Non-perturbative Wavefunctions from Lightcone Conformal Truncation

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LC as the limit of Boosting

Hamiltonian Truncation at finite volume:

$$H = H_0 + V$$

$$|p_1, ...p_n\rangle, \quad E_{state} < \Lambda \ , \quad \sum_i p_i = 0$$

Take large but finite momentum:

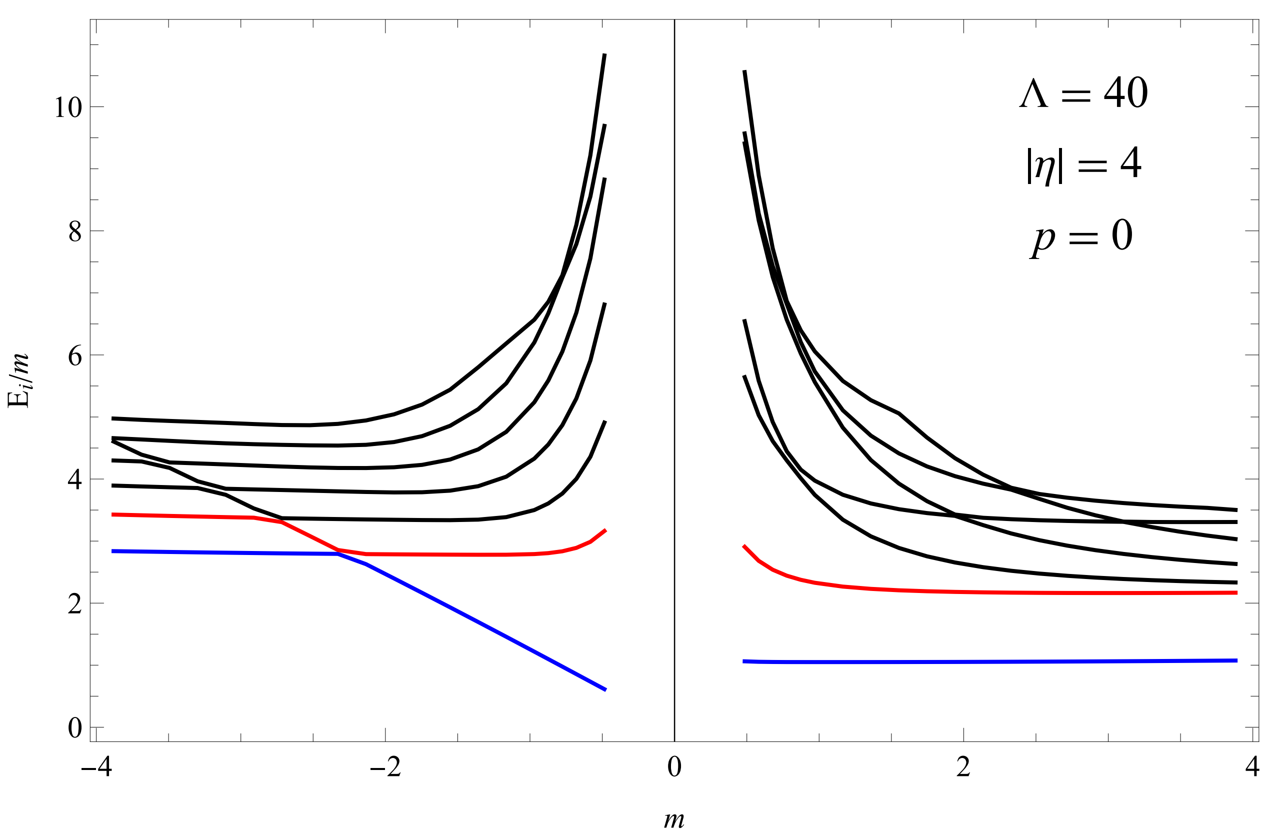
$$|p_1, ..., p_n\rangle, \quad E_{state} < \sqrt{\Lambda^2 + P^2}, \quad \sum_i p_i = P \gg \frac{1}{L}$$

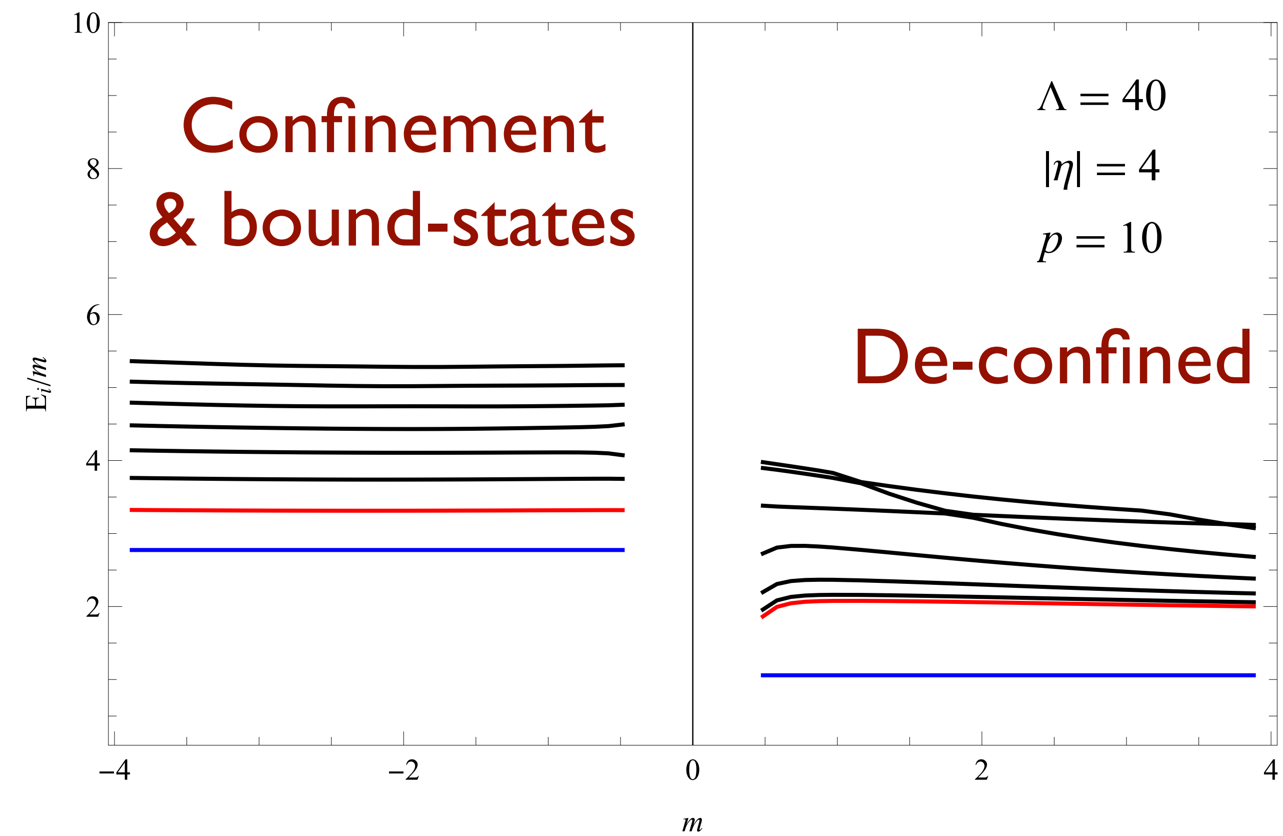
Boosting reduces dependence on the volume!

A 2d example - Ising Field Theory:

$$H = H_0^{fermion} + \int dx\, \left(m\bar{\psi}\psi(x) + g\sigma(x) \right)$$

This theory is labeled by the ratio: $\eta = \frac{m}{g^{\frac{8}{15}}}$





Phase transition at finite boost clearly visible!

Even more true for correlation functions:

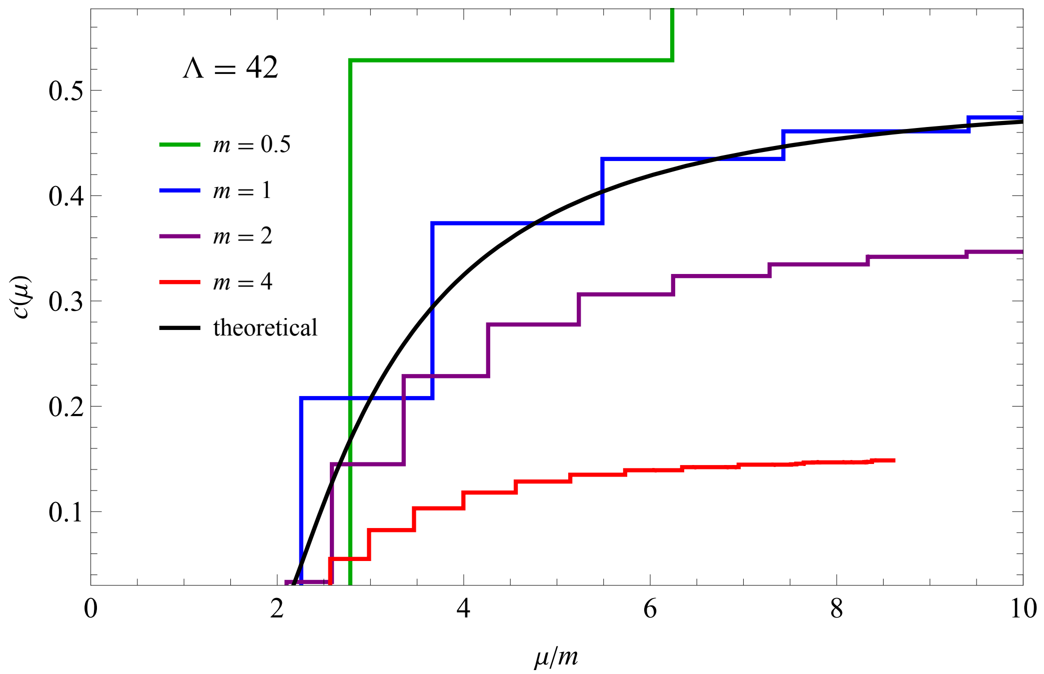
$$\langle T\{\mathcal{O}\mathcal{O}\}\rangle(p) = \int d\mu^2 \frac{i\rho_{\mathcal{O}}(\mu)}{p^2 - \mu^2 + i\epsilon}$$

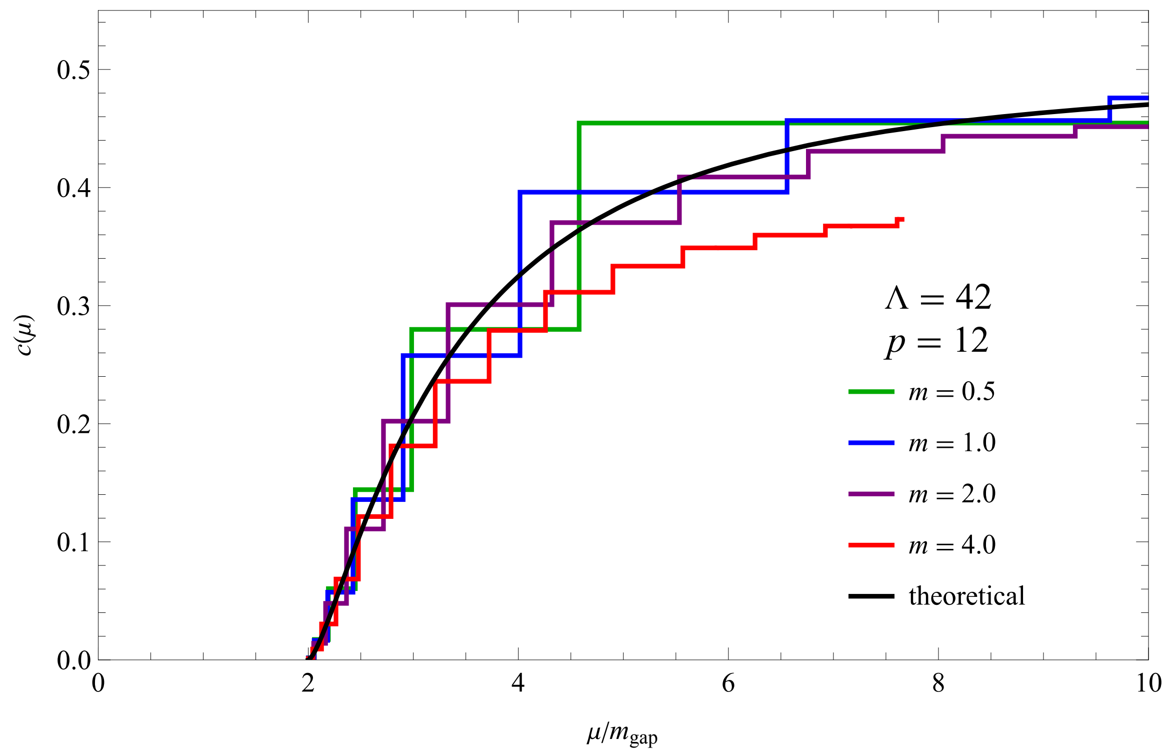
Numerically easier to compute:

$$I_{\mathcal{O}}(\mu) = \int_0^{\mu^2} d\mu'^2 \rho_{\mathcal{O}}(\mu'^2) = \sum_{\mu_i^2 < \mu^2} \langle 0|\mathcal{O}|\mu_i\rangle^2$$

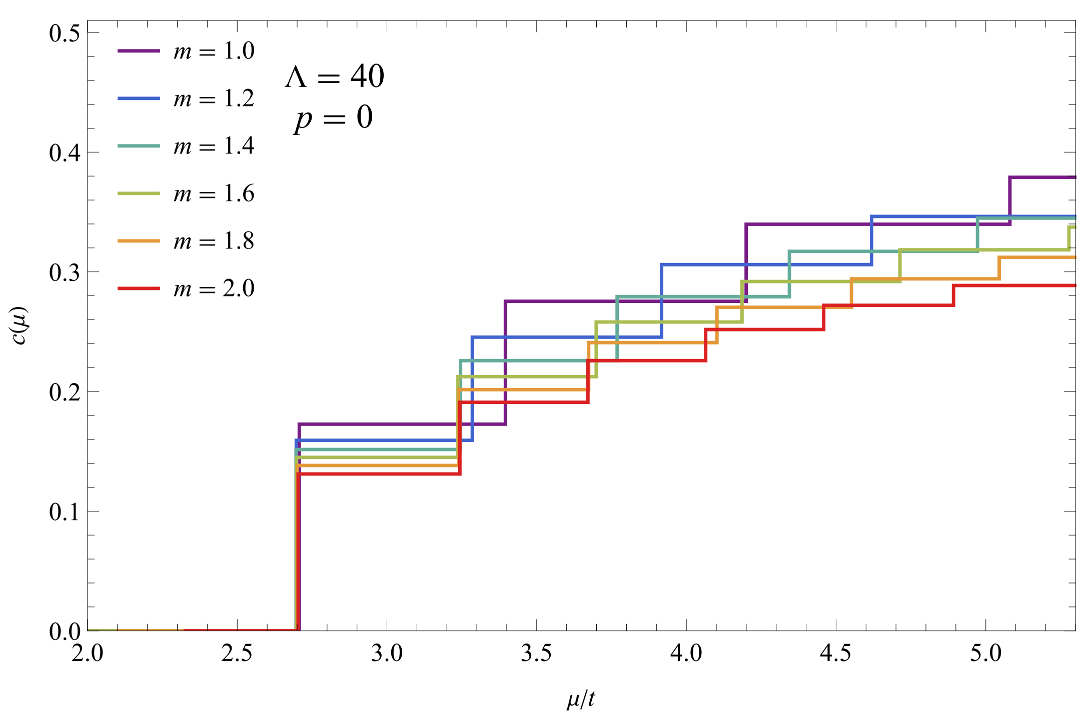
In 2d: $I_T(\mu) = c(\mu)$ (d.o.f. of the QFT)

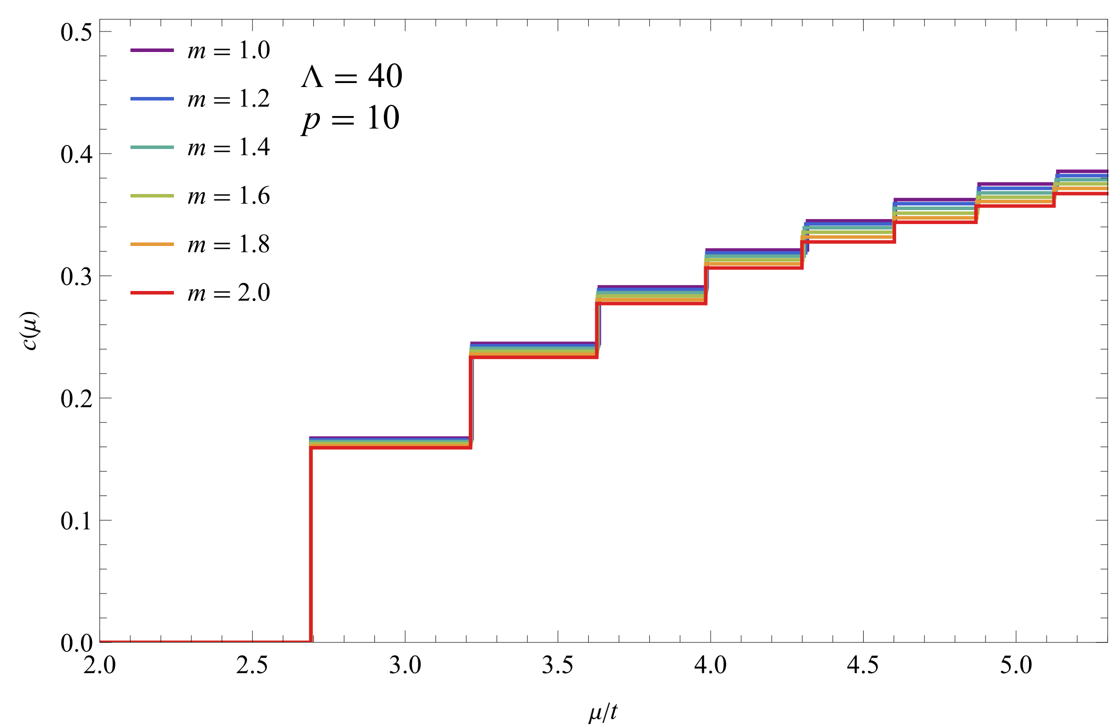
Mass deformation alone:





General strong coupling in the confining phase:





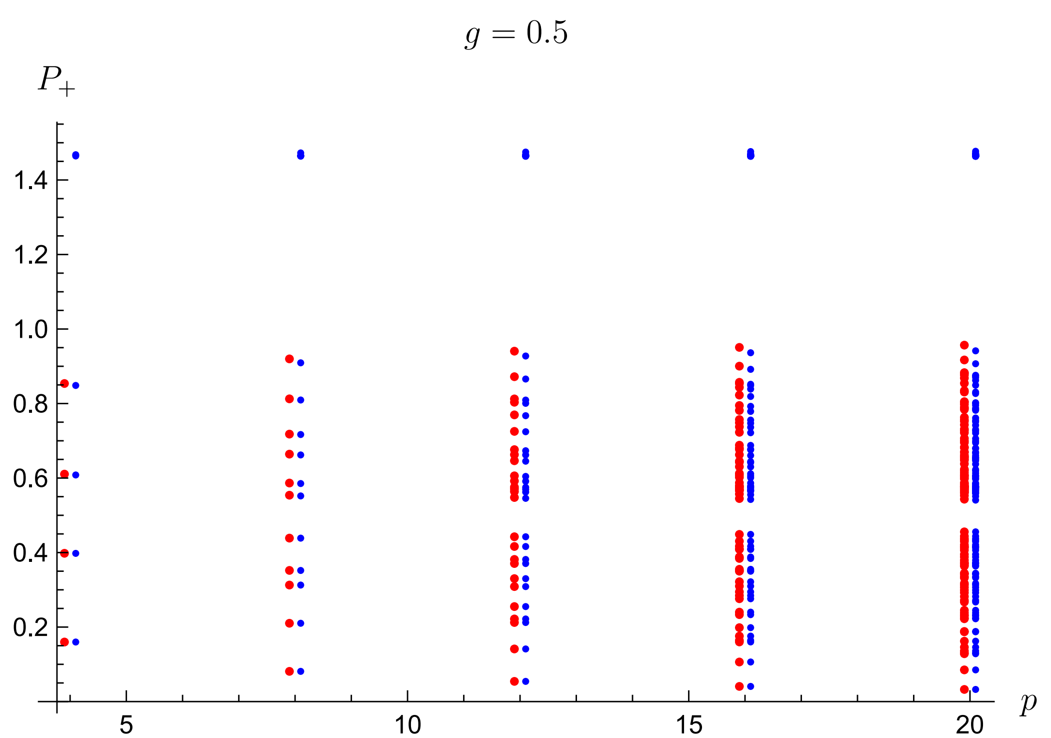
More directly - boosting allows the explicit calculation of the LC Effective Hamiltonian in 2d (including the effects of zero-modes)

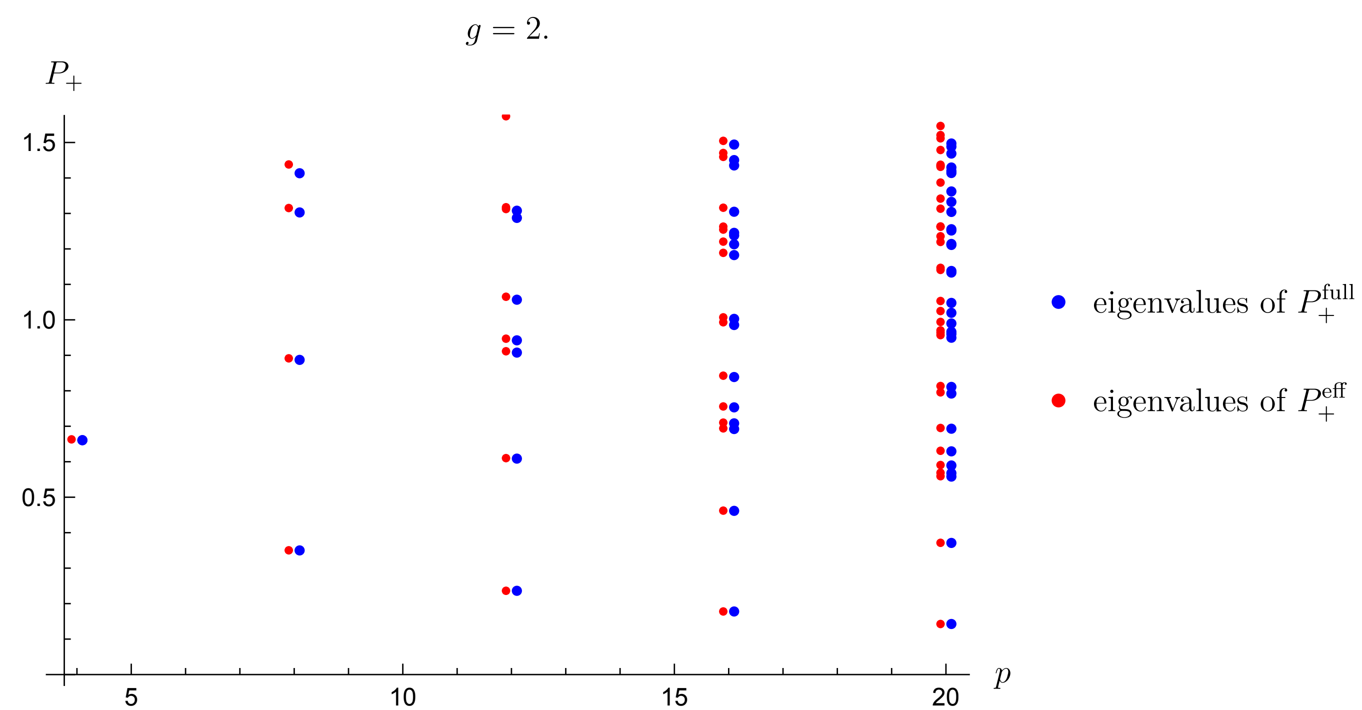
$$H = H_0 + V$$

(finite volume equal-time)

$$P_+^{eff}$$

(acting only on LC modes!)





$$H = H_0 + g \int \sigma(x)$$

The LCT Basis

A conformal basis of the Hilbert space

Can choose CFT "primary "operators s.t. :

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle \sim \delta_{ij}\frac{1}{x^{2\Delta_i}}\,(polarization)$$

CFT basis:

$$|\mathcal{O},\mu\rangle \equiv \int d^3x\, e^{-ip\cdot x}\,\mathcal{O}(x)|0\rangle\ ,\ p^2=\mu^2>0$$

$$\langle \mathcal{O},\mu|\mathcal{O}',\mu'\rangle = \rho_{\mathcal{O}}(\mu)\delta(\mu^2-\mu'^2)\delta_{\mathcal{O},\mathcal{O}'}$$

($\rho_{\mathcal{O}}(\mu)$ is the F.T. of the Wightman 2pt Fcn)

For a Free Scalar CFT:

$$\mathcal{O}(x) = \sum_{\mathbf{k}} C^{\mathcal{O}}_{\mathbf{k}} \partial^{k_1}\phi \cdots \partial^{k_n}\phi, \;\; k_i = (k_{i-}, k_{i\perp})$$

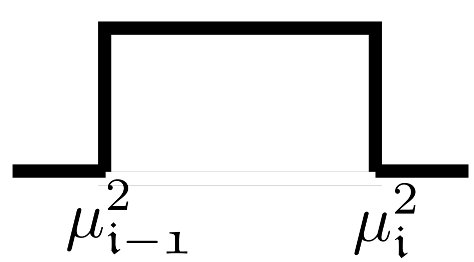
Polynomials on the Fock Space

(also explored by S.S. Chabysheva and J.R. Hiller in 2d)

For 2d free basis states are chiral: $P_+ = 0, \;\; \mu^2 = 2P_+P_- = 0$

In 3d the μ label needs to be discretized:

$$|\mathcal{O}, \mathrm{i}\rangle \equiv \frac{1}{\sqrt{2\pi}} \int_0^{\Lambda^2_{UV}} d\mu^2\, b_{\mathrm{i}}(\mu)\, |\mathcal{O}, \mu\rangle$$

$b_{\mathrm{i}}(\mu) =$ 

$\mathrm{i} = 1, \ldots, \mathrm{i}_{\max}$

Truncation: States $|\mathcal{O}, \mathrm{i}\rangle$ with $\Delta_{\mathcal{O}} \leq \Delta_{max}$

(i.e. we are truncating on a dimensionless parameter!)

1) The dynamic range in energy is determined by Delta Max

2) The size of the basis grows exponentially with Delta Max

(similar to the K parameter of DLCQ)

Crucial to have efficient calculation techniques!

The CFT structure is central to performing efficient calculations!

2d scalar example:

		Wick-Contractions Pedagogical			CFT methods Radial Quantization		
$\Delta_{\max}$	num of states	basis	mass	quartic	basis	mass	quartic
10	42	0.19	0.26	2.36	0.02	0.06	0.07
20	627	3061	170.1	5183	0.46	1.09	3.96
30	5604	weeks?	hours?	weeks?	7.88	17.93	111.9
40	37338	Good luck			231	410	3579

(From our pedagogical review - 2005.13544)

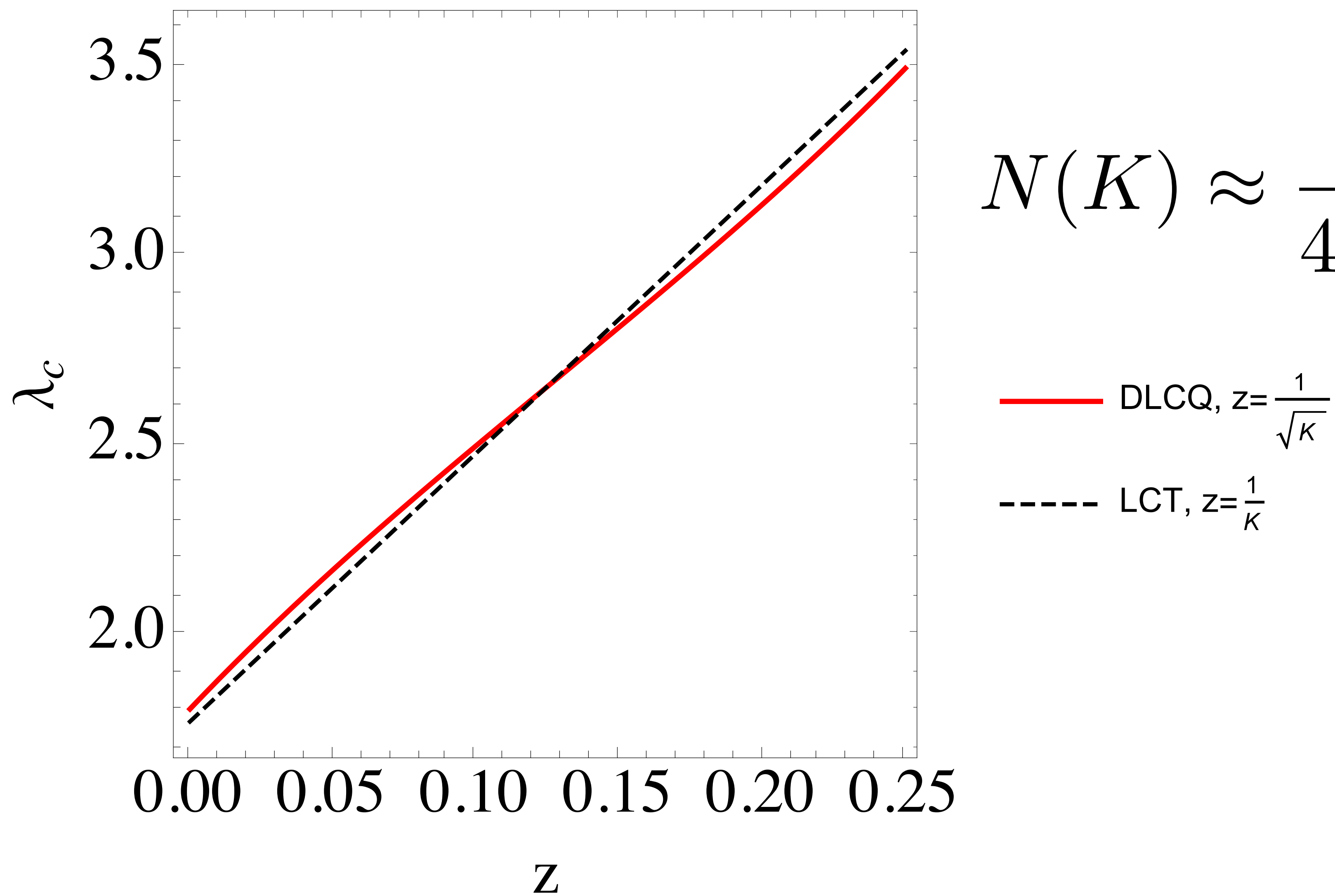
+ Mathematica Package to get started

Why use LCT?

Empirically seems more efficient and flexible than DLCQ

J.P. Vary, M. Huang, S. Jawadekar, M. Sharaf,
A. Harindranath, and D. Chakrabarti

Studied $\lambda\phi^4$ theory in 2d at large K~90



$$N(K) \approx \frac{1}{4K\sqrt{3}} e^{\pi\sqrt{\frac{2}{3}K}}$$

The case of 2d QCD

In the't Hooft model:

$$\psi_{bound-state}(x) \sim x^{\alpha},\ x \to 0,\ \alpha \sim \frac{m_q}{\sqrt{g^2 N_c}}$$

In the chiral limit this leads to very slow convergence for both DLCQ and LCT

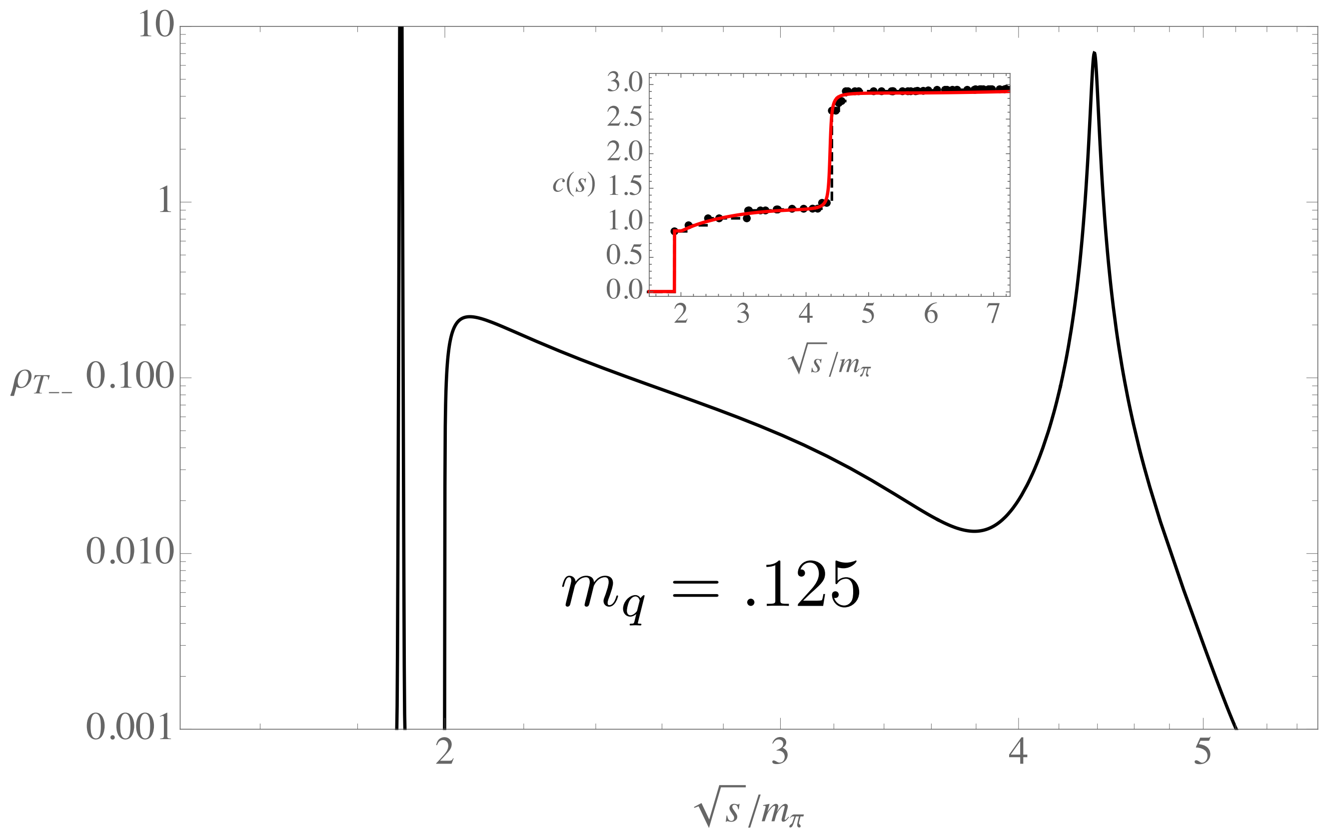
But LCT can be modified:

$$\mathcal{O} \equiv \sum_{\boldsymbol{k}} C_{\boldsymbol{k}}^{\mathcal{O}} (\partial^{k_1+\alpha}\psi^{\dagger}\partial^{k_2+\alpha}\psi)\cdots(\partial^{k_{2n-1}+\alpha}\psi^{\dagger}\partial^{k_{2n}+\alpha}\psi)$$

(see earlier work by T. Sugihara, M. Matsuzaki, and M. Yahiro)

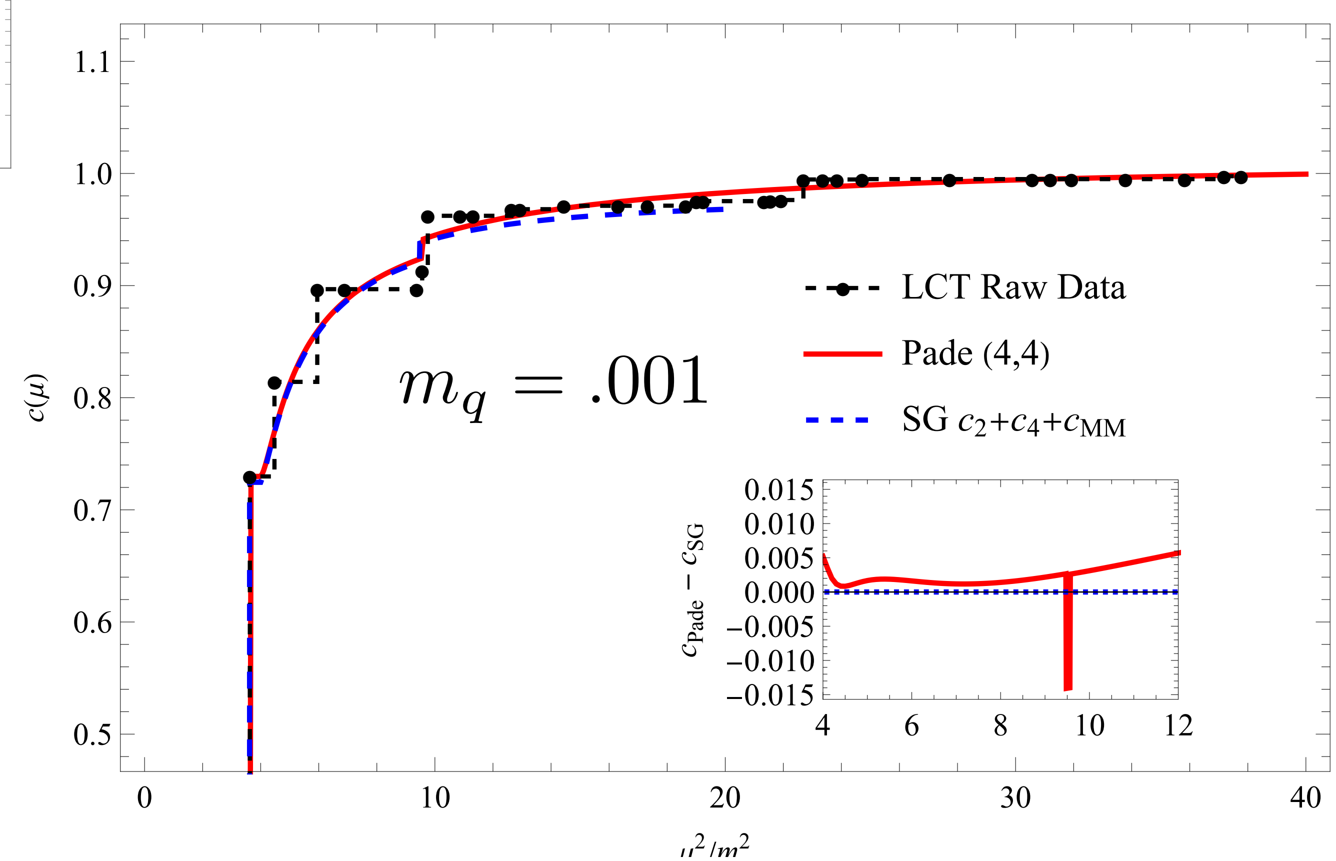
IR of 2d QCD in the chiral limit is Sine-Gordon:

$$\mathcal{L}_{IR} = \frac{1}{2}(\partial\phi)^2 + \lambda\cos(\beta\phi), \quad \Delta_{cos} = \frac{\beta^2}{4\pi} = \frac{1}{N_c}$$



$$"e^+e^- \rightarrow hadrons"$$

a detailed check:



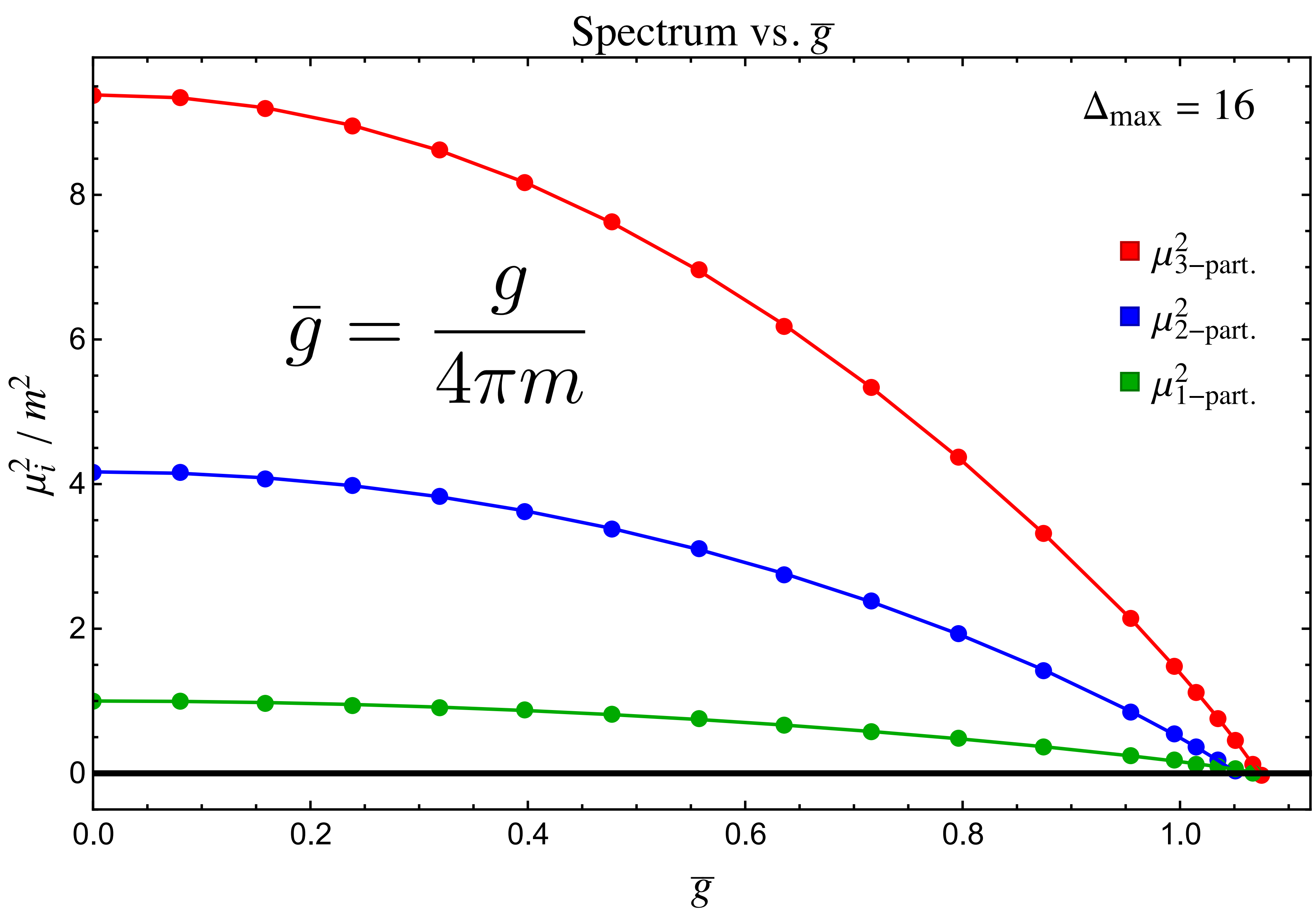
A 3d scalar example:

$$P_+^{(\mathrm{CFT})} + \int d^2\vec{x}\left[\frac{1}{2}(m^2 \boxed{- c_L g^2)\phi^2} + \frac{1}{4!}g\phi^4\right] + \boxed{g^2\delta P_+^{(\text{state-dep.})}}$$

Free massless scalar

local c.t.

non-local c.t.

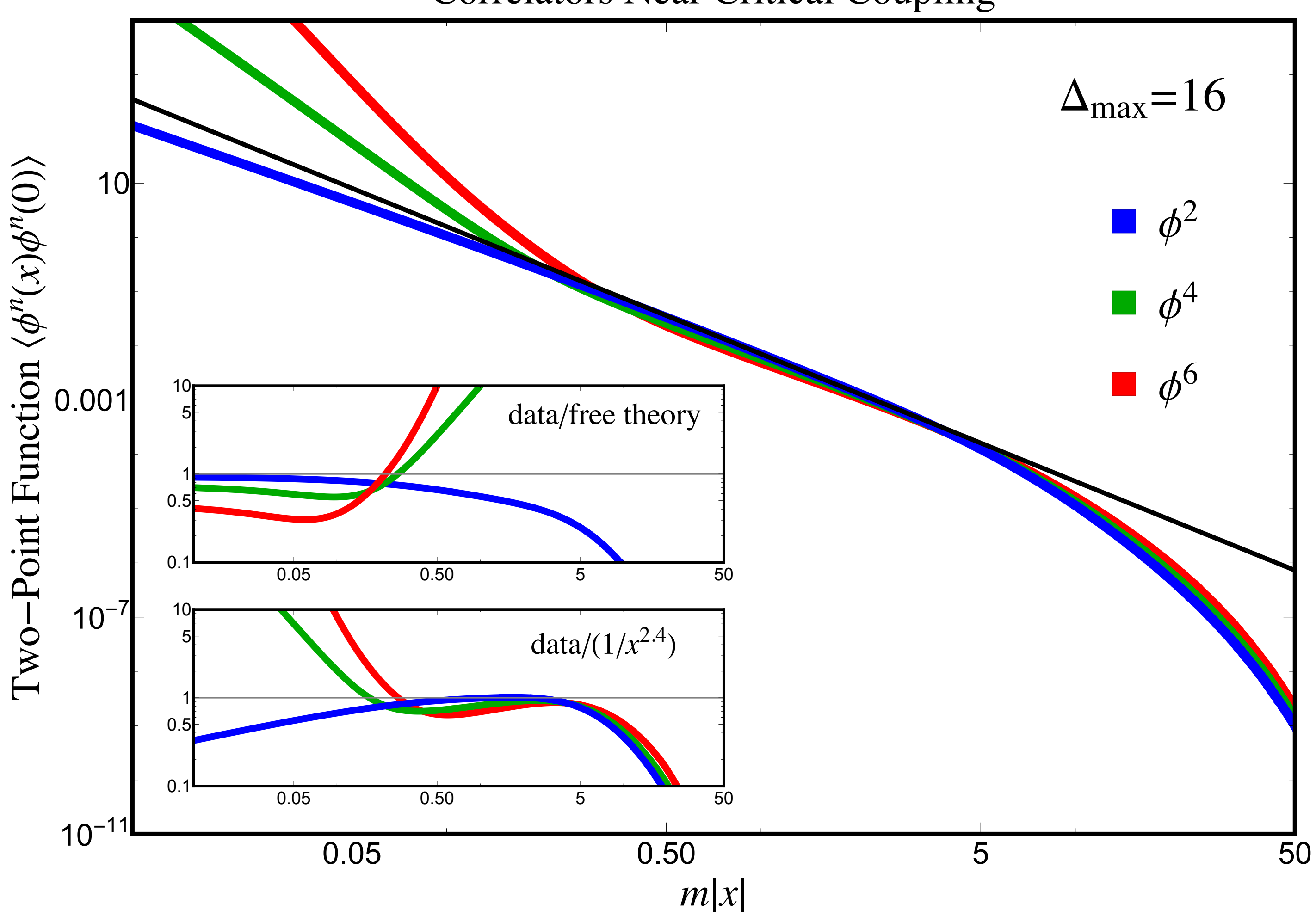


$$|\mathcal{O}, \mathfrak{i}\rangle \equiv \frac{1}{\sqrt{2\pi}} \int_0^{\Lambda_{UV}^2} d\mu^2 \, b_{\mathfrak{i}}(\mu) \, |\mathcal{O}, \mu\rangle$$

545 primary operator states

545 x 65bins = 35,425 total states

Correlators Near Critical Coupling



$$\phi^{2n} \sim \epsilon$$

$$\langle \epsilon(x)\epsilon(0)\rangle \approx \frac{1}{|x|^{2.83}}$$

Thermalization & Chaos

What can we say with LCT about highly excited states?

Naively: $\Delta E \sim e^{-S(E)}$ << Truncation Error

Increasing Delta increases both UV cutoff
and IR resolution

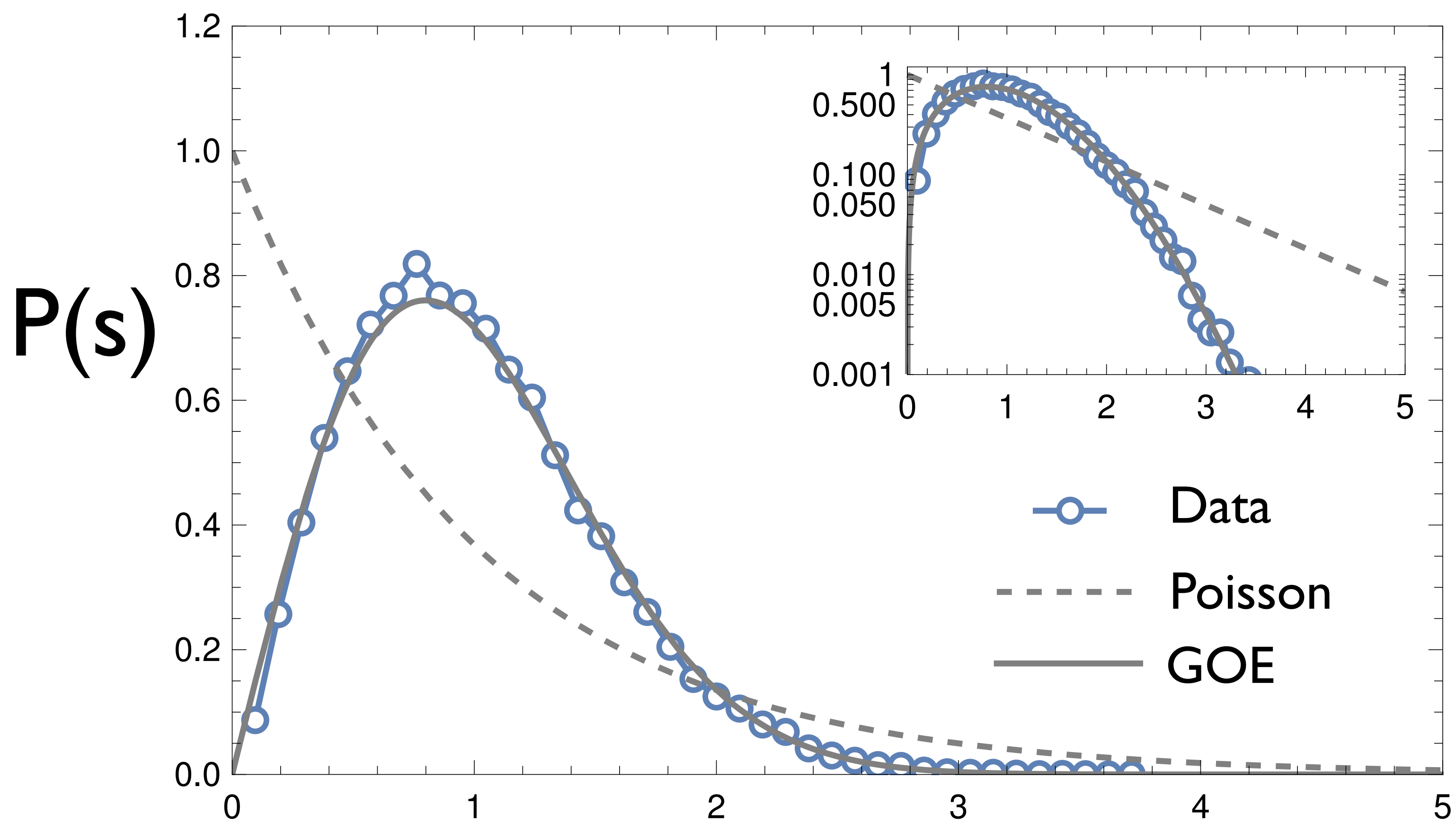
(similar to K for DLCQ)

So how could excited states ever converge?

What information could they possibly contain?

Chaos QFT Lab: 2d $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$

Energy difference stats follow Random Matrix Theory:



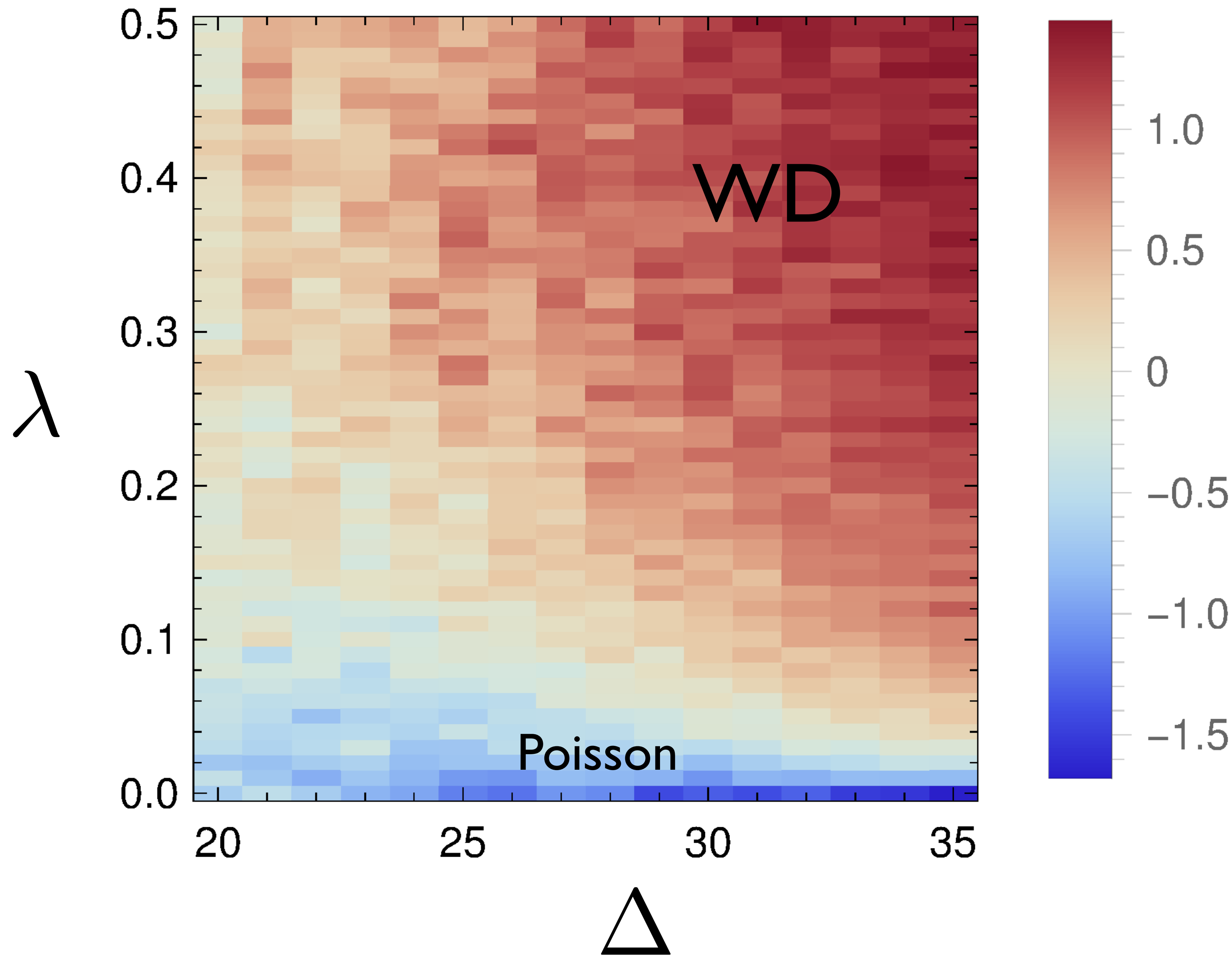
$$P_{GOE}(s) = \frac{\pi}{2} s e^{-\frac{\pi}{4}s^2}$$

Wigner-Dyson

$$s = \frac{E_{i+1} - E_i}{\Delta E_{ave}}$$

For which couplings do we get chaos?

$$\eta \equiv \frac{||P - P_{\text{Poisson}}||}{||P - P_{\text{GOE}}||} = \frac{\int_0^\infty ds |P(s) - e^{-s}|}{\int_0^\infty ds |P(s) - \frac{\pi}{2} s e^{-\frac{\pi}{4} s^2}|}$$



For further measurements we need to know the <u>size</u> of high energy states

States are parameterized by polys with degree controlled by Delta

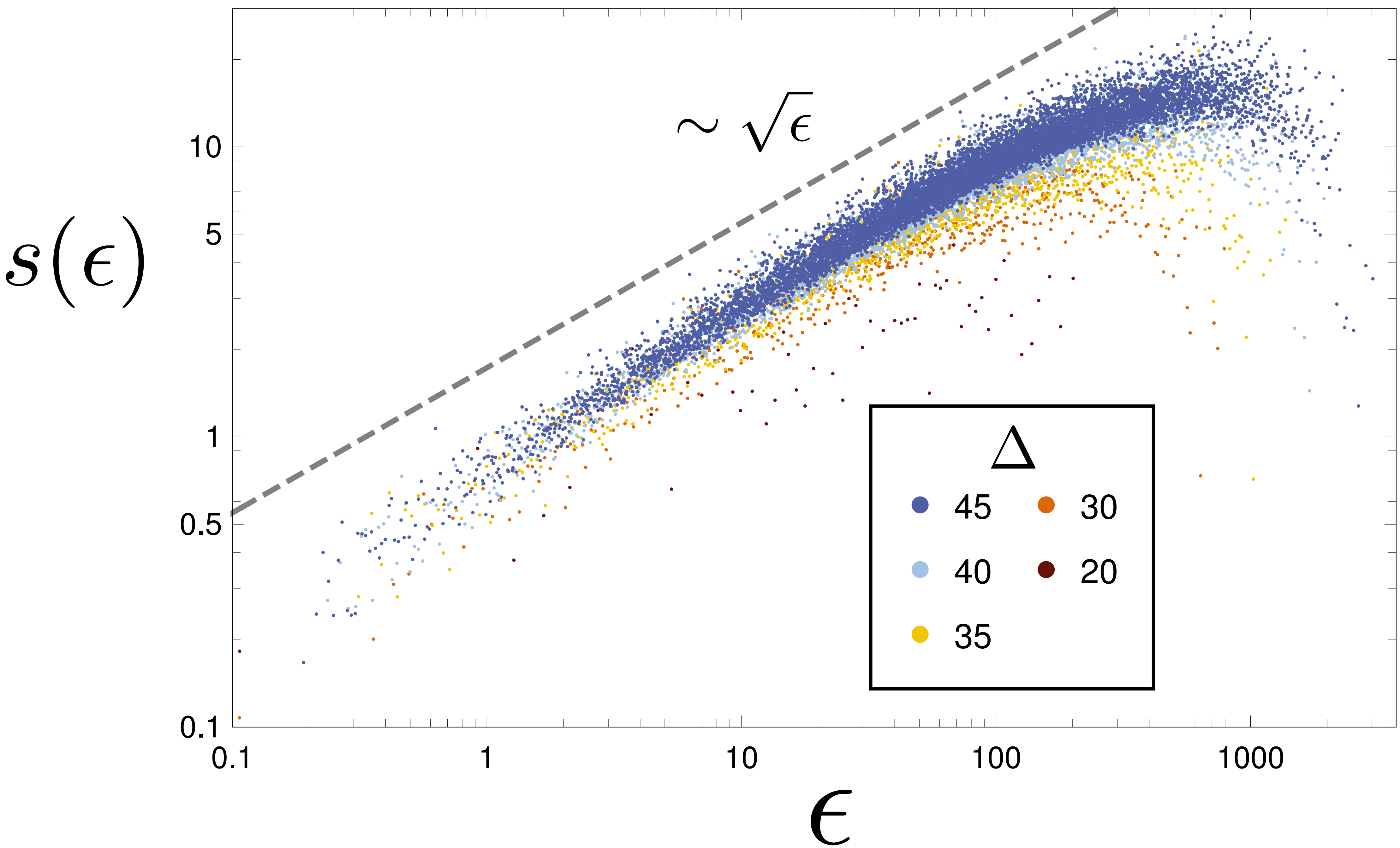
LC momentum resolution: $\delta p_- \sim \frac{1}{\Delta} P_-$

LC IR resolution or "volume": $V^- \sim \frac{\Delta}{P_-}$

Rest-frame volume: $V \sim \frac{\Delta}{E}$

Energy density: $\epsilon = \frac{E}{V} \sim \frac{E^2}{\Delta}$

Entropy Density



Equation of State

Canonical approach requires being careful about energy dependence of the volume

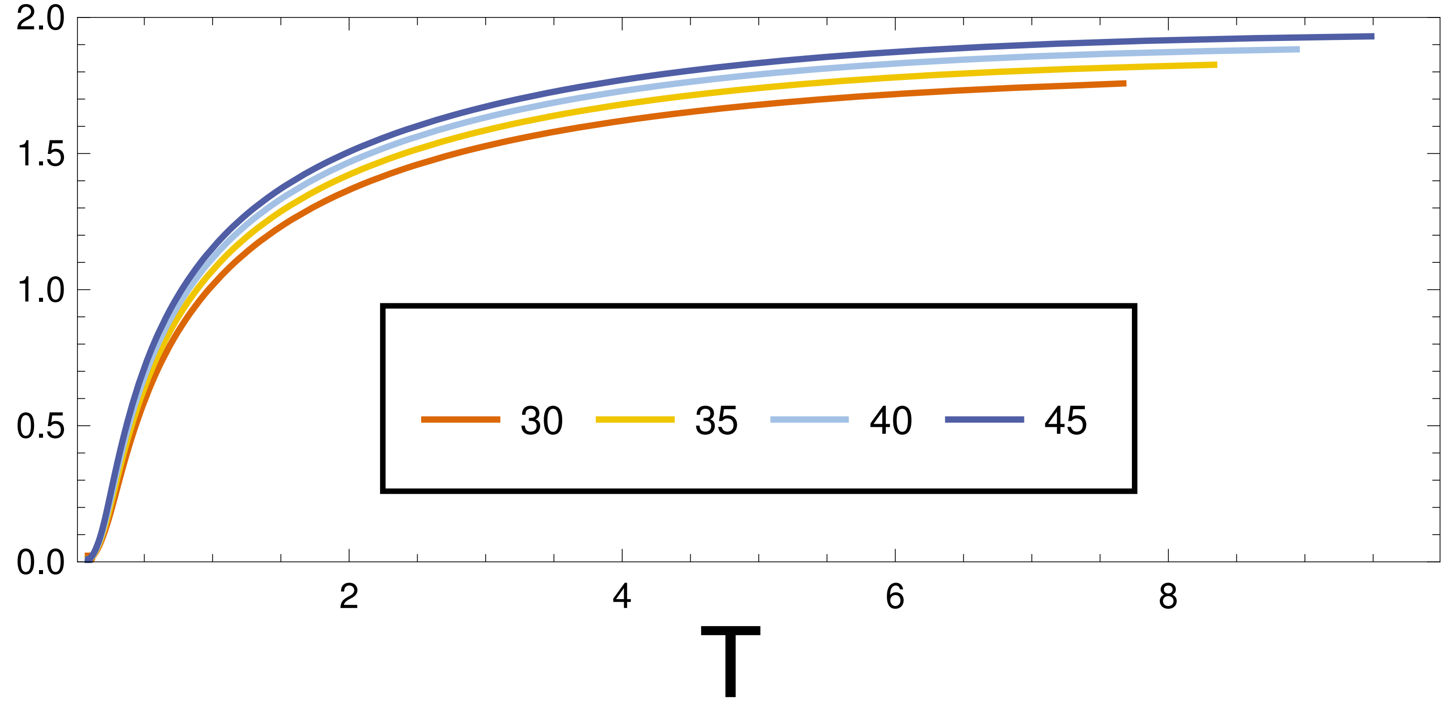
$$TdS = dE + PdV = \left(1 + P\frac{\partial V}{\partial E}\right) dE + P\frac{\partial V}{\partial \Delta} d\Delta$$

$$Z(\tilde{\beta}, \Delta) \equiv \sum_n e^{-\tilde{\beta} E_n}$$

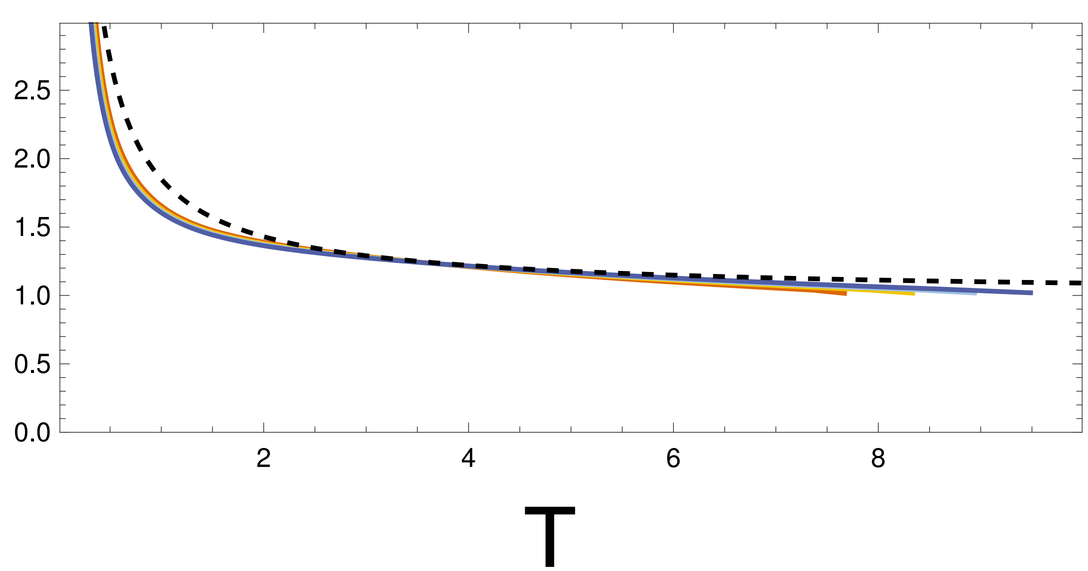
$$\tilde{\beta} = \beta\left(1 - \frac{P}{\varepsilon}\right)$$

Thermodynamics

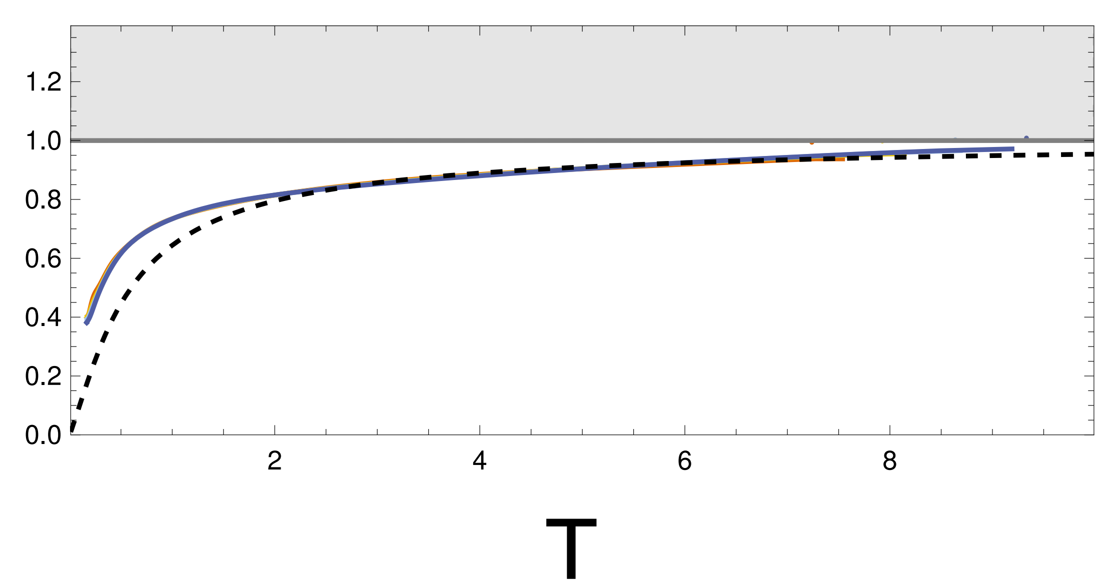
$$s_0 \equiv \frac{1}{T}\frac{dP}{dT}$$



$$\frac{\epsilon}{P}$$



$$c_s^2$$

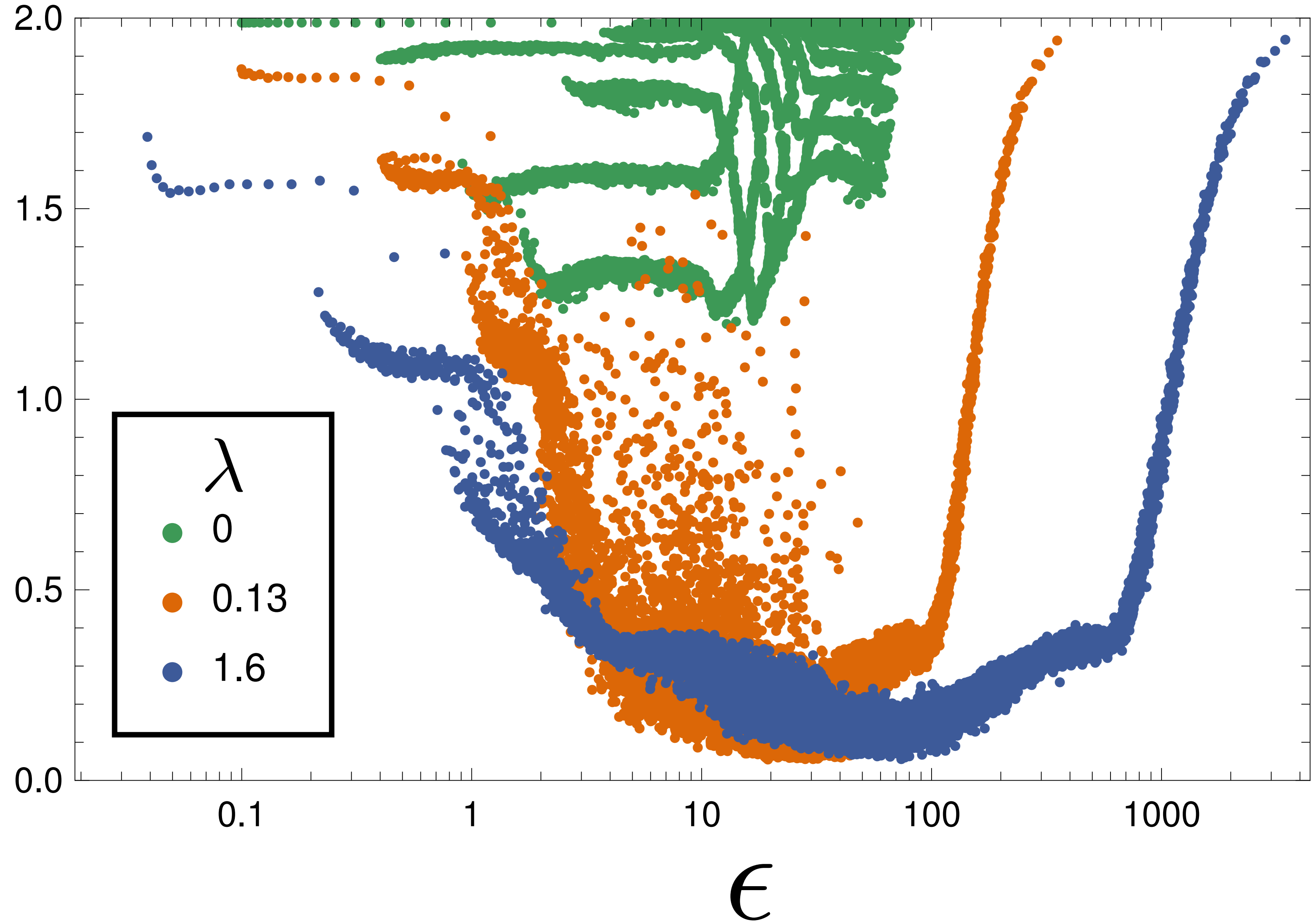


Wavefunction Coefficient Stats

$$|E_n\rangle = \sum_i c_i^{(n)} |\mathcal{O}_i\rangle$$

Random matrix theory: $P_{\mathrm{GOE}}(|c|^2) = \dfrac{1}{\sqrt{2\pi\sigma|c|^2}} e^{-\frac{|c|^2}{2\sigma}}$

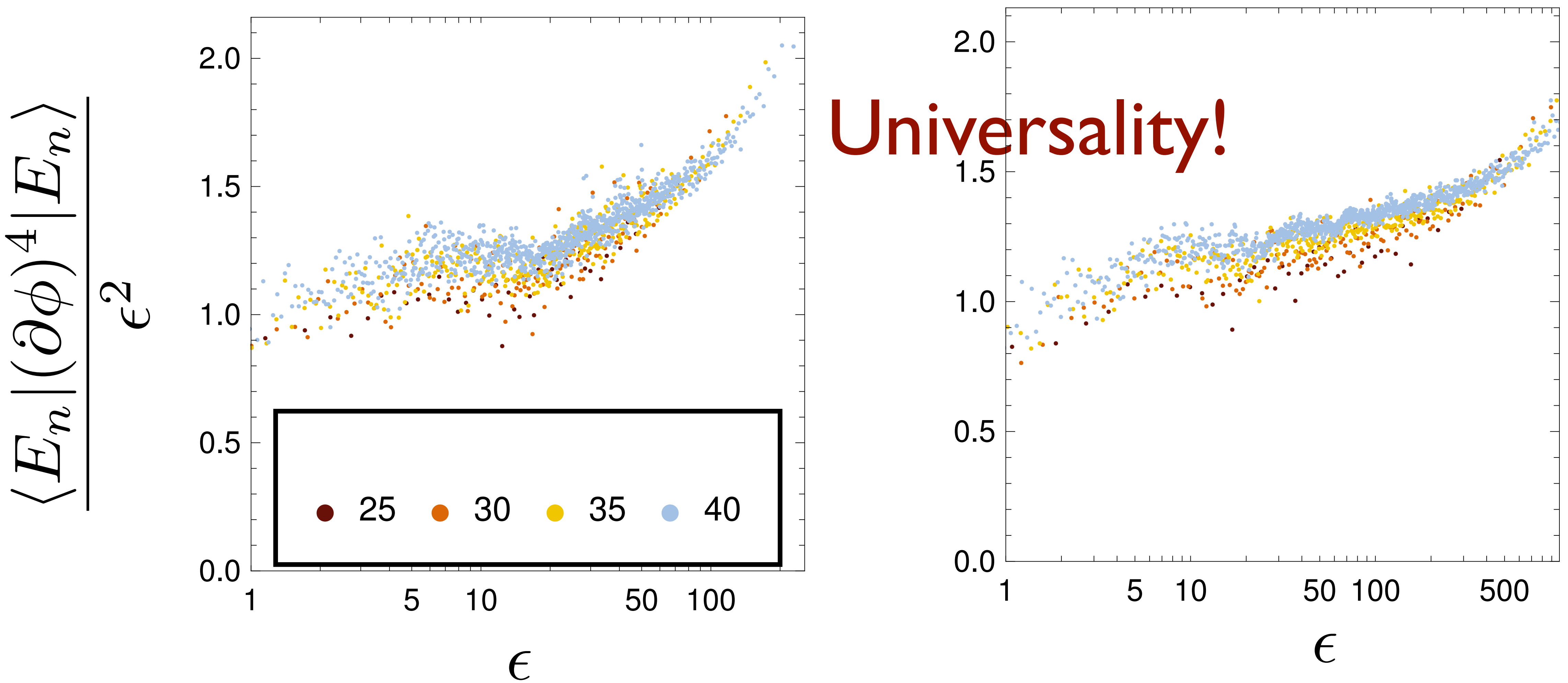
$$||P(c^2) - P_{\mathrm{GOE}}||$$



Eigenstate Thermalization

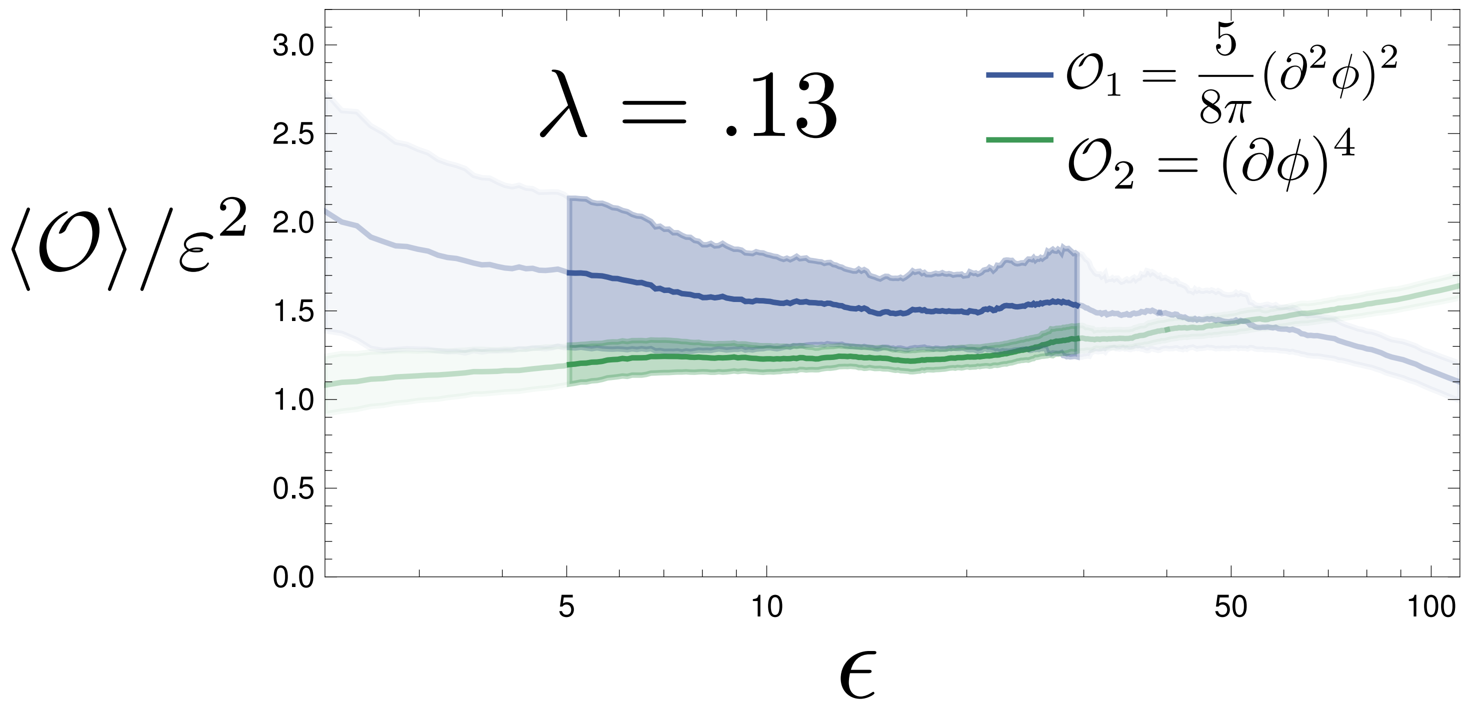
Ergodicity in QM:

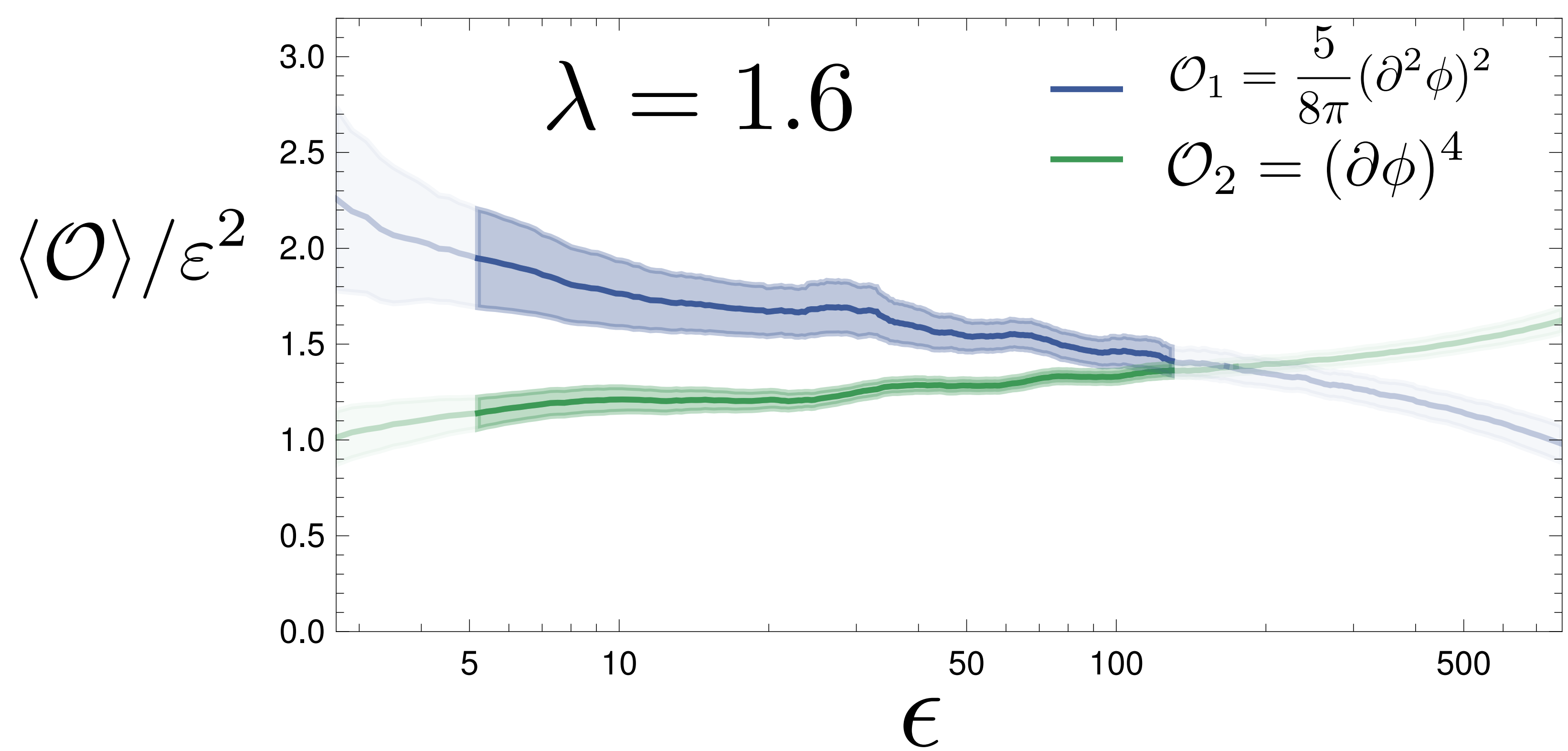
$$\langle E_n|\mathcal{O}|E_n\rangle = \langle \mathcal{O}\rangle_T + e^{-S}$$



Check detailed finite temp. CFT prediction:

$$\langle (\partial_-\phi)^4 - \frac{5}{8\pi}(\partial_-^2\phi)^2\rangle_T = 0 + \mathcal{O}(\frac{1}{T^p})$$





Conclusion

1. Lightcone Conformal Truncation is a robust and flexible numerical tool for non-perturbative 2d and 3d QFTs on the lightcone.
2. At current truncations we already have enough resolution to reconstruct time-dependent correlation functions.
3. LCT can be used to calculate the equation of state.
4. We explored chaos and thermalization, verifying the eigenstate thermalization hypothesis.

A sketch of application to 4d Gauge theories

Traditionally: $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\psi - g A_\mu J^\mu_\psi(x)$

Problem: $g A_\mu J^\mu_\psi(x)$ is not a local gauge-inv. op!

Quantization becomes sensitive choosing an appropriate regulator!

No robust non-perturbative regulator for any known Hamiltonian method respecting space-time symmetries.

Moreover - log running isn't really ever a CFT!

Alternative idea:

1. "Banks-Zaks the theory":

Add vector-like matter to make the theory a weakly coupled CFT.

Ex. QCD w/ some flavors (2-6): Add more flavors.

Nf=16: $\alpha_s * \approx .04$ Nf=15: $\alpha_s * \approx .15$ Nf=14: $\alpha_s * \approx .25$

2. Remove the extra matter:

Ex. QCD: $\delta P_+ = \int d^{d-1}x \; m_q \boxed{\bar{\psi}_i \psi_i(\vec{x})}$

Gauge Inv. local Op.

Chiral Ex. SU(5) w/ 10 + $\bar{5}$

Add 4 Adjoint fermions: $\alpha_* \sim .16$

<u>Tension:</u>

Computability of CFT data:
small coupling.
(conformal bootstrap can help!)

Wanting a smaller basis or shorter RG-flows:
larger coupling.
(but still allowing for separation of scales)

<u>A possible easier start:</u>

$\mathcal{N} = 4$ SYM @ large-N → Large-N Yang-Mills

Size of States Measurement

$$\langle \mu, P_- + q_- | T_{--}(0) | \mu, P_- \rangle = 2P_-^2 \left(1 - \alpha^2 \Delta^2 \frac{q_-^2}{P_-^2} + \cdots \right)$$