Prospects on GPDs and structure functions of spin-1 deuteron

From April 1, 2022

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http://research.kek.jp/people/kumanos/

View of the Ikebukuro downtown from my JWU office

APCTP Workshop on the Physics of Electron Ion Collider
(In-person/Online) Incheon, South Korea, November 2-4, 2022
https://indico.knu.ac.kr/event/592/

September 14, 2022

Contents

Generalized-parton-distribution (GPD) projects (complementary to EIC)

- Introduction to GPDs and gravitational form factors
- Generalized distribution amplitudes (= timelike GPDs) and extraction of gravitational form factors
- GPDs at hadron accelerator facilities, *e.g.* at J-PARC
- GPDs at neutrino facilities

Spin-1 structure functions (directly related to EIC)

- Motivations for spin-1 structure functions: b_1, gluon transversity, TMDs — slightly long introduction
- TMDs, PDFs, multiparton distributions, fragmentation functions up to twist-4
- Useful relations for twist-3 PDFs

(go through quickly (please read original papers for the details))

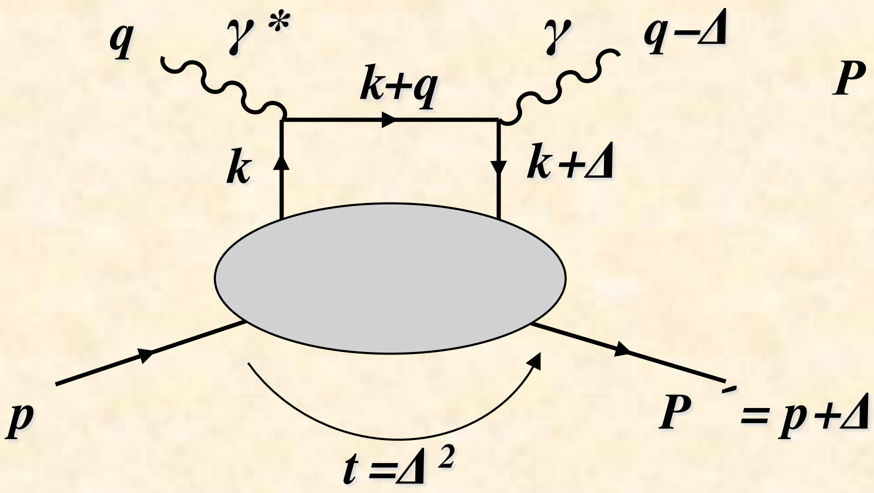
- Experimental prospects

Collaboration opportunities

GPDs

Generalized Parton Distributions (GPDs)

See B. Pasquini's talk at DIS2022 for updated information.



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \dfrac{Q^2}{2p\cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewdness parameter $\xi = \dfrac{p^+ - p'^+}{p^+ + p'^+} = -\dfrac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

Forward limit: PDFs $H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

Second moments: Angular momenta

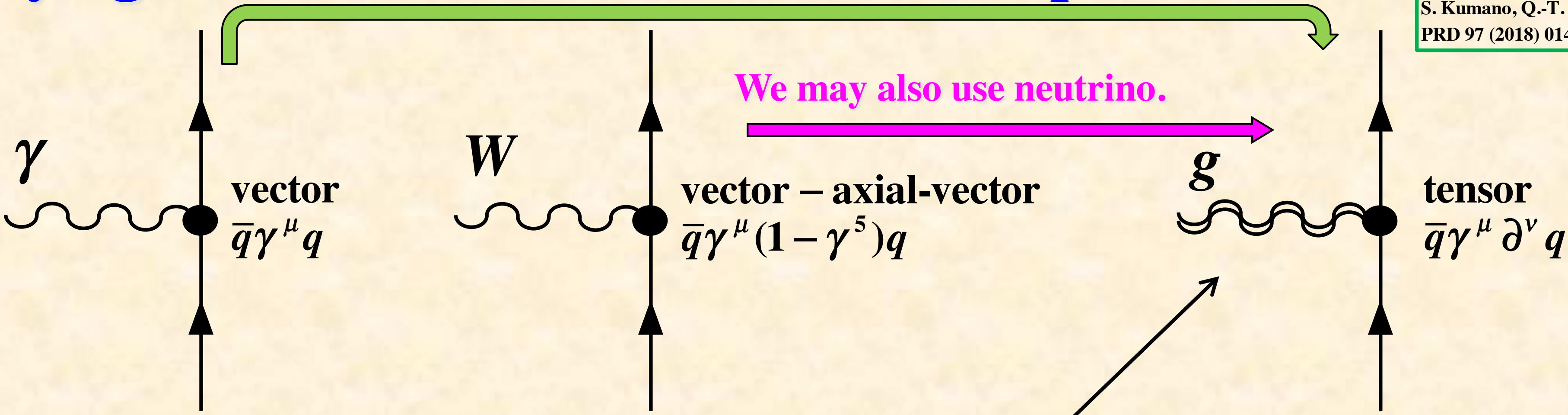
Sum rule: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \frac{1}{2}\Delta q + L_q$

$\Rightarrow$ probe L_q, key quantity to solve the spin puzzle!

Why "gravitational" interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020.



It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2)\gamma^+ q(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

Non-local operator of GPDs/GDAs:

$$(P^+)^n \int dx\, x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z^+=0,\vec{z}_\perp=0}$$

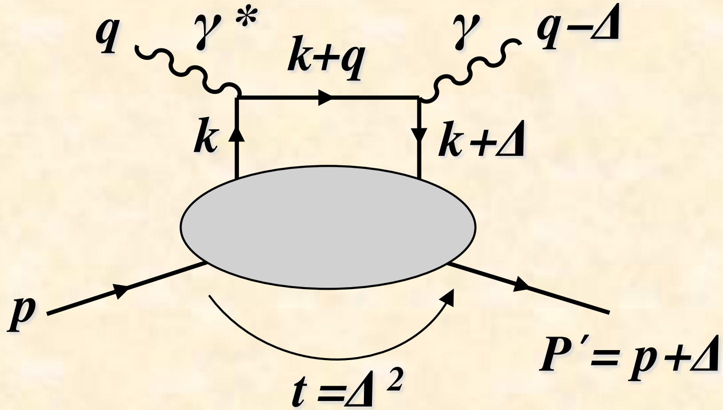
$$= \left(i\frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z=0}$$

$$= \bar{q}(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+ \right)^{n-1} q(0)$$

= energy-momentum tensor of a quark for $n = 2$
(electromagnetic for $n = 1$)

= source of gravitiy

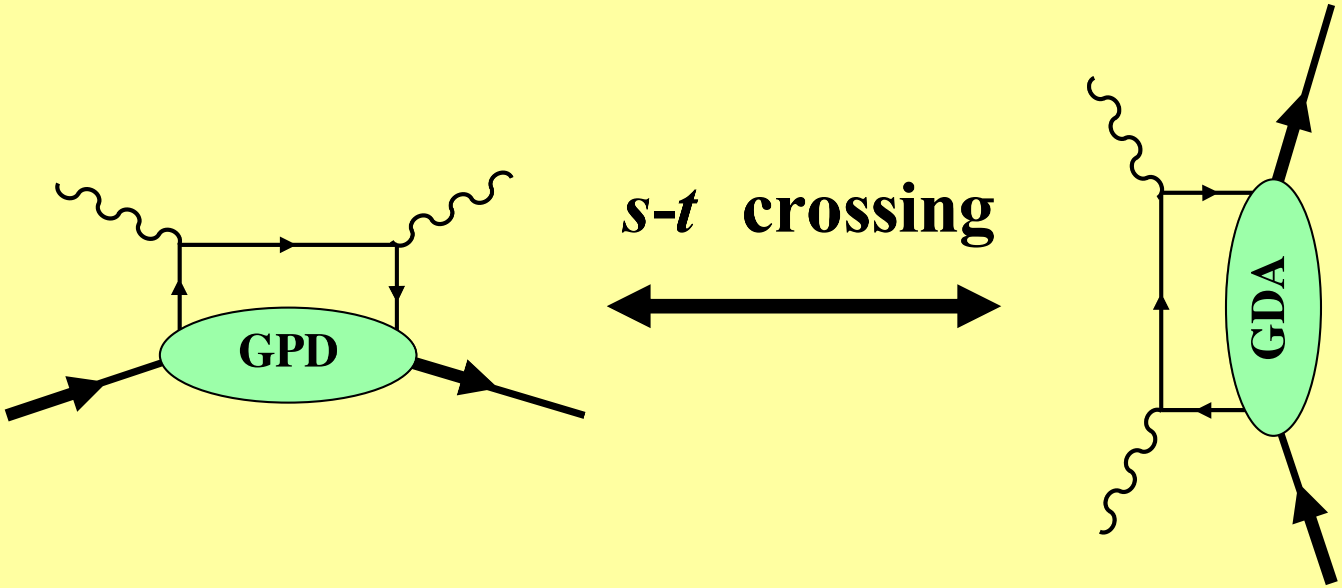
Virtual Compton
or (timelike) two-photon process



Generalized Distribution Amplitudes (GDAs) and extraction of gravitational form factors from KEKB data

Spacelike GPDs

GDA = Timelike GPDs



SK, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

GPD $H_q^h(x,\xi,t)$ and GDA(= timelike GPD) $\Phi_q^{hh}(z,\zeta,W^2)$

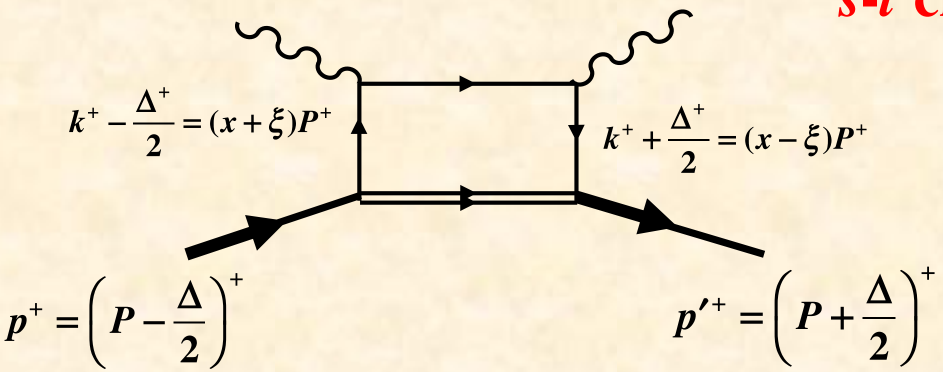
GPD: $H_q(x,\xi,t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | h(p) \rangle \big|_{y^+=0,\vec{y}_\perp=0}$, $\quad P^+ = \frac{(p+p')^+}{2}$

GDA: $\Phi_q(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p)\bar{h}(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | 0 \rangle \big|_{y^+=0,\vec{y}_\perp=0}$

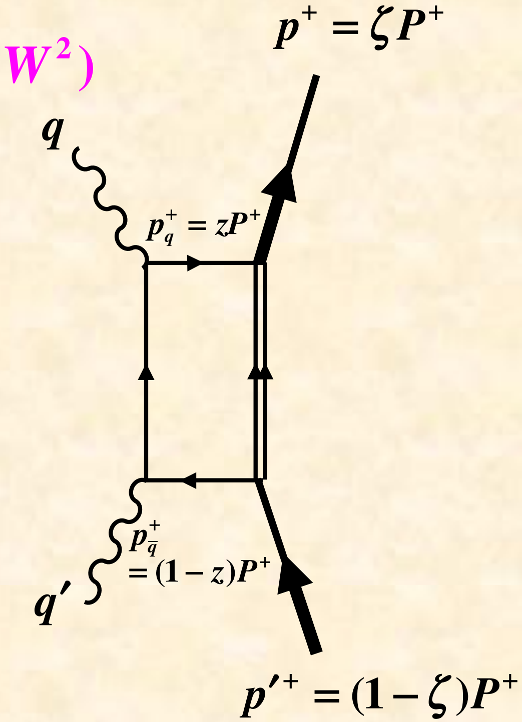
DA: $\Phi_q^\pi(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2)\gamma^+\gamma_5\psi(y/2) | 0 \rangle \big|_{y^+=0,\vec{y}_\perp=0}$

$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

***s-t* crossing**



$$z \Leftrightarrow \frac{1-x/\xi}{2}$$
$$\zeta \Leftrightarrow \frac{1-1/\xi}{2}$$
$$W^2 \Leftrightarrow t$$



JLab / COMPASS | **KEKB**

$P = \frac{p+p'}{2}, \ \Delta = p' - p$

Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

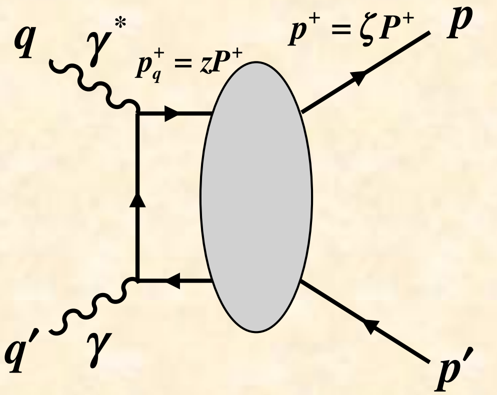
Skewdness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

Bjorken variable for $\gamma\gamma^*$: $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1+\beta\cos\theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Cross section for $\gamma^*\gamma \to \pi^0\pi^0$



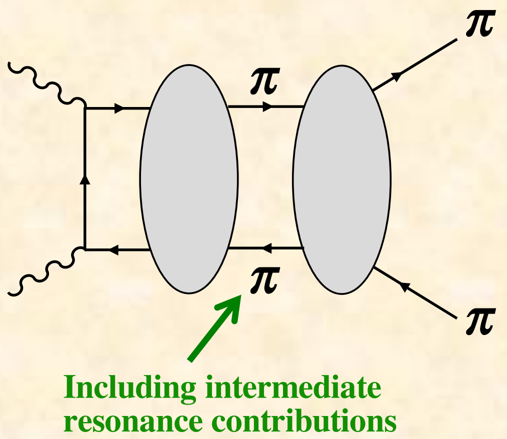
$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}\,\overline{\sum_{\lambda,\lambda'}}|\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i\int d^4\xi\, e^{-i\xi\cdot q}\left\langle \pi(p)\pi(p')\right| T J_{em}^\mu(\xi) J_{em}^\nu(0) \left|0\right\rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2}\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = -\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')g_T^{\mu\nu}\sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

GDA (timelike GPD): $\Phi_q^{\pi\pi}(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \left\langle \pi(p)\pi(p')\right| \bar\psi(-y/2)\gamma^+\psi(y/2)\left|0\right\rangle\Big|_{y^+=0,\vec{y}_\perp=0}$

$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}\,|A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$



- **Continuum: GDAs without intermediate-resonance contribution**

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha(1-z)^\alpha(2z-1)\zeta(1-\zeta)F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{\left[1+(s-4m_\pi^2)/\Lambda^2\right]^{n-1}}, \quad n = 2 \text{ according to constituent counting rule}$$

- **Resonances: There exist resonance contributions to the cross section.**

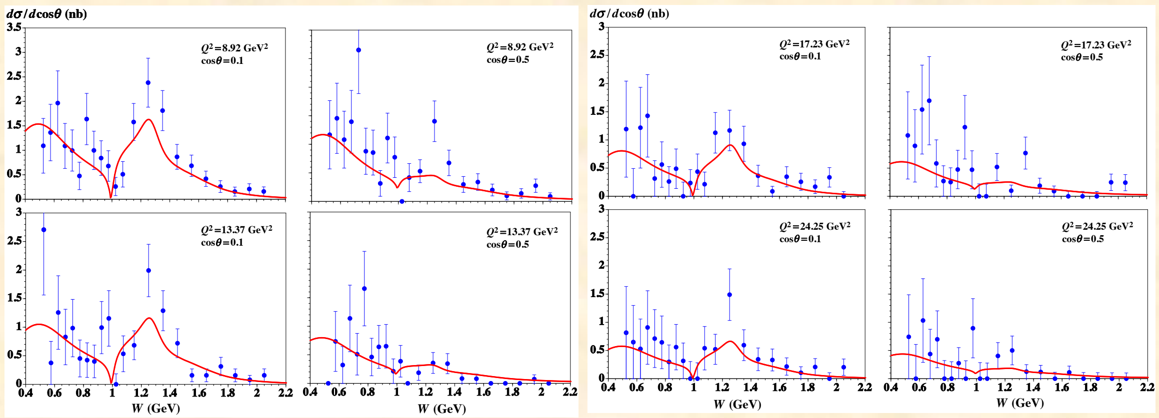
$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18N_f z^\alpha(1-z)^\alpha(2z-1)\left[\tilde B_{10}(W) + \tilde B_{12}(W)P_2(\cos\theta)\right]$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$$\tilde B_{10}(W) = \text{resonance } [f_0(500), f_0(980)] + \text{continuum}$$

$$\tilde B_{12}(W) = \text{resonance} [f_2(1270)] + \text{continuum}$$

$Q^2 = 8.92,\ 13.37\ \mathrm{GeV}^2$ $\qquad$ $Q^2 = 17.23,\ 24.25\ \mathrm{GeV}^2$



Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

Gravitational form factors and radii for pion

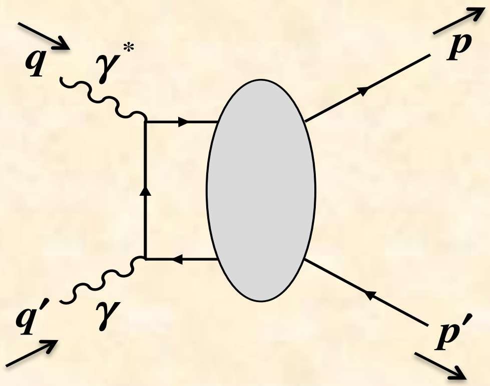
$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\langle \pi^0(p)\pi^0(p')|T_q^{++}(0)|0\rangle$$

$$\langle \pi^0(p)\pi^0(p')|T_q^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^\mu P^\nu\right)\Theta_{1,q}(s) + \Delta^\mu\Delta^\nu\Theta_{2,q}(s)\right]$$

$$P = \frac{p+p'}{2}, \qquad \Delta = p' - p$$

$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}, \Theta_{2,q}$: gravitational form factos for pion



See also Hyeon-Dong Son, Hyun-Chul Kim, PRD90 (2014) 111901.

Analyiss of $\gamma^*\gamma \to \pi^0\pi^0$ cross section

$\Rightarrow$ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z,\zeta,s)$

$\Rightarrow$ Timelike gravitational form factors $\Theta_{1,q}(s)$, $\Theta_{2,q}(s)$

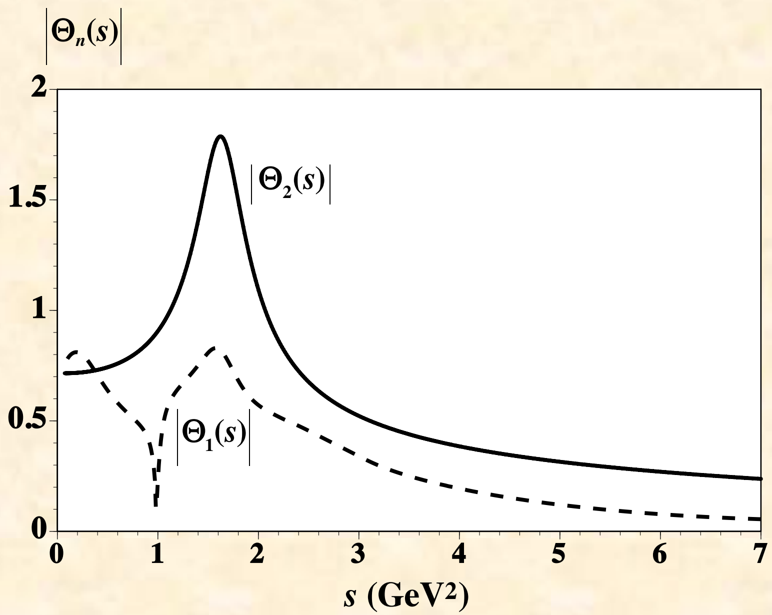
$\Rightarrow$ Spacelike gravitational form factors $\Theta_{1,q}(t)$, $\Theta_{2,q}(t)$

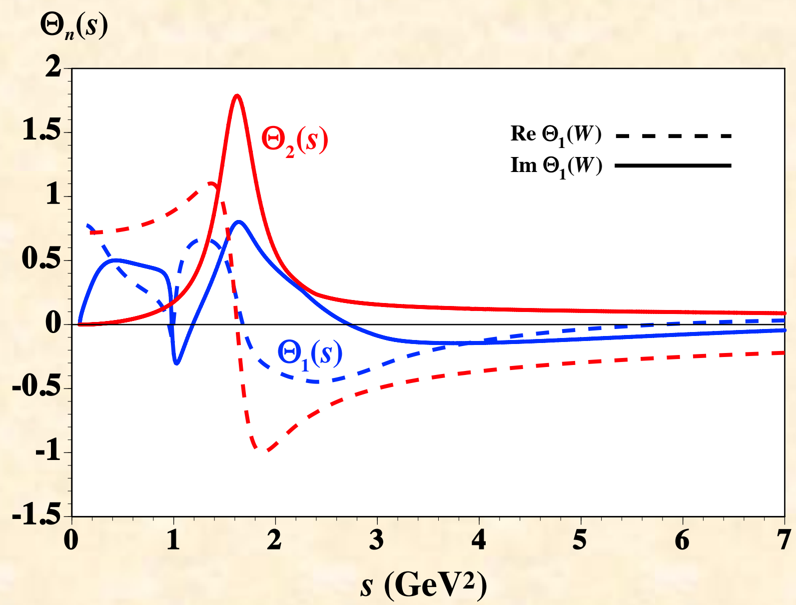
$\Rightarrow$ Gravitational radii of pion

Gravitational form factors:

Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.

Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009;
Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008;
K. Tanaka, JHEP 01 (2019) 120.





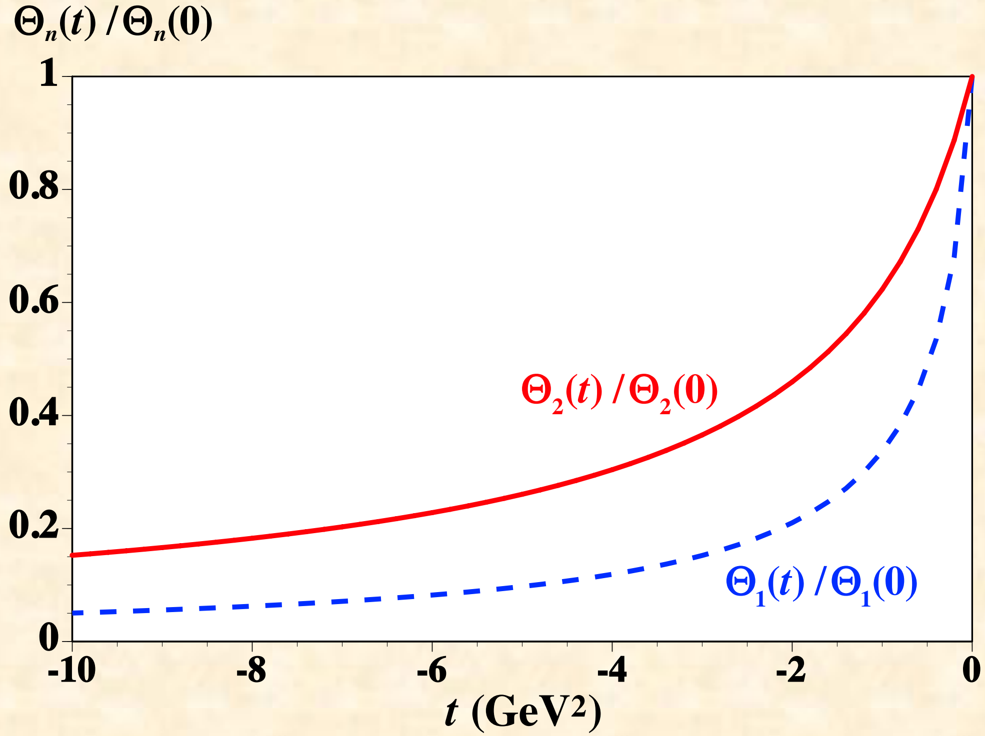
Spacelike gravitational form factors and radii for pion

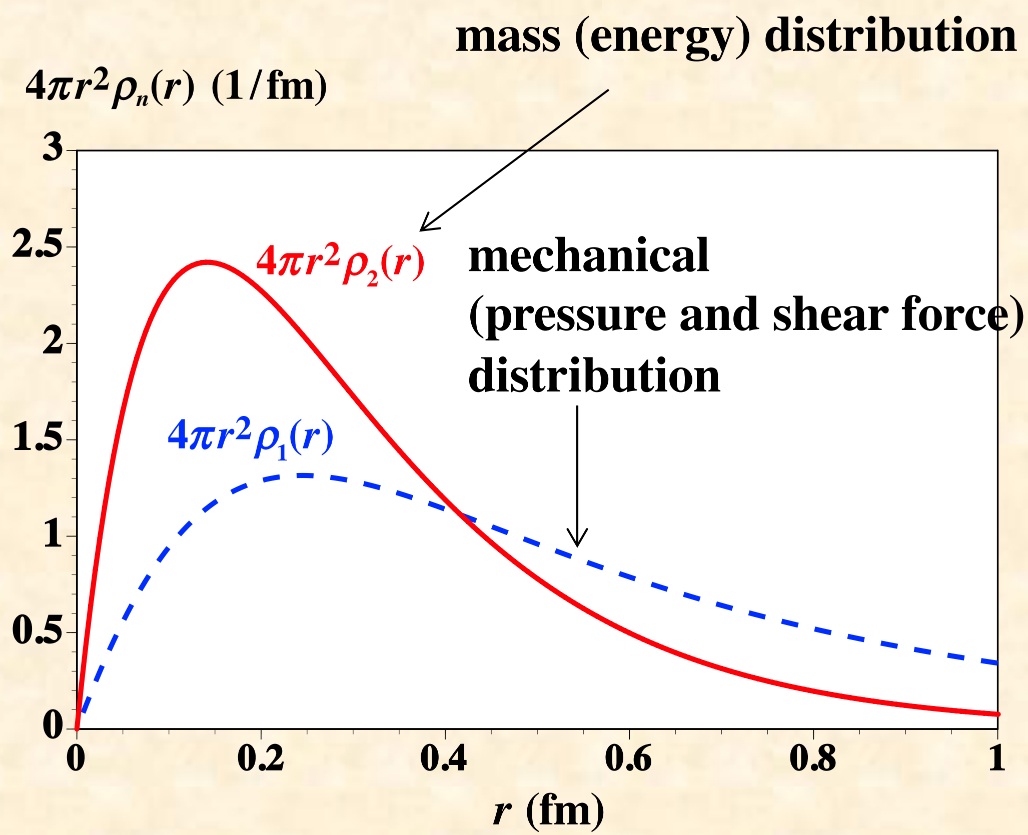
$$F(s)=\Theta_1(s),\ \Theta_1(s),\quad F(t)=\int_{4m_\pi^2}^{\infty} ds\,\frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r)=\frac{1}{(2\pi)^3}\int d^3q\,e^{-i\vec{q}\cdot\vec{r}}F(q)=\frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\,e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

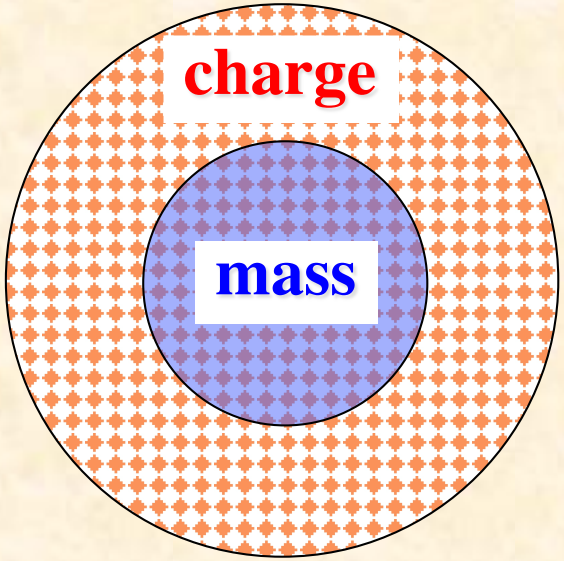
$\sqrt{\langle r^2\rangle_{\text{mass}}}=0.32\sim0.39$ fm, $\sqrt{\langle r^2\rangle_{\text{mech}}}=0.82\sim0.88$ fm ← First finding on gravitational radius from actual experimental measurements

$\Leftrightarrow\ \sqrt{\langle r^2\rangle_{\text{charge}}}=0.672\pm0.008$ fm





Hadron mass radius vs charge radius



For pion

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm} \quad \Leftrightarrow \quad \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020;
Erratum in v3 of arXiv:1711.08088.

Mass radius seems to be much than the charge radius for pion.

This is the first result on the mass radius from actual measurement,
so further studies are needed to find whether there is acutally a significant difference

Quarks contribute to both charge and mass distributions,
but gluons contribute to only the mass distribution.

Electric interactions are repulsive (or could be attractive) and
gravitational interactions are always attractive,
so there would be some differences in both radii.
However, the difference of the factor of 2 may not be expected.

For example, related theoretical studies:
- A. Freeseand I. C. Cloet, Phys. Rev. C 100 (2019) 015201;
- P. E. Shanahan and W. Detmold, Phys. Rev. D 99 (2019) 014511;
- C. D. Roberts, D. G. Richards, T. Horn, and L. Chang, Prog. Part. Nucl. Phys.120 (2021) 103883.

Recent related works:
June-Young Kim and Hyun-Chul Kim, PRD 104 (2021) 074019; Ho-Yeon Won *et al.*, arXiv:2210.03320.

→ See the work
J.-L. Zhang, K. Raya, L. Chang, Z.-F. Cui, J. M. Morgado, C.D. Roberts, J. Rodriguez-Quintero, Phys. Lett. B 815 (2021) 136158

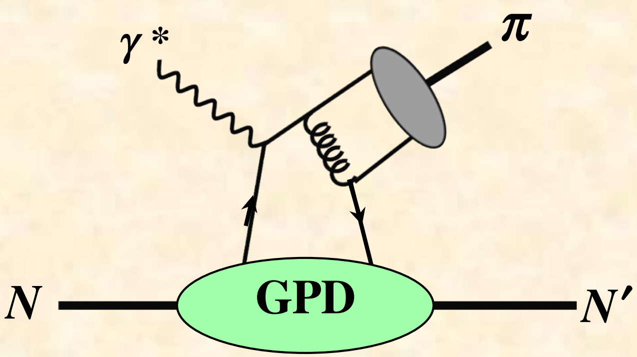
I may miss some of your papers.

Possible studies on GPDs at hadron accelerator facilities

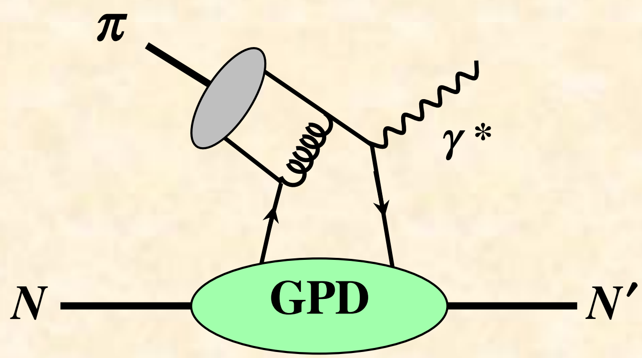
SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003;
T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,
PRD 93 (2016) 114034.
J-PARC LoI 2019-07, J.-K. Ahn *et al*. (2019).
J-PARC proposal under preparation (2022),
Please get in touch with W.-C. Chang if you are interested in this project.

GPD projects at JLab /EIC and J-PARC

JLab / EIC

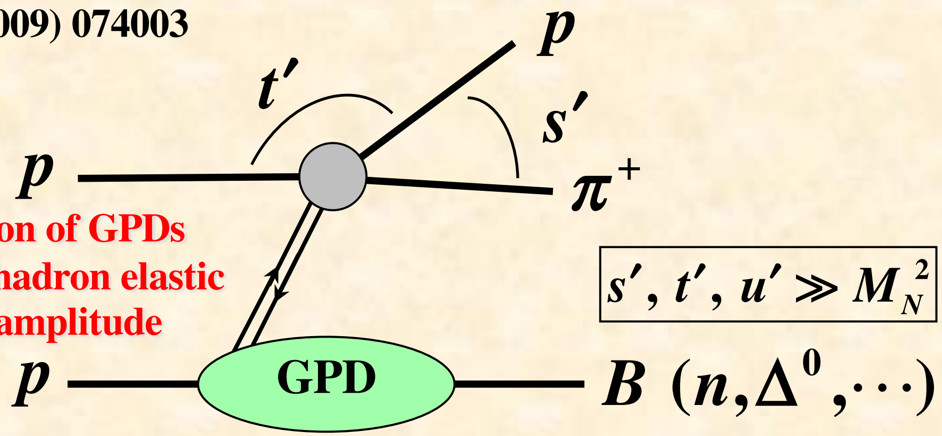


J-PARC



$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2)|p\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



Investigation of GPDs
with 2→2 hadron elastic
scattering amplitude

Exclusive Drell-Yan $\pi^- + p \to \mu^+\mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \,\mathrm{Re}\left\{\tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$

$$Q'^2 = q'^2,\ \ t = (p-p')^2,\ \ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s-m_\pi^2}$$

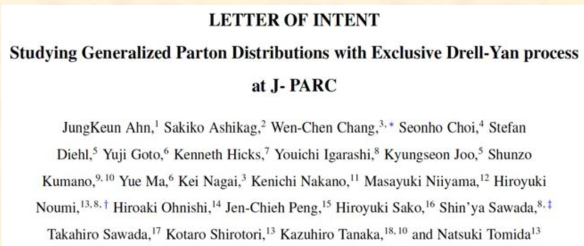
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle p(p')\right| \bar{q}(-z/2)\gamma^+\gamma_5 q(z/2) \left| p(p)\right\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_p^q(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_p^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle n(p')\right| \bar{q}_d(-z/2)\gamma^+\gamma_5 q_u(z/2) \left| p(p)\right\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

$$\tilde{H}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon}\right]\left[\tilde{H}^d(x',\xi,t) - \tilde{H}^u(x',\xi,t)\right]$$

$$\tilde{E}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon}\right]\left[\tilde{E}^d(x',\xi,t) - \tilde{E}^u(x',\xi,t)\right]$$

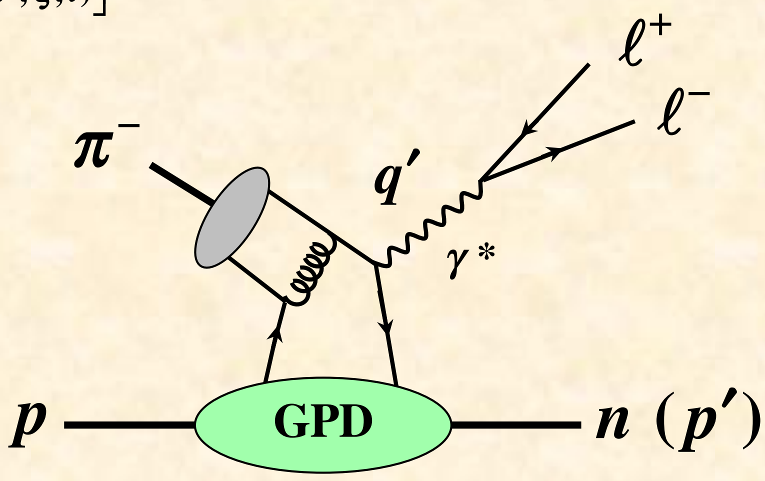
T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka, PRD93 (2016) 114034.


LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,8,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]

LoI for a J-PARC experiment

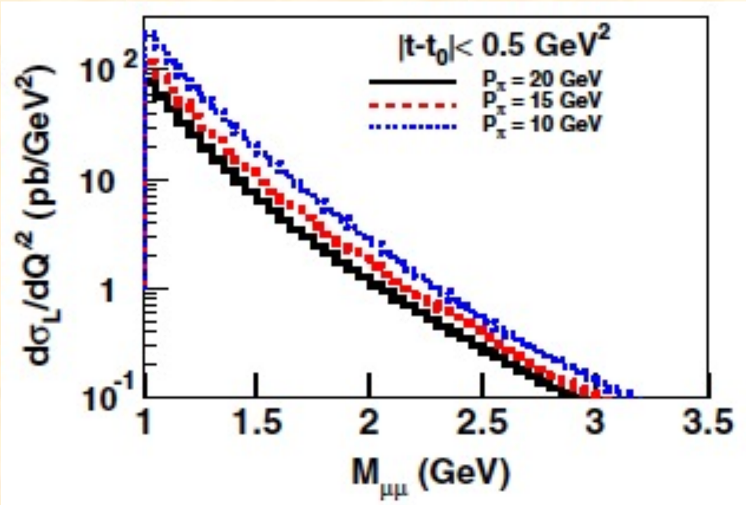


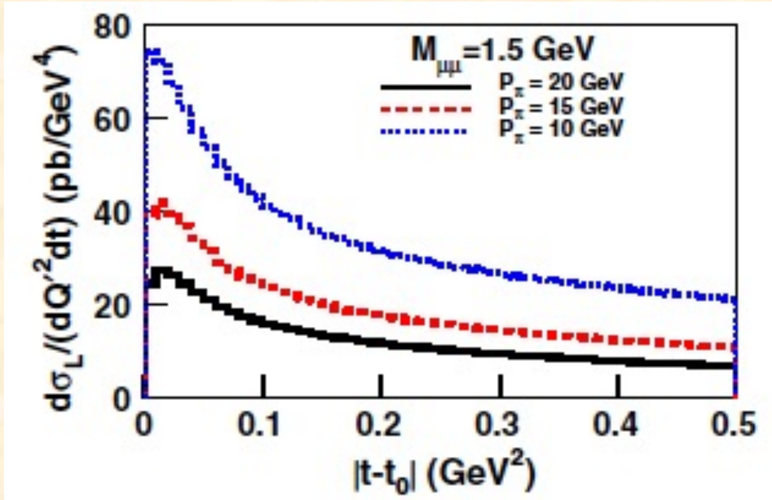
$$\pi^-(\bar{u}d) + p(uud) \to n(udd) + \gamma^*(\to \ell^+\ell^-)$$

Expected Drell-Yan events at J-PARC

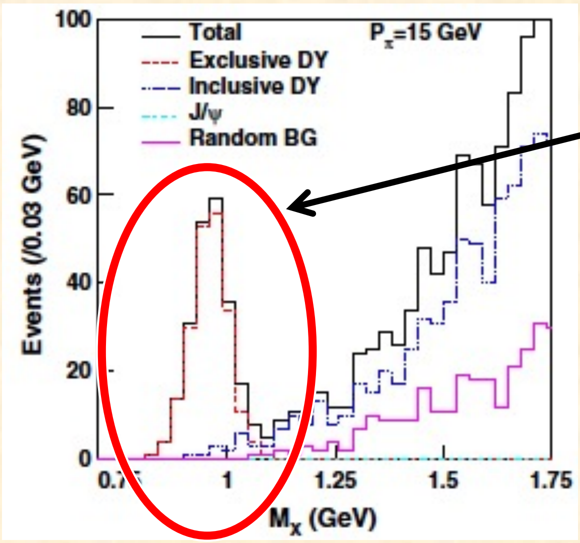
$$Q'^2 = q'^2,\ \ t = (p - p')^2,\ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s - m_N^2}$$

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \,\mathrm{Re}\left\{\tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$





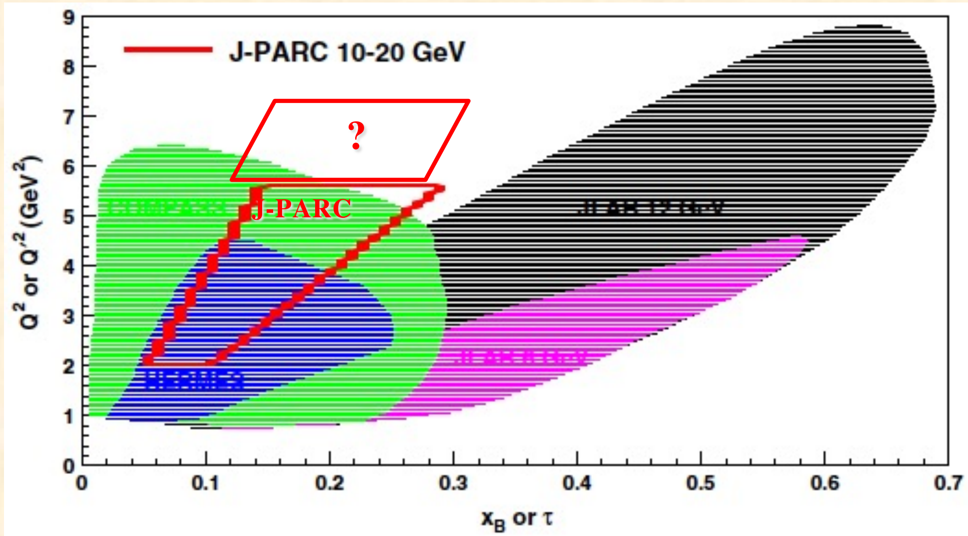
Missing mass



Exclusive Drell-Yan

$$M_X^2 = (q + p - q')^2$$

$$q = p_\pi,\ \ p = p_p,\ \ q = p_{\mu^+\mu^-}$$

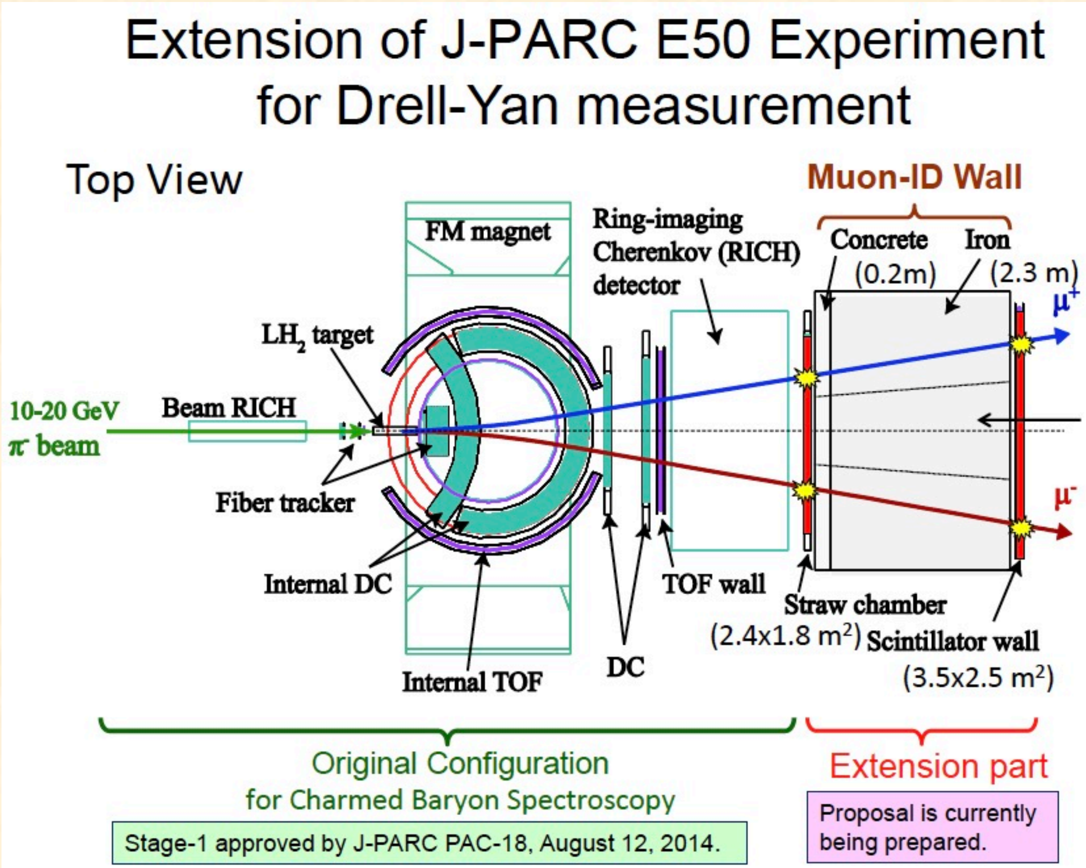


Letter of Intent to join J-PARC-E50 collaboration (Jan. 2019)

LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

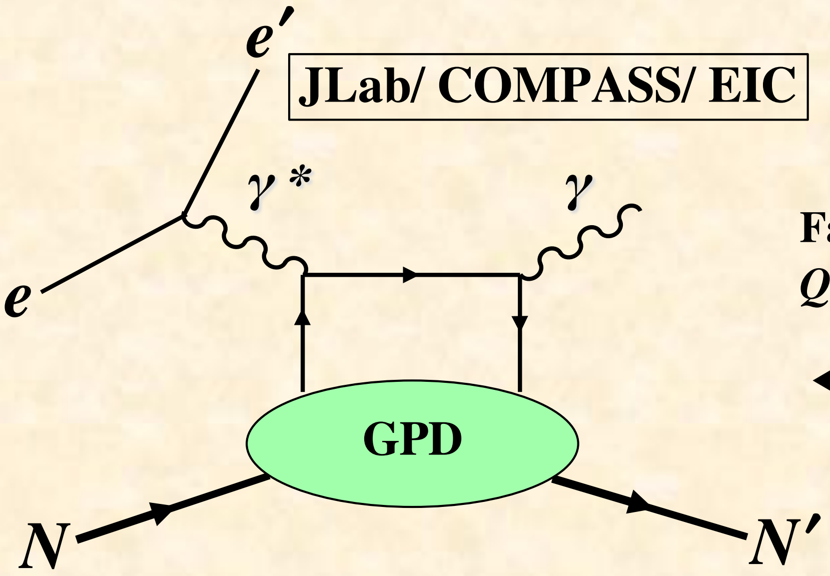
JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,8,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]



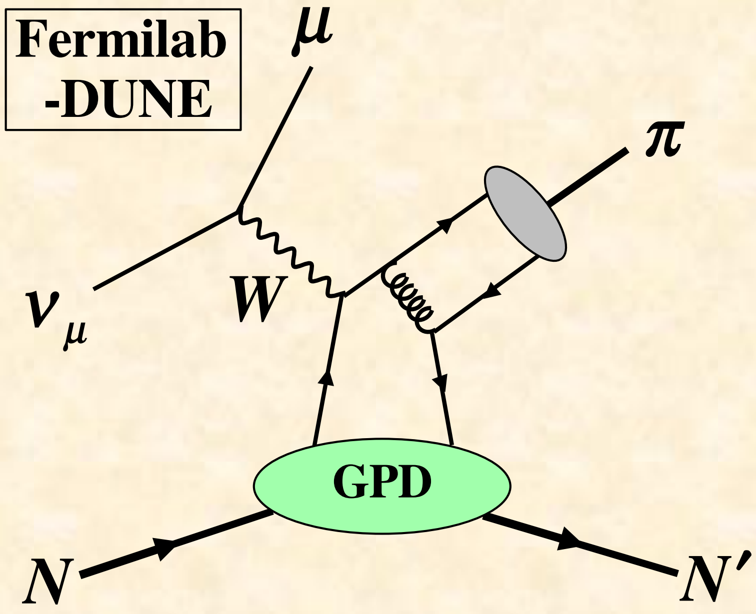
Possible studies on GPDs at neutrino facilities

- SK, EPJ Web Conf. 208 (2019) 07003.
- EIC yellow report, R. Abdul Khalek *et al*., arXiv:2103.05419, Sec. 7.5.2, Neutrino physics by SK and R. Petti.
- SK and R. Petti, PoS (NuFact2021) 092.

Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)

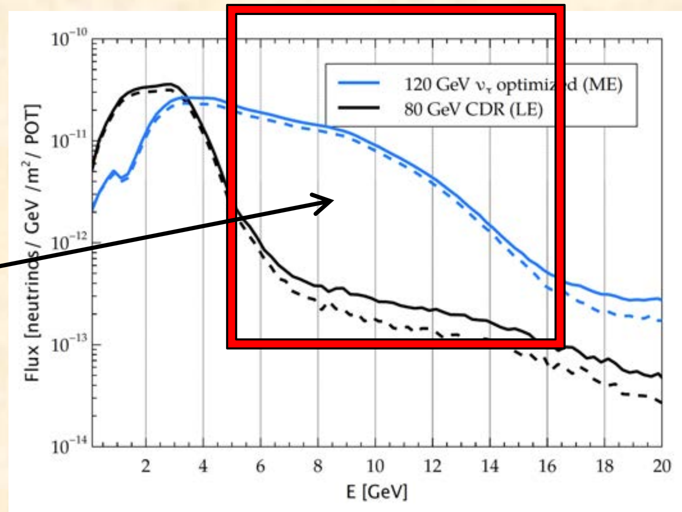


Factorization condition:
$Q^2 \gg |t|, \Lambda^2_{QCD}$



Deep Underground Neutrino Experiment (DUNE) at Long-Baseline Neutrino Facility (LBNF)

High-energy part of the LBNF ν beam can be used for the GPD studies.



J. Rout *et al.*, PRD 102 (2020) 116018

Cross section formalism

B. Pire, L. Szymanowski, J. Wagner,
Phys. Rev. D 95, 114029 (2017).

Cross section

$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\,dQ^2\,dt\,d\phi} = \Gamma\,\varepsilon\,\sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2\,Q^2}{32\,(2\pi)^4 (s-m_N^2)^2\, y\,(1-\varepsilon)\sqrt{1+4x^2 m_N^2/Q^2}}$$

$$\sigma_L = \varepsilon_L^{*\mu} W_{\mu\nu} \varepsilon_L^\nu = \frac{1}{Q^2}\Big[(1-\xi^2)\Big\{\left|C_q\mathcal{H}_q + C_g\mathcal{H}_g\right|^2 + \left|C_q\widetilde{\mathcal{H}}_q\right|^2\Big\} + \frac{\xi^4}{1-\xi^2}\Big\{\left|C_q\mathcal{E}_q + C_g\mathcal{E}_g\right|^2 + \left|C_q\widetilde{\mathcal{E}}_q\right|^2\Big\}$$

$$-2\xi^2\,\mathrm{Re}\Big\{(C_q\mathcal{H}_q + C_g\mathcal{H}_g)(C_q\mathcal{E}_q + C_g\mathcal{E}_g)^*\Big\} - 2\xi^2\,\mathrm{Re}\Big\{C_q\widetilde{\mathcal{H}}_q(C_q\widetilde{\mathcal{E}}_q)^*\Big\}\Big]$$

Quark contributions

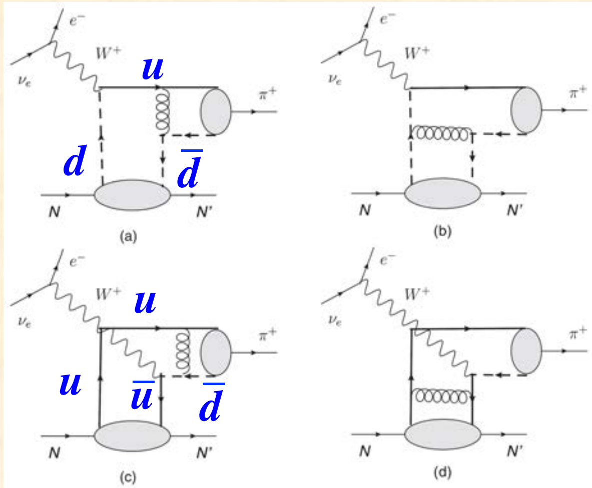
$$T_q = -i\frac{C_q}{2Q}N(p')\left[\mathcal{H}_q\hat{n} + \mathcal{E}_q\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2m_N} - \widetilde{\mathcal{H}}_q\hat{n}\gamma_5 - \widetilde{\mathcal{E}}_q\frac{\gamma_5 n\cdot\Delta}{2m_N}\right]N(p)$$

$$\mathcal{F}_q = 2f_\pi\int\frac{dz\,\phi_\pi(z)}{1-z}\int dx\frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

= (pion distribution amplitude) $\cdot$ (quark GPD)

$$F_q(x,\xi,t) \equiv F_d(x,\xi,t) - F_u(-x,\xi,t)$$

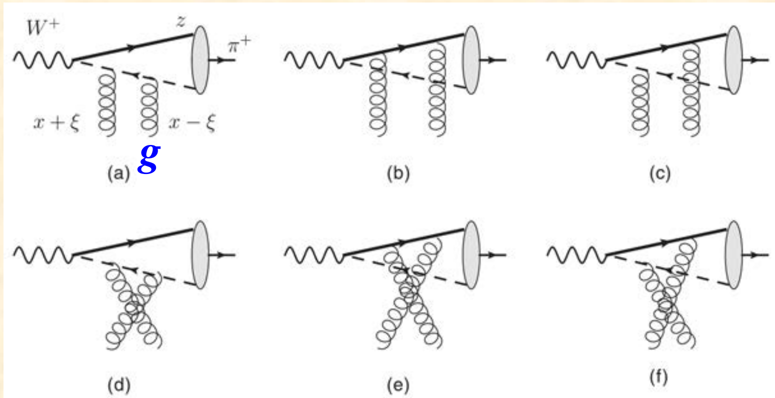
$$F = H,\,E,\,\tilde{H},\,\tilde{E}$$



Gluon contributions

$$T_g = -i\frac{C_g}{2Q}N(p')\left[\mathcal{H}^g\hat{n} + \mathcal{E}^g\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2m_N}\right]N(p)$$

$$\mathcal{F}_g = \frac{8f_\pi}{\xi}\int\frac{dz\,\phi_\pi(z)}{z(1-z)}\int dx\frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$



Cross section estimates

proton: $\nu p \to \ell^- \pi^+ p$ **neutron:** $\nu n \to \ell^- \pi^+ n$

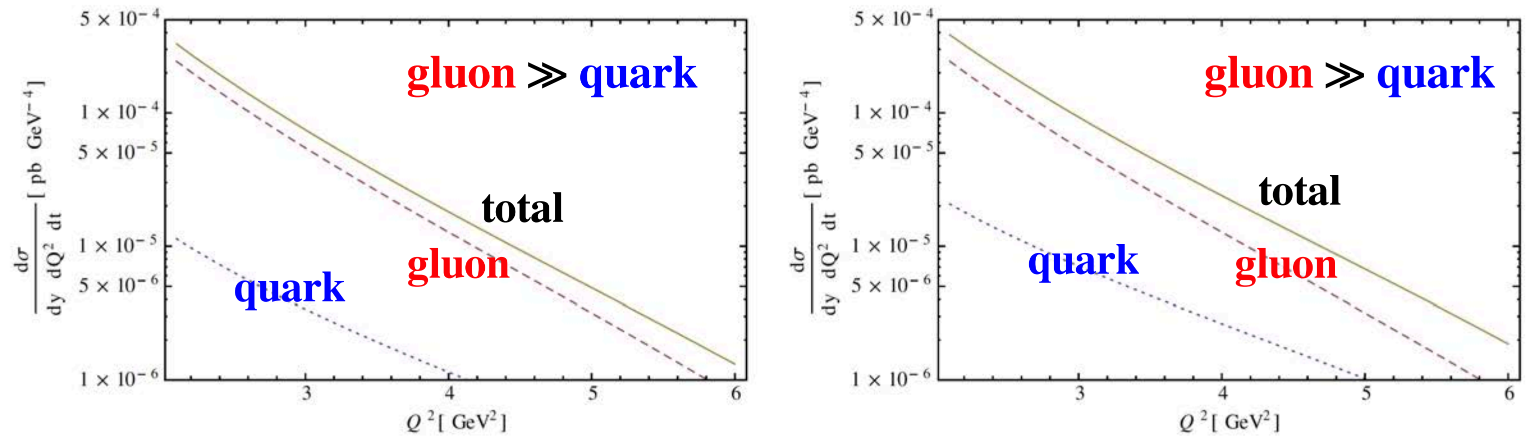


FIG. 3. The Q^2 dependence of the cross section $\frac{d^3\sigma(\nu N \to l^- N \pi^+)}{dy dQ^2 dt}$ (in pb GeV^{-4}) for $y = 0.7$, $\Delta_T = 0$ and $s = 20$ GeV^2, on a proton (left panel) and on a neutron (right panel). The quark contribution (dotted curves) is significantly smaller than the gluon contribution (dashed curves). The solid curves are the sum of the (quark + gluon + interference) contributions.

neutron → proton: $\nu n \to \ell^- \pi^0 p$

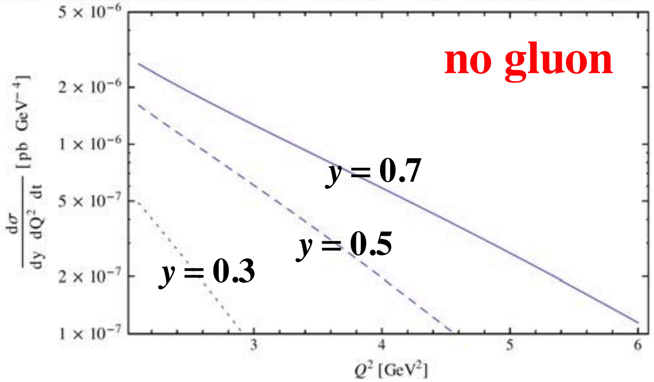
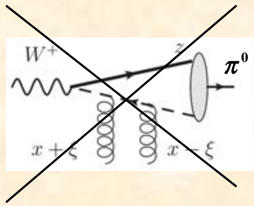


FIG. 6. The Q^2 dependence of the cross section $\frac{d^3\sigma(\nu n \to l^- p \pi^0)}{dy dQ^2 dt}$ (in pb GeV^{-4}) for $\Delta_T = 0$ and $s = 20$ GeV^2. The solid, dashed, and dotted lines correspond to $y = 0.7$, 0.5, and 0.3, respectively. There is no gluon contribution to this amplitude.

Neutrino GPD studies are complementary to the charged-lepton projects.

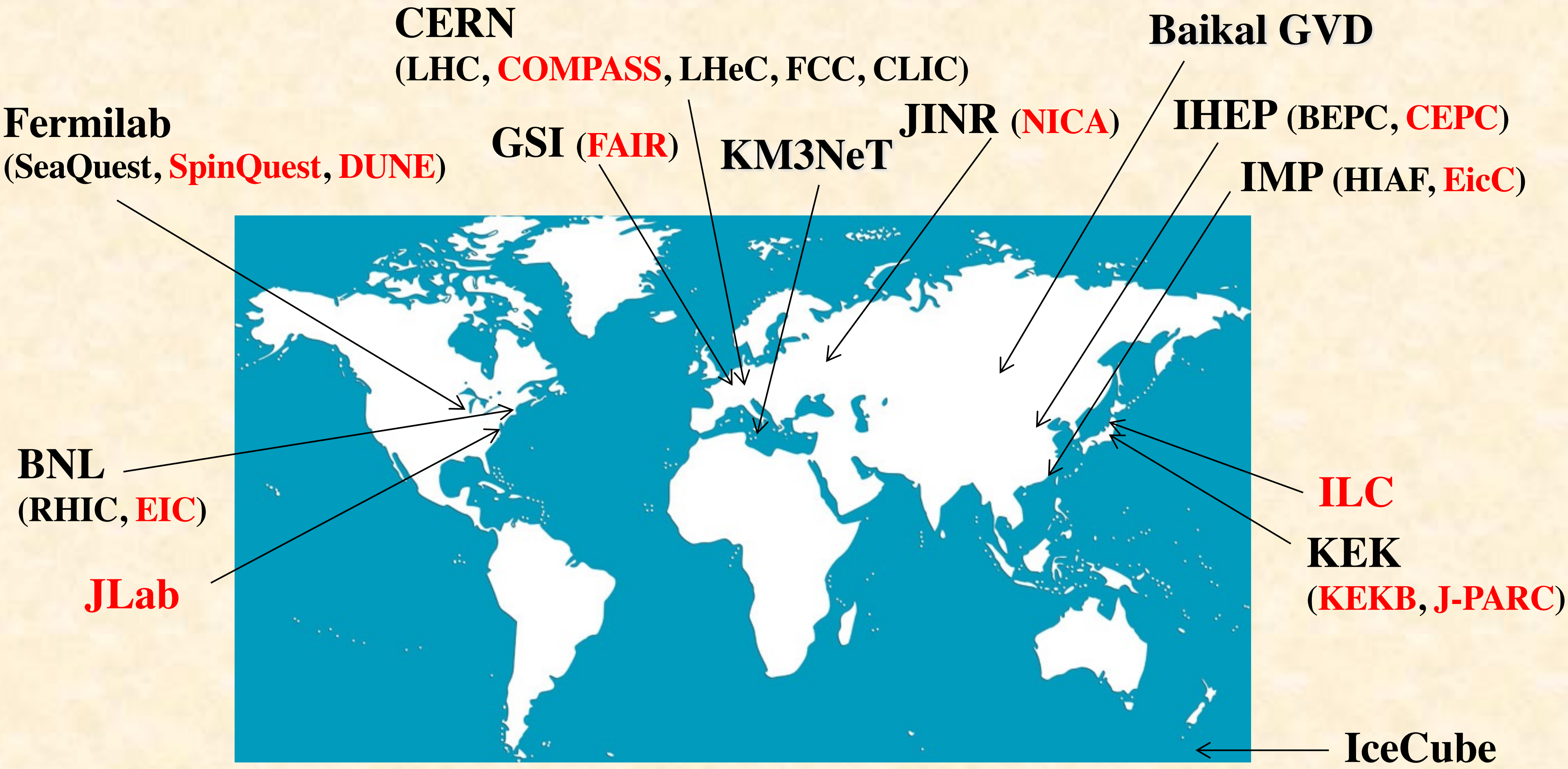
- **Gluon GPDs could be probed in charged-pion production.**
- **Flavor dependece of quark GPDs could be investigated.**



no gluon for π^0

Future prospects

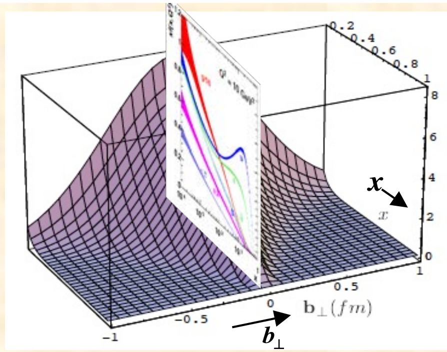
High-energy hadron physics experiments



Facilities on hadron structure functions on GPDs including future possibilities.

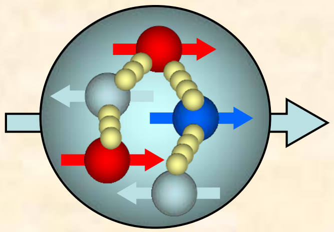
By hadron tomography

3D view of hadrons

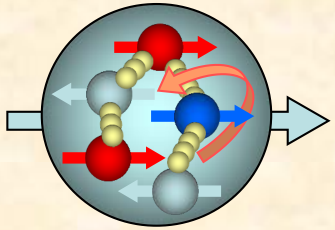


Origin of nucleon spin

By the tomography, we determine



or



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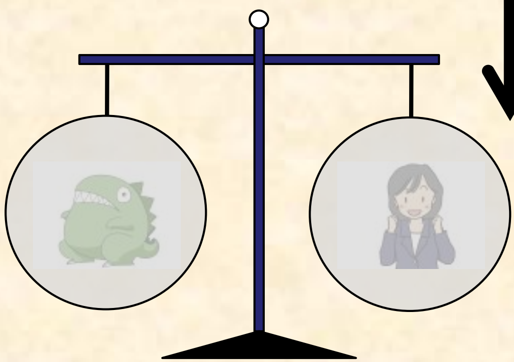
Exotic hadrons



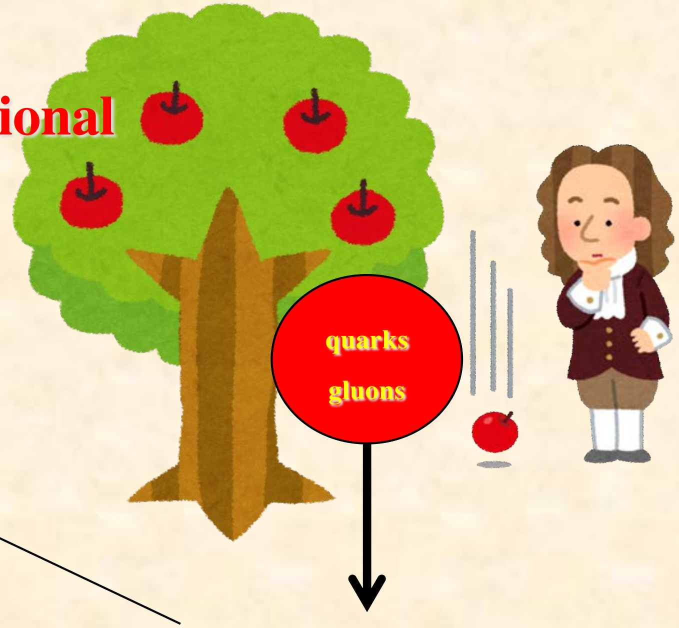
By tomography, we determine

or

.

Origin of gravitational source (mass)

By tomography, we determine gravitational sources in terms of quarks and gluons.



Summary on GPDs

Hadron-tomography and gravitational form factors

- Puzzle to find the origin of hadron masses and pressures
  in terms of quark and gluon degrees of freedom

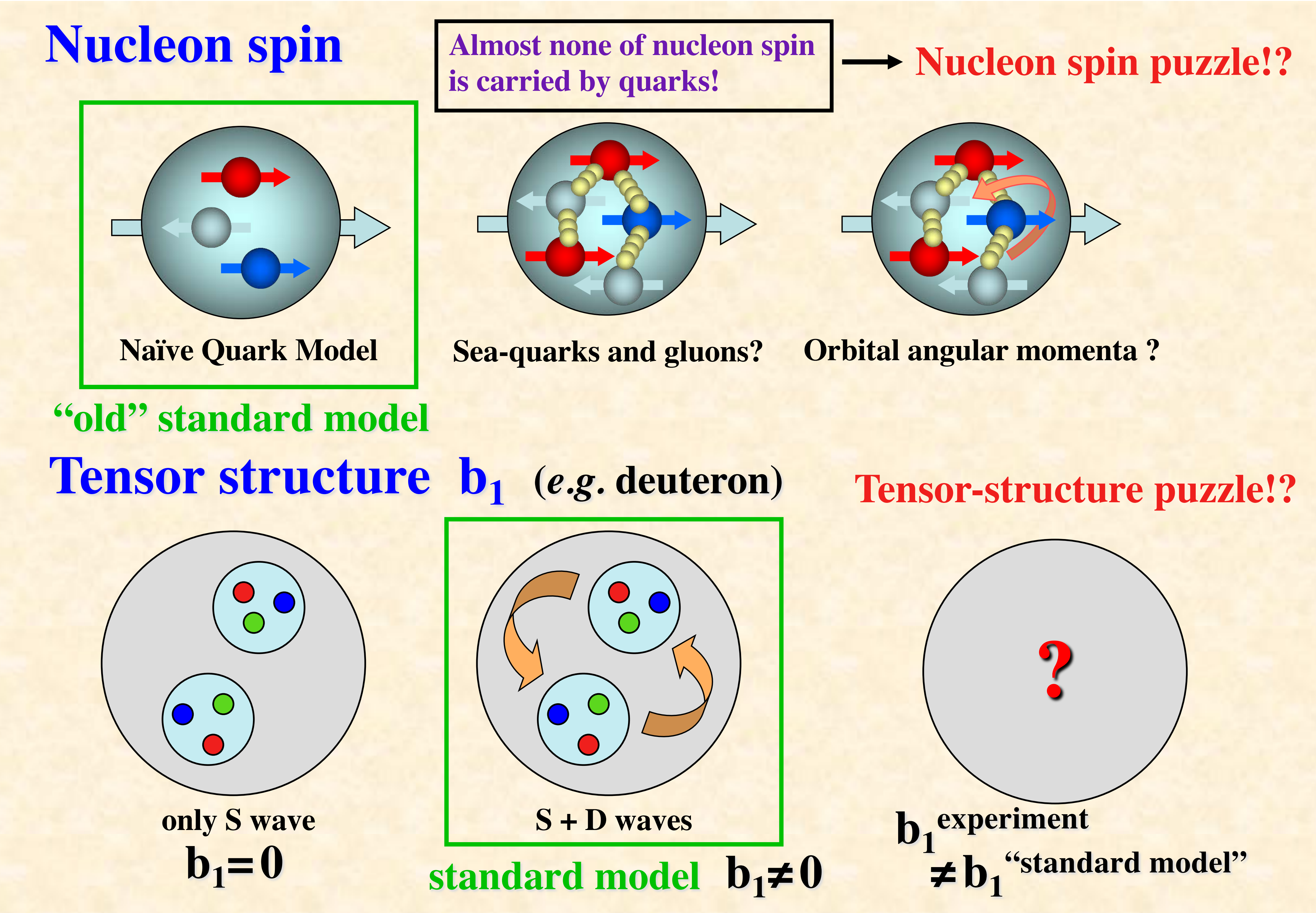
- Puzzle to find the origin of nucleon spin

- Exotic hadron candidates could be studied
  in the same tomography method.

- There are world-wide lepton and hadron accelerator facilities
  which has been used and could be used in future for our studies.

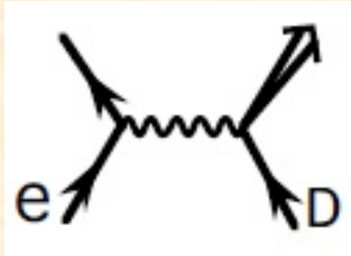
Time has come to understand the gravitational sources in microscopic (instead of usual macroscopic/cosmic) world in terms of quark and gluon degrees of freedom.

Structure functions of spin-1 deuteron

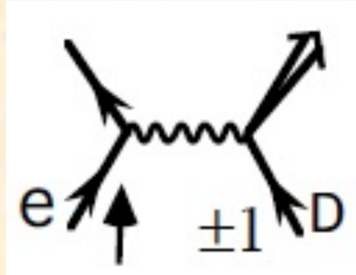

Nucleon spin
Almost none of nucleon spin is carried by quarks!
Nucleon spin puzzle!?
Naïve Quark Model
Sea-quarks and gluons?
Orbital angular momenta ?
"old" standard model
Tensor structure b_1 (*e.g.* deuteron)
Tensor-structure puzzle!?
?
only S wave
$b_1 = 0$
S + D waves
standard model $b_1 \neq 0$
$b_1^{\text{experiment}} \neq b_1^{\text{"standard model"}}$

Structure Functions

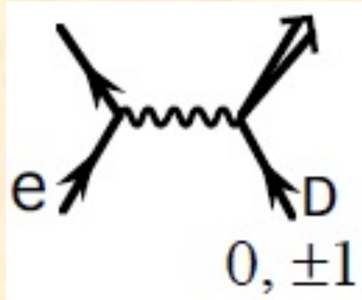
$$F_1 \propto \langle d\sigma \rangle$$



$$g_1 \propto d\sigma(\uparrow,+1) - d\sigma(\uparrow,-1)$$



$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



note: $\sigma(0) - \frac{\sigma(+1)+\sigma(-1)}{2} = 3\langle\sigma\rangle - \frac{3}{2}[\sigma(+1)+\sigma(-1)]$

Parton Model

$$F_1 = \frac{1}{2}\sum_i e_i^2 \left(q_i + \bar{q}_i\right) \qquad q_i = \frac{1}{3}\left(q_i^{+1} + q_i^{0} + q_i^{-1}\right)$$

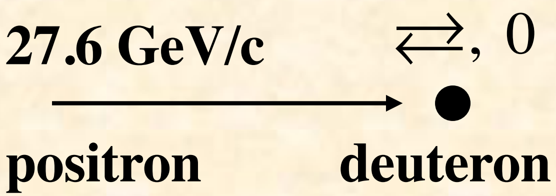
$$g_1 = \frac{1}{2}\sum_i e_i^2 \left(\Delta q_i + \Delta \bar{q}_i\right) \qquad \Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1}$$

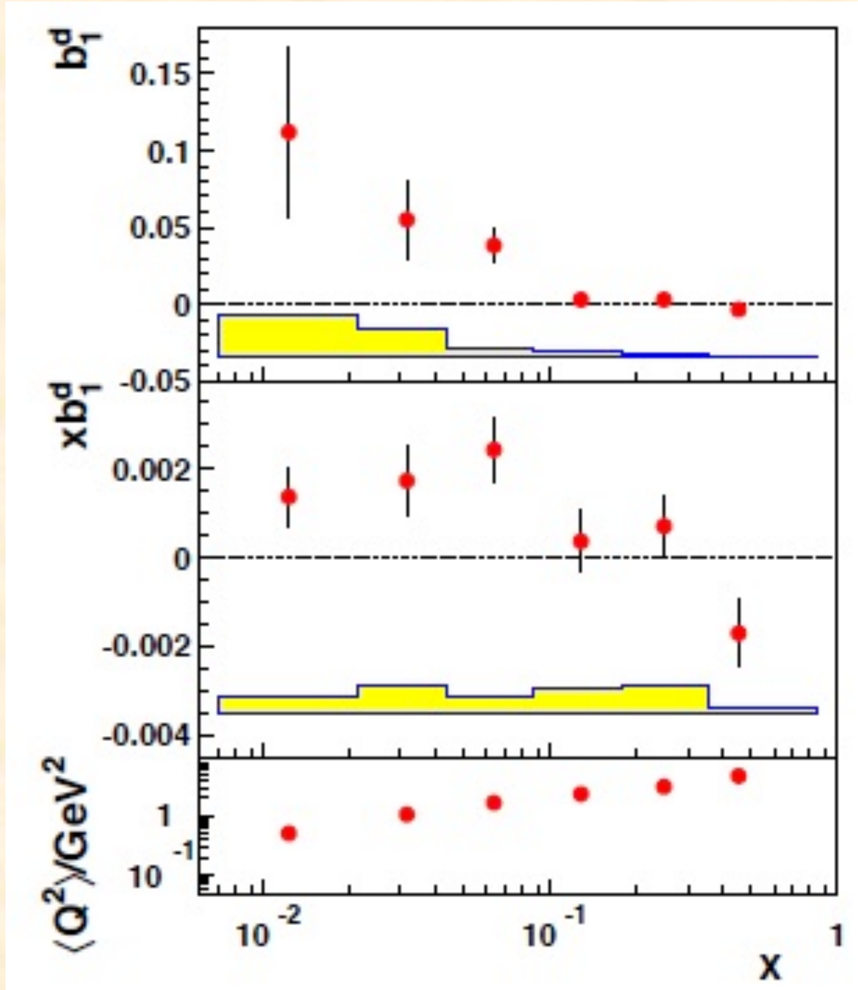
$$\left[q_{\uparrow}^{H}\left(x,Q^2\right)\right]$$

$$b_1 = \frac{1}{2}\sum_i e_i^2 \left(\delta_T q_i + \delta_T \bar{q}_i\right) \qquad \delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.





b_1 measurement in the kinematical region

$$0.01 < x < 0.45,\ \ 0.5\ \text{GeV}^2 < Q^2 < 5\ \text{GeV}^2$$

b_1 sum in the restricted Q^2 range $Q^2 > 1$ GeV^2

$$\int_{0.02}^{0.85} dx\, b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5$ GeV^2

$$\int dx\, b_1^D(x) = \lim_{t \to 0} -\frac{5}{12}\frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx\, \delta_T \bar{q}_i(x) = 0\ ?$$

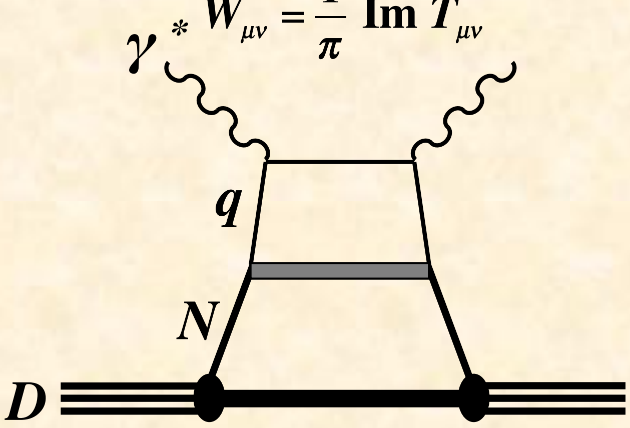
b_1 sum rule: F. E. Close and SK, PRD 42 (1990) 2377.

$$\int \frac{dx}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3}\int dx\left[u_v - d_v\right] + \frac{2}{3}\int dx\left[\bar{u} - \bar{d}\right] \neq 1/3$$

Drell-Yan experiments probe these antiquark distributions.

“Standard-model” prediction for b_1 of deuteron

$$W_{\mu\nu} = \frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}$$



$$b_1(x) = \int \frac{dy}{y}\delta_T f(y) F_1^N(x/y, Q^2), \qquad y = \frac{Mp\cdot q}{M_N P\cdot q} \simeq \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y)+f^-(y)}{2}$$

$$= \int d^3p\, y\left[-\frac{3}{4\sqrt{2}\pi}\phi_0(p)\phi_2(p) + \frac{3}{16\pi}|\phi_2(p)|^2\right](3\cos^2\theta - 1)\delta\left(y - \frac{p\cdot q}{M_N \nu}\right)$$

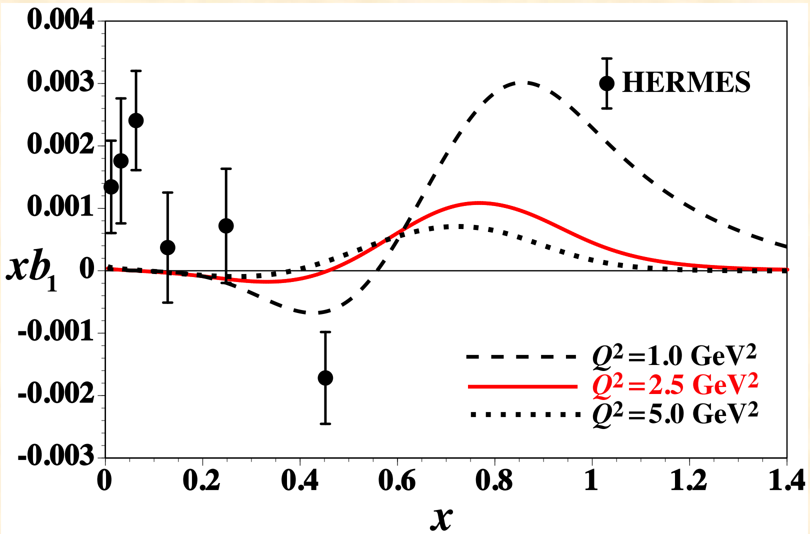
S-D term D-D term

Standard model of the deuteron

Nucleon momentum distribution:

$$f^H(y) \equiv f_\uparrow^H(y) + f_\downarrow^H(y) = \int d^3p\, y\,|\phi^H(\vec{p})|^2 \delta\left(y - \frac{E-p_z}{M_N}\right)$$

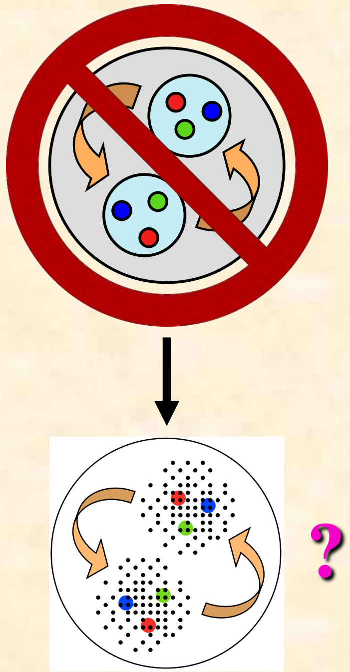
D-state admixture: $\phi^H(\vec{p}) = \phi^H_{\ell=0}(\vec{p}) + \phi^H_{\ell=2}(\vec{p})$



W. Cosyn, Yu-Bing Dong, SK, M. Sargsian, Phys. Rev. D 95 (2017) 074036.

$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$ at $x < 0.5$

Standard convolution model does not work for the deuteron tensor structure!?



G. A. Miller, PRC 89 (2014) 045203,
Interesting suggestions:
hidden-color, 6-quark, ···

$$|6q\rangle = |NN\rangle + |\Delta\Delta\rangle + |CC\rangle + \cdots$$

see also
W. Cosyn and C. Weiss, PRC 102 (2020) 065204.

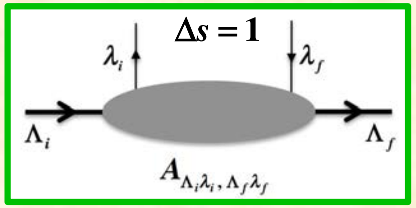
Gluon transversity $\Delta_T g$

Note on our notations:
Tensor-polarized gluon distribution: $\delta_T g$
Gluon transversity: $\Delta_T g$

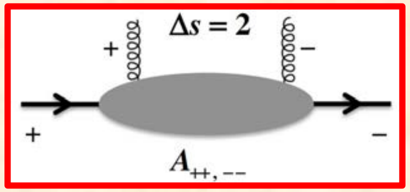
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, **conservation** $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

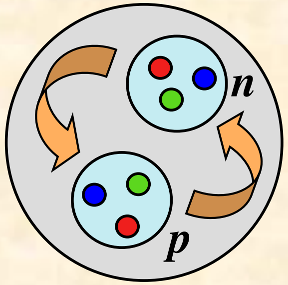
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \to \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, ~~$A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)$~~ not possible for nucleon



Note: Gluon transversity does not exist for spin-1/2 nucleons.



S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \ \Leftrightarrow$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?

Physics beyond "the standard model" in nuclear physics?
(Physics beyond the standard model in particle physics???)

Our recent works on spin-1 hadrons

- Gluon transversity at hadron accelerator facilities by Drell-Yan
  PRD 101 (2020) 054011 & 094013.
- TMDs, PDFs, multiparton distributions,
  fragmentation functions up to twist-4
  PRD 103 (2021) 014025.
- Useful relations similar to Wandzura-Wilczek relation
  and Burkhardt-Cottingham sum rule
  JHEP 09 (2021) 141.
- Relations from equation-of-motion and Lorentz-invariance relations
  PLB 826 (2022) 136908.

Collaborator on recent works:
Qin-Tao Song (Ecole Polytechnique / Zhengzhou University)

Letter of Intent at Jefferson Lab (middle 2020's)

Jefferson Lab,
Electron accelerator ~12 GeV

LoI, arXiv:1803.11206

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan

Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

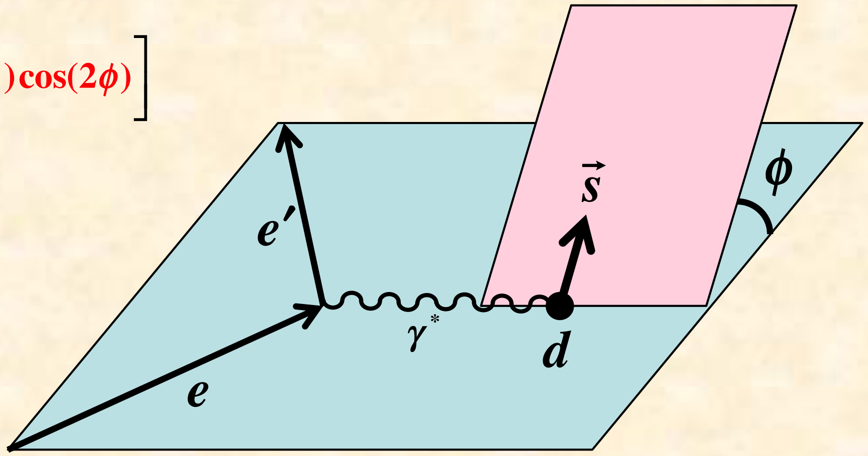
For development of polarized deuteron target, see D. Keller, D. Crabb, D. Day Nucl. Inst. Meth. Phys. Res. A981 (2020) 164504.

Electron scattering with polarized-deuteron target

$$\left.\frac{d\sigma}{dx\,dy\,d\phi}\right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4}\left[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2) - \frac{1}{2}x(1-y)\Delta(x,Q^2)\cos(2\phi)\right]$$

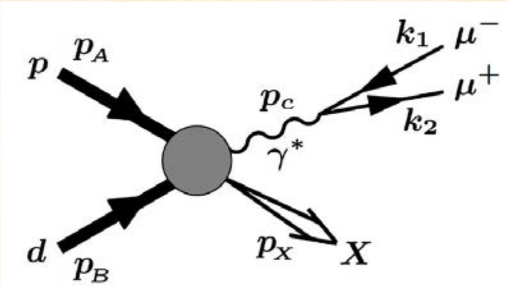
$$\Delta(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3}\Delta_T g(y,Q^2)$$

By looking at the deuteron-polarization angle ϕ, the quark transversty $\Delta_T g$ can be measured.



Theory: J. P. Ma, C. Wang, and G. P. Zhang, arXiv:1306.6693.

Proton-deuteron Drell-Yan cross section



Drell-Yan cross section

$$d\sigma_{pd\to\mu^+\mu^- X} = \int_0^1 dx_a \int_0^1 dx_b f_a(x_a) f_b(x_b) d\hat{\sigma}_{ab\to\mu^+\mu^- d}, \quad M_{ab\to\mu^+\mu^- d} = eM^{\mu}_{\gamma^*\to\mu^+\mu^-} \frac{-1}{Q^2} eM_{ab\to\gamma^* d}$$

In terms of lepton tensor $L^{\mu\nu}$ and hadron tensor $W_{\mu\nu}$

$$\frac{d\sigma_{pd\to\mu^+\mu^- X}}{d\tau\, dq_T^2\, d\phi\, dy} = \frac{\alpha^2}{12\pi^2 Q^4}\left[\int d\Phi_2(q;\, k_1,\, k_2)\; 2L^{\mu\nu}\right] W_{\mu\nu}$$

dilepton phase space: $d\Phi_2(q;\, k_1,\, k_2) = \delta^4(q - k_1 - k_2)\dfrac{d^3k_1}{2E_1(2\pi)^3}\dfrac{d^3k_2}{2E_2(2\pi)^3}$

$$L^{\mu\nu} = 2(k_1^{\mu} k_2^{\nu} + k_1^{\nu} k_2^{\mu} - k_1\cdot k_2 g^{\mu\nu})$$

$$W_{\mu\nu} = \overline{\sum_{\substack{\text{spin,}\\ \text{color}}}} \sum_q e_q^2 \int_{\min(x_a)}^1 dx_a \frac{\pi}{p_g^-(x_a - x_1)} \mathrm{Tr}\left[\Gamma_{\nu\beta}\left\{\Phi_{q/A}(x_a) + \Phi_{\bar{q}/A}(x_a)\right\}\hat{\Gamma}_{\mu\alpha}\Phi^{\alpha\beta}_{g/B}(x_b)\right], \quad \hat{\Gamma}_{\nu\beta} = \gamma^0 \Gamma_{\nu\beta}\gamma^0$$

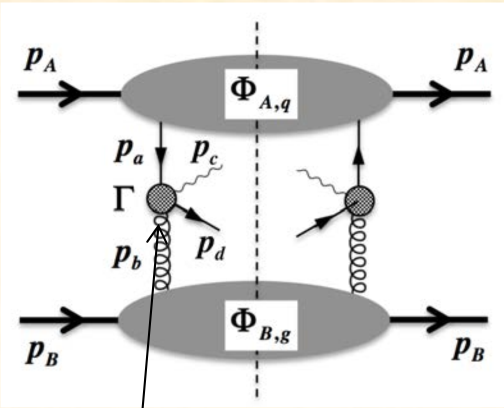
Collinear correlation functions

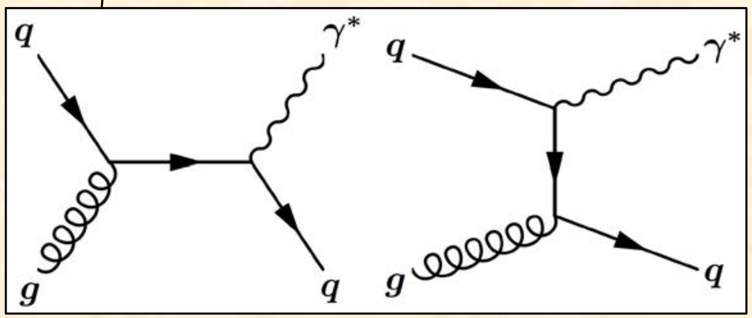
Refs. A. Bacchetta and P. J. Mulders, Phys. Rev. D 62 (2000) 114004,
D. Boer et al., JHEP 10 (2016) 013,
T. van Daal, arXiv:1812.07336 (Ph.D. Thesis).

$$\Phi_{q/A}(x_a) = \frac{1}{2}\left[\bar{n}\!\!\!/ f_{1,q/A}(x_a) + \gamma_5 \bar{n}\!\!\!/ S_{A,L} g_{1,q/A}(x_a) + \bar{n}\!\!\!/ \gamma_5 S\!\!\!/_{A\perp} h_{1,q/A}(x_a)\right]$$

$$\Phi_{q/B}(x_b) = \frac{1}{2}\left[n\!\!\!/ f_{1,q/B}(x_b) + \gamma^5 n\!\!\!/ S_{B,L}\, g_{1,q/B}(x_b) + i\sigma_{\mu\nu}\gamma^5 n^{\mu} S^{\nu}_{B,T} h_{1,q/B}(x_b) + n\!\!\!/ S_{LL} f_{1LL,q/B}(x_b) + \sigma_{\mu\nu} n^{\nu} S^{\mu}_{B,LT} h_{1LT,q/B}(x_b)\right]$$

$$\Phi^{ij}_{g/B}(x_b) = \frac{1}{2}\left[-g_T^{ij} f_{1,g/B}(x_b) + i\varepsilon_T^{ij} S_{B,L} g_{1L,g/B}(x_b) - g_T^{ij} S_{B,LL} f_{1LL,g/B}(x_b) + S^{ij}_{B,TT} h_{1TT,g/B}(x_b)\right]$$





Gluon transversity: $\Delta_T g = h_{1TT,g}$
(Sorry to use two differenent notations in a talk.)

Proton-deuteron Drell-Yan cross section

SK and Qin-Tao Song,
PRD 101 (2020) 054011 & 094013.

Drell-Yan cross section

$$\frac{d\sigma_{pd\to\mu^+\mu^-X}(E_x-E_y)}{d\tau\, dq_T^2\, d\phi\, dy}=\frac{\alpha^2\alpha_s C_F q_T^2}{6\pi s^3}\cos(2\phi)\int_{\min(x_a)}^{1}dx_a\frac{1}{(x_a x_b)^2(x_a-x_1)(\tau-x_a x_2)^2}\sum_q e_q^2 x_a\left[q_A(x_a)+\bar{q}_A(x_a)\right]x_b\Delta_T g_B(x_b)$$

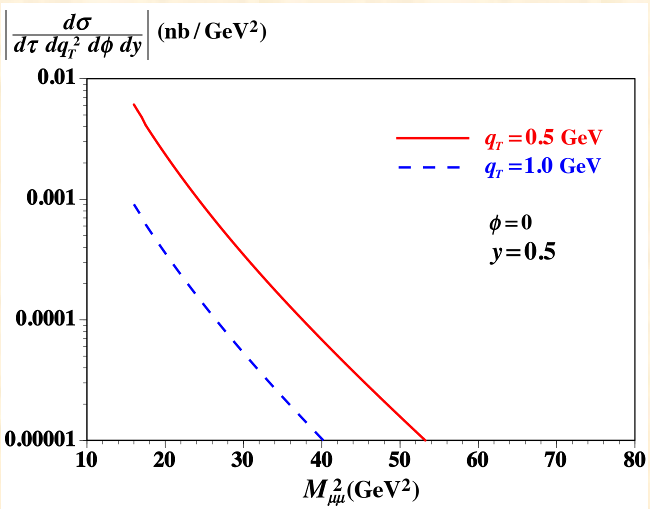
$$C_F=\frac{N_c^2-1}{2N_c},\quad \min(x_a)=\frac{x_1-\tau}{1-x_2},\quad x_b=\frac{x_a x_2-\tau}{x_a-\tau}$$

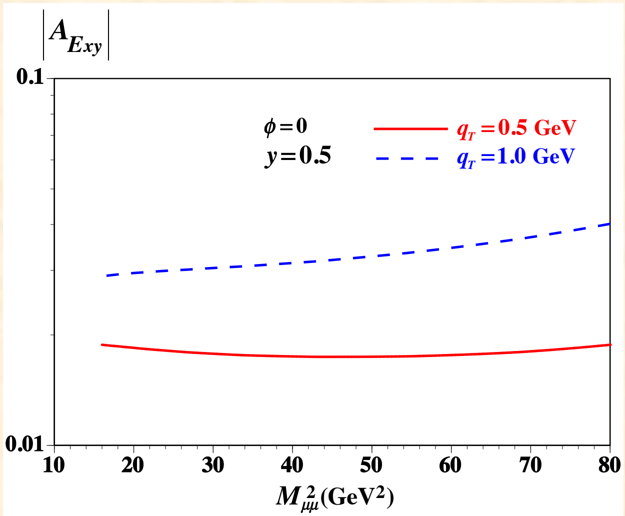
= (unpolarized PDFs of proton) * (gluon transversity distribution in the deuteron)

- Consider the Fermilab-E1039 experiment with the proton beam of $p=120$ GeV
- No available $\Delta_T g$, so we may tentatively assume $\Delta_T g=\Delta g_p+\Delta g_n$ $\left(\text{or } \frac{\Delta g_p+\Delta g_n}{2},\ \frac{\Delta g_p+\Delta g_n}{4}\right)$
- CTEQ14 for $q(x)+\bar{q}(x)$, NNPDFpol1.1 for $\Delta g(x)$

Cross section: Dimuon mass squred ($M_{\mu\mu}^2=Q^2$) dependence

Spin asymmetry: $$A_{E_{xy}}=\frac{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x)-\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x)+\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}$$





New proposal under preparation
at Fermilab-PAC (D. Keller)

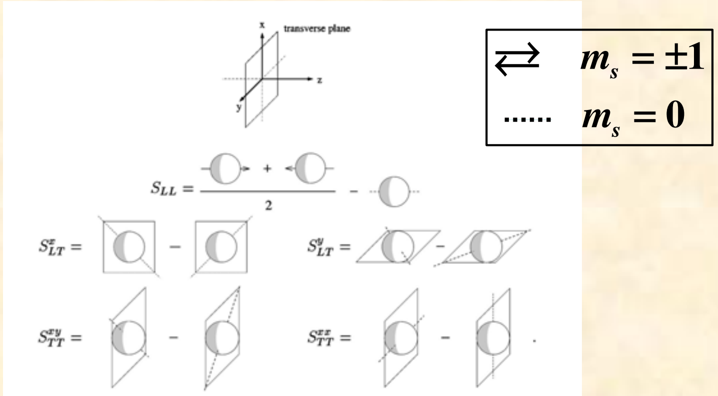
TMDs and their sum rules for spin-1 hadrons

see our PRD paper for the details

Twist-2 TMDs

Bacchetta-Mulders, PRD 62 (2000) 114004.

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1]$, $[h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}]$, $[h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}]$, $[h_{1TT}^\perp]$



Time-reversal invariance in colliear corrlation functions (PDFs)

$$\int d^2k_T \Phi_{\text{T-odd}}(x, k_T^2) = 0$$

Sum rules for the TMDs of spin-1 hadrons

$$\int d^2k_T\, h_{1LT}(x, k_T^2) = 0, \qquad \int d^2k_T\, g_{LT}(x, k_T^2) = 0,$$

$$\int d^2k_T\, h_{LL}(x, k_T^2) = 0, \qquad \int d^2k_T\, h_{3LT}(x, k_T^2) = 0$$

Twist-3 TMDs

SK and Qin-Tao Song, PRD 103 (2021) 014025.

Quark / Hadron	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^+\gamma_5$ T-even	$\gamma^+\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		f_T, $f_T^\perp$ $[e_T, e_T^\perp]$	g_T, $g_T^\perp$		$[h_T]$, $[h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	f_{LT}, $f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			g_{LT}, $g_{LT}^\perp$		$[h_{LT}]$, $[h_{LT}^\perp]$
TT	f_{TT}, $f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			g_{TT}, $g_{TT}^\perp$		$[h_{TT}]$, $[h_{TT}^\perp]$

Twist-4 TMDs

Quark / Hadron	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}]$, $[h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}]$, $[h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}]$, $[h_{3TT}^\perp]$

New fragmentation functions (FFs) for spin-1 hadrons

see arXiv:2201.05397

Corresponding fragmentation functions exist for the spin-1 haddrons
simply by changing function names and kinematical variables.

Collinear FFs:
X. Ji, PRD 49, 114 (1994).

TMD distribution functions: $f,\ g,\ h,\ e\ ;\quad x,\ k_T,\ S,\ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

$$\Downarrow$$

TMD fragmentation functions: $D,\ G,\ H,\ E\ ;\ z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

Collinear FFs, twist 2

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_1					
L			G_{1L}			
T					$[H_1]$	
LL	D_{1LL}					
LT						$[H_{1LT}]$
TT						

Collinear FFs, twist 3

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^i\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[E]$					
L					$[H_L]$	
T			G_T			
LL	$[E_{LL}]$					$[H_{LL}]$
LT	D_{LT}			G_{LT}		
TT						

Collinear FFs, twist 4

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_3					
L			G_{3L}			
T					$[H_{3T}]$	
LL	D_{3LL}					
LT						$[H_{3LT}]$
TT						

[ ] = chiral odd

TMD FFs, twist 2

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_1					$[H_1^\perp]$
L			G_{1L}		$[H_{1L}^\perp]$	
T		$D_{1T}^\perp$	G_{1T}		$[H_1], [H_{1T}^\perp]$	
LL	D_{1LL}					$[H_{1LL}^\perp]$
LT	D_{1LT}			G_{1LT}		$[H_{1LT}], [H_{1LT}^\perp]$
TT	D_{1TT}			G_{1TT}		$[H_{1TT}], [H_{1TT}^\perp]$

TMD FFs, twist 3

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^i\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$D^\perp$ $[E]$			$G^\perp$		$[H]$
L		$D_L^\perp$ $[E_L]$	$G_L^\perp$		$[H_L]$	
T		$D_T, D_T^\perp$ $[E_T, E_T^\perp]$	$G_T, G_T^\perp$		$[H_T], [H_T^\perp]$	
LL	$D_{LL}^\perp$ $[E_{LL}]$			$G_{LL}^\perp$		$[H_{LL}]$
LT	$D_{LT}, D_{LT}^\perp$ $[E_{LT}, E_{LT}^\perp]$			$G_{LT}, G_{LT}^\perp$		$[H_{LT}], [H_{LT}^\perp]$
TT	$D_{TT}, D_{TT}^\perp$ $[E_{TT}, E_{TT}^\perp]$			$G_{TT}, G_{TT}^\perp$		$[H_{TT}], [H_{TT}^\perp]$

TMD FFs, twist 4

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	D_3					$[H_3^\perp]$
L			G_{3L}		$[H_{3L}^\perp]$	
T		$D_{3T}^\perp$	G_{3T}		$[H_{3T}], [H_{3T}^\perp]$	
LL	D_{3LL}					$[H_{3LL}^\perp]$
LT	D_{3LT}			G_{3LT}		$[H_{3LT}], [H_{3LT}^\perp]$
TT	D_{3TT}			G_{3TT}		$[H_{3TT}], [H_{3TT}^\perp]$

New TMD FFs

PDFs for spin-1 hadrons

Twist-2 PDFs

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			$g_{1L}\,(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}\,(b_1)$					
LT						*1
TT						

*1: $h_{1LT}(x)$, *2: $g_{LT}(x)$, *3: $h_{LL}(x)$, *4: $h_{3LT}(x)$

Because of the time-reversal invariance, the collinear PDF vanishes. However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function should exist as a collinear fragmentation function.

[ ] = chiral odd

Twist-3 PDFs

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^+\gamma_5$ T-even	$\gamma^+\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

Twist-4 PDFs

Hadron \ Quark	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

New collinear PDFs

SK and Qin-Tao Song, PRD 103 (2021) 014025.

PDFs for spin-1 hadrons

Twist-2 PDFs

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			g_{1L} (g_1)			
T					[h_1]	
LL	f_{1LL} (b_1)					
LT						*1
TT						

Twist-3 PDFs

[ ] = chiral odd

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^+\gamma_5$ T-even	$\gamma^+\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	[e]					
L					[h_L]	
T			g_T			
LL	[e_{LL}]					*3
LT	f_{LT}			*2		
TT						

We derived analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule for f_{LT} and f_{1LL}.

SK and Qin-Tao Song (2021)

For spin-1/2 nucleons,

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) \text{ (Wandzura-Wilczek relation)}, \qquad \int_0^1 dx\, g_2(x) = 0 \text{ (Burkhardt-Cottingham sum rule)}$$

For tensor-polarized spin-1 hadrons, we obtained

$$f_{2LT}^{+}(x) = -f_{1LL}^{+}(x) + \int_x^1 \frac{dy}{y} f_{1LL}^{+}(y), \qquad \int_0^1 dx\, f_{2LT}^{+}(x) = 0, \qquad f_{2LT}(x) \equiv \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$$

$$\int_0^1 dx\, f_{LT}^{+}(x) = 0 \;\text{ if } \int_0^1 dx\, f_{1LL}^{+}(x) = \frac{2}{3}\int_0^1 dx\, b_1^{+}(x) = 0$$

Existence of multiparton distribution functions: $F_{G,LT}(x_1,x_2)$, $G_{G,LT}(x_1,x_2)$, $H_{G,LL}^{\perp}(x_1,x_2)$, $H_{G,TT}(x_1,x_2)$

Relations from equation of motion and Lorentz-invariance relation for spin-1 hadrons

In the following, I explain derivations on relations from equation of motion for quarks

- $x f_{LT}(x) - \int_{-1}^{+1} dy \left[F_{D,LT}(x,y) + G_{D,LT}(x,y) \right] = 0, \quad x f_{LT}(x) - f_{1LT}^{(1)}(x) - \mathcal{P}\int_{-1}^{+1} dy \frac{F_{G,LT}(x,y) + G_{G,LT}(x,y)}{x-y} = 0$
- $x e_{LL}(x) - 2\int_{-1}^{+1} dy H_{D,LL}^{\perp}(x,y) - \frac{m}{M} f_{1LL}(x) = 0, \quad x e_{LL}(x) - 2\mathcal{P}\int_{-1}^{+1} dy \frac{H_{G,LL}^{\perp}(x,y)}{x-y} - \frac{m}{M} f_{1LL}(x) = 0$

and the Lorentz-invariance relation

- $\frac{d f_{1LT}^{(1)}(x)}{dx} - f_{LT}(x) + \frac{3}{2} f_{1LL}(x) - 2\mathcal{P}\int_{-1}^{+1} dy \frac{F_{G,LT}(x,y)}{(x-y)^2} = 0$

Lorentz invariance
= frame independence of twist-3 observables

transverse-momemtum moment of TMD: $f^{(1)}(x) = \int d^2 k_T \frac{\vec{k}_T^2}{2M^2} f(x, k_T^2)$

Twist-2 PDFs

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			g_{1L} (g_1)			
T					[h_1]	
LL	f_{1LL} (b_1)					
LT						
TT						

Twist-3 PDFs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^+\gamma_5$ T-even	$\gamma^+\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	[e]					
L					[h_L]	
T			g_T			
LL	[e_{LL}]					
LT	f_{LT}					*1
TT						

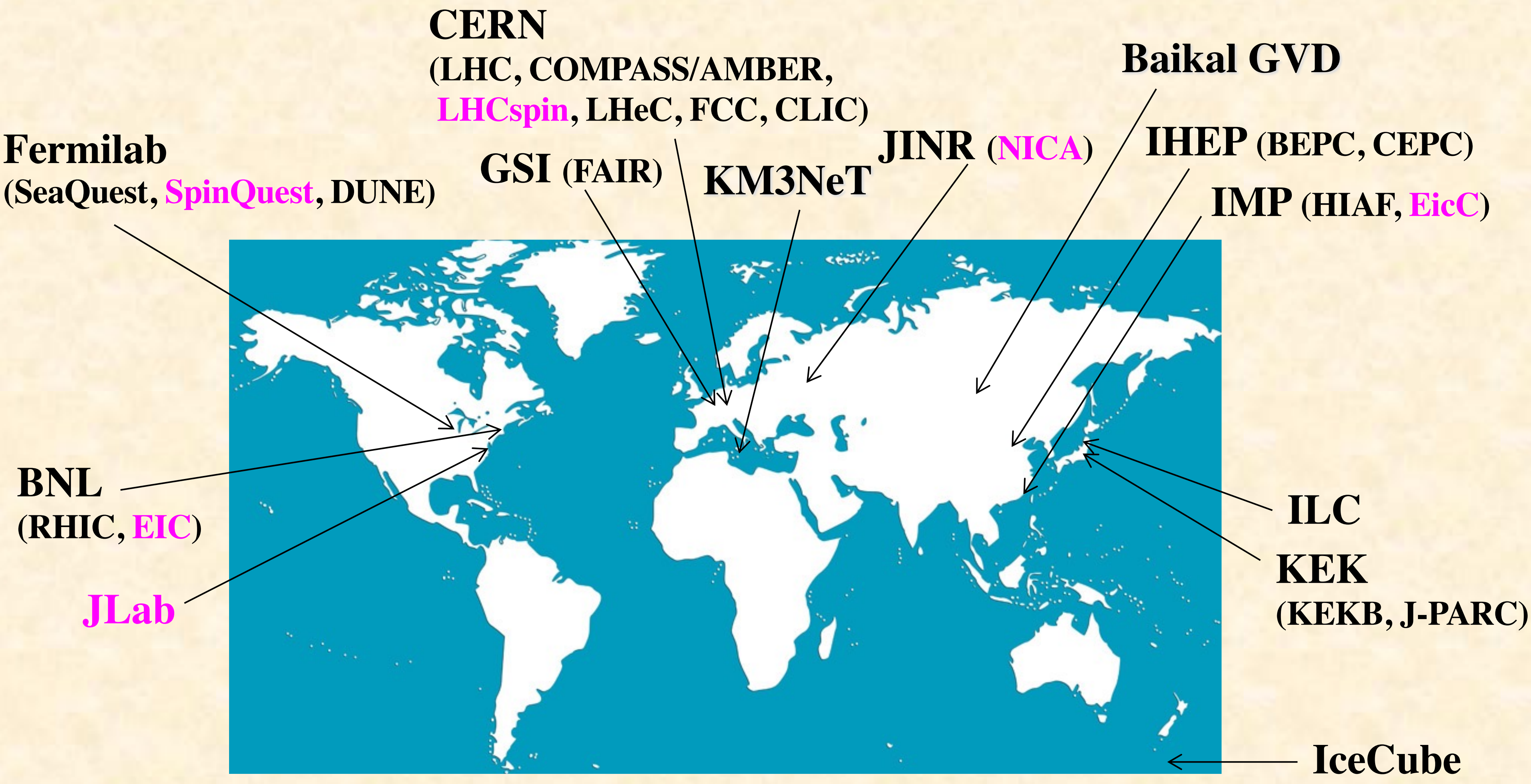
Twist-3 TMDs

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					[$h_1^{\perp}$]
L			g_{1L}		[$h_{1L}^{\perp}$]	
T		$f_{1T}^{\perp}$	g_{1T}		[h_1], [$h_{1T}^{\perp}$]	
LL	f_{1LL}					[$h_{1LL}^{\perp}$]
LT	f_{1LT}			g_{1LT}		[h_{1LT}], [$h_{1LT}^{\perp}$]
TT	f_{1TT}			g_{1TT}		[h_{1TT}], [$h_{1TT}^{\perp}$]

[ ] = chiral odd

Future prospects and summary on spin-1 hadrons

High-energy hadron physics experiments



Facilities on spin-1 hadron structure functions including future possibilities.

JLab PAC-38 (Aug. 22-26, 2011) proposal, PR12-11-110

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

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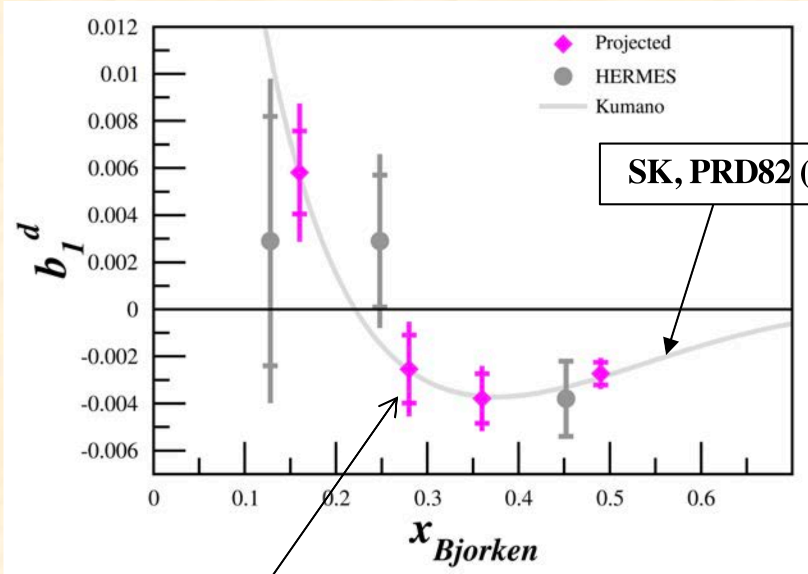

SK, PRD82 (2010) 017501

Expected errors by JLab

Approved!

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

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Experimental possibility at Fermilab in 2020’s

Polarized fixed-target experiments at the Main Injector, Proton beam = 120 GeV

© Fermilab

Fermilab-E1039 (SpinQuest)

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

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D. Crabb, D. Day, D. Keller, O. Rondon
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Fermilab experimentalists are interested in the gluon transversity by replacing the E1039 proton target for the deuteron one. (Spokesperson of E1039: D. Keller) However, there was no theoretical formalism until our work.

SK and Q.-T. Song, PRD 101 (2020) 054011 & 094013

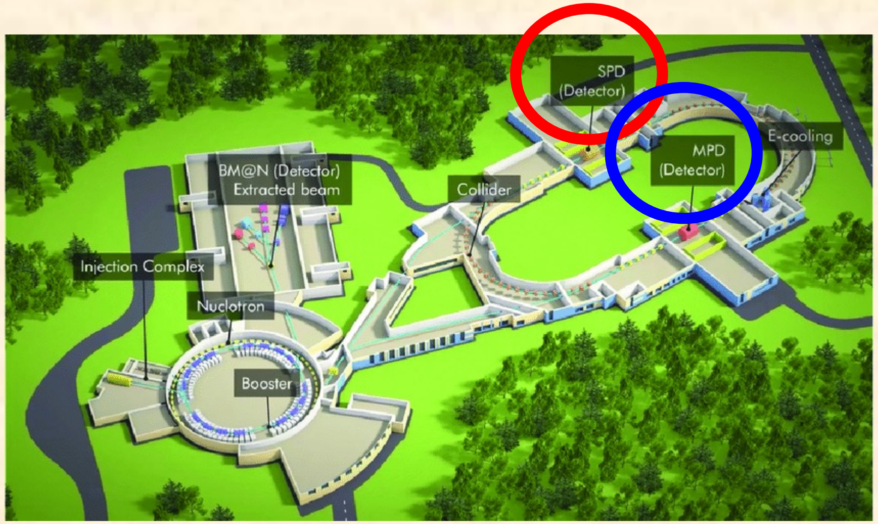
The Transverse Structure of the Deuteron with Drell-Yan

D. Keller[1]

[1] *University of Virginia, Charlottesville, VA 22904*

New proposal for a Fermilab-PAC in 2022.

Nuclotron-based Ion Collider fAcility (NICA)



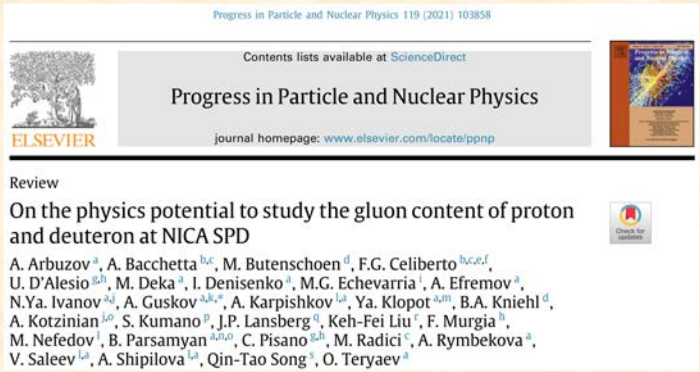
SPD (Spin Physics Detector for physics with polarized beams)
MPD (MultiPurpose Detector for heavy ion physics)

$\vec{p}+\vec{p}: \quad \sqrt{s_{pp}} = 12 \sim 27$ GeV

$\vec{d}+\vec{d}: \quad \sqrt{s_{NN}} = 4 \sim 14$ GeV

$\vec{p}+\vec{d}$ is also possilbe.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al*. (NICA project), Nucl. Part. Phys. 119 (2021) 103858.


Progress in Particle and Nuclear Physics 119 (2021) 103858

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbuzov[a], A. Bacchetta[b,c], M. Butenschoen[d], F.G. Celiberto[b,c,e,f], U. D'Alesio[g,h], M. Deka[a], I. Denisenko[a], M.G. Echevarria[i], A. Efremov[a], N.Ya. Ivanov[a,j], A. Guskov[a,k,*], A. Karpishkov[l,a], Ya. Klopot[a,m], B.A. Kniehl[d], A. Kotzinian[j,o], S. Kumano[p], J.P. Lansberg[q], Keh-Fei Liu[r], F. Murgia[h], M. Nefedov[l], B. Parsamyan[a,n,o], C. Pisano[g,h], M. Radici[c], A. Rymbekova[a], V. Saleev[l,a], A. Shipilova[l,a], Qin-Tao Song[s], O. Teryaev[a]

Unique opportunity in high-energy spin physics, especially on the deuteron spin physics.
→ Theoretical formalisms need to be developed.

Spin-1 deuteron experiments from the middle of 2020's

JLab

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson), K. Allada, A. Camsonne, A. Deur, D. Gaskell, C. Keith, S. Wood, J. Zhang
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K. Slifer[1](co-spokesperson), A. Atkins, T. Badman, J. Calarco, J. Maxwell, S. Phillips, R. Zielinski
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Seonho Choi, Hoyoung Kang, Hyekoo Kang, Yoomin Oh
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Proposal (approved),
Experiment: middle of 2020's

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

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J. Pierce
Oak Ridge National Laboratory, Oak Ridge, TN 37831

Fermilab

The Transverse Structure of the Deuteron with Drell-Yan

D. Keller[1]
[1] *University of Virginia, Charlottesville, VA 22904*

Proposal,
Fermilab-PAC: 2022
Experiment: 2020's

NICA

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Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbuzov[a], A. Bacchetta[b,c], M. Butenschoen[d], F.G. Celiberto[b,c,e,f], U. D'Alesio[g,h], M. Deka[a], I. Denisenko[a], M.G. Echevarria[i], A. Efremov[a], N.Ya. Ivanov[a,j], A. Guskov[a,k,*], A. Karpishkov[l,a], Ya. Klopot[a,m], B.A. Kniehl[d], A. Kotzinian[j,o], S. Kumano[p], J.P. Lansberg[q], Keh-Fei Liu[r], F. Murgia[h], M. Nefedov[l], B. Parsamyan[a,n,o], C. Pisano[g,h], M. Radici[c], A. Rymbekova[a], V. Saleev[l,a], A. Shipilova[l,a], Qin-Tao Song[s], O. Teryaev[a]

Prog. Nucl. Part. Phys.
119 (2021) 103858,
Experiment: middle of 2020's

LHCspin

LHC spin

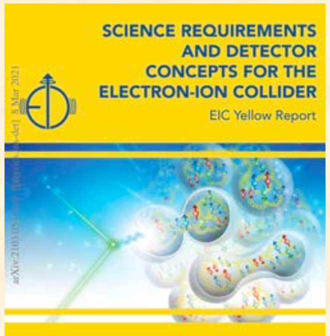
CERN-ESPP-Note-2018-111

The LHCSpin Project

C. A. Aidala[1], A. Bacchetta[2,3], M. Boglione[4,5], G. Bozzi[2,3], V. Carassiti[6,7], M. Chiosso[4,5], R. Cimino[8], G. Ciullo[6,7], M. Contalbrigo[6,7], U. D'Alesio[9,10], P. Di Nezza[8], R. Engels[11], K. Grigoryev[11], D. Keller[12], P. Lenisa[6,7], S. Liuti[12], A. Metz[13], P.J. Mulders[14,15], F. Murgia[10], A. Nass[11], D. Panzieri[5,16], L. L. Pappalardo[6,7], B. Pasquini[2,3], C. Pisano[9,10], M. Radici[3], F. Rathmann[11], D. Reggiani[17], M. Schlegel[18], S. Scopetta[19,20], E. Steffens[21], A. Vasilyev[22]

arXiv:1901.08002,
Experiment: ~2028

2030's EIC/EicC



R. Abdul Khalek *et al*.
arXiv:2103.05419.

D. P. Anderle *et al*.,
Front. Phys. 16 (2021) 64701.

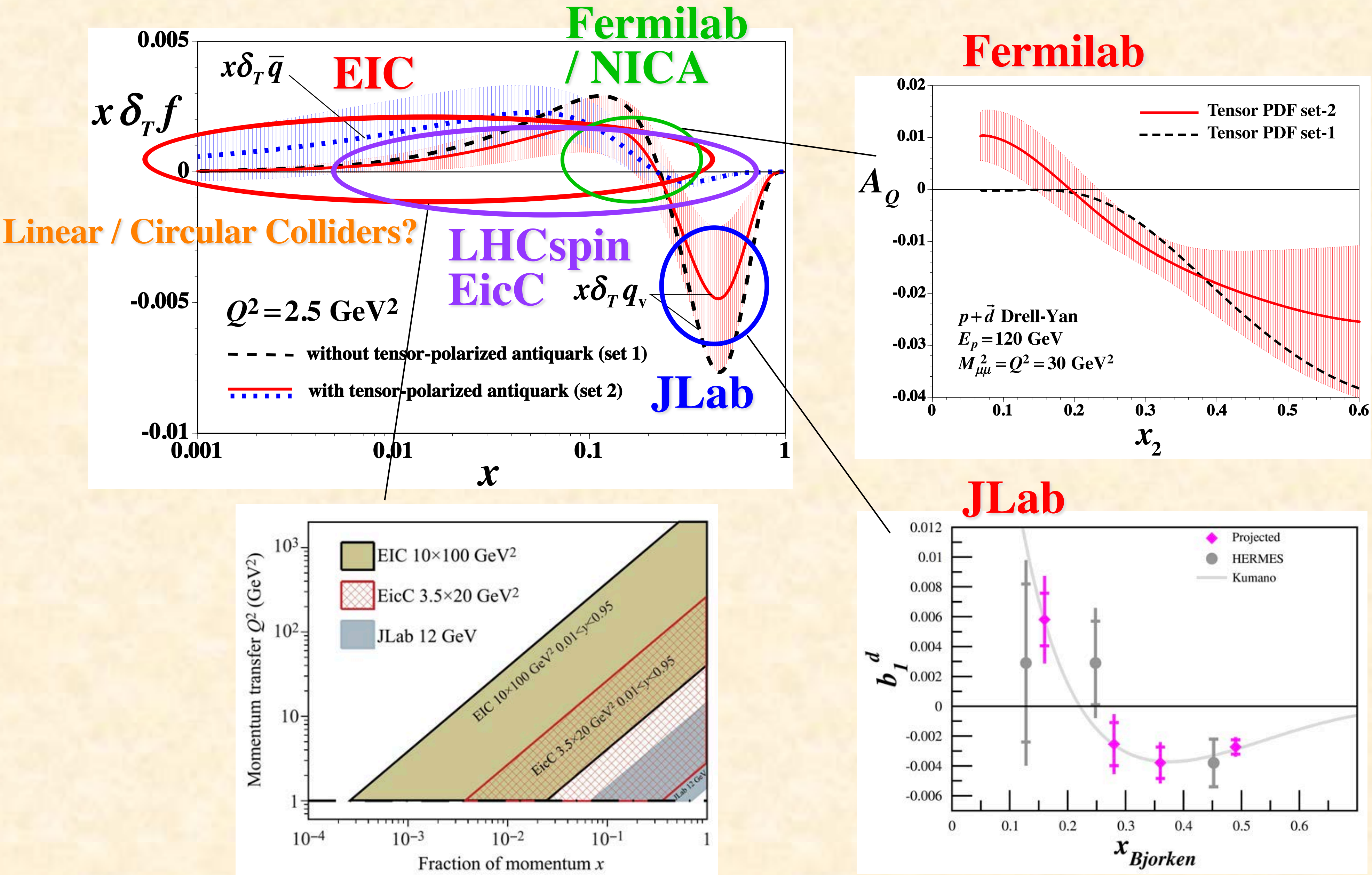
Frontiers of Physics
https://doi.org/10.1007/s11467-021-1062-0
Front. Phys. 16(6), 64701 (2021)

REVIEW ARTICLE

Electron-ion collider in China

Daniele P. Anderle[1], Valerio Bertone[2], Xu Cao[3,4], Lei Chang[5], Ningbo Chang[6], Gu Chen[7], Xurong Chen[3,4], Zhuojun Chen[8], Zhufang Cui[9], Lingyun Dai[8], Weitian Deng[10], Minghui Ding[11], Xu Feng[12], Chang Gong[12], Longcheng Gui[13], Feng-Kun Guo[4,14], Chengdong Han[3,4], Jun He[15], Tie-Jiun Hou[16], Hongxia Huang[15], Yin Huang[17], Krešimir Kumerički[18], L. P. Kaptari[3,19], Demin Li[20], Hengne Li[1], Minxiang Li[3,21], Xueqian Li[5], Yutie Liang[3,4], Zuotang Liang[22], Chen Liu[22], Chuan Liu[12], Guoming Liu[1], Jie Liu[3,4], Liuming Liu[3,4], Xiang Liu[21], Tianbo Liu[22], Xiaofeng Luo[23], Zhun Lyu[24], Boqiang Ma[12], Fu Ma[3,4], Jianping Ma[4,14], Yugang Ma[4,25,26], Lijun Mao[3,4], Cédric Mezrag[2], Hervé Moutarde[2], Jialun Ping[15], Sixue Qin[27], Hang Ren[3,4], Craig D. Roberts[9], Juan Rojo[28,29], Guodong Shen[3,4], Chao Shi[30], Qintao Song[20], Hao Sun[31], Paweł Sznajder[32], Enke Wang[1], Fan Wang[9], Qian Wang[1], Rong Wang[3,4], Ruiru Wang[3,4], Taofeng Wang[33], Wei Wang[34], Xiaoyu Wang[20], Xiaoyun Wang[35], Jiajun Wu[4], Xinggang Wu[27], Lei Xia[36], Bowen Xiao[23,37], Guoqing Xiao[3,4], Ju-Jun Xie[3,4], Yaping Xie[3,4], Hongxi Xing[1], Hushan Xu[3,4], Nu Xu[3,4,23], Shusheng Xu[38], Mengshi Yan[12], Wenbiao Yan[36], Wencheng Yan[20], Xinhu Yan[39], Jiancheng Yang[3,4], Yi-Bo Yang[4,14], Zhi Yang[40], Deliang Yao[5], Zhihong Ye[41], Peilin Yin[38], C.-P. Yuan[42], Wenlong Zhan[3,4], Jianhui Zhang[43], Jinlong Zhang[22], Pengming Zhang[44], Yifei Zhang[36], Chao-Hsi Chang[4,14], Zhenyu Zhang[45], Hongwei Zhao[3,4], Kuang-Ta Chao[12], Qiang Zhao[4,46], Yuxiang Zhao[3,4], Zhengguo Zhao[36], Liang Zheng[47], Jian Zhou[22], Xiang Zhou[45], Xiaorong Zhou[36], Bingsong Zou[4,14], Liping Zou[3,4]

x regions of b_1 in 2020's and 2030's



Summary on spin-1

Spin-1 structure functions of the deuteron (additional spin structure to nucleon spin)

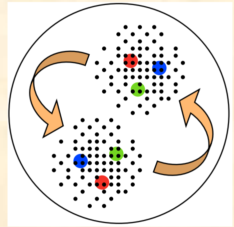
- **Tensor structure in quark-gluon degrees of freedom**
- **Tensor-polarized structure function b_1 and PDFs, gluon transversity**

  Experiments at JLab, Fermilab, NICA, LHCspin/AMBER, EIC/EicC, • • •

- **New signature beyond "standard" hadron physics?**

  (beyond the standard model in particle physics???)

standard model



?

- **TMDs up to twist 4**
- **Higher-twist effects could be sizable at a few GeV^2 Q^2**

  → Our relations (WW-like, BC-like, from eq. of motion, Lorentz invariance) could become valuable for future experimental analyses.

There are various experimental projects on the polarized spin-1 deuteron in 2020's and 2030', and "exotic" hadron structure could be found by focusing on the spin-1 nature.

The US-EIC should play the leading role.

Collaboration opportunities

Collaboration opportunities

Spin-1 deuteron with tensor polarizations (directly related to EIC)

- Most spin-1 studies are for fixed target experiments.
  EIC: polarized-deuteron beam will be available.
- Gluon-transversity estimates
- Antiquark tensor polarization
- Leading-twist distributions from small *x* to large *x*
- Estimates of higher-twist distributions and
  possible methods for measuring higher-twist distributions

GPD (our studies were, so far, indirectly related to EIC)

- Timelike GPDs
- GPDs at hadron accelerator facilities
- Advantages of GPD studies in neutrino reactions
- Model studies on gravitational form factors
- Gluon contributions to gravitational form factors

The End

The End