Getting to grips with GPD extraction

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Outline

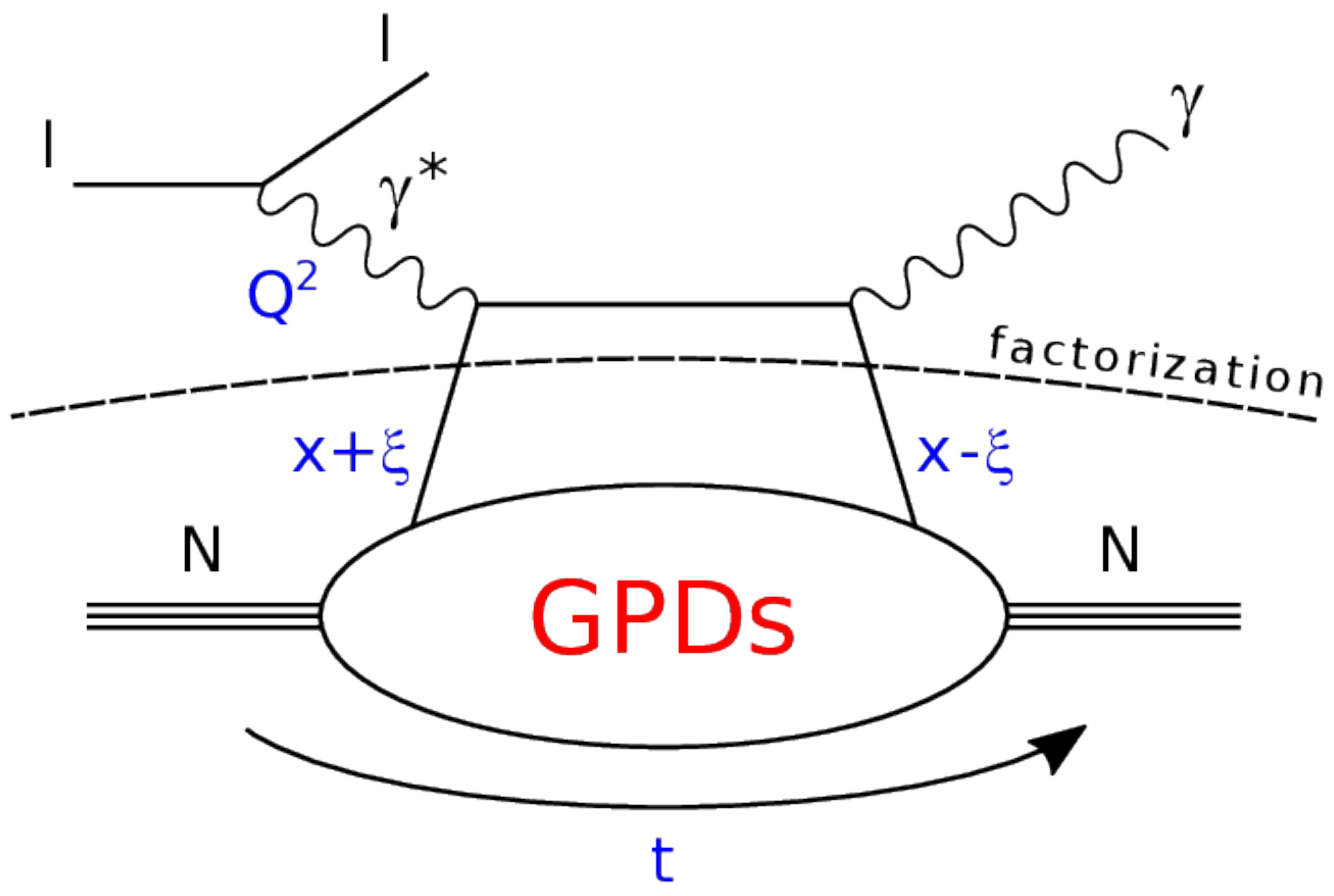
- General introduction

- Model dependency at amplitude and GPD levels
- Software projects
- New channels sensitive to GPDs

- Summary

see Shunzo Kumano's talk for proper definition of GPDs

Deeply Virtual Compton Scattering (DVCS)



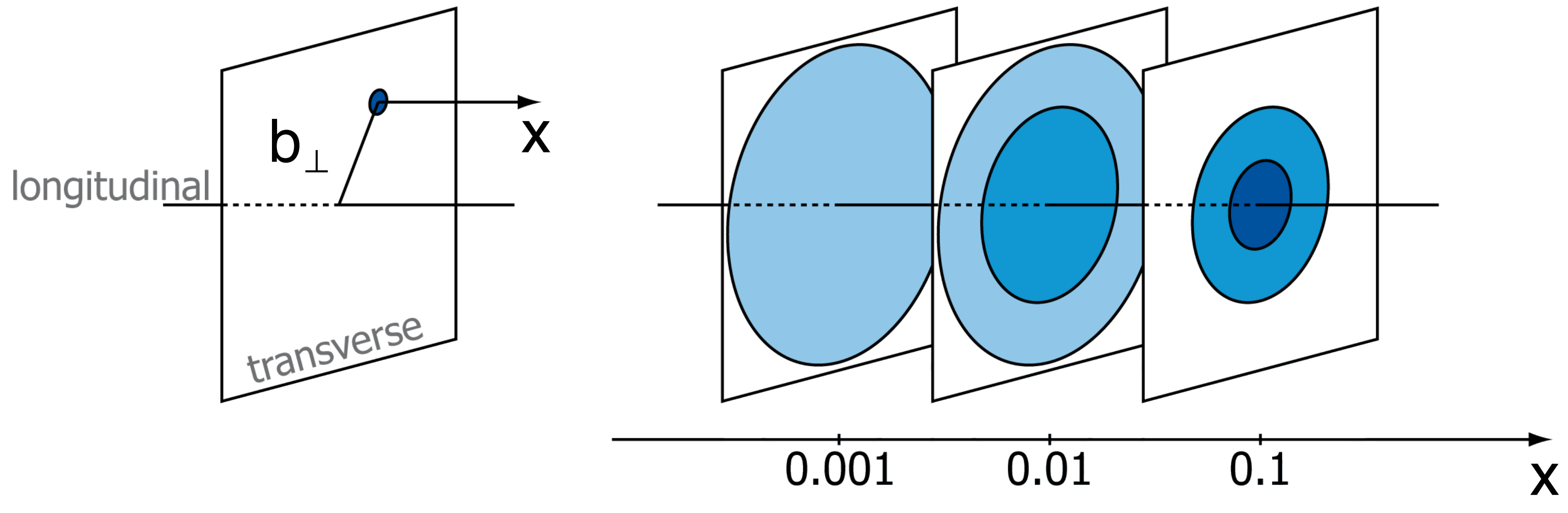
factorisation for $|t|/Q^2 \ll 1$

Chiral-even GPDs:
(helicity of parton conserved)

$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	*for sum over parton helicities*
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	*for difference over parton helicities*
nucleon helicity conserved	*nucleon helicity changed*	

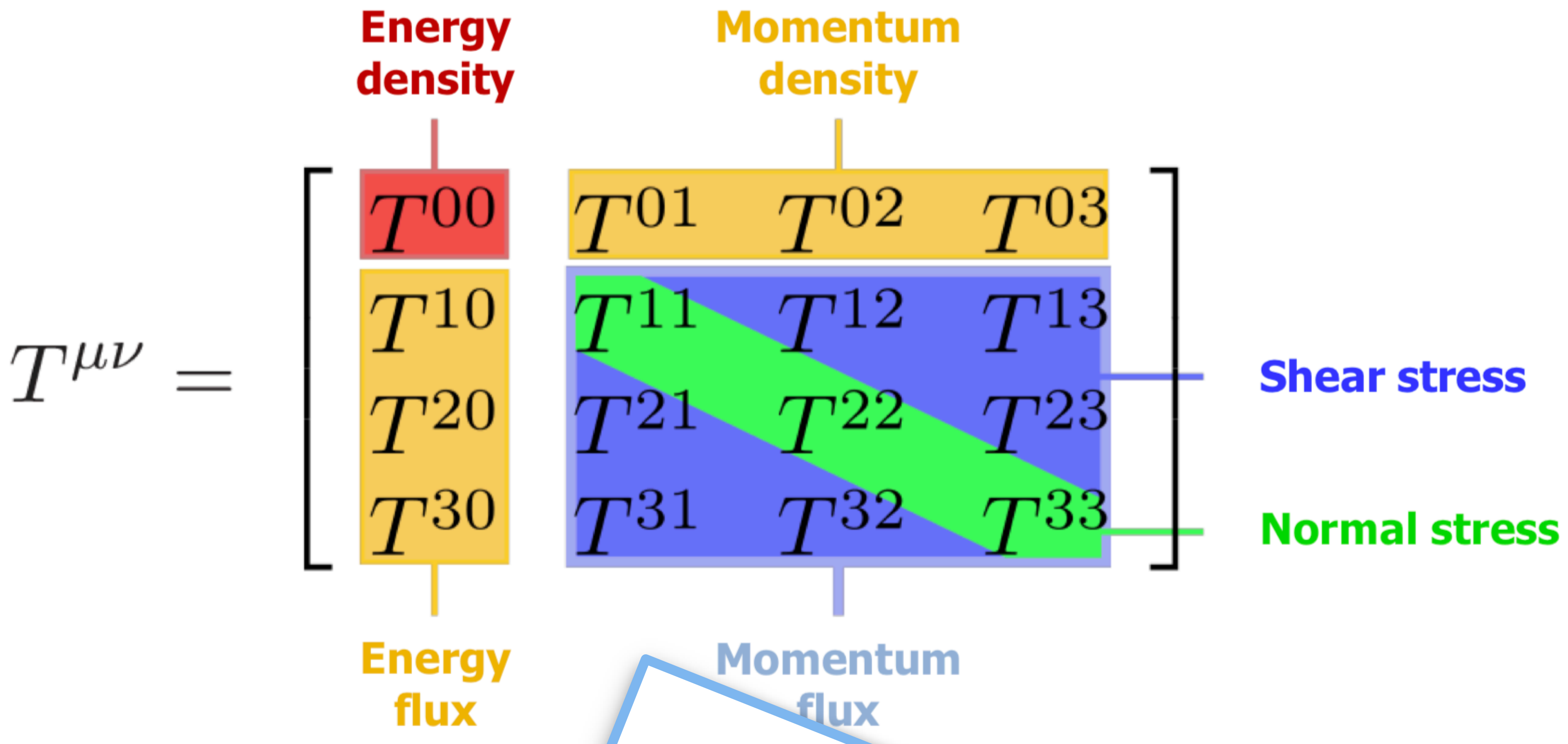
Introduction

Nucleon tomography:



$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\mathbf{\Delta}}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}} H^q(x, 0, t = -\mathbf{\Delta}^2)$$

Energy momentum tensor in terms of form factors (OAM and mechanical forces):



$$\langle p', s' | \widehat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu} \bar{C}(t) + \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M} [A(t) + B(t) + D(t)] + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M} [A(t) + B(t) - D(t)] \right] u(p, s)$$

see Hyun-Chul Kim's talk for more details

Reduction to PDF:

$$H(x, \xi = 0, t = 0) \equiv q(x)$$

Polynomiality - non-trivial consequence of Lorentz invariance:

$$\mathcal{A}_n(\xi, t) = \int_{-1}^{1} \mathrm{d}x x^n H(x, \xi, t) = \sum_{\substack{j=0 \\ \mathrm{even}}}^{n} \xi^j A_{n,j}(t) + \mathrm{mod}(n, 2)\xi^{n+1} A_{n,n+1}(t)$$

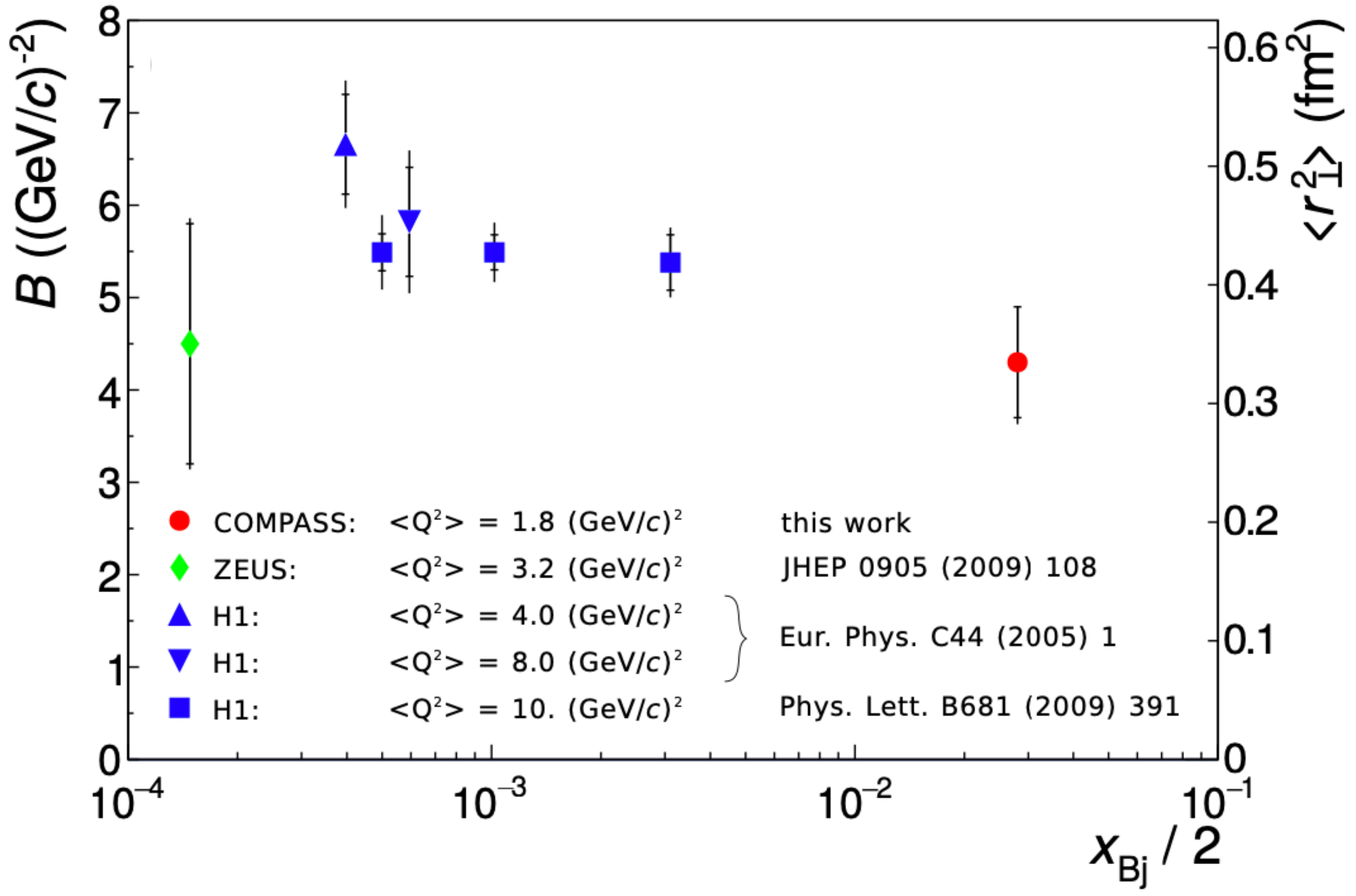
Positivity bounds - positivity of norm in Hilbert space, e.g.:

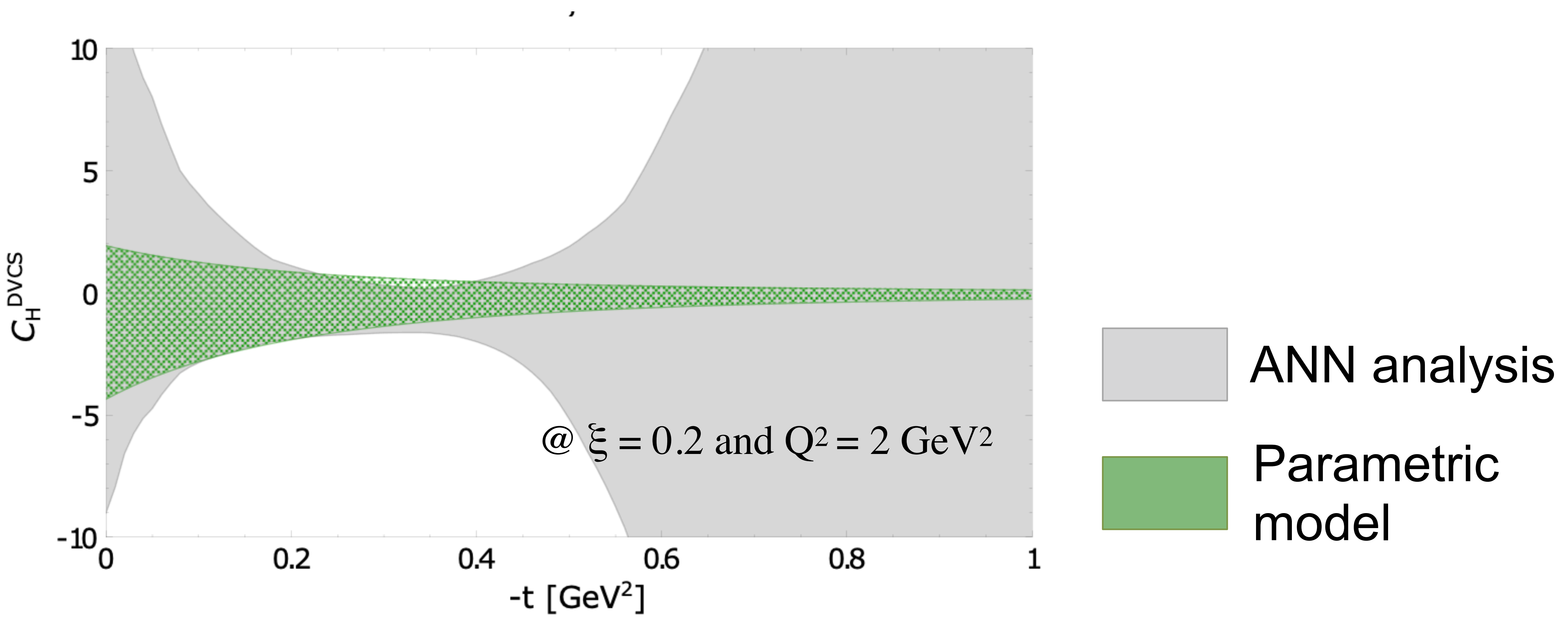
$$|H(x, \xi, t)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right) \frac{1}{1-\xi^2}}$$

Motivation

- Despite a substantial progress in both measurement and description of exclusive processes, and in lattice-QCD the problem of the model dependency of GPDs is still poorly addressed.

- Exceptions:
  probing nucleon tomography at low-xB
  extraction of D-term





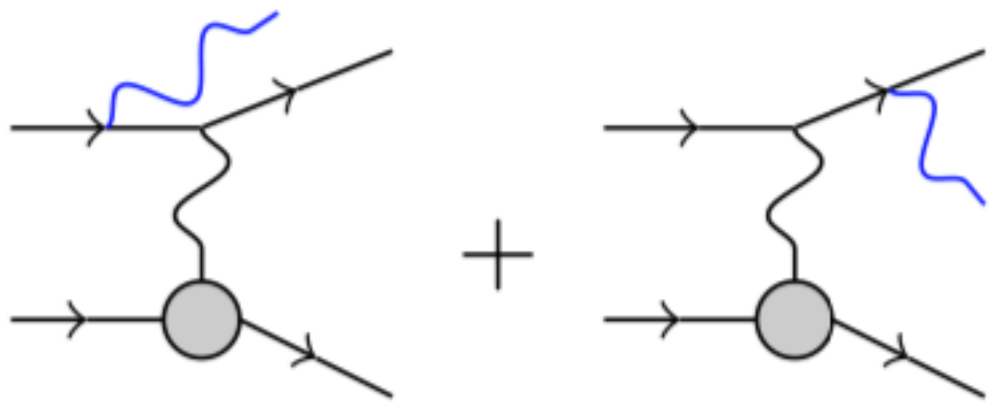
- New tools to study model dependency of GPDs, nucleon tomography and orbital angular momentum appeared just recently.

DVCS Compton Form Factors

Cross-section for single photon production ($l + N \to l + N + \gamma$):

$$\sigma \propto |\mathscr{A}|^2 = |\mathscr{A}_{BH} + \mathscr{A}_{DVCS}|^2 = |\mathscr{A}_{BH}|^2 + |\mathscr{A}_{DVCS}|^2 + \mathscr{I}$$

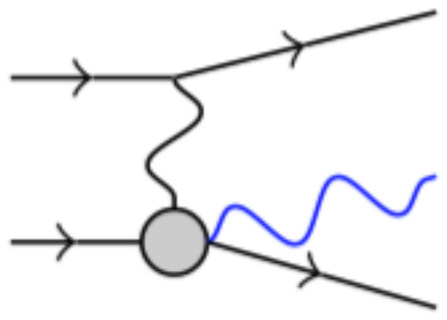
Bethe-Heitler process



calculable within QED

parametrised by elastic FFs

DVCS



calculable within QCD

parametrised by CFFs

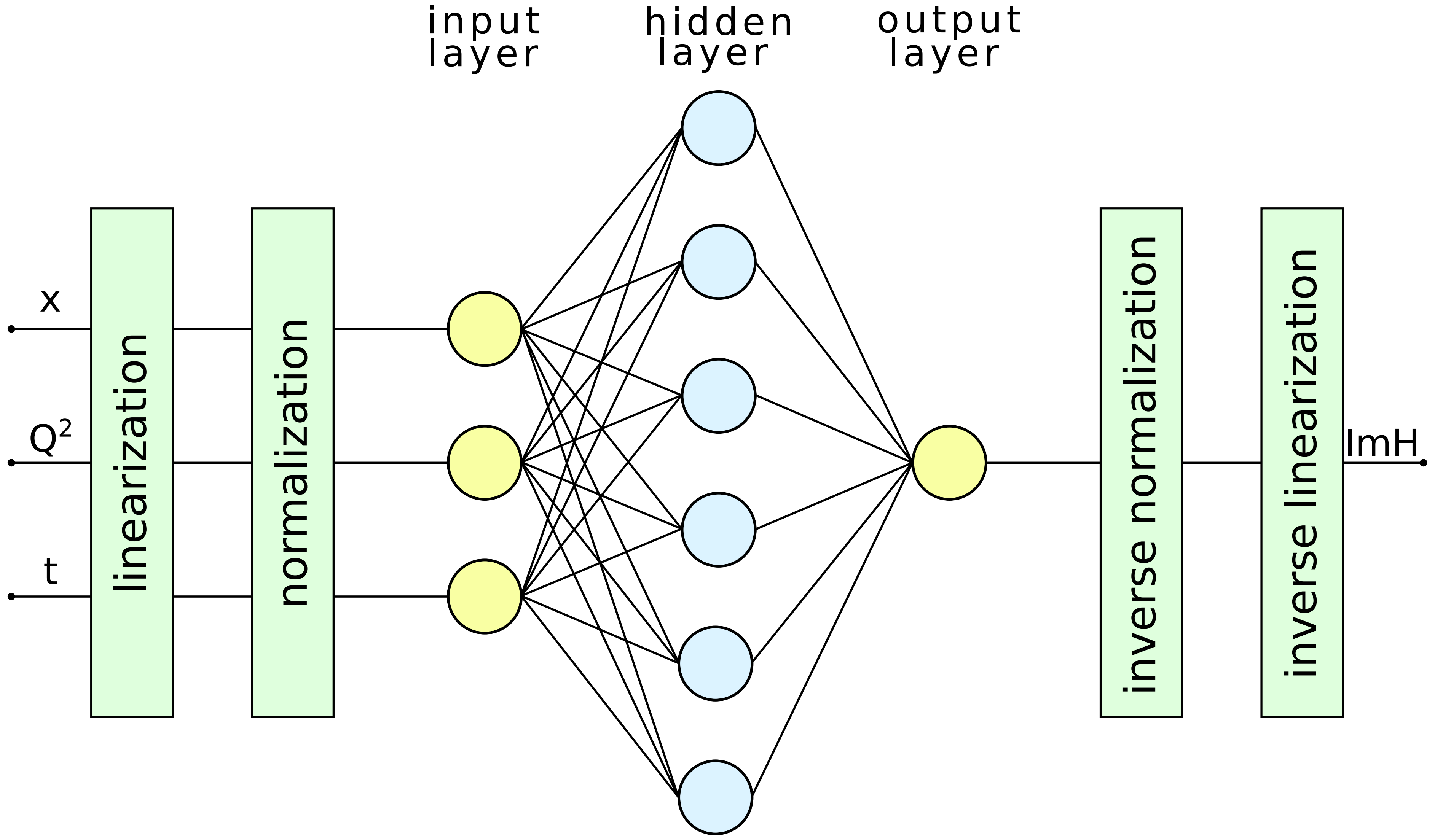
see e.g. NPB 878 (2014) 214 for more details

$$\mathrm{Im}\mathscr{H}(\xi, t) \overset{\text{LO}}{=} \pi \sum_q e_q^2 H^{q(+)}(\xi, \xi, t)$$

$$\mathrm{Re}\mathscr{H}(\xi, t) = \mathrm{PV} \int_0^1 \frac{d\xi'}{\pi} \mathrm{Im}\mathscr{H}(\xi', t) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) + C_H(t)$$

Non-parametric Ansatz of CFFs

H. Moutarde, PS, J. Wagner, Eur. Phys. J. C 79 (2019) 7, 614



Features of analysis:

- Independent artificial neural network for each CFF and Re/Im parts
- Functions of x_B, Q^2 and t
- Network size determined using benchmark sample
- No power-behaviour pre-factors
- Trained with genetic algorithm
- Regularisation method based on early stopping criterion
- Replica method for propagation of experimental uncertainties

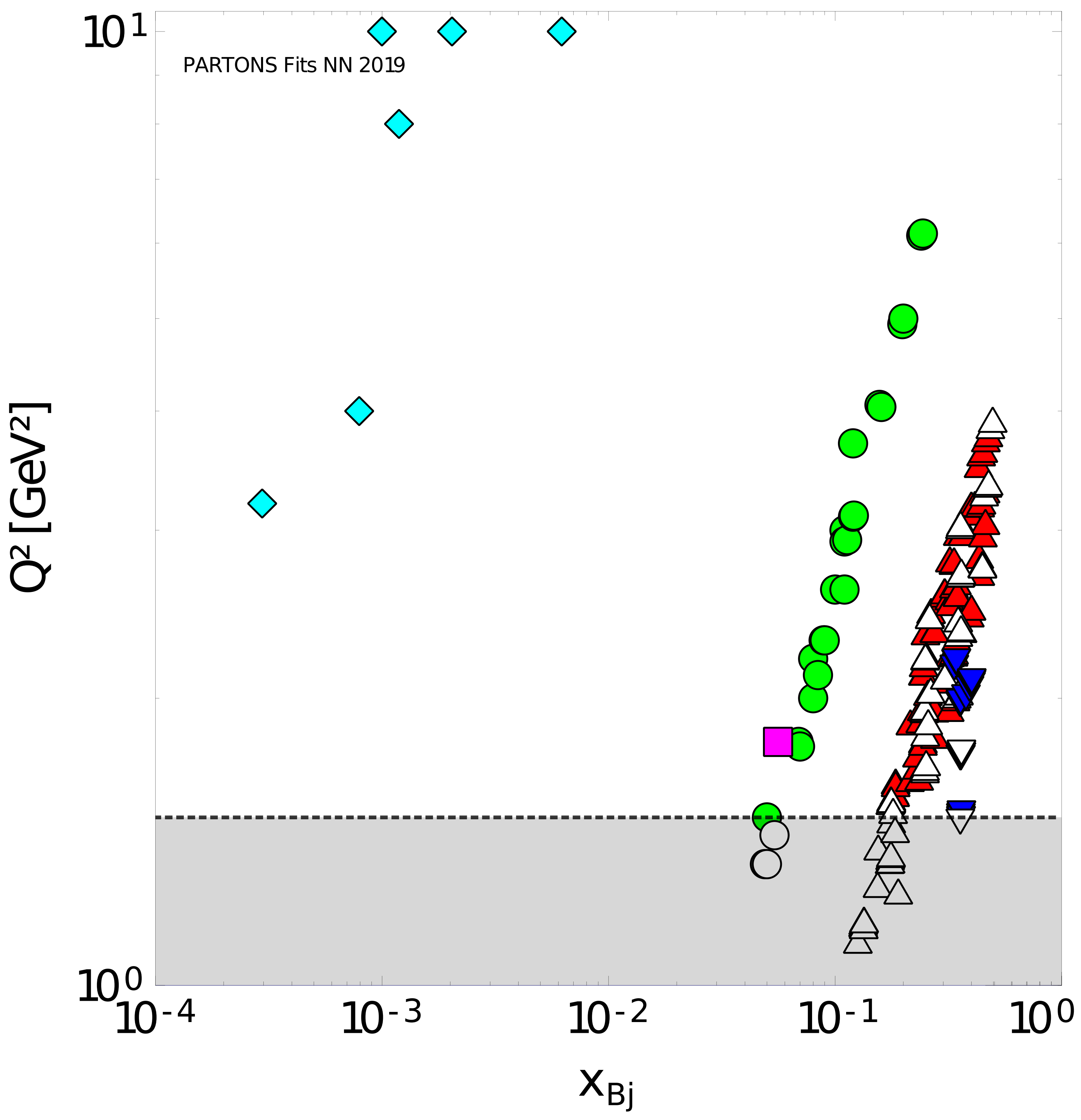
DVCS data

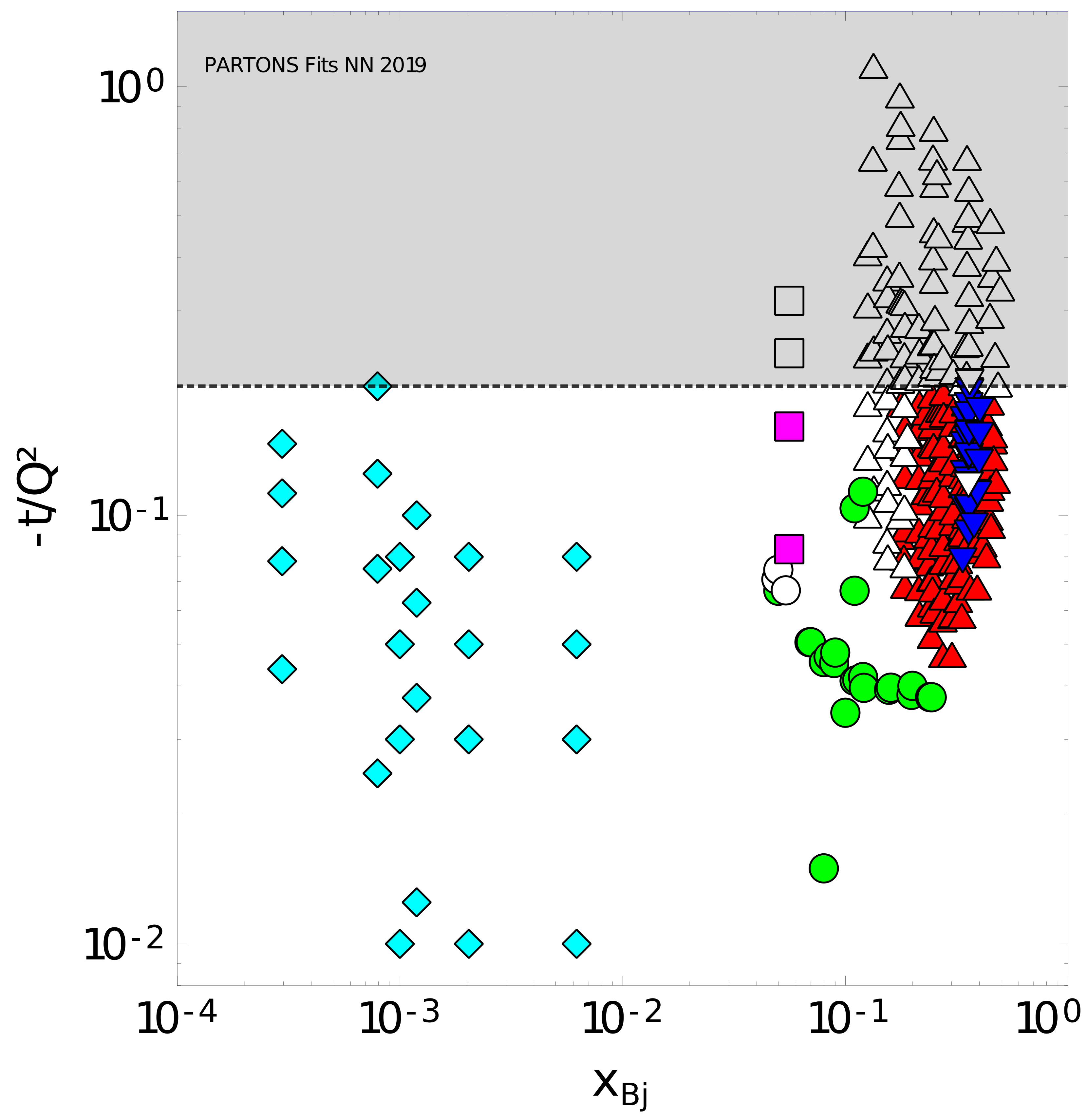
H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 79 (2019) 7, 614

Kinematic cuts
used in our recent analyses:

$$Q^2 > 1.5\ \mathrm{GeV}^2$$

$$-t/Q^2 < 0.2$$





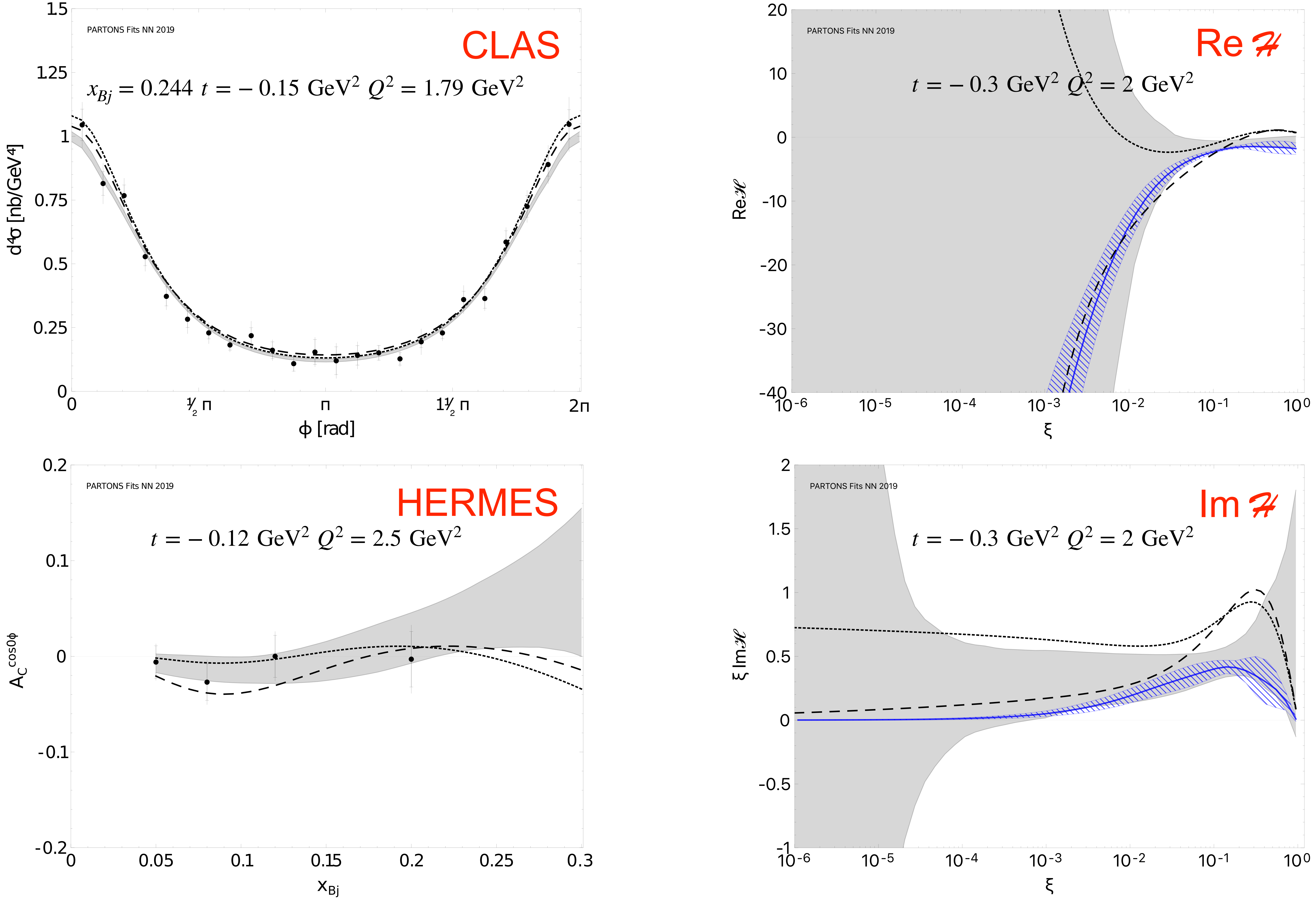
▼ HALL A
▲ CLAS
● HERMES
■ COMPASS
◆ H1 and ZEUS

Results

Performance: $\chi^2/\text{nPoints} = 2243.5/2624 \approx 0.85$

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 78 (2018) 11, 890

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 79 (2019) 7, 614



Non-parametric

Parametric

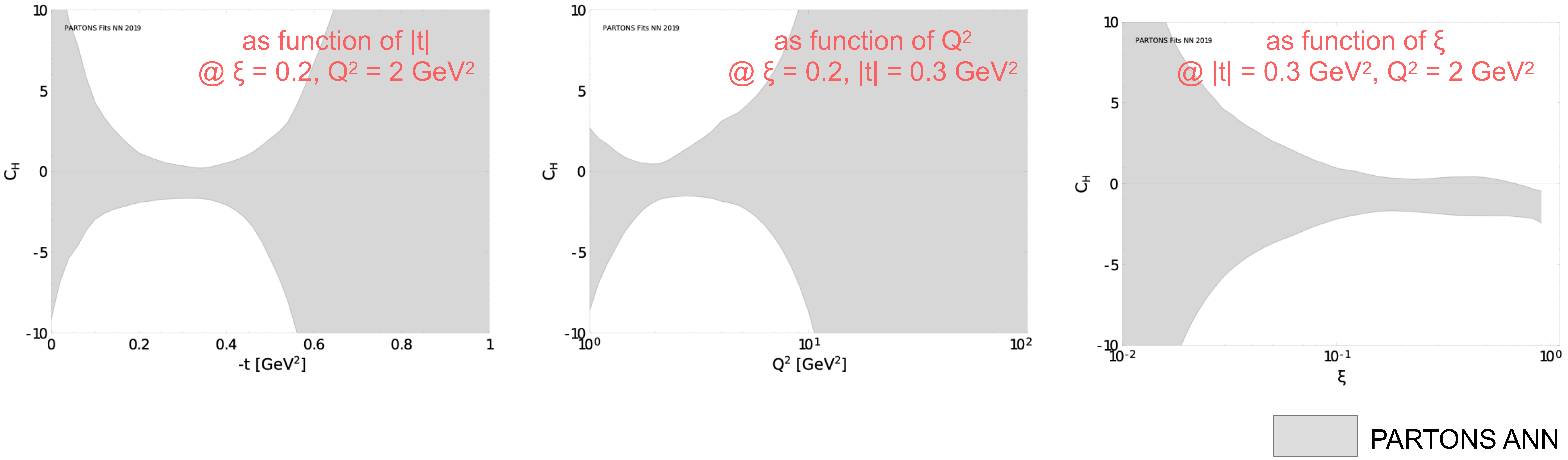
VGG
GK
LO evaluation

$\xi \approx x_{Bj}/(2 - x_{Bj})$

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 79 (2019) 7, 614

Subtraction constant extracted using dispersion relation

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi'\,\mathrm{Im}\,\mathcal{H}(\xi', t, Q^2)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)$$



Subtraction constant

H. Dutrieux et al.,
Eur. Phys. J. C 81 (2021), 300

Relation between subtraction constant and EMT FF C

Dispersion relation:

$$\mathcal{C}_H(t,Q^2) = \mathrm{Re}\,\mathcal{H}(\xi,t,Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi'\,\mathrm{Im}\,\mathcal{H}(\xi',t,Q^2)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)$$

Relation between subtraction constant and D-term (z=x/ξ):

$$\mathcal{C}_H(t,Q^2) \overset{LO}{=} 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z\, \frac{D^q_{\mathrm{term}}(z,t,\mu_F^2 \equiv Q^2)}{1-z}$$

Decomposition into Gegenbauer polynomials:

$$D^q_{\mathrm{term}}(z,t,\mu_F^2) = (1-z^2)\sum_{\mathrm{odd}\ n} d_n^q(t,\mu_F^2)\, C_n^{3/2}(z)$$

Finally:

$$\mathcal{C}_H(t,Q^2) \overset{LO}{=} 4\sum_q e_q^2 \sum_{\mathrm{odd}\ n} d_n^q(t,\mu_F^2 \equiv Q^2)$$

Connection to EMT FF:

$$d_1^q(t,\mu_F^2) = 5C_q(t,\mu_F^2)$$

Subtraction constant

H. Dutrieux et al.,
Eur. Phys. J. C 81 (2021), 300

Master formula:

$$\mathrm{Re}\mathscr{H}(\xi,t,Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \,\mathrm{Im}\,\mathscr{H}(\xi,t,Q^2)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right) \overset{LO}{=} 4\sum_q e_q^2 \sum_{\mathrm{odd}\ n} d_n^q(t,\mu_F^2 \equiv Q^2)$$

Extraction of subtraction constant from DVCS data requires:

- integral over ξ (alternatively: x_{Bj} or ν) between ε and 1

$$\epsilon = 10^{-6}$$

- good knowledge of both Re and Im parts of CFF *H*

Model assumptions to extract EMT FF C from subtraction constant:

- truncation to d1

$$C_H(t,Q^2) = 4\sum_q e_q^2\, d_1^q(t,\mu_F^2 \equiv Q^2)$$

- symmetry of light quark contributions

$$d_1^u(t,\mu_F^2) = d_1^d(t,\mu_F^2) = d_1^s(t,\mu_F^2) \equiv d_1^{uds}(t,\mu_F^2)$$

- sensitivity to gluon contribution via evolution

$$d_1^G(t,\mu_{F,0}^2) = 0 \qquad \mu_{F,0}^2 = 0.1\ \mathrm{GeV}^2$$

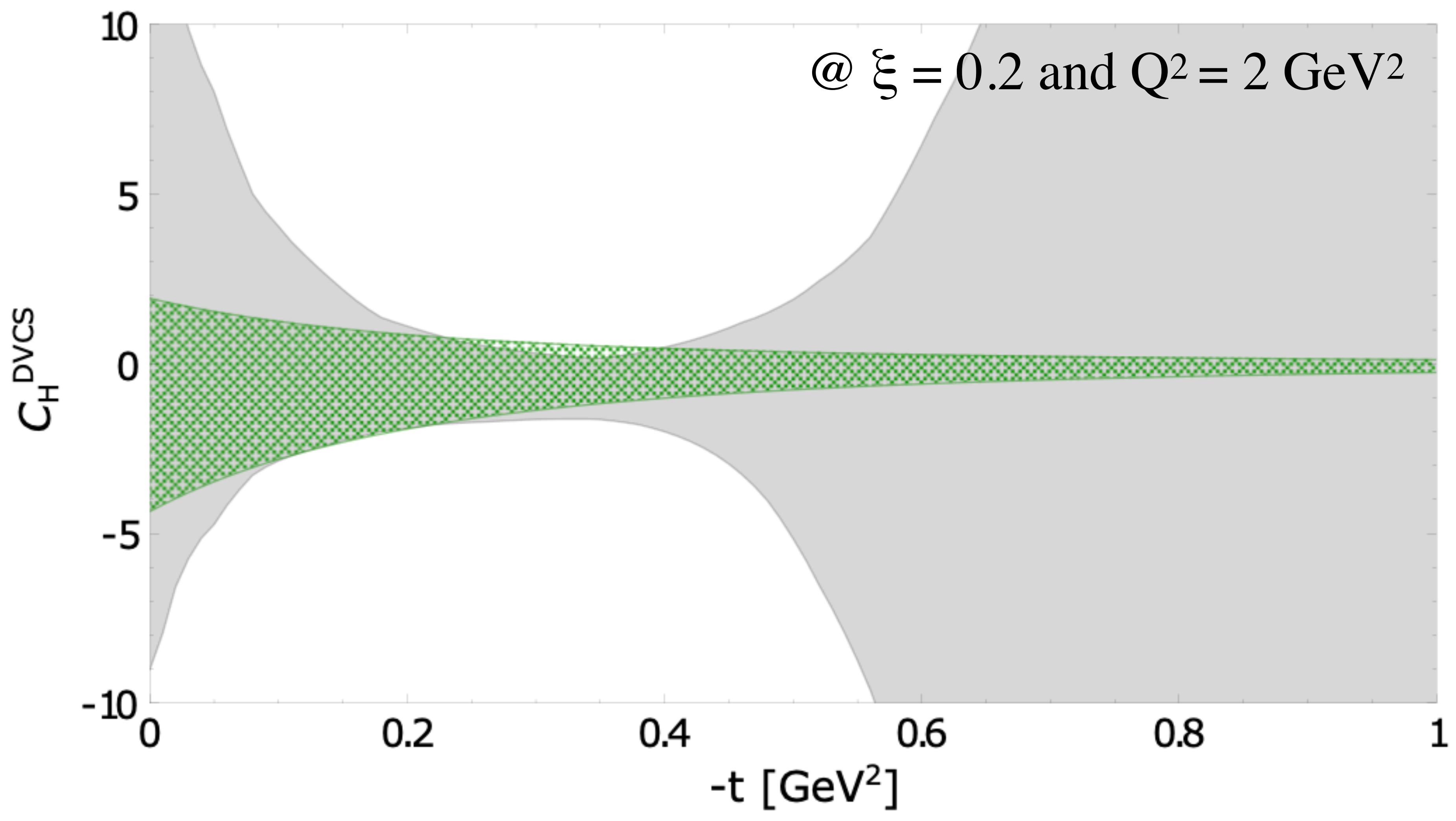
- tripole Ansatz for t-dependence

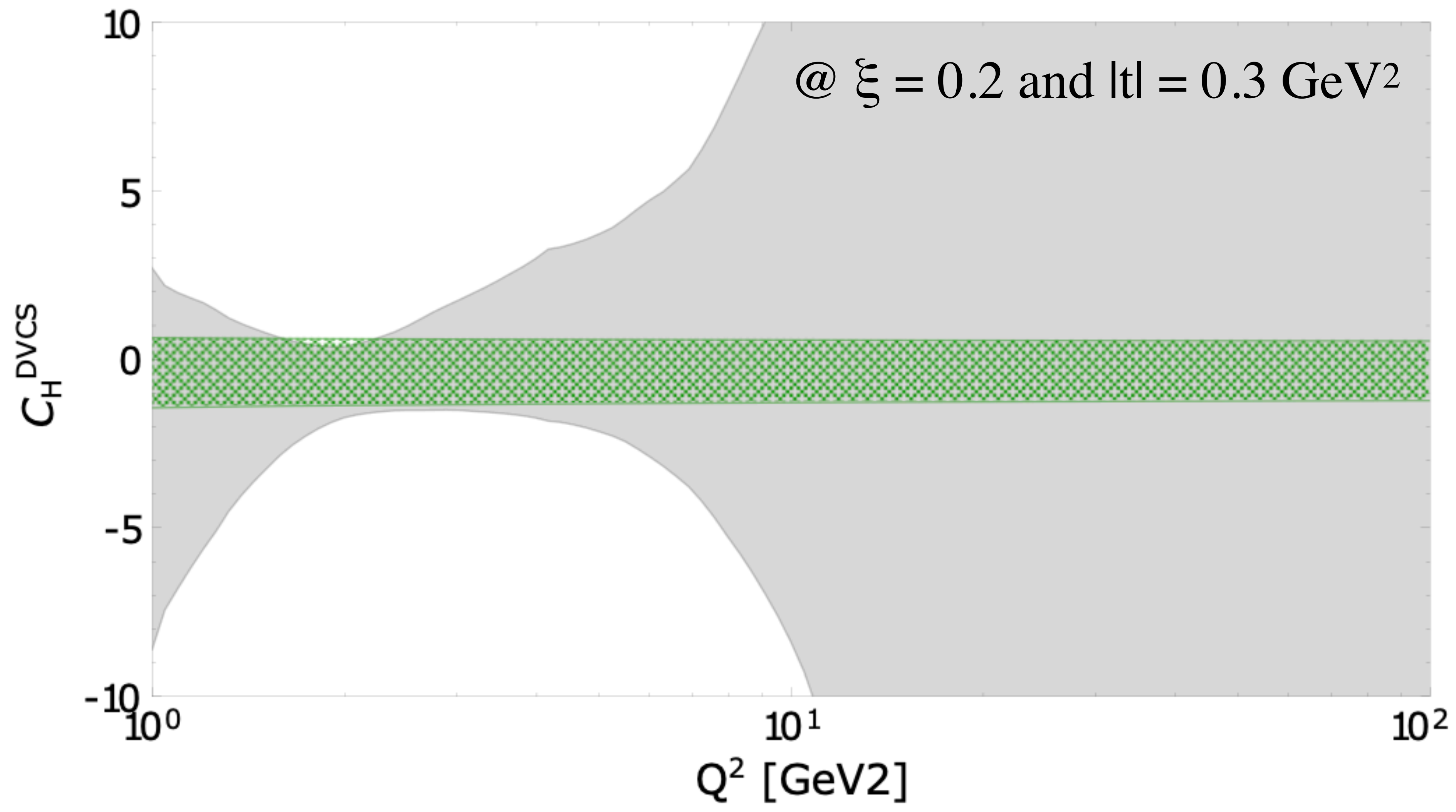
$$d_1^{uds}(t,\mu_F^2) = d_1^{uds}(\mu_F^2)\left(1 - \frac{t}{\Lambda^2}\right)^{-\alpha} \qquad \alpha = 3, \quad \Lambda = 0.8\ \mathrm{GeV}$$

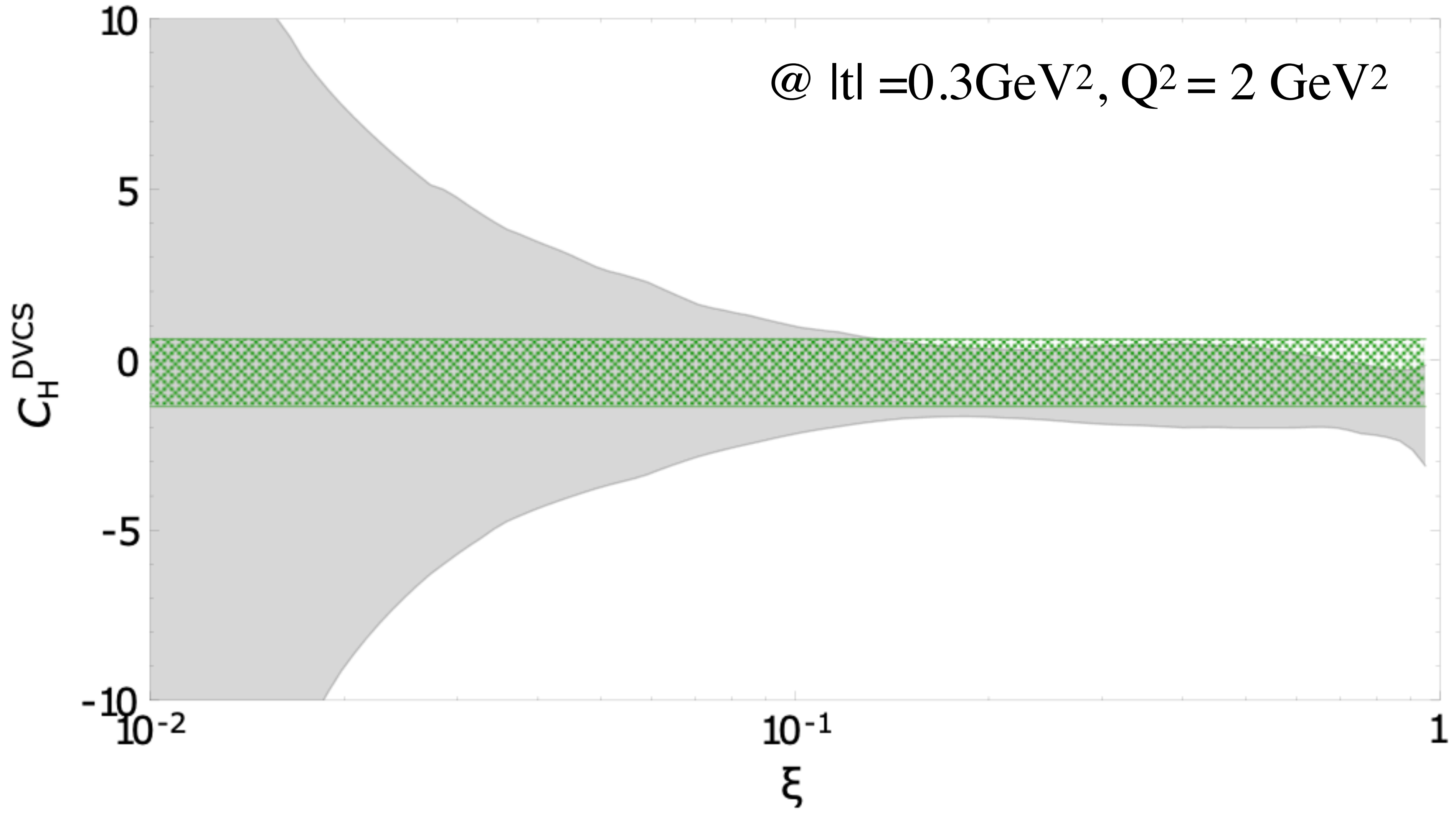
Subtraction constant

H. Dutrieux et al.,
Eur. Phys. J. C 81 (2021), 300

- Subtraction constant:
  - ANN analysis
  - Model dependent extraction







Subtraction constant

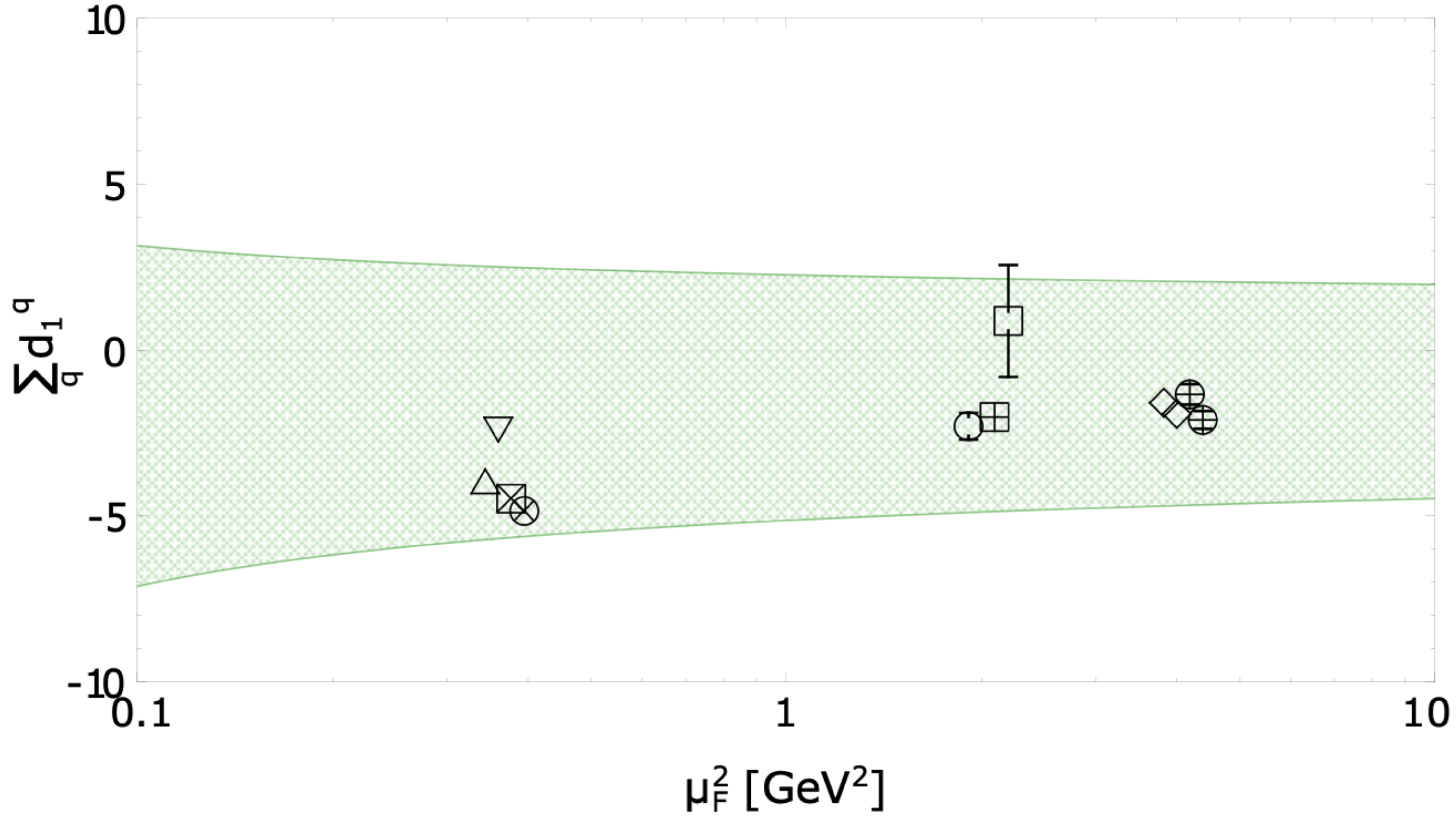
H. Dutrieux et al.,
Eur. Phys. J. C 81 (2021), 300

- Obtained values

Parameter	Value
$d_1^{uds}(\mu_F^2)$	-0.5 ± 1.2
$d_1^c(\mu_F^2)$	-0.0020 ± 0.0053
$d_1^g(\mu_F^2)$	-0.6 ± 1.6

@ $\mu_F^2 = 2\ \mathrm{GeV}^2$

- Comparison with other extractions and theory



No.	Marker	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type
1	○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
2	□	0.88 ± 1.69	2.2	2	from experimental data
3	◇	-1.59	4	2	t-channel saturated model
		-1.92	4	2	t-channel saturated model
4	△	-4	0.36	3	χQSM
5	▽	-2.35	0.36	2	χQSM
6	⊠	-4.48	0.36	2	Skyrme model
7	⊞	-2.02	2	3	LFWF model
8	⊗	-4.85	0.36	2	χQSM
9	⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\mathrm{MS}}$)
		-2.11 ± 0.27	4	2	lattice QCD ($\overline{\mathrm{MS}}$)

Subtraction constant

H. Dutrieux et al.,
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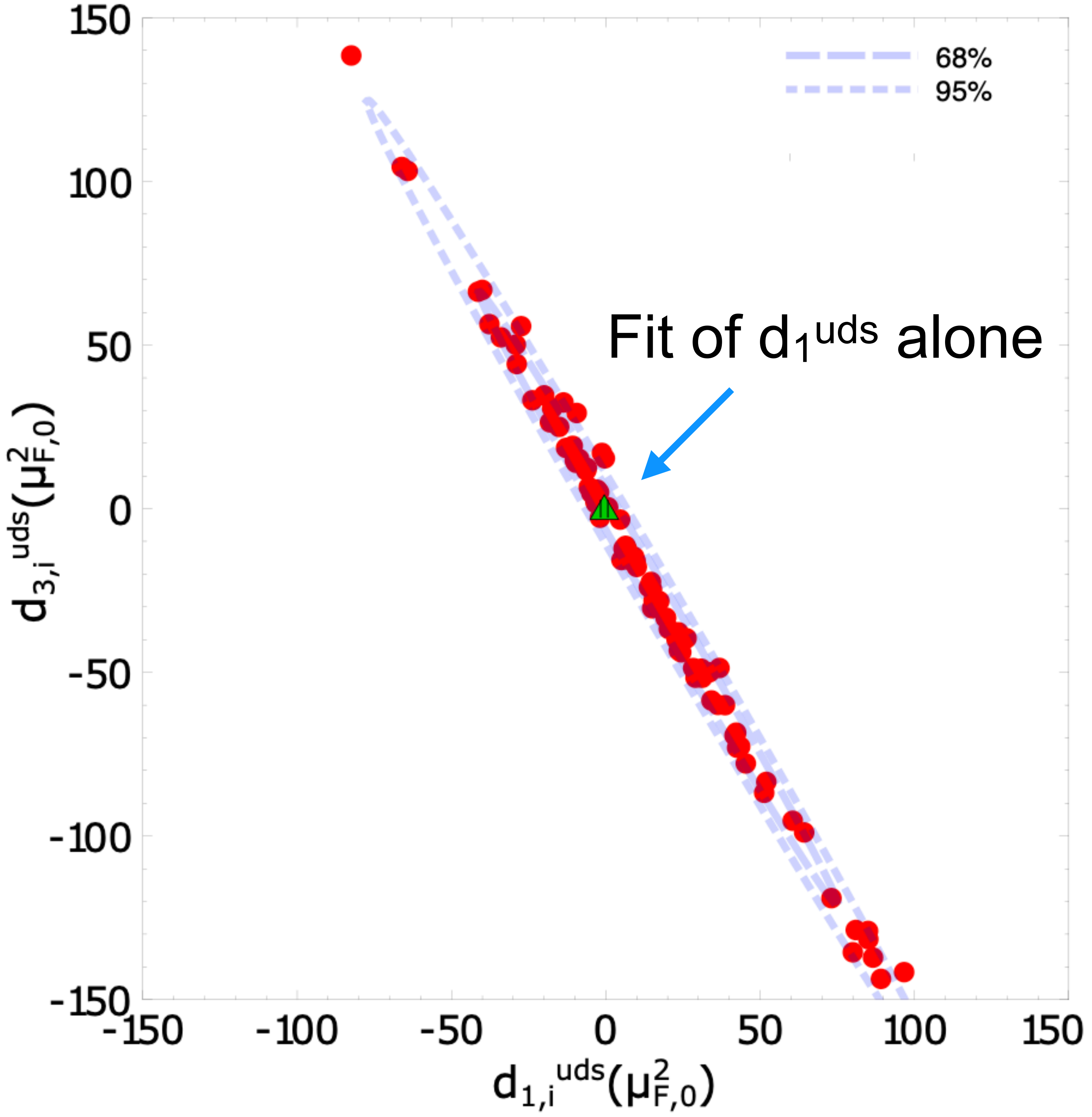
- Alternative fit with d_1 and d_3 extracted together

$$\begin{array}{ll} d_1^{uds}(\mu_F^2) & 11 \pm 25 \\ d_3^{uds}(\mu_F^2) & -11 \pm 26 \end{array}$$

$@\mu_F^2 = 2\ \mathrm{GeV}^2$

- Correlation between d_1^{uds} and d_3^{uds} →



$@\mu_{F,0}^2 = 0.1\ \mathrm{GeV}^2$

Subtraction constant

H. Dutrieux et al.,
Eur. Phys. J. C 81 (2021), 300

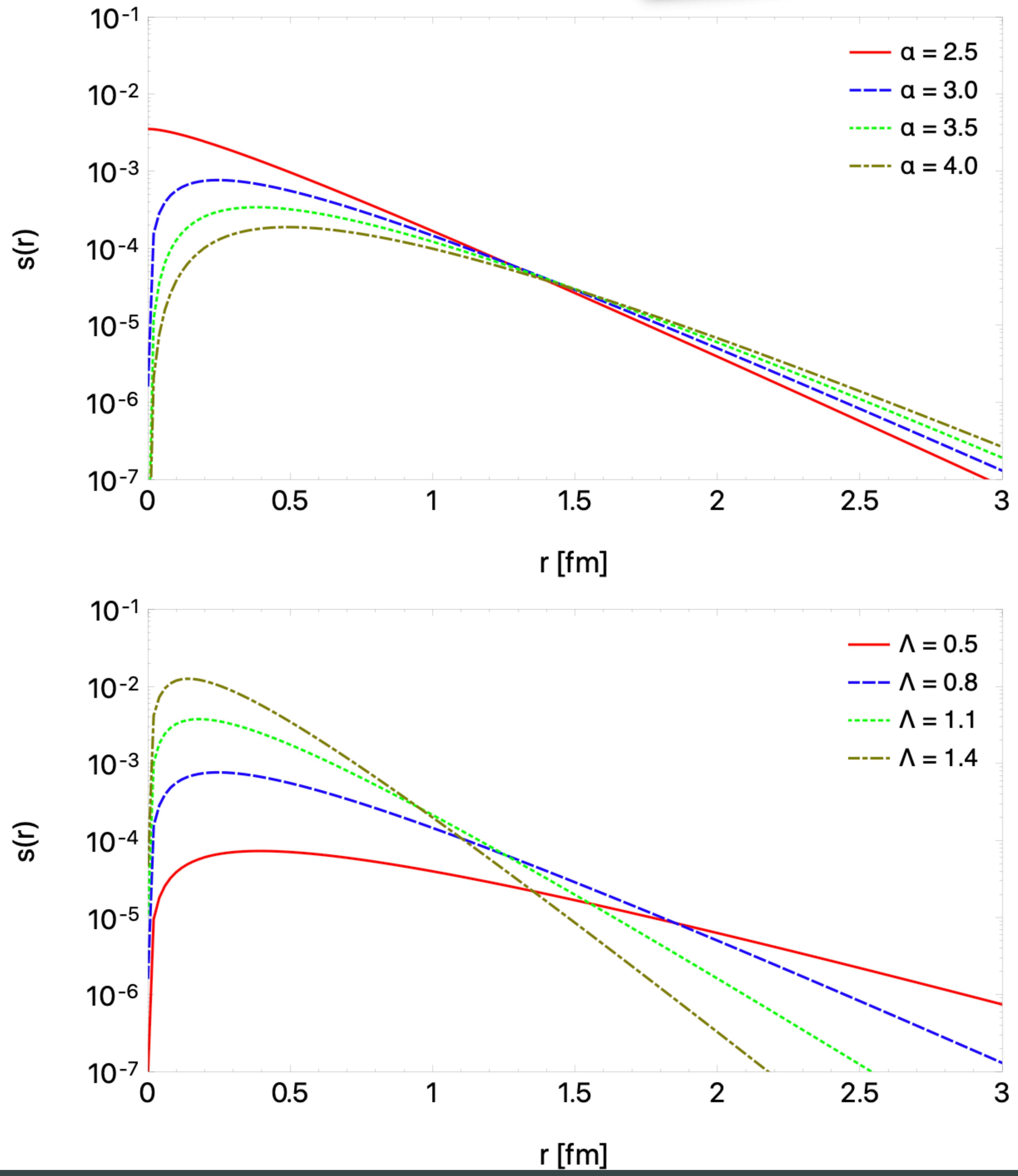
- In this analysis t-dependance of d_1 modelled as:

$$d_1(t) = d_1 \left(1 - \frac{t}{\Lambda^2}\right)^{-\alpha}$$

with

$$\alpha = 3 \qquad \Lambda = 0.8 \text{ GeV}$$

- Impact of parameter values on profiles of pressure anisotropies



@ d_1=-3 Λ=0.8 GeV

@ d_1=-3 α=3

Modelling in (β, α)-space

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

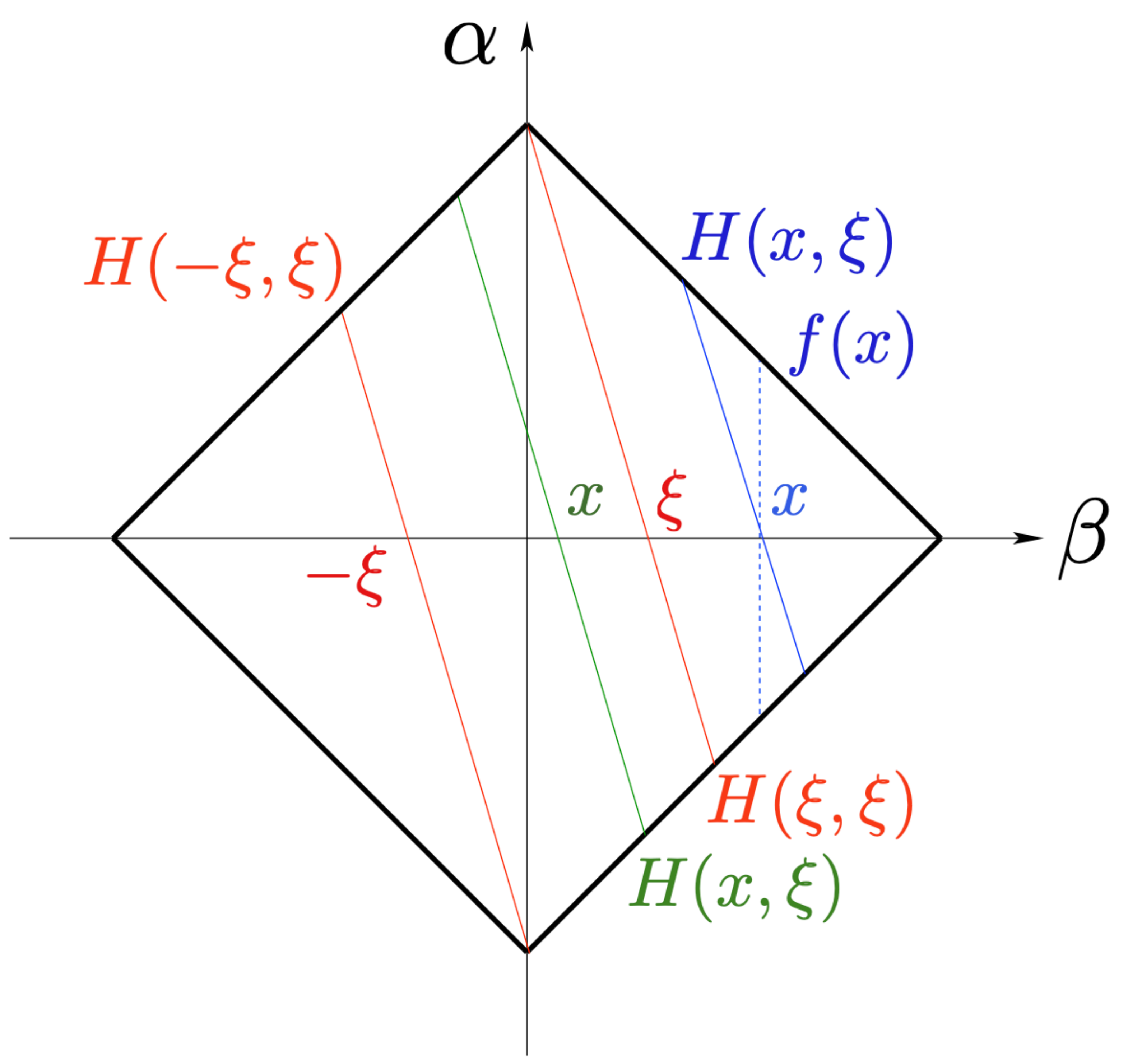
Double distribution:

$$H(x, \xi, t) = \int \mathrm{d}\Omega\, F(\beta, \alpha, t)$$

where:

$$\mathrm{d}\Omega = \mathrm{d}\beta\, \mathrm{d}\alpha\, \delta(x - \beta - \alpha\xi)$$

$$|\alpha| + |\beta| \leq 1$$



from PRD83, 076006, 2011

We also consider non-parametric GPD modelling in (x, ξ)-space, see our paper
The drawback of this modelling is that one can not keep PDF singularity for only x=0 and ξ=0

Modelling in (β, α)-space

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

Double distribution: $(1-x^2)F_C(\beta,\alpha) + (x^2-\xi^2)F_S(\beta,\alpha) + \xi F_D(\beta,\alpha)$

Classical term:

$$F_C(\beta,\alpha) = f(\beta)h_C(\beta,\alpha)\frac{1}{1-\beta^2}$$

$$f(\beta) = \mathrm{sgn}(\beta)q(|\beta|)$$

$$h_C(\beta,\alpha) = \frac{\mathrm{ANN}_C(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_C(|\beta|,\alpha)}$$

Shadow term:

$$F_S(\beta,\alpha) = f(\beta)h_S(\beta,\alpha)$$

$$f(\beta) = \mathrm{sgn}(\beta)q(|\beta|)$$

$$h_S(\beta,\alpha)/N_S = \frac{\mathrm{ANN}_S(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_S(|\beta|,\alpha)} - \frac{\mathrm{ANN}_{S'}(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_{S'}(|\beta|,\alpha)} .$$

$$\mathrm{ANN}_{S'}(|\beta|,\alpha) \equiv \mathrm{ANN}_C(|\beta|,\alpha)$$

D-term:

$$F_D(\beta,\alpha) = \delta(\beta)D(\alpha)$$

$$D(\alpha) = (1-\alpha^2)\sum_{\substack{i=1\\ \mathrm{odd}}} d_i C_i^{3/2}(\alpha)$$

Modelling in (β, α)-space

V. Bertone et al.,
Phys. Rev. D 103 (2021) 11, 114019

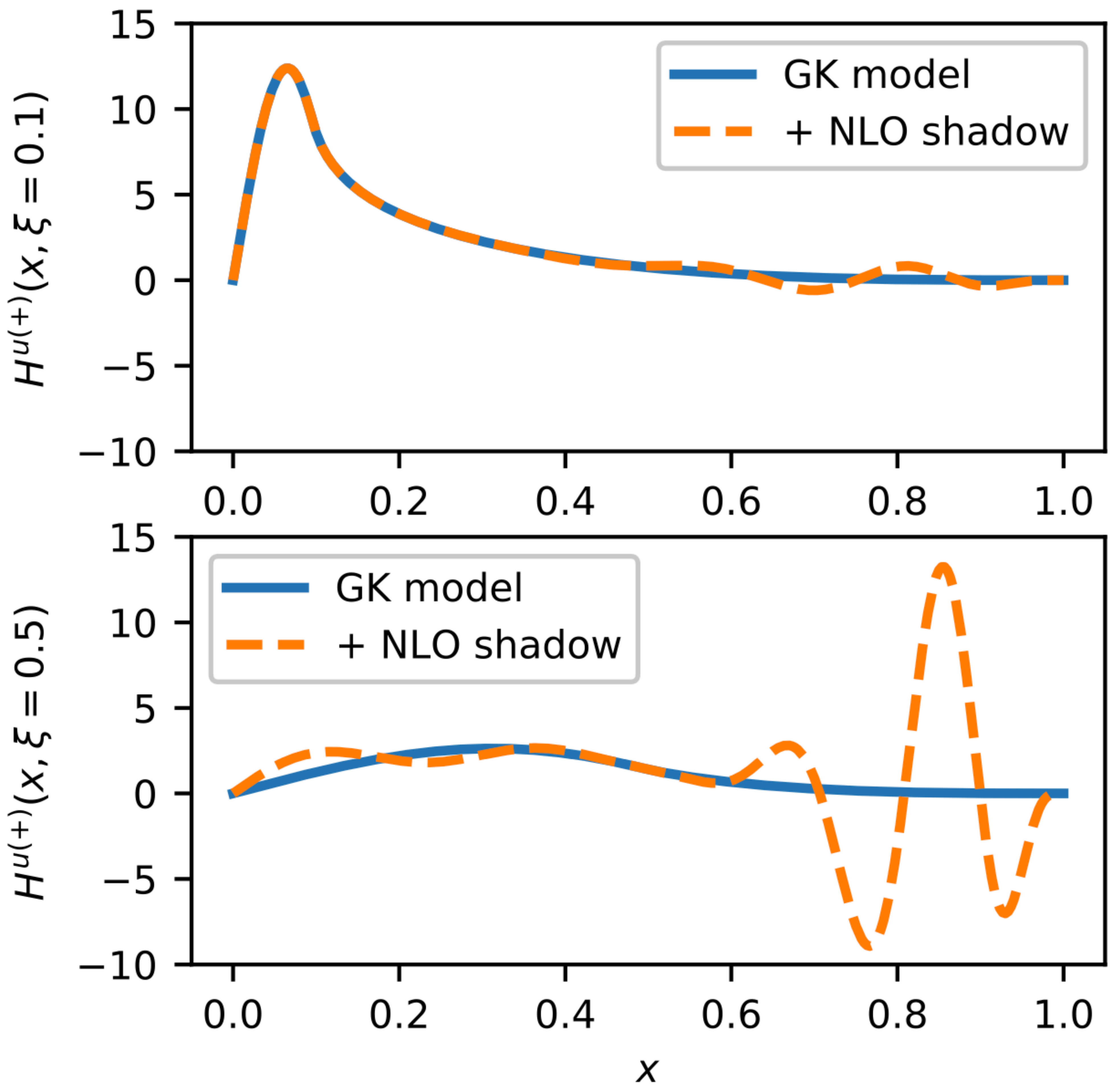
Shadow term is closely related to the so-called shadow GPDs

Shadow GPDs have considerable size and:

- at the initial scale do not contribute to both PDFs and CFFs
- at some other scale they contribute negligibly

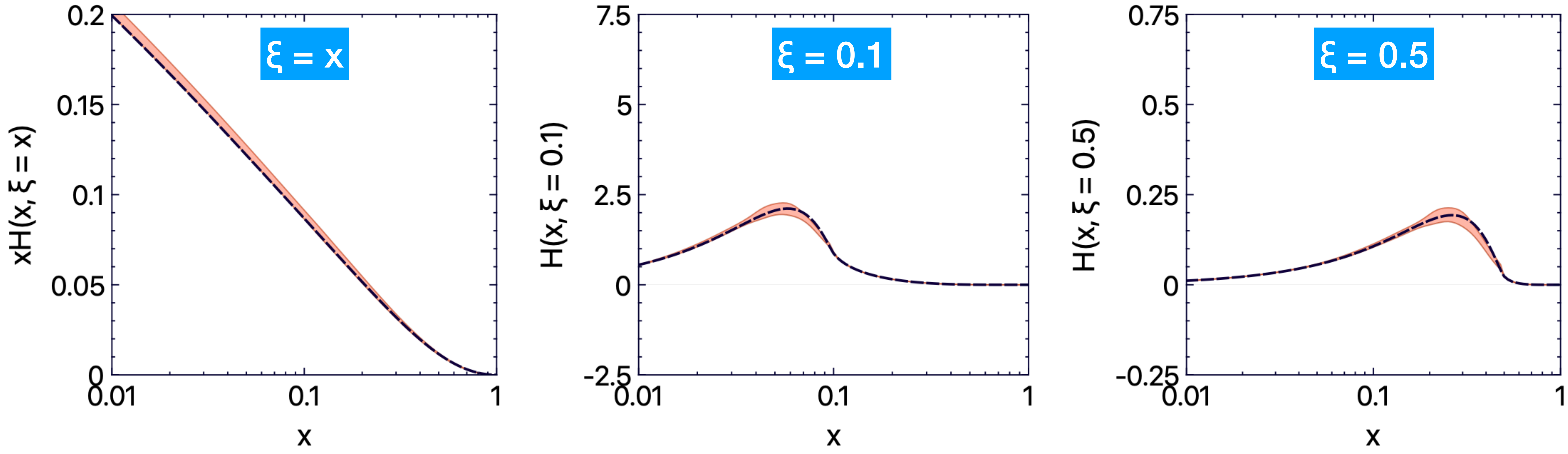
making the deconvolution of CFFs ill-posed

We found such GPDs for both LO and NLO



Demonstration of results

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

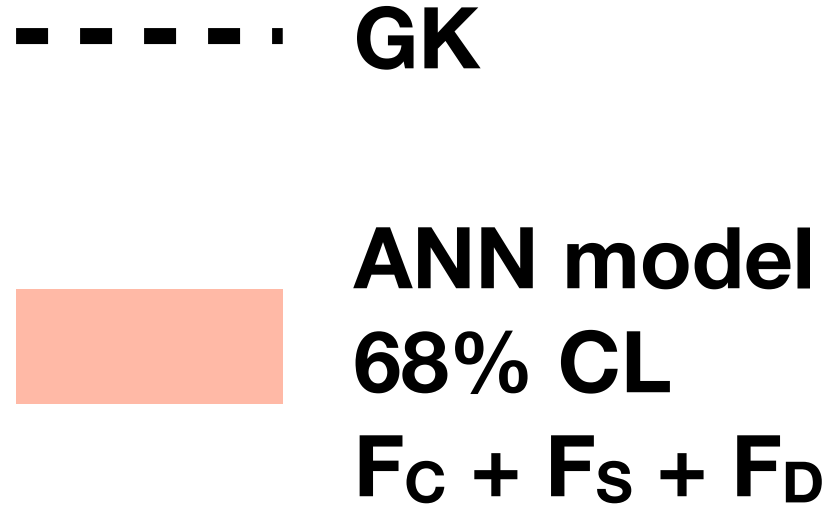


Conditions:

- Input: 400 x ≠ ξ points generated with GK model
- Positivity not forced

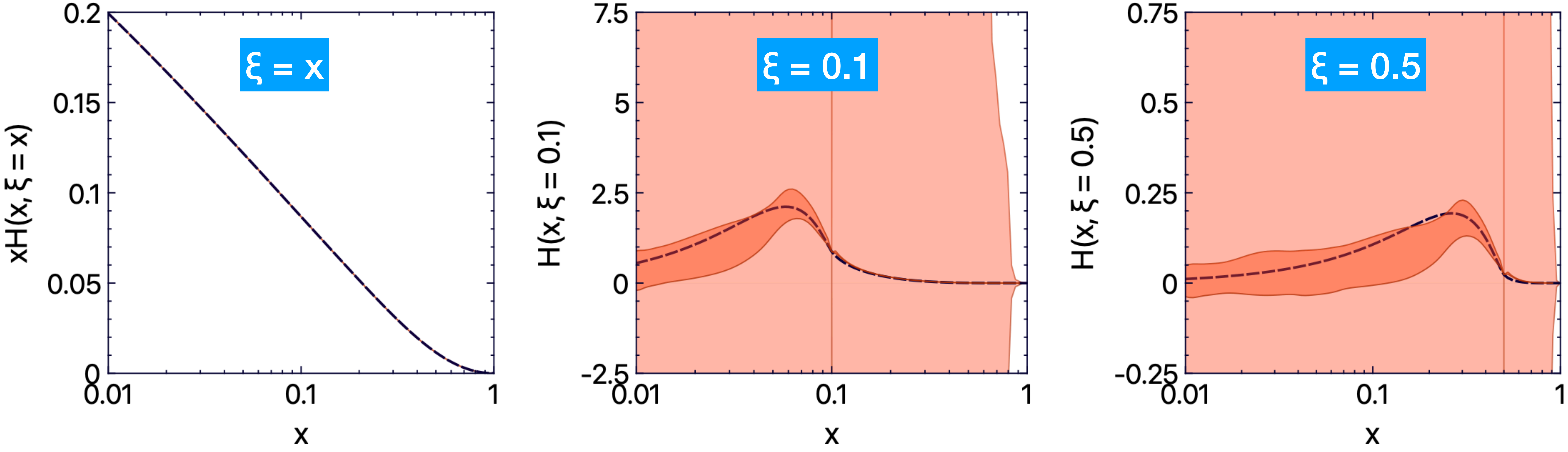
Technical detail of the analysis:

- Minimisation with genetic algorithm
- Replication for estimation of model uncertainties
- “Local” detection of outliers
- Dropout algorithm for regularisation



Demonstration of results

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252



GK

ANN model 68% CL F_C

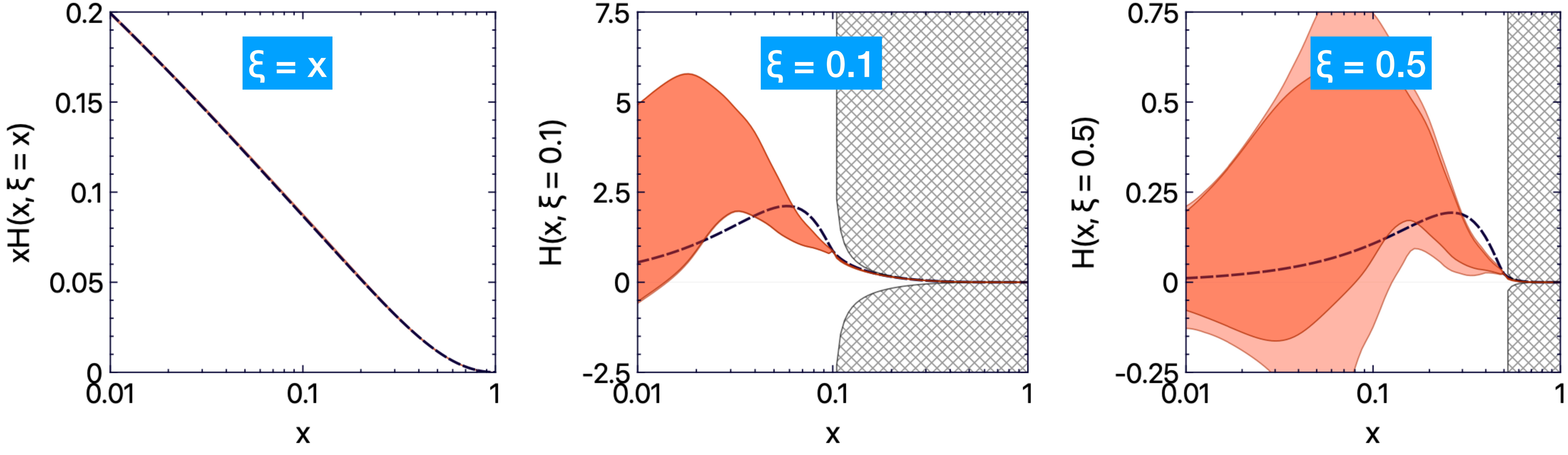
ANN model 68% CL $F_C + F_S$

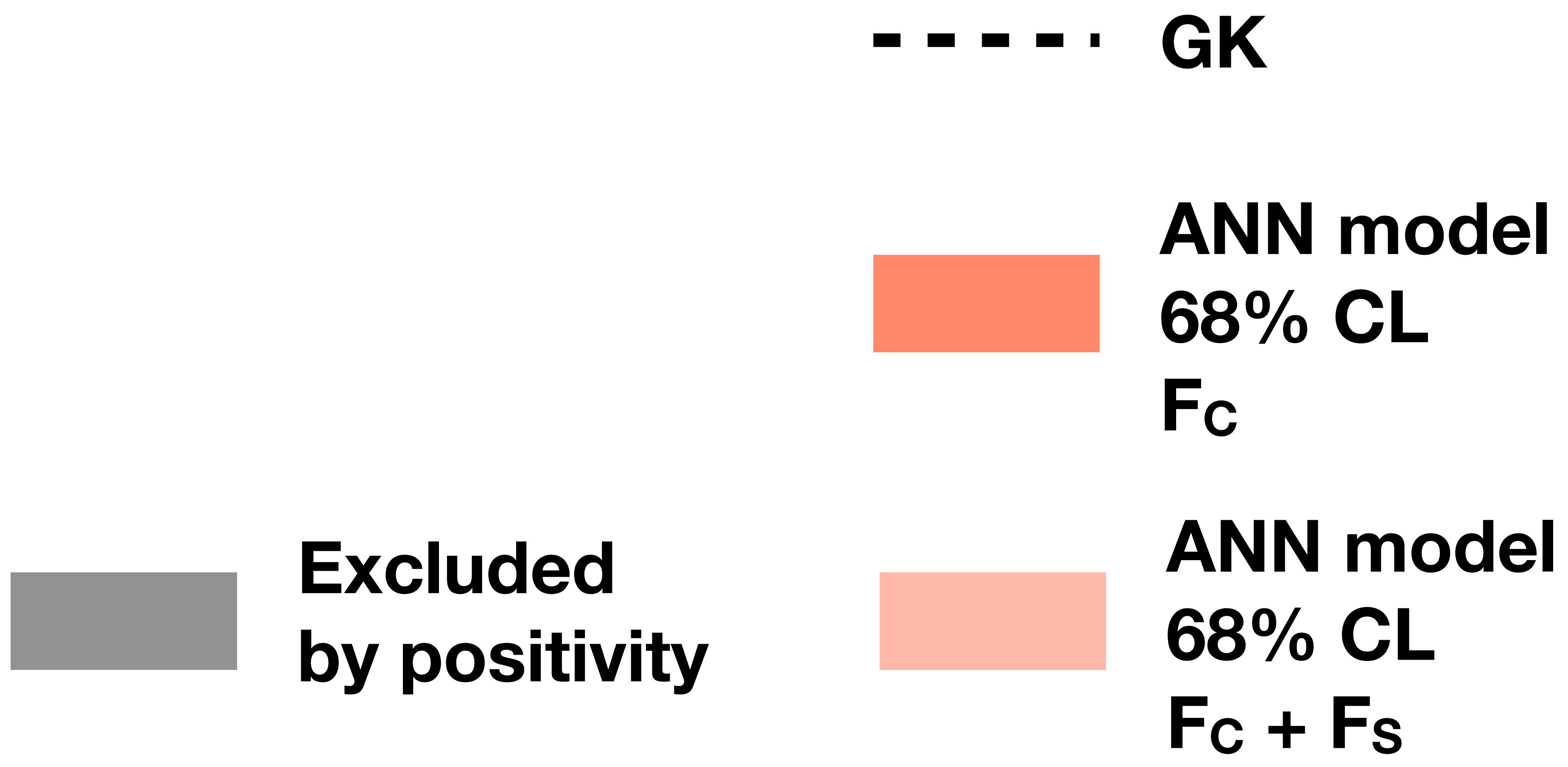
Conditions:

- Input: 200 x = ξ points generated with GK model
- Positivity not forced

Demonstration of results

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252





Conditions:

- Input: 200 x = ξ points generated with GK model
- Positivity **forced**

PARTONS project

B. Berthou et al.,
Eur. Phys. J. C 78 (2018) 6, 478

- PARTONS - open-source framework to study GPDs
  → http://partons.cea.fr
- Come with number of available physics developments implemented
- Written in C++, also available via virtual machines (VirtualBox) and containers (Docker)
- Addition of new developments as easy as possible
- Developed to support effort of GPD community,
  can be used by both theorists and experimentalists
- v3 version of PARTONS is now available!

EpIC MC generator

E. C. Aschenauer et al.,
Eur. Phys. J. C 82 (2022) 9, 819

- Novel MC generator called EpIC released
  → *https://pawelsznajder.github.io/epic*

- EpIC is based on PARTONS

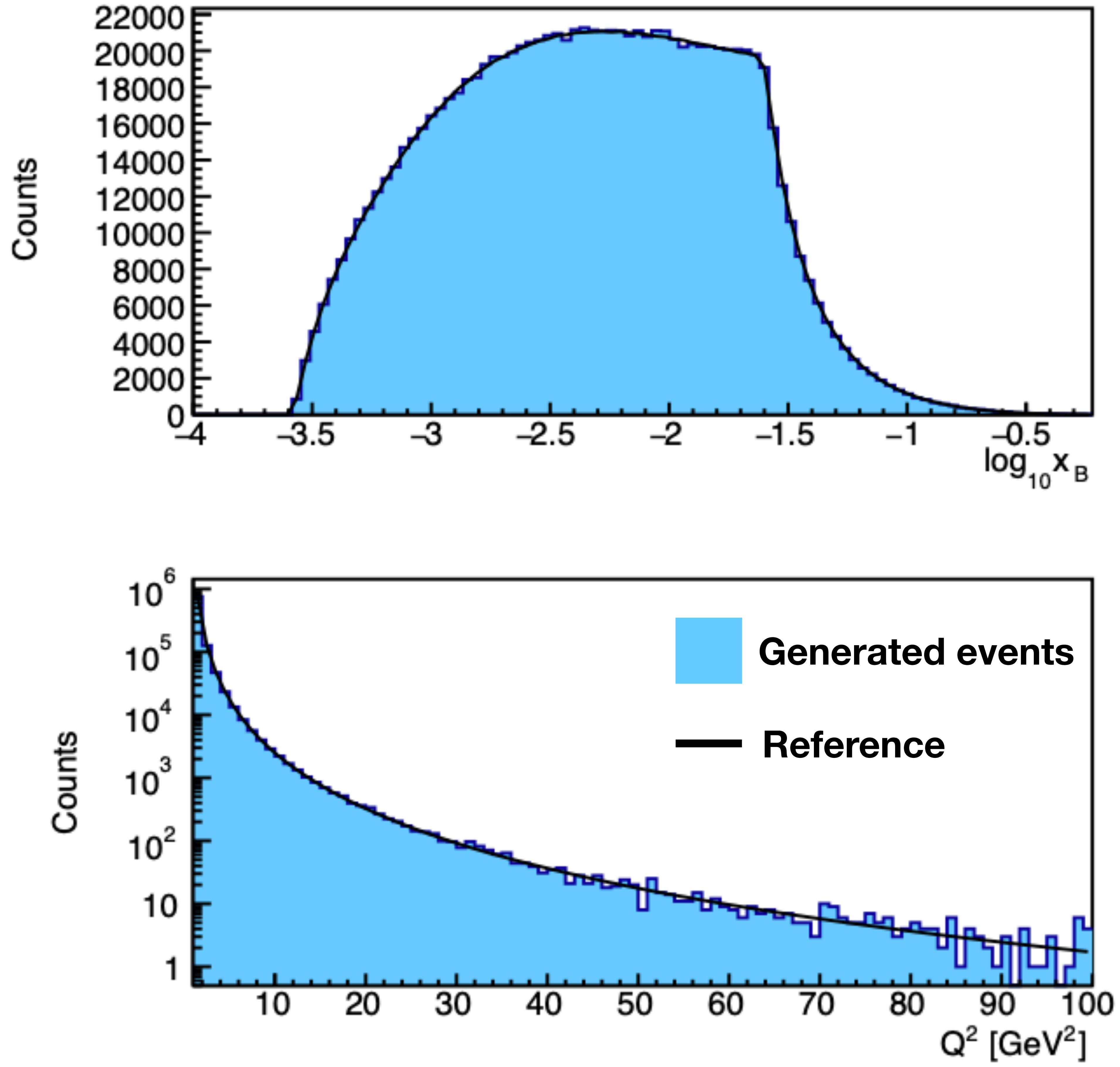
- EpIC is characterised by:
  - flexible architecture that utilises a modular programming paradigm
  - a variety of modelling options, including radiative corrections
  - multichannel capability (now: DVCS, TCS, DVπ^0P, diphoton; coming soon: DDVCS, J/ψ)

- This is the new tool to be use in the precision era commenced by the new generation of experiments

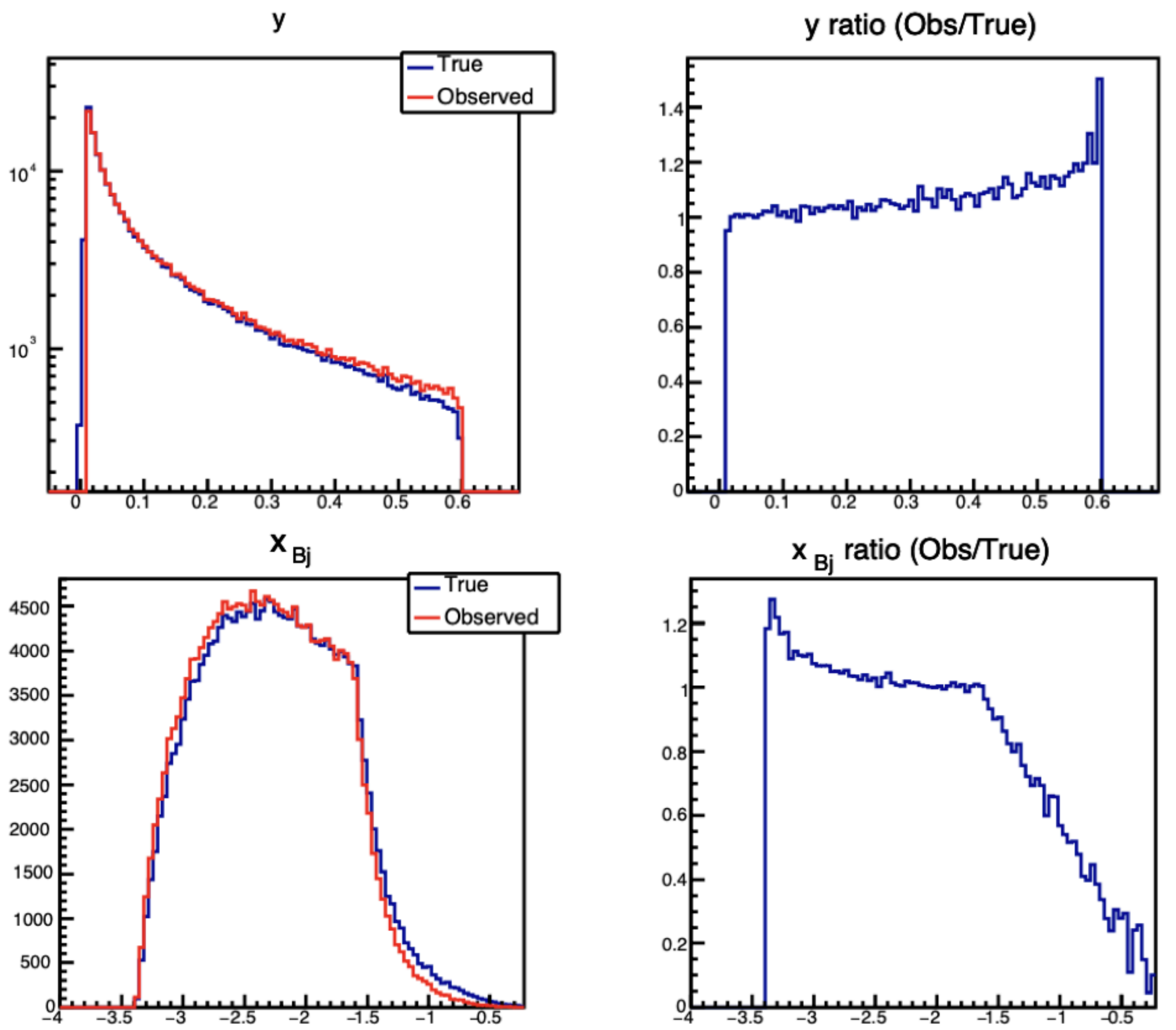
EpIC MC generator

E. C. Aschenauer et al.,
Eur. Phys. J. C 82 (2022) 9, 819

Generated events compared to reference



Effects of radiative corrections



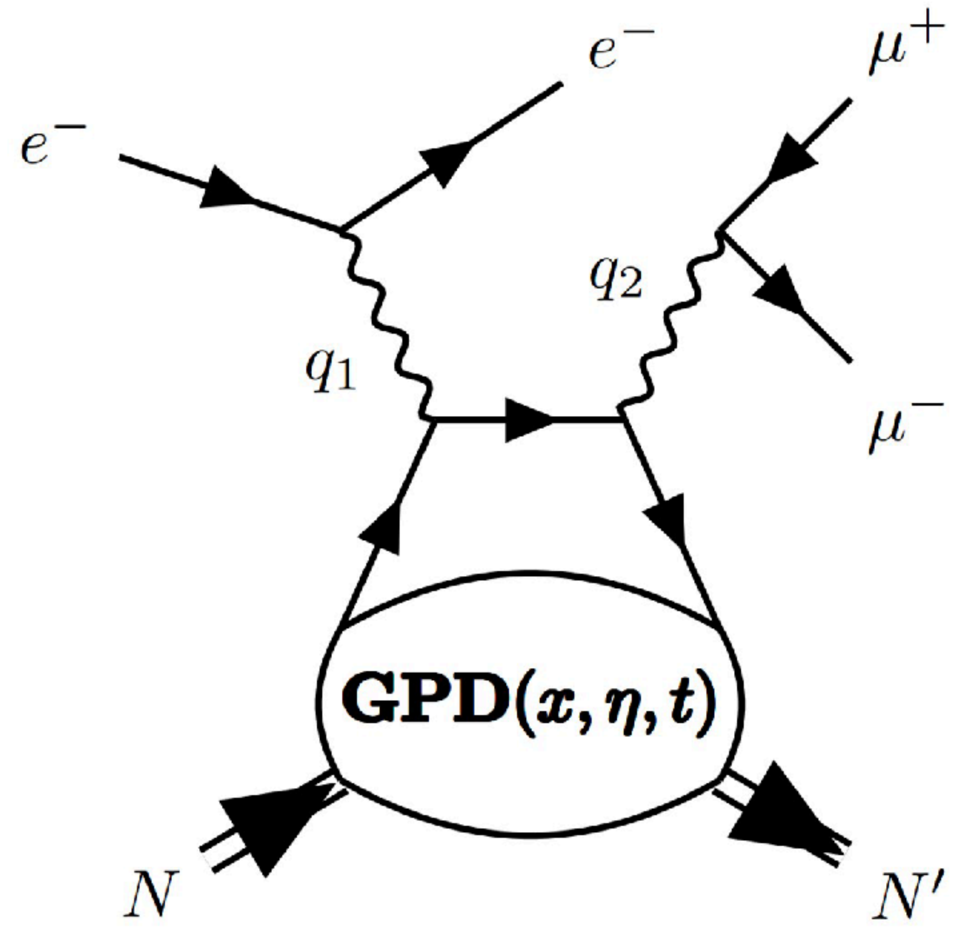
plots for 10 GeV x 100 GeV beam energies, some kinematic cuts included

Double DVCS

K. Deja, V. Martínez-Fernández,
PS, J. Wagner
preliminary

- The process allows to probe GPDs outside x=ξ line, but is much more challenging experimentally

$$\mathcal{A}_{\mathrm{DDVCS}} \overset{LO}{\sim} \int_{-1}^{1} dx \, \frac{1}{x - \xi + i0} \mathrm{GPD}(x, \eta, t)$$

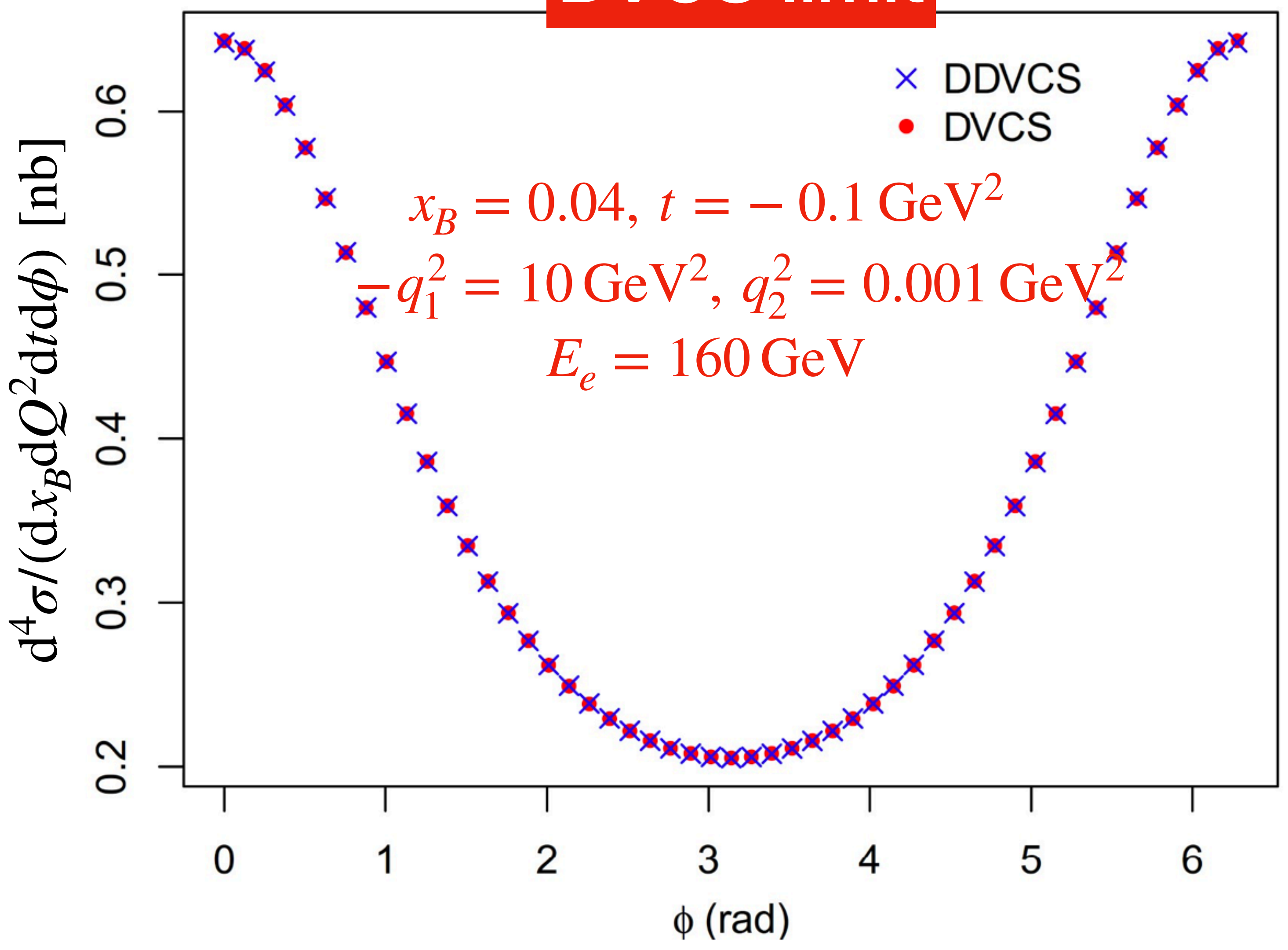


- We are revisiting DDVCS for phenomenological studies, i.e. we reevaluate DDVCS and related BH amplitudes using Kleiss-Stirling technique

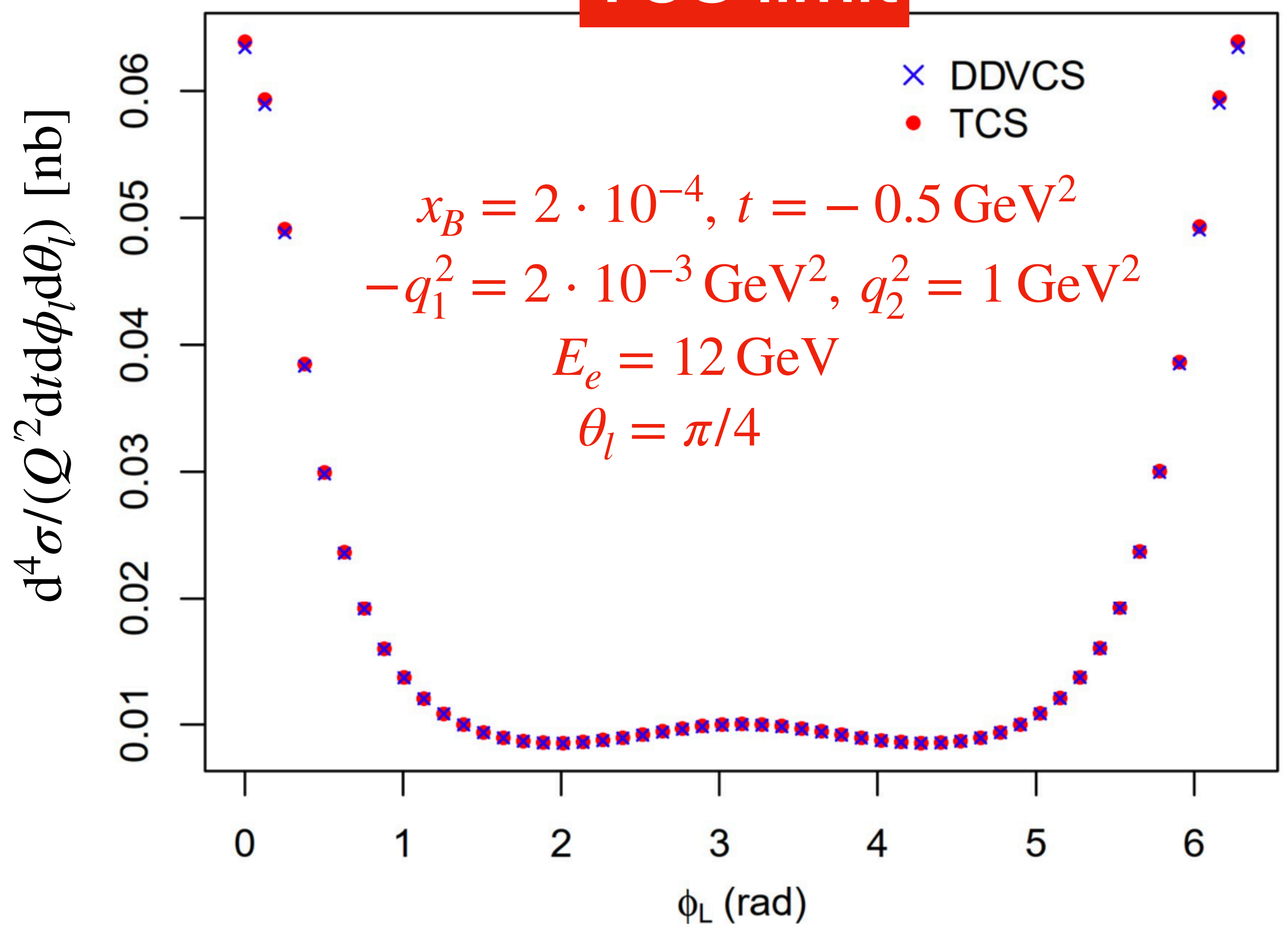
- We plane to release obtained formulae in PARTONS and EpIC MC generator

Preliminary results:
BH cross-section in DVCS and TCS limits

DVCS limit



TCS limit



Summary

- Substantial progress in:
  - understanding of fundamental problems, like deconvolution of CFFs, and analysis methods
    → important for extraction of GPDs
  - modelling of GPD, fulfilling all theory-driven constraints (including positivity)
    → subject not touched enough in the current literature
    → developed in mind for easy inclusion of latticeQCD data
  - addressing the long-standing problem of model dependency of GPDs
    → nontrivial and timely analysis
  - description of exclusive processes
    → new sources of GPD information
  - delivering open-source tools for the community
    → to suport both experimentalists and theoreticians

This progress is important for the precision era of GPD extraction allowed by the new generation of experiments

We are very open for new collaboration opportunities!

next EIC UG meeting in Warsaw (last week of July)

see you there!

The Polish Electron-Ion Collider Group