Global QCD analyses of meson structures

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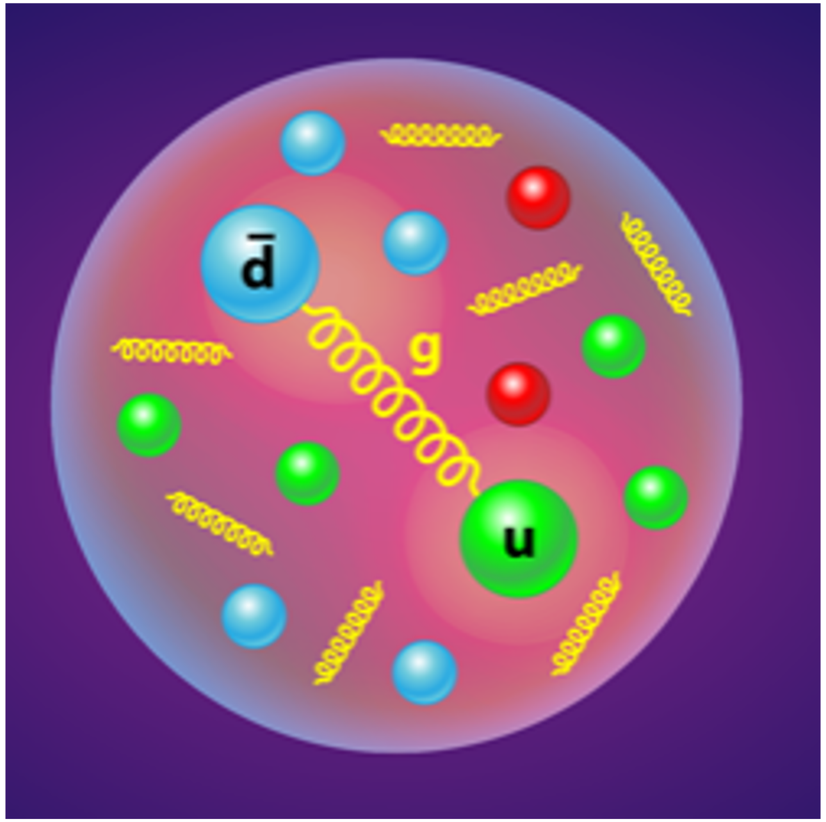
Light Cone 2022, September 19th, 2022

Pions

- Pion presents itself as a dichotomy

1. It is the Goldstone boson associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
2. Made up of quark and antiquark constituents



Large momentum fraction behavior

- Many theoretical papers have studied the behavior of the valence quark distribution as $x \to 1$ and
- Debate whether $q_v^\pi(x \to 1) \sim (1-x)$ or $(1-x)^2$

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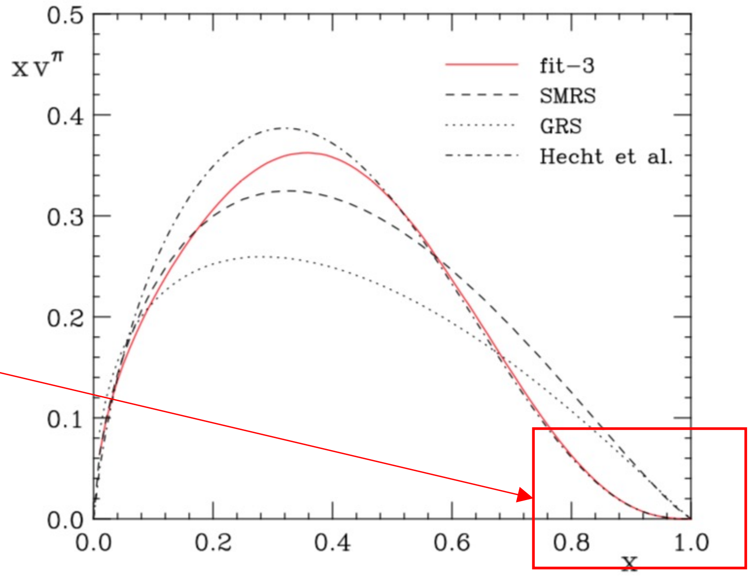
Large-x_π behavior

- Generally, the parametrization lends a behavior as $x \to 1$ of the valence quark PDF of $q_v(x) \propto (1-x)^\beta$
- For a fixed order analysis, find $\beta \approx 1$
- Aicher, Schaefer Vogelsang (ASV) found $\beta = 2$ with threshold resummation

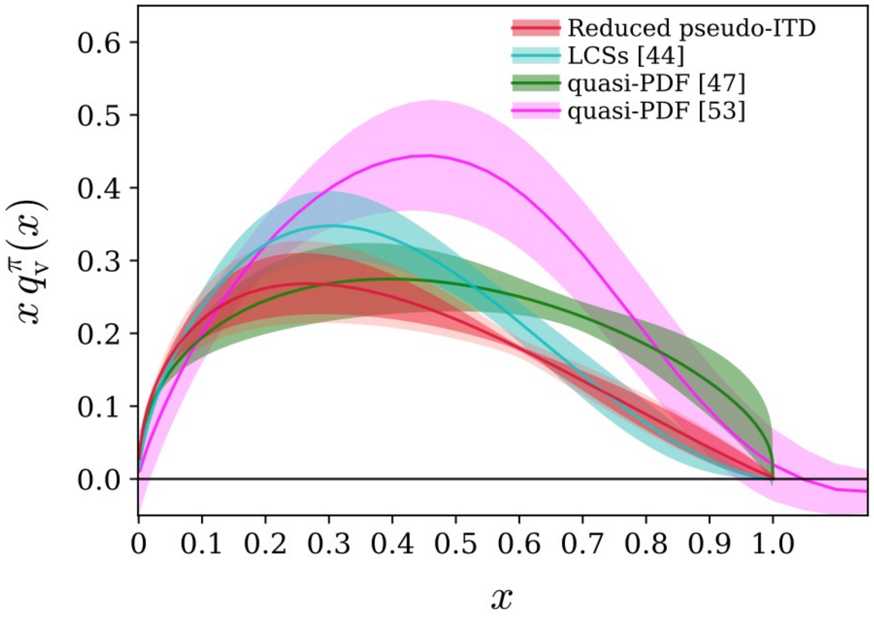


ASV valence PDF

Phys. Rev. Lett. **105**, 114023 (2011).

Lattice QCD Activity

- Simulations on the lattice have been done to investigate this structure



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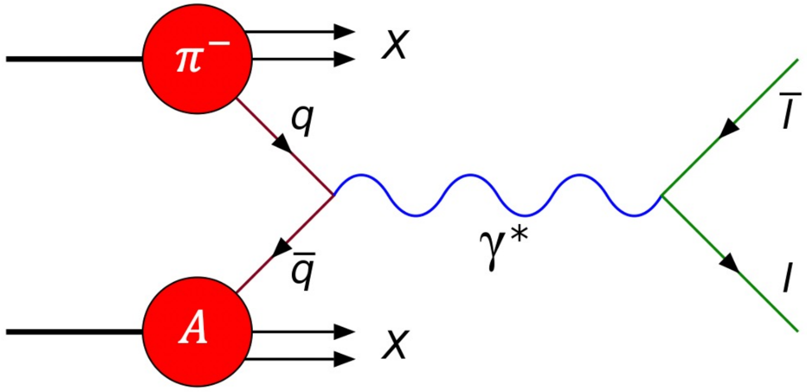
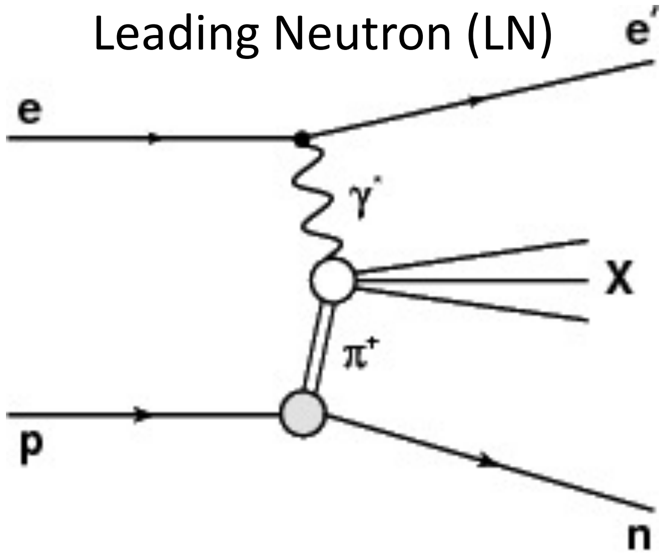
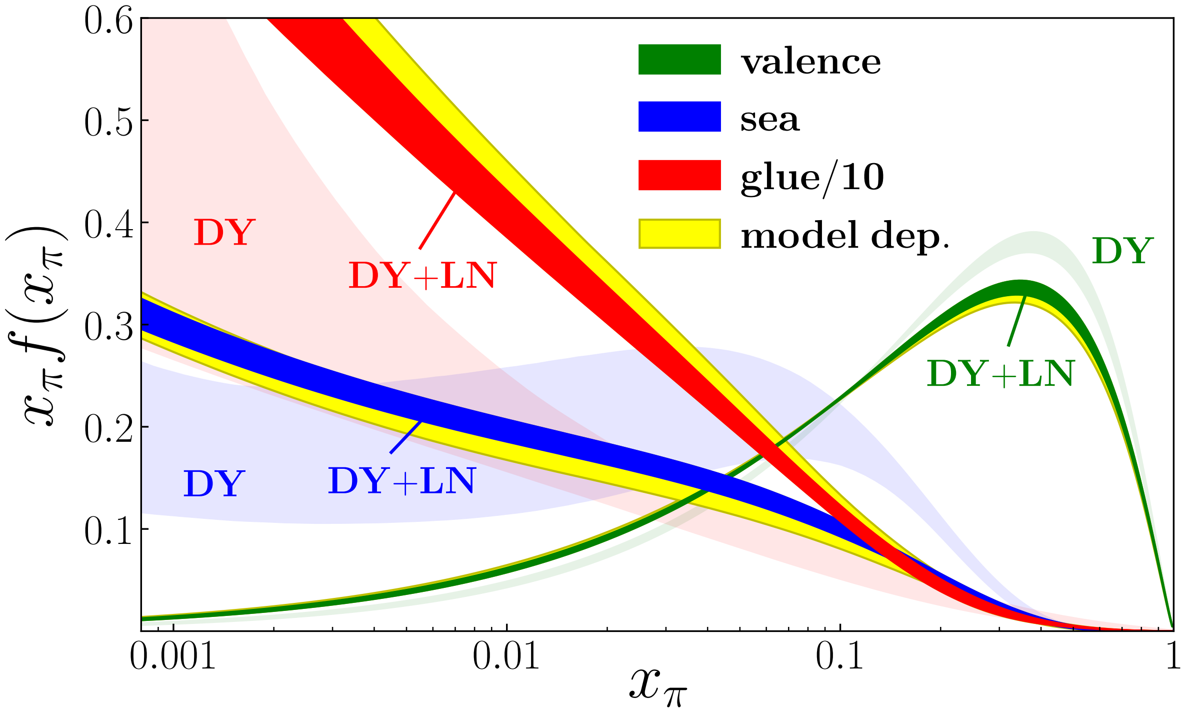
Experiments to probe pion structure

Drell-Yan (DY)



Leading Neutron (LN)





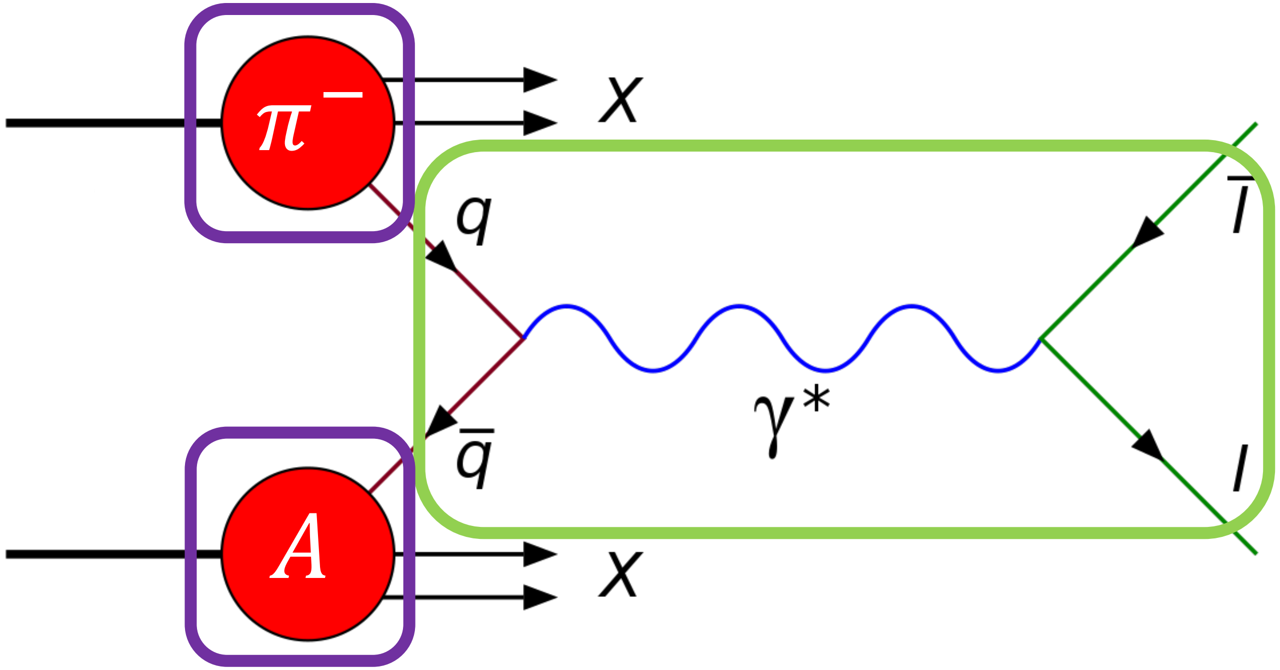
PHYSICAL REVIEW LETTERS **121,** 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

Drell-Yan (DY)



$$\sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi, \mu) \otimes f_j^{A}(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Issues with Perturbative Calculations

$$\hat{\sigma} \sim \delta(1-z) + \alpha_S (\log(1-z))_+$$

$$\hat{\sigma} \sim \delta(1-z)[1 + \alpha_S \log(1-\tau)]$$

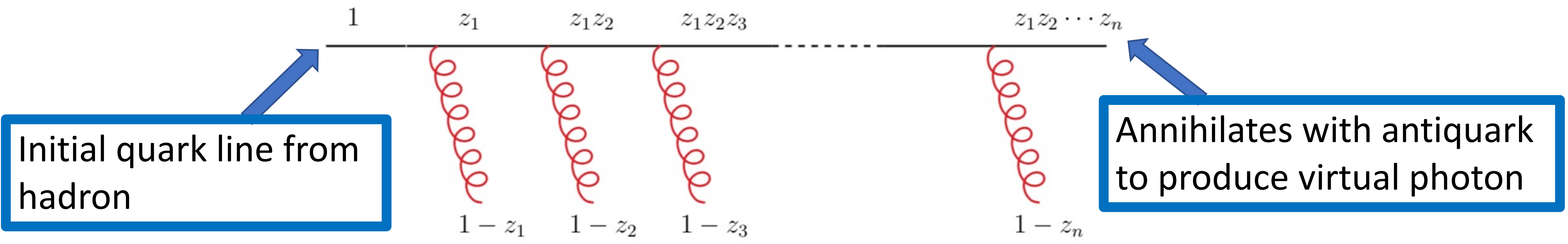
$$\tau = \frac{Q^2}{S}$$

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{\hat{x}_\pi \hat{x}_A}$$

$\hat{S}$ is the center of mass momentum squared of incoming partons

- If τ is large, can potentially spoil the perturbative calculation
- Improvements can be made by resumming $\log(1-z)_+$ terms

Threshold Resummation



Significant contributions to cross section occur in soft gluon emissions and follow the pattern

$$d\hat{\sigma}^{q\bar{q}}_{N^k LO} \propto \alpha_S^k \frac{\ln^{2k-1}(1-z)}{1-z} + \ldots$$

Methods of resummation – Mellin-Fourier

- Threshold resummation is done in conjugate space

$$\sigma_{\rm MF}(N,M) \equiv \int_0^1 {\rm d}\tau\, \tau^{N-1} \int_{\log\sqrt{\tau}}^{\log\frac{1}{\sqrt{\tau}}} {\rm d}Y\, e^{iMY}\, \frac{{\rm d}^2\sigma}{{\rm d}\tau\,{\rm d}Y},$$

Two choices occur when isolating the hard part

$$\hat{\sigma}_{\rm MF}(N,M) = \int_0^1 {\rm d}z\, z^{N-1} \cos\left(\frac{M}{2}\log z\right) \frac{{\rm d}^2\hat{\sigma}}{{\rm d}\tau\,{\rm d}Y}(z)$$

Keep cosine intact – "cosine" method

Keep the first order term in the expansion – $\cos\left(\frac{M}{2}\log z\right) \approx 1$ "expansion" method

Method of resummation – double Mellin

- Alternatively, perform a double Mellin transform

$$\sigma_{\rm DM}(N,M) \equiv \int_0^1 {\rm d}x_\pi^0\, (x_\pi^0)^{N-1} \int_0^1 {\rm d}x_A^0\, (x_A^0)^{M-1} \frac{{\rm d}^2\sigma}{{\rm d}\tau{\rm d}Y}.$$

where $x_\pi^0 = \sqrt{\tau} e^Y, \quad x_A^0 = \sqrt{\tau} e^{-Y}$

- Double Mellin transform is theoretically cleaner and sums up terms appropriately

Deriving resummation expressions – MF

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{\hat{x}_\pi \hat{x}_A}$$

Claim: yellow terms give rise to the resummation expressions

$$y = \frac{\frac{\hat{x}_\pi}{\hat{x}_A} e^{-2Y} - z}{(1-z)(1+\frac{\hat{x}_\pi}{\hat{x}_A} e^{-2Y})}$$

$$\frac{C_{q\bar{q}}}{e_q^2} = \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right]$$
$$+\frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ +1-z\right]\right.$$
$$\left.+\frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right)-2(1-z)\right]\right\}$$

Claim: Red terms are power suppressed in $(1-z)$ and wouldn't contribute to the same order as the yellow terms

Generalized Threshold resummation

G. Lustermans, J. K. L. Michel, and F. J. Tackmann, arXiv:1908.00985 [hep-ph].

- Write the (z, y) coefficients in terms of (z_a, z_b), and for the red terms, you get:

$$z_a = \frac{x_\pi^0}{\hat{x}_\pi}$$

$$z_b = \frac{x_A^0}{\hat{x}_A}$$

$$\mathrm{d}z\,\mathrm{d}y\,\frac{1}{1-z}\left(\frac{1}{y}+\frac{1}{1-y}\right) = \mathrm{d}z_a\mathrm{d}z_b\,\frac{1}{(1-z_a)(1-z_b)}\left[1+\mathcal{O}(1-z_a, 1-z_b)\right].$$

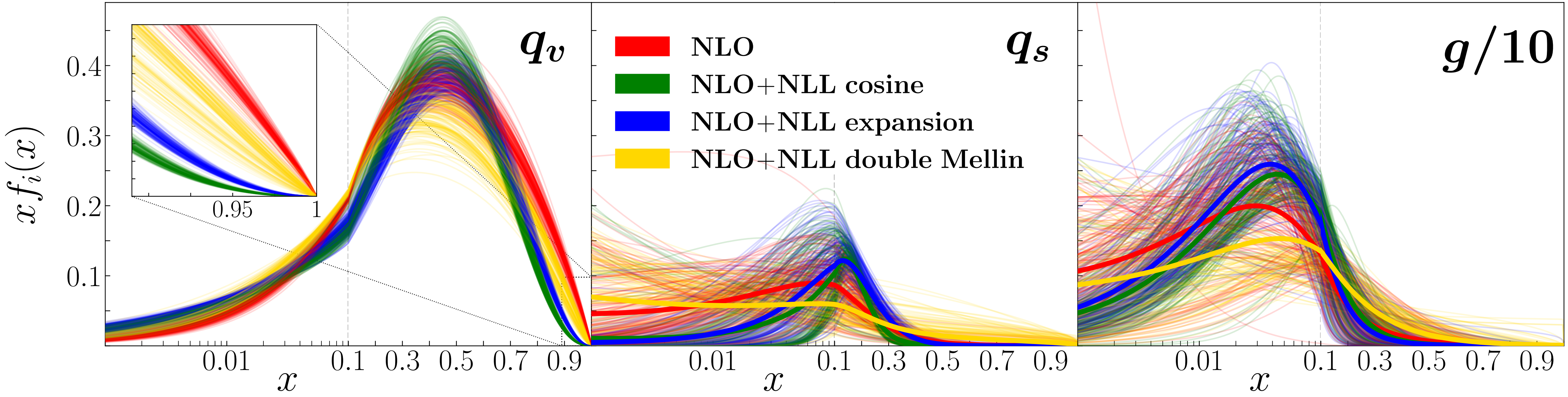
- This is *not* power suppressed in $(1 - z_a)$ or $(1 - z_b)$ but instead the same order as the leading power in the soft limit
- Generalized threshold resummation in the soft limit does not agree with the MF methods

Including threshold resummation in DY - Resulting PDFs

PHYSICAL REVIEW LETTERS **127**, 232001 (2021)

Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]

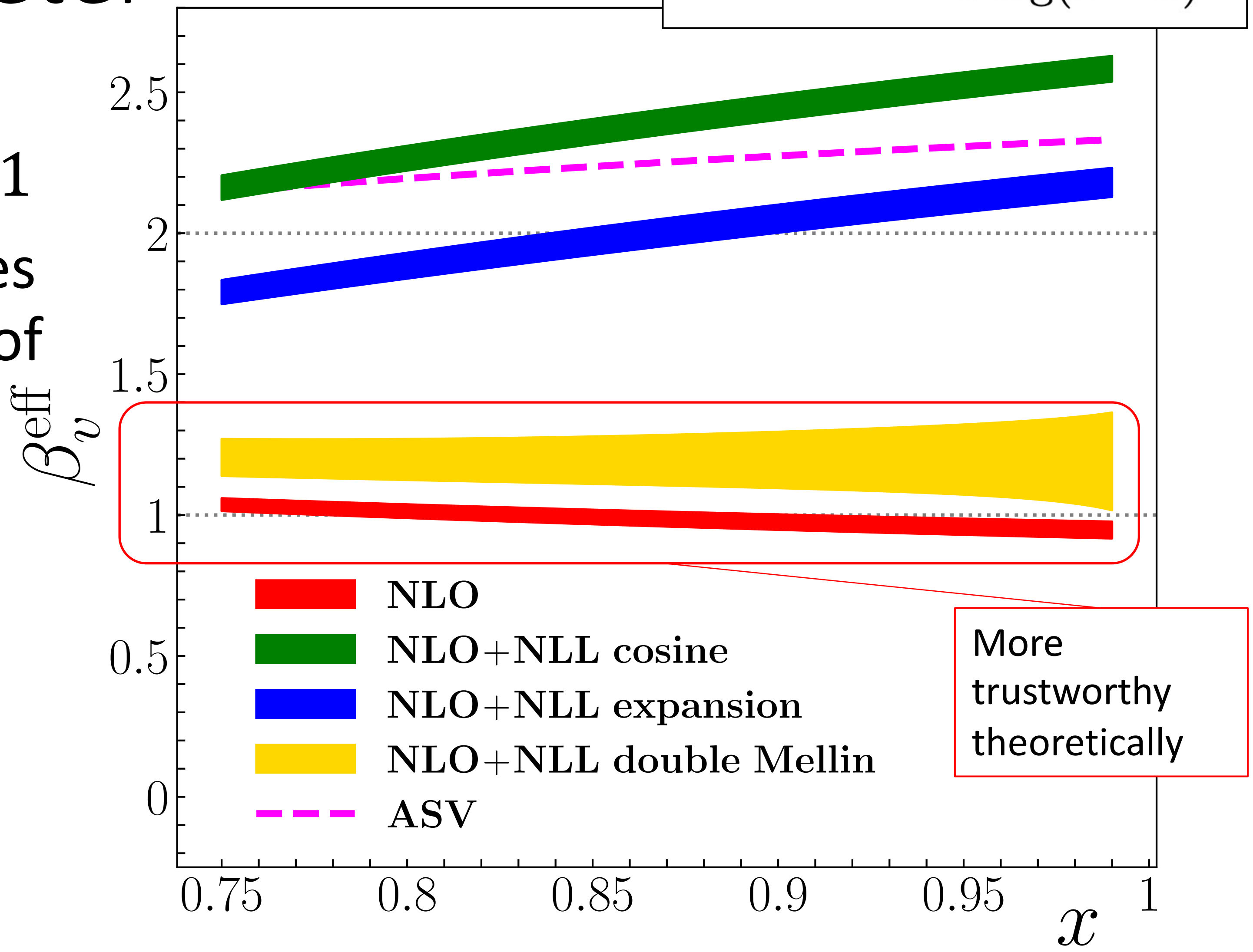


- Large x behavior of q_v **highly sensitive** to method of resummation

Effective β_v parameter

$$\beta_v^{\text{eff}}(x,\mu) = \frac{\partial \log |q_v(x,\mu)|}{\partial \log(1-x)}.$$

- $q_v(x) \sim (1-x)^{\beta_v^{\text{eff}}}$ as $x \to 1$
- Threshold resummation does not give universal behavior of β_v^{eff}
- NLO and double Mellin give $\beta_v^{\text{eff}} \approx 1$ – theoretically cleaner
- Cosine and Expansion give $\beta_v^{\text{eff}} > 2$



Fitting the Data and Systematic Corrections

Valence quark distribution in pion

Wilson coefficients for matching

$$\mathrm{Re}\,\mathfrak{M}(\nu, z^2) = \int_0^1 \mathrm{d}x \; q_v(x, \mu_{\mathrm{lat}}) \, \mathcal{C}^{\mathrm{Rp-ITD}}\left(x\nu, z^2, \mu_{\mathrm{lat}}\right) + z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu) + \ldots$$

Integration lower bound is 0

Other potential systematic corrections the data is not sensitive to

<u>Systematic corrections to parametrize</u>

- $z^2 B_1(\nu)$: power corrections
- $\frac{a}{|z|} P_1(\nu)$: lattice spacing errors
- $e^{-m_\pi (L-z)} F_1(\nu)$: finite volume corrections

PHYSICAL REVIEW D **105**, 114051 (2022)

Complementarity of experimental and lattice QCD data on pion parton distributions

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(Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations)

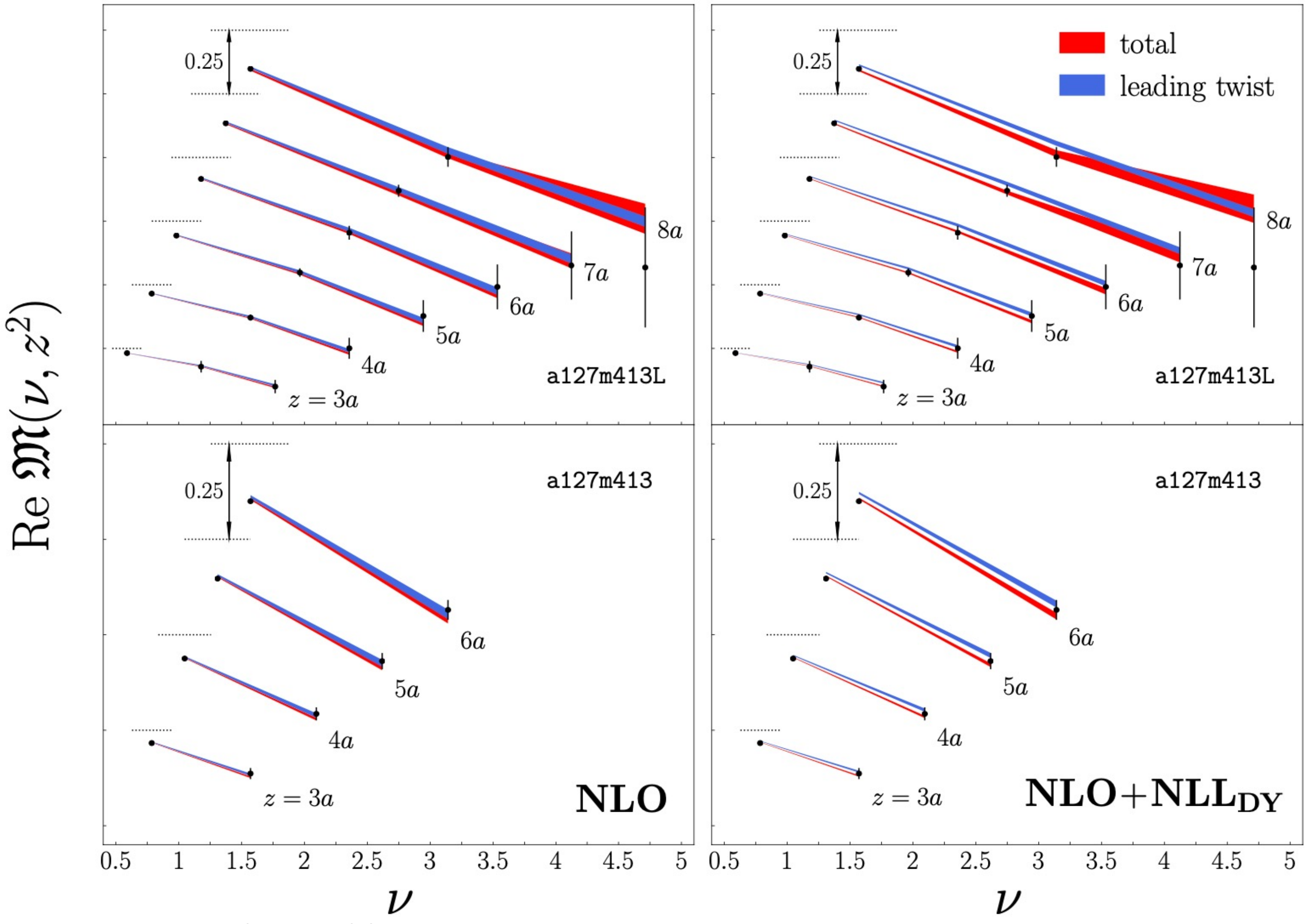
Goodness of fit

- Scenario A: experimental data alone
- Scenario B: experimental + lattice, no systematics
- Scenario C: experimental + lattice, with systematics

			Scenario A		Scenario B		**Scenario C**	
			NLO	+NLL$_{\rm DY}$	NLO	+NLL$_{\rm DY}$	NLO	+NLL$_{\rm DY}$
Process	Experiment	$N_{\rm dat}$	$\overline{\chi}^2$		$\overline{\chi}^2$		$\overline{\chi}^2$	
DY	E615	61	0.84	0.82	0.83	0.82	**0.84**	**0.82**
	NA10 (194 GeV)	36	0.53	0.53	0.52	0.54	**0.52**	**0.55**
	NA10 (286 GeV)	20	0.80	0.81	0.78	0.79	**0.78**	**0.87**
LN	H1	58	0.36	0.35	0.39	0.39	**0.37**	**0.37**
	ZEUS	50	1.56	1.48	1.62	1.69	**1.58**	**1.60**
Rp-ITD	a127m413L	18	–	–	1.04	1.06	**1.04**	**1.06**
	a127m413	8	–	–	1.98	2.63	**1.14**	**1.42**
Total		**251**	**0.82**	**0.80**	**0.89**	**0.92**	**0.85**	**0.87**

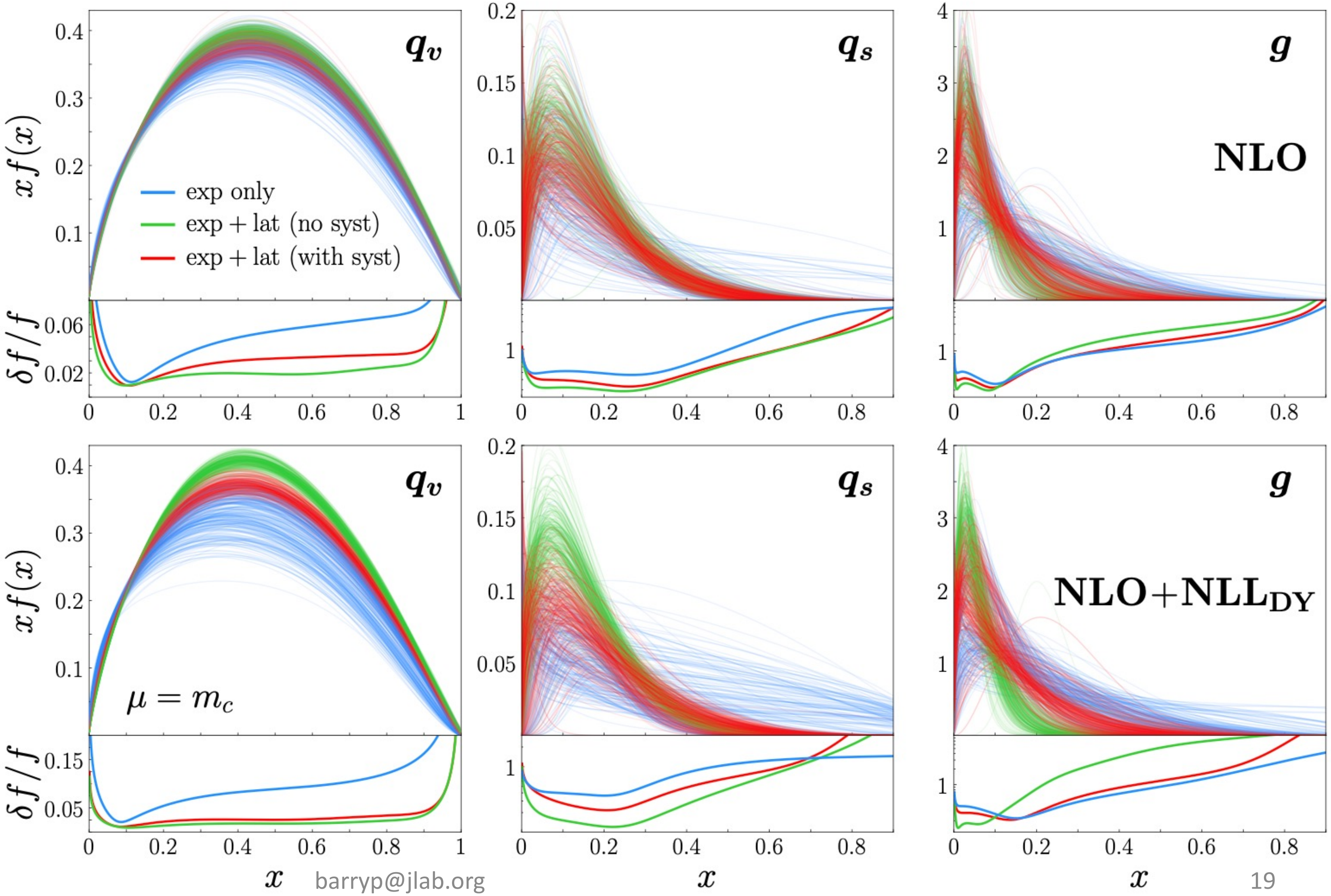
Agreement with the data

- Results from the full fit and isolating the leading twist term
- Difference between bands is the systematic correction



Resulting PDFs

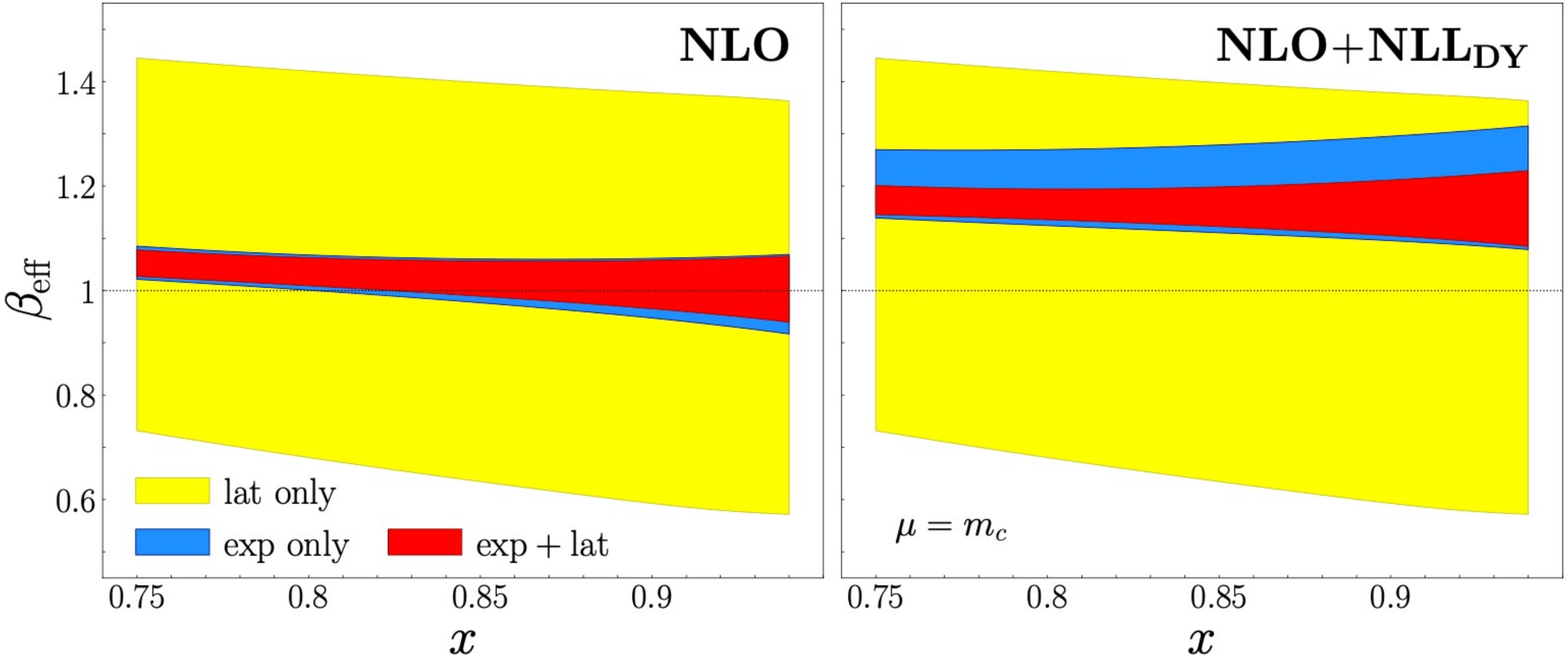
- PDFs and relative uncertainties
- Including lattice reduces uncertainties
- NLO+NLL$_{DY}$ changes a lot – unstable under new data



Effective β from $(1-x)^{\beta_{\mathrm{eff}}}$

$$\beta_{\mathrm{eff}}(x,\mu) = \frac{\partial \log |q_v(x,\mu)|}{\partial \log(1-x)}$$

Calculations from QCD do not predict $\beta_{\mathrm{eff}} = 2$



Another direction – small-q_T data

- In small-$q_{\rm T}$ region, use the Collins-Soper-Sterman (CSS) formalism and b_* prescription

Can these data constrain the pion collinear PDF?

$$\frac{d\sigma}{dQ^2\,dy\,dq_{\rm T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} H_{j\bar{j}}^{\rm DY}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{d^2\boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}}$$
$$\times\, e^{-g_{j/A}(x_A,b_{\rm T};b_{\rm max})} \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\rm PDF}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\, e^{-g_{\bar{j}/B}(x_B,b_{\rm T};b_{\rm max})} \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\rm PDF}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\, \exp\left\{-g_K(b_{\rm T};b_{\rm max})\ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

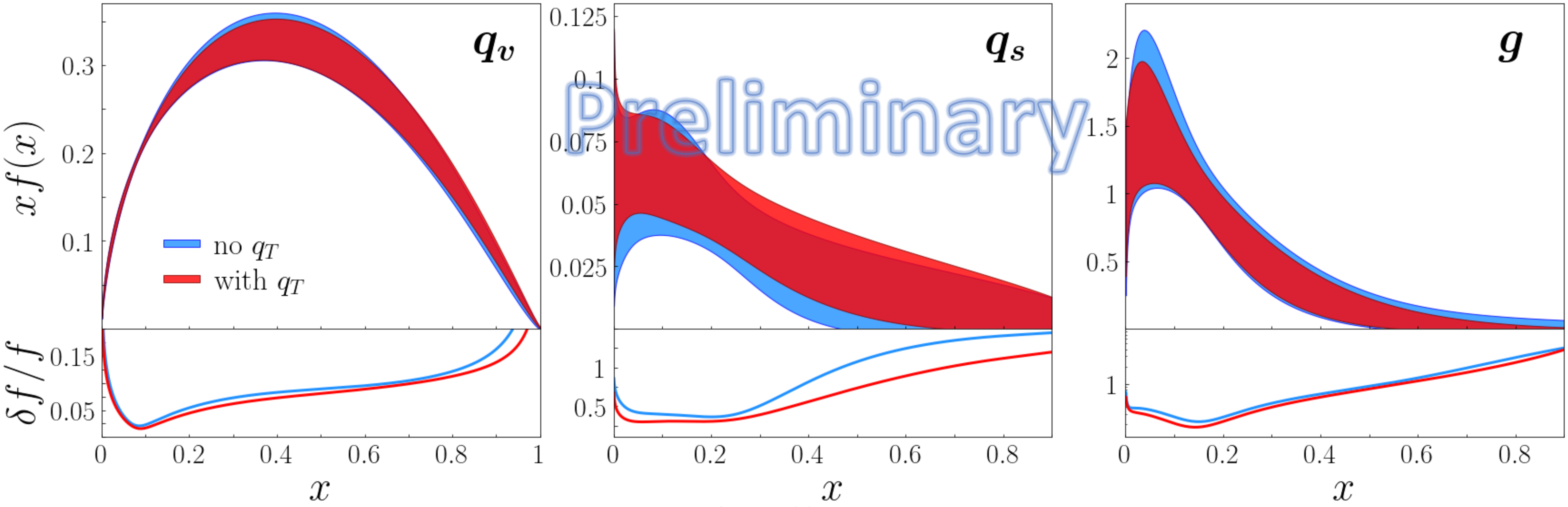
Non-perturbative pieces

Perturbative pieces

Non-perturbative piece of the CS kernel

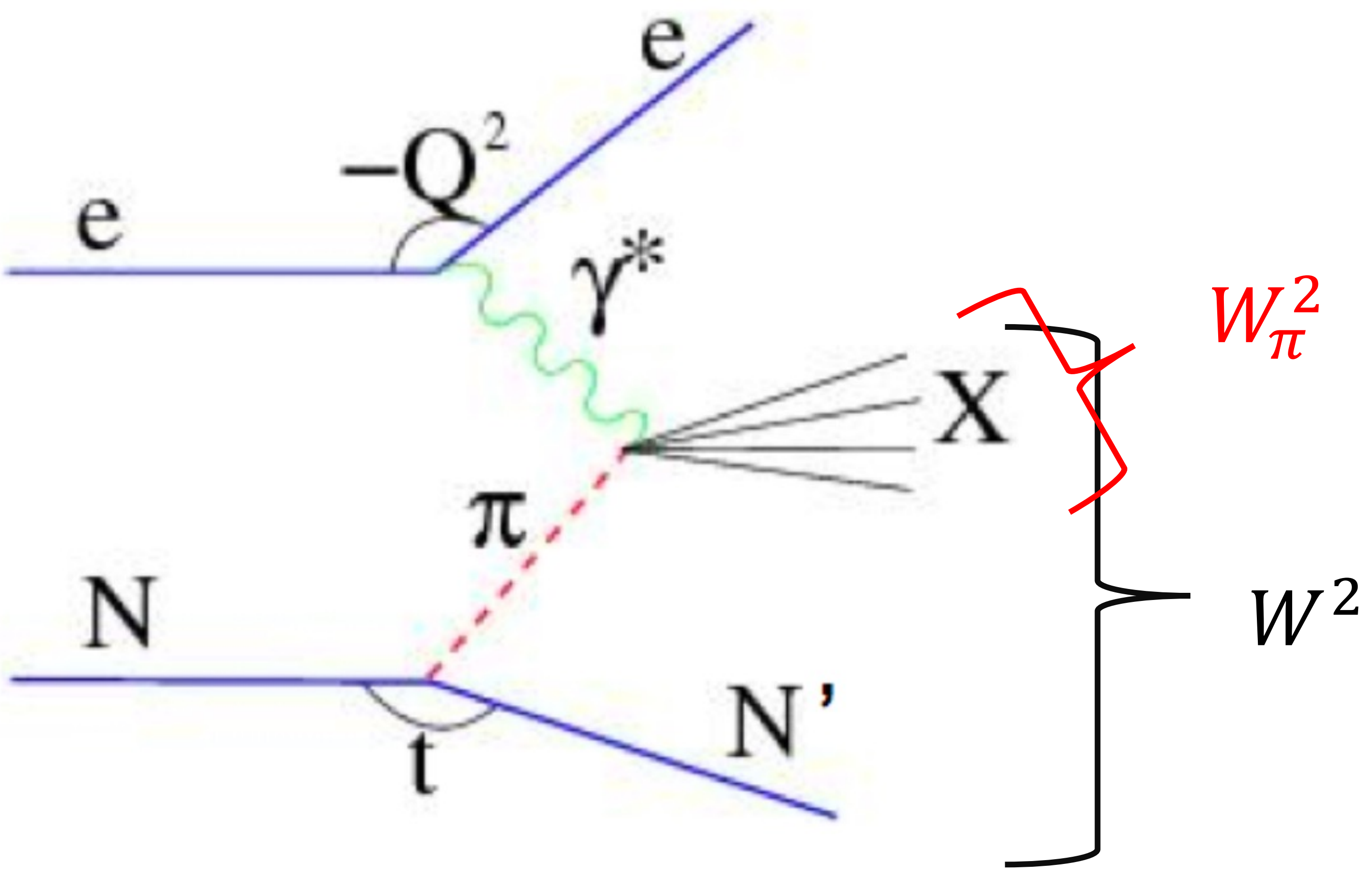
Impact on PDFs using double Mellin PDFs

- Slight reduction in uncertainties
- Overall very consistent with totally collinear analysis

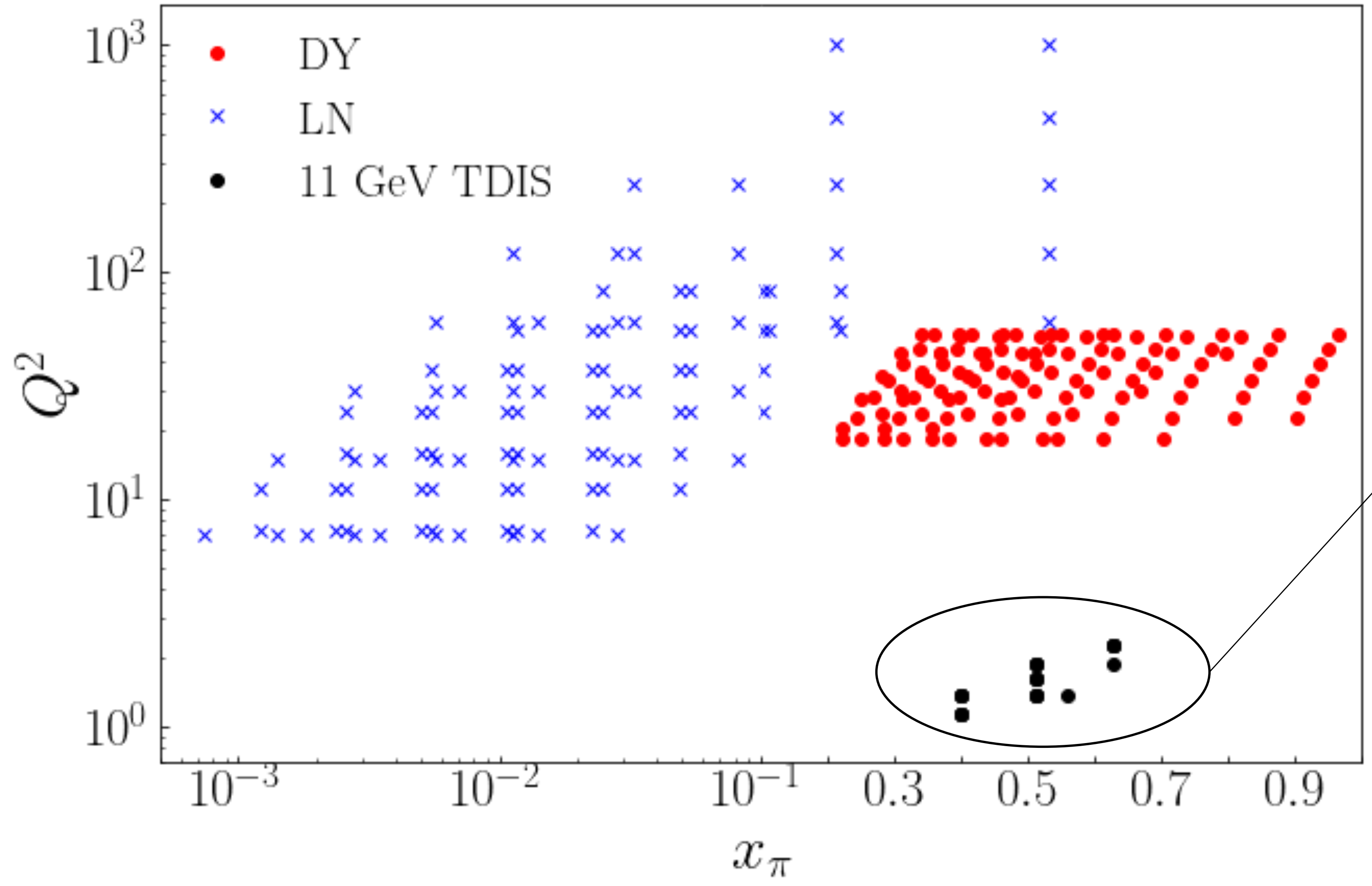


Future JLab TDIS: Sullivan process and W_π^2

- Impose kinematic cuts on experimental data such as lower limit on the totally *inclusive* W^2
- What about the W_π^2?



Total pion kinematics

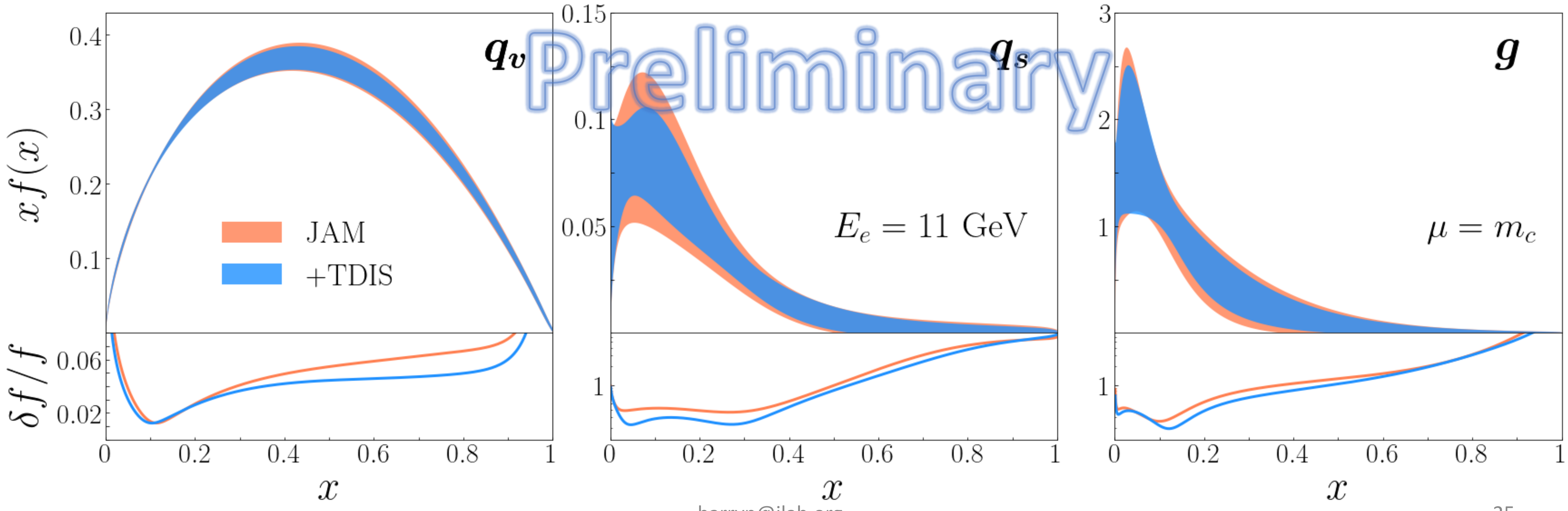


Potential kinematic region for JLab 11 GeV TDIS after cut on W_π^2

Higher twist effects and potentially non-perturbative effects could be relevant

Performing impact study with 11 GeV

- Create pseudodata from these points and perform global analysis with available experimental data



Conclusions

- Behavior of large-x valence distribution with double Mellin threshold resummation $q_v(x \to 1) \propto (1-x)^{\sim 1.2}$
- The complementarity between lattice and experimental data sheds light on the pion PDF itself as well as systematics associated with the lattice
- Other processes such as p_T-dependent cross sections are sensitive to PDFs
- Future experimental and lattice data are needed to further pin down large-x behavior of the valence quark distribution

Backup Slides

Reduced Ioffe time pseudo-distribution (Rp-ITD)

- Lorentz-invariant Ioffe time pseudo-distribution:

$$\mathcal{M}(\nu, z^2) = \frac{1}{2p^0} \langle p | \bar{\psi}(0) \gamma^0 \mathcal{W}(z;0) \psi(z) | p \rangle$$

Quark and antiquark fields ($\bar{\psi}(0)$, $\psi(z)$)

Gauge link ($\mathcal{W}(z;0)$)

"Ioffe time"

$$\nu = p \cdot z$$

$$z = (0,0,0,z_3)$$

Observable is the *reduced* Ioffe time pseudo-distribution (Rp-ITD)

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

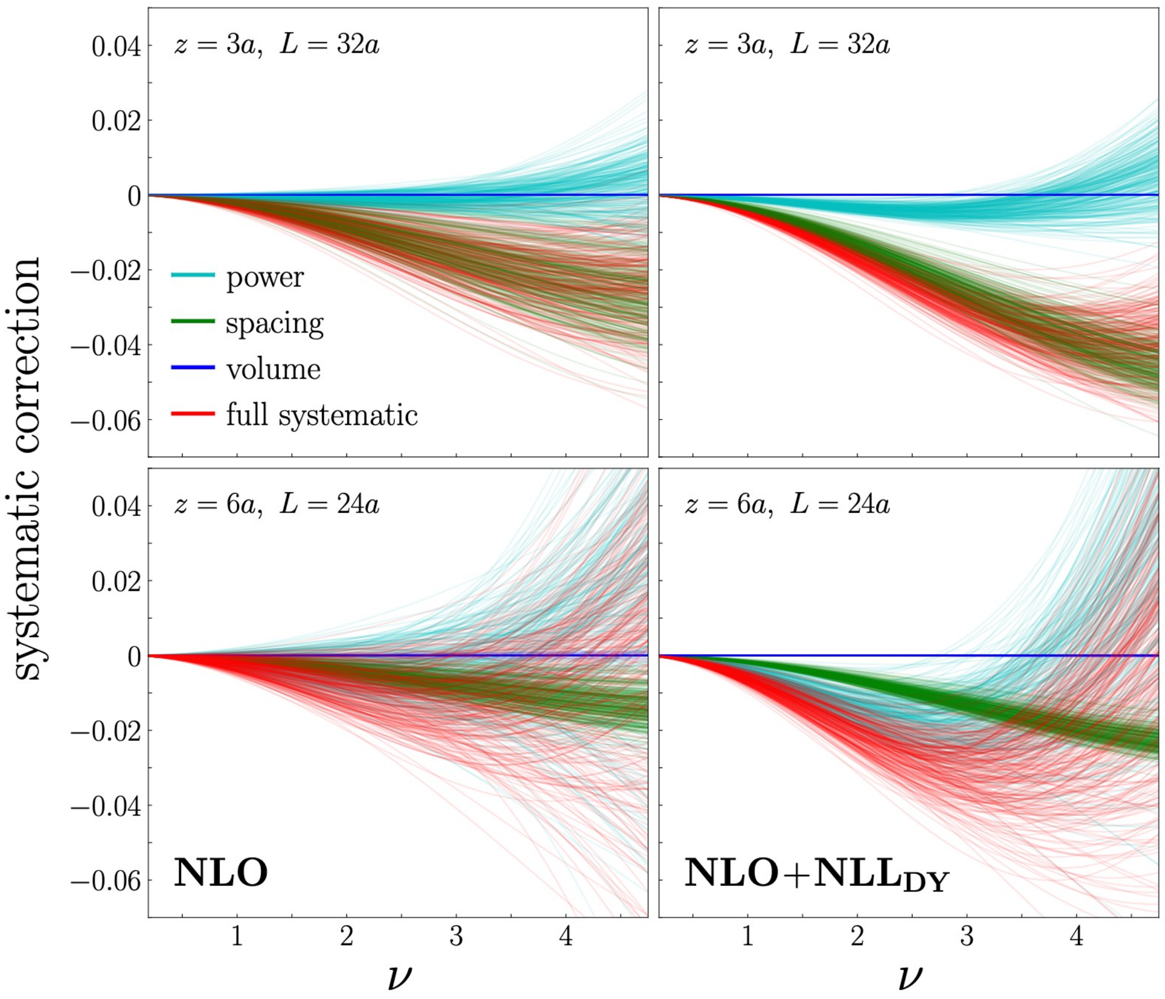
Ratio cancels UV divergences

Quantifying individual systematic corrections on the lattice

- Breaking down by the 3 systematics

$$z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu)$$

- Dominance of power or spacing corrections depends on z
- Finite volume corrections don't matter



Critiques suggested $(1-x)^2$ is a fact of QCD

$$u^\pi(x;\zeta) \overset{x\simeq 1}{\approx} (1-x)^{\beta = 2+\gamma(\zeta)}$$

T1: If QCD describes the pion, then at any scale for which an analysis of data using known techniques is valid, the form extracted for the pion's valence-quark DF must behave as $(1-x)^\beta$, $\beta > 2$, on $x \gtrsim 0.9$ [10, 59, 73, 74].

the associated disagreement with Eq. (27) requires explanation; and these are the only possibilities: [a] the dM scheme is incomplete, omitting or misrepresenting some aspect or aspects of the hard processes involved; [b] (some of) the data being considered in the analysis are not a true expression of a quality intrinsic to the pion; or [c] QCD, as it is currently understood, is not the theory of strong interactions.

- T1: There is no proof of this in QCD
- [a] The double Mellin method is more rigorous than Mellin-Fourier
- [b] We carefully apply factorization; lattice QCD data prefer a linear falloff; there is no evidence to suggest these data are wrong
- [c] There is no indication to insinuate QCD is not the theory of strong interactions

Ezawa

Wide-Angle Scattering in Softened Field Theory.

Z. F. Ezawa

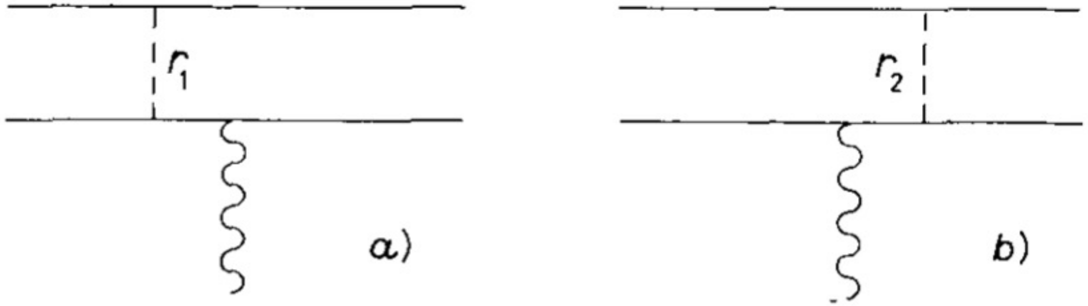
Department of Applied Mathematics and Theoretical Physics
University of Cambridge - Cambridge

(ricevuto il 25 Marzo 1974)

Not QCD

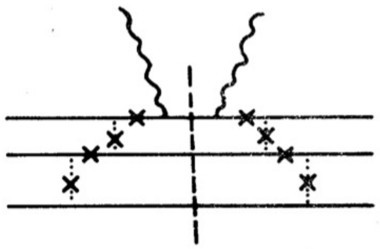
Summary. — The picture of Brodsky and Farrar for scattering processes at large transverse momentum is formulated in softened field theory. A modest softening of the quark-quark-gluon vertex is introduced to suppress unwanted logarithms in the formalism. It is shown that the electromagnetic form factors of the proton and the pion yield asymptotically behaviours which agree with the result of simple dimensional counting. The threshold behaviours of the deep inelastic structure functions are calculated for the proton and the pion to give $\sim(1-\omega)^3$ and $\sim(1-\omega)^2$, respectively. Thus the Drell-Yan-West relation holds in the case of the proton target but is violated in the case of the pion target. It is also proved that the asymptotic behaviours of wide-angle elastic $\pi\pi$ and pp scattering naively predicted by dimensional counting and conjectured by Brodsky and Farrar on the basis of simple Born diagrams are actually the next-to-leading-order terms. The highest-order terms come from a certain set of diagrams that Landshoff studied.

- No explicit proof of nonperturbative $q_v^\pi(x \to 1) \sim (1-x)^2$
- Assumes one hard gluon exchange dominance



Farrar and Jackson

- Assumption made that the below diagram dominates the structure



(a)

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Pion and Nucleon Structure Functions near $x = 1$*

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In a colored-quark and vector-gluon model of hadrons we show that a quark carrying nearly all the momentum of a nucleon ($x \approx 1$) must have the same helicity as the nucleon; consequently $\nu W_2^n/\nu W_2^p \to \frac{3}{7}$ as $x \to 1$, not $\frac{2}{3}$ as might naively have been expected. Furthermore as $x \to 1$, $\nu W_2^\pi \sim (1-x)^2$ and $(\sigma_L/\sigma_T)^\pi \sim \mu^2 Q^{-2}(1-x)^{-2} + O(g^2)$; the resulting angular dependence for $e^+e^- \to h^\pm + X$ is consistent with present data and has a distinctive form which can be easily tested when better data are available.

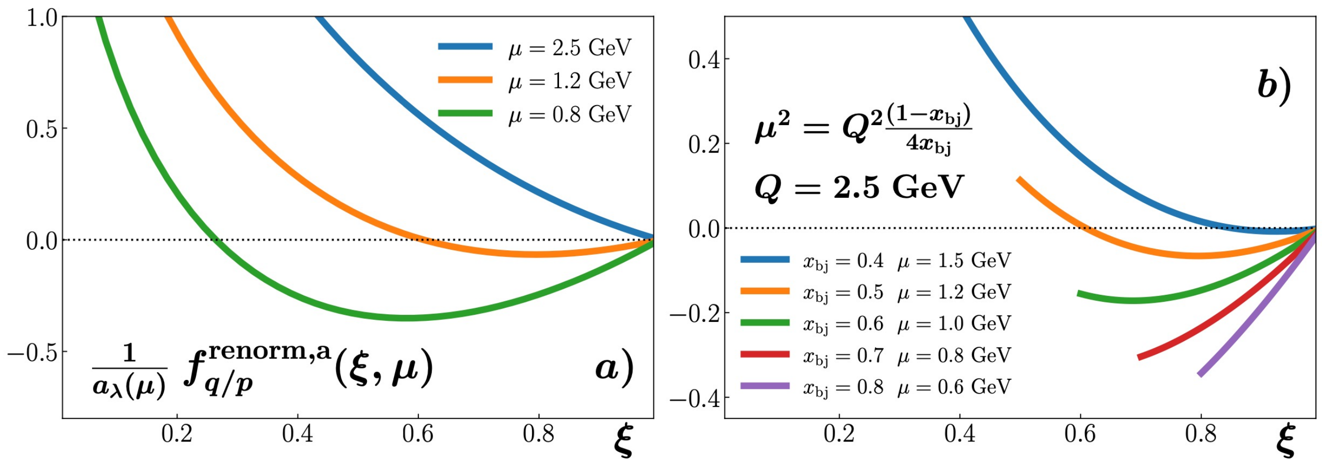
Assumption

go from the normal to "exceptional" (one quark having large p^2) wave functions. We assume that (a) the normal wave function is sufficiently damped at large p^2's that the convolution is dominated by the region in which the p^2's of the incoming quarks are finite, and (b) the spin and

- This is a *perturbative* assumption – we cannot say that higher order terms or soft gluons do not contribute to the *nonperturbative* structure of the hadron in QCD
- First principles QCD does not prove this behavior for PDF

Not necessary to have $(1-x)^{\beta}$ behavior

- A recent work by Collins, Rogers, and Sato proved that $\overline{\mathrm{MS}}$ PDFs were not necessarily positive as long as *cross section was positive.*



Phys. Rev. D **105**, 076010 (2022).

- PDFs do not have to have a large-x behavior associated with the counting rules

QCD does not fail if $\beta_v^\pi \neq 2$

- The perturbative expansion performed in Ezawa and Farrar & Jackson does not capture nonperturbative effects
- Like in threshold resummation, the buildup of very soft gluon exchanges between quark states may be non-negligible contributions to the perturbation
- When $(1-x) \to 0$, the light front zero mode could play a non-trivial role, which cannot be calculated perturbatively

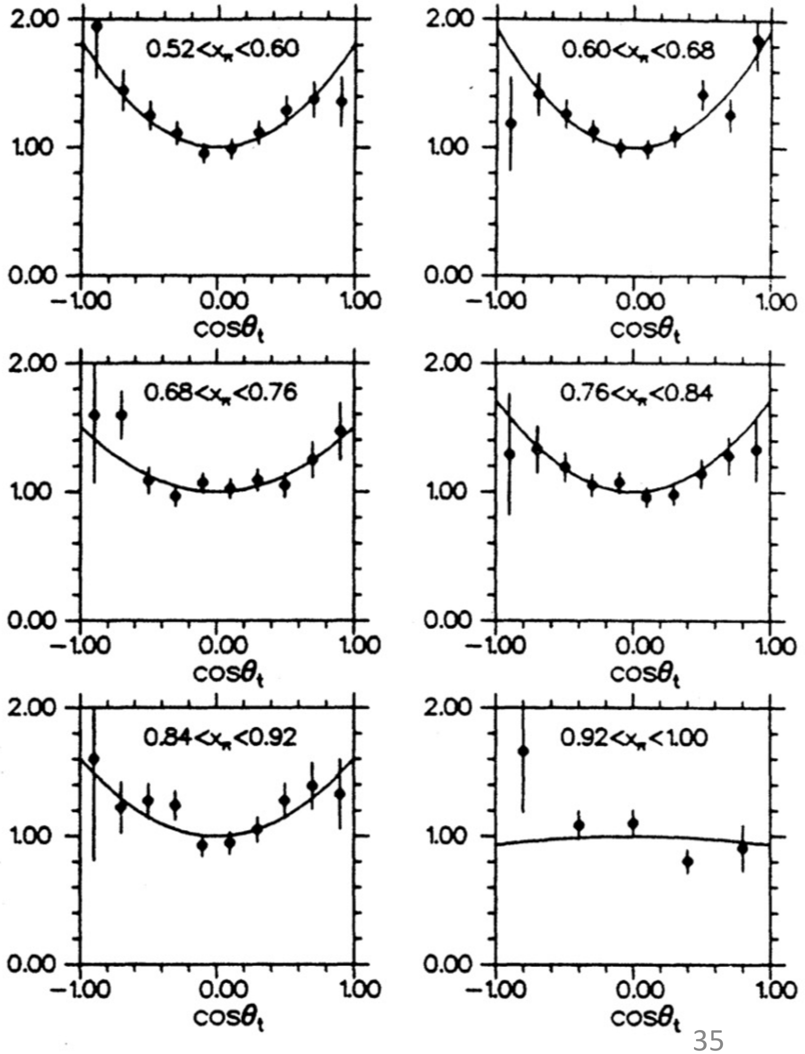
Angular dependence in E615 DY data

- Expected behavior of the cross section

$$d\sigma \propto (1-x_\pi)^2(1+\cos^2\theta)+\frac{4x_\pi^2\langle k_T^2\rangle}{9m_{\mu\mu}^2}\sin^2\theta\,.$$

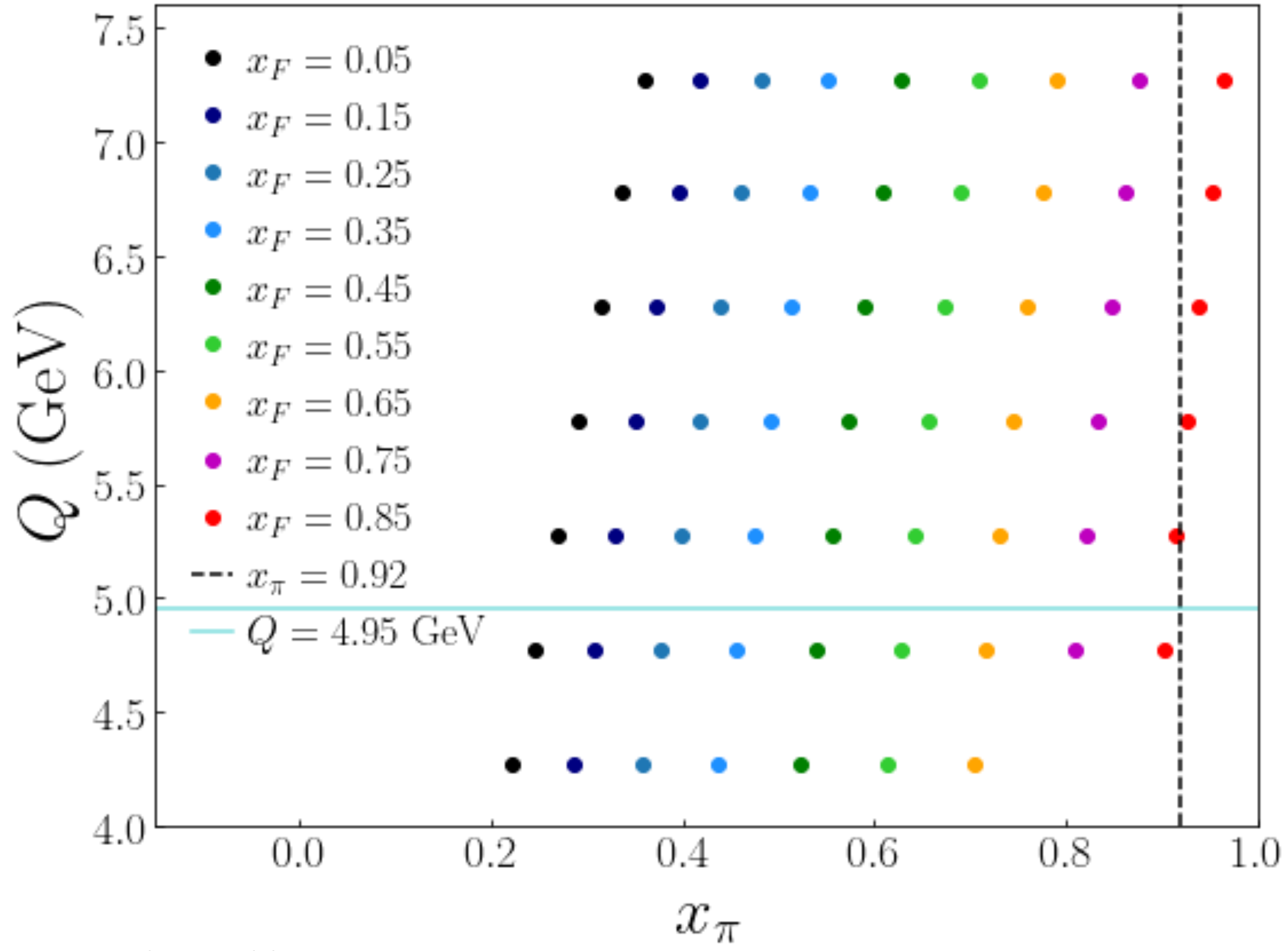
higher twist

- Parabolic = leading twist
- Each range of x_π follows the parabolic behavior except $0.92 < x_\pi < 1$ for shown $4.05 < M_{\mu^+\mu^-} < 4.95$ GeV where higher twist is expected to be most dominant



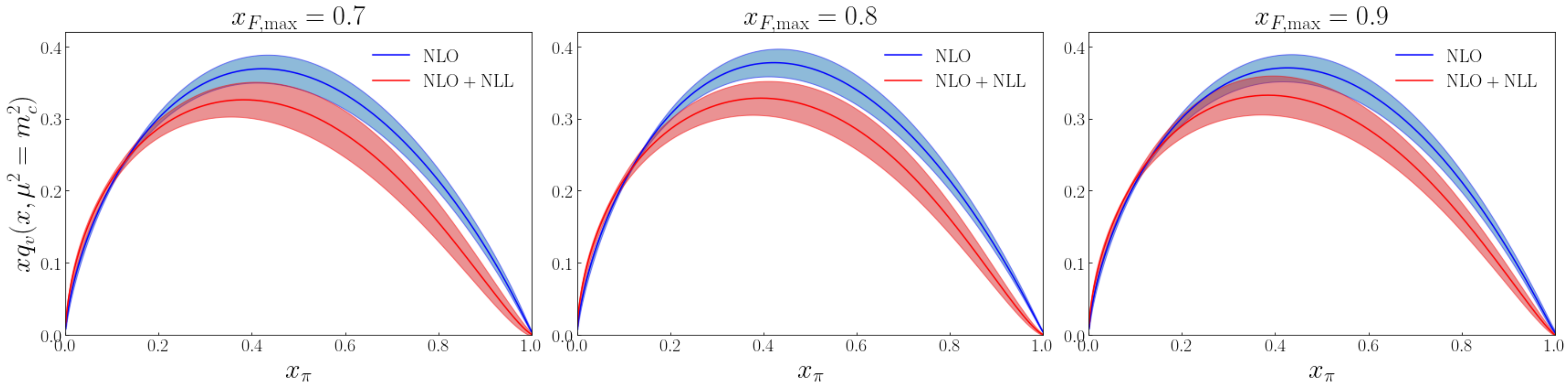
Kinematics of E615

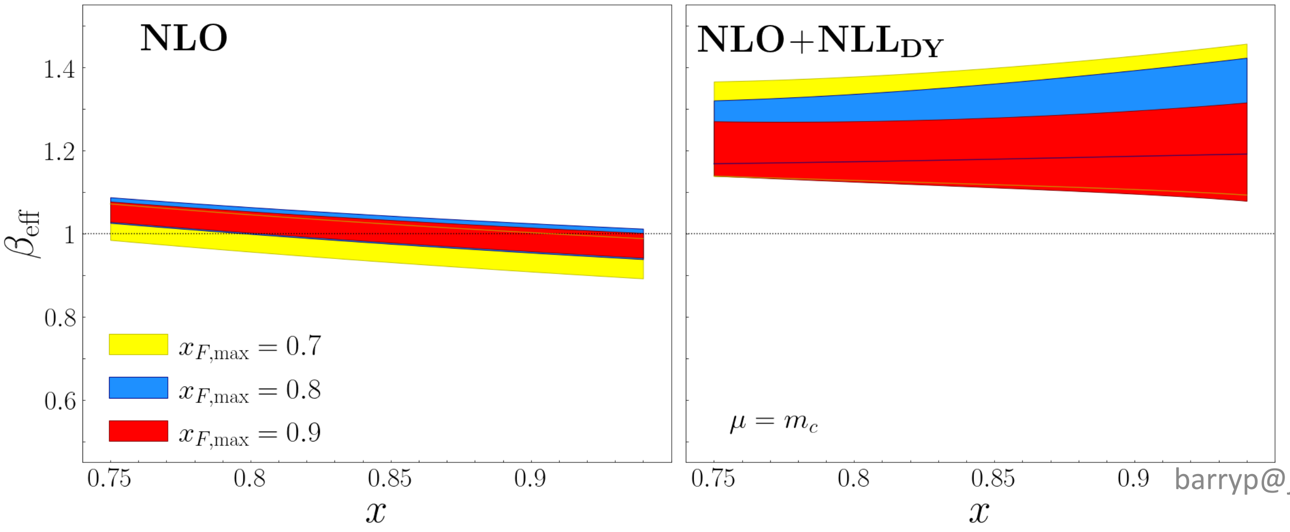
- Each of these points is included in the global analysis
- For small Q, we only have $x_\pi < 0.92$ points



Studying cuts in x_F

- To ensure the leading twist formalism, we also modify the $x_{F,\max}$





Large x behavior is conserved, albeit with larger uncertainties