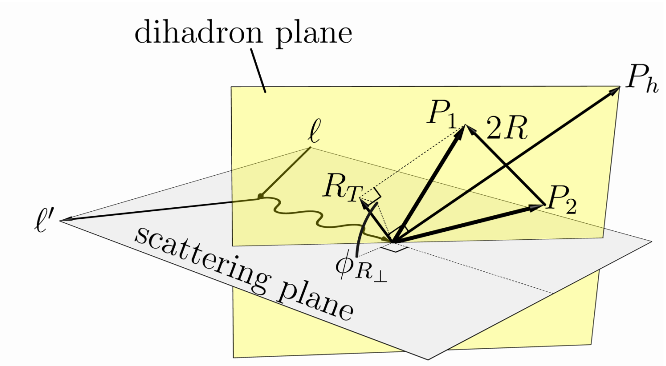
cf. single-hadron ϕ_h

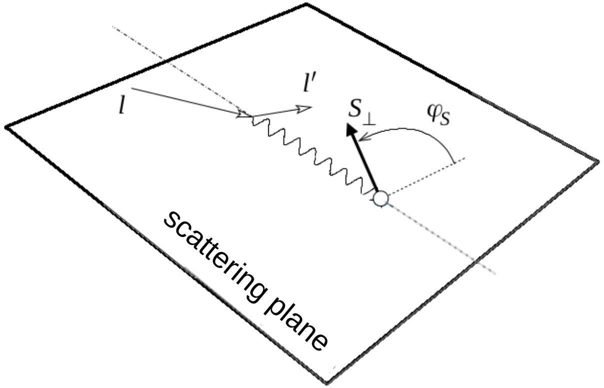


Inclusive:

$$x_B = \frac{Q^2}{2\,P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}$$

$$\gamma = \frac{2Mx_B}{Q}$$





Online 3D View:

https://c-dilks.github.io/dihadronAngleDefs/dihadronAngleDefs.html

SIDIS Dihadron Cross Section

UU

$$d\sigma_{UU} = \frac{\alpha^2}{4\pi xyQ^2}\left(1+\frac{\gamma^2}{2x}\right) \times \sum_{\ell=0}^{\ell_{\max}} \left\{ A(x,y)\sum_{m=0}^{\ell}\left[P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))\left(F_{UU,T}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}+\epsilon F_{UU,L}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}\right)\right] + B(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\cos((2-m)\phi_h+m\phi_{R_\perp})F_{UU}^{P_{\ell,m}\cos((2-m)\phi_h+m\phi_{R_\perp})} + V(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp})F_{UU}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp})}\right\}.$$

LU

$$d\sigma_{LU} = \frac{\alpha^2}{4\pi xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\lambda_e \times \sum_{\ell=0}^{\ell_{\max}} \left\{ C(x,y)\sum_{m=1}^{\ell}\left[P_{\ell,m}\sin(m(\phi_h-\phi_{R_\perp}))\,2\left(F_{LU,T}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}+\epsilon F_{LU,L}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}\right)\right] + W(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})F_{LU}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})}\right\}.$$

UL

$$d\sigma_{UL} = \frac{\alpha^2}{4\pi xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)S_L \times \left\{ A(x,y)\sum_{\ell=1}^{\ell_{\max}}\sum_{m=1}^{\ell}P_{\ell,m}\sin(-m\phi_h+m\phi_{R_\perp})F_{UL}^{P_{\ell,m}\sin(-m\phi_h+m\phi_{R_\perp})} + B(x,y)\sum_{\ell=0}^{\ell_{\max}}\sum_{m=-\ell}^{\ell}P_{\ell,m}\sin((2-m)\phi_h+m\phi_{R_\perp})F_{UL}^{P_{\ell,m}\sin((2-m)\phi_h+m\phi_{R_\perp})} + V(x,y)\sum_{\ell=0}^{\ell_{\max}}\sum_{m=-\ell}^{\ell}P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})F_{UL}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})}\right\}.$$

LL

$$d\sigma_{LL} = \frac{\alpha^2}{4\pi xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\lambda_e S_L \times \sum_{\ell=0}^{\ell_{\max}} \left\{ C(x,y)\sum_{m=0}^{\ell}2^{2-\delta_{m0}}\,P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))F_{LL}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))} + W(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp})F_{LL}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp})}\right\}.$$

UT

$$d\sigma_{UT} = \frac{\alpha^2}{4\pi xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)|\boldsymbol{S}_\perp| \times \sum_{\ell=0}^{\ell_{\max}}\sum_{m=-\ell}^{\ell}\left\{ A(x,y)\Big[P_{\ell,m}\sin((m+1)\phi_h-m\phi_{R_\perp}-\phi_S)) \times\left(F_{UT,T}^{P_{\ell,m}\sin((m+1)\phi_h-m\phi_{R_\perp}-\phi_S)}+\epsilon F_{UT,L}^{P_{\ell,m}\sin((m+1)\phi_h-m\phi_{R_\perp}-\phi_S)}\right)\Big] + B(x,y)\Big[P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp}+\phi_S)F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp}+\phi_S)} + P_{\ell,m}\sin((3-m)\phi_h+m\phi_{R_\perp}-\phi_S)F_{UT}^{P_{\ell,m}\sin((3-m)\phi_h+m\phi_{R_\perp}-\phi_S)}\Big] + V(x,y)\Big[P_{\ell,m}\sin(-m\phi_h+m\phi_{R_\perp}+\phi_S)F_{UT}^{P_{\ell,m}\sin(-m\phi_h+m\phi_{R_\perp}+\phi_S)} + P_{\ell,m}\sin((2-m)\phi_h+m\phi_{R_\perp}-\phi_S)F_{UT}^{P_{\ell,m}\sin((2-m)\phi_h+m\phi_{R_\perp}-\phi_S)}\Big]\right\}.$$

LT

$$d\sigma_{LT} = \frac{\alpha^2}{4\pi xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\lambda_e|\boldsymbol{S}_\perp|\sum_{\ell=0}^{\ell_{\max}}\sum_{m=-\ell}^{\ell}\left\{ C(x,y)\,2\,P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp}-\phi_S))F_{LT}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp}-\phi_S))} + W(x,y)\Big[P_{\ell,m}\cos(-m\phi_h+m\phi_{R_\perp}+\phi_S)F_{LT}^{P_{\ell,m}\cos(-m\phi_h+m\phi_{R_\perp}+\phi_S)} + P_{\ell,m}\cos((2-m)\phi_h+m\phi_{R_\perp}-\phi_S)F_{LT}^{P_{\ell,m}\cos((2-m)\phi_h+m\phi_{R_\perp}-\phi_S)}\Big]\right\}.$$

Phys.Rev.D 90 (2014) 11, 114027

SIDIS Dihadron Cross Section

General form of each term:

$$d\sigma_{XY} \propto D(x, y, Q^2) \cdot S(\phi_h, \phi_R, \phi_S, \theta) \cdot F_{XY}^{S(\phi, \dots)} + \dots$$

Depolarization **Sinusoidal modulation** **Structure Function**

- Several of these terms per polarization configuration ‘XY’
  - X = electron polarization, Y = proton polarization $X, Y \in \{U, L, T\}$
- Separate terms at twist-2 and twist-3
  - Twist-3 asymmetries ~1/Q

SIDIS Dihadron Cross Section

General form of each term:

$$d\sigma_{XY} \propto D(x,y,Q^2) \cdot S(\phi_h,\phi_R,\phi_S,\theta) \cdot F_{XY}^{S(\phi,\dots)} + \dots$$

Depolarization

Sinusoidal modulation

Structure Function

Ratio of longitudinal and transverse photon flux

$$\epsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

Depolarization factors at twist-2

$$\boxed{A(\epsilon,y)} = \frac{y^2}{2(1-\epsilon)} \qquad \boxed{B(\epsilon,y)} = \frac{y^2}{2(1-\epsilon)}\epsilon$$

$$\boxed{C(\epsilon,y)} = \frac{y^2}{2(1-\epsilon)}\sqrt{1-\epsilon^2}$$

at twist-3

$$\boxed{V(\epsilon,y)} = \frac{y^2}{2(1-\epsilon)}\sqrt{2\epsilon(1+\epsilon)}$$

$$\boxed{W(\epsilon,y)} = \frac{y^2}{2(1-\epsilon)}\sqrt{2\epsilon(1-\epsilon)}$$

SIDIS Dihadron Cross Section

General form of each term:

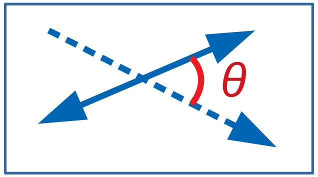
$$d\sigma_{XY} \propto D(x,y,Q^2) \cdot \boxed{S(\phi_h,\phi_R,\phi_S,\theta)} \cdot F_{XY}^{S(\phi,\dots)} + \dots$$

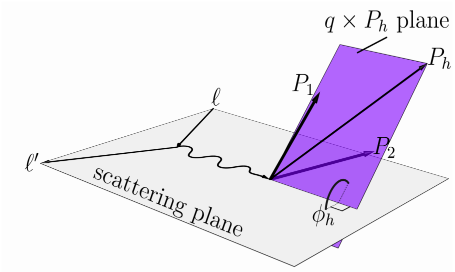
Depolarization

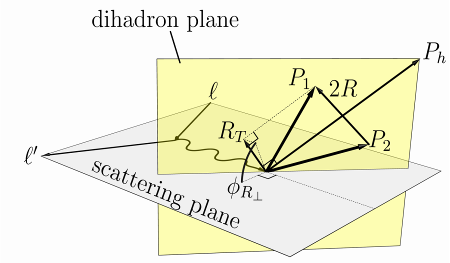
Sinusoidal modulation

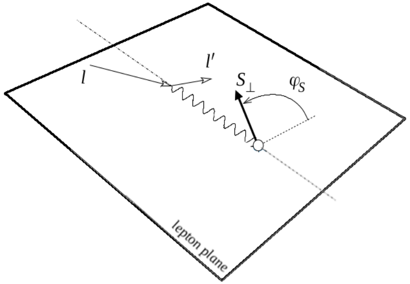
Structure Function

Legendre Polynomial x Sine (Cosine) azimuthal modulation









SIDIS Dihadron Cross Section

General form of each term:

$$d\sigma_{XY} \propto D(x,y,Q^2)\cdot S(\phi_h,\phi_R,\phi_S,\theta)\cdot F_{XY}^{S(\phi,\dots)} + \dots$$

Depolarization

Sinusoidal modulation

Structure Function

Structure Function

$$F_{XY} = \mathcal{I}\left[w(\mathbf{k}_T,\mathbf{p}_T,x,z,M_h,\dots)\cdot f(x,k_T)\cdot D(z,M_h,pT)+\dots\right]$$

$$\mathcal{I}\left[wfD\right]$$

quark transverse
momentum <u>convolution</u>

kinematic factor
(usually quark momentum
vector products)

TMD
PDF

DiFF

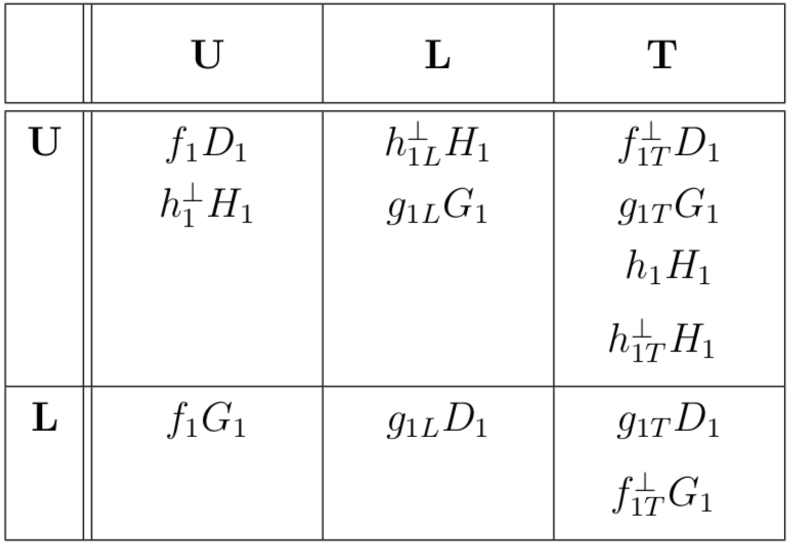
Dihadron
Fragmentation Function

Dihadron Access to PDFs x DiFFs

Twist 2

Nucleon Polarization

Electron Polarization

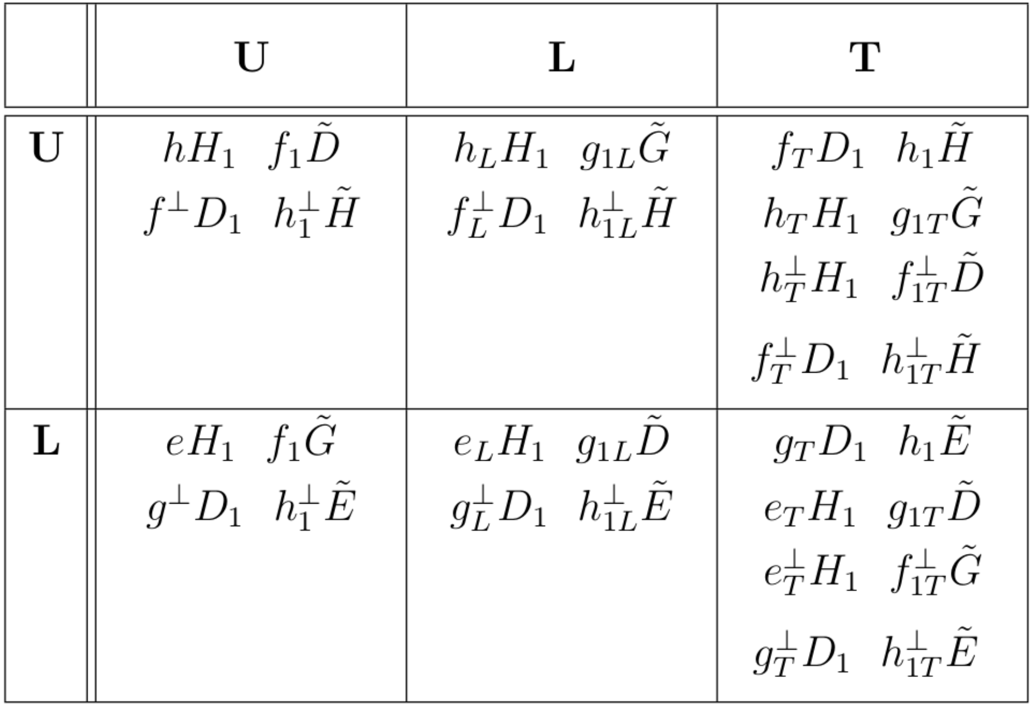


	U	**L**	**T**
U	$f_1 D_1$ $h_1^\perp H_1$	$h_{1L}^\perp H_1$ $g_{1L} G_1$	$f_{1T}^\perp D_1$ $g_{1T} G_1$ $h_1 H_1$ $h_{1T}^\perp H_1$
L	$f_1 G_1$	$g_{1L} D_1$	$g_{1T} D_1$ $f_{1T}^\perp G_1$

Twist 3

Nucleon Polarization

Electron Polarization

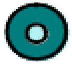
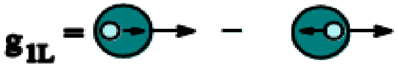
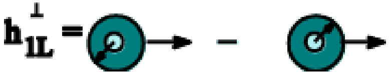
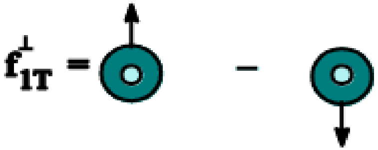
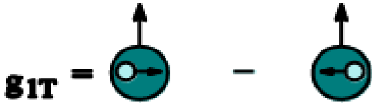
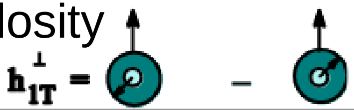


	U	**L**	**T**
U	$h H_1 \quad f_1 \tilde{D}$ $f^\perp D_1 \quad h_1^\perp \tilde{H}$	$h_L H_1 \quad g_{1L} \tilde{G}$ $f_L^\perp D_1 \quad h_{1L}^\perp \tilde{H}$	$f_T D_1 \quad h_1 \tilde{H}$ $h_T H_1 \quad g_{1T} \tilde{G}$ $h_T^\perp H_1 \quad f_{1T}^\perp \tilde{D}$ $f_T^\perp D_1 \quad h_{1T}^\perp \tilde{H}$
L	$e H_1 \quad f_1 \tilde{G}$ $g^\perp D_1 \quad h_1^\perp \tilde{E}$	$e_L H_1 \quad g_{1L} \tilde{D}$ $g_L^\perp D_1 \quad h_{1L}^\perp \tilde{E}$	$g_T D_1 \quad h_1 \tilde{E}$ $e_T H_1 \quad g_{1T} \tilde{D}$ $e_T^\perp H_1 \quad f_{1T}^\perp \tilde{G}$ $g_T^\perp D_1 \quad h_{1T}^\perp \tilde{E}$

Outline

- SIDIS Dihadron Kinematics and Cross Section
- **TMD PDFs and EIC Impact**
- Dihadron Fragmentation Functions and Partial Waves
- Lambdas and TMD Fragmentation

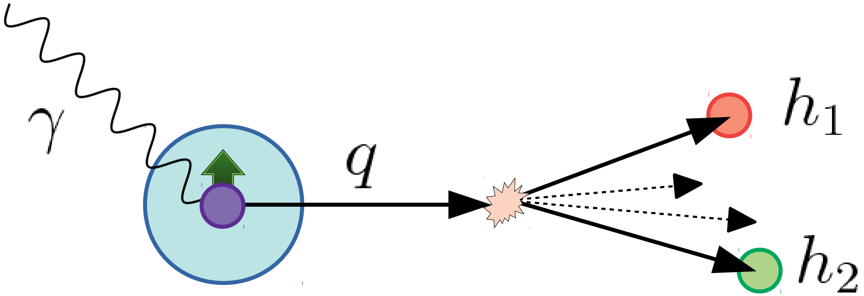
Transverse Momentum Dependent (TMD) PDFs [twist 2]

		Quark Polarization		
		U	**L**	**T**
Nucleon Polarization	**U**	Unpolarized PDF f = 		Boer-Mulders 
	L		Helicity 	Kotzinian-Mulders Wormgear 
	T	Sivers 	Kotzinian-Mulders Wormgear 	Transversity  Pretzelosity 

Dihadrons → Spin-Orbit Correlations in Hadronization

Unpolarized SIDIS:

- **Cahn Effect**: quark transverse momentum leads to azimuthal modulations of SIDIS cross section
- **Boer-Mulders Effect:** Non-collinear quarks in an unpolarized proton can have transverse polarization, also contributing azimuthal modulations



- **Boer-Mulders and Cahn effects are comparable in single hadron production**
  - HERMES and COMPASS data, e.g. Phys.Rev.D 81 (2010) 114026
- **Dihadrons can help decouple BM from Cahn**
  - Extra degree of freedom in dihadrons
    - Cahn effect impacts dihadron total momentum direction P_h
    - Utilize azimuthal angle about P_h, in addition to the azimuth about the virtual photon
- **Advantages from a broader and higher Q^2 range at an EIC**
  - Broader Q^2 range probes evolution effects
  - Higher Q^2 suppresses Cahn effect in single-hadron asymmetries (Cahn is twist-4)
  - Lower Q^2 for overlap with other SIDIS experiments

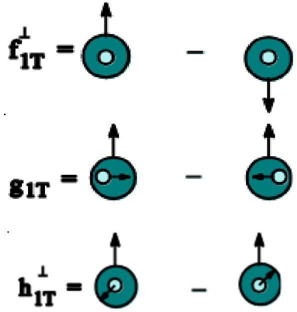
Transversely Polarized Nucleons

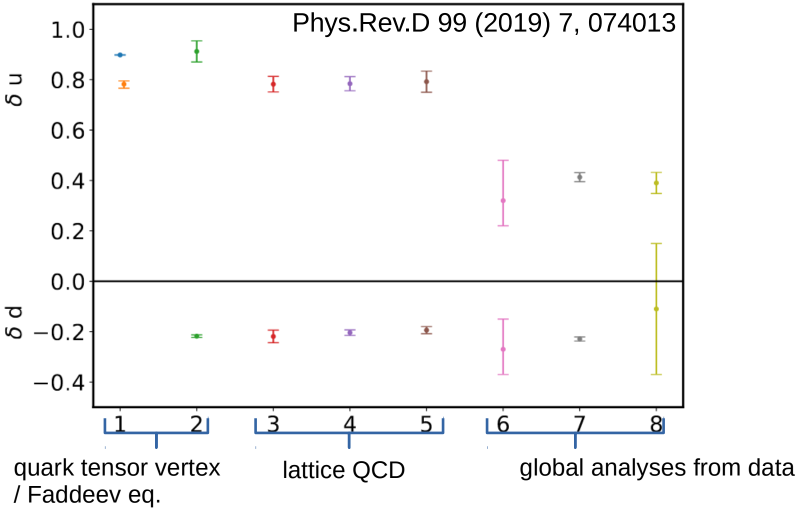
Transversely polarized SIDIS:

- Access to several additional **TMDs**:
  - **Transversity** → Tensor Charge

$$\delta q = \int_{-1}^{1} dx\, h(x) = \int_{0}^{1} dx \left[h(x) - \bar{h}(x) \right]$$

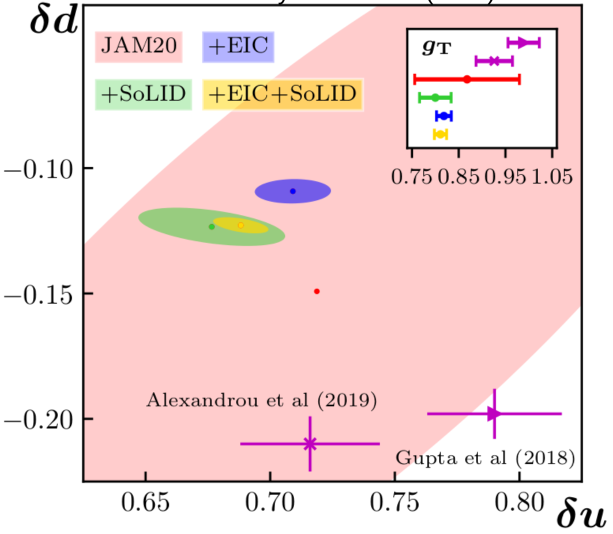
    - Quark EDM contribution to nucleon EDM
    - Comparisons with lattice QCD calculation
  - **Sivers Function**
  - **Kotzinian-Mulders (wormgear) Function**
  - **Pretzelocity**
  - **Twist-3 TMDs**





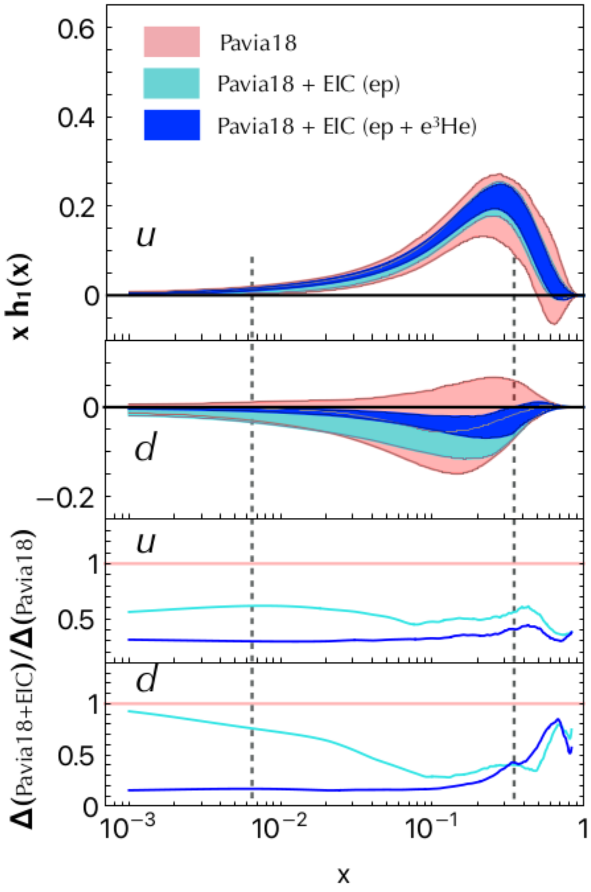
Phys.Rev.D 99 (2019) 7, 074013

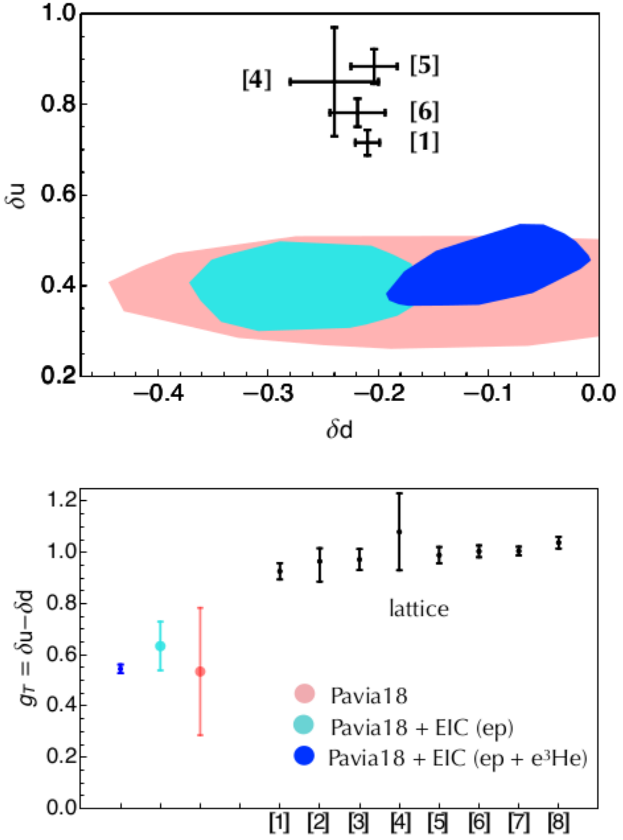
Phys.Lett.B 816 (2021) 136255



Dihadron Impact on Transversity

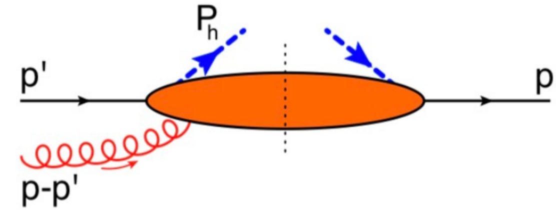
- Complementary to single-hadron SIDIS and hadrons in jets
- Complementarity reduces systematic uncertainties overall
- Additional advantages from dihadrons:
  - Expect little contribution from twist-3 FFs
  - Acceptance effects tend to “average out” between the two hadrons, which is especially good for F_{UU} measurements (Boer-Mulders function)





EIC Yellow Report

Collinear Twist-3 PDFs

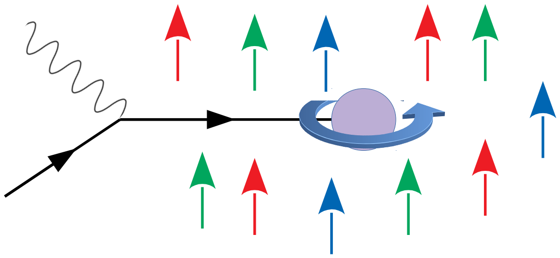


Semi-classical interpretation via x-moments

e(x)

- Pion-nucleon σ term: $m_q \to m_N$
- “Boer-Mulders Force”: Transverse force exerted by color field on q↑ after scattering, in an unpolarized nucleon

Phys.Rev.D 88 (2013) 114502



g_T(x)

- “Average transverse force that acts on an unpolarized quark in a transversely polarized nucleon”

Phys.Rev.D 94 (2016) 9, 094040

h_L(x)

- “Average longitudinal gradient of the transverse force that acts on transversely polarized [struck] quarks in longitudinally polarized nucleons”

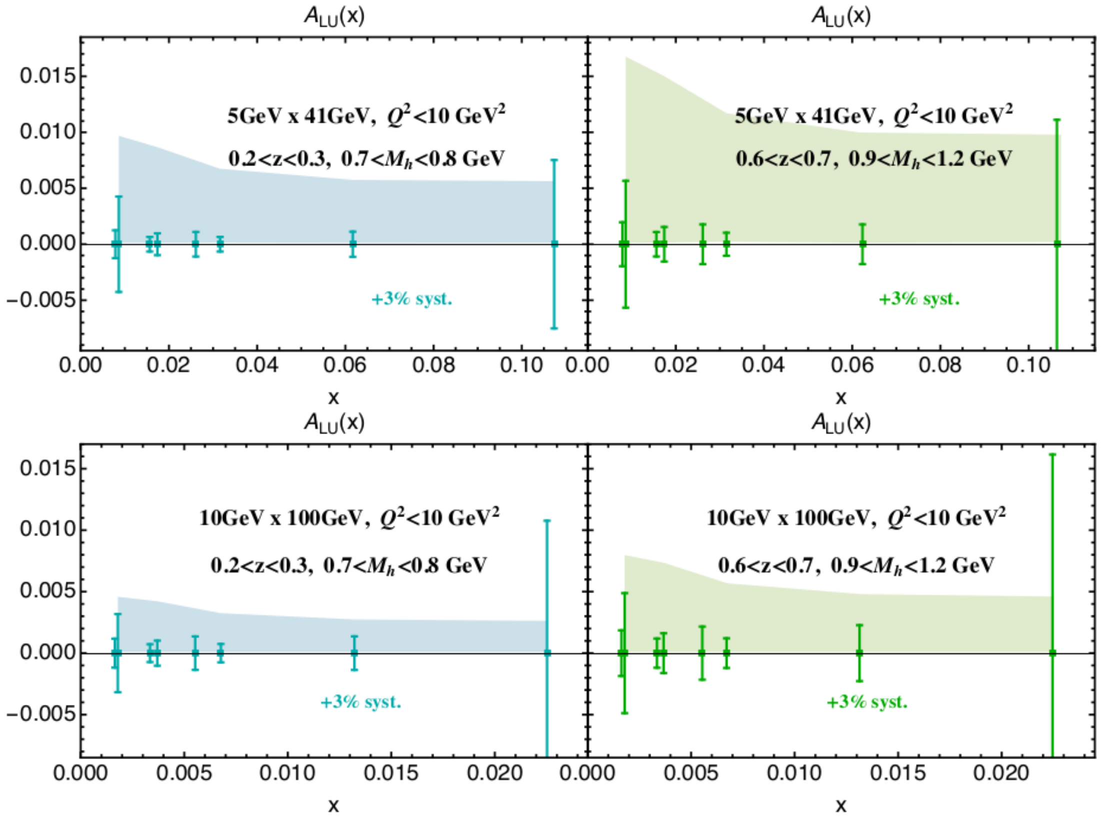
$$\mathcal{L}^q_{\rm JM} - L^q_{\rm Ji} = \Delta L^q_{\rm FSI}$$

Expressible in terms of the change in quark OAM as it leaves the target

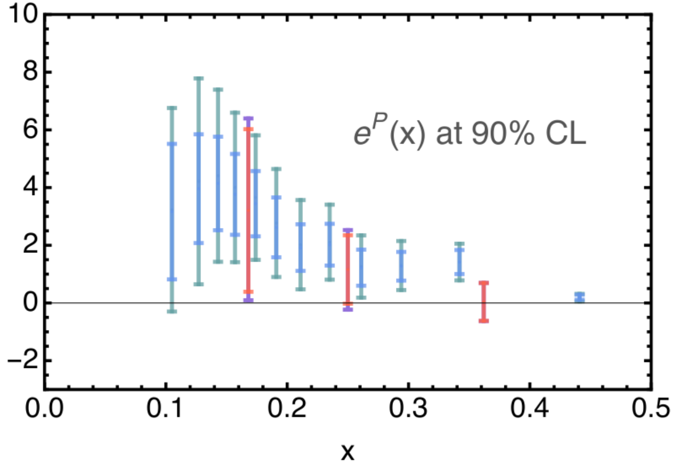
- Phys.Rev.D 94 (2016) 9, 094040
- Phys.Rev.D 66 (2002) 114005
- Nucl.Phys.B 461 (1996) 197-237

EIC Impact on Collinear Twist-3 PDFs: e(x)

- e(x) is accessible in beam spin asymmetry A_{LU}
- Cleaner access in dihadrons, compared to single-hadron SIDIS (which involves additional unknowns)
- Caveat: depolarization for A_{LU} favors high y



EIC Yellow Report

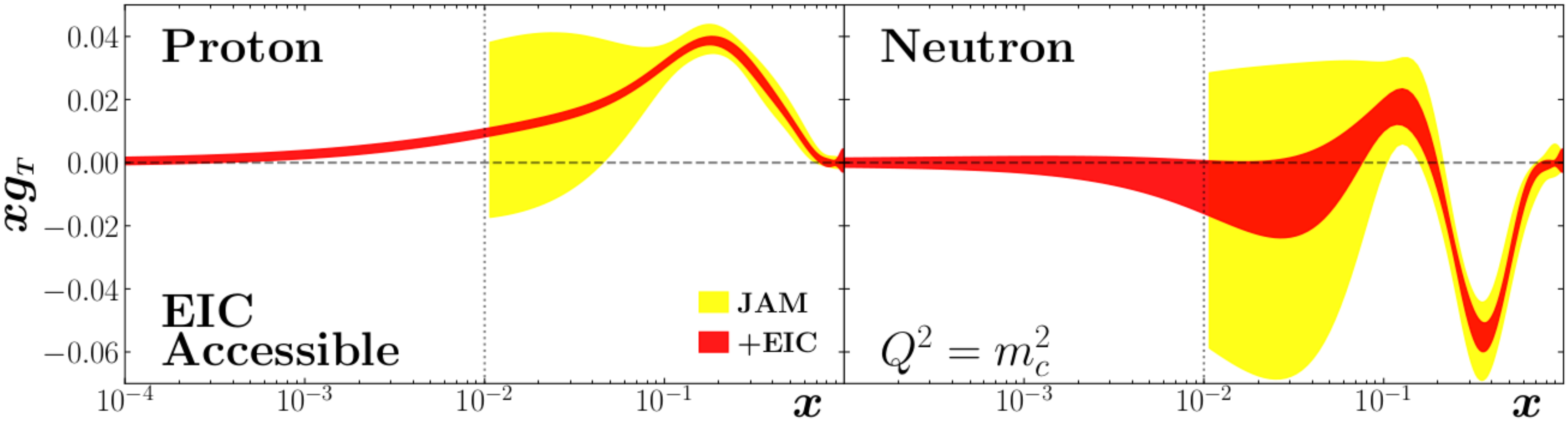


Extraction from CLAS dihadrons

Courtoy, Aurore, et al. e-Print: 2203.14975 [hep-ph]

Courtoy, Aurore – CPHI 2022

EIC Impact on Collinear Twist-3 PDFs: $g_T(x)$



EIC Yellow Report

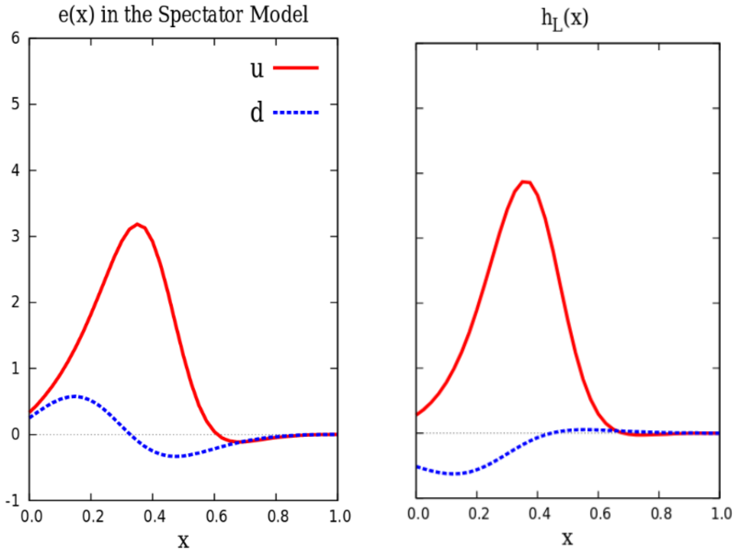
- Accessible in inclusive measurements: $\vec{e}p^{\uparrow} \rightarrow e'X$
- $g_T(x)$ is also accessible in double spin asymmetry A_{LT} in semi-inclusive dihadrons
  - Caveat: depolarization for A_{LT} favors high y…

EIC Impact on Collinear Twist-3 PDFs: $h_L(x)$

Any impact studies for the EIC?

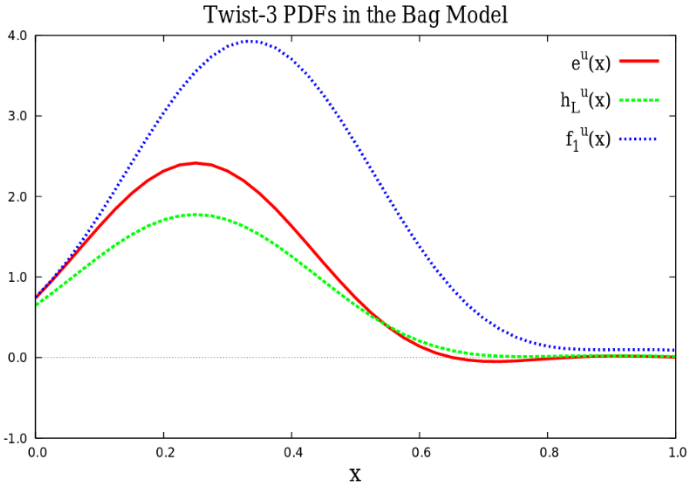
- Accessible in target spin asymmetry A_{UL}
  - Depolarization for allows broad coverage
  - Ongoing experiment at CLAS (RG-C)

Spectator Model



Jakob, Mulders, and Rodrigues, Nucl.Phys. A626 (1997) 937-965

Bag Model



Figures from JLab Proposal E12-06-112B/E12-09-008B

Jaffe and Ji, Nucl.Phys. B375 (1992) 527-560

See also:

– Chiral Quark Soliton Model — Cebulla et al., Acta Phys.Polon. B39 (2008) 609-640

– Light Front Constituent Quark Model — Lorcé , Pasquini, Schweitzer, JHEP 1501 (2015) 103

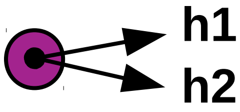
Outline

- SIDIS Dihadron Kinematics and Cross Section
- TMD PDFs and EIC Impact
- **Dihadron Fragmentation Functions and Partial Waves**
- Lambdas and TMD Fragmentation

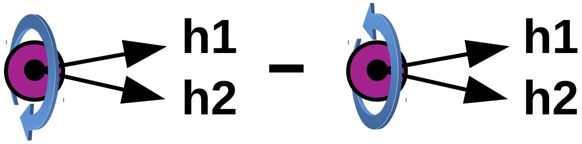
Dihadron Fragmentation Functions

Twist 2

$D_1 =$



$G_1^\perp =$



$H_1^\perp, \; H_1^\sphericalangle =$

Twist 3

$\tilde{D}^\perp$ $\tilde{G}^\perp$

$\tilde{H}$ $\tilde{E}$

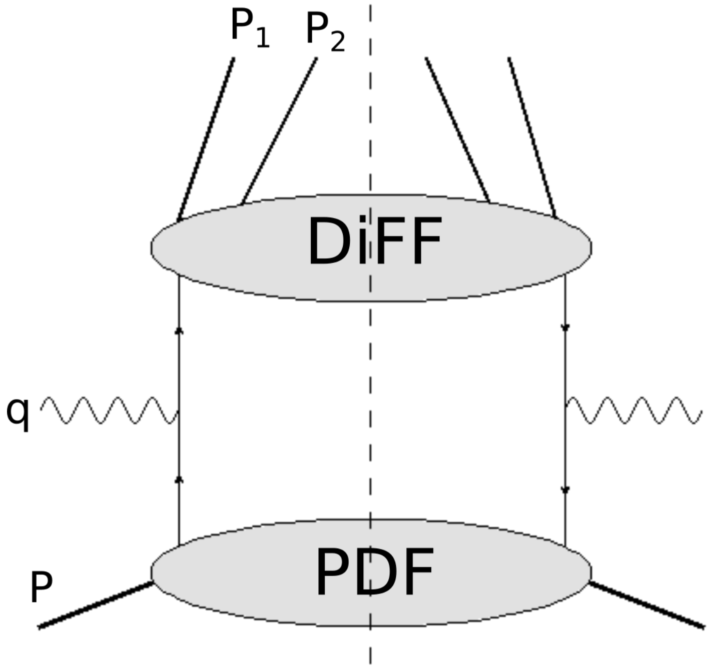
Thought to be small…
see, for example:

PoS DIS2014 (2014) 231

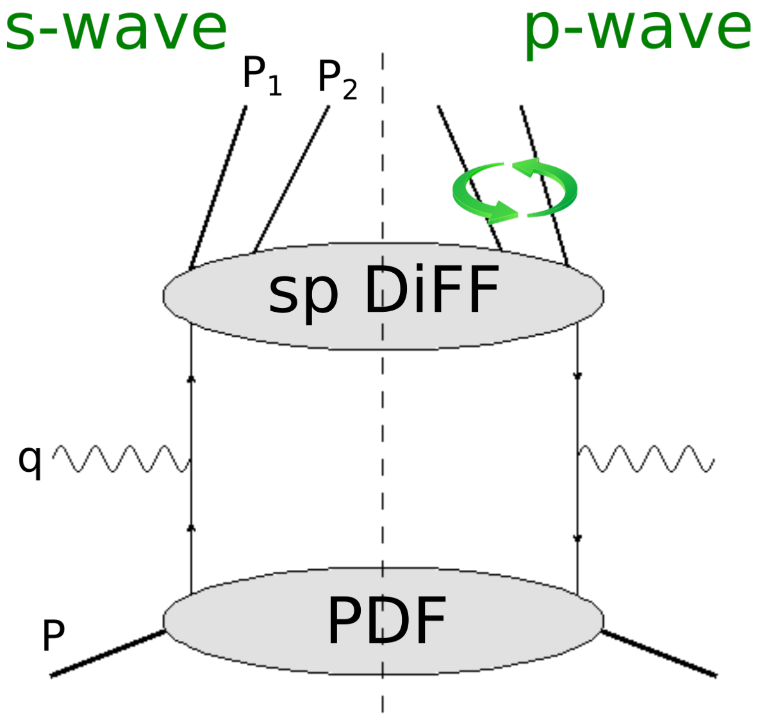
Phys.Rev.D 99 (2019) 5, 054003

arXiv: 1405.7659 [hep-ph]

Dihadron Partial Waves



Dihadron Partial Waves

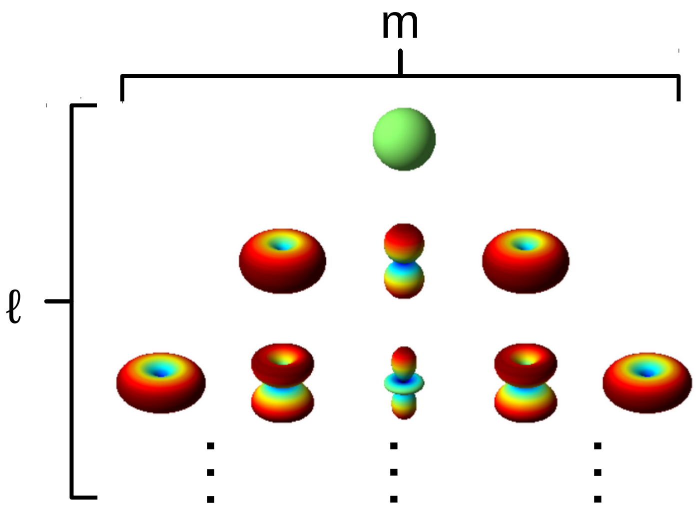


Dihadron Partial Waves

$$D_1 = \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos\vartheta)\ \cos\left(m\left(\phi_{R_\perp}-\phi_p\right)\right)\ D_1^{|\ell,m\rangle}(z, M_h, |\boldsymbol{p}_T|),$$

$$G_1 = \sum_{\ell=1}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos\vartheta)\ \sin\left(m\left(\phi_{R_\perp}-\phi_p\right)\right)\ G_1^{|\ell,m\rangle}(z, M_h, |\boldsymbol{p}_T|),$$

$$H_1^{\perp} = \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos\vartheta)\, e^{im\left(\phi_{R_\perp}-\phi_p\right)}\ H_1^{\perp|\ell,m\rangle}(z, M_h, |\boldsymbol{p}_T|)\ ,$$



- Expand DiFFs into spherical harmonics (Legendre Polynomials ‘$P_{l,m}$’)
- Angular Momentum (AM) eigenvalues |ℓ,m>
- Correlations of dihadron AM with fragmenting quark AM

Dihadron Partial Waves

ℓ = 0

ss

$|0,0\rangle$

U U

m = 0

Dihadron Partial Waves

ℓ = 0 *ss*		$\lvert 0,0 \rangle$ **U U**	
ℓ = 1 *sp*	$\lvert 1,-1 \rangle$ **U T**	$\lvert 1,0 \rangle$ **U L**	$\lvert 1,1 \rangle$ **U T**
	m = -1		*m = +1*

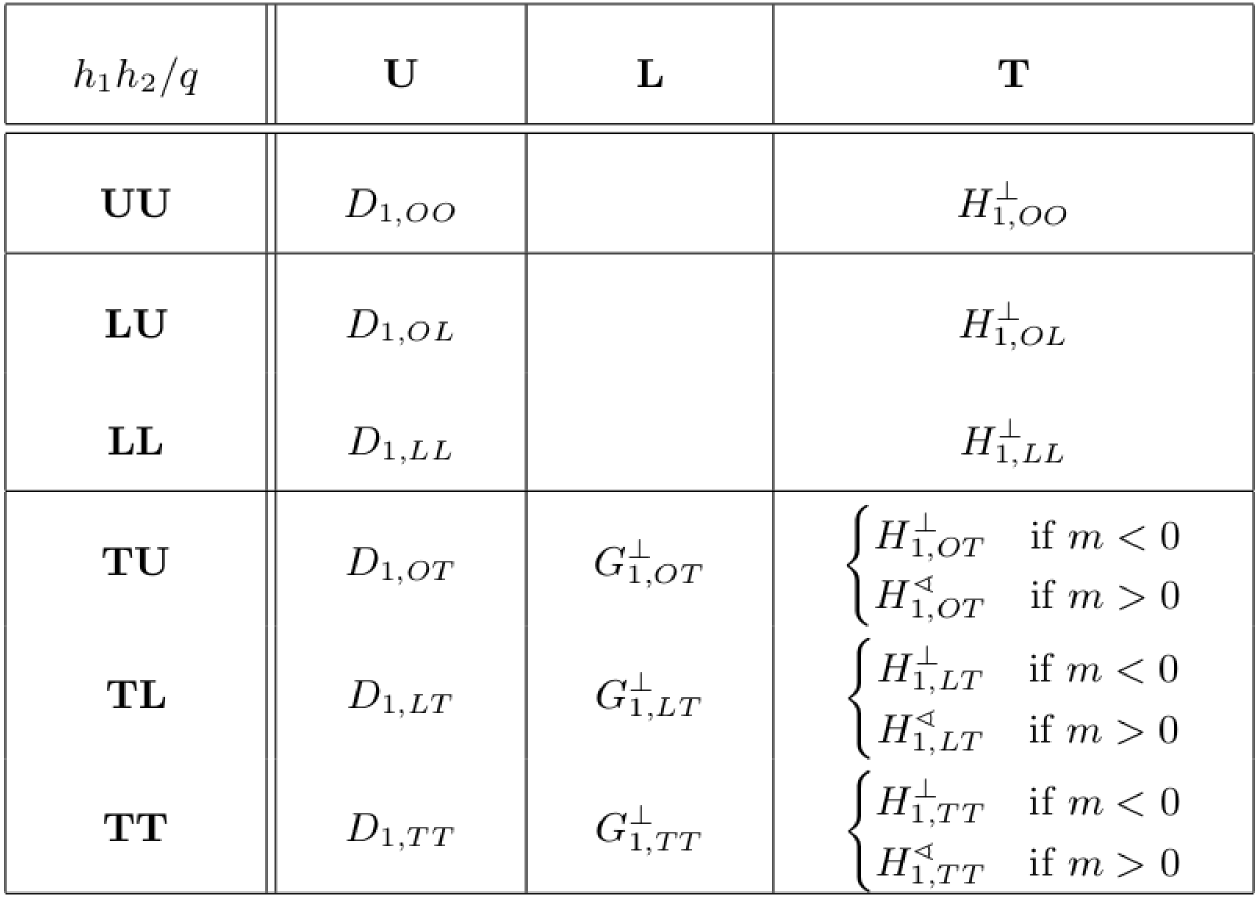
Dihadron Partial Waves

ℓ = 0 *ss*			$\lvert 0,0\rangle$ **U U**		
ℓ = 1 *sp*		$\lvert 1,-1\rangle$ **U T**	$\lvert 1,0\rangle$ **U L**	$\lvert 1,1\rangle$ **U T**	
	$\lvert 2,-2\rangle$ **T T**	$\lvert 2,-1\rangle$ **L T**	$\lvert 2,0\rangle$ **L L**	$\lvert 2,1\rangle$ **L T**	$\lvert 2,2\rangle$ **T T**
	m = -2				*m = +2*

Dihadron Partial Waves

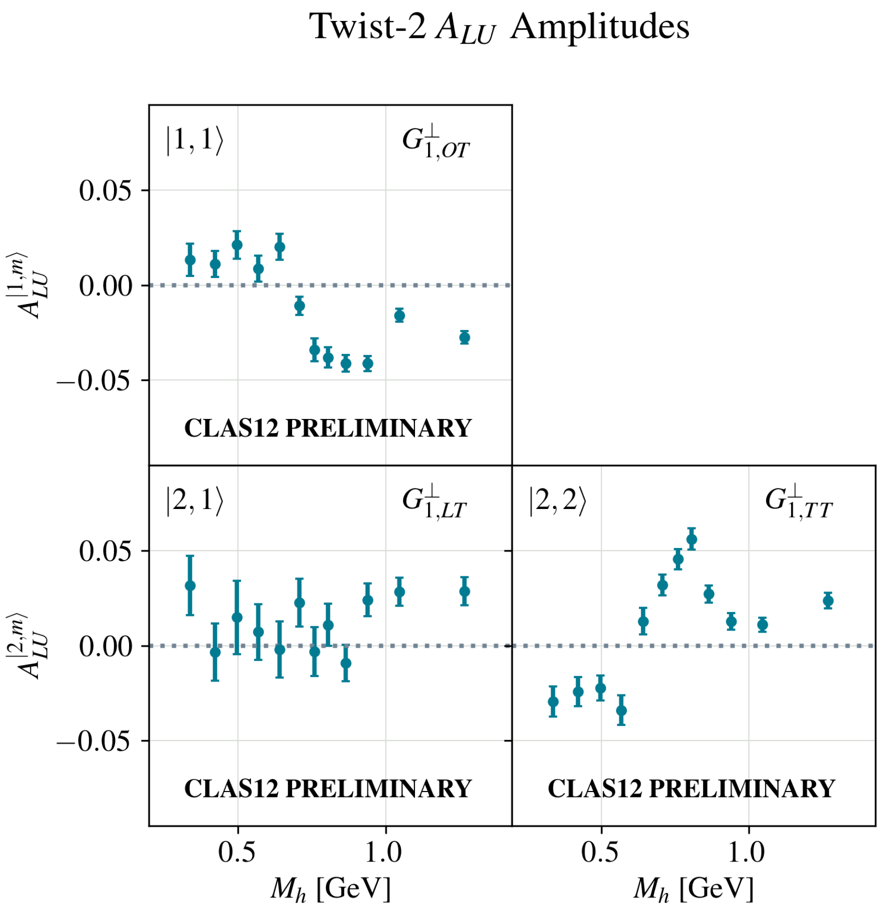
ℓ = 0 ***ss***			$\vert 0,0\rangle$ **U U** $H_{1,OO}^{\perp}$		
ℓ = 1 ***sp***		$\vert 1,-1\rangle$ **U T** $H_{1,OT}^{\perp}$	$\vert 1,0\rangle$ **U L** $H_{1,OL}^{\perp}$	$\vert 1,1\rangle$ **U T** $H_{1,OT}^{\sphericalangle}$	
	$\vert 2,-2\rangle$ **T T** $H_{1,TT}^{\perp}$	$\vert 2,-1\rangle$ **L T** $H_{1,LT}^{\perp}$	$\vert 2,0\rangle$ **L L** $H_{1,LL}^{\perp}$	$\vert 2,1\rangle$ **L T** $H_{1,LT}^{\sphericalangle}$	$\vert 2,2\rangle$ **T T** $H_{1,TT}^{\sphericalangle}$
	m = -2				***m = +2***

Dihadron Partial Waves



h_1h_2/q	**U**	**L**	**T**
UU	$D_{1,OO}$		$H^{\perp}_{1,OO}$
LU	$D_{1,OL}$		$H^{\perp}_{1,OL}$
LL	$D_{1,LL}$		$H^{\perp}_{1,LL}$
TU	$D_{1,OT}$	$G^{\perp}_{1,OT}$	$\begin{cases} H^{\perp}_{1,OT} & \text{if } m < 0 \\ H^{\sphericalangle}_{1,OT} & \text{if } m > 0 \end{cases}$
TL	$D_{1,LT}$	$G^{\perp}_{1,LT}$	$\begin{cases} H^{\perp}_{1,LT} & \text{if } m < 0 \\ H^{\sphericalangle}_{1,LT} & \text{if } m > 0 \end{cases}$
TT	$D_{1,TT}$	$G^{\perp}_{1,TT}$	$\begin{cases} H^{\perp}_{1,TT} & \text{if } m < 0 \\ H^{\sphericalangle}_{1,TT} & \text{if } m > 0 \end{cases}$

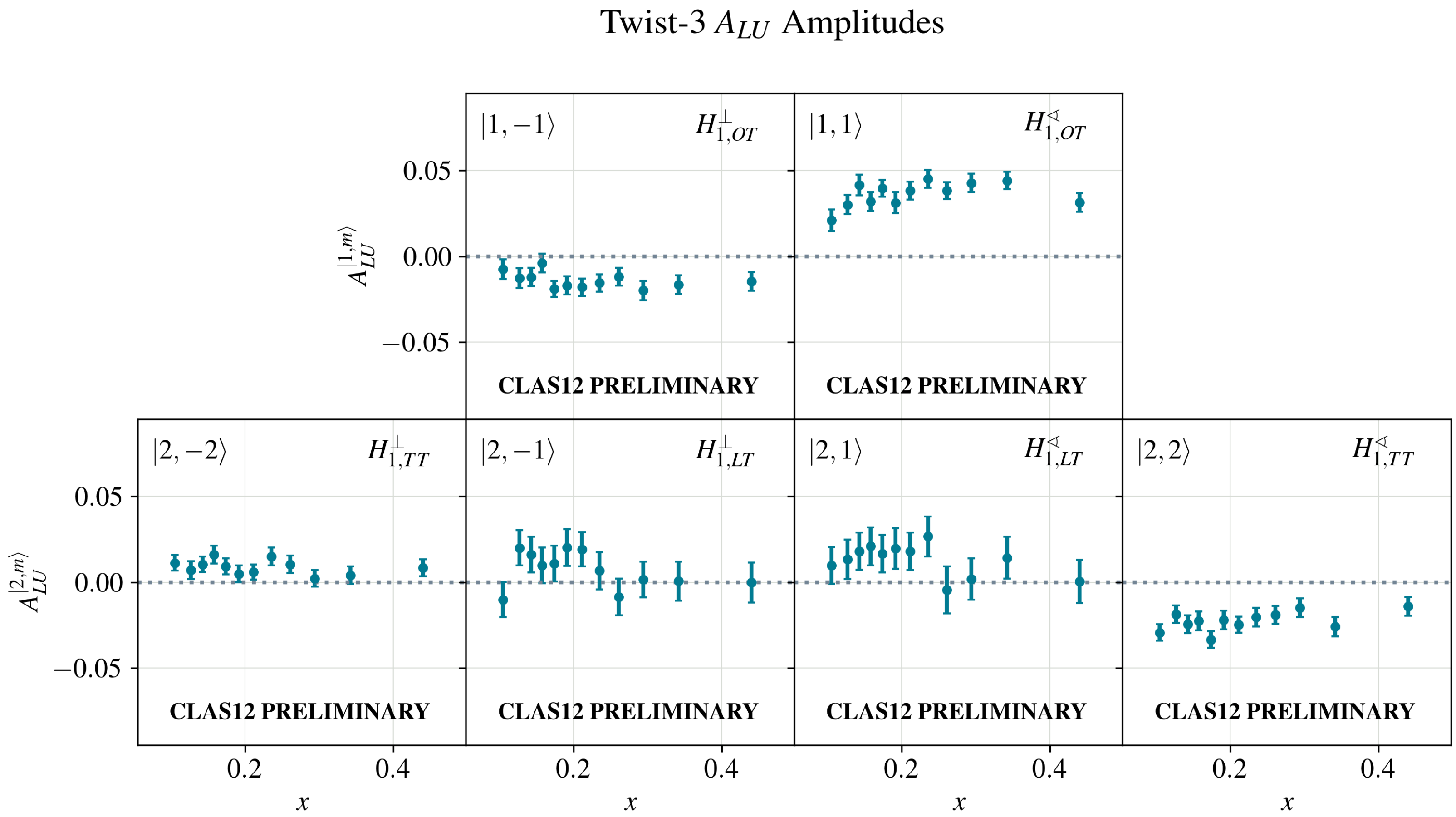
Dihadron Partial Waves at CLAS



CD 2107.12965 [hep-ex]

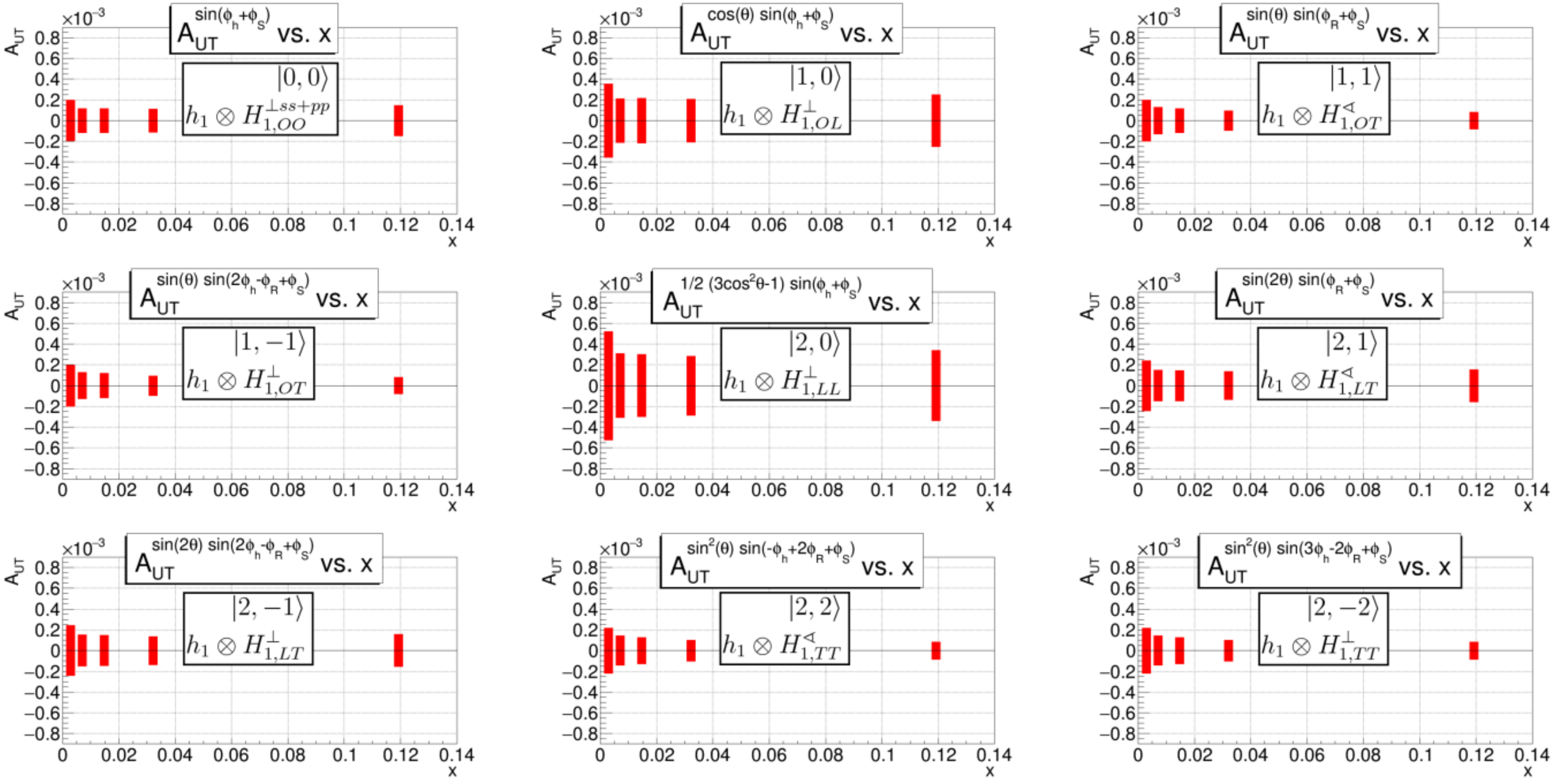
- Sign change near ρ mass in $G_{1,OT}$
- Enhancement at ρ mass in $G_{1,TT}$

Dihadron Partial Waves at CLAS



CD 2107.12965 [hep-ex]

Partial Wave Projections



- Significant impact on DiFF partial waves
- Relative differences in uncertainties comes partial wave correlations and phase space limitations

$ep^{\uparrow} \rightarrow e\pi^{+}\pi^{-}X$	5×41 GeV	$10\ \mathrm{fb}^{-1}$

EIC Yellow Report

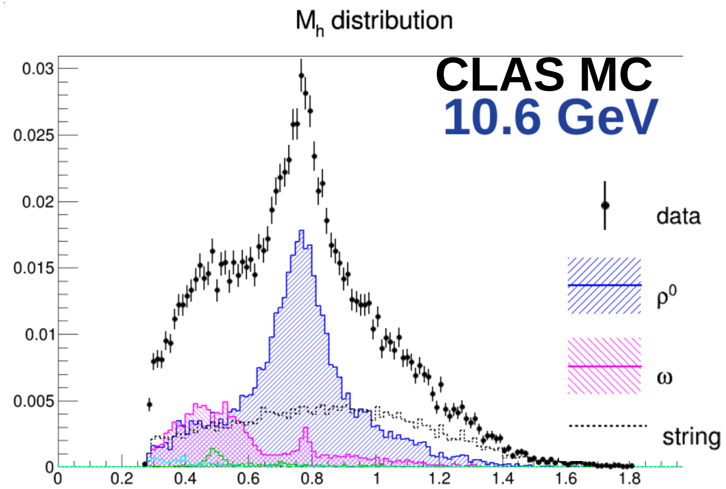
Even more from Dihadrons...

Vector Mesons: a significant fraction of dihadrons

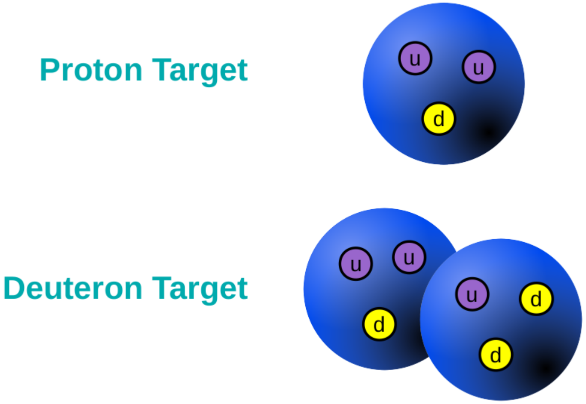
$$\rho \rightarrow \pi\pi$$

$$K^* \rightarrow \pi K$$

$$\phi \rightarrow KK$$



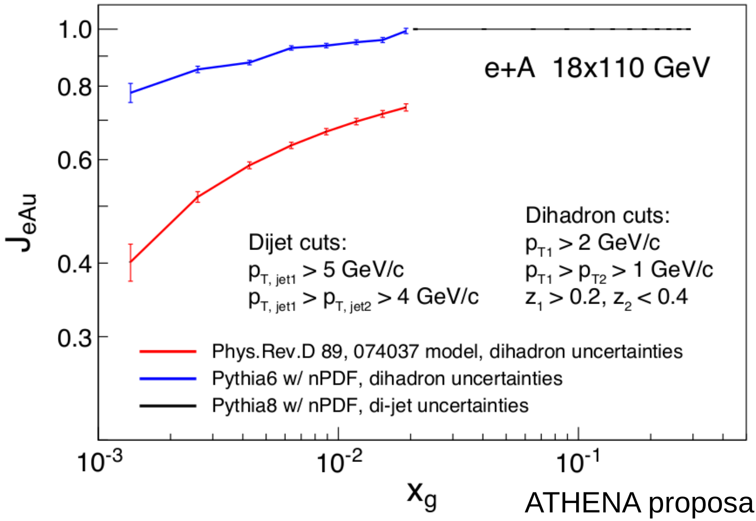
Flavor-dependence of twist-3 PDFs



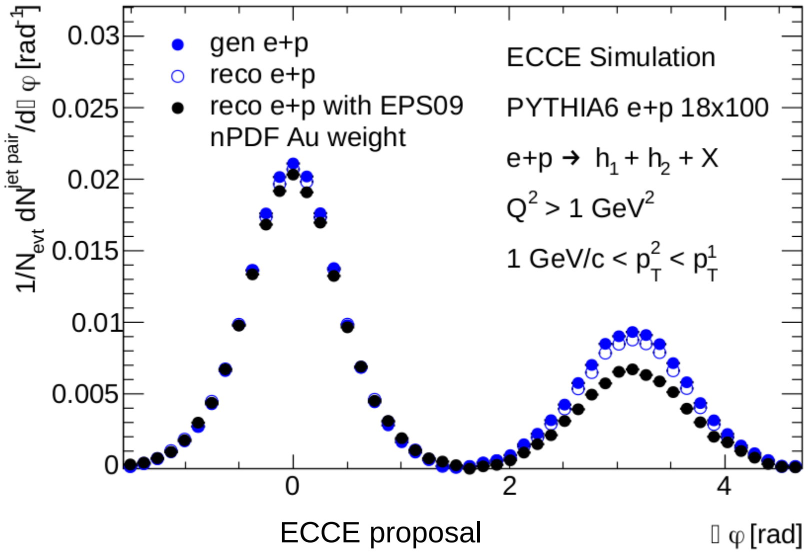
Channel dependence of DiFFs

$$\begin{matrix} D_1^{q/\pi^+\pi^-} & & D_1^{q/\pi^\pm\pi^0} \\ G_1^{q/\pi^+\pi^-} & \neq & G_1^{q/\pi^\pm\pi^0} \\ H_1^{q/\pi^+\pi^-} & & H_1^{q/\pi^\pm\pi^0} \end{matrix}$$

Dihadrons for Gluon Saturation



ATHENA proposal



ECCE proposal

- Away-side peak in Δϕ de-correlates when non-linear QCD effects set in
- Sensitive to gluon TMDs
- Measure suppression J_{eAu}, the relative e+Au to e+p back-to-back dihadron yields
  - Scaled by $A^{1/3}$
  - J_{eAu}~1 if no collective nuclear effects

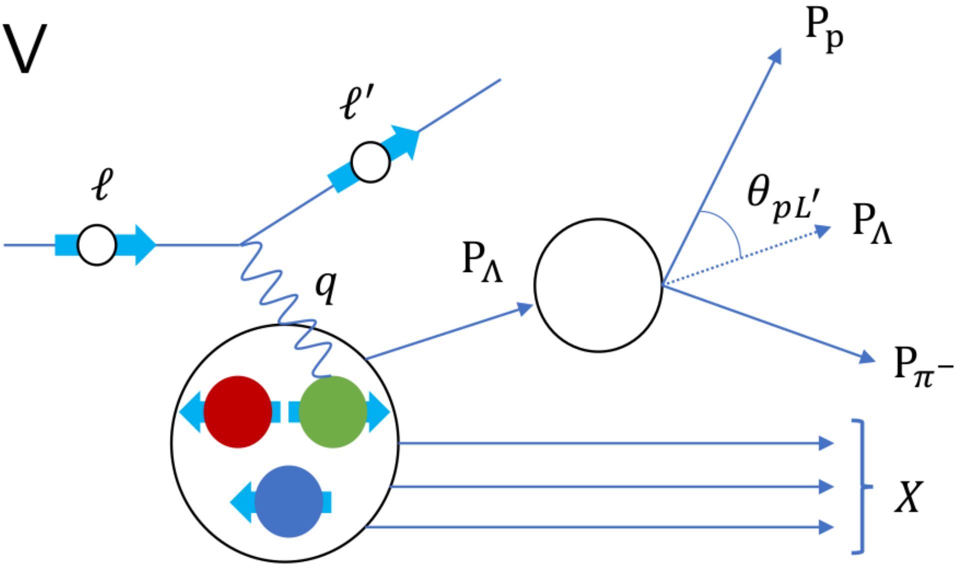
Outline

- SIDIS Dihadron Kinematics and Cross Section
- TMD PDFs and EIC Impact
- Dihadron Fragmentation Functions and Partial Waves
- **Lambdas and TMD Fragmentation**

Λ Decays → Nπ "Dihadrons"

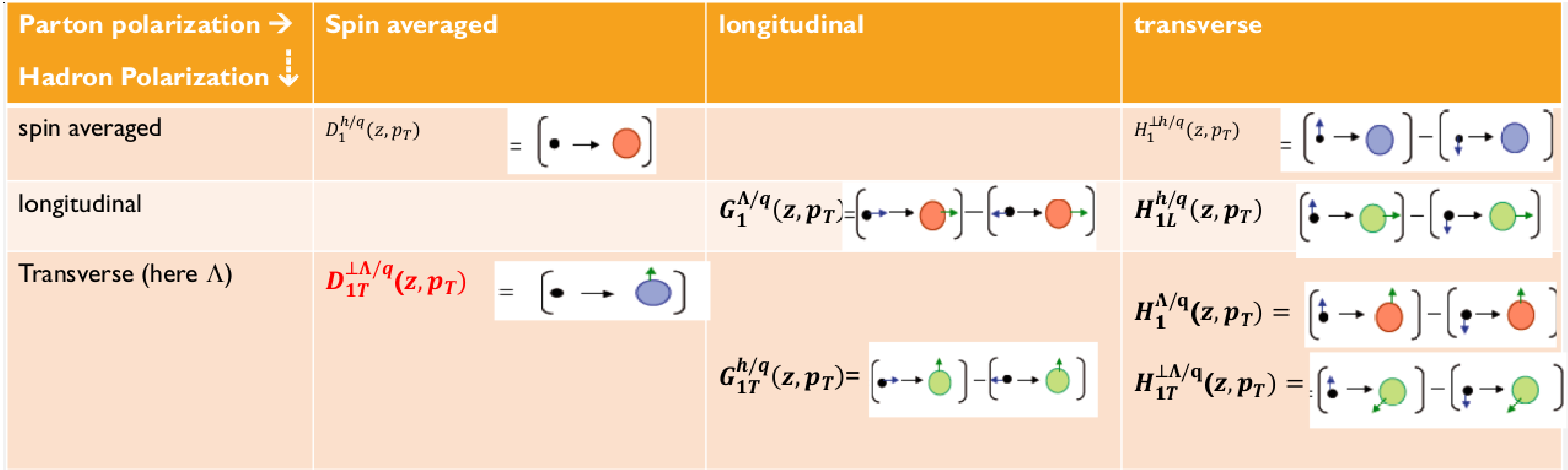
Λ DECAY MODES	Fraction (Γ_i/Γ)
$p\pi^-$	(63.9 ± 0.5) %
$n\pi^0$	(35.8 ± 0.5) %

Mass $m = 1115.683 \pm 0.006$ MeV



TMD Fragmentation Functions

- Needs knowledge of final state hadron polarization → “self-analyzing” Λ decay
- TMD and spin-dependent fragmentation → Analogous to TMD PDFs
  - TMD Polarizing Fragmentation Function (TMD PFF): $D_{1T}^{\perp\Lambda/q}$
  - Transversity TMD FF: $H_1^{\Lambda/q}$
  - Spin transfer S_Λ and spontaneous polarization P_Λ from structure function ratios



Parton polarization → Hadron Polarization ↓	Spin averaged	longitudinal	transverse
spin averaged	$D_1^{h/q}(z,p_T)$ =		$H_1^{\perp h/q}(z,p_T)$ =
longitudinal		$G_1^{\Lambda/q}(z,p_T)$=	$H_{1L}^{h/q}(z,p_T)$
Transverse (here Λ)	$D_{1T}^{\perp\Lambda/q}(z,p_T)$ =	$G_{1T}^{h/q}(z,p_T)$=	$H_1^{\Lambda/q}(z,p_T)$ = $H_{1T}^{\perp\Lambda/q}(z,p_T)$ =

Table from A. Vossen, INT-18-3

Transverse Lambdas

Spontaneous Polarization: $P_\Lambda = \dfrac{F_{UT}^{\sin(\phi_S-\phi_\Lambda)}}{F_{UU}}$

Spin Transfer: $S_\Lambda = D(y)\,\dfrac{F_{TT}^{\cos(\varphi_S-\phi_S)}}{F_{UU}}$

F_{XY}

X = proton polarization
Y = Λ polarization

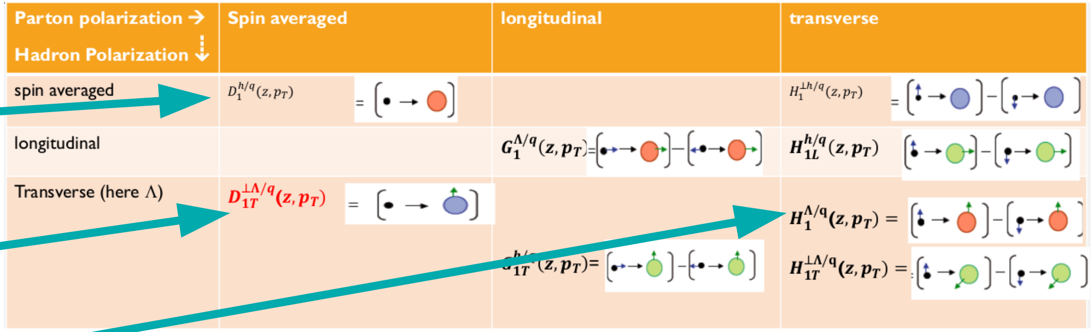
Accessible via cosθ distribution of protons in Λ → pπ

$$\frac{\mathrm{d}N_{\mathrm{p}(\bar{\mathrm{p}})}}{\mathrm{d}\cos\theta} \propto 1 + \alpha_{\Lambda(\bar{\Lambda})} P_{\Lambda(\bar{\Lambda})} \cos\theta$$

$$F_{UU} \sim f^{q/p} \otimes D_1^{\Lambda/q}$$

$$F_{UT}^{\sin(\phi_S-\phi_\Lambda)} \sim f^{q/p} \otimes D_{1T}^{\perp\Lambda/q}$$

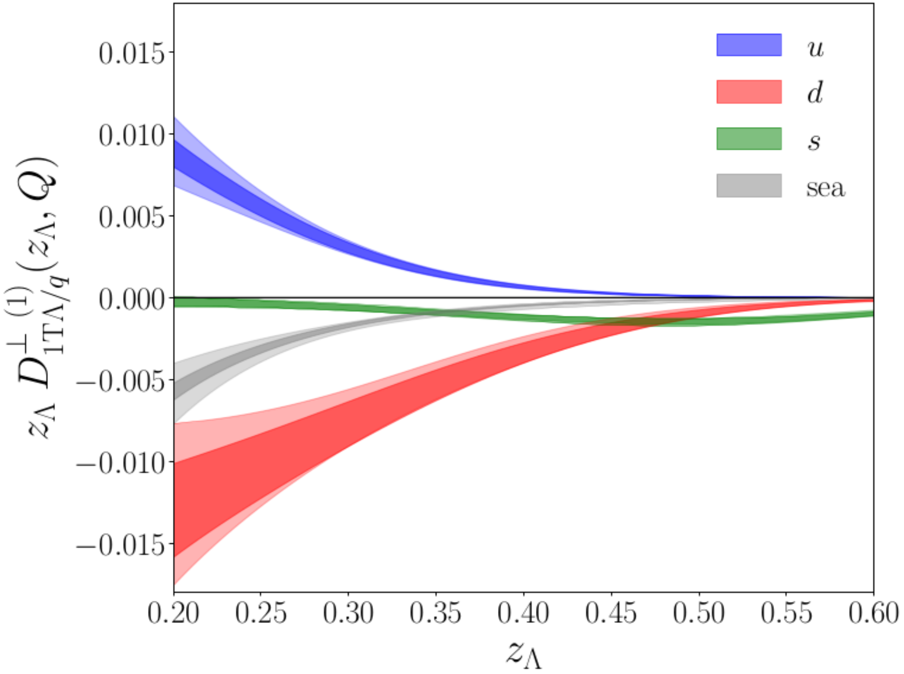
$$F_{TT}^{\cos(\varphi_S-\phi_S)} \sim h^{q/p} \otimes H_1^{\Lambda/q}$$



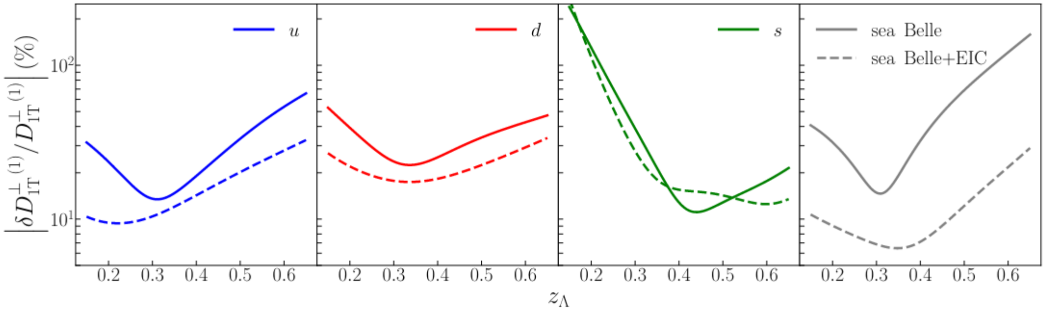
Parton polarization → Hadron Polarization ↓	Spin averaged	longitudinal	transverse
spin averaged	$D_1^{h/q}(z,p_T)$ =		$H_1^{\perp h/q}(z,p_T)$ =
longitudinal		$G_1^{\Lambda/q}(z,p_T)$=	$H_{1L}^{h/q}(z,p_T)$
Transverse (here Λ)	$D_{1T}^{\perp\Lambda/q}(z,p_T)$ =	$G_{1T}^{h/q}(z,p_T)$=	$H_1^{\Lambda/q}(z,p_T)$ = $H_{1T}^{\perp\Lambda/q}(z,p_T)$ =

Impact on TMD PFF

Extracted TMD PFF moment



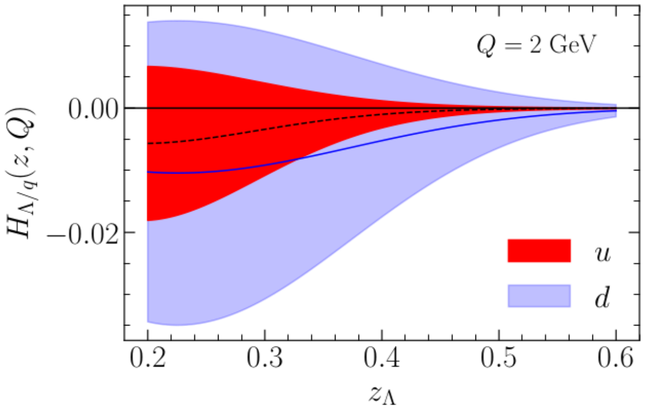
Theoretical Uncertainty Impact



Figures from
Phys.Rev.D 105 (2022) 9, 094033

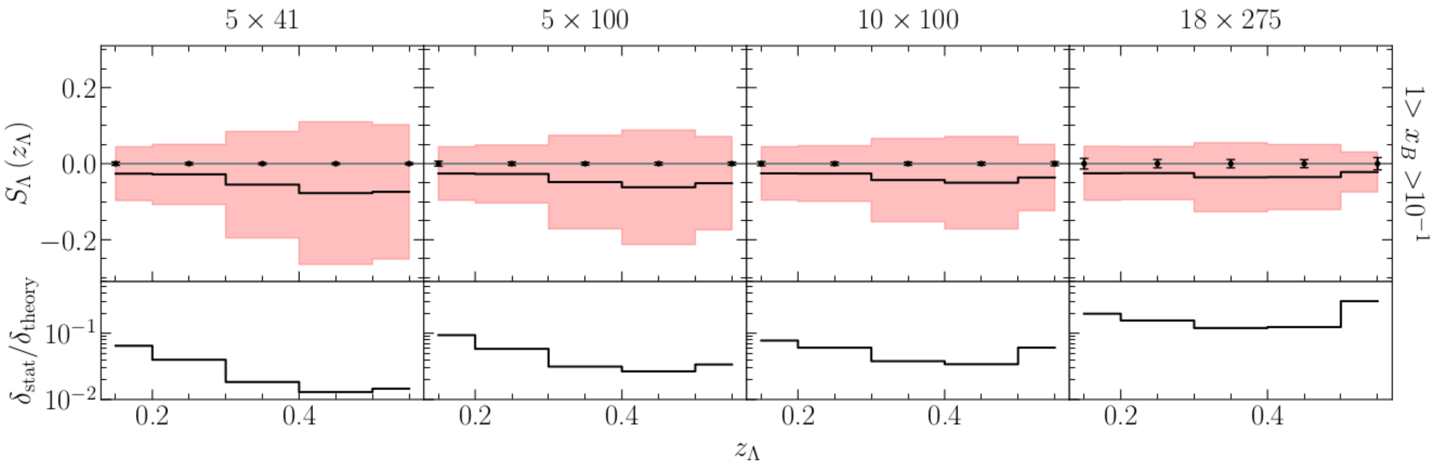
- Larger bands: from Belle [Phys.Rev.D 102 (2020) 9, 096007]
- Smaller bands: Belle + EIC pseudodata

Impact on Spin Transfer



- Transversity FF extracted from COMPASS [Phys.Lett.B 824 (2022) 136834]

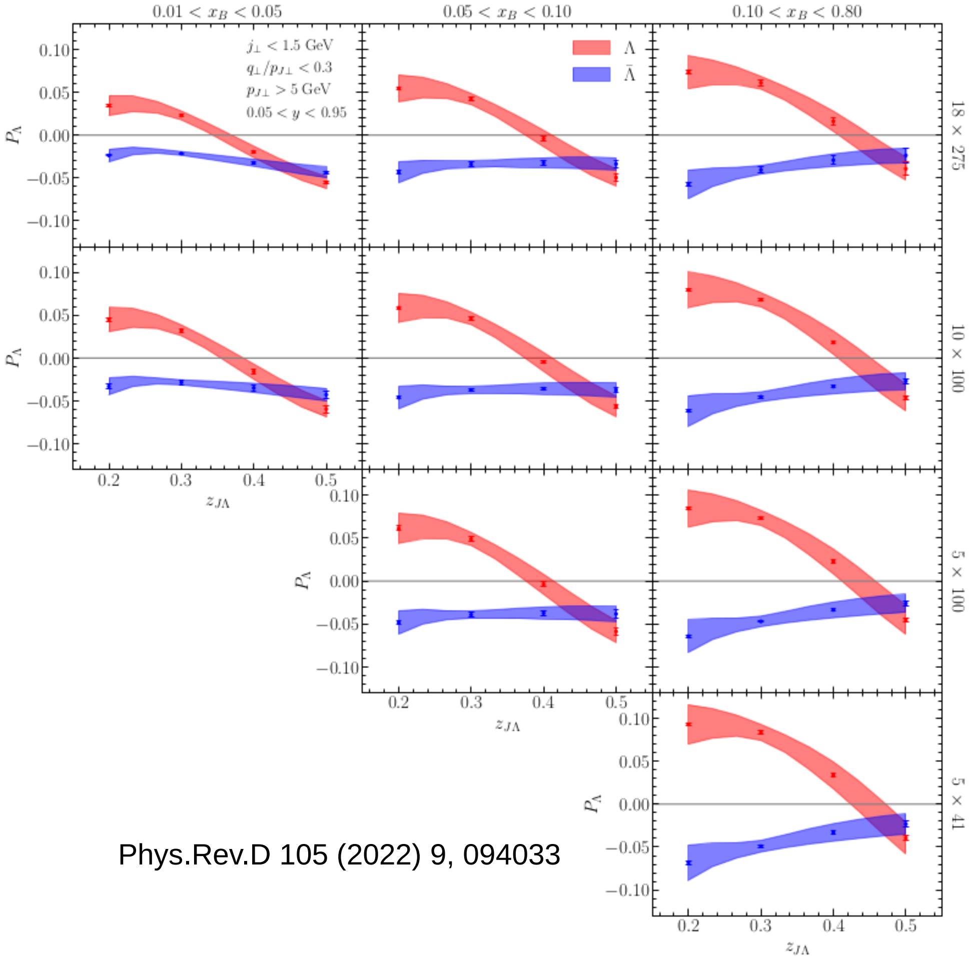
- EIC Impact on Spin Transfer
    - Red bands: theoretical uncertainty
    - Black error bars: projected statistical uncertainty (40 fb^{-1})



Figures from
Phys.Rev.D 105 (2022) 9, 094033

Spontaneous Polarization Impact from Λs in Jets

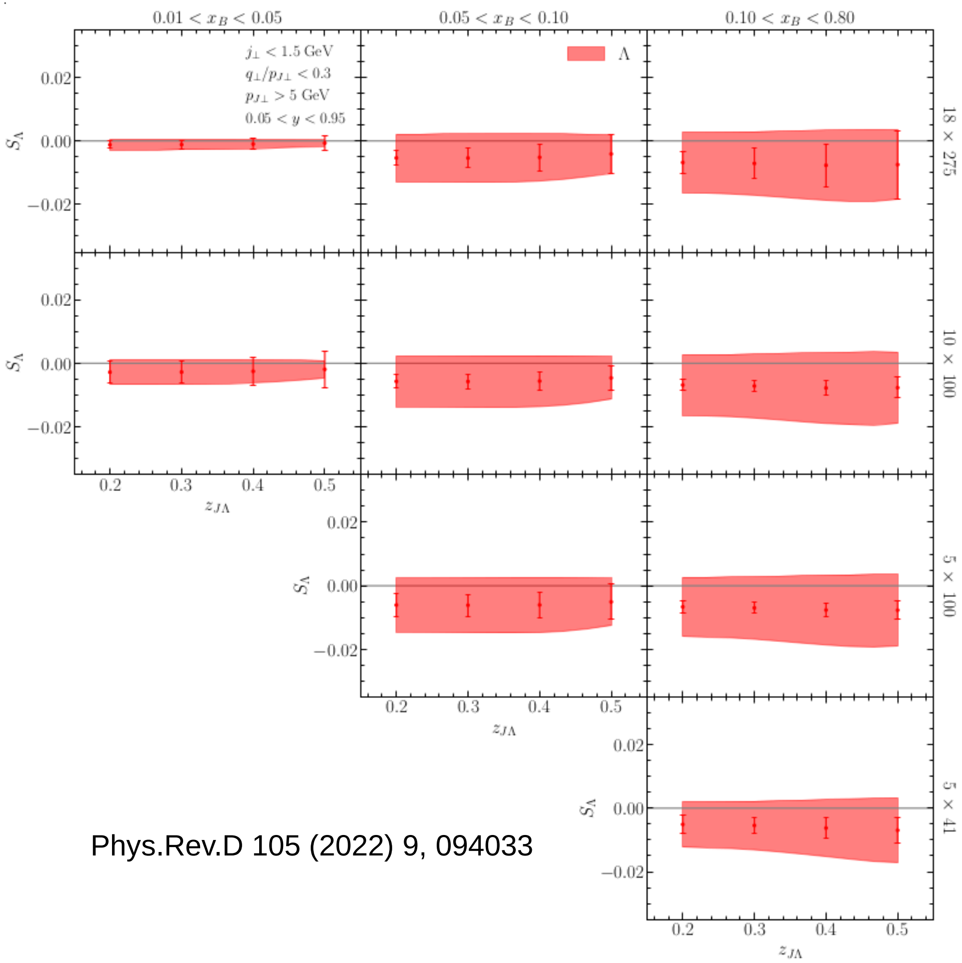
- Measuring Λs in jets provides another probe for TMD FFs
- Distribution of hadrons relative to jet axis allows for decorrelation of TMD FFs and PDFs
- Impact on spontaneous polarization:
  - Bands: theoretical uncertainty
  - Error bars: projection from 100 fb^{-1}



Phys.Rev.D 105 (2022) 9, 094033

Spin Transfer Impact from Λs in Jets

- Impact on spin transfer:
  - Bands: theoretical uncertainty
  - Error bars: projection from 100 fb^{-1}



Phys.Rev.D 105 (2022) 9, 094033

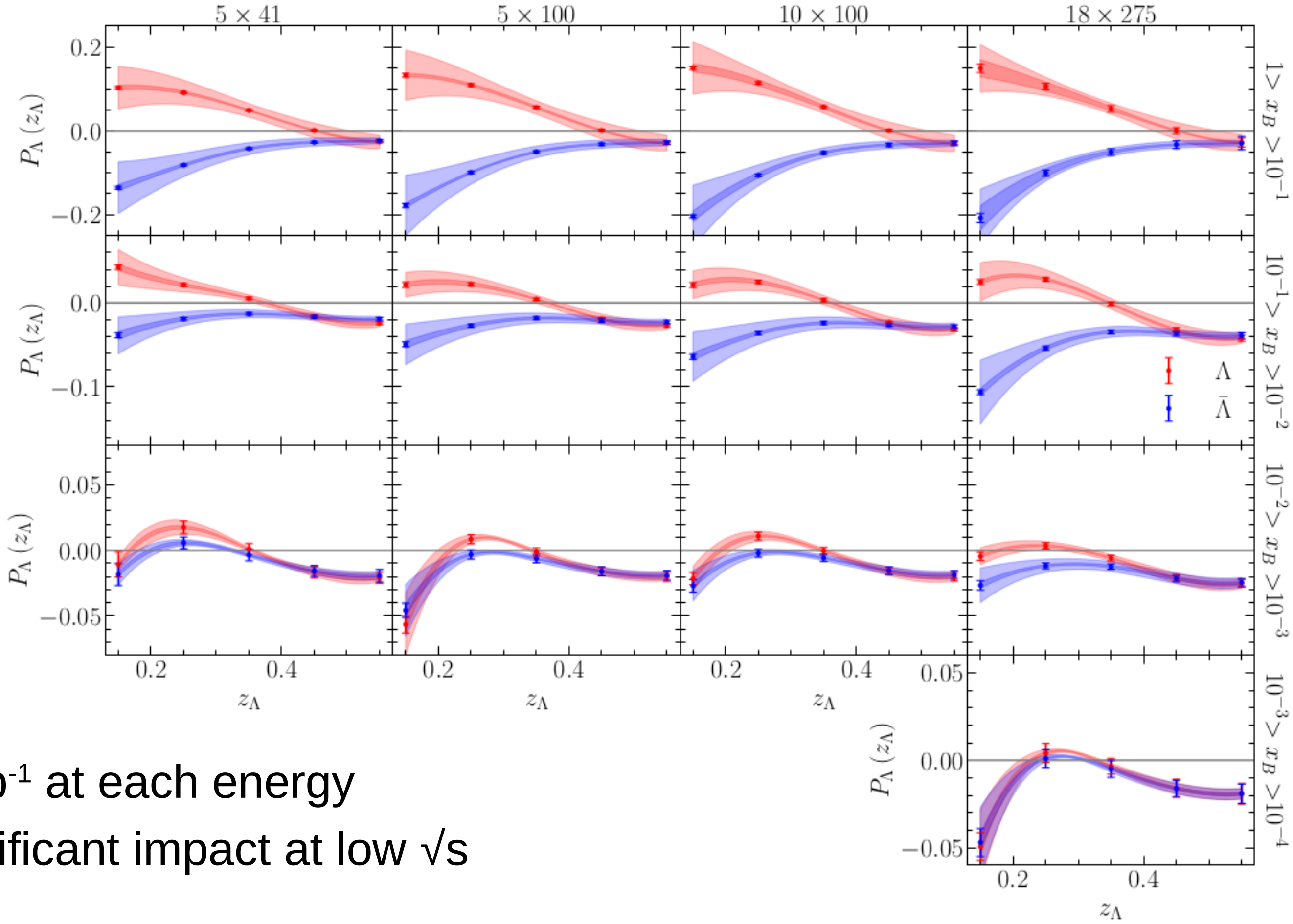
Summary

- **Dihadrons**
  - TMD parton distribution functions
  - Collinear Twist-3 PDFs
  - Dihadron Fragmentation Functions
  - Partial waves → spin/orbit correlations in hadronization
  - Vector meson decay
  - Gluon saturation
- **Lambdas**
  - TMD fragmentation
  - Lambda polarization and spin transfer

Many analysis opportunities will be available, for both experiment and theory!

BACKUP

Impact on Spontaneous Polarization



- 40 fb^{-1} at each energy
- Significant impact at low √s

Dihadron Access to PDFs x DiFFs

Twist 2

Target Polarization

Beam Polarization

	U	L	T
U	$f_1 D_1$ $h_1^\perp H_1$	$h_{1L}^\perp H_1$ $g_{1L} G_1$	$f_{1T}^\perp D_1$ $g_{1T} G_1$ $h_1 H_1$ $h_{1T}^\perp H_1$
L	$f_1 G_1$	$g_{1L} D_1$	$g_{1T} D_1$ $f_{1T}^\perp G_1$

Twist 3

Target Polarization

Beam Polarization

	U	L	T
U	$h H_1 \quad f_1 \tilde{D}$ $f^\perp D_1 \quad h_1^\perp \tilde{H}$	$h_L H_1 \quad g_{1L} \tilde{G}$ $f_L^\perp D_1 \quad h_{1L}^\perp \tilde{H}$	$f_T D_1 \quad h_1 \tilde{H}$ $h_T H_1 \quad g_{1T} \tilde{G}$ $h_T^\perp H_1 \quad f_{1T}^\perp \tilde{D}$ $f_T^\perp D_1 \quad h_{1T}^\perp \tilde{H}$
L	$e H_1 \quad f_1 \tilde{G}$ $g^\perp D_1 \quad h_1^\perp \tilde{E}$	$e_L H_1 \quad g_{1L} \tilde{D}$ $g_L^\perp D_1 \quad h_{1L}^\perp \tilde{E}$	$g_T D_1 \quad h_1 \tilde{E}$ $e_T H_1 \quad g_{1T} \tilde{D}$ $e_T^\perp H_1 \quad f_{1T}^\perp \tilde{G}$ $g_T^\perp D_1 \quad h_{1T}^\perp \tilde{E}$

Dihadron Access to PDFs x DiFFs

Twist 2

Target Polarization

Beam Polarization

	U	L	T
U	A $f_1 D_1$ B $h_1^\perp H_1$	B $h_{1L}^\perp H_1$ A $g_{1L} G_1$	A $f_{1T}^\perp D_1$ A $g_{1T} G_1$ B $h_1 H_1$ B $h_{1T}^\perp H_1$
L	C $f_1 G_1$	C $g_{1L} D_1$	C $g_{1T} D_1$ $f_{1T}^\perp G_1$

Depolarization Factors

Twist 3

Target Polarization

Beam Polarization

	U	L	T
U	V hH_1 $f_1 \tilde{D}$ $f^\perp D_1$ $h_1^\perp \tilde{H}$	V $h_L H_1$ $g_{1L} \tilde{G}$ $f_L^\perp D_1$ $h_{1L}^\perp \tilde{H}$	V $f_T D_1$ $h_1 \tilde{H}$ $h_T H_1$ $g_{1T} \tilde{G}$ $h_T^\perp H_1$ $f_{1T}^\perp \tilde{D}$ $f_T^\perp D_1$ $h_{1T}^\perp \tilde{H}$
L	W eH_1 $f_1 \tilde{G}$ $g^\perp D_1$ $h_1^\perp \tilde{E}$	W $e_L H_1$ $g_{1L} \tilde{D}$ $g_L^\perp D_1$ $h_{1L}^\perp \tilde{E}$	W $g_T D_1$ $h_1 \tilde{E}$ $e_T H_1$ $g_{1T} \tilde{D}$ $e_T^\perp H_1$ $f_{1T}^\perp \tilde{G}$ $g_T^\perp D_1$ $h_{1T}^\perp \tilde{E}$

Depolarization Factors

- Depolarization factors (and ε) depend on (x,y,Q^2)
- Asymmetry denominator:

$$\int d\sigma_{UU} \sim A$$

Depolarization Factors

	Twist 2	Twist 3
Unpolarized Beam	***A, B***	***V***
Longitudinal Beam	***C***	***W***

Asymmetry, for modulation $M(\theta, \phi_h, \phi_R, \phi_S)$

$$D \in \{A, B, C, V, W\}$$

Structure Functions

$$A_{XY}^{M} \propto \frac{D_{XY}^{M}}{A} \cdot \frac{F_{XY}^{M}}{F_{UU,T}^{\text{const}} + \epsilon F_{UU,L}^{\text{const}}}$$

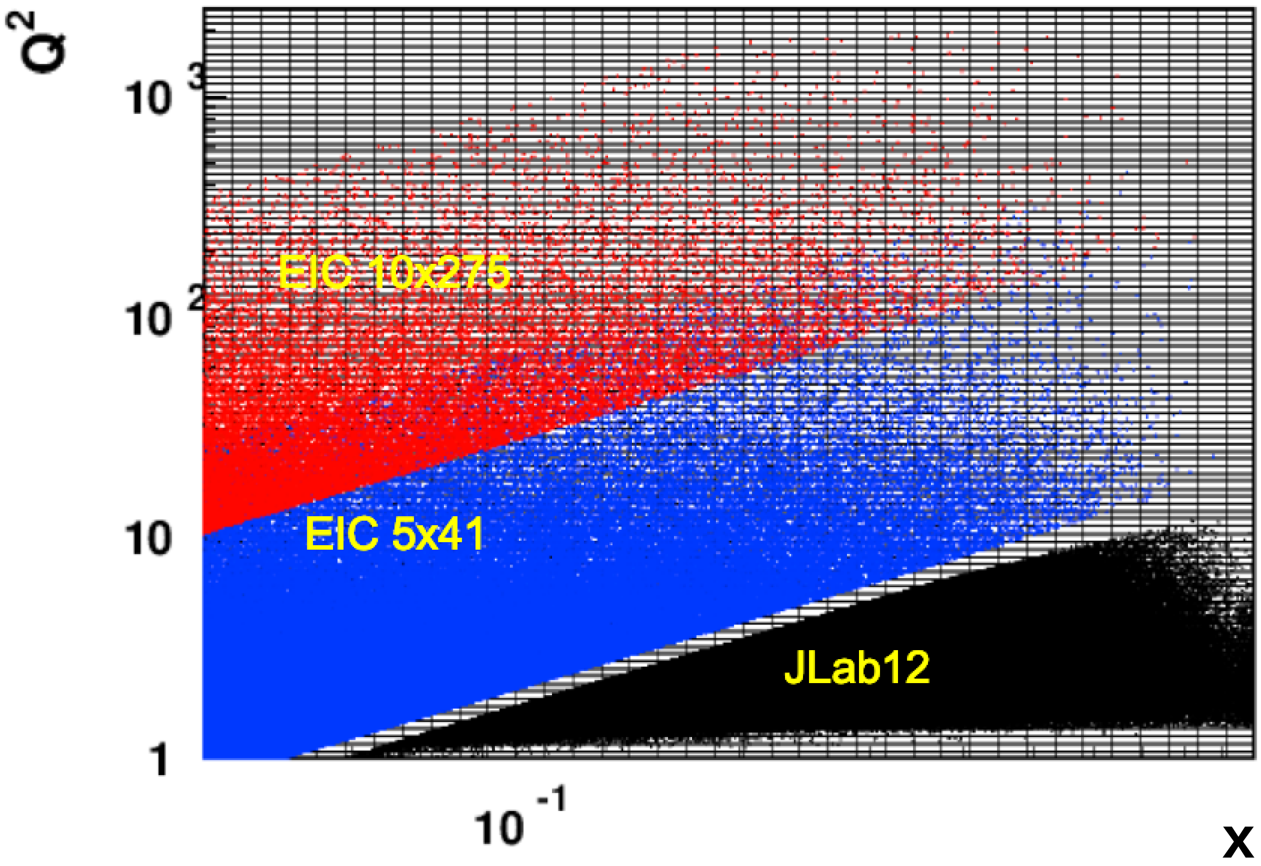
Depolarization Factors

Twist 2

Twist 3

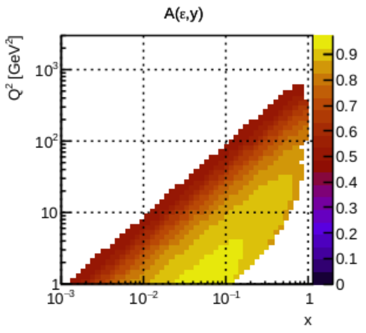
	Polarization	Depolarization
Boer-Mulders	UU	B
Sivers	UT	1
Transversity	UT	B/A
Kotzinian-Mulders	UL	B/A
Wormgear (LT)	LT	C/A
Helicity DiFF $G_1^\perp$	LU	C/A
	UL	1
e(x)	LU	W/A
$h_L(x)$	UL	V/A
$g_T(x)$	LT	W/A

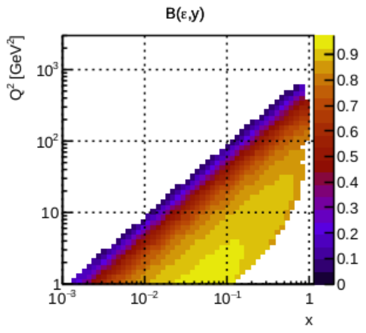
Kinematic Coverage

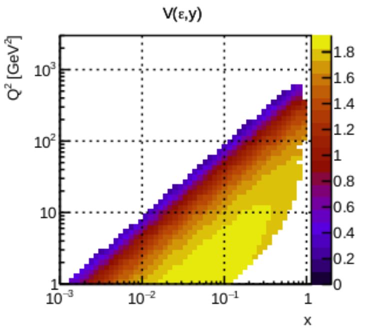


Depolarization Factors

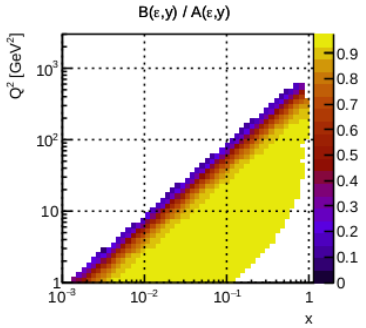
UU

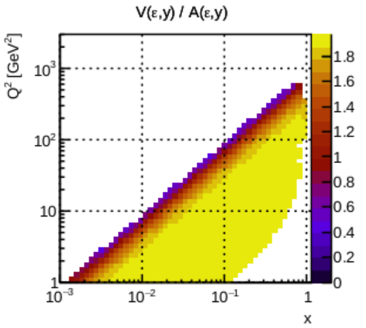

A(ε,y)


B(ε,y)

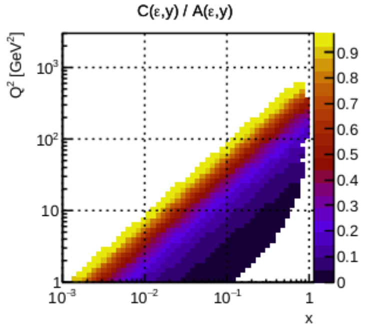

V(ε,y)

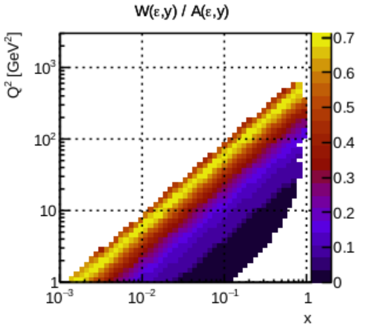
UL, UT


B(ε,y) / A(ε,y)


V(ε,y) / A(ε,y)

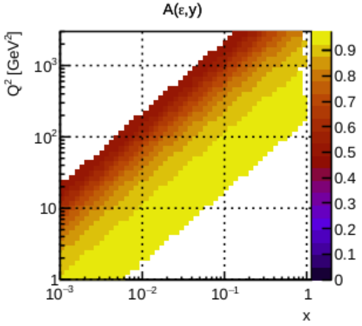
LU, LL, LT

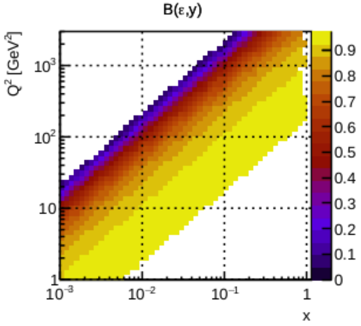

C(ε,y) / A(ε,y)

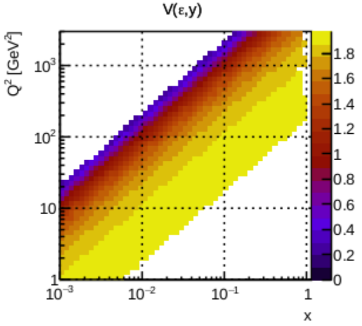

W(ε,y) / A(ε,y)

Depolarization Factors

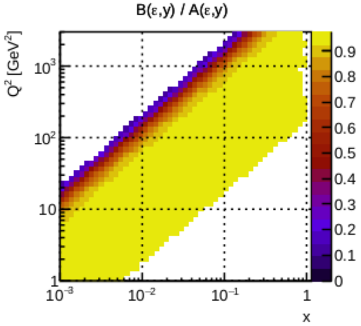
UU

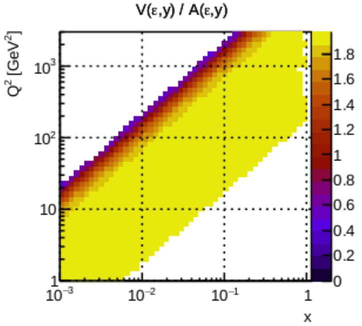

A(ε,y)


B(ε,y)

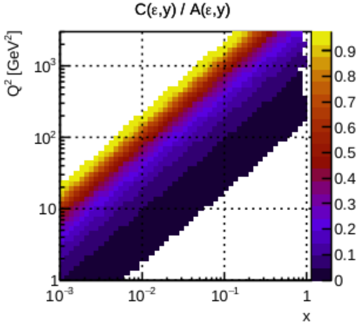

V(ε,y)

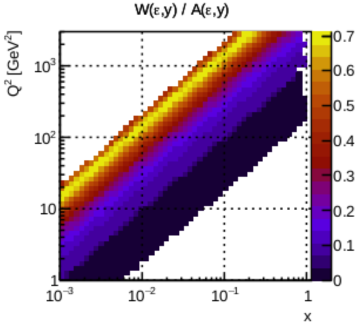
UL, UT


B(ε,y) / A(ε,y)

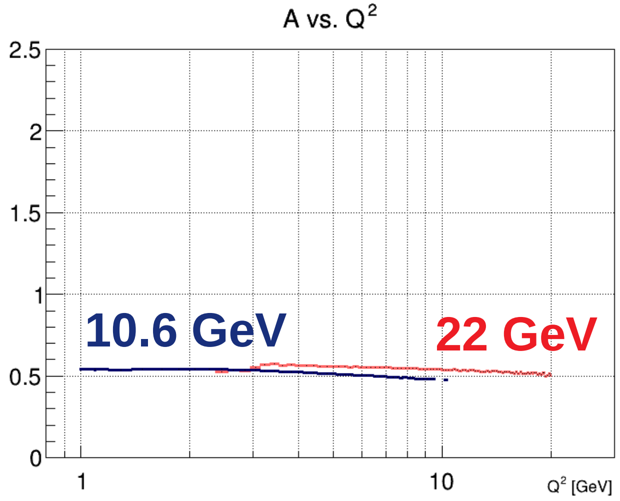

V(ε,y) / A(ε,y)

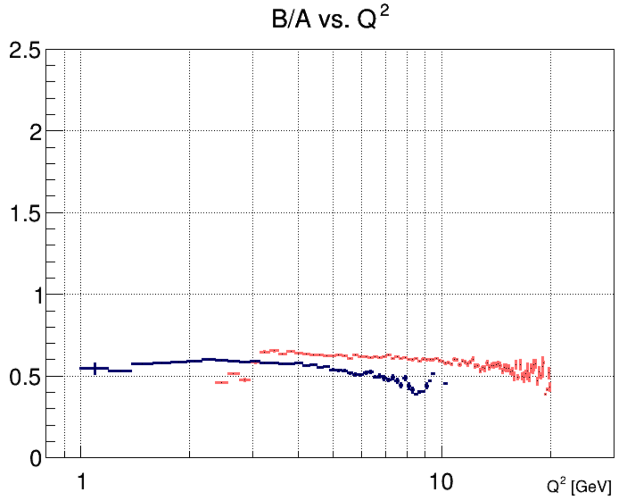
LU, LL, LT

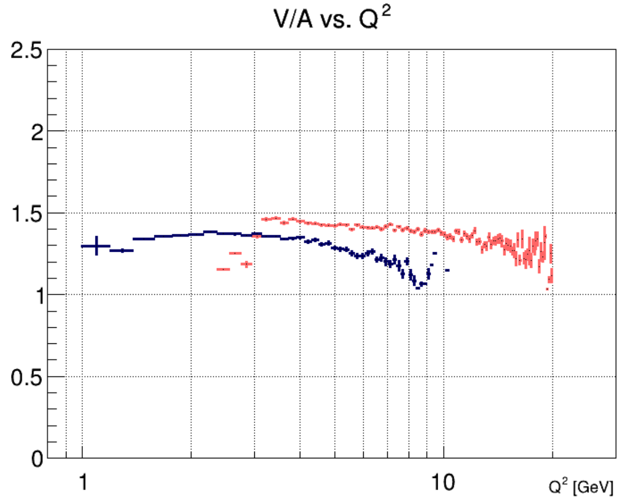

C(ε,y) / A(ε,y)

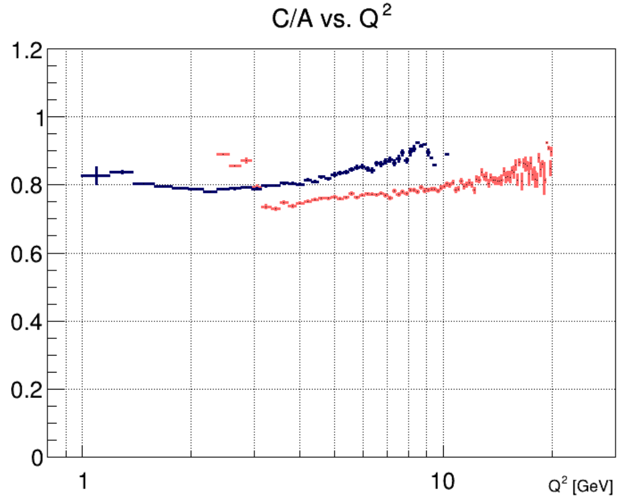

W(ε,y) / A(ε,y)

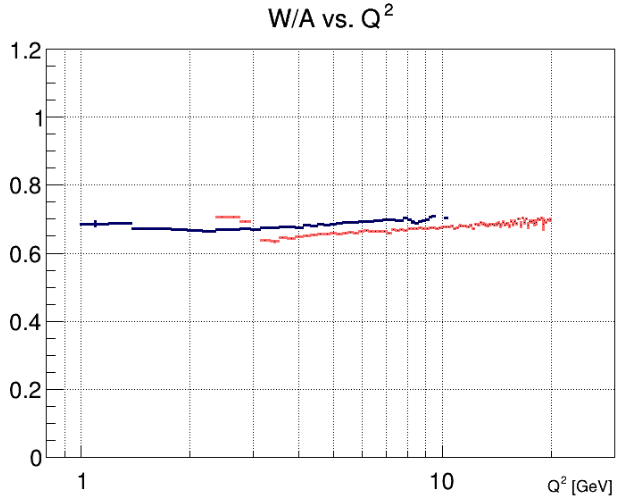
Depolarization at CLAS





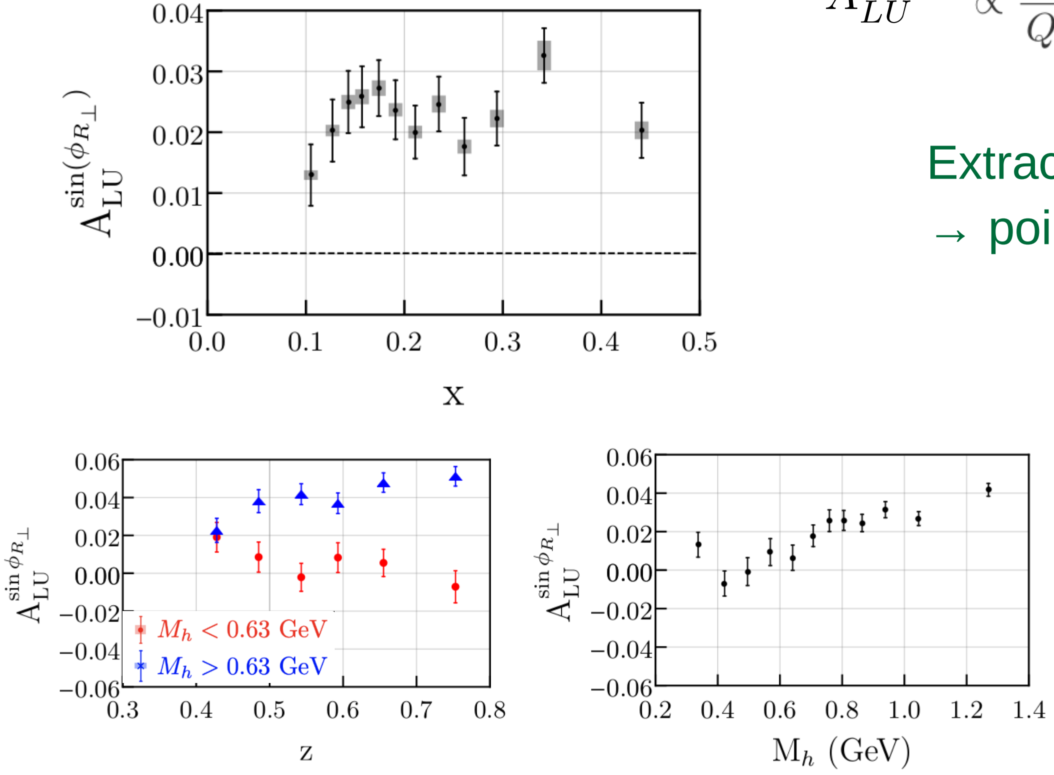






CLAS Dihadron A_{LU} Measurements for e(x)

CLAS12 $\pi^+\pi^-$ $A_{LU}^{\sin\phi_R}$



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$$A_{LU}^{\sin\phi_R} \propto \frac{M}{Q}\,\frac{\sum_q e_q^2\left[x e^q(x)\, H_{1,sp}^{\sphericalangle,q}(z,m_{\pi\pi}) + \frac{m_{\pi\pi}}{zM}\, f_1^q(x)\, \tilde{G}_{sp}^{\sphericalangle,q}(z,m_{\pi\pi})\right]}{\sum_q e_q^2\, f_1^q(x)\, D_{1,ss+pp}^q(z,m_{\pi\pi})}$$

twist-3 DiFF

Extraction of $H_1^{<}$ from Belle Data

→ point-by-point extraction of e(x)

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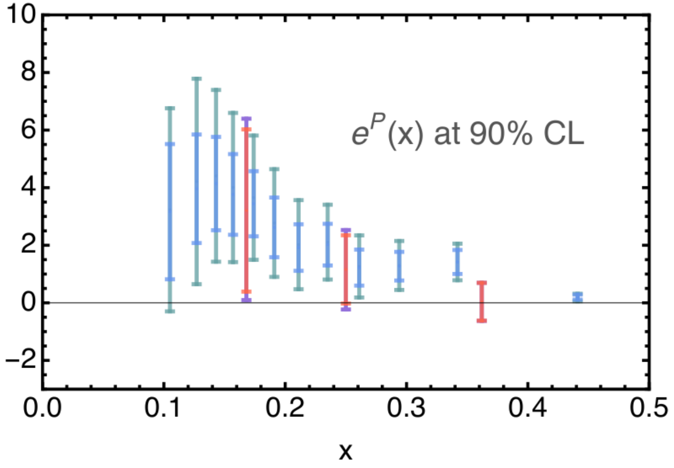

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