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Construction of some higher spin currents in the enhanced N=3 Kazama-Suzuki model

Department of Physics, Major in Particle Physics
The Graduate School

Jaesung Hong

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The Graduate School
Kyungpook National University

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Department of Physics, Major in Particle Physics
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Supervised by Professor Changhyun Ahn

Approved as a qualified thesis of Jaesung Hong
for the degree of Ph.D. by the Evaluation Committee

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1 Introduction

Various minimal models can be obtained from coset construction. One simple example is [1]

$$\frac{\widehat{su}(2)_k \times \widehat{su}(2)_1}{\widehat{su}(2)_{k+1}}, \tag{1.1}$$

where k is a level. The central charges of this model is [1]

$$1 - \frac{6}{(k+2)(k+3)}, \tag{1.2}$$

which is the exactly same as central charges of unitary minimal models when the $k \geq 1 (k \in \mathbb{Z})$. There is a more interesting example, which is [1]

$$\frac{\widehat{su}(2)_k \times \widehat{su}(2)_2}{\widehat{su}(2)_{k+2}}. \tag{1.3}$$

This coset realization leads to the series [1]

$$c = \frac{3}{2}\left(1 - \frac{8}{(n+2)(n+3)}\right), \tag{1.4}$$

which is the central charges of $\mathcal{N}$=1 super Virasoro algebras.

Higher spin algebras(W-algebras and super W-algebras), which play a crucial role in string theory, can be realized by coset construction. It follows that various coset models appear in the CFT side of the AdS_3/CFT_2 holography which was suggested by M. Gaberdiel and R. Gopakumar in [2]. We have a special interest in the enhanced $\mathcal{N}$=3 Kazama-Suzuki model, which have the coset [3, 4]

$$\frac{\widehat{su}(N+M)_k \times \widehat{so}(2NM)_1}{\widehat{su}(N)_{k+M} \times \widehat{su}(M)_{k+N} \times \widehat{u}(1)_{NM(N+M)(N+M+k)}}, \qquad c = \frac{3kNM}{(k+N+M)}, \tag{1.5}$$

among the coset models. The model was proposed by T. Creutzig, Y. Hikida, and P. B. Rønne in [3]. In [4, 5] and this thesis, $k = N + M$, i.e. critical level, to construct a supersymmetric model [3]. It is already introduced some low higher spin currents for this model in [4, 5]. In [4], the SOPE between the lowest higer spin(superspin-$\frac{3}{2}$) supermultiplet and itself is found.

The outline of this thesis is as follows: In section 2, we briefly review the Sugawara construction, coset theroy, and extensions of conformal symmetry.

In section 3, the four kinds of OPEs between component currents of the first and the second $\mathcal{N}$=3 supermultiplets in the enhanced $\mathcal{N}$=3 Kazama-Suzuki model with fixed (N,M) are observed [5]. The right hand side of those OPEs have component currents of seven new supermultiplets. One of the new supermultiplets have the lowest higher spin.

In section 4, we describe the SOPE between the first and the second $\mathcal{N}$=3 supermultiplets from the OPEs constructed in section 3 [5]. The right hand side of the SOPE contains four new SO(3) singlets and three new SO(3) triplets. We also study some structure of component OPEs between the two lowest higher spin supermultiplets.

2 Preliminary

2.1 Sugawara construction

Let us begin with a Kac-Moody algebra $\hat{\mathfrak{g}}_k$ [1,6],

$$\left[j_m^\alpha, j_n^\beta\right] = \imath \sum_\gamma f^{\alpha\beta\gamma} j_{m+n}^\gamma + km\delta^{\alpha\beta}\delta_{m+n,0}, \tag{2.1}$$

where k is the level of $\hat{\mathfrak{g}}_k$ and $f^{\alpha\beta\gamma}$ are the structure constants of the Lie algebra $\mathfrak{g}$. One can construct the ansatz for the energy-momentum tensor [1,6]

$$T(z) = d \sum_{\alpha=1}^{dim\,\mathfrak{g}} (j^\alpha j^\alpha)(z), \tag{2.2}$$

where $(j^\alpha j^\alpha)$ denotes the normal ordering of two operators, via the currents j^α of a Kac-Moody algebra $\hat{\mathfrak{g}}_k$. We use the commutation relation

$$[L_m, j_n^\alpha] = ((\Delta - 1)m - n)\, j_{m+n}^\alpha, \tag{2.3}$$

where L_m is the Laurent mode of $T(z)$ and Δ is the conformal dimension of the currents consisting of $\hat{\mathfrak{g}}_k$, to determine d. Notice that the conformal dimension of the currents $j^\alpha(z)$ is one. With the equation (2.2) and simple calculation, one finds [1]

$$[L_m, j_n^\alpha] = -2dn(k + C_{\mathfrak{g}}) j_{m+n}^\alpha, \tag{2.4}$$

where $C_{\mathfrak{g}}$ denotes the dual Coxeter number of the Lie algebra $\mathfrak{g}$. It follows from the relation (2.3) that $d = \frac{1}{2(k+C_{\mathfrak{g}})}$. We, then, have the explicit form of the energy-momentum tensor [1,6],

$$T(z) = \frac{1}{2(k + C_{\mathfrak{g}})} \sum_{\alpha=1}^{dim\,\mathfrak{g}} (j^\alpha j^\alpha)(z), \tag{2.5}$$

which is called the Sugawara construction. We have, now, the extended conformal field theory from the Kac-Moody algebra $\hat{\mathfrak{g}}_k$ via the Sugawara construction. Notice that the central charge of this extended conformal field theory is obtained by a simple algebraic calculation(See [1,6] for details). The central charge [1,6]

$$c = \frac{k\, dim\,\mathfrak{g}}{k + C_{\mathfrak{g}}}. \tag{2.6}$$

The currents j^α can be constructed from fermion fields in the $\widehat{so}(N)_1$ algebra. In this theory, the currents are [1]

$$j^\alpha(z) = \frac{1}{2}(\psi^i t_{ij}^\alpha \psi^j)(z), \tag{2.7}$$

where t_{ij}^α are the matrices of the SO(N) vector representation and ψ^i are N real fermions. The index α runs over 1 to $\frac{N(N-1)}{2}$, and i, j run over 1 to N. According to the equation (2.6), the central charge of the $\widehat{so}(N)_1$ algebra realized by N free fermions is $\frac{N}{2}$.

2.2 Coset theory

If there exists the subalgebra $\mathfrak{h}$ of the Lie algebra $\mathfrak{g}$, one can construct the Sugawara energy-momentum tensor, which is [1]

$$T_{\mathfrak{h}}(z) = \frac{1}{2(k_{\mathfrak{h}} + C_{\mathfrak{h}})} \sum_{\alpha=1}^{dim\,\mathfrak{h}} (j_{\mathfrak{h}}^{\alpha} j_{\mathfrak{h}}^{\alpha})(z), \tag{2.8}$$

where $j_{\mathfrak{h}}^{\alpha}$ denotes the currents of the subalgebra $\hat{\mathfrak{h}}_{k_{\mathfrak{h}}}$. One can seperate $T_{\mathfrak{g}}$ and define the energy-momentum tensor $T_{\mathfrak{g}/\mathfrak{h}}$ of the coset theory $\hat{\mathfrak{g}}_{k_{\mathfrak{g}}}/\hat{\mathfrak{h}}_{k_{\mathfrak{h}}}$ as follows: [1,6]

$$T_{\mathfrak{g}} = (T_{\mathfrak{g}} - T_{\mathfrak{h}}) + T_{\mathfrak{h}} \quad and \quad T_{\mathfrak{g}/\mathfrak{h}} \equiv T_{\mathfrak{g}} - T_{\mathfrak{h}} \tag{2.9}$$

because the OPEs between $(T_{\mathfrak{g}} - T_{\mathfrak{h}})(z)$ and $j_{\mathfrak{h}}^{\alpha}(w)$, and $(T_{\mathfrak{g}} - T_{\mathfrak{h}})(z)$ and $T_{\mathfrak{h}}$ are regular. Since the OPE between $T_{\mathfrak{g}/\mathfrak{h}}$ and $T_{\mathfrak{h}}$ is regular, we have

$$T_{\mathfrak{g}/\mathfrak{h}} T_{\mathfrak{g}/\mathfrak{h}} = T_{\mathfrak{g}} T_{\mathfrak{g}} - T_{\mathfrak{h}} T_{\mathfrak{h}} + \cdots . \tag{2.10}$$

We, then, have the relation [1]

$$c_{\mathfrak{g}/\mathfrak{h}} = c_{\mathfrak{g}} - c_{\mathfrak{h}} = \frac{k_{\mathfrak{g}}\, dim\,\mathfrak{g}}{k_{\mathfrak{g}} + C_{\mathfrak{g}}} - \frac{k_{\mathfrak{h}}\, dim\,\mathfrak{h}}{k_{\mathfrak{h}} + C_{\mathfrak{h}}}. \tag{2.11}$$

One can obtain a large variety of minimal models from coset constructions. Some of those constructions provide important minimal models. We introduce one of those constructions, so called diagonal cosets which contains crucial constructions that describe W_N minimal models. The coset construction having the structure $\frac{\hat{\mathfrak{g}}_{k_1} \times \hat{\mathfrak{g}}_{k_2}}{\hat{\mathfrak{g}}_{k_3}}$ with the currents $j_{(1)}^{\alpha}$, $j_{(2)}^{\beta}$, and $j_{(3)}^{\gamma}$, where $j_{(3)}^{\alpha} = j_{(1)}^{\alpha} + j_{(2)}^{\alpha}$ and $\left[j_{(1),m}^{\alpha}, j_{(2),n}^{\beta}\right] = 0$ by definition, have the commutation relation for the current $j_{(3)}^{\gamma}$ [1]

$$\left[j_{(3),m}^{\alpha}, j_{(3),n}^{\beta}\right] = \imath \sum_{\gamma} f_{(3)}^{\alpha\beta\gamma} j_{(3),m+n}^{\gamma} + k_3 m \delta^{\alpha\beta} \delta_{m+n,0}, \tag{2.12}$$

where $f_{(3)}^{\alpha\beta\gamma} = f_{(1)}^{\alpha\beta\gamma} + f_{(2)}^{\alpha\beta\gamma}$ and $k_3 = k_1 + k_2$. The cosets described above are called diagonal cosets. The energy-momentum tensor for those cosets is [1]

$$T_{(\mathfrak{g}_{k_1} \times \mathfrak{g}_{k_2})/\mathfrak{g}_{k_3}} = T_{\mathfrak{g}_{k_1}} + T_{\mathfrak{g}_{k_2}} - T_{\mathfrak{g}_{k_3}}. \tag{2.13}$$

2.3 Extension of Virasoro algebra

A plentiful number of the extended conformal symmetries are obtained from Kac-Moody algebras via the Sugawara constructions and their coset constructions. Especially, W symmetries have great importance in string theory, which was firstly proposed by A. B. Zamolodchikov in [7,8][1]. W_N algebra is obtained by the coset construction [2,9],

$$\frac{\widehat{su}(N)_k \times \widehat{su}(N)_1}{\widehat{su}(N)_{k+1}}. \tag{2.14}$$

[1] W_3 symmetry was proposed.

$$[L_m, L_n] = (m-n)L_{m+n} + \tfrac{c}{12}(m^3-m)\delta_{m+n,0},$$

$$\left[L_m, G^i_r\right] = \left(\tfrac{m}{2}-r\right)G^i_{m+r}, \quad \left[L_m, J^i_n\right] = -nJ^i_{m+n}, \quad [L_m, \Gamma_s] = -\left(\tfrac{m}{2}+s\right)\Gamma_{m+s},$$

$$\left\{G^i_r, G^j_s\right\} = 2\delta^{ij}L_{r+s} + \imath(r-s)\varepsilon^{ijk}J^k_{r+s} + \tfrac{c}{3}\left(m^2-\tfrac{1}{4}\right)\delta^{ij}\delta_{m+n,0},$$

$$\left[J^i_m, G^j_r\right] = \imath\varepsilon^{ijk}G^k_{m+r} + m\delta^{ij}\Gamma_{m+r}, \quad \left\{\Gamma_r, G^i_s\right\} = J^i_{r+s},$$

$$\left[J^i_m, J^j_n\right] = \imath\varepsilon^{ijk}J^k_{m+n} + \tfrac{c}{3}m\delta^{ij}\delta_{m+n,0},$$

$$\{\Gamma_r, \Gamma_s\} = \tfrac{c}{3}\delta_{m+n,0}.$$

Table 2.1: *The*$\mathscr{N}$=3 super Virasoro algebra

This is the diagonal coset which is studied in Sec. 2.2. The central charge of this theory has the form

$$c\left[\frac{\widehat{su}(N)_k \times \widehat{su}(N)_1}{\widehat{su}(N)_{k+1}}\right] = c\left[\widehat{su}(N)_k\right] + c\left[\widehat{su}(N)_1\right] - c\left[\widehat{su}(N)_{k+1}\right] \tag{2.15}$$
$$= \frac{k(N^2-1)}{k+N} + \frac{(N^2-1)}{N+1} - \frac{(k+1)(N^2-1)}{k+N+1} = (N-1)\left(1 - \frac{N(N+1)}{(k+N)(k+N+1)}\right).$$

Note that the form $c[\mathfrak{g}_k] \equiv c_{\mathfrak{g}_k}$.

Supersymmetric extensions are another way to extend Virasoro symmetry. We are not going to study the details of superconformal algebras. Instead, we will focus on the $\mathscr{N}$=3 superconformal algebra which we mainly deal with. The table 2.1 describes the $\mathscr{N}$=3 super Virasoro algebra. The following SOPE between the $\mathscr{N}$=3 stress energy tensor and itself induces the $\mathscr{N}$=3 super Virasoro algebra in the table 2.1 [4, 10–13]:

$$\mathbf{J}(Z_1)\mathbf{J}(Z_2) = -\frac{1}{z_{12}}\frac{c}{12} + \frac{\theta_{12}^{3-0}}{z_{12}^2}\frac{1}{2}\mathbf{J}(Z_2) + \frac{\theta_{12}^{3-i}}{z_{12}^2}\frac{1}{2}D^i\mathbf{J}(Z_2) + \frac{\theta_{12}^{3-0}}{z_{12}^2}\partial\mathbf{J}(Z_2) + \cdots, \tag{2.16}$$

where [4, 12]

$$\mathbf{J}(Z) = \frac{\imath}{2}\Psi(z) + \theta^i\frac{\imath}{2}J^i(z) + \theta^{3-i}\frac{1}{2}G^i(z) + \theta^{3-0}T(z). \tag{2.17}$$

We used the notations of [4][2]. The super derivative $D^i = \frac{\partial}{\partial\theta^i} + \theta^i\frac{\partial}{\partial z}$.

[2] $Z_k = (z_k, \theta^i_k)$, $\theta^i_{12} = \theta^i_1 - \theta^i_2$, $z_{12} = z_1 - z_2 - \theta^i_1\theta^i_2$, $\theta^{3-0} = \theta^1\theta^2\theta^3$, and $\theta^{3-i}\theta^i = \theta^1\theta^2\theta^3$(no summation over i).

3 The OPEs between the component currents of the first and the second supermultiplets in the enhanced $\mathscr{N}$=3 Kazama-Suzuki model

In this section, we will review the results of [5]. The basic information such as the explicit construction of the $\mathscr{N}$=3 stress energy tensor and its components in the enhanced $\mathscr{N}$=3 Kazama-Suzuki model, the primary conditions for superspin-Δ supermultiplets in the $\mathscr{N}$=3 supersymmetric theories and their component approach [12, 13], the SOPE between the first $\mathscr{N}$=3 supermultiplet and itself, and the several super primary supermultiplets in the SOPE in the enhanced $\mathscr{N}$=3 Kazama-Suzuki model is explicitly described in [3, 4].

Based on [3, 4], we compute the OPEs between the component currents of the first and the second $\mathscr{N}$=3 supermultiplets by using Thielemans package [15] with fixed (N,M)=(3,2) [5]. Therefore, we can obtain the explicit form of the currents appeared in the right hand side of the OPEs to the coset(The equation (1.5)) field level(See [4,5] for computation details). We will not, however, specifically describe those currents because those are extremely complex and only for fixed (N,M) [5].

3.1 The OPE between the lowest spin component current of the first $\mathscr{N}$=3 supermultiplet and the lowest spin component of the second $\mathscr{N}$=3 supermultiplet

The OPE between the spin-$\frac{3}{2}$ current of the first $\mathscr{N}$=3 supermultiplet and the spin-2 current of the $\mathscr{N}$=3 second supermultiplet is [5]

$$\psi_{a_1}^{(\frac{3}{2})}(z)\psi_{a_1}^{(2)}(w) \sim \frac{1}{(z-w)^2}\left[-2iC_{a_2,(\frac{3}{2})(2)}^{(\frac{3}{2})}\psi_{a_2}^{(\frac{3}{2})}\right](w) + \frac{1}{(z-w)}\left[\frac{1}{3}\partial(pole-2)\right](w). \tag{3.1}$$

We follow the notations in [4,5] except the subscript of the currents to distinguish those currents[3].

There is a new spin-$\frac{3}{2}$ primary current $\psi_{a_2}^{(\frac{3}{2})}$ with the structure constant $C_{a_2,(\frac{3}{2})(2)}^{(\frac{3}{2})}$ in the second order pole of the OPE [5]. The new primary current is the lowest spin component of a new superspin-$\frac{3}{2}$ $\mathscr{N}$=3 supermultiplet(SO(3) singlet). The other component currents of this supermultiplet are obtained by applying the primary condition for the $\mathscr{N}$=3 higher spin currents(See the appendix B of [4] for details) [4, 12]. The descendant with a proper constant is determined by the formula of the OPE between two quasi-primary fields which is well explained in [1].

[3] a_1 represents the currents appeared in [4], and a_2 and a_3 represent the currents appeared in [5]

3.2 The OPEs between the three second lowest spin component currents of the first $\mathcal{N}=3$ supermultiplet and the lowest spin component of the second $\mathcal{N}=3$ supermultiplet

The OPEs between the spin-2 currents of the first $\mathcal{N}=3$ supermultiplet and the spin-2 current of the second $\mathcal{N}=3$ supermultiplet are [5]

$$\begin{aligned}
\phi_{a_1}^{(2),i}(z)\psi_{a_1}^{(2)}(w) \sim & \frac{1}{(z-w)^2}\left[-\frac{2}{3}iC_{a_2,(\frac{3}{2})(2)}^{(\frac{3}{2})}\phi_{a_2}^{(2),i}\right](w) \\
& +\frac{1}{(z-w)}\left[\frac{1}{2}\partial\,(pole-2)+C_{a_2,(\frac{3}{2})(2)}^{(\frac{3}{2})}\left(\frac{27i(c+1)}{c(c+3)(c-3)}\Psi J^i\psi_{a_2}^{(\frac{3}{2})}\right.\right. \\
& \quad -\frac{9i(c+1)}{(c+3)(c-3)}\left(G^i\psi_{a_2}^{(\frac{3}{2})}-\frac{1}{2}\partial\phi_{a_2}^{(2),i}\right) \\
& \quad \left.-\frac{(c-15)}{(c+3)(c-3)}\varepsilon^{ijk}\left(J^j\phi_{a_2}^{(2),k}-\frac{i}{4}\varepsilon^{jkl}\partial\phi_{a_2}^{(2),l}\right)\right) \\
& \quad +C_{a_2,(3)(2)}^{(2)}\left(\frac{c}{10(c-3)}\psi_{a_2}^{(3),i}-\frac{3}{10(c-3)}\Psi\phi_{a_2}^{(\frac{5}{2}),i}-\frac{6}{5(c-3)}J^i\psi_{a_2}^{(2)}\right) \\
& \quad -C_{a_2,(\frac{5}{2})(2)}^{(\frac{5}{2})}\left(\frac{c}{3(c-1)}\delta^{i\alpha}(T^j)^{\alpha\beta}\phi_{a_2}^{(3),j,\beta}+\frac{2}{c-1}\Psi\psi_{a_2}^{(\frac{5}{2}),\alpha=i}\right) \\
& \quad \left.-2iC_{a_2,(2)(2)}^{(3)}\psi_{a_2}^{(3),\alpha=i}\right](w) \qquad (3.2)
\end{aligned}$$

Note that T^i are SO(3) generators [4,5]. Spin-2 primary fields $\phi_{a_2}^{(2),i}$ appear in the second order pole and are the spin-2 components of the superspin-$\frac{3}{2}$ $\mathcal{N}=3$ supermultiplet which is stated in the previous subsection. There are five primary fields[4] related to the structure constants $C_{a_2,(3)(2)}^{(2)}$ and $C_{a_2,(\frac{5}{2})(2)}^{(\frac{5}{2})}$ in the first order pole of the OPE. Those currents contain new primary currents which are not defined yet [5]. However, they will be defined in the following subsections. New primary currents $\psi_{a_2}^{(3),\alpha=i}$ also emerged in the first order pole with the structure constant $C_{a_2,(2)(2)}^{(3)}$ [5]. Those are the spin-3 components of a new superspin-3 $\mathcal{N}=3$ supermultiplet(SO(3) triplet). There are three more primary currents[5] with the structure constant $C_{a_2,(\frac{3}{2})(2)}^{(\frac{3}{2})}$ in the first order pole.

3.3 The OPEs between the three second highest spin component currents of the first $\mathcal{N}=3$ supermultiplet and the lowest spin component of the second $\mathcal{N}=3$ supermultiplet

The OPEs between the spin-$\frac{5}{2}$ currents of the first $\mathcal{N}=3$ supermultiplet and the spin-2 current of the second $\mathcal{N}=3$ supermultiplet are [5]

$$\begin{aligned}
\psi_{a_1}^{\frac{5}{2},i}(z)\psi_{a_1}^{2}(w) \sim & \frac{1}{(z-w)^2}\left[-C_{a_2,(\frac{3}{2})(2)}^{(\frac{3}{2})}\left(\frac{i(c-15)}{3(c-3)}\psi_{a_2}^{(\frac{5}{2}),i}+\frac{i(c+9)}{c(c-3)}\Psi\phi_{a_2}^{(2),i}+\frac{3i(5c-3)}{c(c-3)}J^i\psi_{a_2}^{(\frac{3}{2})}\right)\right. \\
& \left.+\frac{1}{2}C_{a_2,(3)(2)}^{(2)}\phi_{a_2}^{(\frac{5}{2}),i}+2C_{a_2,(\frac{5}{2})(2)}^{(\frac{5}{2})}\psi_{a_2}^{(\frac{5}{2}),\alpha=i}\right](w)
\end{aligned}$$

[4] Those are $\psi_{a_2}^{(3),i}$, $\Psi\phi_{a_2}^{(\frac{5}{2}),i}$, $J^i\psi_{a_2}^{(2)}$, $\delta^{i\alpha}(T^j)^{\alpha\beta}\phi_{a_2}^{(3),j,\beta}$, $\Psi\psi_{a_2}^{(\frac{5}{2}),\alpha=i}$.

[5] Those are $\Psi J^i\psi_{a_2}^{(\frac{3}{2})}$, $G^i\psi_{a_2}^{(\frac{3}{2})}-\frac{1}{2}\partial\phi_{a_2}^{(2),i}$, $\varepsilon^{ijk}\left(J^j\phi_{a_2}^{(2),k}-\frac{i}{4}\varepsilon^{jkl}\partial\phi_{a_2}^{(2),l}\right)$.

$$
\begin{aligned}
&+\frac{1}{(z-w)}\Bigg[\frac{3}{5}\partial\,(pole-2)\\
&\quad + C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\Bigg(-\frac{27i(c+1)}{c(c+3)(c-3)}\left(\Psi G^i\psi^{(\frac{3}{2})}_{a_2}-\frac{1}{2}\Psi\partial\phi^{(2),i}_{a_2}\right)\\
&\quad - \frac{18(c+1)(c-1)}{(5c+6)(c+3)(c-3)}\varepsilon^{ijk}\left(G^j\phi^{(2),k}_{a_2}-\frac{2}{5}i\varepsilon^{jkl}\partial\psi^{(\frac{5}{2}),l}_{a_2}\right)\\
&\quad - \frac{2\left(c^2-18c-27\right)}{(5c+6)(c+3)(c-3)}\varepsilon^{ijk}\left(J^j\psi^{(\frac{5}{2}),k}_{a_2}-\frac{i}{5}\varepsilon^{jkl}\partial\psi^{(\frac{5}{2}),l}_{a_2}\right)\\
&\quad - \frac{i(5c^3-72c^2-99c-54)}{c(c+3)(c-3)(5c+6)}\left(\partial\Psi\phi^{(2),i}_{a_2}-\frac{1}{5}\partial(\Psi\phi^{(2),i}_{a_2})\right)\\
&\quad - \frac{12i(5c^3-18c-9)}{c(c+3)(c-3)(5c+6)}\left(\partial J^i\psi^{(\frac{3}{2})}_{a_2}-\frac{2}{5}\partial(J^i\psi^{(\frac{3}{2})}_{a_2})\right)\\
&\quad + \frac{9(c+1)}{c(c+3)(c-3)}\varepsilon^{ijk}\left(\Psi J^j\phi^{(2),k}_{a_2}-\frac{i}{4}\varepsilon^{jkl}\Psi\partial\phi^{(2),l}_{a_2}\right)\Bigg)\\
&\quad + C^{(2)}_{a_2,(3)(2)}\Bigg(-\frac{18}{5(c-3)(2c+9)}\Psi J^i\psi^{(2)}_{a_2}+\frac{3}{10(c-3)}\Psi\psi^{(3),i}_{a_2}\\
&\quad - \frac{6(c+9)}{5(c-3)(2c+9)}\left(G^i\psi^{(2)}_{a_2}-\frac{2}{5}\partial\phi^{(\frac{5}{2}),i}_{a_2}\right)\\
&\quad - \frac{3i(2c+3)}{5(c-3)(2c+9)}\varepsilon^{ijk}\left(J^j\phi^{(\frac{5}{2}),k}_{a_2}-\frac{i}{5}\varepsilon^{jkl}\partial\phi^{(\frac{5}{2}),l}_{a_2}\right)\Bigg)\\
&\quad - C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}\Bigg(\frac{1}{c-1}\delta^{i\alpha}(T^j)^{\alpha\beta}\Psi\phi^{(3),j,\beta}_{a_2}\\
&\quad + \frac{1}{7}\delta^{i\alpha}(T^j)^{\alpha\beta}\left(\psi^{(\frac{7}{2}),j,\beta}_{a_2}+\frac{1}{5}(T^j)^{\beta\gamma}\partial\psi^{(\frac{5}{2}),\gamma}_{a_2}\right)\Bigg)\\
&\quad + C^{(3)}_{a_2,(2)(2)}\left(\frac{6i(c-1)}{7c-6}\delta^{i\alpha}(T^j)^{\alpha\beta}\phi^{(\frac{7}{2}),j,\beta}_{a_2}-\frac{6i}{7c-6}\Psi\psi^{(3),\alpha=i}_{a_2}\right)\\
&\quad + \frac{1}{3}C^{(3)}_{a_3,(3)(2)}\phi^{(\frac{7}{2}),i}_{a_3}+2C^{(\frac{7}{2})}_{a_2,(\frac{5}{2})(2)}\psi^{(\frac{7}{2}),\alpha=i}_{a_2}\Bigg](w)
\end{aligned}
\tag{3.3}
$$

In the second order pole, we have new primary currents $\psi^{(\frac{5}{2}),\alpha=i}_{a_2}$ with the structure constant $C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}$ which explains the two terms with $C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}$ appeared in the previous subsection clearly. Those currents are the spin-$\frac{5}{2}$ components of a new superspin-$\frac{5}{2}$ $\mathscr{N}=3$ supermultiplet(SO(3) triplet). Another new primary currents $\phi^{(\frac{5}{2}),i}_{a_2}$ with the structure constant $C^{(2)}_{a_2,(3)(2)}$ are still undefined at the moment. There are three more primary currents[6] in the second order pole. There exist new spin-$\frac{7}{2}$ primary currents $\psi^{(\frac{7}{2}),\alpha=i}_{a_2}$ which are the spin-$\frac{7}{2}$ components of a new superspin-$\frac{7}{2}$ $\mathscr{N}=3$ supermultiplet(SO(3) triplet) in the first order pole [5]. Five primary currents[7] with the structure contants $C^{(2)}_{a_2,(3)(2)}$ and $C^{(3)}_{a_3,(3)(2)}$ are not determined yet. Those will

[6] Those are $\psi^{(\frac{5}{2}),i}_{a_2}$, $\Psi\phi^{(2),i}_{a_2}$, $J^i\psi^{(\frac{3}{2})}_{a_2}$.

[7] Those are $\Psi J^i\psi^{(2)}_{a_2}$, $\Psi\psi^{(3),i}_{a_2}$, $G^i\psi^{(2)}_{a_2}-\frac{2}{5}\partial\phi^{(\frac{5}{2}),i}_{a_2}$, $\varepsilon^{ijk}\left(J^j\phi^{(\frac{5}{2}),k}_{a_2}-\frac{i}{5}\varepsilon^{jkl}\partial\phi^{(\frac{5}{2}),l}_{a_2}\right)$, $\delta^{i\alpha}(T^j)^{\alpha\beta}\Psi\phi^{(3),j,\beta}_{a_2}$.

be defined in the next subsection. Ten more primary currents[8] with the structure constants $C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}$, $C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}$, and $C^{(3)}_{a_2,(2)(2)}$ appear in the first order pole.

3.4 The OPE between the highest spin component current of the first $\mathcal{N}=3$ supermultiplet and the lowest spin component of the second $\mathcal{N}=3$ supermultiplet

The OPE between the spin-3 current of the first $\mathcal{N}=3$ supermultiplet and the spin-2 current of the second $\mathcal{N}=3$ supermultiplet is [5]

$$\begin{aligned}
\phi^{(3)}_{a_1}(z)\psi^{(2)}_{a_1}(w) \sim & \frac{1}{(z-w)^3}\left[-\frac{6i}{c}C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\Psi\psi^{(\frac{3}{2})}_{a_2} + C^{(2)}_{a_2,(3)(2)}\psi^{(2)}_{a_2}\right](w) \\
& + \frac{1}{(z-w)^2}\left[\frac{3}{4}\partial\,(pole-3) - C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\left(\frac{2i(7c+15)}{(c-3)(5c+6)}J^i\phi^{(2),i}_{a_2}\right.\right. \\
& \left. + \frac{8i(8c+3)}{(c-3)(5c+6)}\left(\partial\Psi\psi^{(\frac{3}{2})}_{a_2} - \frac{1}{4}\partial\left(\Psi\psi^{(\frac{3}{2})}_{a_2}\right)\right) + \frac{i\left(c^2-25c-42\right)}{(c-3)(5c+6)}\phi^{(3)}_{a_2}\right) \\
& \left. + \frac{3}{7}C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}\phi^{(3),i,\alpha=i}_{a_2} + C^{(3)}_{a_3,(3)(2)}\psi^{(3)}_{a_3}\right](w) \\
& + \frac{1}{(z-w)}\left[\frac{3}{10}\partial^2\,(pole-3) + \frac{2}{3}\partial\left(pole-2-\frac{3}{4}\partial(pole-3)\right)\right. \\
& + C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\left(\frac{3i(c-12)(c+1)}{c(c-3)(c+3)^2}\Psi J^i\psi^{(\frac{5}{2}),i}_{a_2}\right. \\
& - \frac{3i(c-12)(c+1)}{c(c-3)(c+3)^2}\left(\Psi G^i\phi^{(2),i}_{a_2} - \frac{1}{4}\delta^{ii}\Psi\partial^2\psi^{(\frac{3}{2})}_{a_2}\right) \\
& + \frac{135i(c+1)}{c(c-3)(c+3)^2}\Psi J^iJ^i\psi^{(\frac{3}{2})}_{a_2} - \frac{i(c-12)(c+1)}{(c-3)(c+3)^2}\left(G^i\psi^{(\frac{5}{2}),i}_{a_2} - \frac{2}{3}\delta^{ii}\partial\phi^{(3)}_{a_2}\right) \\
& - \frac{45i(c+1)}{(c-3)(c+3)^2}\left(J^iG^i\psi^{(\frac{3}{2})}_{a_2} - \frac{1}{2}J^i\partial\phi^{(2),i}_{a_2}\right) \\
& - \frac{i\left(4c^2-3c-135\right)}{2(c-3)(c+3)^2}\left(\partial J^i\phi^{(2),i}_{a_2} - \frac{1}{3}\partial\left(J^i\phi^{(2),i}_{a_2}\right)\right) \\
& - \frac{i(32c^3+51c^2+207c-324)}{2c(c-3)(c+3)^2}\left(\frac{1}{2}\partial^2\Psi\psi^{(\frac{3}{2})}_{a_2}\right. \\
& \left.\left. - \frac{1}{3}\partial\left(\partial\Psi\psi^{(\frac{3}{2})}_{a_2} - \frac{1}{4}\partial\left(\Psi\psi^{(\frac{3}{2})}_{a_2}\right)\right) - \frac{1}{20}\partial^2\left(\Psi\psi^{(\frac{3}{2})}_{a_2}\right)\right)\right) \\
& + C^{(2)}_{a_2,(3)(2)}\left(-\frac{18\left(8c^2+53c+57\right)}{5(c-3)(2c+9)\left(c^2+26c+9\right)}J^iJ^i\psi^{(2)}_{a_2}\right. \\
& + \frac{3\left(2c^3+7c^2+63c+90\right)}{5(c-3)(2c+9)\left(c^2+26c+9\right)}J^i\psi^{(3),i}_{a_2}
\end{aligned}$$

[8] Those are $\Psi G^i\psi^{(\frac{3}{2})}_{a_2} - \frac{1}{2}\Psi\partial\phi^{(2),i}_{a_2}$, $\varepsilon^{ijk}\left(G^j\phi^{(2),k}_{a_2} - \frac{2}{5}i\varepsilon^{jkl}\partial\psi^{(\frac{5}{2}),l}_{a_2}\right)$, $\varepsilon^{ijk}\left(J^j\psi^{(\frac{5}{2}),k}_{a_2} - \frac{i}{5}\varepsilon^{jkl}\partial\psi^{(\frac{5}{2}),l}_{a_2}\right)$, $\partial\Psi\phi^{(2),i}_{a_2} - \frac{1}{5}\partial(\Psi\phi^{(2),i}_{a_2})$, $\partial J^i\psi^{(\frac{3}{2})}_{a_2} - \frac{2}{5}\partial(J^i\psi^{(\frac{3}{2})}_{a_2})$, $\varepsilon^{ijk}\left(\Psi J^j\phi^{(2),k}_{a_2} - \frac{i}{4}\varepsilon^{jkl}\Psi\partial\phi^{(2),l}_{a_2}\right)$, $\delta^{i\alpha}(T^j)^{\alpha\beta}\Psi\phi^{(3),j,\beta}_{a_2}$, $\delta^{i\alpha}(T^j)^{\alpha\beta}\left(\psi^{(\frac{7}{2}),j,\beta}_{a_2} + \frac{1}{5}(T^j)^{\beta\gamma}\partial\psi^{(\frac{5}{2}),\gamma}_{a_2}\right)$, $\delta^{i\alpha}(T^j)^{\alpha\beta}\phi^{(\frac{7}{2}),j,\beta}_{a_2}$, $\Psi\psi^{(3),\alpha=i}_{a_2}$.

$$
\begin{aligned}
&+\frac{12c\left(4c^2+9c-99\right)}{5(c-3)(2c+9)\left(c^2+26c+9\right)}\left(T\psi_{a_2}^{(2)}-\frac{3}{10}\partial^2\psi_{a_2}^{(2)}\right)\\
&-\frac{27\left(6c^2+21c-1\right)}{10(c-3)(2c+9)\left(c^2+26c+9\right)}\Psi J^i\phi_{a_2}^{(\frac{5}{2}),i}\\
&+\frac{3\left(14c^3+49c^2-129c-180\right)}{10(c-3)(2c+9)\left(c^2+26c+9\right)}\left(G^i\phi_{a_2}^{(\frac{5}{2}),i}-\frac{1}{5}\partial^2\psi_{a_2}^{(2)}\right)\\
&-\frac{18\left(4c^2+9c-99\right)}{5(c-3)(2c+9)\left(c^2+26c+9\right)}\partial\Psi\Psi\psi_{a_2}^{(2)}\Bigg)\\
&+C_{a_2,(\frac{5}{2})(2)}^{(\frac{5}{2})}\Bigg(-\frac{2c}{(c-1)(c+15)}\delta^{i\alpha}(T^j)^{\alpha\beta}J^i\phi_{a_2}^{(3),j,\beta}\\
&-\frac{30}{(c-1)(c+15)}\Psi J^i\psi_{a_2}^{(\frac{5}{2}),\alpha=i}\\
&+\frac{6c}{(c-1)(c+15)}\left(G^i\psi_{a_2}^{(\frac{5}{2}),\alpha=i}-\frac{1}{3}\partial\phi_{a_2}^{(3),i,\alpha=i}\right)\Bigg)\\
&-C_{a_2,(2)(2)}^{(3)}\Bigg(\frac{3ic(c-2)}{2(c+3)(7c-6)}\psi_{a_2}^{(4),i,\alpha=i}+\frac{9i(c-2)}{2(c+3)(7c-6)}\Psi\phi_{a_2}^{(\frac{7}{2}),i,\alpha=i}\\
&+\frac{12i(4c-3)}{(c+3)(7c-6)}J^i\psi_{a_2}^{(3),\alpha=i}\Bigg)+\frac{4}{9}C_{a_2,(\frac{5}{2})(2)}^{(\frac{7}{2})}\phi_{a_2}^{(4),i,\alpha=i}+C_{a_2,(3)(2)}^{(4)}\psi_{a_2}^{(4)}\Bigg](w)
\end{aligned}
\tag{3.4}
$$

There exists a new primary current $\psi_{a_2}^{(2)}$ with the structure constant $C_{a_2,(3)(2)}^{(2)}$ which is the lowest spin component of a new superspin-2 $\mathscr{N}$=3 supermultiplet(SO(3) singlet) in the third order pole. This current explains every undetermined terms with the structure constant $C_{a_2,(3)(2)}^{(2)}$ in the previous two subsections. The OPEs are described in spin-ascending order. However, the computation is proceeded in the descending order of the degree of poles. This explains the appearance of the undefined currents in the previous sections. The other primary current $\Psi\psi_{a_2}^{(\frac{3}{2})}$ also appears in the third order pole. A primary current $\psi_{a_3}^{(3)}$ with the structure constant $C_{a_3,(3)(2)}^{(3)}$ which is the lowest spin component of a superspin-3 $\mathscr{N}$=3 supermultiplet(SO(3) singlet) is newly appeared in the second order pole. The current explains the term with the structure constant $C_{a_3,(3)(2)}^{(3)}$ in subsection 3.3. Four more primary currents[9] with the structure constants $C_{a_2,(\frac{3}{2})(2)}^{(\frac{3}{2})}$ and $C_{a_2,(\frac{5}{2})(2)}^{(\frac{5}{2})}$ emerged in the second order pole. There is a new primary current $\psi_{a_2}^{(4)}$ with the structure constant $C_{a_2,(3)(2)}^{(4)}$ which is the lowest spin component of a new superspin-4 $\mathscr{N}$=3 supermultiplet(SO(3) singlet) in the first order pole. We have ten more primary currents[10] and ten more quasi-primary currents[11] with the structure constants $C_{a_2,(\frac{3}{2})(2)}^{(\frac{3}{2})}$, $C_{a_2,(3)(2)}^{(2)}$, $C_{a_2,(\frac{5}{2})(2)}^{(\frac{5}{2})}$,

[9] Those are $J^i\phi_{a_2}^{(2),i}$, $\partial\Psi\psi_{a_2}^{(\frac{3}{2})}-\frac{1}{4}\partial\left(\Psi\psi_{a_2}^{(\frac{3}{2})}\right)$, $\phi_{a_2}^{(3)}$, $\phi_{a_2}^{(3),i,\alpha=i}$

[10] Those are $\Psi J^i\phi_{a_2}^{(\frac{5}{2}),i}$, $\Psi J^i\psi_{a_2}^{(\frac{5}{2}),\alpha=i}$, $\Psi\phi_{a_2}^{(\frac{7}{2}),i,\alpha=i}$, $G^i\psi_{a_2}^{(\frac{5}{2}),\alpha=i}-\frac{1}{3}\partial\phi_{a_2}^{(3),i,\alpha=i}$, $G^i\psi_{a_2}^{(\frac{5}{2}),i}-\frac{2}{3}\delta^{ii}\partial\phi_{a_2}^{(3)}$, $\delta^{i\alpha}(T^j)^{\alpha\beta}J^i\phi_{a_2}^{(3),j,\beta}$, $J^i\psi_{a_2}^{(3),\alpha=i}$, $\partial J^i\phi_{a_2}^{(2),i}-\frac{1}{3}\partial\left(J^i\phi_{a_2}^{(2),i}\right)$, $\psi_{a_2}^{(4),i,\alpha=i}$, $\phi_{a_2}^{(4),i,\alpha=i}$.

[11] Those are $T\psi_{a_2}^{(2)}-\frac{3}{10}\partial^2\psi_{a_2}^{(2)}$, $\Psi G^i\phi_{a_2}^{(2),i}-\frac{1}{4}\delta^{ii}\Psi\partial^2\psi_{a_2}^{(\frac{3}{2})}$, $\Psi J^iJ^i\psi_{a_2}^{(\frac{3}{2})}$, $\Psi J^i\psi_{a_2}^{(\frac{5}{2}),i}$, $G^i\phi_{a_2}^{(\frac{5}{2}),i}-\frac{1}{5}\partial^2\psi_{a_2}^{(2)}$, $J^iG^i\psi_{a_2}^{(\frac{3}{2})}-\frac{1}{2}J^i\partial\phi_{a_2}^{(2),i}$, $J^iJ^i\psi_{a_2}^{(2)}$, $J^i\psi_{a_2}^{(3),i}$, $\partial\Psi\Psi\psi_{a_2}^{(2)}$, $\frac{1}{2}\partial^2\Psi\psi_{a_2}^{(\frac{3}{2})}-\frac{1}{3}\partial\left(\partial\Psi\psi_{a_2}^{(\frac{3}{2})}-\frac{1}{4}\partial\left(\Psi\psi_{a_2}^{(\frac{3}{2})}\right)\right)-\frac{1}{20}\partial^2\left(\Psi\psi_{a_2}^{(\frac{3}{2})}\right)$

$C^{(3)}_{a_2,(2)(2)}$, and $C^{(\frac{7}{2})}_{a_2,(\frac{5}{2})(2)}$ in the first order pole.

4 The SOPE between the first and the second $\mathcal{N}$=3 supermultiplets, and some component OPEs between the two lowest higher spin supermultiplets

4.1 The SOPE between the first and the second $\mathcal{N}$=3 supermultiplets

It is obtained the structure of the SOPE between the first and the second $\mathcal{N}$=3 supermultiplets with undetermined coefficients from the component OPEs which are described in the previous section by applying the $\mathcal{N}$=3 superspace formalism [5, 17, 18]. To be more specific, applying the replacement [5]

$$\begin{aligned} \psi^{(\Delta),\alpha}_{a_i} &= -2i\mathbf{\Phi}^{(\Delta),\alpha}_{a_i}|_{\theta^i=0}, & \phi^{(\Delta+\frac{1}{2}),i,\alpha}_{a_i} &= -2iD^i\mathbf{\Phi}^{(\Delta),\alpha}_{a_i}|_{\theta^i=0}, \\ \psi^{(\Delta+1),i,\alpha}_{a_i} &= -2D^{3-i}\mathbf{\Phi}^{(\Delta),\alpha}_{a_i}|_{\theta^i=0}, & \phi^{(\Delta+\frac{3}{2}),\alpha}_{a_i} &= -D^{3-0}\mathbf{\Phi}^{(\Delta),\alpha}_{a_i}|_{\theta^i=0} \end{aligned} \tag{4.1}$$

to the OPEs described in section 3 gives us the structure of the SOPE between the first and second $\mathcal{N}$=3 supermultiplets with undetermined coefficients. The $\mathcal{N}$=3 supermultiplets appeared in the right hand side of this SOPE and the SOPE between the first $\mathcal{N}$=3 supermultiplet and itself[12] are as follows [4, 5]:

$$\begin{aligned}
\mathbf{\Phi}^{(\frac{3}{2})}_{a_1}(Z) &= \frac{i}{2}\psi^{(\frac{3}{2})}_{a_1}(z) + \theta^i\frac{i}{2}\phi^{(2),i}_{a_1}(z) + \theta^{3-i}\frac{1}{2}\psi^{(\frac{5}{2}),i}_{a_1}(z) + \theta^{3-0}\phi^{(3)}_{a_1}(z), \\
\mathbf{\Phi}^{(2)}_{a_1}(Z) &= \frac{i}{2}\psi^{(2)}_{a_1}(z) + \theta^i\frac{i}{2}\phi^{(\frac{5}{2}),i}_{a_1}(z) + \theta^{3-i}\frac{1}{2}\psi^{(3),i}_{a_1}(z) + \theta^{3-0}\phi^{(\frac{7}{2})}_{a_1}(z), \\
\mathbf{\Phi}^{(\frac{5}{2})}_{a_1}(Z) &= \frac{i}{2}\psi^{(\frac{5}{2})}_{a_1}(z) + \theta^i\frac{i}{2}\phi^{(3),i}_{a_1}(z) + \theta^{3-i}\frac{1}{2}\psi^{(\frac{7}{2}),i}_{a_1}(z) + \theta^{3-0}\phi^{(4)}_{a_1}(z), \\
\mathbf{\Phi}^{(3),\alpha}_{a_1}(Z) &= \frac{i}{2}\psi^{(3),\alpha}_{a_1}(z) + \theta^i\frac{i}{2}\phi^{(\frac{7}{2}),i,\alpha}_{a_1}(z) + \theta^{3-i}\frac{1}{2}\psi^{(4),i,\alpha}_{a_1}(z) + \theta^{3-0}\phi^{(\frac{9}{2}),\alpha}_{a_1}(z), \\
\\
\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}(Z) &= \frac{i}{2}\psi^{(\frac{3}{2})}_{a_2}(z) + \theta^i\frac{i}{2}\phi^{(2),i}_{a_2}(z) + \theta^{3-i}\frac{1}{2}\psi^{(\frac{5}{2}),i}_{a_2}(z) + \theta^{3-0}\phi^{(3)}_{a_2}(z), \\
\mathbf{\Phi}^{(2)}_{a_2}(Z) &= \frac{i}{2}\psi^{(2)}_{a_2}(z) + \theta^i\frac{i}{2}\phi^{(\frac{5}{2}),i}_{a_2}(z) + \theta^{3-i}\frac{1}{2}\psi^{(3),i}_{a_2}(z) + \theta^{3-0}\phi^{(\frac{7}{2})}_{a_2}(z), \\
\mathbf{\Phi}^{(\frac{5}{2}),\alpha}_{a_2}(Z) &= \frac{i}{2}\psi^{(\frac{5}{2}),\alpha}_{a_2}(z) + \theta^i\frac{i}{2}\phi^{(3),i,\alpha}_{a_2}(z) + \theta^{3-i}\frac{1}{2}\psi^{(\frac{7}{2}),i,\alpha}_{a_2}(z) + \theta^{3-0}\phi^{(4),\alpha}_{a_2}(z), \\
\mathbf{\Phi}^{(3),\alpha}_{a_2}(Z) &= \frac{i}{2}\psi^{(3),\alpha}_{a_2}(z) + \theta^i\frac{i}{2}\phi^{(\frac{7}{2}),i,\alpha}_{a_2}(z) + \theta^{3-i}\frac{1}{2}\psi^{(4),i,\alpha}_{a_2}(z) + \theta^{3-0}\phi^{(\frac{9}{2}),\alpha}_{a_2}(z), \\
\mathbf{\Phi}^{(3)}_{a_3}(Z) &= \frac{i}{2}\psi^{(3)}_{a_3}(z) + \theta^i\frac{i}{2}\phi^{(\frac{7}{2}),i}_{a_3}(z) + \theta^{3-i}\frac{1}{2}\psi^{(4),i}_{a_3}(z) + \theta^{3-0}\phi^{(\frac{9}{2})}_{a_3}(z), \\
\mathbf{\Phi}^{(\frac{7}{2}),\alpha}_{a_2}(Z) &= \frac{i}{2}\psi^{(\frac{7}{2}),\alpha}_{a_2}(z) + \theta^i\frac{i}{2}\phi^{(4),i,\alpha}_{a_2}(z) + \theta^{3-i}\frac{1}{2}\psi^{(\frac{9}{2}),i,\alpha}_{a_2}(z) + \theta^{3-0}\phi^{(5),\alpha}_{a_2}(z), \\
\mathbf{\Phi}^{(4)}_{a_2}(Z) &= \frac{i}{2}\psi^{(4)}_{a_2}(z) + \theta^i\frac{i}{2}\phi^{(\frac{9}{2}),i}_{a_2}(z) + \theta^{3-i}\frac{1}{2}\psi^{(5),i}_{a_2}(z) + \theta^{3-0}\phi^{(\frac{11}{2})}_{a_2}(z).
\end{aligned} \tag{4.2}$$

[12] As we obtain the structure of the SOPE between the first and the second $\mathcal{N}$=3 supermultiplets, one can find out this SOPE structure from the OPEs between the component currents of the first $\mathcal{N}$=3 supermultiplet which are described in [4]

The first to the fourth $\mathscr{N}$=3 supermultiplets are described in [4]. The fifth to the last $\mathscr{N}$=3 supermultiplets are specified in this this thesis and [5]. Note that the general form of a =3 superfield we are dealing with as follow [4,5]:

$$\Phi^{\Delta,\alpha}_{a_i} = \frac{\imath}{2}\psi^{(\Delta),\alpha}_{a_i} + \theta^i \frac{\imath}{2}\phi^{(\Delta+\frac{1}{2}),i,\alpha}_{a_i} + \theta^{3-i}\frac{1}{2}\psi^{(\Delta+1),i,\alpha}_{a_i} + \theta^{3-0}\frac{\imath}{2}\phi^{(\Delta+\frac{3}{2}),\alpha}_{a_i}, \tag{4.3}$$

where α-index relates with the representation of SO(3) and i-index relates with the SO(3) vector representation.

Actually, the OPEs in section 3 have already determined coefficients because those OPEs are final results. However, one can only obtain the structure of those OPEs in the first place because the OPE calculations are conducted at the coset field level with the fixed value of (N,M). Concretely, the coefficients of the currents in the right hand side of the OPEs in section 3 are specific numbers, which means that we need to change those numbers into the languae of the central charge of the theory(The enhanced $\mathscr{N}$=3 Kazama-Suzuki model). Therefore, we replace those specific numbers with undetermined coefficients. The OPEs between the components of the $\mathscr{N}$=3 supercurrent $\boldsymbol{J}$ and the component currents of all $\mathscr{N}$=3 supermultiplets emerged in the right hand side of the SOPE between the first and the second $\mathscr{N}$=3 supermultiplets can be also acquired by using the superprimary conditions described in the appendix B of [4](See also [12,13]). Therefore, one can find out the undetermined coefficients of the SOPE between the first and the second $\mathscr{N}$=3 supermultiplets by using Jacobi identities between the components of the $\mathscr{N}$=3 supercurrent $\boldsymbol{J}$ and the component currents of the first and the second $\mathscr{N}$=3 supermultiplets. Those procedures are conducted by Mathematica. We build the code for those procedures to obtain the SOPE in the $\mathscr{N}$=3 superspace. The code is described in the appendix A. We, then, have the SOPE [5]

$$\begin{aligned}
\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_1}(Z_1)\boldsymbol{\Phi}^{(2)}_{a_1}(Z_2) \sim\ & \frac{\theta^{3-0}}{z_{12}^3}\left[-\frac{12}{c}C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2} + C^{(2)}_{a_2,(3)(2)}\boldsymbol{\Phi}^{(2)}_{a_2}\right](Z_2) \\
& + \frac{\theta^{3-0}}{z_{12}^2}\left[\frac{3}{4}\partial\left(\frac{\theta^{3-0}}{z_{12}^3} - term\right) - C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\left(\frac{4(7c+15)}{(c-3)(5c+6)}D^i\boldsymbol{J}D^i\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right.\right. \\
& \qquad + \frac{16(8c+3)}{(c-3)(5c+6)}\left(\partial\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2} - \frac{1}{4}\partial(\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2})\right) \\
& \qquad \left. + \frac{c^2-25c-42}{2(c-3)(5c+6)}D^{3-0}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right) \\
& \qquad \left. + \frac{3}{7}C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}\delta^{i\alpha}D^i\boldsymbol{\Phi}^{(\frac{5}{2}),\alpha}_{a_2} + C^{(3)}_{a_3,(3)(2)}\boldsymbol{\Phi}^{(3)}_{a_3}\right](Z_2) \\
& + \frac{\theta^{3-i}}{z_{12}^2}\left[C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\left(-\frac{(c-15)}{6(c-3)}D^{3-i}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2} - \frac{c+9}{c(c-3)}\boldsymbol{J}D^i\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2} - \frac{3(5c-3)}{c(c-3)}D^i\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right)\right. \\
& \qquad \left. + \frac{1}{4}C^{(2)}_{a_2,(3)(2)}D^i\boldsymbol{\Phi}^{(2)}_{a_2} + C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}\delta^{i\alpha}\boldsymbol{\Phi}^{(\frac{5}{2}a_2),\alpha}\right](Z_2) \\
& + \frac{\theta^i}{z_{12}^2}\left[\frac{1}{3}C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}D^i\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right](Z_2)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{z_{12}^2}\left[C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right](Z_2)\\
&+\frac{\theta^{3-0}}{z_{12}}\left[\frac{3}{10}\partial^2\left(\frac{\theta^{3-0}}{z_{12}^3}-term\right)+\frac{2}{3}\partial\left((\frac{\theta^{3-0}}{z_{12}^2}-term)-\frac{3}{4}\partial(\frac{\theta^{3-0}}{z_{12}^3}-term)\right)\right.\\
&\quad + C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\left(-\frac{12(c+1)(c-12)}{c(c-3)(c+3)^2}\boldsymbol{J}D^i\boldsymbol{J}D^{3-i}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right.\\
&\quad -\frac{12(c+1)(c-12)}{c(c-3)(c+3)^2}\left(-\boldsymbol{J}D^{3-i}\boldsymbol{J}D^i\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}-\frac{1}{4}\delta^{ii}\boldsymbol{J}\partial^2\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right)\\
&\quad -\frac{1080(c+1)}{c(c-3)(c+3)^2}\boldsymbol{J}D^i\boldsymbol{J}D^i\boldsymbol{J}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\\
&\quad +\frac{2(c+1)(c-12)}{(c-3)(c+3)^2}\left(D^{3-i}\boldsymbol{J}D^{3-i}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}+\frac{1}{6}\delta^{ii}\partial D^{3-0}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right)\\
&\quad -\frac{180(c+1)}{(c-3)(c+3)^2}\left(-D^i\boldsymbol{J}D^{3-i}\boldsymbol{J}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}-\frac{1}{4}D^i\boldsymbol{J}\partial D^i\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right)\\
&\quad -\frac{(4c^2-3c-135)}{(c+3)^2(c-3)}\left(\partial D^i\boldsymbol{J}D^i\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}-\frac{1}{3}\partial\left(D^i\boldsymbol{J}D^i\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right)\right)\\
&\quad -\frac{(32c^3+51c^2+207c-324)}{c(c-3)(c+3)^2}\left(\frac{1}{2}\partial^2\boldsymbol{J}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right.\\
&\quad \left.\left.-\frac{1}{3}\partial\left(\partial\boldsymbol{J}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}-\frac{1}{4}\partial\left(\boldsymbol{J}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right)\right)-\frac{1}{20}\partial^2\left(\boldsymbol{J}\mathbf{\Phi}^{(\frac{3}{2})}_{a_2}\right)\right)\right)\\
&\quad + C^{(2)}_{a_2,(3)(2)}\left(\frac{72(8c^2+53c+57)}{5(c^2+26c+9)(2c+9)(c-3)}D^i\boldsymbol{J}D^i\boldsymbol{J}\mathbf{\Phi}^{(2)}_{a_2}\right.\\
&\quad -\frac{6(2c^3+7c^2+63c+90)}{5(c^2+26c+9)(2c+9)(c-3)}D^i\boldsymbol{J}D^{3-i}\mathbf{\Phi}^{(2)}_{a_2}\\
&\quad +\frac{12c(4c^2+9c-99)}{5(c^2+26c+9)(2c+9)(c-3)}\left(-D^{3-0}\boldsymbol{J}\mathbf{\Phi}^{(2)}_{a_2}-\frac{3}{10}\partial^2\mathbf{\Phi}^{(2)}_{a_2}\right)\\
&\quad +\frac{54(6c^2+21c-1)}{5(c^2+26c+9)(2c+9)(c-3)}\boldsymbol{J}D^i\boldsymbol{J}D^i\mathbf{\Phi}^{(2)}_{a_2}\\
&\quad +\frac{3(14c^3+49c^2-129c-180)}{5(c^2+26c+9)(2c+9)(c-3)}\left(-D^{3-i}\boldsymbol{J}D^i\mathbf{\Phi}^{(2)}_{a_2}-\frac{1}{10}\partial^2\mathbf{\Phi}^{(2)}_{a_2}\right)\\
&\quad \left.+\frac{72(4c^2+9c-99)}{5(c^2+26c+9)(2c+9)(c-3)}\partial\boldsymbol{J}\boldsymbol{J}\mathbf{\Phi}^{(2)}_{a_2}\right)\\
&\quad + C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}\left(\frac{4ic}{(c+15)(c-1)}\delta^{i\alpha}(T^j)^{\alpha\beta}D^i\boldsymbol{J}D^j\mathbf{\Phi}^{(\frac{5}{2}),\beta}_{a_2}\right.\\
&\quad +\frac{120}{(c+15)(c-1)}\delta^{i\alpha}\boldsymbol{J}D^i\boldsymbol{J}\mathbf{\Phi}^{(\frac{5}{2}),\alpha}_{a_2}\\
&\quad \left.+\delta^{i\alpha}\frac{12c}{(c+15)(c-1)}\left(-D^{3-i}\boldsymbol{J}\mathbf{\Phi}^{(\frac{5}{2}),\alpha}_{a_2}-\frac{1}{6}\partial D^i\mathbf{\Phi}^{(\frac{5}{2}),\alpha}_{a_2}\right)\right)\\
&\quad + C^{(3)}_{a_2,(2)(2)}\left(-\frac{3c(c-2)}{2(c+3)(7c-6)}\delta^{i\alpha}D^{3-i}\mathbf{\Phi}^{(3),\alpha}_{a_2}\right.\\
&\quad \left.-\frac{9(c-2)}{(c+3)(7c-6)}\delta^{i\alpha}\boldsymbol{J}D^i\mathbf{\Phi}^{(3),\alpha}_{a_2}-\frac{24(4c-3)}{(c+3)(7c-6)}\delta^{i\alpha}D^i\boldsymbol{J}\mathbf{\Phi}^{(3),\alpha}_{a_2}\right)\\
&\quad \left.+\frac{4}{9}C^{(\frac{7}{2})}_{a_2,(\frac{5}{2})(2)}\delta^{i\alpha}D^i\mathbf{\Phi}^{(\frac{7}{2}),\alpha}_{a_2}+C^{(4)}_{a_2,(3)(2)}\mathbf{\Phi}^{(4)}_{a_2}\right](Z_2)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\theta^{3-i}}{z_{12}}\Bigg[\frac{3}{5}\partial\left(\frac{\theta^{3-i}}{z_{12}^2}-term\right)\\
&\quad+C^{(\frac{3}{2})}_{(a_2,\frac{3}{2})(2)}\Bigg(-\frac{54(c+1)}{c(c+3)(c-3)}\left(-\boldsymbol{J}D^{3-i}\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}-\frac{1}{4}\boldsymbol{J}\partial D^i\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right)\\
&\quad+\frac{18(c+1)(c-1)}{(c+3)(c-3)(5c+6)}\varepsilon^{ijk}\left(D^{3-j}\boldsymbol{J}D^k\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}+\frac{1}{5}\varepsilon^{jkl}\partial D^{3-l}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right)\\
&\quad+\frac{2(c^2-18c-27)}{(c+3)(c-3)(5c+6)}\varepsilon^{ijk}\left(D^j\boldsymbol{J}D^{3-k}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}+\frac{1}{10}\varepsilon^{jkl}D^{3-k}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right)\\
&\quad-\frac{5c^3-72c^2-99c-54}{c(c+3)(c-3)(5c+6)}\left(\partial\boldsymbol{J}D^i\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}-\frac{1}{5}\partial(\boldsymbol{J}D^i\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2})\right)\\
&\quad-\frac{12(5c^3-18c-9)}{c(c+3)(c-3)(5c+6)}\left(\partial D^i\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}-\frac{2}{5}\partial(D^i\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2})\right)\\
&\quad-\frac{18(c+1)}{c(c+3)(c-3)}\varepsilon^{ijk}\left(\boldsymbol{J}D^j\boldsymbol{J}D^k\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}+\frac{1}{8}\varepsilon^{jkl}\boldsymbol{J}\partial D^l\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right)\Bigg)\\
&\quad+C^{(2)}_{a_2,(3)(2)}\Bigg(\frac{36}{5(c-3)(2c+9)}\boldsymbol{J}D^i\boldsymbol{J}\boldsymbol{\Phi}^{(2)}_{a_2}-\frac{3}{10(c-3)}\boldsymbol{J}D^{3-i}\boldsymbol{\Phi}^{(2)}_{a_2}\\
&\quad-\frac{6(c+9)}{5(2c+9)(c-3)}\left(-D^{3-i}\boldsymbol{J}\boldsymbol{\Phi}^{(2)}_{a_2}-\frac{1}{5}\partial D^i\boldsymbol{\Phi}^{(2)}_{a_2}\right)\\
&\quad-\frac{3(2c+3)}{5(2c+9)(c-3)}\varepsilon^{ijk}\left(D^j\boldsymbol{J}D^k\boldsymbol{\Phi}^{(2)}_{a_2}+\frac{1}{10}\varepsilon^{jkl}\partial D^l\boldsymbol{\Phi}^{(2)}_{a_2}\right)\Bigg)\\
&\quad+C^{(\frac{5}{2})}_{(a_2,\frac{5}{2})(2)}\Bigg(\frac{i}{c-1}\delta^{i\alpha}(T^j)^{\alpha\beta}\boldsymbol{J}D^j\boldsymbol{\Phi}^{(\frac{5}{2}),\beta}_{a_2}\\
&\quad-\frac{i}{14}\delta^{i\alpha}(T^j)^{\alpha\beta}\left(-D^{3-j}\boldsymbol{\Phi}^{(\frac{5}{2}),\beta}_{a_2}-\frac{i}{5}(T^j)^{\beta\gamma}\partial\boldsymbol{\Phi}^{(\frac{5}{2}),\gamma}_{a_2}\right)\Bigg)\\
&\quad+C^{(3)}_{a_2,(2)(2)}\left(\frac{3i(c-1)}{7c-6}\delta^{i\alpha}(T^j)^{\alpha\beta}D^j\boldsymbol{\Phi}^{(3),\beta}_{a_2}-\frac{6}{7c-6}\delta^{i\alpha}\boldsymbol{J}\boldsymbol{\Phi}^{(3),\alpha}_{a_2}\right)\\
&\quad+\frac{1}{6}C^{(3)}_{a_3,(3)(2)}D^i\boldsymbol{\Phi}^{(3)}_{a_3}+C^{(\frac{7}{2})}_{a_2,(\frac{5}{2})(2)}\delta^{i\alpha}\boldsymbol{\Phi}^{(\frac{7}{2}),\alpha}_{a_2}\;](Z_2)\\
&+\frac{\theta^i}{z_{12}}\Bigg[\frac{1}{2}\partial\left(\frac{\theta^i}{z_{12}^2}-term\right)+C^{(\frac{3}{2})}_{a_2,(\frac{3}{2})(2)}\Bigg(\frac{54(c+1)}{c(c+3)(c-3)}\boldsymbol{J}D^i\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\\
&\quad+\frac{9(c+1)}{(c+3)(c-3)}\left(-D^{3-i}\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}-\frac{1}{4}\partial D^i\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right)\\
&\quad-\frac{(c-15)}{(c+3)(c-3)}\varepsilon^{ijk}D^j\left(\boldsymbol{J}D^k\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}+\frac{1}{8}\varepsilon^{jkl}\partial D^l\boldsymbol{\Phi}^{(\frac{3}{2})}_{a_2}\right)\Bigg)\\
&\quad-C^{(2)}_{a_2,(3)(2)}\left(-\frac{c}{20(c-3)}D^{3-i}\boldsymbol{\Phi}^{(2)}_{a_2}+\frac{3}{10(c-3)}\boldsymbol{J}D^i\boldsymbol{\Phi}^{(2)}_{a_2}+\frac{6}{5(c-3)}D^i\boldsymbol{J}\boldsymbol{\Phi}^{(2)}_{a_2}\right)\\
&\quad-C^{(\frac{5}{2})}_{a_2,(\frac{5}{2})(2)}\left(\frac{ic}{6(c-1)}\delta^{i\alpha}(T^j)^{\alpha\beta}D^j\boldsymbol{\Phi}^{(\frac{5}{2}),\beta}_{a_2}+\frac{2}{c-1}\delta^{i\alpha}\boldsymbol{J}\boldsymbol{\Phi}^{(\frac{5}{2}),\alpha}_{a_2}\right)\\
&\quad+C^{(3)}_{a_2,(2)(2)}\delta^{i\alpha}\boldsymbol{\Phi}^{(3),\alpha}_{a_2}\Bigg](Z_2)\\
&+\frac{1}{z_{12}}\left[\frac{1}{3}\partial(\frac{1}{z_{12}}-term)\right](Z_2).
\end{aligned}
\tag{4.4}
$$

One can recover the OPEs between the component currents of the first and the second $\mathscr{N}=3$ supermultiplets by applying superderivatives to the SOPE (4.4).

4.2 Some component OPEs between the two lowest higher spin supermultiplets

We have found another lowest higher spin current $\mathbf{\Phi}_{a_2}^{(\frac{3}{2})}$. One can consider the OPEs between those currents, i.e. $\mathbf{\Phi}_{a_1}^{(\frac{3}{2})}(Z_1)\mathbf{\Phi}_{a_2}^{(\frac{3}{2})}(Z_2)$ and $\mathbf{\Phi}_{a_2}^{(\frac{3}{2})}(Z_1)\mathbf{\Phi}_{a_2}^{(\frac{3}{2})}(Z_2)$ [5]. We will describe the structure of some component OPEs between those currents. The structure of those OPEs are as follows:

$$
\begin{aligned}
[\psi_{a_1}^{(\frac{3}{2})}]\cdot[\psi_{a_2}^{(\frac{3}{2})}] &\sim [I]+[\psi_{a_1}^{(2)}]+\delta^{i\alpha}[\phi_{a_3}^{(2),i,\alpha}],\\
[\phi_{a_1}^{(2),i}]\cdot[\psi_{a_2}^{(\frac{3}{2})}] &\sim [I]+[\psi_{a_3}^{(\frac{3}{2}),\alpha=i}]+[\phi_{a_1}^{(\frac{5}{2}),i}]+(T^i)^{\alpha\beta}\delta^{\alpha k}[\psi_{a_3}^{(\frac{5}{2}),k,\beta}],\\
[\psi_{a_1}^{(\frac{5}{2}),i}]\cdot[\psi_{a_2}^{(\frac{3}{2})}] &\sim [I]+((T^i)^{\alpha\beta}\delta^{\alpha k}+\delta^{\beta i})[\psi_{a_3}^{(\frac{3}{2}),\beta}]+((T^i)^{\alpha\beta}\delta^{\alpha k}+\varepsilon^{ijk}\delta^{j\beta}+\delta^{i\beta})[\phi_{a_3}^{(2),k,\beta}]\\
&\quad+((T^j)^{\alpha\beta}\delta^{i\alpha}+\delta^{j\beta})[\phi_{a_3}^{(2),j,\beta}]+\delta^{j\alpha}[\phi_{a_3}^{(2),i,\alpha}]+[\phi_{a_1}^{(\frac{5}{2}),i}]+(T^i)^{\alpha\beta}\delta^{\alpha k}[\psi_{a_3}^{(\frac{5}{2}),k,\beta}]\\
&\quad+[\psi_{a_1}^{(2)}]+[\phi_{a_3}^{(3),\alpha=i}]+[\phi_{a_3}^{(3),i}]+[\psi_{a_1}^{(3),i}]+[\psi_{a_4}^{(3),\alpha=i}],\\
[\psi_{a_2}^{(\frac{3}{2})}]\cdot[\psi_{a_2}^{(\frac{3}{2})}] &\sim [I]+[\psi_{a_1}^{(2)}]+\delta^{i\alpha}[\phi_{a_3}^{(2),i,\alpha}]\\
[\phi_{a_2}^{(2),i}]\cdot[\psi_{a_2}^{(\frac{3}{2})}] &\sim [I]+[\psi_{a_3}^{(\frac{3}{2}),\alpha=i}]+[\phi_{a_1}^{(\frac{5}{2}),i}]+(T^i)^{\alpha\beta}\delta^{\alpha k}[\psi_{a_3}^{(\frac{5}{2})k,\beta}],\\
[\psi_{a_2}^{(\frac{5}{2}),i}]\cdot[\psi_{a_2}^{(\frac{3}{2})}] &\sim [I]+((T^i)^{\alpha\beta}\delta^{\alpha k}+\delta^{\beta i})[\psi_{a_3}^{(\frac{3}{2}),\beta}]+((T^i)^{\alpha\beta}\delta^{\alpha k}+\varepsilon^{ijk}\delta^{j\beta}+\delta^{i\beta})[\phi_{a_3}^{(2),k,\beta}]\\
&\quad+((T^j)^{\alpha\beta}\delta^{i\alpha}+\delta^{j\beta})[\phi_{a_3}^{(2),j,\beta}]+\delta^{j\alpha}[\phi_{a_3}^{(2),i,\alpha}]+[\phi_{a_1}^{(\frac{5}{2}),i}]+(T^i)^{\alpha\beta}\delta^{\alpha k}[\psi_{a_3}^{(\frac{5}{2}),k,\beta}]\\
&\quad+[\psi_{a_1}^{(2)}]+[\phi_{a_3}^{(3),\alpha=i}]+[\phi_{a_4}^{(3),i}]+[\psi_{a_1}^{(3),i}]+[\psi_{a_5}^{(3),\alpha=i}].
\end{aligned} \tag{4.5}
$$

We followed the notations in [4]. The notation $[A]$ represents the superconformal family of the field A. One can observe that new primaries are emerged. The new fields $\psi_{a_3}^{(\frac{3}{2}),\alpha}$, $\phi_{a_3}^{(2),i,\alpha}$, $\psi_{a_3}^{(\frac{5}{2}),i,\alpha}$, and $\phi_{a_3}^{(3),\alpha=i}$ are components of $\mathbf{\Phi}_{a_3}^{(\frac{3}{2})}$. The new primaries $\phi_{a_3}^{(3),i}$, $\phi_{a_4}^{(3),i}$, $\psi_{a_4}^{(3),\alpha=i}$, and $\psi_{a_5}^{(3),\alpha=i}$ are component of $\mathbf{\Phi}_{a_3}^{(\frac{5}{2})}$, $\mathbf{\Phi}_{a_4}^{(\frac{5}{2})}$, $\mathbf{\Phi}_{a_4}^{(3)}$, and $\mathbf{\Phi}_{a_5}^{(3)}$, respectively. Therefore, we newly have

$$
\begin{aligned}
\mathbf{\Phi}_{a_3}^{(\frac{3}{2}),\alpha}(Z) &= \frac{i}{2}\psi_{a_3}^{(\frac{3}{2})}(z)+\theta^i\frac{i}{2}\phi_{a_3}^{(2),i,\alpha}(z)+\theta^{3-i}\frac{1}{2}\psi_{a_3}^{(\frac{5}{2}),i,\alpha}(z)+\theta^{3-0}\phi_{a_3}^{(3),\alpha}(z),\\
\mathbf{\Phi}_{a_3}^{(\frac{5}{2})}(Z) &= \frac{i}{2}\psi_{a_3}^{(\frac{5}{2})}(z)+\theta^i\frac{i}{2}\phi_{a_3}^{(3),i}(z)+\theta^{3-i}\frac{1}{2}\psi_{a_3}^{(\frac{7}{2}),i}(z)+\theta^{3-0}\phi_{a_3}^{(4)}(z),\\
\mathbf{\Phi}_{a_4}^{(\frac{5}{2})}(Z) &= \frac{i}{2}\psi_{a_4}^{(\frac{5}{2})}(z)+\theta^i\frac{i}{2}\phi_{a_4}^{(3),i}(z)+\theta^{3-i}\frac{1}{2}\psi_{a_4}^{(\frac{7}{2}),i}(z)+\theta^{3-0}\phi_{a_4}^{(4)}(z),\\
\mathbf{\Phi}_{a_4}^{(3),\alpha}(Z) &= \frac{i}{2}\psi_{a_4}^{(3),\alpha}(z)+\theta^i\frac{i}{2}\phi_{a_4}^{(\frac{7}{2}),i,\alpha}(z)+\theta^{3-i}\frac{1}{2}\psi_{a_4}^{(4),i,\alpha}(z)+\theta^{3-0}\phi_{a_4}^{(\frac{9}{2}),\alpha}(z),\\
\mathbf{\Phi}_{a_5}^{(3),\alpha}(Z) &= \frac{i}{2}\psi_{a_5}^{(3),\alpha}(z)+\theta^i\frac{i}{2}\phi_{a_5}^{(\frac{7}{2}),i,\alpha}(z)+\theta^{3-i}\frac{1}{2}\psi_{a_5}^{(4),i,\alpha}(z)+\theta^{3-0}\phi_{a_5}^{(\frac{9}{2}),\alpha}(z).
\end{aligned} \tag{4.6}
$$

Some of new primaries are not determined in the above OPEs. Specifically, $\phi_{a_3}^{(3),i}$ and $\phi_{a_4}^{(3),i}$ are obtained from $\psi_{a_3}^{(\frac{5}{2})}$ and $\psi_{a_4}^{(\frac{5}{2})}$, respectively, by using the primary conditions. The new primaries $\psi_{a_3}^{(\frac{5}{2})}$ and $\psi_{a_4}^{(\frac{5}{2})}$ are determined at the second order pole of the OPEs $\phi_{a_1}^{(3)}\psi_{a_2}^{(\frac{3}{2})}$ and $\phi_{a_2}^{(3)}\psi_{a_2}^{(\frac{3}{2})}$, which are not described in this thesis, respectively.

5 Conclusion

We specified the SOPE between the first and the second $\mathcal{N}=3$ supermultiplets in the enhanced Kazama-Suzuki model which have the coset $\frac{\widehat{su}(N+M)_k \times \widehat{so}(2NM)_1}{\widehat{su}(N)_{k+M} \times \widehat{su}(M)_{k+N} \times \widehat{u}(1)_{NM(N+M)(N+M+k)}}$ with $k = N + M$(Critical level). One can have the component approach of this SOPE by conducting superderivatives on it.

On the right hand side of the SOPE relation (4.4), various supermultiplets are newly appeared. Those are listed on (4.2)(The fifth to the last supermultiplets). As we have observed in the previous section, there are many structure constants with various higher spin multiplets. It follows that we need more OPE relations to obtain more Jacobi identies [5]. To be specific, it is needed to compute the OPEs between the two superspin-$\frac{3}{2}$ supermultiplets, i.e. $\mathbf{\Phi}_{a_1}^{(\frac{3}{2})}(Z_1)\mathbf{\Phi}_{a_2}^{(\frac{3}{2})}(Z_2)$ and $\mathbf{\Phi}_{a_2}^{(\frac{3}{2})}(Z_1)\mathbf{\Phi}_{a_2}^{(\frac{3}{2})}(Z_2)$ [5]. The brief structure of the component approach for those OPEs are proposed in (4.5). In the equation (4.5), we have another new spin-$\frac{3}{2}$ primary. It follows that we can consider the OPEs $\mathbf{\Phi}_{a_1}^{(\frac{3}{2})}(Z_1)\mathbf{\Phi}_{a_3}^{(\frac{3}{2}),\alpha}(Z_2)$, $\mathbf{\Phi}_{a_2}^{(\frac{3}{2})}(Z_1)\mathbf{\Phi}_{a_3}^{(\frac{3}{2}),\alpha}(Z_2)$, and $\mathbf{\Phi}_{a_3}^{(\frac{3}{2}),\alpha}(Z_1)\mathbf{\Phi}_{a_3}^{(\frac{3}{2}),\beta}(Z_2)$. Obtaining the complete structure of those OPEs, however, will not be easy because the computation of the OPEs described in the section 3 already have huge complexity. Even though the (N,M)=(3,2) case, the number of the operators of spin-4 currents are more than 180000. The worse part is that the more we find new multiplets, the more possible combinations of operators with undetermined coefficients, which slow down the computation, we need to compute OPEs. The capacity of the memory of computer is also problem. Although, we detoured it by using the piece by piece strategy, this takes too much time.

On the other hand, because we considered a two dimensional conformal field theory, one can study the corresponding gravity theory, i.e. AdS_3 gravity. According to [3], the enhanced $\mathcal{N}=3$ Kazama-Suzuki model, which is specified with the coset $\frac{\widehat{su}(N+M)_k \times \widehat{so}(2NM)_1}{\widehat{su}(N)_{k+M} \times \widehat{su}(M)_{k+N} \times \widehat{u}(1)_{NM(N+M)(N+M+k)}}$ with the critical level $k = N + M$, have the corresponding higher spin gravity with extended supersymmtry. One may study the explicit structure of this gravity theory for low higer spin fields.

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확장된 N=3 Kazama-Suzuki 모델에서 고스핀 전류 만들기

홍 재 성

경북대학교 대학원 물리학과 입자물리학전공
(지도교수 안창현)

(초 록)
이 논문에서는 이전에 연구되었던 크리티컬 레벨(k=N+M)에서 coset $\frac{\widehat{su}(N+M)_k \times \widehat{so}(2NM)_1}{\widehat{su}(N)_{k+M} \times \widehat{su}(M)_{k+N} \times \widehat{u}(1)_{NM(N+M)(N+M+k)}}$ 으로 기술되는 확장된 N=3 Kazama-Suzuki 모형에서의 첫 번째와 두 번째 N=3 초 multiplet 사이의 OPE를 살펴볼 것이다. 이 두 초 multiplet 사이의 초 OPE는 고정된 (N,M)에서 첫 번째 N=3 초 multiplet을 구성하는 네 종류의 구성요소 전류와 두 번째 N=3 초 multiplet의 가장 낮은 스핀을 가지는 구성요소 전류와의 OPE들에 의해 결정된다. 그 초 OPE의 우변에는 여러 새로운 N=3 초 multiplet들이 발견된다. 이들 초 multipelt들은 SO(3) singlet들과 triplet들을 포함한다. 이 새로 발견된 초 multiplet들 중 하나는 가장 낮은 고스핀을 가진다. 여기서는 가장 낮은 스핀을 가지는 두 초 multiplet들의 몇몇 구성요소 전류사이의 OPE를 간략하게 보일 것이다.

Appendix A Mathematica source: SOPE package

We provide mathematica code for the SOPE between the first and the second $\mathscr{N}=3$ multiplets. One can obtain not only the SOPE between the first and the second $\mathscr{N}=3$ multiplets but also all OPEs from the component approach of the SOPE by following the procedures from subsection A.1 to A.3. This package is the deformation of the SOPE package for $\mathscr{N}=4$ case which is coded by Man Hea Kim.

A.1 Defining fields and OPEs for Jacobi identities

<< OPEdefs.m

OPEdefs Version 3.1 (beta 4) by Kris Thielemans
Type ?OPEdefsHelp for a primer on OPEdefs.

$RecursionLimit = Infinity

∞

Phi3Hcp1[$\frac{3}{2}$]=$\psi^{\left(\frac{3}{2}\right)}$,
Phi3Hcp2[$i,2$]=$\phi^{(2),i}$,
Phi3Hcp3[i,$\frac{5}{2}$]=$\psi^{\left(\frac{5}{2}\right),i}$,
Phi3Hcp4[3]=$\phi^{(3)}$,

Phi2cp1[2]=$\psi^{(2)}$,
Phi2cp2[$i, \frac{5}{2}$]=$\phi^{\left(\frac{5}{2}\right),i}$,
Phi2cp3[i,3]=$\psi^{(3),i}$,
Phi2cp4[$\frac{7}{2}$]=$\phi^{\left(\frac{7}{2}\right)}$,

Phi5Hcp1[$\frac{5}{2}$]=$\psi^{\left(\frac{5}{2}\right)}$,
Phi5Hcp2[$i,3$]=$\phi^{(3),i}$,
Phi5Hcp3[i,$\frac{7}{2}$]=$\psi^{\left(\frac{7}{2}\right),i}$,
Phi5Hcp4[4]=$\phi^{(4)}$,

Phi3cp1[$3,\{a\}$]=$\psi^{(3),a}$,
Phi3cp2[$i, \frac{7}{2}$,{a}]=$\phi^{\left(\frac{7}{2}\right),i,a}$,
Phi3cp3[i,4,{a}]=$\psi^{(4),i,a}$,
Phi3cp4[$\frac{9}{2}, \{a\}$]=$\phi^{\left(\frac{9}{2}\right),a}$,

NewPhi3Hcp1[$\frac{3}{2}$]=$\bar{\psi}^{(\frac{3}{2})}$,
NewPhi3Hcp2[$i, 2$]=$\bar{\phi}^{(2),i}$,
NewPhi3Hcp3[i,$\frac{5}{2}$]=$\bar{\psi}^{(\frac{5}{2}),i}$,
NewPhi3Hcp4[3]=$\bar{\phi}^{(3)}$,

NNewPhi3Hcp1[$\frac{3}{2}, \{a\}$]=$\tilde{\psi}^{(\frac{3}{2}),a}$,
NNewPhi3Hcp2[$i, 2, \{a\}$]=$\tilde{\phi}^{(2),i,a}$,
NNewPhi3Hcp3[i,$\frac{5}{2}, \{a\}$]=$\tilde{\psi}^{(\frac{5}{2}),i,a}$,
NNewPhi3Hcp4[3,{a}]=$\tilde{\phi}^{(3),a}$,

NewPhi2cp1[2]=$\bar{\psi}^{(2)}$,
NewPhi2cp2[$i, \frac{5}{2}$]=$\bar{\phi}^{(\frac{5}{2}),i}$,
NewPhi2cp3[i,3]=$\bar{\psi}^{(3),i}$,
NewPhi2cp4[$\frac{7}{2}$]=$\bar{\phi}^{(\frac{7}{2})}$,

NewPhi5Hcp1[$\frac{5}{2}, \{a\}$]=$\bar{\psi}^{(\frac{5}{2}),a}$,
NewPhi5Hcp2[$i, 3$,{a}]=$\bar{\phi}^{(3),i,a}$,
NewPhi5Hcp3[i,$\frac{7}{2}$,{a}]=$\bar{\psi}^{(\frac{7}{2}),i,a}$,
NewPhi5Hcp4[4,{a}]=$\bar{\phi}^{(4),a}$,

NewPhi3cp1[$3, \{a\}$]=$\bar{\psi}^{(3),a}$,
NewPhi3cp2[$i, \frac{7}{2}, \{a\}$]=$\bar{\phi}^{(\frac{7}{2}),i,a}$,
NewPhi3cp3[i,4,{a}]=$\bar{\psi}^{(4),i,a}$,
NewPhi3cp4[$\frac{9}{2}$,{a}]=$\bar{\phi}^{(\frac{9}{2}),a}$,

NNewPhi5Hcp1[$\frac{5}{2}$]=$\tilde{\psi}^{(\frac{5}{2})}$,
NNewPhi5Hcp2[$i, 3$]=$\tilde{\phi}^{(3),i}$,
NNewPhi5Hcp3[i,$\frac{7}{2}$]=$\tilde{\psi}^{(\frac{7}{2}),i}$,
NNewPhi5Hcp4[4]=$\tilde{\phi}^{(4)}$,

NNewPhi3cp1[3]=$\tilde{\psi}^{(3)}$,
NNewPhi3cp2[$i, \frac{7}{2}$]=$\tilde{\phi}^{(\frac{7}{2}),i}$,

NNewPhi3cp3[i,4]=$\tilde{\psi}^{(4),i}$,
NNewPhi3cp4[$\frac{9}{2}$]=$\tilde{\phi}^{(\frac{9}{2})}$,

NewPhi7Hcp1[$\frac{7}{2}, \{a\}$]=$\bar{\psi}^{(\frac{7}{2}),a}$,
NewPhi7Hcp2[$i, 4$,{a}]=$\bar{\phi}^{(4),i,a}$,
NewPhi7Hcp3[i,$\frac{9}{2}$,{a}]=$\bar{\psi}^{(\frac{9}{2}),i,a}$,
NewPhi7Hcp4[5,{a}]=$\bar{\phi}^{(5),a}$,

NewPhi4cp1[4]=$\bar{\psi}^{(4)}$,
NewPhi4cp2[$i, \frac{9}{2}$]=$\bar{\phi}^{(\frac{9}{2}),i}$,
NewPhi4cp3[i,5]=$\bar{\psi}^{(5),i}$,
NewPhi4cp4[$\frac{11}{2}$]=$\bar{\phi}^{(\frac{11}{2})}$,

Bosonic[Ts]

Fermionic[Ψ]

Bosonic[*J*[1], *J*[2], *J*[3]]

Fermionic[*G*[1], *G*[2], *G*[3]]

Fermionic[Phi3Hcp1[3/2]]

Bosonic[Phi3Hcp2[1, 2], Phi3Hcp2[2, 2], Phi3Hcp2[3, 2]]

Fermionic[Phi3Hcp3[1, 5/2], Phi3Hcp3[2, 5/2], Phi3Hcp3[3, 5/2]]

Bosonic[Phi3Hcp4[3]]

Bosonic[Phi2cp1[2]]

Fermionic[Phi2cp2[1, 5/2], Phi2cp2[2, 5/2], Phi2cp2[3, 5/2]]

Bosonic[Phi2cp3[1, 3], Phi2cp3[2, 3], Phi2cp3[3, 3]]

Fermionic[Phi2cp4[7/2]]

Fermionic[Phi5Hcp1[5/2]]

Bosonic[Phi5Hcp2[1, 3], Phi5Hcp2[2, 3], Phi5Hcp2[3, 3]]

Fermionic[Phi5Hcp3[1, 7/2], Phi5Hcp3[2, 7/2], Phi5Hcp3[3, 7/2]]

Bosonic[Phi5Hcp4[4]]

Bosonic[Phi3cp1[3, {1}]]

Bosonic[Phi3cp1[3, {2}]]

Bosonic[Phi3cp1[3, {3}]]

Fermionic[Phi3cp2[1, 7/2, {1}], Phi3cp2[2, 7/2, {1}],
Phi3cp2[3, 7/2, {1}]]

Fermionic[Phi3cp2[1, 7/2, {2}], Phi3cp2[2, 7/2, {2}],
Phi3cp2[3, 7/2, {2}]]

Fermionic[Phi3cp2[1, 7/2, {3}], Phi3cp2[2, 7/2, {3}],
Phi3cp2[3, 7/2, {3}]]

Bosonic[Phi3cp3[1, 4, {1}], Phi3cp3[2, 4, {1}], Phi3cp3[3, 4, {1}]]

Bosonic[Phi3cp3[1, 4, {2}], Phi3cp3[2, 4, {2}], Phi3cp3[3, 4, {2}]]

Bosonic[Phi3cp3[1, 4, {3}], Phi3cp3[2, 4, {3}], Phi3cp3[3, 4, {3}]]

Fermionic[Phi3cp4[9/2, {1}]]

Fermionic[Phi3cp4[9/2, {2}]]

Fermionic[Phi3cp4[9/2, {3}]]

Fermionic[NewPhi3Hcp1[3/2]]

Bosonic[NewPhi3Hcp2[1, 2], NewPhi3Hcp2[2, 2], NewPhi3Hcp2[3, 2]]

Fermionic[NewPhi3Hcp3[1, 5/2], NewPhi3Hcp3[2, 5/2],
NewPhi3Hcp3[3, 5/2]]

Bosonic[NewPhi3Hcp4[3]]

Fermionic[NNewPhi3Hcp1[3/2, {1}]]

Fermionic[NNewPhi3Hcp1[3/2, {2}]]

Fermionic[NNewPhi3Hcp1[3/2, {3}]]

Bosonic[NNewPhi3Hcp2[1, 2, {1}], NNewPhi3Hcp2[2, 2, {1}], NNewPhi3Hcp2[3, 2, {1}]]

Bosonic[NNewPhi3Hcp2[1, 2, {2}], NNewPhi3Hcp2[2, 2, {2}], NNewPhi3Hcp2[3, 2, {2}]]

Bosonic[NNewPhi3Hcp2[1, 2, {3}], NNewPhi3Hcp2[2, 2, {3}], NNewPhi3Hcp2[3, 2, {3}]]

Fermionic[NNewPhi3Hcp3[1, 5/2, {1}], NNewPhi3Hcp3[2, 5/2, {1}], NNewPhi3Hcp3[3, 5/2, {1}]]

Fermionic[NNewPhi3Hcp3[1, 5/2, {2}], NNewPhi3Hcp3[2, 5/2, {2}], NNewPhi3Hcp3[3, 5/2, {2}]]

Fermionic[NNewPhi3Hcp3[1, 5/2, {3}], NNewPhi3Hcp3[2, 5/2, {3}], NNewPhi3Hcp3[3, 5/2, {3}]]

Bosonic[NNewPhi3Hcp4[3, {1}]]

Bosonic[NNewPhi3Hcp4[3, {2}]]

Bosonic[NNewPhi3Hcp4[3, {3}]]

Bosonic[NewPhi2cp1[2]]

Fermionic[NewPhi2cp2[1, 5/2], NewPhi2cp2[2, 5/2], NewPhi2cp2[3, 5/2]]

Bosonic[NewPhi2cp3[1, 3], NewPhi2cp3[2, 3], NewPhi2cp3[3, 3]]

Fermionic[NewPhi2cp4[7/2]]

Fermionic[NNewPhi5Hcp1[5/2]]

Bosonic[NNewPhi5Hcp2[1, 3], NNewPhi5Hcp2[2, 3], NNewPhi5Hcp2[3, 3]]

Fermionic[NNewPhi5Hcp3[1, 7/2], NNewPhi5Hcp3[2, 7/2],

NNewPhi5Hcp3[3, 7/2]]

Bosonic[NNewPhi5Hcp4[4]]

Fermionic[NewPhi5Hcp1[5/2, {1}]]

Fermionic[NewPhi5Hcp1[5/2, {2}]]

Fermionic[NewPhi5Hcp1[5/2, {3}]]

Bosonic[NewPhi5Hcp2[1, 3, {1}], NewPhi5Hcp2[2, 3, {1}],

NewPhi5Hcp2[3, 3, {1}]]

Bosonic[NewPhi5Hcp2[1, 3, {2}], NewPhi5Hcp2[2, 3, {2}],

NewPhi5Hcp2[3, 3, {2}]]

Bosonic[NewPhi5Hcp2[1, 3, {3}], NewPhi5Hcp2[2, 3, {3}],

NewPhi5Hcp2[3, 3, {3}]]

Fermionic[NewPhi5Hcp3[1, 7/2, {1}], NewPhi5Hcp3[2, 7/2, {1}],

NewPhi5Hcp3[3, 7/2, {1}]]

Fermionic[NewPhi5Hcp3[1, 7/2, {2}], NewPhi5Hcp3[2, 7/2, {2}],

NewPhi5Hcp3[3, 7/2, {2}]]

Fermionic[NewPhi5Hcp3[1, 7/2, {3}], NewPhi5Hcp3[2, 7/2, {3}],

NewPhi5Hcp3[3, 7/2, {3}]]

Bosonic[NewPhi5Hcp4[4, {1}]]

Bosonic[NewPhi5Hcp4[4, {2}]]

Bosonic[NewPhi5Hcp4[4, {3}]]

Bosonic[NewPhi3cp1[3, {1}]]

Bosonic[NewPhi3cp1[3, {2}]]

Bosonic[NewPhi3cp1[3, {3}]]

Fermionic[NewPhi3cp2[1, 7/2, {1}], NewPhi3cp2[2, 7/2, {1}],
NewPhi3cp2[3, 7/2, {1}]]

Fermionic[NewPhi3cp2[1, 7/2, {2}], NewPhi3cp2[2, 7/2, {2}],
NewPhi3cp2[3, 7/2, {2}]]

Fermionic[NewPhi3cp2[1, 7/2, {3}], NewPhi3cp2[2, 7/2, {3}],
NewPhi3cp2[3, 7/2, {3}]]

Bosonic[NewPhi3cp3[1, 4, {1}], NewPhi3cp3[2, 4, {1}],
NewPhi3cp3[3, 4, {1}]]

Bosonic[NewPhi3cp3[1, 4, {2}], NewPhi3cp3[2, 4, {2}],
NewPhi3cp3[3, 4, {2}]]

Bosonic[NewPhi3cp3[1, 4, {3}], NewPhi3cp3[2, 4, {3}],
NewPhi3cp3[3, 4, {3}]]

Fermionic[NewPhi3cp4[9/2, {1}]]

Fermionic[NewPhi3cp4[9/2, {2}]]

Fermionic[NewPhi3cp4[9/2, {3}]]

Bosonic[NNewPhi3cp1[3]]

Fermionic[NNewPhi3cp2[1, 7/2], NNewPhi3cp2[2, 7/2],
NNewPhi3cp2[3, 7/2]]

Bosonic[NNewPhi3cp3[1, 4], NNewPhi3cp3[2, 4], NNewPhi3cp3[3, 4]]

Fermionic[NNewPhi3cp4[9/2]]

Fermionic[NewPhi7Hcp1[7/2, {1}]]

Fermionic[NewPhi7Hcp1[7/2, {2}]]

Fermionic[NewPhi7Hcp1[7/2, {3}]]

Bosonic[NewPhi7Hcp2[1, 4, {1}], NewPhi7Hcp2[2, 4, {1}],
NewPhi7Hcp2[3, 4, {1}]]

Bosonic[NewPhi7Hcp2[1, 4, {2}], NewPhi7Hcp2[2, 4, {2}],
NewPhi7Hcp2[3, 4, {2}]]

Bosonic[NewPhi7Hcp2[1, 4, {3}], NewPhi7Hcp2[2, 4, {3}],
NewPhi7Hcp2[3, 4, {3}]]

Fermionic[NewPhi7Hcp3[1, 9/2, {1}], NewPhi7Hcp3[2, 9/2, {1}],
NewPhi7Hcp3[3, 9/2, {1}]]

Fermionic[NewPhi7Hcp3[1, 9/2, {2}], NewPhi7Hcp3[2, 9/2, {2}],
NewPhi7Hcp3[3, 9/2, {2}]]

Fermionic[NewPhi7Hcp3[1, 9/2, {3}], NewPhi7Hcp3[2, 9/2, {3}],
NewPhi7Hcp3[3, 9/2, {3}]]

Bosonic[NewPhi7Hcp4[5, {1}]]

Bosonic[NewPhi7Hcp4[5, {2}]]

Bosonic[NewPhi7Hcp4[5, {3}]]

Bosonic[NewPhi4cp1[4]]

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Fermionic[NewPhi4cp2[1, 9/2], NewPhi4cp2[2, 9/2],
NewPhi4cp2[3, 9/2]]
Bosonic[NewPhi4cp3[1, 5], NewPhi4cp3[2, 5], NewPhi4cp3[3, 5]]
Fermionic[NewPhi4cp4[11/2]]
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$$\text{Tmatirx}[1,1,1] = 0;$$
$$\text{Tmatirx}[1,1,2] = 0;$$
$$\text{Tmatirx}[1,1,3] = 0;$$
$$\text{Tmatirx}[1,2,1] = 0;$$
$$\text{Tmatirx}[1,2,2] = 0;$$
$$\text{Tmatirx}[1,2,3] = -i;$$
$$\text{Tmatirx}[1,3,1] = 0;$$
$$\text{Tmatirx}[1,3,2] = i;$$
$$\text{Tmatirx}[1,3,3] = 0;$$

$$\text{Tmatirx}[2,1,1] = 0;$$
$$\text{Tmatirx}[2,1,2] = 0;$$
$$\text{Tmatirx}[2,1,3] = i;$$
$$\text{Tmatirx}[2,2,1] = 0;$$
$$\text{Tmatirx}[2,2,2] = 0;$$
$$\text{Tmatirx}[2,2,3] = 0;$$
$$\text{Tmatirx}[2,3,1] = -i;$$
$$\text{Tmatirx}[2,3,2] = 0;$$
$$\text{Tmatirx}[2,3,3] = 0;$$

$$\text{Tmatirx}[3,1,1] = 0;$$
$$\text{Tmatirx}[3,1,2] = -i;$$
$$\text{Tmatirx}[3,1,3] = 0;$$
$$\text{Tmatirx}[3,2,1] = i;$$
$$\text{Tmatirx}[3,2,2] = 0;$$
$$\text{Tmatirx}[3,2,3] = 0;$$
$$\text{Tmatirx}[3,3,1] = 0;$$
$$\text{Tmatirx}[3,3,2] = 0;$$

$\mathrm{Tmatirx}[3,3,3]=0;$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{Ts}]=\mathrm{MakeOPE}[\{(c\mathrm{One})/2,0,2\mathrm{Ts},\mathrm{Ts}'\}];$

$\mathrm{OPE}[\mathrm{Ts},\Psi]=\mathrm{MakeOPE}[\{(1/2)\Psi,\Psi'\}];$

$\mathrm{OPE}[\mathrm{Ts},J[\mathrm{i\_}]]=\mathrm{MakeOPE}[\{J[i],J[i]'\}];$

$\mathrm{OPE}[\mathrm{Ts},G[\mathrm{i\_}]]=\mathrm{MakeOPE}[\{(3/2)G[i],G[i]'\}];$

$\mathrm{OPE}[\Psi,\Psi]=\mathrm{MakeOPE}[\{(c\mathrm{One})/3\}];$

$\mathrm{OPE}[\Psi,J[\mathrm{i\_}]]=\mathrm{MakeOPE}[\{0\}];$

$\mathrm{OPE}[\Psi,G[\mathrm{i\_}]]=\mathrm{MakeOPE}[\{J[i]\}];$

$\mathrm{OPE}[J[\mathrm{i\_}],J[\mathrm{j\_}]]=$

$\mathrm{MakeOPE}\left[\left\{((c\mathrm{One})/3)\mathrm{Delta}[i,j],I*\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]J[\mathrm{x1}]\right\}\right];$

$\mathrm{OPE}[J[\mathrm{i\_}],G[\mathrm{j\_}]]=$

$\mathrm{MakeOPE}\left[\left\{\mathrm{Delta}[i,j]\Psi,I*\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]G[\mathrm{x1}]\right\}\right];$

$\mathrm{OPE}[G[\mathrm{i\_}],G[\mathrm{j\_}]]=$

$\mathrm{MakeOPE}\left[\left\{((2c\mathrm{One})/3)\mathrm{Delta}[i,j],2I*\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]J[\mathrm{x1}],\right.\right.$

$\left.\left.2\mathrm{Delta}[i,j]\mathrm{Ts}+I*\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]J[\mathrm{x1}]'\right\}\right];$

$c=(3*\mathrm{NN}*\mathrm{MM})/2;$

$J\Phi^{\left(\frac{3}{2}\right)}$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{Phi3Hcp1}[3/2]]=$

$\mathrm{MakeOPE}[\{(3/2)\mathrm{Phi3Hcp1}[3/2],\mathrm{Phi3Hcp1}[3/2]'\}];$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{Phi3Hcp2}[\mathrm{i\_},2]]=\mathrm{MakeOPE}[\{2\mathrm{Phi3Hcp2}[i,2],\mathrm{Phi3Hcp2}[i,2]'\}];$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{Phi3Hcp3}[\mathrm{i\_},5/2]]=$

$\mathrm{MakeOPE}[\{(5/2)\mathrm{Phi3Hcp3}[i,5/2],\mathrm{Phi3Hcp3}[i,5/2]'\}];$

$\text{OPE}[\text{Ts}, \text{Phi3Hcp4}[3]] = \text{MakeOPE}[\{3\text{Phi3Hcp4}[3], \text{Phi3Hcp4}[3]'\}];$

$\text{OPE}[G[\text{i\_}], \text{Phi3Hcp1}[3/2]] = \text{MakeOPE}[\{\text{Phi3Hcp2}[i, 2]\}];$

$\text{OPE}[G[\text{i\_}], \text{Phi3Hcp2}[\text{j\_}, 2]] =$

$\text{MakeOPE}[\{\text{Delta}[i, j]3\text{Phi3Hcp1}[3/2],$

$\text{Delta}[i, j]\text{Phi3Hcp1}[3/2]' +$

$i \sum_{\text{x1}=1}^{3} \text{Epsilon}[i, j, \text{x1}]\text{Phi3Hcp3}[\text{x1}, 5/2]\Big\}\Big];$

$\text{OPE}[G[\text{i\_}], \text{Phi3Hcp3}[\text{j\_}, 5/2]] =$

$\text{MakeOPE}\Big[\Big\{4i \sum_{\text{x1}=1}^{3} \text{Epsilon}[i, j, \text{x1}]\text{Phi3Hcp2}[\text{x1}, 2],$

$2\text{Delta}[i, j]\text{Phi3Hcp4}[3] + i \sum_{\text{x1}=1}^{3} \text{Epsilon}[i, j, \text{x1}]\text{Phi3Hcp2}[\text{x1}, 2]'\Big\}\Big];$

$\text{OPE}[G[\text{i\_}], \text{Phi3Hcp4}[3]] =$

$\text{MakeOPE}[\{(5/2)\text{Phi3Hcp3}[i, 5/2], (1/2)\text{Phi3Hcp3}[i, 5/2]'\}];$

$\text{OPE}[J[\text{i\_}], \text{Phi3Hcp2}[\text{j\_}, 2]] =$

$\text{MakeOPE}\Big[\Big\{i \sum_{\text{x1}=1}^{3} \text{Epsilon}[i, j, \text{x1}]\text{Phi3Hcp2}[\text{x1}, 2]\Big\}\Big];$

$\text{OPE}[J[\text{i\_}], \text{Phi3Hcp3}[\text{j\_}, 5/2]] =$

$\text{MakeOPE}[\{3\text{Delta}[i, j]\text{Phi3Hcp1}[3/2],$

$i \sum_{\text{x1}=1}^{3} \text{Epsilon}[i, j, \text{x1}]\text{Phi3Hcp3}[\text{x1}, 5/2]\Big\}\Big];$

$\text{OPE}[J[\text{i\_}], \text{Phi3Hcp4}[3]] = \text{MakeOPE}[\{2\text{Phi3Hcp2}[i, 2], 0\}];$

$\text{OPE}[\Psi, \text{Phi3Hcp3}[\text{i\_}, 5/2]] = \text{MakeOPE}[\{\text{Phi3Hcp2}[i, 2]\}];$

$\text{OPE}[\Psi, \text{Phi3Hcp4}[3]] =$

$\text{MakeOPE}[\{(3/2)\text{Phi3Hcp1}[3/2], -(1/2)\text{Phi3Hcp1}[3/2]'\}];$

$J\Phi^{(2)}$

$\text{OPE}[\text{Ts}, \text{Phi2cp1}[2]] = \text{MakeOPE}[\{2\text{Phi2cp1}[2], \text{Phi2cp1}[2]'\}];$

$\text{OPE}[\text{Ts}, \text{Phi2cp2}[\text{i\_}, 5/2]] =$

$\text{MakeOPE}[\{(5/2)\text{Phi2cp2}[i, 5/2], \text{Phi2cp2}[i, 5/2]'\}];$

$\text{OPE}[\text{Ts}, \text{Phi2cp3}[\text{i\_}, 3]] = \text{MakeOPE}[\{3\text{Phi2cp3}[i, 3], \text{Phi2cp3}[i, 3]'\}];$

$\text{OPE}[\text{Ts}, \text{Phi2cp4}[7/2]] = \text{MakeOPE}[\{(7/2)\text{Phi2cp4}[7/2], \text{Phi2cp4}[7/2]'\}];$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{Phi2cp1}[2]] = \mathrm{MakeOPE}[\{\mathrm{Phi2cp2}[i, 5/2]\}];$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{Phi2cp2}[\mathrm{j\_}, 5/2]] =$

$\mathrm{MakeOPE}[\{\mathrm{Delta}[i, j]4\mathrm{Phi2cp1}[2],$

$\mathrm{Delta}[i, j]\mathrm{Phi2cp1}[2]' + i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{Phi2cp3}[\mathrm{x1}, 3]\}\Big];$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{Phi2cp3}[\mathrm{j\_}, 3]] =$

$\mathrm{MakeOPE}\Big[\Big\{5i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{Phi2cp2}[\mathrm{x1}, 5/2],$

$2\mathrm{Delta}[i, j]\mathrm{Phi2cp4}[7/2]+$

$i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{Phi2cp2}[\mathrm{x1}, 5/2]'\Big\}\Big];$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{Phi2cp4}[7/2]] =$

$\mathrm{MakeOPE}[\{3\mathrm{Phi2cp3}[i, 3], (1/2)\mathrm{Phi2cp3}[i, 3]'\}];$

$\mathrm{OPE}[J[\mathrm{i\_}], \mathrm{Phi2cp2}[\mathrm{j\_}, 5/2]] =$

$\mathrm{MakeOPE}\Big[\Big\{i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{Phi2cp2}[\mathrm{x1}, 5/2]\Big\}\Big];$

$\mathrm{OPE}[J[\mathrm{i\_}], \mathrm{Phi2cp3}[\mathrm{j\_}, 3]] =$

$\mathrm{MakeOPE}[\{4\mathrm{Delta}[i, j]\mathrm{Phi2cp1}[2],$

$i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{Phi2cp3}[\mathrm{x1}, 3]\Big\}\Big];$

$\mathrm{OPE}[J[\mathrm{i\_}], \mathrm{Phi2cp4}[7/2]] = \mathrm{MakeOPE}[\{(5/2)\mathrm{Phi2cp2}[i, 5/2], 0\}];$

$\mathrm{OPE}[\Psi, \mathrm{Phi2cp3}[\mathrm{i\_}, 3]] = \mathrm{MakeOPE}[\{\mathrm{Phi2cp2}[i, 5/2]\}];$

$\mathrm{OPE}[\Psi, \mathrm{Phi2cp4}[7/2]] = \mathrm{MakeOPE}[\{2\mathrm{Phi2cp1}[2], -(1/2)\mathrm{Phi2cp1}[2]'\}];$

$J\Phi^{(\frac{5}{2})}$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{Phi5Hcp1}[5/2]] =$

$\mathrm{MakeOPE}[\{(5/2)\mathrm{Phi5Hcp1}[5/2], \mathrm{Phi5Hcp1}[5/2]'\}];$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{Phi5Hcp2}[\mathrm{i\_}, 3]] = \mathrm{MakeOPE}[\{3\mathrm{Phi5Hcp2}[i, 3], \mathrm{Phi5Hcp2}[i, 3]'\}];$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{Phi5Hcp3}[\mathrm{i\_}, 7/2]] =$

$\mathrm{MakeOPE}[\{(7/2)\mathrm{Phi5Hcp3}[i, 7/2], \mathrm{Phi5Hcp3}[i, 7/2]'\}];$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{Phi5Hcp4}[4]] = \mathrm{MakeOPE}[\{4\mathrm{Phi5Hcp4}[4], \mathrm{Phi5Hcp4}[4]'\}];$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{Phi5Hcp1}[5/2]] = \mathrm{MakeOPE}[\{\mathrm{Phi5Hcp2}[i, 3]\}];$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{Phi5Hcp2}[\mathrm{j\_}, 3]] =$

$\mathrm{MakeOPE}[\{\mathrm{Delta}[i, j]5\mathrm{Phi5Hcp1}[5/2],$

$\mathrm{Delta}[i,j]\mathrm{Phi5Hcp1}[5/2]'+$

$i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]\mathrm{Phi5Hcp3}[\mathrm{x1},7/2]\Big\}\Big]$;

$\mathrm{OPE}[G[\mathrm{i\_}],\mathrm{Phi5Hcp3}[\mathrm{j\_},7/2]]=$

$\mathrm{MakeOPE}\Big[\Big\{6i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]\mathrm{Phi5Hcp2}[\mathrm{x1},3],$

$2\mathrm{Delta}[i,j]\mathrm{Phi5Hcp4}[4]+i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]\mathrm{Phi5Hcp2}[\mathrm{x1},3]'\Big\}\Big]$;

$\mathrm{OPE}[G[\mathrm{i\_}],\mathrm{Phi5Hcp4}[4]]=$

$\mathrm{MakeOPE}[\{(7/2)\mathrm{Phi5Hcp3}[i,7/2],(1/2)\mathrm{Phi5Hcp3}[i,7/2]'\}];$

$\mathrm{OPE}[J[\mathrm{i\_}],\mathrm{Phi5Hcp2}[\mathrm{j\_},3]]=$

$\mathrm{MakeOPE}\Big[\Big\{i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]\mathrm{Phi5Hcp2}[\mathrm{x1},3]\Big\}\Big]$;

$\mathrm{OPE}[J[\mathrm{i\_}],\mathrm{Phi5Hcp3}[\mathrm{j\_},7/2]]=$

$\mathrm{MakeOPE}[\{5\mathrm{Delta}[i,j]\mathrm{Phi5Hcp1}[5/2],$

$i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]\mathrm{Phi5Hcp3}[\mathrm{x1},7/2]\Big\}\Big]$;

$\mathrm{OPE}[J[\mathrm{i\_}],\mathrm{Phi5Hcp4}[4]]=\mathrm{MakeOPE}[\{3\mathrm{Phi5Hcp2}[i,3],0\}];$

$\mathrm{OPE}[\Psi,\mathrm{Phi5Hcp3}[\mathrm{i\_},7/2]]=\mathrm{MakeOPE}[\{\mathrm{Phi5Hcp2}[i,3]\}];$

$\mathrm{OPE}[\Psi,\mathrm{Phi5Hcp4}[4]]=$

$\mathrm{MakeOPE}[\{(5/2)\mathrm{Phi5Hcp1}[5/2],-(1/2)\mathrm{Phi5Hcp1}[5/2]'\}];$

$J\bar{\Phi}^{(\frac{5}{2})}$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{NNewPhi5Hcp1}[5/2]]=$

$\mathrm{MakeOPE}[\{(5/2)\mathrm{NNewPhi5Hcp1}[5/2],\mathrm{NNewPhi5Hcp1}[5/2]'\}];$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{NNewPhi5Hcp2}[\mathrm{i\_},3]]=$

$\mathrm{MakeOPE}[\{3\mathrm{NNewPhi5Hcp2}[i,3],\mathrm{NNewPhi5Hcp2}[i,3]'\}];$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{NNewPhi5Hcp3}[\mathrm{i\_},7/2]]=$

$\mathrm{MakeOPE}[\{(7/2)\mathrm{NNewPhi5Hcp3}[i,7/2],\mathrm{NNewPhi5Hcp3}[i,7/2]'\}];$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{NNewPhi5Hcp4}[4]]=$

$\mathrm{MakeOPE}[\{4\mathrm{NNewPhi5Hcp4}[4],\mathrm{NNewPhi5Hcp4}[4]'\}];$

$\mathrm{OPE}[G[\mathrm{i\_}],\mathrm{NNewPhi5Hcp1}[5/2]]=\mathrm{MakeOPE}[\{\mathrm{NNewPhi5Hcp2}[i,3]\}];$

$\text{OPE}[G[\text{i\_}], \text{NNewPhi5Hcp2}[\text{j\_}, 3]] =$

$\text{MakeOPE}[\{\text{Delta}[i, j]5\text{NNewPhi5Hcp1}[5/2],$

$\text{Delta}[i, j]\text{NNewPhi5Hcp1}[5/2]'+$

$i\sum_{\text{x1}=1}^{3}\text{Epsilon}[i, j, \text{x1}]\text{NNewPhi5Hcp3}[\text{x1}, 7/2]\}\Big];$

$\text{OPE}[G[\text{i\_}], \text{NNewPhi5Hcp3}[\text{j\_}, 7/2]] =$

$\text{MakeOPE}\Big[\Big\{6i\sum_{\text{x1}=1}^{3}\text{Epsilon}[i, j, \text{x1}]\text{NNewPhi5Hcp2}[\text{x1}, 3],$

$2\text{Delta}[i, j]\text{NNewPhi5Hcp4}[4]+$

$i\sum_{\text{x1}=1}^{3}\text{Epsilon}[i, j, \text{x1}]\text{NNewPhi5Hcp2}[\text{x1}, 3]'\Big\}\Big];$

$\text{OPE}[G[\text{i\_}], \text{NNewPhi5Hcp4}[4]] =$

$\text{MakeOPE}[\{(7/2)\text{NNewPhi5Hcp3}[i, 7/2], (1/2)\text{NNewPhi5Hcp3}[i, 7/2]'\}];$

$\text{OPE}[J[\text{i\_}], \text{NNewPhi5Hcp2}[\text{j\_}, 3]] =$

$\text{MakeOPE}\Big[\Big\{i\sum_{\text{x1}=1}^{3}\text{Epsilon}[i, j, \text{x1}]\text{NNewPhi5Hcp2}[\text{x1}, 3]\Big\}\Big];$

$\text{OPE}[J[\text{i\_}], \text{NNewPhi5Hcp3}[\text{j\_}, 7/2]] =$

$\text{MakeOPE}[\{5\text{Delta}[i, j]\text{NNewPhi5Hcp1}[5/2],$

$i\sum_{\text{x1}=1}^{3}\text{Epsilon}[i, j, \text{x1}]\text{NNewPhi5Hcp3}[\text{x1}, 7/2]\}\Big];$

$\text{OPE}[J[\text{i\_}], \text{NNewPhi5Hcp4}[4]] = \text{MakeOPE}[\{3\text{NNewPhi5Hcp2}[i, 3], 0\}];$

$\text{OPE}[\Psi, \text{NNewPhi5Hcp3}[\text{i\_}, 7/2]] = \text{MakeOPE}[\{\text{NNewPhi5Hcp2}[i, 3]\}];$

$\text{OPE}[\Psi, \text{NNewPhi5Hcp4}[4]] =$

$\text{MakeOPE}[\{(5/2)\text{NNewPhi5Hcp1}[5/2], -(1/2)\text{NNewPhi5Hcp1}[5/2]'\}];$

$J\Phi^{(3),a}$

$\text{OPE}[\text{Ts}, \text{Phi3cp1}[3, \{\text{a\_}\}]] =$

$\text{MakeOPE}[\{3\text{Phi3cp1}[3, \{a\}], \text{Phi3cp1}[3, \{a\}]'\}];$

$\text{OPE}[\text{Ts}, \text{Phi3cp2}[\text{i\_}, 7/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}[\{(7/2)\text{Phi3cp2}[i, 7/2, \{a\}], \text{Phi3cp2}[i, 7/2, \{a\}]'\}];$

$\text{OPE}[\text{Ts}, \text{Phi3cp3}[\text{i\_}, 4, \{\text{a\_}\}]] =$

$\text{MakeOPE}\Big[\Big\{-\sum_{b=1}^{3}(\text{Tmatirx}[i, a, b]\text{Phi3cp1}[3, \{b\}]), 4\text{Phi3cp3}[i, 4, \{a\}],$

$\text{Phi3cp3}[i, 4, \{a\}]'\}];$

$\text{OPE}[\text{Ts}, \text{Phi3cp4}[9/2, \{\text{a\_}\}]] =$

MakeOPE $\left[\left\{-(1/2)\sum_{b=1}^{3}\left(\sum_{\mathrm{id1}=1}^{3}(\mathrm{Tmatirx}[\mathrm{id1},a,b]\mathrm{Phi3cp2}[\mathrm{id1},7/2,\{b\}])\right),\right.\right.$

(9/2)Phi3cp4[9/2, $\{a\}$], Phi3cp4[9/2, $\{a\}$]$'$}];

OPE[G[id_], Phi3cp1[3, {a_}]] = MakeOPE[{Phi3cp2[id, 7/2, $\{a\}$]}];

OPE[G[id1_], Phi3cp2[id2_, 7/2, {a_}]] =

MakeOPE[

{6Delta[id1, id2]Phi3cp1[3, $\{a\}$]−

$i\sum_{\mathrm{x1}=1}^{3}\left(\sum_{b=1}^{3}\mathrm{Epsilon}[\mathrm{id1},\mathrm{id2},\mathrm{x1}]\mathrm{Tmatirx}[\mathrm{x1},a,b]\mathrm{Phi3cp1}[3,\{b\}]\right),$

Delta[id1, id2]Phi3cp1[3, $\{a\}$]$'$+

$\left.\left. i\sum_{\mathrm{x1}=1}^{3}(\mathrm{Epsilon}[\mathrm{id1},\mathrm{id2},\mathrm{x1}]\mathrm{Phi3cp3}[\mathrm{x1},4,\{a\}])\right\}\right];$

OPE[G[id1_], Phi3cp3[id2_, 4, {a_}]] =

MakeOPE $\left[\left\{7i\sum_{\mathrm{x1}=1}^{3}(\mathrm{Epsilon}[\mathrm{id1},\mathrm{id2},\mathrm{x1}]\mathrm{Phi3cp2}[\mathrm{x1},7/2,\{a\}])+\right.\right.$

$\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id2},a,b]\mathrm{Phi3cp2}[\mathrm{id1},7/2,\{b\}]-$

$\sum_{b=1}^{3}\left(\sum_{\mathrm{x1}=1}^{3}\mathrm{Delta}[\mathrm{id1},\mathrm{id2}](\mathrm{Tmatirx}[\mathrm{x1},a,b]\mathrm{Phi3cp2}[\mathrm{x1},7/2,\{b\}])\right),$

2Delta[id1, id2]Phi3cp4[9/2, $\{a\}$]+

$\left.\left. i\sum_{\mathrm{x1}=1}^{3}(\mathrm{Epsilon}[\mathrm{id1},\mathrm{id2},\mathrm{x1}]\mathrm{Phi3cp2}[\mathrm{x1},7/2,\{a\}]')\right\}\right];$

OPE[G[id1_], Phi3cp4[9/2, {a_}]] =

MakeOPE $\left[\left\{-\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id1},a,b]\mathrm{Phi3cp1}[3,\{b\}],\right.\right.$

4Phi3cp3[id1, 4, $\{a\}$]+

(i/2)

$\sum_{b=1}^{3}\left(\sum_{\mathrm{x1}=1}^{3}\left(\sum_{\mathrm{x2}=1}^{3}\mathrm{Epsilon}[\mathrm{id1},\mathrm{x1},\mathrm{x2}]\right.\right.$

(Tmatirx[x1, a, b]Phi3cp3[x2, 4, $\{b\}$]))),

(1/2)Phi3cp3[id1, 4, $\{a\}$]$'$}];

OPE[J[id1_], Phi3cp1[3, {a_}]] =

MakeOPE $\left[\left\{-\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id1},a,b]\mathrm{Phi3cp1}[3,\{b\}]\right\}\right];$

OPE[J[id1_], Phi3cp2[id2_, 7/2, {a_}]] =

MakeOPE $\left[\left\{i\left(\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[\mathrm{id1},\mathrm{id2},\mathrm{x1}](\mathrm{Phi3cp2}[\mathrm{x1},7/2,\{a\}])\right)-\right.\right.$

$\left.\left.\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id1},a,b]\mathrm{Phi3cp2}[\mathrm{id2},7/2,\{b\}]\right\}\right];$

OPE[J[id1_], Phi3cp3[id2_, 4, {a_}]] =

MakeOPE[

$\{6\text{Delta}[\text{id1}, \text{id2}]\text{Phi3cp1}[3, \{a\}]-$

$i \sum_{\text{x1}=1}^{3} \text{Epsilon}[\text{id1}, \text{id2}, \text{x1}] \left(\sum_{b=1}^{3} \text{Tmatirx}[\text{x1}, a, b]\text{Phi3cp1}[3, \{b\}]\right),$

$i \sum_{\text{x1}=1}^{3} (\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{Phi3cp3}[\text{x1}, 4, \{a\}])-$

$\left(\sum_{b=1}^{3} \text{Tmatirx}[\text{id1}, a, b]\text{Phi3cp3}[\text{id2}, 4, \{b\}]\right)\Big\}\Big];$

$\text{OPE}[J[\text{id1\_}], \text{Phi3cp4}[9/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}[$

$\{(7/2)\text{Phi3cp2}[\text{id1}, 7/2, \{a\}]+$

$(i/2)$

$\sum_{b=1}^{3} \left(\sum_{\text{x1}=1}^{3} \left(\sum_{\text{x2}=1}^{3} \text{Epsilon}[\text{id1}, \text{x1}, \text{x2}]\right.\right.$

$(\text{Tmatirx}[\text{x1}, a, b]\text{Phi3cp2}[\text{x2}, 7/2, \{b\}]))),$

$-\sum_{b=1}^{3} \text{Tmatirx}[\text{id1}, a, b]\text{Phi3cp4}[9/2, \{b\}]\Big\}\Big];$

$\text{OPE}[\Psi, \text{Phi3cp2}[\text{id1\_}, 7/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}\left[\left\{-\sum_{b=1}^{3} \text{Tmatirx}[\text{id1}, a, b]\text{Phi3cp1}[3, \{b\}]\right\}\right];$

$\text{OPE}[\Psi, \text{Phi3cp3}[\text{id1\_}, 4, \{\text{a\_}\}]] =$

$\text{MakeOPE}[$

$\{\text{Phi3cp2}[\text{id1}, 7/2, \{a\}]+$

i

$\sum_{b=1}^{3} \left(\sum_{\text{x1}=1}^{3} \left(\sum_{\text{x2}=1}^{3} \text{Epsilon}[\text{id1}, \text{x1}, \text{x2}]\right.\right.$

$(\text{Tmatirx}[\text{x1}, a, b]\text{Phi3cp2}[\text{x2}, 7/2, \{b\}])))\}];$

$\text{OPE}[\Psi, \text{Phi3cp4}[9/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}[\{3\text{Phi3cp1}[3, \{a\}],$

$-(1/2)\text{Phi3cp1}[3, \{a\}]'-$

$(1/2) \sum_{\text{id1}=1}^{3} \left(\sum_{b=1}^{3} \text{Tmatirx}[\text{id1}, a, b]\text{Phi3cp3}[\text{id1}, 4, \{b\}]\right)\Big\}\Big];$

$J\bar{\Phi}^{\left(\frac{3}{2}\right)}$

$\text{OPE}[\text{Ts}, \text{NewPhi3Hcp1}[3/2]] =$

$\text{MakeOPE}[\{(3/2)\text{NewPhi3Hcp1}[3/2], \text{NewPhi3Hcp1}[3/2]'\}];$

$\text{OPE}[\text{Ts}, \text{NewPhi3Hcp2}[\text{i\_}, 2]] =$

$\text{MakeOPE}[\{2\text{NewPhi3Hcp2}[i, 2], \text{NewPhi3Hcp2}[i, 2]'\}];$

OPE[Ts, NewPhi3Hcp3[i_, 5/2]] =

MakeOPE[{(5/2)NewPhi3Hcp3[i, 5/2], NewPhi3Hcp3[i, 5/2]′}];

OPE[Ts, NewPhi3Hcp4[3]] = MakeOPE[{3NewPhi3Hcp4[3], NewPhi3Hcp4[3]′}];

OPE[G[i_], NewPhi3Hcp1[3/2]] = MakeOPE[{NewPhi3Hcp2[i, 2]}];

OPE[G[i_], NewPhi3Hcp2[j_, 2]] =

MakeOPE[{Delta[i, j]3NewPhi3Hcp1[3/2],

Delta[i, j]NewPhi3Hcp1[3/2]′+

$i\sum_{\mathrm{x1}=1}^{3}$ Epsilon[i, j, x1]NewPhi3Hcp3[x1, 5/2]}] ;

OPE[G[i_], NewPhi3Hcp3[j_, 5/2]] =

MakeOPE [{4$i\sum_{\mathrm{x1}=1}^{3}$ Epsilon[i, j, x1]NewPhi3Hcp2[x1, 2],

2Delta[i, j]NewPhi3Hcp4[3]+

$i\sum_{\mathrm{x1}=1}^{3}$ Epsilon[i, j, x1]NewPhi3Hcp2[x1, 2]′}] ;

OPE[G[i_], NewPhi3Hcp4[3]] =

MakeOPE[{(5/2)NewPhi3Hcp3[i, 5/2], (1/2)NewPhi3Hcp3[i, 5/2]′}];

OPE[J[i_], NewPhi3Hcp2[j_, 2]] =

MakeOPE [{$i\sum_{\mathrm{x1}=1}^{3}$ Epsilon[i, j, x1]NewPhi3Hcp2[x1, 2]}] ;

OPE[J[i_], NewPhi3Hcp3[j_, 5/2]] =

MakeOPE[{3Delta[i, j]NewPhi3Hcp1[3/2],

$i\sum_{\mathrm{x1}=1}^{3}$ Epsilon[i, j, x1]NewPhi3Hcp3[x1, 5/2]}] ;

OPE[J[i_], NewPhi3Hcp4[3]] = MakeOPE[{2NewPhi3Hcp2[i, 2], 0}];

OPE[Ψ, NewPhi3Hcp3[i_, 5/2]] = MakeOPE[{NewPhi3Hcp2[i, 2]}];

OPE[Ψ, NewPhi3Hcp4[3]] =

MakeOPE[{(3/2)NewPhi3Hcp1[3/2], −(1/2)NewPhi3Hcp1[3/2]′}];

$J\tilde{\Phi}^{(\frac{3}{2}),a}$

$\text{OPE}[\text{Ts}, \text{NNewPhi3Hcp1}[3/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}[\{(3/2)\text{NNewPhi3Hcp1}[3/2, \{a\}], \text{NNewPhi3Hcp1}[3/2, \{a\}]'\}];$

$\text{OPE}[\text{Ts}, \text{NNewPhi3Hcp2}[\text{i\_}, 2, \{\text{a\_}\}]] =$

$\text{MakeOPE}[\{2\text{NNewPhi3Hcp2}[i, 2, \{a\}], \text{NNewPhi3Hcp2}[i, 2, \{a\}]'\}];$

$\text{OPE}[\text{Ts}, \text{NNewPhi3Hcp3}[\text{i\_}, 5/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}\left[\left\{-\sum_{b=1}^{3}(\text{Tmatirx}[i, a, b]\text{NNewPhi3Hcp1}[3/2, \{b\}]),\right.\right.$

$(5/2)\text{NNewPhi3Hcp3}[i, 5/2, \{a\}], \text{NNewPhi3Hcp3}[i, 5/2, \{a\}]'\}];$

$\text{OPE}[\text{Ts}, \text{NNewPhi3Hcp4}[3, \{\text{a\_}\}]] =$

$\text{MakeOPE}[$

$\left\{-(1/2)\sum_{b=1}^{3}\left(\sum_{\text{id1}=1}^{3}(\text{Tmatirx}[\text{id1}, a, b]\text{NNewPhi3Hcp2}[\text{id1}, 2, \{b\}])\right),\right.$

$3\text{NNewPhi3Hcp4}[3, \{a\}], \text{NNewPhi3Hcp4}[3, \{a\}]'\}];$

$\text{OPE}[G[\text{id\_}], \text{NNewPhi3Hcp1}[3/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}[\{\text{NNewPhi3Hcp2}[\text{id}, 2, \{a\}]\}];$

$\text{OPE}[G[\text{id1\_}], \text{NNewPhi3Hcp2}[\text{id2\_}, 2, \{\text{a\_}\}]] =$

$\text{MakeOPE}[\{3\text{Delta}[\text{id1}, \text{id2}]\text{NNewPhi3Hcp1}[3/2, \{a\}]-$

i

$\sum_{\text{x1}=1}^{3}\left(\sum_{b=1}^{3}\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{Tmatirx}[\text{x1}, a, b]\right.$

$\text{NNewPhi3Hcp1}[3/2, \{b\}]),$

$\text{Delta}[\text{id1}, \text{id2}]\text{NNewPhi3Hcp1}[3/2, \{a\}]'+$

$\left.\left. i\sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{NNewPhi3Hcp3}[\text{x1}, 5/2, \{a\}])\right\}\right];$

$\text{OPE}[G[\text{id1\_}], \text{NNewPhi3Hcp3}[\text{id2\_}, 5/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}\left[\left\{4i\sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{NNewPhi3Hcp2}[\text{x1}, 2, \{a\}])+\right.\right.$

$\sum_{b=1}^{3}$ Tmatirx[id2, a, b]NNewPhi3Hcp2[id1, 2, {b}]−

$\sum_{b=1}^{3}\left(\sum_{x1=1}^{3}\right.$ Delta[id1, id2](Tmatirx[x1, a, b]NNewPhi3Hcp2[x1, 2, {b}])$\Big)$,

2Delta[id1, id2]NNewPhi3Hcp4[3, {a}]+

$i\sum_{x1=1}^{3}$(Epsilon[id1, id2, x1]NNewPhi3Hcp2[x1, 2, {a}]′)$\Big\}\Big]$;

OPE[G[id1_], NNewPhi3Hcp4[3, {a_}]] =

MakeOPE $\Big[\Big\{-\sum_{b=1}^{3}$ Tmatirx[id1, a, b]NNewPhi3Hcp1[3/2, {b}],

(5/2)NNewPhi3Hcp3[id1, 5/2, {a}]+

(i/2)

$\sum_{b=1}^{3}\left(\sum_{x1=1}^{3}\left(\sum_{x2=1}^{3}\right.\right.$ Epsilon[id1, x1, x2]

(Tmatirx[x1, a, b]NNewPhi3Hcp3[x2, 5/2, {b}]))),

(1/2)NNewPhi3Hcp3[id1, 5/2, {a}]′}];

OPE[J[id1_], NNewPhi3Hcp1[3/2, {a_}]] =

MakeOPE $\Big[\Big\{-\sum_{b=1}^{3}$ Tmatirx[id1, a, b]NNewPhi3Hcp1[3/2, {b}]$\Big\}\Big]$;

OPE[J[id1_], NNewPhi3Hcp2[id2_, 2, {a_}]] =

MakeOPE $\Big[\Big\{i\Big(\sum_{x1=1}^{3}$ Epsilon[id1, id2, x1](NNewPhi3Hcp2[x1, 2, {a}])$\Big)$ −

$\sum_{b=1}^{3}$ Tmatirx[id1, a, b]NNewPhi3Hcp2[id2, 2, {b}]$\Big\}\Big]$;

OPE[J[id1_], NNewPhi3Hcp3[id2_, 5/2, {a_}]] =

MakeOPE[{3Delta[id1, id2]NNewPhi3Hcp1[3/2, {a}]−

$i\sum_{x1=1}^{3}$ Epsilon[id1, id2, x1]

$\Big(\sum_{b=1}^{3}$ Tmatirx[x1, a, b]NNewPhi3Hcp1[3/2, {b}]$\Big)$,

$i\sum_{x1=1}^{3}$(Epsilon[id1, id2, x1]NNewPhi3Hcp3[x1, 5/2, {a}])−

$\Big(\sum_{b=1}^{3}$ Tmatirx[id1, a, b]NNewPhi3Hcp3[id2, 5/2, {b}]$\Big)\Big\}\Big]$;

OPE[J[id1_], NNewPhi3Hcp4[3, {a_}]] =

MakeOPE[

{2NNewPhi3Hcp2[id1, 2, {a}]+

(i/2)

$\sum_{b=1}^{3}\left(\sum_{x1=1}^{3}\left(\sum_{x2=1}^{3}\right.\right.$ Epsilon[id1, x1, x2]

(Tmatirx[x1, a, b]NNewPhi3Hcp2[x2, 2, {b}]))),

$-\sum_{b=1}^{3}$ Tmatirx[id1, a, b]NNewPhi3Hcp4[3, {b}]$\Big\}\Big]$;

OPE[Ψ, NNewPhi3Hcp2[id1_, 2, {a_}]] =

$\mathrm{MakeOPE}\left[\left\{-\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id1},a,b]\mathrm{NNewPhi3Hcp1}[3/2,\{b\}]\right\}\right];$

$\mathrm{OPE}[\Psi,\mathrm{NNewPhi3Hcp3}[\mathrm{id1\_},5/2,\{\mathrm{a\_}\}]]=$

MakeOPE[

$\{\mathrm{NNewPhi3Hcp2}[\mathrm{id1},2,\{a\}]+$

i

$\sum_{b=1}^{3}\left(\sum_{\mathrm{x1}=1}^{3}\left(\sum_{\mathrm{x2}=1}^{3}\mathrm{Epsilon}[\mathrm{id1},\mathrm{x1},\mathrm{x2}]\right.\right.$

$(\mathrm{Tmatirx}[\mathrm{x1},a,b]\mathrm{NNewPhi3Hcp2}[\mathrm{x2},2,\{b\}])))\}];$

$\mathrm{OPE}[\Psi,\mathrm{NNewPhi3Hcp4}[3,\{\mathrm{a\_}\}]]=$

$\mathrm{MakeOPE}[\{(3/2)\mathrm{NNewPhi3Hcp1}[3/2,\{a\}],$

$-(1/2)\mathrm{NNewPhi3Hcp1}[3/2,\{a\}]'-$

$\left.\left.(1/2)\sum_{\mathrm{id1}=1}^{3}\left(\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id1},a,b]\mathrm{NNewPhi3Hcp3}[\mathrm{id1},5/2,\{b\}]\right)\right\}\right];$

$J\bar{\Phi}^{(2)}$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{NewPhi2cp1}[2]]=\mathrm{MakeOPE}[\{2\mathrm{NewPhi2cp1}[2],\mathrm{NewPhi2cp1}[2]'\}];$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{NewPhi2cp2}[\mathrm{i\_},5/2]]=$

$\mathrm{MakeOPE}[\{(5/2)\mathrm{NewPhi2cp2}[i,5/2],\mathrm{NewPhi2cp2}[i,5/2]'\}];$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{NewPhi2cp3}[\mathrm{i\_},3]]=$

$\mathrm{MakeOPE}[\{3\mathrm{NewPhi2cp3}[i,3],\mathrm{NewPhi2cp3}[i,3]'\}];$

$\mathrm{OPE}[\mathrm{Ts},\mathrm{NewPhi2cp4}[7/2]]=$

$\mathrm{MakeOPE}[\{(7/2)\mathrm{NewPhi2cp4}[7/2],\mathrm{NewPhi2cp4}[7/2]'\}];$

$\mathrm{OPE}[G[\mathrm{i\_}],\mathrm{NewPhi2cp1}[2]]=\mathrm{MakeOPE}[\{\mathrm{NewPhi2cp2}[i,5/2]\}];$

$\mathrm{OPE}[G[\mathrm{i\_}],\mathrm{NewPhi2cp2}[\mathrm{j\_},5/2]]=$

$\mathrm{MakeOPE}[\{\mathrm{Delta}[i,j]4\mathrm{NewPhi2cp1}[2],$

$\mathrm{Delta}[i,j]\mathrm{NewPhi2cp1}[2]'+$

$\left.\left.i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]\mathrm{NewPhi2cp3}[\mathrm{x1},3]\right\}\right];$

$\mathrm{OPE}[G[\mathrm{i\_}],\mathrm{NewPhi2cp3}[\mathrm{j\_},3]]=$

$\mathrm{MakeOPE}\left[\left\{5i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]\mathrm{NewPhi2cp2}[\mathrm{x1},5/2],\right.\right.$

$2\mathrm{Delta}[i,j]\mathrm{NewPhi2cp4}[7/2]+$

$\left.\left.i\sum_{\mathrm{x1}=1}^{3}\mathrm{Epsilon}[i,j,\mathrm{x1}]\mathrm{NewPhi2cp2}[\mathrm{x1},5/2]'\right\}\right];$

$\mathrm{OPE}[G[\mathrm{i\_}],\mathrm{NewPhi2cp4}[7/2]]=$

MakeOPE[{3NewPhi2cp3[i, 3], (1/2)NewPhi2cp3[i, 3]′}];

OPE[J[i_], NewPhi2cp2[j_, 5/2]] =

MakeOPE $\left[\left\{i \sum_{\text{x1}=1}^{3} \text{Epsilon}[i, j, \text{x1}]\text{NewPhi2cp2}[\text{x1}, 5/2]\right\}\right]$;

OPE[J[i_], NewPhi2cp3[j_, 3]] =

MakeOPE[{4Delta[i, j]NewPhi2cp1[2],

$i \sum_{\text{x1}=1}^{3} \text{Epsilon}[i, j, \text{x1}]\text{NewPhi2cp3}[\text{x1}, 3]\}\Big]$;

OPE[J[i_], NewPhi2cp4[7/2]] = MakeOPE[{(5/2)NewPhi2cp2[i, 5/2], 0}];

OPE[Ψ, NewPhi2cp3[i_, 3]] = MakeOPE[{NewPhi2cp2[i, 5/2]}];

OPE[Ψ, NewPhi2cp4[7/2]] =

MakeOPE[{2NewPhi2cp1[2], −(1/2)NewPhi2cp1[2]′}];

$J\bar{\Phi}^{\left(\frac{5}{2}\right),a}$

OPE[Ts, NewPhi5Hcp1[5/2, {a_}]] =

MakeOPE[{(5/2)NewPhi5Hcp1[5/2, {a}], NewPhi5Hcp1[5/2, {a}]′}];

OPE[Ts, NewPhi5Hcp2[i_, 3, {a_}]] =

MakeOPE[{3NewPhi5Hcp2[i, 3, {a}], NewPhi5Hcp2[i, 3, {a}]′}];

OPE[Ts, NewPhi5Hcp3[i_, 7/2, {a_}]] =

MakeOPE $\Big[\Big\{- \sum_{b=1}^{3}(\text{Tmatirx}[i, a, b]\text{NewPhi5Hcp1}[5/2, \{b\}])$,

(7/2)NewPhi5Hcp3[i, 7/2, {a}], NewPhi5Hcp3[i, 7/2, {a}]′}];

OPE[Ts, NewPhi5Hcp4[4, {a_}]] =

MakeOPE $\Big[\Big\{-(1/2) \sum_{b=1}^{3} \left(\sum_{\text{id1}=1}^{3}(\text{Tmatirx}[\text{id1}, a, b]\text{NewPhi5Hcp2}[\text{id1}, 3, \{b\}])\right)$,

4NewPhi5Hcp4[4, {a}], NewPhi5Hcp4[4, {a}]′}];

OPE[G[id_], NewPhi5Hcp1[5/2, {a_}]] = MakeOPE[{NewPhi5Hcp2[id, 3, {a}]}];

OPE[G[id1_], NewPhi5Hcp2[id2_, 3, {a_}]] =

MakeOPE[{5Delta[id1, id2]NewPhi5Hcp1[5/2, {a}]−

i

$\sum_{\text{x1}=1}^{3} \Big(\sum_{b=1}^{3} \text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{Tmatirx}[\text{x1}, a, b]$

NewPhi5Hcp1[5/2, {b}]),

Delta[id1, id2]NewPhi5Hcp1[5/2, {a}]′+

$i \sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{NewPhi5Hcp3}[\text{x1}, 7/2, \{a\}])\Big\}\Big]$;

$\text{OPE}[G[\text{id1\_}], \text{NewPhi5Hcp3}[\text{id2\_}, 7/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}\left[\left\{6i\sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{NewPhi5Hcp2}[\text{x1}, 3, \{a\}])+\right.\right.$

$\sum_{b=1}^{3}\text{Tmatirx}[\text{id2}, a, b]\text{NewPhi5Hcp2}[\text{id1}, 3, \{b\}]-$

$\sum_{b=1}^{3}\left(\sum_{\text{x1}=1}^{3}\text{Delta}[\text{id1}, \text{id2}](\text{Tmatirx}[\text{x1}, a, b]\text{NewPhi5Hcp2}[\text{x1}, 3, \{b\}])\right),$

$2\text{Delta}[\text{id1}, \text{id2}]\text{NewPhi5Hcp4}[4, \{a\}]+$

$\left.\left. i\sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{NewPhi5Hcp2}[\text{x1}, 3, \{a\}]')\right\}\right];$

$\text{OPE}[G[\text{id1\_}], \text{NewPhi5Hcp4}[4, \{\text{a\_}\}]] =$

$\text{MakeOPE}\left[\left\{-\sum_{b=1}^{3}\text{Tmatirx}[\text{id1}, a, b]\text{NewPhi5Hcp1}[5/2, \{b\}],\right.\right.$

$(7/2)\text{NewPhi5Hcp3}[\text{id1}, 7/2, \{a\}]+$

$(i/2)$

$\sum_{b=1}^{3}\left(\sum_{\text{x1}=1}^{3}\left(\sum_{\text{x2}=1}^{3}\text{Epsilon}[\text{id1}, \text{x1}, \text{x2}]\right.\right.$

$\left.\left.(\text{Tmatirx}[\text{x1}, a, b]\text{NewPhi5Hcp3}[\text{x2}, 7/2, \{b\}])\right)\right),$

$(1/2)\text{NewPhi5Hcp3}[\text{id1}, 7/2, \{a\}]'\}];$

$\text{OPE}[J[\text{id1\_}], \text{NewPhi5Hcp1}[5/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}\left[\left\{-\sum_{b=1}^{3}\text{Tmatirx}[\text{id1}, a, b]\text{NewPhi5Hcp1}[5/2, \{b\}]\right\}\right];$

$\text{OPE}[J[\text{id1\_}], \text{NewPhi5Hcp2}[\text{id2\_}, 3, \{\text{a\_}\}]] =$

$\text{MakeOPE}\left[\left\{i\left(\sum_{\text{x1}=1}^{3}\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}](\text{NewPhi5Hcp2}[\text{x1}, 3, \{a\}])\right)-\right.\right.$

$\left.\left.\sum_{b=1}^{3}\text{Tmatirx}[\text{id1}, a, b]\text{NewPhi5Hcp2}[\text{id2}, 3, \{b\}]\right\}\right];$

$\text{OPE}[J[\text{id1\_}], \text{NewPhi5Hcp3}[\text{id2\_}, 7/2, \{\text{a\_}\}]] =$

$\text{MakeOPE}[\{5\text{Delta}[\text{id1}, \text{id2}]\text{NewPhi5Hcp1}[5/2, \{a\}]-$

$i\sum_{\text{x1}=1}^{3}\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]$

$\left(\sum_{b=1}^{3}\text{Tmatirx}[\text{x1}, a, b]\text{NewPhi5Hcp1}[5/2, \{b\}]\right),$

$i\sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1}, \text{id2}, \text{x1}]\text{NewPhi5Hcp3}[\text{x1}, 7/2, \{a\}])-$

$\left.\left.\left(\sum_{b=1}^{3}\text{Tmatirx}[\text{id1}, a, b]\text{NewPhi5Hcp3}[\text{id2}, 7/2, \{b\}]\right)\right\}\right];$

$\text{OPE}[J[\text{id1\_}], \text{NewPhi5Hcp4}[4, \{\text{a\_}\}]] =$

$\text{MakeOPE}[$

$\{3\text{NewPhi5Hcp2}[\text{id1}, 3, \{a\}]+$

$(i/2)$

$\sum_{b=1}^{3}\left(\sum_{\text{x1}=1}^{3}\left(\sum_{\text{x2}=1}^{3}\text{Epsilon}[\text{id1}, \text{x1}, \text{x2}]\right.\right.$

$\left.\left.(\text{Tmatirx}[\text{x1}, a, b]\text{NewPhi5Hcp2}[\text{x2}, 3, \{b\}])\right)\right),$

$-\sum_{b=1}^{3}$ Tmatirx[id1, a, b]NewPhi5Hcp4[4, {b}]}] ;

OPE[Ψ, NewPhi5Hcp2[id1_, 3, {a_}]] =

MakeOPE [{$-\sum_{b=1}^{3}$ Tmatirx[id1, a, b]NewPhi5Hcp1[5/2, {b}]}] ;

OPE[Ψ, NewPhi5Hcp3[id1_, 7/2, {a_}]] =

MakeOPE[

{NewPhi5Hcp2[id1, 3, {a}]+

i

$\sum_{b=1}^{3}$ ($\sum_{x1=1}^{3}$ ($\sum_{x2=1}^{3}$ Epsilon[id1, x1, x2]

(Tmatirx[x1, a, b]NewPhi5Hcp2[x2, 3, {b}])))}];

OPE[Ψ, NewPhi5Hcp4[4, {a_}]] =

MakeOPE[{(5/2)NewPhi5Hcp1[5/2, {a}],

−(1/2)NewPhi5Hcp1[5/2, {a}]′−

(1/2) $\sum_{id1=1}^{3}$ ($\sum_{b=1}^{3}$ Tmatirx[id1, a, b]NewPhi5Hcp3[id1, 7/2, {b}])}] ;

$J \bar{\Phi}^{(3),a}$

OPE[Ts, NewPhi3cp1[3, {a_}]] =

MakeOPE[{3NewPhi3cp1[3, {a}], NewPhi3cp1[3, {a}]′}];

OPE[Ts, NewPhi3cp2[i_, 7/2, {a_}]] =

MakeOPE[{(7/2)NewPhi3cp2[i, 7/2, {a}], NewPhi3cp2[i, 7/2, {a}]′}];

OPE[Ts, NewPhi3cp3[i_, 4, {a_}]] =

MakeOPE [{$-\sum_{b=1}^{3}$(Tmatirx[i, a, b]NewPhi3cp1[3, {b}]),

4NewPhi3cp3[i, 4, {a}], NewPhi3cp3[i, 4, {a}]′}];

OPE[Ts, NewPhi3cp4[9/2, {a_}]] =

MakeOPE[

{−(1/2) $\sum_{b=1}^{3}$ ($\sum_{id1=1}^{3}$(Tmatirx[id1, a, b]NewPhi3cp2[id1, 7/2, {b}])) ,

(9/2)NewPhi3cp4[9/2, {a}], NewPhi3cp4[9/2, {a}]′}];

OPE[G[id_], NewPhi3cp1[3, {a_}]] = MakeOPE[{NewPhi3cp2[id, 7/2, {a}]}];

OPE[G[id1_], NewPhi3cp2[id2_, 7/2, {a_}]] =

MakeOPE[

{6Delta[id1, id2]NewPhi3cp1[3, {a}]−

$$i \sum_{\text{x1}=1}^{3} \left(\sum_{b=1}^{3} \text{Epsilon[id1, id2, x1]Tmatirx[x1}, a, b\text{]NewPhi3cp1[3}, \{b\}] \right),$$

$$\text{Delta[id1, id2]NewPhi3cp1[3}, \{a\}]' +$$

$$i \sum_{\text{x1}=1}^{3} (\text{Epsilon[id1, id2, x1]NewPhi3cp3[x1, 4}, \{a\}]) \Big\} \Big];$$

$$\text{OPE}[G\text{[id1\_], NewPhi3cp3[id2\_, 4, \{a\_\}]]} =$$

$$\text{MakeOPE} \Big[\Big\{ 7i \sum_{\text{x1}=1}^{3} (\text{Epsilon[id1, id2, x1]NewPhi3cp2[x1, 7/2}, \{a\}]) +$$

$$\sum_{b=1}^{3} \text{Tmatirx[id2}, a, b\text{]NewPhi3cp2[id1, 7/2}, \{b\}] -$$

$$\sum_{b=1}^{3} \left(\sum_{\text{x1}=1}^{3} \text{Delta[id1, id2]} \right.$$

$$(\text{Tmatirx[x1}, a, b\text{]NewPhi3cp2[x1, 7/2}, \{b\}])),$$

$$\text{2Delta[id1, id2]NewPhi3cp4[9/2}, \{a\}] +$$

$$i \sum_{\text{x1}=1}^{3} (\text{Epsilon[id1, id2, x1]NewPhi3cp2[x1, 7/2}, \{a\}]') \Big\} \Big];$$

$$\text{OPE}[G\text{[id1\_], NewPhi3cp4[9/2, \{a\_\}]]} =$$

$$\text{MakeOPE} \Big[\Big\{ - \sum_{b=1}^{3} \text{Tmatirx[id1}, a, b\text{]NewPhi3cp1[3}, \{b\}],$$

$$\text{4NewPhi3cp3[id1, 4}, \{a\}] +$$

$$(i/2)$$

$$\sum_{b=1}^{3} \left(\sum_{\text{x1}=1}^{3} \left(\sum_{\text{x2}=1}^{3} \text{Epsilon[id1, x1, x2]} \right. \right.$$

$$(\text{Tmatirx[x1}, a, b\text{]NewPhi3cp3[x2, 4}, \{b\}]))),$$

$$\text{(1/2)NewPhi3cp3[id1, 4}, \{a\}]' \}];$$

$$\text{OPE}[J\text{[id1\_], NewPhi3cp1[3, \{a\_\}]]} =$$

$$\text{MakeOPE} \Big[\Big\{ - \sum_{b=1}^{3} \text{Tmatirx[id1}, a, b\text{]NewPhi3cp1[3}, \{b\}] \Big\} \Big];$$

$$\text{OPE}[J\text{[id1\_], NewPhi3cp2[id2\_, 7/2, \{a\_\}]]} =$$

$$\text{MakeOPE} \Big[\Big\{ i \left(\sum_{\text{x1}=1}^{3} \text{Epsilon[id1, id2, x1](NewPhi3cp2[x1, 7/2}, \{a\}]) \right) -$$

$$\sum_{b=1}^{3} \text{Tmatirx[id1}, a, b\text{]NewPhi3cp2[id2, 7/2}, \{b\}] \Big\} \Big];$$

$$\text{OPE}[J\text{[id1\_], NewPhi3cp3[id2\_, 4, \{a\_\}]]} =$$

$$\text{MakeOPE[}$$

$$\{\text{6Delta[id1, id2]NewPhi3cp1[3}, \{a\}] -$$

$$i \sum_{\text{x1}=1}^{3} \text{Epsilon[id1, id2, x1]} \left(\sum_{b=1}^{3} \text{Tmatirx[x1}, a, b\text{]NewPhi3cp1[3}, \{b\}] \right),$$

$$i \sum_{\text{x1}=1}^{3} (\text{Epsilon[id1, id2, x1]NewPhi3cp3[x1, 4}, \{a\}]) -$$

$$\left(\sum_{b=1}^{3} \text{Tmatirx[id1}, a, b\text{]NewPhi3cp3[id2, 4}, \{b\}] \right) \Big\} \Big];$$

$$\text{OPE}[J\text{[id1\_], NewPhi3cp4[9/2, \{a\_\}]]} =$$

MakeOPE[

$\{(7/2)\mathrm{NewPhi3cp2}[\mathrm{id1}, 7/2, \{a\}]+$

$(i/2)$

$\sum_{b=1}^{3}\left(\sum_{\mathrm{x1}=1}^{3}\left(\sum_{\mathrm{x2}=1}^{3}\mathrm{Epsilon}[\mathrm{id1}, \mathrm{x1}, \mathrm{x2}]\right.\right.$

$(\mathrm{Tmatirx}[\mathrm{x1}, a, b]\mathrm{NewPhi3cp2}[\mathrm{x2}, 7/2, \{b\}]))),$

$\left.\left.-\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id1}, a, b]\mathrm{NewPhi3cp4}[9/2, \{b\}]\right\}\right];$

$\mathrm{OPE}[\Psi, \mathrm{NewPhi3cp2}[\mathrm{id1\_}, 7/2, \{\mathrm{a\_}\}]] =$

$\mathrm{MakeOPE}\left[\left\{-\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id1}, a, b]\mathrm{NewPhi3cp1}[3, \{b\}]\right\}\right];$

$\mathrm{OPE}[\Psi, \mathrm{NewPhi3cp3}[\mathrm{id1\_}, 4, \{\mathrm{a\_}\}]] =$

MakeOPE[

$\{\mathrm{NewPhi3cp2}[\mathrm{id1}, 7/2, \{a\}]+$

i

$\sum_{b=1}^{3}\left(\sum_{\mathrm{x1}=1}^{3}\left(\sum_{\mathrm{x2}=1}^{3}\mathrm{Epsilon}[\mathrm{id1}, \mathrm{x1}, \mathrm{x2}]\right.\right.$

$(\mathrm{Tmatirx}[\mathrm{x1}, a, b]\mathrm{NewPhi3cp2}[\mathrm{x2}, 7/2, \{b\}])))\}];$

$\mathrm{OPE}[\Psi, \mathrm{NewPhi3cp4}[9/2, \{\mathrm{a\_}\}]] =$

$\mathrm{MakeOPE}[\{3\mathrm{NewPhi3cp1}[3, \{a\}],$

$-(1/2)\mathrm{NewPhi3cp1}[3, \{a\}]'-$

$\left.\left.(1/2)\sum_{\mathrm{id1}=1}^{3}\left(\sum_{b=1}^{3}\mathrm{Tmatirx}[\mathrm{id1}, a, b]\mathrm{NewPhi3cp3}[\mathrm{id1}, 4, \{b\}]\right)\right\}\right];$

$J\tilde{\Phi}^{(3)}$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{NNewPhi3cp1}[3]] = \mathrm{MakeOPE}[\{3\mathrm{NNewPhi3cp1}[3], \mathrm{NNewPhi3cp1}[3]'\}];$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{NNewPhi3cp2}[\mathrm{i\_}, 7/2]] =$

$\mathrm{MakeOPE}[\{(7/2)\mathrm{NNewPhi3cp2}[i, 7/2], \mathrm{NNewPhi3cp2}[i, 7/2]'\}];$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{NNewPhi3cp3}[\mathrm{i\_}, 4]] =$

$\mathrm{MakeOPE}[\{4\mathrm{NNewPhi3cp3}[i, 4], \mathrm{NNewPhi3cp3}[i, 4]'\}];$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{NNewPhi3cp4}[9/2]] =$

$\mathrm{MakeOPE}[\{(9/2)\mathrm{NNewPhi3cp4}[9/2], \mathrm{NNewPhi3cp4}[9/2]'\}];$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{NNewPhi3cp1}[3]] = \mathrm{MakeOPE}[\{\mathrm{NNewPhi3cp2}[i, 7/2]\}];$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{NNewPhi3cp2}[\mathrm{j\_}, 7/2]] =$

$\mathrm{MakeOPE}[\{\mathrm{Delta}[i, j] 6 \mathrm{NNewPhi3cp1}[3],$

$\mathrm{Delta}[i, j]\mathrm{NNewPhi3cp1}[3]' +$

$i \sum_{\mathrm{x1}=1}^{3} \mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{NNewPhi3cp3}[\mathrm{x1}, 4]\Big\}\Big]\,;$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{NNewPhi3cp3}[\mathrm{j\_}, 4]] =$

$\mathrm{MakeOPE}\Big[\Big\{7i \sum_{\mathrm{x1}=1}^{3} \mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{NNewPhi3cp2}[\mathrm{x1}, 7/2],$

$2\mathrm{Delta}[i, j]\mathrm{NNewPhi3cp4}[9/2] +$

$i \sum_{\mathrm{x1}=1}^{3} \mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{NNewPhi3cp2}[\mathrm{x1}, 7/2]'\Big\}\Big]\,;$

$\mathrm{OPE}[G[\mathrm{i\_}], \mathrm{NNewPhi3cp4}[9/2]] =$

$\mathrm{MakeOPE}[\{4\mathrm{NNewPhi3cp3}[i, 4], (1/2)\mathrm{NNewPhi3cp3}[i, 4]'\}];$

$\mathrm{OPE}[J[\mathrm{i\_}], \mathrm{NNewPhi3cp2}[\mathrm{j\_}, 7/2]] =$

$\mathrm{MakeOPE}\Big[\Big\{i \sum_{\mathrm{x1}=1}^{3} \mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{NNewPhi3cp2}[\mathrm{x1}, 7/2]\Big\}\Big]\,;$

$\mathrm{OPE}[J[\mathrm{i\_}], \mathrm{NNewPhi3cp3}[\mathrm{j\_}, 4]] =$

$\mathrm{MakeOPE}[\{6\mathrm{Delta}[i, j]\mathrm{NNewPhi3cp1}[3],$

$i \sum_{\mathrm{x1}=1}^{3} \mathrm{Epsilon}[i, j, \mathrm{x1}]\mathrm{NNewPhi3cp3}[\mathrm{x1}, 4]\Big\}\Big]\,;$

$\mathrm{OPE}[J[\mathrm{i\_}], \mathrm{NNewPhi3cp4}[9/2]] = \mathrm{MakeOPE}[\{(7/2)\mathrm{NNewPhi3cp2}[i, 7/2], 0\}];$

$\mathrm{OPE}[\Psi, \mathrm{NNewPhi3cp3}[\mathrm{i\_}, 4]] = \mathrm{MakeOPE}[\{\mathrm{NNewPhi3cp2}[i, 7/2]\}];$

$\mathrm{OPE}[\Psi, \mathrm{NNewPhi3cp4}[9/2]] =$

$\mathrm{MakeOPE}[\{3\mathrm{NNewPhi3cp1}[3], -(1/2)\mathrm{NNewPhi3cp1}[3]'\}];$

$J\bar{\Phi}^{(\frac{7}{2}),a}$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{NewPhi7Hcp1}[7/2, \{\mathrm{a\_}\}]] =$

$\mathrm{MakeOPE}[\{(7/2)\mathrm{NewPhi7Hcp1}[7/2, \{a\}], \mathrm{NewPhi7Hcp1}[7/2, \{a\}]'\}];$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{NewPhi7Hcp2}[\mathrm{i\_}, 4, \{\mathrm{a\_}\}]] =$

$\mathrm{MakeOPE}[\{4\mathrm{NewPhi7Hcp2}[i, 4, \{a\}], \mathrm{NewPhi7Hcp2}[i, 4, \{a\}]'\}];$

$\mathrm{OPE}[\mathrm{Ts}, \mathrm{NewPhi7Hcp3}[\mathrm{i\_}, 9/2, \{\mathrm{a\_}\}]] =$

MakeOPE $\left[\left\{-\sum_{b=1}^{3}(\text{Tmatirx}[i,a,b]\text{NewPhi7Hcp1}[7/2,\{b\}]),\right.\right.$

$(9/2)\text{NewPhi7Hcp3}[i,9/2,\{a\}],\text{NewPhi7Hcp3}[i,9/2,\{a\}]'\}];$

OPE[Ts, NewPhi7Hcp4[5, {a_}]] =

MakeOPE $\left[\left\{-(1/2)\sum_{b=1}^{3}\left(\sum_{\text{id1}=1}^{3}(\text{Tmatirx}[\text{id1},a,b]\text{NewPhi7Hcp2}[\text{id1},4,\{b\}])\right),\right.\right.$

$5\text{NewPhi7Hcp4}[5,\{a\}],\text{NewPhi7Hcp4}[5,\{a\}]'\}];$

OPE[G[id_], NewPhi7Hcp1[7/2, {a_}]] = MakeOPE[{NewPhi7Hcp2[id, 4, {a}]}];

OPE[G[id1_], NewPhi7Hcp2[id2_, 4, {a_}]] =

MakeOPE[{7Delta[id1, id2]NewPhi7Hcp1[7/2, {a}]−

i

$\sum_{\text{x1}=1}^{3}\left(\sum_{b=1}^{3}\text{Epsilon}[\text{id1},\text{id2},\text{x1}]\text{Tmatirx}[\text{x1},a,b]\right.$

NewPhi7Hcp1[7/2, {b}]),

Delta[id1, id2]NewPhi7Hcp1[7/2, {a}]′+

$\left.\left.i\sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1},\text{id2},\text{x1}]\text{NewPhi7Hcp3}[\text{x1},9/2,\{a\}])\right\}\right];$

OPE[G[id1_], NewPhi7Hcp3[id2_, 9/2, {a_}]] =

MakeOPE $\left[\left\{8i\sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1},\text{id2},\text{x1}]\text{NewPhi7Hcp2}[\text{x1},4,\{a\}])+\right.\right.$

$\sum_{b=1}^{3}\text{Tmatirx}[\text{id2},a,b]\text{NewPhi7Hcp2}[\text{id1},4,\{b\}]-$

$\sum_{b=1}^{3}\left(\sum_{\text{x1}=1}^{3}\text{Delta}[\text{id1},\text{id2}](\text{Tmatirx}[\text{x1},a,b]\text{NewPhi7Hcp2}[\text{x1},4,\{b\}])\right),$

2Delta[id1, id2]NewPhi7Hcp4[5, {a}]+

$\left.\left.i\sum_{\text{x1}=1}^{3}(\text{Epsilon}[\text{id1},\text{id2},\text{x1}]\text{NewPhi7Hcp2}[\text{x1},4,\{a\}]')\right\}\right];$

OPE[G[id1_], NewPhi7Hcp4[5, {a_}]] =

MakeOPE $\left[\left\{-\sum_{b=1}^{3}\text{Tmatirx}[\text{id1},a,b]\text{NewPhi7Hcp1}[7/2,\{b\}],\right.\right.$

(9/2)NewPhi7Hcp3[id1, 9/2, {a}]+

(i/2)

$\sum_{b=1}^{3}\left(\sum_{\text{x1}=1}^{3}\left(\sum_{\text{x2}=1}^{3}\text{Epsilon}[\text{id1},\text{x1},\text{x2}]\right.\right.$

(Tmatirx[x1, a, b]NewPhi7Hcp3[x2, 9/2, {b}]))),

(1/2)NewPhi7Hcp3[id1, 9/2, {a}]′}];

OPE[J[id1_], NewPhi7Hcp1[7/2, {a_}]] =

MakeOPE $\left[\left\{-\sum_{b=1}^{3}\text{Tmatirx}[\text{id1},a,b]\text{NewPhi7Hcp1}[7/2,\{b\}]\right\}\right];$

OPE[J[id1_], NewPhi7Hcp2[id2_, 4, {a_}]] =

$\text{MakeOPE}\left[\left\{i\left(\sum_{\text{x1}=1}^{3}\text{Epsilon[id1, id2, x1](NewPhi7Hcp2[x1, 4, }\{a\}])\right)-\right.\right.$
$\left.\left.\sum_{b=1}^{3}\text{Tmatirx[id1, }a, b\text{]NewPhi7Hcp2[id2, 4, }\{b\}]\right\}\right];$

$\text{OPE}[J\text{[id1\_], NewPhi7Hcp3[id2\_, 9/2, \{a\_\}]]} =$

$\text{MakeOPE[\{7Delta[id1, id2]NewPhi7Hcp1[7/2, }\{a\}]-$

$i\sum_{\text{x1}=1}^{3}\text{Epsilon[id1, id2, x1]}$

$\left(\sum_{b=1}^{3}\text{Tmatirx[x1, }a, b\text{]NewPhi7Hcp1[7/2, }\{b\}]\right),$

$i\sum_{\text{x1}=1}^{3}\text{(Epsilon[id1, id2, x1]NewPhi7Hcp3[x1, 9/2, }\{a\}])-$

$\left.\left.\left(\sum_{b=1}^{3}\text{Tmatirx[id1, }a, b\text{]NewPhi7Hcp3[id2, 9/2, }\{b\}]\right)\right\}\right];$

$\text{OPE}[J\text{[id1\_], NewPhi7Hcp4[5, \{a\_\}]]} =$

MakeOPE[

$\{\text{4NewPhi7Hcp2[id1, 4, }\{a\}]+$

$(i/2)$

$\sum_{b=1}^{3}\left(\sum_{\text{x1}=1}^{3}\left(\sum_{\text{x2}=1}^{3}\text{Epsilon[id1, x1, x2]}\right.\right.$

$\text{(Tmatirx[x1, }a, b\text{]NewPhi7Hcp2[x2, 4, }\{b\}]))),$

$\left.\left.-\sum_{b=1}^{3}\text{Tmatirx[id1, }a, b\text{]NewPhi7Hcp4[5, }\{b\}]\right\}\right];$

$\text{OPE}[\Psi\text{, NewPhi7Hcp2[id1\_, 4, \{a\_\}]]} =$

$\text{MakeOPE}\left[\left\{-\sum_{b=1}^{3}\text{Tmatirx[id1, }a, b\text{]NewPhi7Hcp1[7/2, }\{b\}]\right\}\right];$

$\text{OPE}[\Psi\text{, NewPhi7Hcp3[id1\_, 9/2, \{a\_\}]]} =$

MakeOPE[

$\{\text{NewPhi7Hcp2[id1, 4, }\{a\}]+$

i

$\sum_{b=1}^{3}\left(\sum_{\text{x1}=1}^{3}\left(\sum_{\text{x2}=1}^{3}\text{Epsilon[id1, x1, x2]}\right.\right.$

$\text{(Tmatirx[x1, }a, b\text{]NewPhi7Hcp2[x2, 4, }\{b\}])))\}];$

$\text{OPE}[\Psi\text{, NewPhi7Hcp4[5, \{a\_\}]]} =$

$\text{MakeOPE[\{(7/2)NewPhi7Hcp1[7/2, }\{a\}],$

$-\text{(1/2)NewPhi7Hcp1[7/2, }\{a\}]'-$

$\left.\left.(1/2)\sum_{\text{id1}=1}^{3}\left(\sum_{b=1}^{3}\text{Tmatirx[id1, }a, b\text{]NewPhi7Hcp3[id1, 9/2, }\{b\}]\right)\right\}\right];$

$J\bar{\Phi}^{(4)}$

OPE[Ts, NewPhi4cp1[4]] = MakeOPE[{4NewPhi4cp1[4], NewPhi4cp1[4]′}];

OPE[Ts, NewPhi4cp2[i_, 9/2]] =

MakeOPE[{(9/2)NewPhi4cp2[*i*, 9/2], NewPhi4cp2[*i*, 9/2]′}];

OPE[Ts, NewPhi4cp3[i_, 5]] =

MakeOPE[{5NewPhi4cp3[*i*, 5], NewPhi4cp3[*i*, 5]′}];

OPE[Ts, NewPhi4cp4[11/2]] =

MakeOPE[{(11/2)NewPhi4cp4[11/2], NewPhi4cp4[11/2]′}];

OPE[*G*[i_], NewPhi4cp1[4]] = MakeOPE[{NewPhi4cp2[*i*, 9/2]}];

OPE[*G*[i_], NewPhi4cp2[j_, 9/2]] =

MakeOPE[{Delta[*i*, *j*]8NewPhi4cp1[4],

Delta[*i*, *j*]NewPhi4cp1[4]′+

$i\sum_{x1=1}^{3}$ Epsilon[*i*, *j*, x1]NewPhi4cp3[x1, 5]}];

OPE[*G*[i_], NewPhi4cp3[j_, 5]] =

MakeOPE[{9$i\sum_{x1=1}^{3}$ Epsilon[*i*, *j*, x1]NewPhi4cp2[x1, 9/2],

2Delta[*i*, *j*]NewPhi4cp4[11/2]+

$i\sum_{x1=1}^{3}$ Epsilon[*i*, *j*, x1]NewPhi4cp2[x1, 9/2]′}];

OPE[*G*[i_], NewPhi4cp4[11/2]] =

MakeOPE[{5NewPhi4cp3[*i*, 5], (1/2)NewPhi4cp3[*i*, 5]′}];

OPE[*J*[i_], NewPhi4cp2[j_, 9/2]] =

MakeOPE[{$i\sum_{x1=1}^{3}$ Epsilon[*i*, *j*, x1]NewPhi4cp2[x1, 9/2]}];

OPE[*J*[i_], NewPhi4cp3[j_, 5]] =

MakeOPE[{8Delta[*i*, *j*]NewPhi4cp1[4],

$i\sum_{x1=1}^{3}$ Epsilon[*i*, *j*, x1]NewPhi4cp3[x1, 5]}];

OPE[*J*[i_], NewPhi4cp4[11/2]] = MakeOPE[{(9/2)NewPhi4cp2[*i*, 9/2], 0}];

OPE[Ψ, NewPhi4cp3[i_, 5]] = MakeOPE[{NewPhi4cp2[*i*, 9/2]}];

OPE[Ψ, NewPhi4cp4[11/2]] =

MakeOPE[{4NewPhi4cp1[4], −(1/2)NewPhi4cp1[4]′}];

A.2 The package for $\mathscr{N}$=3 superspace formalism

Delta and Epsilon from Thielemans: [15]

```
Clear[Delta]

Delta[a_, a_] = 1

Delta[i_, j_]:=0/;i!=j

SetAttributes[Delta, {Orderless}];

1

Epsilon[___, a_, ___, a_, ___] = 0;

Epsilon[a__]:=Signature[{a}]Apply[Epsilon, Sort[{a}]]/;
 !OrderedQ[{a}]

Epsilon[x__]:=1/;{x}==Range[Length[{x}]]

Clear[EpsilonRule]

EpsilonRule[n_] = Literal[Epsilon[ints : (_Integer).., rest : (_)..]]:>
 Block[{remlist =
   Permutations[
 Complement[Range[Length[{ints}] + Length[{rest}]], {ints}]
   ]
   },
 Apply[Plus,
```

```
    Function[rem,
      Epsilon[ints, Sequence@@rem]*
    Inner[Delta, {rest}, rem, Times]
    ]/@remlist
   ]
 ]/;small[{ints, rest}, Length[{ints, rest}]]&&
 Length[{rest}]<=n;
(*smalltestsifanyoftheargumentsislargerthanthetotallength.
 Note : donottestonints}alone, otherwisethelargeiwillbe
 shiftedinto'rest'.
*)

small[l_List, n_Integer]:=
 And@@Map[#<=n&, Select[l, IntegerQ]]
```

.

ComponentOPE of SuperOPE

The portion of Grassman Variable rule.

```
Clear[CGSign]
CGSign[v1_Integer, v2_Integer]:=1
CGSign[v1_Integer, v2_Integer]:= - 1/;OddQ[v1]&&OddQ[v2]
Clear[RLT]
RLT[A_List]:=Length[A]
RLT[{ZR}]:=2
Clear[BElz]
BElz[x1__]:=Delete[{x1}, Position[{x1}, ZR]]
Clear[Elz]
Elz[x1__]:=BElz[x1]/;BElz[x1] ≠ {}
Elz[x1__] = {ZR};
```

```
Clear[CθQ]

CθQ[Cθ[A_List, B_List]] = True
```

True

```
Clear[GrM]


HoldPattern[GrM[A___, B_ + C_, D___]]:=
 GrM[A, B, D] + GrM[A, C, D]


HoldPattern[GrM[A___, s_B_, C___]]:=
 sGrM[A, B, C]/;CθQ[B]&&NumberQ[s]


GrM[Cθ[{x1__}, {y1__}], Cθ[{x2__}, {y2__}]]:=
CGSign[RLT[Elz[y1]], RLT[Elz[x2]]]Cθ[Elz[x1, x2], Elz[y1, y2]]
GrM[Cθ[{x1__}, {y1__}], 0]:=Cθ[{x1}, {y1}]
GrM[0, Cθ[{x1__}, {y1__}]]:=Cθ[{x1}, {y1}]
```

Cθ[{Z1},{Z2}]

Z1 grassman component

Z2 grassman component

```
Chcg1[1, 0]:=Cθ[{ZR}, {ZR}]
Chcg2[1, 0] = Chcg1[1, 0];


Chcg1[2, i_]:=Cθ[{i}, {ZR}]
Chcg2[2, i_]:=Cθ[{ZR}, {i}]


Chcg1[3, 1] = Cθ[{2, 3}, {ZR}];
Chcg1[3, 2] = Cθ[{3, 1}, {ZR}];
Chcg1[3, 3] = Cθ[{1, 2}, {ZR}];
```

$\text{Chcg2}[3,1] = \text{C}\theta[\{\text{ZR}\},\{2,3\}];$

$\text{Chcg2}[3,2] = \text{C}\theta[\{\text{ZR}\},\{3,1\}];$

$\text{Chcg2}[3,3] = \text{C}\theta[\{\text{ZR}\},\{1,2\}];$

$\text{Chcg1}[4,0] = \text{C}\theta[\{1,2,3\},\{\text{ZR}\}];$

$\text{Chcg2}[4,0] = \text{C}\theta[\{\text{ZR}\},\{1,2,3\}];$

Clear[BLHSign]

OddQ EvenQ

EvenQOddQ

$\text{BLHSign}[\text{co1}\_,\text{co2}\_] = 2;$

BLHSign[co1_, co2_]:=1/;OddQ[co1]&&EvenQ[co2]

Clear[LHSign]

$\text{LHSign}[\text{co1}\_,\text{co2}\_]{:=}(-1)^{\text{BLHSign}[\text{co1},\text{co2}]}$

(θ_12)^(3-i)

Clear[B1SExpension]

B1SExpension[3, i_]:=

$\sum_{j=1}^{3}\left(\sum_{\text{kay}=1}^{3}(\text{Epsilon}[i,j,\text{kay}]\text{GrM}[\text{C}\theta[\{j\},\{\text{ZR}\}]-\text{C}\theta[\{\text{ZR}\},\{j\}],\right.$
$\text{C}\theta[\{\text{kay}\},\{\text{ZR}\}]-\text{C}\theta[\{\text{ZR}\},\{\text{kay}\}]]))$

$\text{B1SExpension}[2,\text{i}\_]{:=}\text{C}\theta[\{i\},\{\text{ZR}\}]-\text{C}\theta[\{\text{ZR}\},\{i\}]$

Clear[B2SExpension]

B2SExpension[3, 1]:=

$\text{GrM}\left[\text{B1SExpension}[3,1],\sum_{i=1}^{3}\text{GrM}[\text{C}\theta[\{i\},\{\text{ZR}\}],\text{C}\theta[\{\text{ZR}\},\{i\}]]\right]$

B2SExpension[3, 2]:=

GrM $\left[\text{B1SExpension}[3,2], \sum_{i=1}^{3} \text{GrM}[\text{C}\theta[\{i\},\{\text{ZR}\}], \text{C}\theta[\{\text{ZR}\},\{i\}]]\right]$

B2SExpension[3, 3]:=

GrM $\left[\text{B1SExpension}[3,3], \sum_{i=1}^{3} \text{GrM}[\text{C}\theta[\{i\},\{\text{ZR}\}], \text{C}\theta[\{\text{ZR}\},\{i\}]]\right]$

B2SExpension[2, i_]:=

GrM $\left[\text{B1SExpension}[2,i], \sum_{\text{b2e}=1}^{3} \text{GrM}[\text{C}\theta[\{\text{b2e}\},\{\text{ZR}\}], \text{C}\theta[\{\text{ZR}\},\{\text{b2e}\}]]- \right.$

$\left.\text{GrM}[\text{C}\theta[\{i\},\{\text{ZR}\}], \text{C}\theta[\{\text{ZR}\},\{i\}]]\right]$

Clear[B3SExpension]

B3SExpension[2, i_]:=

GrM[B1SExpension[2, i],

$\sum_{\text{b31e}=1}^{3} \left(\sum_{\text{b32e}=1}^{3}(\text{GrM}[\text{GrM}[(1/2!)\text{C}\theta[\{\text{b31e}\},\{\text{ZR}\}], \text{C}\theta[\{\text{ZR}\},\{\text{b31e}\}]],\right.$

$\left.\text{GrM}[\text{C}\theta[\{\text{b32e}\},\{\text{ZR}\}], \text{C}\theta[\{\text{ZR}\},\{\text{b32e}\}]]])\right)-$

$\sum_{\text{b33e}=1}^{3}(\text{GrM}[\text{GrM}[\text{C}\theta[\{i\},\{\text{ZR}\}], \text{C}\theta[\{\text{ZR}\},\{i\}]],$

$\text{GrM}[\text{C}\theta[\{\text{b33e}\},\{\text{ZR}\}], \text{C}\theta[\{\text{ZR}\},\{\text{b33e}\}]]])]$

Clear[SGrM]

HoldPattern[SGrM[A_ + B_, C___, D___, E___, F___]]:=

SGrM[A, C, D, E, F] + SGrM[B, C, D, E, F]

HoldPattern[SGrM[s_A_, C___, D___, E___, F___]]:=

sSGrM[A, C, D, E, F]/;CθQ[A]&&NumberQ[s]

SGrM[Cθ[x1_List, y1_List], gn_, comp_, pole_, {i_}]:=

SuperInfo[GrM[Cθ[x1, y1], Theta[i]], SP[gn, comp, pole, i]]

Theta[0]:=Cθ[{ZR}, {ZR}]

Theta[1]:=Cθ[{ZR}, {1}]

Theta[2]:=Cθ[{ZR}, {2}]

Theta[3]:=Cθ[{ZR}, {3}]

Theta[4]:=Cθ[{ZR}, {2, 3}]

Theta[5]:=Cθ[{ZR}, {3, 1}]

Theta[6]:=Cθ[{ZR}, {1, 2}]

Theta[7]:=Cθ[{ZR}, {1, 2, 3}]

SuperExpension

SExpension[4,0,pole_,i_]= $\frac{\theta_{12}^{3-0}}{\boldsymbol{Z}_{\mathbf{12}}^{\mathbf{pole}}} = \frac{1}{\boldsymbol{Z}_{\mathbf{12}}^{\mathbf{pole}}} \left(\theta_1^1 - \theta_2^1\right)\left(\theta_1^2 - \theta_2^2\right)\left(\theta_1^3 - \theta_2^3\right)$

SExpension[3, 1 ,pole_,i_]= $\frac{\theta_{12}^{3-1}}{\boldsymbol{Z}_{\mathbf{12}}^{\mathbf{pole}}} = \frac{1}{2!\boldsymbol{Z}_{\mathbf{12}}^{\mathbf{pole}}} \left\{\left(\theta_1^2 - \theta_2^2\right)\left(\theta_1^3 - \theta_2^3\right) - \left(\theta_1^3 - \theta_2^3\right)\left(\theta_1^2 - \theta_2^2\right)\right\}$

expand $\frac{1}{\boldsymbol{Z}_{\mathbf{12}}^{\mathbf{pole}}}$ about θ

$\frac{1}{\boldsymbol{Z}_{\mathbf{12}}^{\mathbf{pole}}} = \frac{1}{(z_1 - z_2)^{\mathrm{pole}}} + f\frac{\theta_1^i \theta_1^i}{(z_1 - z_2)^{\mathrm{pole}}} + \ldots$

Clear[SExpension]

SExpension[4, 0, pole_, i_]:=

SGrM[GrM[Cθ[{1}, {ZR}] − Cθ[{ZR}, {1}],

GrM[Cθ[{2}, {ZR}] − Cθ[{ZR}, {2}], Cθ[{3}, {ZR}] − Cθ[{ZR}, {3}]]],

4, 0, pole, {i}]

SExpension[3, cp_, pole_, i_]:=

(1/2!)∗

(SGrM[B1SExpension[3, cp], 3, cp, pole, {i}]+

poleSGrM[B2SExpension[3, cp], 3, cp, pole + 1i, {i}])

SExpension[2, cp_, pole_, i_]:=

SGrM[B1SExpension[2, cp], 2, cp, pole, {i}]+

poleSGrM[B2SExpension[2, cp], 2, cp, pole + 1i, {i}]+

pole(pole + 1)SGrM[B3SExpension[2, cp], 2, cp, pole + 2i, {i}]

SExpension[1, 0, pole_, i_]:=

```
SuperInfo[GrM[Cθ[{ZR}, {ZR}], Theta[i]], SP[1, 0, pole, i]]+
poleSGrM [∑_{sx01=1}^3 GrM[Cθ[{sx01}, {ZR}], Cθ[{ZR}, {sx01}]], 1, 0,
pole + 1i, {i}]+
pole(pole + 1)(1/2!)
SGrM[
∑_{sx102=1}^3 (∑_{sx202=1}^3 (GrM[GrM[Cθ[{sx102}, {ZR}], Cθ[{ZR}, {sx102}]],
GrM[Cθ[{sx202}, {ZR}], Cθ[{ZR}, {sx202}]]])), 1, 0,
pole + 2i, {i}]+
pole(pole + 1)(pole + 2)
SGrM[GrM[Cθ[{1}, {ZR}],
GrM[Cθ[{ZR}, {1}], GrM[Cθ[{2}, {ZR}],
GrM[Cθ[{ZR}, {2}], GrM[Cθ[{3}, {ZR}], Cθ[{ZR}, {3}]]]]]],
1, 0, pole + 3i, {i}]
(****)
Clear[ZeroQ]
ZeroQ[0]:=True
ZeroQ[x_]:=False/;x ≠ 0
Clear[θNSort]

θNSort[x1_List]:=(1 + Wrong * Count[x1, 0])Sort[x1]

Clear[θ1]
Clear[θ2]

θ1[x1__]:=Signature[{x1}]θ1@@θNSort[{x1}]/;
 !OrderedQ[{x1}]
θ2[x1__]:=Signature[{x1}]θ2@@θNSort[{x1}]/;
 !OrderedQ[{x1}]
θ1[x1__]:=0/;ZeroQ[Signature[{x1}]]
θ2[x1__]:=0/;ZeroQ[Signature[{x1}]]
```

θ1[x1__, ZR]:=θ1[x1]

θ1[ZR, x1__]:=θ1[x1]

θ2[x1__, ZR]:=θ2[x1]

θ2[ZR, x1__]:=θ2[x1]

Clear[TRABSCM]

TRABSCM[XX_]:=XX/.{Cθ → Mθ}

Clear[Mθ]

Mθ[{A__}, {B__}]:=θ1[A]θ2[B]

*SuperOPE max change

SuperOPEnonzerothebiggestpole(max)
max4– $> \theta^{3-0}$
max3– $> \theta^{3-i}$
max2– $> \theta^{i}$
max1– $> \theta^{0}$

max4 = 3;

max3 = 2;

max2 = 2;

max1 = 2;

SUPEROPEframe =
$$\sum_{\text{gindex}=0}^{7}\left(\left(\sum_{\text{a1}=1}^{\text{max4}}(\text{SExpension}[4,0,\text{a1},\text{gindex}])\right)+\sum_{\text{a2}=1}^{3}\left(\sum_{\text{a1}=1}^{\text{max3}}(\text{SExpension}[3,\text{a2},\text{a1},\text{gindex}])\right)+\sum_{\text{a2}=1}^{3}\left(\sum_{\text{a1}=1}^{\text{max2}}(\text{SExpension}[2,\text{a2},\text{a1},\text{gindex}])\right)+\sum_{\text{a1}=1}^{\text{max1}}\text{SExpension}[1,0,\text{a1},\text{gindex}]\right);$$

θDelta

Clear[θTrNumber]

θTrNumber[{ZR}]:=0

θTrNumber[{1}]:=1

```
θTrNumber[{2}]:=2
θTrNumber[{3}]:=3
θTrNumber[{2, 3}]:=4
θTrNumber[{1, 3}]:=5
θTrNumber[{1, 2}]:=6
θTrNumber[{1, 2, 3}]:=7
Clear[θDelta]
θDelta[θ1[z__], θ1[w__]]:=Delta[θTrNumber[{z}], θTrNumber[{w}]]
θDelta[θ1[z__], −θ1[w__]]:= − Delta[θTrNumber[{z}], θTrNumber[{w}]]
θDelta[−θ1[z__], θ1[w__]]:= − Delta[θTrNumber[{z}], θTrNumber[{w}]]
θDelta[−θ1[z__], −θ1[w__]]:=Delta[θTrNumber[{z}], θTrNumber[{w}]]

θDelta[θ2[z__], θ2[w__]]:=Delta[θTrNumber[{z}], θTrNumber[{w}]]
θDelta[θ2[z__], −θ2[w__]]:= − Delta[θTrNumber[{z}], θTrNumber[{w}]]
θDelta[−θ2[z__], θ2[w__]]:= − Delta[θTrNumber[{z}], θTrNumber[{w}]]
θDelta[−θ2[z__], −θ2[w__]]:=Delta[θTrNumber[{z}], θTrNumber[{w}]]

θDelta[θ1[z__], θ2[w__]]:=0
θDelta[θ1[z__], −θ2[w__]]:=0
θDelta[−θ1[z__], θ2[w__]]:=0
θDelta[−θ1[z__], −θ2[w__]]:=0

θDelta[A_, 0]:=0
Clear[θ1Derivative]
HoldPattern[θ1Derivative[Di_, A_ + B_]]:=
θ1Derivative[Di, A] + θ1Derivative[Di, B]
HoldPattern[θ1Derivative[Di_, s_A_]]:=
 sθ1Derivative[Di, A]/;NumberQ[s]

θ1Derivative[Di_, SuperInfo[Cθ[{th1__}, {th2__}], SP[a_, b_, c_, d_]]]:=
```

```
θDelta[Di, θ1[th1]]SuperInfo[Cθ[{ZR}, {th2}], SP[a, b, c, d]]
θ1Derivative[Di_, 0]:=0
Clear[θ2Derivative]
HoldPattern[θ2Derivative[Di_, A_ + B_]]:=
θ2Derivative[Di, A] + θ2Derivative[Di, B]
HoldPattern[θ2Derivative[Di_, s_A_]]:=
 sθ2Derivative[Di, A]/;NumberQ[s]

θ2Derivative[Di_, SuperInfo[Cθ[{th1__}, {th2__}], SP[a_, b_, c_, d_]]]:=
θDelta[Di, θ2[th2]]SuperInfo[Cθ[{th1}, {ZR}], SP[a, b, c, d]]
θ2Derivative[Di_, 0]:=0
```

Select Pole

```
Clear[CSPQ]
CSPQ[SP[a_, b_, c_, d_]]:=True
Clear[SelPole]

HoldPattern[SelPole[A_ + B_, p_]]:=
SelPole[A, p] + SelPole[B, p]
HoldPattern[SelPole[s_A_, p_]]:=
 sSelPole[A, p]/;CSPQ[A]&&NumberQ[s]

SelPole[SP[a_, b_, c_, d_], wantPole_]:=
Delta[Re[c] + Im[c], wantPole] * SuperOPE[a, b, Re[c], d]
```

Result

```
$RecursionLimit = Infinity
∞
SChcg1[1, 0]:=θ1[ZR]
SChcg2[1, 0] = θ2[ZR];
```

```
SChcg1[2, i_]:=θ1[i]

SChcg2[2, i_]:=θ2[i]


SChcg1[3, 1] = θ1[2, 3];

SChcg1[3, 2] = θ1[3, 1];

SChcg1[3, 3] = θ1[1, 2];


SChcg2[3, 1] = θ2[2, 3];

SChcg2[3, 2] = θ2[3, 1];

SChcg2[3, 3] = θ2[1, 2];


SChcg1[4, 0] = θ1[1, 2, 3];


SChcg2[4, 0] = θ2[1, 2, 3];

Clear[DelZR]

HoldPattern[DelZR[A_ + B_]]:=

DelZR[A] + DelZR[B]

HoldPattern[DelZR[s_A_]]:=

 sDelZR[A]/;NumberQ[s]

DelZR[SuperInfo[Cθ[{ZR}, {ZR}], SP[a_, b_, c_, d_]]]:=SP[a, b, c, d]

Clear[normalcoeff]

normalcoeff = {(i/2)^-1, (i/2)^-1, (1/2)^-1, 1};
```

SuperOPE[3,2,1,6]; 2-> Phi_{4,2}

That is -> SuperOPE[, , ,x] –>Theta[x]

```
Clear[DeSuperSpace]

DeSuperSpace[{comp1_, subcomp1_}, {comp2_, subcomp2_}, wantPole_]:=

normalcoeff[[comp1]] * normalcoeff[[comp2]] * LHSign[comp1, comp2]*

SelPole[DelZR[θ2Derivative[SChcg2[comp2, subcomp2],

θ1Derivative[SChcg1[comp1, subcomp1], SUPEROPEframe]]], wantPole]
```

normalcoeff

convention SpJ3 $= \frac{i}{2}\Psi+\ \theta^i\ \frac{i}{2}J[1]+\theta^{3-i}\frac{1}{2}G[i]+\theta^{3-0}\mathrm{Ts}$ compensate$\frac{i}{2}$ $\frac{1}{2}$

SuperSpace Library Construction

SpJ3 $= \frac{i}{2}\Psi+\ \theta^i\ \frac{i}{2}J[1]+\theta^{3-i}\frac{1}{2}G[i]+\theta^{3-0}\mathrm{Ts}$

GrassB= even number of θ, GrassF= odd number of θ

FieldB= Boson, FieldF= Fermion,

SpJ3 = {{{ZR}, (*i*/2)Ψ, GrassB, FieldF}, {{1}, (*i*/2)*J*[1], GrassF, FieldB},

{{2}, (*i*/2)*J*[2], GrassF, FieldB}, {{3}, (*i*/2)*J*[3], GrassF, FieldB},

{{1, 2}, (1/2)*G*[3], GrassB, FieldF}, {{3, 1}, (1/2)*G*[2], GrassB, FieldF},

{{2, 3}, (1/2)*G*[1], GrassB, FieldF}, {{1, 2, 3}, Ts, GrassF, FieldB}};

SpPhi3H = {{{ZR}, (*i*/2)Phi3Hcp1[3/2], GrassB, FieldF},

{{1}, (*i*/2)Phi3Hcp2[1, 2], GrassF, FieldB},

{{2}, (*i*/2)Phi3Hcp2[2, 2], GrassF, FieldB},

{{3}, (*i*/2)Phi3Hcp2[3, 2], GrassF, FieldB},

{{2, 3}, (1/2)Phi3Hcp3[1, 5/2], GrassB, FieldF},

{{3, 1}, (1/2)Phi3Hcp3[2, 5/2], GrassB, FieldF},

{{1, 2}, (1/2)Phi3Hcp3[3, 5/2], GrassB, FieldF},

{{1, 2, 3}, Phi3Hcp4[3], GrassF, FieldB}};

SpPhi2 = {{{ZR}, (*i*/2)Phi2cp1[2], GrassB, FieldB},

{{1}, (*i*/2)Phi2cp2[1, 5/2], GrassF, FieldF},

{{2}, (*i*/2)Phi2cp2[2, 5/2], GrassF, FieldF},

{{3}, (*i*/2)Phi2cp2[3, 5/2], GrassF, FieldF},

{{2, 3}, (1/2)Phi2cp3[1, 3], GrassB, FieldB},

{{3, 1}, (1/2)Phi2cp3[2, 3], GrassB, FieldB},

{{1, 2}, (1/2)Phi2cp3[3, 3], GrassB, FieldB},

{{1, 2, 3}, Phi2cp4[7/2], GrassF, FieldF}};

SpPhi5H = {{{ZR}, (*i*/2)Phi5Hcp1[5/2], GrassB, FieldF},

$\{\{1\},(i/2)\mathrm{Phi5Hcp2}[1,3],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{2\},(i/2)\mathrm{Phi5Hcp2}[2,3],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{3\},(i/2)\mathrm{Phi5Hcp2}[3,3],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{2,3\},(1/2)\mathrm{Phi5Hcp3}[1,7/2],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{3,1\},(1/2)\mathrm{Phi5Hcp3}[2,7/2],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{1,2\},(1/2)\mathrm{Phi5Hcp3}[3,7/2],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{1,2,3\},\mathrm{Phi5Hcp4}[4],\mathrm{GrassF},\mathrm{FieldB}\}\};$

$\mathrm{SpPhi3}[\mathrm{orth\_}]{:=}\{\{\{\mathrm{ZR}\},(i/2)\mathrm{Phi3cp1}[3,\{\mathrm{orth}\}],\mathrm{GrassB},\mathrm{FieldB}\},$

$\{\{1\},(i/2)\mathrm{Phi3cp2}[1,7/2,\{\mathrm{orth}\}],\mathrm{GrassF},\mathrm{FieldF}\},$

$\{\{2\},(i/2)\mathrm{Phi3cp2}[2,7/2,\{\mathrm{orth}\}],\mathrm{GrassF},\mathrm{FieldF}\},$

$\{\{3\},(i/2)\mathrm{Phi3cp2}[3,7/2,\{\mathrm{orth}\}],\mathrm{GrassF},\mathrm{FieldF}\},$

$\{\{2,3\},(1/2)\mathrm{Phi3cp3}[1,4,\{\mathrm{orth}\}],\mathrm{GrassB},\mathrm{FieldB}\},$

$\{\{3,1\},(1/2)\mathrm{Phi3cp3}[2,4,\{\mathrm{orth}\}],\mathrm{GrassB},\mathrm{FieldB}\},$

$\{\{1,2\},(1/2)\mathrm{Phi3cp3}[3,4,\{\mathrm{orth}\}],\mathrm{GrassB},\mathrm{FieldB}\},$

$\{\{1,2,3\},\mathrm{Phi3cp4}[9/2,\{\mathrm{orth}\}],\mathrm{GrassF},\mathrm{FieldF}\}\};$

$\mathrm{SpNewPhi3H}=\{\{\{\mathrm{ZR}\},(i/2)\mathrm{NewPhi3Hcp1}[3/2],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{1\},(i/2)\mathrm{NewPhi3Hcp2}[1,2],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{2\},(i/2)\mathrm{NewPhi3Hcp2}[2,2],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{3\},(i/2)\mathrm{NewPhi3Hcp2}[3,2],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{2,3\},(1/2)\mathrm{NewPhi3Hcp3}[1,5/2],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{3,1\},(1/2)\mathrm{NewPhi3Hcp3}[2,5/2],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{1,2\},(1/2)\mathrm{NewPhi3Hcp3}[3,5/2],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{1,2,3\},\mathrm{NewPhi3Hcp4}[3],\mathrm{GrassF},\mathrm{FieldB}\}\};$

$\mathrm{SpNNewPhi3H}[\mathrm{orth\_}]{:=}$

$\{\{\{\mathrm{ZR}\},(i/2)\mathrm{NNewPhi3cp1}[3/2,\{\mathrm{orth}\}],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{1\},(i/2)\mathrm{NNewPhi3cp2}[1,2,\{\mathrm{orth}\}],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{2\},(i/2)\mathrm{NNewPhi3cp2}[2,2,\{\mathrm{orth}\}],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{3\},(i/2)\mathrm{NNewPhi3cp2}[3,2,\{\mathrm{orth}\}],\mathrm{GrassF},\mathrm{FieldB}\},$

$\{\{2,3\},(1/2)\mathrm{NNewPhi3cp3}[1,5/2,\{\mathrm{orth}\}],\mathrm{GrassB},\mathrm{FieldF}\},$

$\{\{3,1\},(1/2)\mathrm{NNewPhi3cp3}[2,5/2,\{\mathrm{orth}\}],\mathrm{GrassB},\mathrm{FieldF}\},$

{{1, 2}, (1/2)NNewPhi3cp3[3, 5/2, {orth}], GrassB, FieldF},

{{1, 2, 3}, NNewPhi3cp4[3, {orth}], GrassF, FieldB}};

SpNewPhi2 = {{{ZR}, (i/2)NewPhi2cp1[2], GrassB, FieldB},

{{1}, (i/2)NewPhi2cp2[1, 5/2], GrassF, FieldF},

{{2}, (i/2)NewPhi2cp2[2, 5/2], GrassF, FieldF},

{{3}, (i/2)NewPhi2cp2[3, 5/2], GrassF, FieldF},

{{2, 3}, (1/2)NewPhi2cp3[1, 3], GrassB, FieldB},

{{3, 1}, (1/2)NewPhi2cp3[2, 3], GrassB, FieldB},

{{1, 2}, (1/2)NewPhi2cp3[3, 3], GrassB, FieldB},

{{1, 2, 3}, NewPhi2cp4[7/2], GrassF, FieldF}};

SpNNewPhi5H = {{{ZR}, (i/2)NNewPhi5Hcp1[5/2], GrassB, FieldF},

{{1}, (i/2)NNewPhi5Hcp2[1, 3], GrassF, FieldB},

{{2}, (i/2)NNewPhi5Hcp2[2, 3], GrassF, FieldB},

{{3}, (i/2)NNewPhi5Hcp2[3, 3], GrassF, FieldB},

{{2, 3}, (1/2)NNewPhi5Hcp3[1, 7/2], GrassB, FieldF},

{{3, 1}, (1/2)NNewPhi5Hcp3[2, 7/2], GrassB, FieldF},

{{1, 2}, (1/2)NNewPhi5Hcp3[3, 7/2], GrassB, FieldF},

{{1, 2, 3}, NNewPhi5Hcp4[4], GrassF, FieldB}};

SpNewPhi5H[orth_]:=

{{{ZR}, (i/2)NewPhi5Hcp1[5/2, {orth}], GrassB, FieldF},

{{1}, (i/2)NewPhi5Hcp2[1, 3, {orth}], GrassF, FieldB},

{{2}, (i/2)NewPhi5Hcp2[2, 3, {orth}], GrassF, FieldB},

{{3}, (i/2)NewPhi5Hcp2[3, 3, {orth}], GrassF, FieldB},

{{2, 3}, (1/2)NewPhi5Hcp3[1, 7/2, {orth}], GrassB, FieldF},

{{3, 1}, (1/2)NewPhi5Hcp3[2, 7/2, {orth}], GrassB, FieldF},

{{1, 2}, (1/2)NewPhi5Hcp3[3, 7/2, {orth}], GrassB, FieldF},

{{1, 2, 3}, NewPhi5Hcp4[4, {orth}], GrassF, FieldB}};

SpNewPhi3[orth_]:={{{ZR}, (i/2)NewPhi3cp1[3, {orth}], GrassB, FieldB},

{{1}, (i/2)NewPhi3cp2[1, 7/2, {orth}], GrassF, FieldF},

{{2}, (i/2)NewPhi3cp2[2, 7/2, {orth}], GrassF, FieldF}, {{3}, (i/2)NewPhi3cp2[3, 7/2, {orth}], GrassF, FieldF},

{{2, 3}, (1/2)NewPhi3cp3[1, 4, {orth}], GrassB, FieldB},

{{3, 1}, (1/2)NewPhi3cp3[2, 4, {orth}], GrassB, FieldB},

{{1, 2}, (1/2)NewPhi3cp3[3, 4, {orth}], GrassB, FieldB},

{{1, 2, 3}, NewPhi3cp4[9/2, {orth}], GrassF, FieldF}};

SpNNewPhi3 = {{{ZR}, (i/2)NNewPhi3cp1[3], GrassB, FieldB},

{{1}, (i/2)NNewPhi3cp2[1, 7/2], GrassF, FieldF},

{{2}, (i/2)NNewPhi3cp2[2, 7/2], GrassF, FieldF},

{{3}, (i/2)NNewPhi3cp2[3, 7/2], GrassF, FieldF},

{{2, 3}, (1/2)NNewPhi3cp3[1, 4], GrassB, FieldB},

{{3, 1}, (1/2)NNewPhi3cp3[2, 4], GrassB, FieldB},

{{1, 2}, (1/2)NNewPhi3cp3[3, 4], GrassB, FieldB},

{{1, 2, 3}, NNewPhi3cp4[9/2], GrassF, FieldF}};

SpNewPhi7H[orth_]:=

{{{ZR}, (i/2)NewPhi7Hcp1[7/2, {orth}], GrassB, FieldF},

{{1}, (i/2)NewPhi7Hcp2[1, 4, {orth}], GrassF, FieldB},

{{2}, (i/2)NewPhi7Hcp2[2, 4, {orth}], GrassF, FieldB},

{{3}, (i/2)NewPhi7Hcp2[3, 4, {orth}], GrassF, FieldB},

{{2, 3}, (1/2)NewPhi7Hcp3[1, 9/2, {orth}], GrassB, FieldF},

{{3, 1}, (1/2)NewPhi7Hcp3[2, 9/2, {orth}], GrassB, FieldF},

{{1, 2}, (1/2)NewPhi7Hcp3[3, 9/2, {orth}], GrassB, FieldF},

{{1, 2, 3}, NewPhi7Hcp4[5, {orth}], GrassF, FieldB}};

SpNewPhi4 = {{{ZR}, (i/2)NewPhi4cp1[4], GrassB, FieldB},

{{1}, (i/2)NewPhi4cp2[1, 9/2], GrassF, FieldF},

{{2}, (i/2)NewPhi4cp2[2, 9/2], GrassF, FieldF},

{{3}, (i/2)NewPhi4cp2[3, 9/2], GrassF, FieldF},

{{2, 3}, (1/2)NewPhi4cp3[1, 5], GrassB, FieldB},

{{3, 1}, (1/2)NewPhi4cp3[2, 5], GrassB, FieldB},

{{1, 2}, (1/2)NewPhi4cp3[3, 5], GrassB, FieldB},

{{1, 2, 3}, NewPhi4cp4[11/2], GrassF, FieldF}};

```
Clear[ZeroQ]
ZeroQ[0]:=True
ZeroQ[i_]:=False/;i ≠ 0

Clear[OneQ]
OneQ[1]:=True
OneQ[i_]:=False/;i ≠ 1
Clear[CDerQ]
CDerQ[A_List, B_List]:=True
CDerQ[A_List, B_List]:=False/;Intersection[A, B] ≠ {}
Clear[TransG]
TransG[GrassB]:=GrassF
TransG[GrassF]:=GrassB
Clear[GrMW]
GrMW[{w1_}, {w2__}]:={w1, w2}
GrMW[{w1_}, {ZR}]:={w1}
GrMW[{ZR}, {w2__}]:={w2}
GrMW[{w1_}, {w1_}]:=Wrong

Clear[DGrMW]
DGrMW[i_, A_List]:=
Delete[A[[1]], Position[A[[1]], Intersection[{i}, A[[1]]][[1]]][[1]]]/;
Delete[A[[1]], Position[A[[1]], Intersection[{i}, A[[1]]][[1]]][[1]]] ≠
{}
DGrMW[i_, A_List]:={ZR}
Clear[DOPpt]
DOPpt[i_, A_List]:=
{GrMW[{i}, A[[1]]], Derivative[1][A[[2]]], TransG[A[[3]]], A[[4]]}/;
CDerQ[{i}, A[[1]]]
DOPpt[i_, A_List]:=
```

```
{DGrMW[i, A], (-1)^(1+Position[A[[1]],i][[1]][[1]]) A[[2]], TransG[A[[3]]], A[[4]]} /;
OneQ[Count[A[[1]], i]]

Clear[DOP]

DOP[i_, A_List]:={DOPpt[i, A[[1]]], DOPpt[i, A[[2]]],
DOPpt[i, A[[3]]], DOPpt[i, A[[4]]], DOPpt[i, A[[5]]], DOPpt[i, A[[6]]],
DOPpt[i, A[[7]]], DOPpt[i, A[[8]]]}

Clear[TRT]

TRT[{n1__}]:=θ2[n1]

Clear[SF]

SF[A_List]:= Sum[(TRT[A[[x1]][[1]]] * A[[x1]][[2]]), {x1, 1, 8}];
```

DJ[i___] $\sim D^{\mathrm{i}}$-J

DSpPhi3H[i___] $\sim D^{\mathrm{i}}$-$\Phi^{(3/2)}$

```
Clear[DJ]

DJ[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpJ3]]]]

DJ[i_, j_]:=SF[DOP[i, DOP[j, SpJ3]]]

DJ[i_]:=SF[DOP[i, SpJ3]]

DJ[0]:=SF[SpJ3]

Clear[DSpPhi3H]

DSpPhi3H[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpPhi3H]]]]

DSpPhi3H[i_, j_]:=SF[DOP[i, DOP[j, SpPhi3H]]]

DSpPhi3H[i_]:=SF[DOP[i, SpPhi3H]]

DSpPhi3H[0]:=SF[SpPhi3H]

Clear[DSpPhi2]

DSpPhi2[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpPhi2]]]]

DSpPhi2[i_, j_]:=SF[DOP[i, DOP[j, SpPhi2]]]

DSpPhi2[i_]:=SF[DOP[i, SpPhi2]]

DSpPhi2[0]:=SF[SpPhi2]

Clear[DSpPhi5H]

DSpPhi5H[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpPhi5H]]]]

DSpPhi5H[i_, j_]:=SF[DOP[i, DOP[j, SpPhi5H]]]
```

```
DSpPhi5H[i_]:=SF[DOP[i, SpPhi5H]]

DSpPhi5H[0]:=SF[SpPhi5H]

Clear[DSpPhi3]

DSpPhi3[i_, j_, kay_, {orth_}]:=

SF[DOP[i, DOP[j, DOP[kay, SpPhi3[orth]]]]]

DSpPhi3[i_, j_, {orth_}]:=SF[DOP[i, DOP[j, SpPhi3[orth]]]]

DSpPhi3[i_, {orth_}]:=SF[DOP[i, SpPhi3[orth]]]

DSpPhi3[0, {orth_}]:=SF[SpPhi3[orth]]

Clear[DSpNewPhi3H]

DSpNewPhi3H[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpNewPhi3H]]]]

DSpNewPhi3H[i_, j_]:=SF[DOP[i, DOP[j, SpNewPhi3H]]]

DSpNewPhi3H[i_]:=SF[DOP[i, SpNewPhi3H]]

DSpNewPhi3H[0]:=SF[SpNewPhi3H]

Clear[DSpNNewPhi3H]

DSpNNewPhi3H[i_, j_, kay_, {orth_}]:=

SF[DOP[i, DOP[j, DOP[kay, SpNNewPhi3H[orth]]]]]

DSpNNewPhi3H[i_, j_, {orth_}]:=SF[DOP[i, DOP[j, SpNNewPhi3H[orth]]]]

DSpNNewPhi3H[i_, {orth_}]:=SF[DOP[i, SpNNewPhi3H[orth]]]

DSpNNewPhi3H[0, {orth_}]:=SF[SpNNewPhi3H[orth]]

Clear[DSpNewPhi2]

DSpNewPhi2[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpNewPhi2]]]]

DSpNewPhi2[i_, j_]:=SF[DOP[i, DOP[j, SpNewPhi2]]]

DSpNewPhi2[i_]:=SF[DOP[i, SpNewPhi2]]

DSpNewPhi2[0]:=SF[SpNewPhi2]

Clear[DSpNNewPhi5H]

DSpNNewPhi5H[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpNNewPhi5H]]]]

DSpNNewPhi5H[i_, j_]:=SF[DOP[i, DOP[j, SpNNewPhi5H]]]

DSpNNewPhi5H[i_]:=SF[DOP[i, SpNNewPhi5H]]

DSpNNewPhi5H[0]:=SF[SpNNewPhi5H]

Clear[DSpNewPhi5H]
```

```
DSpNewPhi5H[i_, j_, kay_, {orth_}]:=
SF[DOP[i, DOP[j, DOP[kay, SpNewPhi5H[orth]]]]]
DSpNewPhi5H[i_, j_, {orth_}]:=SF[DOP[i, DOP[j, SpNewPhi5H[orth]]]]
DSpNewPhi5H[i_, {orth_}]:=SF[DOP[i, SpNewPhi5H[orth]]]
DSpNewPhi5H[0, {orth_}]:=SF[SpNewPhi5H[orth]]

Clear[DSpNewPhi3]
DSpNewPhi3[i_, j_, kay_, {orth_}]:=
SF[DOP[i, DOP[j, DOP[kay, SpNewPhi3[orth]]]]]
DSpNewPhi3[i_, j_, {orth_}]:=SF[DOP[i, DOP[j, SpNewPhi3[orth]]]]
DSpNewPhi3[i_, {orth_}]:=SF[DOP[i, SpNewPhi3[orth]]]
DSpNewPhi3[0, {orth_}]:=SF[SpNewPhi3[orth]]

Clear[DSpNNewPhi3]
DSpNNewPhi3[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpNNewPhi3]]]]
DSpNNewPhi3[i_, j_]:=SF[DOP[i, DOP[j, SpNNewPhi3]]]
DSpNNewPhi3[i_]:=SF[DOP[i, SpNNewPhi3]]
DSpNNewPhi3[0]:=SF[SpNNewPhi3]

Clear[DSpNewPhi7H]
DSpNewPhi7H[i_, j_, kay_, {orth_}]:=
SF[DOP[i, DOP[j, DOP[kay, SpNewPhi7H[orth]]]]]
DSpNewPhi7H[i_, j_, {orth_}]:=SF[DOP[i, DOP[j, SpNewPhi7H[orth]]]]
DSpNewPhi7H[i_, {orth_}]:=SF[DOP[i, SpNewPhi7H[orth]]]
DSpNewPhi7H[0, {orth_}]:=SF[SpNewPhi7H[orth]]

Clear[DSpNewPhi4]
DSpNewPhi4[i_, j_, kay_]:=SF[DOP[i, DOP[j, DOP[kay, SpNewPhi4]]]]
DSpNewPhi4[i_, j_]:=SF[DOP[i, DOP[j, SpNewPhi4]]]
DSpNewPhi4[i_]:=SF[DOP[i, SpNewPhi4]]
DSpNewPhi4[0]:=SF[SpNewPhi4]


Clear[TLDJ]
```

```
TLDJ[1] = DJ[3, 2];
TLDJ[2] = DJ[1, 3];
TLDJ[3] = DJ[2, 1];


Clear[TLDSpPhi3H]
TLDSpPhi3H[1] = DSpPhi3H[3, 2];
TLDSpPhi3H[2] = DSpPhi3H[1, 3];
TLDSpPhi3H[3] = DSpPhi3H[2, 1];
Clear[TLDSpPhi2]
TLDSpPhi2[1] = DSpPhi2[3, 2];
TLDSpPhi2[2] = DSpPhi2[1, 3];
TLDSpPhi2[3] = DSpPhi2[2, 1];
Clear[TLDSpPhi5H]
TLDSpPhi5H[1] = DSpPhi5H[3, 2];
TLDSpPhi5H[2] = DSpPhi5H[1, 3];
TLDSpPhi5H[3] = DSpPhi5H[2, 1];
Clear[TLDSpPhi3]
TLDSpPhi3[1, {orth_}]:=DSpPhi3[3, 2, {orth}];
TLDSpPhi3[2, {orth_}]:=DSpPhi3[1, 3, {orth}];
TLDSpPhi3[3, {orth_}]:=DSpPhi3[2, 1, {orth}];
Clear[TLDSpNewPhi3H]
TLDSpNewPhi3H[1] = DSpNewPhi3H[3, 2];
TLDSpNewPhi3H[2] = DSpNewPhi3H[1, 3];
TLDSpNewPhi3H[3] = DSpNewPhi3H[2, 1];
Clear[TLDSpNNewPhi3H]
TLDSpNNewPhi3H[1, {orth_}]:=DSpNNewPhi3H[3, 2, {orth}];
TLDSpNNewPhi3H[2, {orth_}]:=DSpNNewPhi3H[1, 3, {orth}];
TLDSpNNewPhi3H[3, {orth_}]:=DSpNNewPhi3H[2, 1, {orth}];
Clear[TLDSpNewPhi2]
```

```
TLDSpNewPhi2[1] = DSpNewPhi2[3, 2];

TLDSpNewPhi2[2] = DSpNewPhi2[1, 3];

TLDSpNewPhi2[3] = DSpNewPhi2[2, 1];

Clear[TLDSpNewPhi5H]

TLDSpNNewPhi5H[1] = DSpNNewPhi5H[3, 2];

TLDSpNNewPhi5H[2] = DSpNNewPhi5H[1, 3];

TLDSpNNewPhi5H[3] = DSpNNewPhi5H[2, 1];

Clear[TLDSpNewPhi5H]

TLDSpNewPhi5H[1, {orth_}]:=DSpNewPhi5H[3, 2, {orth}];

TLDSpNewPhi5H[2, {orth_}]:=DSpNewPhi5H[1, 3, {orth}];

TLDSpNewPhi5H[3, {orth_}]:=DSpNewPhi5H[2, 1, {orth}];

Clear[TLDSpNewPhi3]

TLDSpNewPhi3[1, {orth_}]:=DSpNewPhi3[3, 2, {orth}];

TLDSpNewPhi3[2, {orth_}]:=DSpNewPhi3[1, 3, {orth}];

TLDSpNewPhi3[3, {orth_}]:=DSpNewPhi3[2, 1, {orth}];

Clear[TLDSpNNewPhi3]

TLDSpNNewPhi3[1] = DSpNNewPhi3[3, 2];

TLDSpNNewPhi3[2] = DSpNNewPhi3[1, 3];

TLDSpNNewPhi3[3] = DSpNNewPhi3[2, 1];

Clear[TLDSpNewPhi7H]

TLDSpNewPhi7H[1, {orth_}]:=DSpNewPhi7H[3, 2, {orth}];

TLDSpNewPhi7H[2, {orth_}]:=DSpNewPhi7H[1, 3, {orth}];

TLDSpNewPhi7H[3, {orth_}]:=DSpNewPhi7H[2, 1, {orth}];

Clear[TLDSpNewPhi4]

TLDSpNewPhi4[1] = DSpNewPhi4[3, 2];

TLDSpNewPhi4[2] = DSpNewPhi4[1, 3];

TLDSpNewPhi4[3] = DSpNewPhi4[2, 1];

Derivative[n_][0]:=0

NO[0, X_]:=0
```

```
NO[X_, 0]:=0
```

SUPERD[SpD[A,B]] Leibniz's rule, Single D^i derivative not D^ij...

Add sc11 sc12 sc22 if needed

Shoud be CoeffQ = True

```
Clear[CoeffQ]

HoldPattern[CoeffQ[A_ + B_]]:=CoeffQ[A]

HoldPattern[CoeffQ[s_A_]]:=CoeffQ[A]/;NumberQ[s]


CoeffQ[sc11[x_]]:=True

CoeffQ[sc12[x_]]:=True

CoeffQ[sc22[x_]]:=True

CoeffQ[newsc12[x_]]:=True

CoeffQ[0]:=True

Clear[ConfD]

ConfD[OP_]:=
False/;Factor[Solve[GetCoefficients[OPEPole[2][Ts, OP] - hxOP] == 0, hx]][[
1, 1, 2]] == 0

ConfD[Derivative[n_][OP_]]:=
False/;Factor[Solve[GetCoefficients[OPEPole[2][Ts, OP] - hxOP] == 0, hx]][[
1, 1, 2]] == 0

ConfD[OP_]:=
Factor[Solve[GetCoefficients[OPEPole[2][Ts, OP] - hxOP] == 0, hx]][[1, 1, 2]]

ConfD[Derivative[n_][OP_]]:=
Factor[Solve[GetCoefficients[OPEPole[2][Ts, OP] - hxOP] == 0, hx]][[1, 1, 2]]

ConfD[One]:=0

Clear[BFOP]

HoldPattern[BFOP[s_B_]]:=BFOP[B]/;NumberQ[s]

HoldPattern[BFOP[Derivative[i_][B_]]]:=BFOP[B]


BFOP[Ts]:=1

BFOP[J[x__]]:=1
```

```
BFOP[G[x_]]:= - 1
BFOP[Ψ]:= - 1

BFOP[OP_]:= - 1/;OddQ[2ConfD[OP]]
BFOP[OP_]:= + 1/;EvenQ[2ConfD[OP]]
Clear[Subθ2ZR]
Subθ2ZR[X__]:=
X/.{θ2[ZR] → 1, θ2[1] → 0, θ2[2] → 0, θ2[3] → 0, θ2[1, 2] → 0,
θ2[1, 3] → 0, θ2[2, 3] → 0, θ2[1, 2, 3] → 0}
Clear[DtransSign]
DtransSign[X__]:=BFOP[Subθ2ZR[X]]
Clear[Mθ2]
Mθ2[{B__}]:=θ2[B]
Clear[θ2zeroQ]
θ2zeroQ[θ2[ZR]]:=True
Clear[θ2notzeroQ]
θ2notzeroQ[θ2[ZR]]:=False
θ2notzeroQ[θ2[X__]]:=True
Clear[SUPERD]

HoldPattern[SUPERD[Di_, A_ + B_]]:=
SUPERD[Di, A] + SUPERD[Di, B]
HoldPattern[SUPERD[Di_, ce_A_]]:=
 ceSUPERD[Di, A]/;CoeffQ[ce]
HoldPattern[SUPERD[Di_, A_ce_]]:=
 ceSUPERD[Di, A]/;CoeffQ[ce]
HoldPattern[SUPERD[Di_, s_A_]]:=
 sSUPERD[Di, A]/;NumberQ[s]
HoldPattern[SUPERD[Di_, Derivative[i_][A__]]]:=
 Derivative[i][SUPERD[Di, A]]
```

```
HoldPattern[SUPERD[Di_, SpD[A__, B__]]]:=
SpD[SUPERD[Di, A], B] + DtransSign[A] * SpD[A, SUPERD[Di, B]]


SUPERD[Di_, X__θ2[y__]]:=
θ2[Di, y]X /;Intersection[{Di}, {y}] ≠ {Di}&&θ2notzeroQ[θ2[y]]
SUPERD[Di_, X__θ2[y__]]:=
(−1)^(1+Position[{y},Di][[1]][[1]]) Mθ2[Complement[{y}, {Di}]]X/;
Complement[{y}, {Di}] ≠ {}&&θ2notzeroQ[θ2[y]]
SUPERD[Di_, X__θ2[Di_]]:=θ2[ZR]X/;θ2notzeroQ[θ2[y]]
SUPERD[Di_, X__θ2[y_]]:=θ2[Di]Derivative[1][X]
SUPERD[Di_, 0]:=0
SUPERD[Di_, One]:=0
Clear[TrNO]
TrNO[X_]:=
X/.{θ2[ZR] → 1, θ2[1] → 0, θ2[2] → 0, θ2[3] → 0, θ2[1, 2] → 0,
θ2[1, 3] → 0, θ2[2, 3] → 0, θ2[1, 2, 3] → 0}/.{SpD → NO}


Clear[SCPOPE]


HoldPattern[SCPOPE[A_ + B_]]:=SCPOPE[A] + SCPOPE[B]
HoldPattern[SCPOPE[s_A_]]:=sSCPOPE[A]/;NumberQ[s]


SCPOPE[SuperOPE[a_, b_, c_, 0]]:=TrNO[SOPE[a, b, c]]


SCPOPE[SuperOPE[a_, b_, c_, 1]]:=TrNO[SUPERD[1, SOPE[a, b, c]]]
SCPOPE[SuperOPE[a_, b_, c_, 2]]:=TrNO[SUPERD[2, SOPE[a, b, c]]]
SCPOPE[SuperOPE[a_, b_, c_, 3]]:=TrNO[SUPERD[3, SOPE[a, b, c]]]
```

SCPOPE[SuperOPE[a_, b_, c_, 4]]:=TrNO[SUPERD[3, SUPERD[2, SOPE[a, b, c]]]]

SCPOPE[SuperOPE[a_, b_, c_, 5]]:=TrNO[SUPERD[1, SUPERD[3, SOPE[a, b, c]]]]

SCPOPE[SuperOPE[a_, b_, c_, 6]]:=TrNO[SUPERD[2, SUPERD[1, SOPE[a, b, c]]]]

SCPOPE[SuperOPE[a_, b_, c_, 7]]:=

TrNO[SUPERD[3, SUPERD[2, SUPERD[1, SOPE[a, b, c]]]]]

SCPOPE[0]:=0

A.3 The package for SOPE and its component approach

SpD = Super normal ordering

DJ[x]= D^xJ= J^x
TLDJ[x]= D^{3-x}J= J^{3-x}
DJ[3,2,1]=D^{3-0}J=J^{3-0}

DspPhiX[i]=$D^i\Phi^{(x)}$
Dsp New PhiX[i]=$D^i\bar{\Phi}^{(x)}$ (New=bar)
Dsp NNew PhiX[i]=$D^i\tilde{\Phi}^{(x)}$ (NNew= tilde)
TLDsp ... [i] —> D^{3-i}

SOPE[number of theta +1 , i , pole] <—— $\frac{\theta^{[i]}}{Z_2{}^{\text{pole}}}(...)$

SOPE[4 , 0 , pole] <— $\frac{\theta^{3-0}}{Z_2{}^{\text{pole}}}(...)$
SOPE[3 , i, pole] <— $\frac{\theta^{3-i}}{Z_2{}^{\text{pole}}}(...)$
SOPE[2 , i, pole] <— $\frac{\theta^{i}}{Z_2{}^{\text{pole}}}(...)$
SOPE[1 , 0, pole] <— $\frac{1}{Z_2{}^{\text{pole}}}(...)$

SOPE[4, 0, 3]:=newsc12[1]DSpNewPhi2[0] + sc12[2]SpD[DJ[0], DSpNewPhi3H[0]]

SOPE[4, 0, 2]:=$\frac{3}{4}$(newsc12[1]DSpNewPhi2[0] + sc12[2]SpD[DJ[0], DSpNewPhi3H[0]])′+

sc12[3] $\sum_{i=1}^{3}$ SpD[DJ[i], DSpNewPhi3H[i]] + sc12[4]SpD[DJ[0], DSpNewPhi3H[0]′]+

sc12[5]SpD[DJ[0]′, DSpNewPhi3H[0]]+

sc12[6] $\sum_{\alpha=1}^{3}\sum_{i=1}^{3}$ Delta[i, α]DSpNewPhi5H[i, {α}] + sc12[7]DSpNewPhi3H[3, 2, 1]+

$\text{newsc12[8]DSpNNewPhi3[0]}$

$\text{SOPE}[4,0,1]{:=}\text{sc12[9]SpD}[\text{DJ}[3,2,1],\text{DSpNewPhi2}[0]]+\text{sc12[10]DSpNewPhi2}[0]''+$

$\sum_{i=1}^{3}(\text{sc12[11]SpD}[\text{DJ}[0],\text{SpD}[\text{TLDJ}[i],\text{DSpNewPhi3H}[i]]])+$

$\text{sc12[12]}\sum_{i=1}^{3}(\text{SpD}[\text{DJ}[0],\text{SpD}[\text{DJ}[i],\text{SpD}[\text{DJ}[i],\text{DSpNewPhi3H}[0]]]])+$

$\text{sc12[13]}\sum_{i=1}^{3}(\text{SpD}[\text{DJ}[0],\text{SpD}[\text{DJ}[i],\text{DSpNewPhi2}[i]]])+$

$\text{sc12[14]}\sum_{\alpha=1}^{3}\sum_{i=1}^{3}(\text{Delta}[i,\alpha]\text{SpD}[\text{DJ}[0],\text{SpD}[\text{DJ}[i],\text{DSpNewPhi5H}[0,\{\alpha\}]]])+$

$\text{sc12[15]}\sum_{i=1}^{3}(\text{SpD}[\text{DJ}[0],\text{SpD}[\text{DJ}[i],\text{TLDSpNewPhi3H}[i]]])+$

$\text{sc12[16]}\sum_{\alpha=1}^{3}\sum_{i=1}^{3}(\text{Delta}[i,\alpha]\text{SpD}[\text{DJ}[0],\text{DSpNewPhi3}[i,\{\alpha\}]])+$

$\sum_{i=1}^{3}\text{sc12[17]SpD}[\text{TLDJ}[i],\text{DSpNewPhi2}[i]]+$

$\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\text{Delta}[i,\alpha]$

$(\text{sc12[18]SpD}[\text{TLDJ}[i],\text{DSpNewPhi5H}[0,\{\alpha\}]]+\text{sc12[19]DSpNewPhi5H}[i,\{\alpha\}]')+$

$\sum_{i=1}^{3}\text{sc12[20]SpD}[\text{TLDJ}[i],\text{TLDSpNewPhi3H}[i]]+\text{sc12[21]DSpNewPhi3H}[3,2,1]'+$

$\sum_{i=1}^{3}(\text{sc12[22]SpD}[\text{DJ}[i],\text{SpD}[\text{TLDJ}[i],\text{DSpNewPhi3H}[0]]]+$

$\text{sc12[23]SpD}\,[\text{DJ}[i],\text{DSpNewPhi3H}[i]'])+$

$\text{sc12[24]}\sum_{i=1}^{3}(\text{SpD}[\text{DJ}[i],\text{SpD}[\text{DJ}[i],\text{DSpNewPhi2}[0]]])+$

sc12[25]

$\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{\beta=1}^{3}\text{Delta}[i,\alpha]\text{Tmatirx}[j,\alpha,\beta](\text{SpD}[\text{DJ}[i],\text{DSpNewPhi5H}[j,\{\beta\}]])+$

$\text{sc12[26]}\sum_{i=1}^{3}(\text{SpD}[\text{DJ}[i],\text{TLDSpNewPhi2}[i]])+$

$\text{sc12[27]}\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\text{Delta}[i,\alpha](\text{SpD}[\text{DJ}[i],\text{DSpNewPhi3}[0,\{\alpha\}]])+$

$\text{sc12[28]SpD}\,[\text{DJ}[0]',\text{SpD}[\text{DJ}[0],\text{DSpNewPhi2}[0]]]+$

$\sum_{i=1}^{3}(\text{sc12[29]SpD}\,[\text{DJ}[i]',\text{DSpNewPhi3H}[i]])+\text{sc12[30]SpD}\,[\text{DJ}[0]'',\text{DSpNewPhi3H}[0]]+$

$\text{sc12[31]SpD}\,[\text{DJ}[0]',\text{DSpNewPhi3H}[0]]'+\text{sc12[32]SpD}[\text{DJ}[0],\text{DSpNewPhi3H}[0]]''+$

$\text{sc12[33]}\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\text{Delta}[i,\alpha]\text{DSpNewPhi7H}[i,\{\alpha\}]+$

$\text{sc12[34]}\sum_{\alpha=1}^{3}\sum_{i=1}^{3}(\text{Delta}[i,\alpha]\text{TLDSpNewPhi3}[i,\{\alpha\}])+\text{newsc12[35]DSpNewPhi4}[0]+$

$\frac{3}{10}(\text{newsc12[1]DSpNewPhi2}[0]+\text{sc12[2]SpD}[\text{DJ}[0],\text{DSpNewPhi3H}[0]])''+$

$\frac{2}{3}$

$\Big(\text{sc12[3]}\sum_{i=1}^{3}\text{SpD}[\text{DJ}[i],\text{DSpNewPhi3H}[i]]+\text{sc12[4]SpD}[\text{DJ}[0],\text{DSpNewPhi3H}[0]']+$

$\text{sc12[5]SpD}[\text{DJ}[0]',\text{DSpNewPhi3H}[0]]+$

$\text{sc12[6]}\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\text{Delta}[i,\alpha]\text{DSpNewPhi5H}[i,\{\alpha\}]+\text{sc12[7]DSpNewPhi3H}[3,2,1]+$

$\text{newsc12[8]DSpNNewPhi3}[0])'$

$SOPE[3, id\_, 2]{:=}newsc12[36] \sum_{\alpha=1}^{3} Delta[id, \alpha]DSpNewPhi5H[0, \{\alpha\}]+$

$sc12[37]DSpNewPhi2[id] + sc12[38]TLDSpNewPhi3H[id]+$

$sc12[39]SpD[DJ[0], DSpNewPhi3H[id]] + sc12[40]SpD[DJ[id], DSpNewPhi3H[0]]$

$SOPE[3, id\_, 1]{:=}sc12[41]SpD[DJ[0], SpD[TLDJ[id], DSpNewPhi3H[0]]]$

$+sc12[42]SpD[DJ[0], SpD[DJ[id], DSpNewPhi2[0]]]+$

$\sum_{j=1}^{3} \sum_{k=1}^{3} Epsilon[id, j, k](sc12[43]SpD[DJ[0], SpD[DJ[j], DSpNewPhi3H[k]]])-$

$sc12[44] * \sum_{\alpha=1}^{3} \sum_{j=1}^{3} \sum_{\beta=1}^{3}(Delta[id, \alpha] * Tmatirx[j, \alpha, \beta]SpD[DJ[0], DSpNewPhi5H[j, \{\beta\}]])+$

$sc12[45]SpD[DJ[0], TLDSpNewPhi2[id]]+$

$sc12[46] * \sum_{\alpha=1}^{3} Delta[id, \alpha] * SpD[DJ[0], DSpNewPhi3[0, \{\alpha\}]]+$

$sc12[47]SpD[TLDJ[id], DSpNewPhi2[0]] + sc12[48]DSpNewPhi2[id]'+$

$\sum_{j=1}^{3} \sum_{k=1}^{3} Epsilon[id, j, k]$

$(sc12[49]SpD[TLDJ[j], DSpNewPhi3H[k]]+$

$sc12[50] \sum_{l=1}^{3} Epsilon[j, k, l]TLDSpNewPhi3H[l]'\Big) +$

$\sum_{j=1}^{3} \sum_{k=1}^{3} Epsilon[id, j, k](sc12[51]SpD[DJ[j], DSpNewPhi2[k]])+$

$\sum_{j=1}^{3} \sum_{k=1}^{3} Epsilon[id, j, k](sc12[52] * SpD[DJ[j], TLDSpNewPhi3H[k]])+$

$(sc12[53]SpD\,[DJ[0]', DSpNewPhi3H[id]] + sc12[54]SpD[DJ[0], DSpNewPhi3H[id]]') +$

$(sc12[55]SpD\,[DJ[id]', DSpNewPhi3H[0]] + sc12[56]SpD[DJ[id], DSpNewPhi3H[0]]') +$

$sc12[57] * \sum_{\alpha=1}^{3} \sum_{j=1}^{3} \sum_{\beta=1}^{3} Delta[id, \alpha]Tmatirx[j, \alpha, \beta]DSpNewPhi3[j, \{\beta\}]+$

$sc12[58]DSpNNewPhi3[id]-$

$\sum_{\alpha=1}^{3} \sum_{j=1}^{3} \sum_{\beta=1}^{3} Delta[id, \alpha]Tmatirx[j, \alpha, \beta]$

$(sc12[59]TLDSpNewPhi5H[j, \{\beta\}]+$

$sc12[60] \sum_{\gamma=1}^{3} Tmatirx[j, \beta, \gamma] * DSpNewPhi5H[0, \{\gamma\}]'\Big) +$

$newsc12[61] \sum_{\alpha=1}^{3} Delta[id, \alpha]DSpNewPhi7H[0, \{\alpha\}]+$

$\frac{3}{5} \Big(newsc12[36] \sum_{\alpha=1}^{3} Delta[id, \alpha]DSpNewPhi5H[0, \{\alpha\}] + sc12[37]DSpNewPhi2[id]$

$+sc12[38]TLDSpNewPhi3H[id] + sc12[39]SpD[DJ[0], DSpNewPhi3H[id]]+$

$sc12[40]SpD[DJ[id], DSpNewPhi3H[0]])'$

$SOPE[2, id\_, 2]{:=}sc12[62]DSpNewPhi3H[id];$

$SOPE[2, id\_, 1]{:=}newsc12[63] \sum_{\alpha=1}^{3}(Delta[id, \alpha]DSpNewPhi3[0, \{\alpha\}])+$

$sc12[64] \Big(\sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \sum_{j=1}^{3}(Delta[id, \alpha]Tmatirx[j, \alpha, \beta]DSpNewPhi5H[j, \{\beta\}])\Big) +$

$\text{sc12[65]TLDSpNewPhi2[id]} + \text{sc12[66]SpD[DJ[0], DSpNewPhi2[id]]}+$

$\text{sc12[67]} \sum_{\alpha=1}^{3} \text{Delta}[\text{id}, \alpha]\text{SpD[DJ[0], DSpNewPhi5H[0}, \{\alpha\}]]+$

$\text{sc12[68]SpD[DJ[0], SpD[DJ[id], DSpNewPhi3H[0]]]}+$

$\text{sc12[69]SpD[TLDJ[id], DSpNewPhi3H[0]]} + \text{sc12[70]SpD[DJ[id], DSpNewPhi2[0]]}+$

$\text{sc12[71]} \sum_{j=1}^{3} \left(\sum_{k=1}^{3} \text{Epsilon}[\text{id}, j, k](\text{SpD[DJ}[j], \text{DSpNewPhi3H}[k]])\right) +$

$\text{sc12[72]DSpNewPhi3H[id]}';$

$\text{SOPE}[1, 0, 2] = \text{newsc12[73]DSpNewPhi3H[0]};$

$\text{SOPE}[1, 0, 1] = \text{sc12[74]DSpNewPhi3H[0]}';$

$\Phi^{\left(\frac{3}{2}\right)}\Phi^{(2)}$, Jacobi Id.

$\text{Xn} = \{\{1, 0\}, \{2, 1\}, \{2, 2\}, \{2, 3\}, \{3, 1\}, \{3, 2\}, \{3, 3\}, \{4, 0\}\};$

$\text{COM1sp} = \left\{\text{Phi3Hcp1}\left[\frac{3}{2}\right], \text{Phi3Hcp2}[1, 2], \text{Phi3Hcp2}[2, 2], \text{Phi3Hcp2}[3, 2],\right.$

$\left.\text{Phi3Hcp3}\left[1, \frac{5}{2}\right], \text{Phi3Hcp3}\left[2, \frac{5}{2}\right], \text{Phi3Hcp3}\left[3, \frac{5}{2}\right], \text{Phi3Hcp4}[3]\right\};$

$\text{COM2sp} = \left\{\text{Phi2cp1}[2], \text{Phi2cp2}\left[1, \frac{5}{2}\right], \text{Phi2cp2}\left[2, \frac{5}{2}\right], \text{Phi2cp2}\left[3, \frac{5}{2}\right],\right.$

$\left.\text{Phi2cp3}[1, 3], \text{Phi2cp3}[2, 3], \text{Phi2cp3}[3, 3], \text{Phi2cp4}\left[\frac{7}{2}\right]\right\};$

Do[OPE[COM1sp[[Z1]], COM2sp[[Z2]]] =

MakeOPE[{OPESimplify[SCPOPE[DeSuperSpace[Xn[[Z1]], Xn[[Z2]], 7]], Simplify],

OPESimplify[SCPOPE[DeSuperSpace[Xn[[Z1]], Xn[[Z2]], 6]], Simplify],

OPESimplify[SCPOPE[DeSuperSpace[Xn[[Z1]], Xn[[Z2]], 5]], Simplify],

OPESimplify[SCPOPE[DeSuperSpace[Xn[[Z1]], Xn[[Z2]], 4]], Simplify],

OPESimplify[SCPOPE[DeSuperSpace[Xn[[Z1]], Xn[[Z2]], 3]], Simplify],

OPESimplify[SCPOPE[DeSuperSpace[Xn[[Z1]], Xn[[Z2]], 2]], Simplify],

OPESimplify[SCPOPE[DeSuperSpace[Xn[[Z1]], Xn[[Z2]], 1]], Simplify]}];

Print[COM1sp[[Z1]], COM2sp[[Z2]]], {Z1, 8}, {Z2, 8}]

$\text{J3Com} = \{\Psi, J[1], J[2], J[3], G[1], G[2], G[3], \text{Ts}\};$

Clear[TCH1, UTCH1]

$\text{TCH1} = \text{Table}[x, \{\text{n1}, 8\}, \{\text{n2}, 1\}, \{\text{n3}, 8\}];$

Do[TCH1[[n1, n2, n3]] =

Simplify[GetCoefficients[OPEJacobi[J3Com[[n1]], COM1sp[[n2]], COM2sp[[n3]]]]],

{n1, 8}, {n2, 1}, {n3, 8}]

```
UTCH1 = Union[Flatten[TCH1]]

SUB = Simplify[Solve[UTCH1 == 0, Flatten[{Array[sc12, 74]}]]]
```

Jacobi check

{{{0}}} —> Ok

```
Clear[TCH2, UTCH2]

TCH2 = Table[x, {n1, 8}, {n2, 8}, {n3, 8}];

Do[TCH2[[n1, n2, n3]] =

Simplify[GetCoefficients[OPEJacobi[J3Com[[n1]], COM1sp[[n2]], COM2sp[[n3]]]/.

SUB]];

If[n3 == 8,

Print[

{{Union[

Flatten[

Simplify[GetCoefficients[

OPEJacobi[J3Com[[n1]], COM1sp[[n2]], COM2sp[[n3]]]/.SUB]]]]},

n1, n2, n3}]], {n1, 8}, {n2, 8}, {n3, 8}]

UTCH2 = Union[Flatten[TCH2]]
```