Compton scattering on ^{4}He: Rosenbluth formulae

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Outline

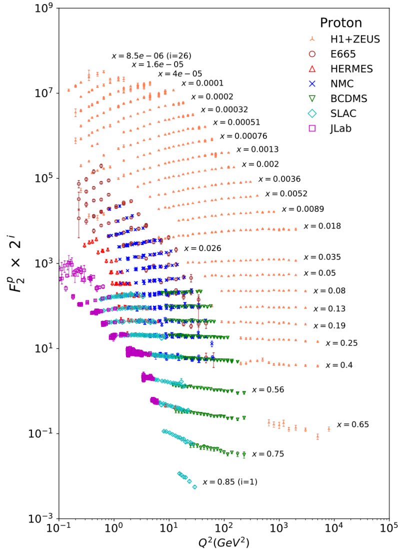
Motivation

The traditional motivation for the Parton Distribution approach to the study of hadronic structure is based on the ideas of factorization and scaling. These ideas have worked well in DIS, where the PDFs are determined, which are Lorentz scalars.

For large enough Q, scaling is seen as a weak dependence of the PDFs on Q as illustrated by the compilation by

the Particle Data Group.

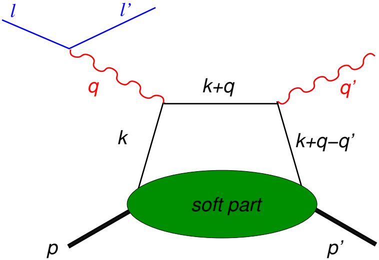
18. Structure Functions



Source: PDG 2020

Deeply-virtual Compton scattering (DVCS) has been proposed to determine the generalized-parton distributions (GPDs) of hadrons.

A hard, virtual photon with momentum q, $q^2 = -Q^2$, with Q much larger than the characteristic hadronic scales, probes the quark content of the hadronic target. The detection of the outgoing, real photon provides information not contained in DIS.



Handbag diagram for VCS, including the leptonic part

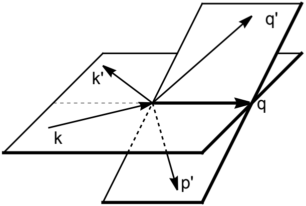
It is usually assumed that to allow for the extraction of the GPDs, the experiments should be set-up in (approximately) collinear kinematics. Such kinematics may not always be realized in concrete experiments.

We propose to first analyze the experimental data in terms of Lorentz-invariant amplitudes, Compton form factors (CFFs).

By definition, the CFFs can be determined in any suitable kinematics. Once they are measured, theorists may use them to extract the GPDs.

Here, we present our work on VCS off the ^{4}He nucleus, motivated by a considerable numbers of experiments about VCS on ^{4}He, one of the most recent examples is the work of R. Dupré et al., CLAS collaboration at Jefferson Lab.[1]

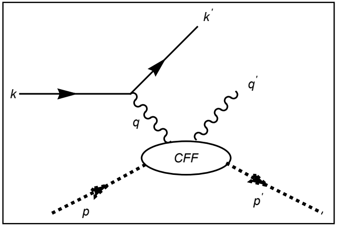
We shall work in the target rest frame (TRF) with the z-axis along the three momentum $\boldsymbol{q}$ of the virtual photon. The amplitudes can be expressed in terms of three invariants and the azimuthal angle ϕ, which is the angle between the *leptonic plane*, defined by the momenta $\boldsymbol{k}$ and $\boldsymbol{k}'$ and the *hadronic plane* defined by $\boldsymbol{q}'$ and $\boldsymbol{p}'$. The momentum $\overline{\boldsymbol{P}} = \boldsymbol{p}' + \boldsymbol{p}$ as well as the momentum $\boldsymbol{\Delta} = \boldsymbol{p}' - \boldsymbol{p}$ are in the hadronic plane, while $\boldsymbol{q}$ defines the intersection line of the two planes.



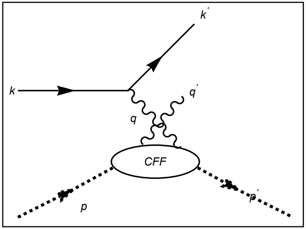
[1] R. Dupré et al., Phys. Rev. C **104**, 025203 (2021))

Formal Framework

In Compton scattering the physical amplitudes can be written in terms of a leptonic and a hadronic part.



hadronic s-channel



hadronic u-channel

For the VCS amplitude this form is

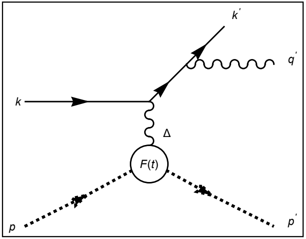
$$\mathcal{M}_{VCS}(\lambda', \lambda, h') = \sum_h L^{\rho}_{\mathrm{VCS}}(\lambda', \lambda)\epsilon^*_{\rho}(q, h)\frac{1}{q^2}\epsilon^*_{\mu}(q', h')T^{\mu\nu}\epsilon_{\nu}(q, h).$$

The tensor $T^{\mu\nu}$ is the **Compton tensor.** It must be transverse to q'_{μ} and q_{ν}. $T^{\mu\nu}$ depends linearly on the CFFs.

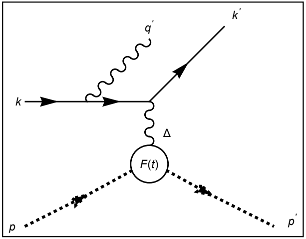
In order not to introduce **unwarranted restrictions**, it is important to use the most general form of that tensor operator consistent with EM gauge invariance.

The leptonic part of the VCS amplitude is given by the current

$$L^{\rho}_{\mathrm{VCS}}(\lambda', \lambda) = \bar{u}(k', \lambda')\gamma^{\rho}u(k, \lambda).$$



leptonic s-channel



leptonic u-channel

The Bethe-Heitler amplitude is given by

$$\mathcal{M}_{\rm BH}(\lambda', h', \lambda) = \sum_h A_{\rm BH}(\lambda', h', \lambda, h),$$

where

$$A_{\rm BH}(\lambda', h', \lambda, h) = \frac{F(t)}{t} \bar{P}_\mu \epsilon^\mu(\Delta, h) \epsilon^{*\nu}(\Delta, h) L_\nu(\lambda, \lambda', h').$$

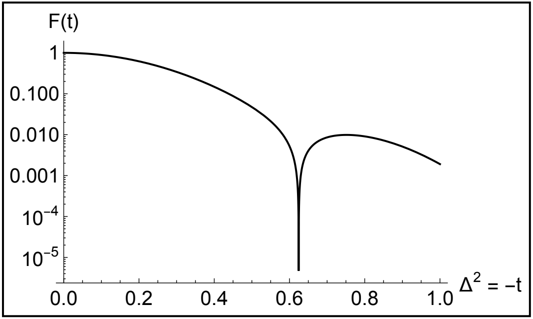
where we use the notation $\Delta^2 = (p - p')^2 = t$. $F(t)$ is the EM form factor of the scalar target.
The leptonic current is given by (s_ℓ and u_ℓ are the leptonic Mandelstam variables):

$$L^\nu(\lambda', \lambda, h') = \bar{u}(k', \lambda') \left[\gamma^\mu \frac{\not{k} - \not{\Delta}}{s_\ell} \gamma^\nu + \gamma^\nu \frac{\not{k}' + \not{\Delta}}{u_\ell} \gamma^\mu \right] u(k, \lambda)\, \epsilon^*_\mu(q', h'). \tag{1}$$

The hadronic part of the BH amplitude is given by the current

$$L^\rho_{\rm BH}(h) = \bar{P}^\rho, \quad \bar{P} = p' + p.$$

The EM form factor of the ^{4}He nucleus is the only phenomenological element. We use the parameterized form R.F. Frosch, J.S. McCarthy, R.E. Rand, and M.R. Yearian, Phys. Rev. **180**, 874 (1967).



Note the node in the ^{4}He form factor at $Q = 0.624$ GeV$/c$. This node is important, because it marks the point where the contribution of the BH process changes sign. At this point both the BH amplitude and its interference with the hadronic amplitude vanish.

Because the Bethe-Heitler and the *coherent* VCS processes are coherent, their amplitudes must be added when the cross section for the process $e + {}^4\text{He} \to e' + {}^4\text{He} + \gamma$ is calculated. Then the complete squared amplitudes can be split into a Bethe-Heitler part, a VCS part and a part that is obtained by the interference of the two amplitudes:

$$|A_{\text{tot}}|^2 = |A_{\text{BH}}|^2 + |A_{\text{VCS}}|^2 + A^*_{\text{BH}} A_{\text{VCS}} + A_{\text{BH}} A^*_{\text{VCS}}.$$

These amplitudes can be written as the convolution of the leptonic (QED) amplitude and a hadronic amplitude, which involves the electro-magnetic form factor of the ^{4}He nucleus.

The Bethe-Heitler amplitudes follow directly from QED. We shall see that in the hadronic target rest frame kinematics, only the amplitude where the virtual photon has helicty $h = 0$ contributes,

Our main point, however, is the question what is the most general form of the Compton tensor $T^{\mu\nu}$ and the importance of including the fully general form of this tensor.

The second point is the relative importance of the contribution of the different CFFs to the amplitudes.

Hadronic Compton tensor

In order for the hadronic Compton tensor $T^{\mu\nu}$ to be acceptable, it must be transverse to the photon momenta to guarantee charge conservation. We have proposed a method[2] that we dubbed the DNA method. The back bone of the Compton tensor is

$$d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\beta} g^{\nu\alpha}.$$

We note that $d^{\mu\nu\alpha\beta}$ is symmetric under the simultaneous interchange $\mu \leftrightarrow \nu,\ \alpha \leftrightarrow \beta$ and changes sign by the interchanges $\mu \leftrightarrow \alpha$, and $\nu \leftrightarrow \beta$.

Using this back bone we construct pieces of "DNA" by adding "base pairs", *i.e.* contracting it with three basis four vectors, q, q', and $\bar{P}$. With an obvious notation we write them as follows:

$$\begin{aligned}
G^{\mu\nu}(q'q) &= q'_\alpha d^{\mu\nu\alpha\beta} q_\beta = q' \cdot q\, g^{\mu\nu} - q^\mu q'^\nu, \\
G^{\mu\nu}(qq) &= q_\alpha d^{\mu\nu\alpha\beta} q_\beta = q^2\, g^{\mu\nu} - q^\mu q^\nu, \\
G^{\mu\nu}(q'q') &= q'_\alpha d^{\mu\nu\alpha\beta} q'_\beta = q'^2\, g^{\mu\nu} - q'^\mu q'^\nu, \\
G^{\mu\nu}(\overline{P}q) &= \overline{P}_\alpha d^{\mu\nu\alpha\beta} q_\beta = \overline{P} \cdot q\, g^{\mu\nu} - q^\mu \overline{P}^\nu, \\
G^{\mu\nu}(q'\overline{P}) &= q'_\alpha d^{\mu\nu\alpha\beta} \overline{P}_\beta = \overline{P} \cdot q'\, g^{\mu\nu} - \overline{P}^\mu q'^\nu.
\end{aligned}$$

The momentum $\overline{P}$ is the sum of the hadron momenta: $\overline{P} = p' + p$.

[2] B.L.G. Bakker and C.-R. Ji, Few-Body Syst., **58**, 1 (2017)

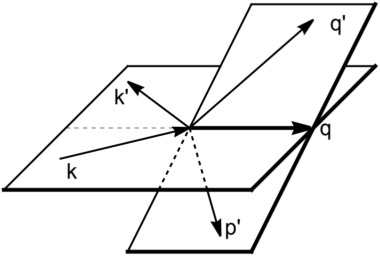
Given these building blocks we write the transverse tensor as

$$
\begin{aligned}
T^{\mu\nu}_{\mathrm{DNA}} &:= \sum_{i=1}^{5} \mathcal{S}_i \bar{C}_i^{\mu\nu} \\
&= \mathcal{S}_1 G^{\mu\nu}(q', q) + \mathcal{S}_2 G^{\mu\lambda}(q', q') G_\lambda^{\ \nu}(q, q) + \mathcal{S}_3 G^{\mu\lambda}(q', \bar{P}) G_\lambda^{\ \nu}(\bar{P}, q) \\
&\quad + \mathcal{S}_4 \left(G^{\mu\lambda}(q', \bar{P}) G_\lambda^{\ \nu}(q, q) + G^{\mu\lambda}(q', q')) G_\lambda^{\ \nu}(\bar{P}, q) \right) + \mathcal{S}_5 G^{\mu\lambda}(q', q') \bar{P}_\lambda \bar{P}_{\lambda'} G^{\lambda'\nu}(q, q).
\end{aligned}
$$

The $\mathcal{S}_i$ are the CFFs in the DNA construction.

Note that for the case ${q'}^2 = 0$, the CFFs $\mathcal{S}_2$ and $\mathcal{S}_5$ do not contribute to the hadronic amplitude, because $G^{\mu\nu}(q', q')$ annihilates the polarization vectors $\epsilon_\mu(q', h')$ and $G^{\mu\nu}(q, q)$ annihilates the polarization vectors $\epsilon_\nu(q, h)$.

Kinematics



It is relevant to discuss the kinematics, because it is important for an answer to the question whether or not the conditions needed for an interpretation of DVCS in terms of GPDs can be realized in practice.

We shall work in the target rest frame (TRF) with the z-axis along the three momentum $\boldsymbol{q}$ of the virtual photon. The amplitudes can be expressed in terms of three invariants and the azimuthal angle ϕ, which is the angle between the *leptonic plane*, defined by the momenta $\boldsymbol{k}$ and $\boldsymbol{k}'$ and the *hadronic plane* defined by $\boldsymbol{p}$, $\boldsymbol{q}$, and $\boldsymbol{p}'$. The momentum $\overline{\boldsymbol{P}} = \boldsymbol{p}' + \boldsymbol{p}$ as well as the momentum $\boldsymbol{\Delta} = \boldsymbol{p}' - \boldsymbol{p}$, which plays a role in the BH proces, are in the hadronic plane, while $\boldsymbol{q}$ defines the intersection line of the two planes.

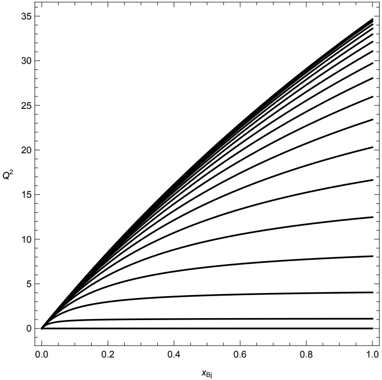
The relevant invariants are the mass M of the hadronic target and

$$\begin{aligned} Q^2 &= -q^2, \quad x_{\mathrm{Bj}} = \frac{Q^2}{2p\cdot q}, \quad y = \frac{p\cdot q}{p\cdot k} = \frac{Q^2}{2E_b M x_{\mathrm{Bj}}}, \\ s_{\mathrm{had}} &= (p+q)^2 = M^2 + \frac{1-x_{\mathrm{Bj}}}{x_{\mathrm{Bj}}}\, Q^2, \\ t_{\mathrm{had}} &= (p-p')^2, \quad u_{\mathrm{had}} = (p-q')^2. \end{aligned}$$

E_b is the energy of the incoming electron; it determines the overall energy and momentum scales. The invariants t_{had} and u_{had} depend on the azimuthal angle ϕ. We shall use the notation t for t_{had} where it does not lead to confusion.

The invariants x_{Bj} and y are both limited to the interval $[0, 1]$.

The kinematical domain for fixed M and E_b is parametrized by the scattering angle $\theta_{\rm e}$ of the electron.
The plots below are for $M = 3.7373$ GeV$/c^2$ and $E_{\rm b} = 6.064$ GeV$/c^2$. Q^2 in GeV$^2/c^2$.



The curves are lines of constant electron scattering angle $\theta_{\rm e}$. This angle runs from $\theta_{\rm e} = 0$, the lowest curve, to $\theta_{\rm e} = \pi$, the highest, in steps of $\frac{\pi}{18}$. Q^2 is largest for $x_{\rm Bj} \to 1$.

It is interesting to consider the asymptotic behaviour of $t = t_{\text{had}}$ for large Q.

$$\lim_{Q\to\infty} t = -M^2 \frac{x_{\mathrm{Bj}}^2}{1-x_{\mathrm{Bj}}}, \text{ for } \vartheta = 0, \tag{2}$$
$$= -Q^2 \frac{1-\cos\vartheta}{2x_{\mathrm{Bj}}}, \text{ for } \vartheta \neq 0.$$

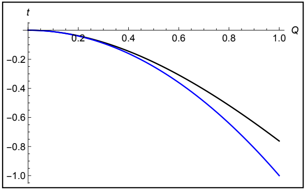
The quantity ϑ is the photon scattering angle in the hadronic CMF, where $\boldsymbol{q}$ defines the z-axis. For small values of ϑ, which are relevant here, it is close to the scattering angle in the TRF.

If $\vartheta \neq 0$, $t_{\text{had}} \propto -Q^2$ independent of the angle ϑ. In fact, $t_{\text{had}}/Q^2 \to -1$ for $Q \to 0$.

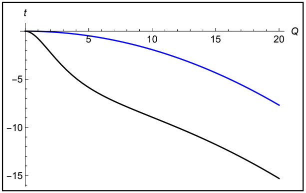
Because the Mandelstam variable t plays a special role, we consider its behaviour at large Q^2 in more detail. Its expression in terms of the other invariants is

$$t = -Q^2 \frac{Q^2(1-x_{\mathrm{Bj}}) + 2M^2 x_{\mathrm{Bj}}^2 - Q(1-x_{\mathrm{Bj}})\sqrt{Q^2+4M^2x_{\mathrm{Bj}}^2}\cos\vartheta}{2x_{\mathrm{Bj}}(Q^2(1-x_{\mathrm{Bj}}) + M^2 x_{\mathrm{Bj}})}.$$

For a value of $x_{\mathrm{Bj}} = 1/2$ and the photon scattering angle $\vartheta = \pi/16$ we find the behaviour:

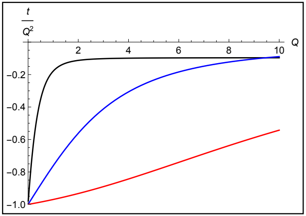


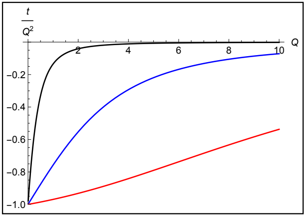
t, $-1/\ Q^2$



t, asymptotic form

The truly asymptotic regime where t/Q^2 is much smaller than 1 is only reached for $Q \gg M$. In the kinematical domain mentioned, where $M \approx 4$ and $Q < 6$, we find that for ϑ above a minimal value of $\pi/16$, the minimal value of $|t/Q^2|$ can be as large as 13% for $x_{\mathrm{Bj}} = 0.3$ increasing to 30% for $x_{\mathrm{Bj}} = 0.5$ for Q = M.
For $x_{\mathrm{Bj}} = 1$ this ratio is 1.





lhs: $\vartheta = \pi/16$ $\quad$ $x_{\mathrm{Bj}} = 1/10$, $x_{\mathrm{Bj}} = 5/10$, $x_{\mathrm{Bj}} = 9/10$ $\quad$ rhs: $\vartheta = 0$

Rosenbluth form of the squared VCS amplitude

To calculate the cross sections, one needs the squares of the amplitudes. We write them in the Rosenbluth form.

$$\mathcal{A}_{VCS}(\lambda,\lambda',h,h') = \sum_{i=1,3,4} \frac{\mathcal{S}_i}{q^2} H_i^{\mu}(q',h')\epsilon_{\mu}(q,h)\epsilon^*_{\nu}(q,h)L^{\nu}_{VCS}(\lambda,\lambda')$$

with

$$H_i^{\mu}(q',h') = \overline{C}_i^{\mu\nu}\epsilon^*_{\nu}(q',h'), \quad L^{\nu}_{VCS}(\lambda,\lambda') = \bar{u}(k',\lambda')\gamma^{\nu}u(k,\lambda).$$

and the CFFs $\mathcal{S}_i$ as defined in the Compton tensor:

$$T^{\mu\nu} = \sum_{i=1}^{5} \mathcal{S}_i \bar{C}^{(i)\,\mu\nu}.$$

The squared amplitude is then

$$\begin{aligned}\left|\sum_h \mathcal{A}_{\mathrm{VCS}}(\lambda',\lambda,h',h)\right|^2 \quad &= \quad \sum_h \sum_{\tilde{h}} \frac{\mathcal{S}_i\mathcal{S}_j^*}{(q^2)^2} H_i^{\mu}(q',h')H^*_{j\tilde{\mu}}(q',h') \\ &\times \quad \Pi_{\mu\tilde{\mu}}(q,q,\tilde{h},h)\Pi^*_{\nu\tilde{\nu}}(q,q,\tilde{h},h)L^{*\nu}_{\mathrm{VCS}}(\lambda',\lambda)L^{*\nu}_{\mathrm{VCS}}(\lambda',\lambda)\end{aligned}$$

with

$$\Pi_{\mu\tilde{\mu}}(q,q,\tilde{h},h) = \epsilon_{\mu}(q,h)\epsilon^*_{\tilde{\mu}}(q,\tilde{h})$$

Rosenbluth form of the squared BH amplitude

$$\mathcal{A}_{\mathrm{BH}}(\lambda, \lambda', h') = \frac{F(t)}{t} \sum_h \bar{P}_\mu \epsilon^\mu(\Delta, h) \epsilon^{*\nu}(\Delta, h) L_\nu(\lambda', \lambda, h')$$

as written before. The leptonic current is given by

$$L^\nu(\lambda', \lambda, h') = \bar{u}(k', \lambda') \left[\gamma^\mu \frac{\not{k} - \not{\Delta}}{s_\ell} \gamma^\nu + \gamma^\nu \frac{\not{k}' + \not{\Delta}}{u_\ell} \gamma^\mu \right] u(k, \lambda)\ \epsilon^*_\mu(q', h').$$

Then the squared amplitude has the form

$$|\mathcal{A}|^2 = \sum_h \sum_{\tilde{h}} \left| \frac{F(t)}{t} \right|^2 \bar{P}_\mu \bar{P}_{\tilde{\mu}} \epsilon^\mu(\Delta, h) \epsilon^{*\tilde{\mu}}(\Delta, \tilde{h}) \epsilon^\nu(\Delta, h) \epsilon^{*\tilde{\nu}}(\Delta, \tilde{h}) L_\nu(\lambda', \lambda, h') L^*_{\tilde{\nu}}(\lambda', \lambda, h')$$

We notice two structures, the leptonic and hadronic densitiy matrices:

$$\rho_{\mathrm{L}}(h, \tilde{h}) = \epsilon^\nu(\Delta, h) \epsilon^{*\tilde{\nu}}(\Delta, \tilde{h}) L_\nu L^*_{\tilde{\nu}},$$

and

$$\rho_{\mathrm{H}}(h, \tilde{h}) = \left| \frac{F(t)}{t} \right|^2 \bar{P}_\mu \bar{P}_{\tilde{\mu}} \epsilon^\mu(\Delta, h) \epsilon^{*\tilde{\mu}}(\Delta, \tilde{h}).$$

In the hadronic target restframe (TRF) where $\boldsymbol{\Delta} = \bar{\boldsymbol{P}} = \boldsymbol{p}'$ and in the gauge we use where $\epsilon^0(\Delta, \pm) = 0$, the hadronic polarization tensor reduces to the simple form

$$\rho_{\mathrm{H}}(h, \tilde{h}) = \rho_{\mathrm{H}}(0,0)\delta_{h,0}\delta_{\tilde{h},0}.$$

with

$$\rho_{\mathrm{H}}(0,0) = \left|\frac{F(t)}{t}\right|^2 (\bar{P}\cdot\epsilon^0(\Delta,0))^2 = \left|\frac{F(t)}{t}\right|^2 (t - 4M^2).$$

This result implies, incidentally, that all BH amplitudes calculated in the TRF are proportional to $\bar{P}\cdot\epsilon^0(\Delta,0)$. Owing to this simplification, the spin sum of the squared BH amplitudes is simplified to:

$$\sum_{\lambda', h', \lambda} |\mathcal{A}_{\mathrm{BH}}(\lambda, \lambda', h')|^2 = 16\,\frac{t_\ell}{t}\,\frac{4M^2 - t}{s_\ell - t}.$$

(Note that the polarization vector with with $h = 0$ is proportional to $1/\sqrt{t}$ and thus both the hadronic and the leptonic density matrices are negative. Their product is positive as it must be.)

Benchmark Calculation

As a **benchmark model** one may consider the tree-level case, which of course describes completely structureless particles.

The tree-level DVCS amplitude corresponds to the CFFs

$$\mathcal{S}_1^{\text{tree}} = -\left(\frac{1}{s_{\text{had}} - M^2} + \frac{1}{u_{\text{had}} - M^2}\right), \ \mathcal{S}_3^{\text{tree}} = \frac{2}{(s_{\text{had}} - M^2)(u_{\text{had}} - M^2)}.$$

Thus, only 2 out of 5 CFFs contribute. We note that at large Q, $\mathcal{S}_3$ is of relative order $1/Q^2$ compared to $\mathcal{S}_1$. This suppression by the factor $1/Q^2$ is in line with the hierarchy predicted by the operator-product expansion.

Because we study the relative importance of the CFFs, we do not include the factors $-e$ and $2e$ for the charges of the elektron and the ^{4}He nucleus, respectively.

Mass dimensions and large-Q behaviour

The relative importance of the CCFs not only depend on their Q-scaling, but also on the scaling of the Compton tensor.

The mass dimension of $\mathcal{A}_{\mathrm{VCS}}$ can be found as follows. $[\mathcal{A}_{\mathrm{VCS}}] = [L_\nu][\epsilon_\nu][\bar{C}_i^{\mu\nu} S_i][q^{-2}]$. The polarization vectors ϵ^μ are dimensionless. We know the dimensions $[L_\nu] = [M]$ and $[q^{-2}] = [M^{-2}]$. Finally the mass dimensions $[\bar{C}_i^{\mu\nu} S_i]$ must be identical with the mass dimension of this product in the case $i = 1$.
$[\bar{C}_1^{\mu\nu}] = [M^2]$ and $[S_1^{\mathrm{Tree}}] = [M^{-2}]$. Thus the Compton tensor in our definition is dimensionless. The final result is thus that the amplitude has mass dimension $[A_{\mathrm{VCS}}] = [M^{-1}]$, and obviously its square has mass dimension $[M^{-2}]$.
Similarly, we find that $\mathcal{A}_{\mathrm{BH}}$ has the same mass dimension.

The CFFs have different mass dimensions, related on the mass dimensions of the effective tensors $\bar{C}_i^{\mu\nu}$.
The mass dimensions for the complete coefficient functions thus produces a homogeneous value for the mass dimension of the Compton tensor

Mass and Q dimensions of $\bar{C}_i$ and $\mathcal{S}_i$

$\mathcal{S}_1$	$\mathcal{S}_3$	$\bar{C}_1$	$\bar{C}_3$	$\mathcal{S}_1 \times \bar{C}_1$	$\mathcal{S}_3 \times \bar{C}_3$
m^{-2}	m^{-4}	m^2	m^4	m^0	m^0
Q^{-2}	Q^{-4}	Q^4/M^2	Q^6/M^2	Q^2/M^2	Q^2/M^2

We see that although the CFF $\mathcal{S}_3^{\mathrm{tree}}$ is of order Q^{-2} suppressed compared to $\mathcal{S}_1^{\mathrm{tree}}$, the large-$Q$ behaviour of the tensors $\bar{C}_1$ and $\bar{C}_3$ compensate the factor Q^{-2}.

Large-Q behaviour of the tensors: details

The two parts of the Compton tensor at tree level turn out to be remarkably similar for large Q:

$$\mathcal{S}_1 \times \bar{C}_1 = Q^2 \frac{(1+\cos(\vartheta))}{8M^2(1-xBj))} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

and

$$\mathcal{S}_3 \times \bar{C}_3 = Q^2 \frac{(2-xBj)\sin(\vartheta])^2}{8M^2(1-xBj)xBj} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

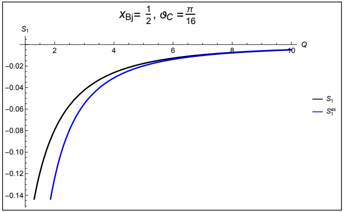
Consequently, at very large Q, one cannot distinguish between a situation where the scalar target has only a single CFF or has more than one.

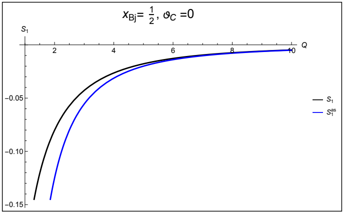
The big question is for which values of Q one may rely on the asymptotic form of the hadronic Compton tensor in the analysis of the data.

The fact that the target mass $M = M(^4\text{He})$ may not be small in an actual experiment compared to Q, is a point of concern.

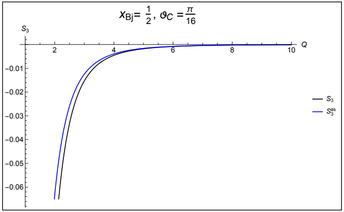
Large-Q behaviour of the CFFs: details

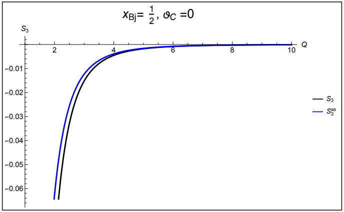
S_1





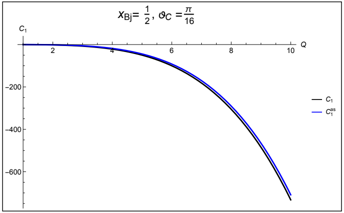
S_3

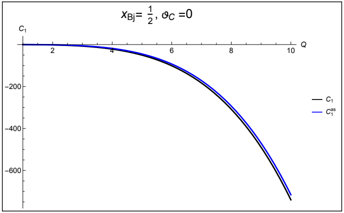




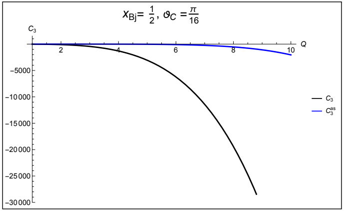
Large-Q behaviour of the basis tensors

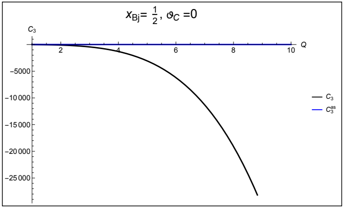
$\bar{C}_1^{\mu\nu}$, $\mu = \nu = 1$





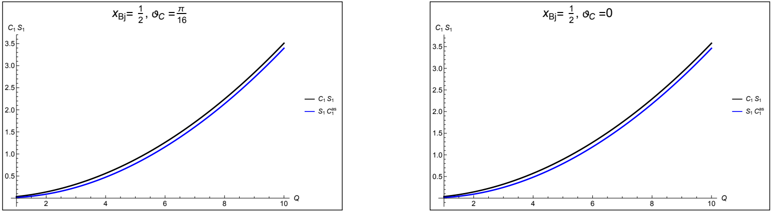
$\bar{C}_3^{\mu\nu}$, $\mu = \nu = 1$



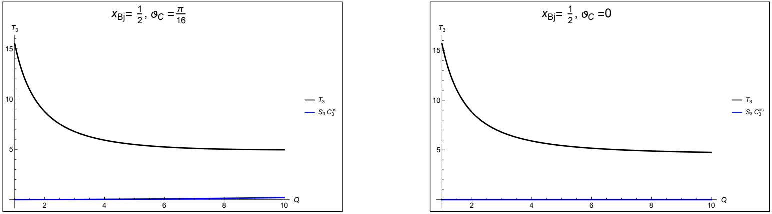


Large-Q behaviour of the partial tensors

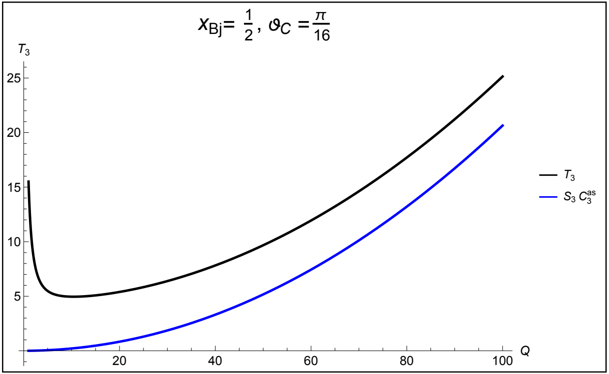
$$T_1 = S_1 \bar{C}_1^{\mu\nu},\ \mu = \nu = 1$$



$$T_3 = S_3 \bar{C}_3^{\mu\nu},\ \mu = \nu = 1$$

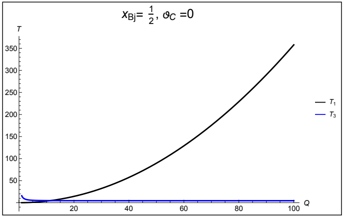


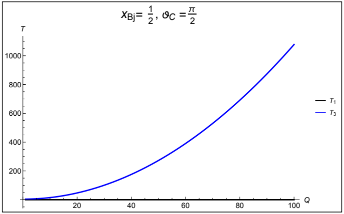
Very large-Q behaviour of the partial tensor T_3

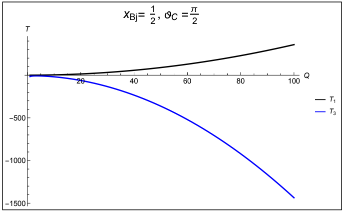


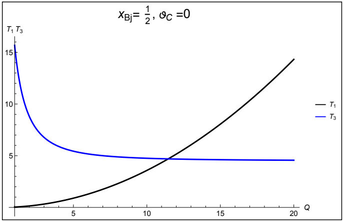
Comparisons of partial amplitudes

For larger scattering angles, ϑ, the behaviour of the two parts is spectacularly different.









Crucial test: Beam spin asymmetry

It is clear from these results that, when extracting the CFFs from the data, it is dangerous to rely on what has been considered the dominant CFF, in this case $\mathcal{S}_1$.

The two CFFs we have included are not realistic. To begin with, they are both *real*, while there is no reason for the CFFs to be real. For a ^{4}He nucleus, one cam be sure that the CFFs are *complex*.

When the CFFs are complex, a beam spin asymmetry may show up in the VCS cross section. The common understanding is that the beam spin asymmetry is due to the interference part of the cross section proportional to:

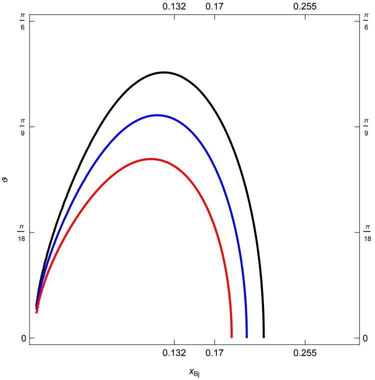
$$A^*_{\mathrm{BH}} A_{\mathrm{VCS}} + A_{\mathrm{BH}} A^*_{\mathrm{VCS}}.$$

However, since A_{BH} is proportional to the ^{4}He form factor, which has a node at $Q = 0.624$ GeV$/c$ (in the low-Q part of the kinematic domain), one may perform a **crucial experiment** by measuring the beam spin asymmetry checking the minimum number of CFFs.

If no beam spin asymmetry is measured, the minimal number of CFFs may be 1. If the beam spin asymmetry does not vanish, it is proof that at least two CFFs are involved and one of them must be complex.

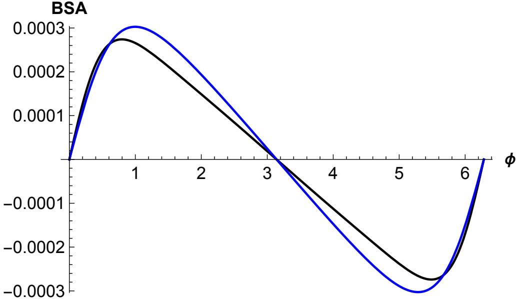
Kinematics for the node in the ^{4}He form factor

The nodal position can be reached for small values of x_{Bj} and Q^2, as in the CLAS experiment.



The nodal position is $\Delta^2 = 0.389941\mathrm{GeV}^2/c^2$; the angle ϑ is the polar angle of the emitted photon in the CMF.

Beam Spin Asymmetry



$x_{Bj} = 0.132$, $x_{Bj} = 0.170$. (For $x_{Bj} = 0.255$ the node cannot be reached).

For the two kinematics from the CLAS experiment the BSA is tiny. Remarkably, the form of the BSA is not a pure sine, because the coefficient of sinϕ depends cosϕ and cos2ϕ.

Summary and conclusions

- Our treatment of Virtual Compton Scattering is entirely phenomenological.
- We have discussed the number of Compton Form factors for a scalar target. This number is **three** if the emitted photon is **real**.
- **We used a benchmark form of the Compton tensor, containing two CFFs**.
- We have demonstrated that the partial tensors $\bar{C}_i^{\mu\nu}$, $(i = 1, 3)$ have different asymptotic behaviour as functions of Q^2. This behaviour compensates for the behaviour of the CFFs for large Q^2.
- At very large values of Q^2, the tensors $\bar{C}_1$ and $\bar{C}_3$ become proportional. The Q-values at which this phenomenon occurs is very large. For our benchmark the ratio of the partial tensors is

$$\frac{S_3\,\bar{C}_3}{S_1\,\bar{C}_1} \to \frac{(2 - xBj)\sin^2\left(\frac{\vartheta_C}{2}\right)}{x_{\mathrm{Bj}}}$$

- We found that for the kinematics in the CLAS experiment at $E_b = 6$ GeV, the relative magnitude of the contribution of the two parts, $T_1 = \mathcal{S}_1\bar{C}_1^{\mu\nu}$ and $T_3 = \mathcal{S}_3\bar{C}_3^{\mu\nu}$ depends strongly on the kinematics. For the values of x_{Bj} and Q that characterise the CLAS experiment, T_3 dominates.
- Even without interference of the Bethe-Heitler process, there may occur a single-spin symmetry in VCS. This result is obtained because the VCS amplitude is the coherent sum of two parts, one related to the CFF $\mathcal{S}_1$, the other to $\mathcal{S}_3$.