The EMC effect within the Light-Front Hamiltonian dynamics

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in collaboration with

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STRONG 2020

Based on

R. Alessandro, A. Del Dotto, E. Pace, G. Perna, G. Salmè and S. Scopetta,
Light-Front Transverse Momentum Distributions for $\mathcal{J} = 1/2$ Hadronic Systems in Valence Approximation
Phys.Rev.C 104 (2021) 6, 065204

A. Del Dotto, E. Pace, G. Salmè, and S. Scopetta,
Light-Front spin-dependent Spectral Function and Nucleon Momentum Distributions for a Three-Body System, **Phys. Rev. C 95, 014001 (2017)**

E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta,
EMC effect, few-nucleon systems and Poincaré covariance,
Phys. Scr. 95, 064008 (2020)

MARATHON Coll.
Measurement of the Nucleon Fn2/Fp2 Structure Function Ratio by the Jefferson Lab MARATHON Tritium/Helium-3 Deep Inelastic Scattering Experiment,
Phys.Rev.Lett 128 (2022) 13, 132003

E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta,
The European Muon Collaboration effect in Light-Front Hamiltonian Dynamics
ArXiv: 2206.05484

Outline

- **Motivations**
- The Light-Front Poincaré covariant approach
- The Light-Front spectral function
- Application: “TMDs” of ^{3}He
- The EMC effect within the Light-Front approach: the ^{3}He case
- Conclusions and Perspectives

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1 Motivations: why the ^{3}He

- **Phenomenological:** a reliable flavor decomposition needs the neutron parton structure (PDFs, GPDs TMDs....)

Accurate and long-lasting experimental efforts in developing effective neutron targets to carefully investigate its electromagnetic responses. **$^3\vec{He}$ is SPECIAL**

the polarized ^{3}He target, 90% neutron target (e.g. H. Gao et al, PR12-09-014, Chen et al, PR12-11-007, @JLab12)

- Due to the experimental energies, the accurate theoretical description (of a polarized ^{3}He) has ***to relativistic***

- **Theoretical:** a LF description of three body interacting systems! Bonus: Transverse-Momentum Distributions (**TMDs**) for addressing in a novel way the nuclear dynamics

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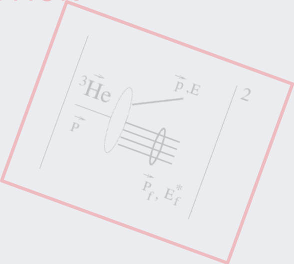
2 Nuclear dynamics and Spectral Function

From a theoretical point of view, we need:

- a description of the nuclear dynamics which retains as many general properties as possible...
- ... leading to realistic procedures to *extract* the Nucleon (neutron) structure

In the presented approach the key quantity is the nuclear *SPECTRAL FUNCTION*
(Nucleon Green's function in the medium)



$$P_{\sigma'\sigma}(k,E) = -\frac{1}{\pi}\Im m\left\{\langle\Psi_{gr}|a^\dagger_{\mathbf{k},\sigma'}\ \frac{1}{E-H+i\epsilon}\ a_{\mathbf{k},\sigma}|\Psi_{gr}\rangle\right\}$$

$$\boxed{H = \sum_{n}^{\infty}\frac{1}{n}\sum_{\substack{\alpha_1,\ldots,\alpha_n\\ \beta_1,\ldots,\beta_n}}\langle\alpha_1\ldots\alpha_n|H_n|\beta_1\ldots\beta_n\rangle\prod_{i=1}^{n}a^\dagger(\alpha_i)a(\beta_i)}$$

Diagonal terms: probability density to find a constituent with σ , k with an energy E of the remaining system in the ground state of the bound system.

Quite familiar in nuclear Physics;in hadron physics one introduces the LC correlator:

$$\Phi^\tau(x,y) = \langle\Psi_{gr}|\bar{\psi}_\tau(x)\mathcal{W}(\hat{n}\cdot A)\psi_\tau(y)|\Psi_{gr}\rangle$$

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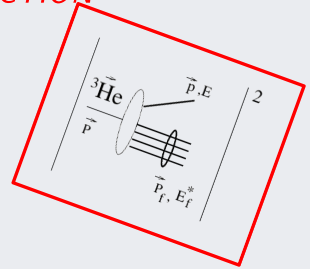

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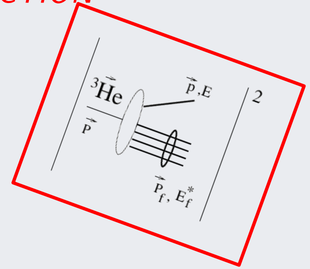

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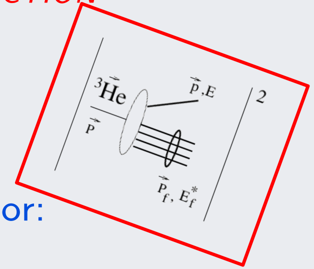

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Our point: in valence approximation, one can relate $P_{\sigma'\sigma}(k,E)$ (given in a Poincaré covariant framework) and $\Phi^{\tau}(x,y)$

[Alessandro, Del Dotto, Pace, Perna, Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204]

3 The relativistic Hamiltonian dynamics framework

Why do we need a relativistic treatment ?

General answer: to develop an advanced scheme, appropriate for the kinematics of JLAB12 and of EIC

- The Standard Model of Few-Nucleon Systems, with nucleon and meson degrees of freedom within a non relativistic (NR) framework, has achieved high sophistication
  [e.g. the NR ^{3}He and ^{3}H Spectral Functions in Kievsky, Pace, Salmè, Viviani PRC 56, 64 (1997)].
- Covariance wrt the Poincaré Group, $\mathcal{G}_P$, needed for nucleons at large 4-momenta and pointing to high precision measurements. Necessary if one studies, e.g., i) nucleon structure functions; ii) nucleon GPDs and TMDs, iii) signatures of short-range correlations; iv) exotics (e.g. 6-bag quarks in ^{2}H), etc
- At least, one should carefully treat the boosts of the nuclear states, $|\Psi_i\rangle$ and $|\Psi_f\rangle$!

Our definitely preferred **framework for embedding** the successful **NR** phenomenology:

Light-front Relativistic Hamiltonian Dynamics (RHD, fixed dof) +Bakamjian-Thomas (BT) construction of the Poincaré generators for an interacting theory.

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In RHD+BT, one can address both Poincaré covariance and locality, general principles to be implemented in presence of interaction:

- Poincaré covariance → The 10 generators, $P^\mu \rightarrow$ 4D displacements and $M^{\nu\mu} \rightarrow$ *Lorentz* transformations, have to fulfill

$$[P^\mu, P^\nu] = 0, \quad [M^{\mu\nu}, P^\rho] = -\imath(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -\imath(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

Also $\mathcal{P}$ and $\mathcal{T}$ have to be taken into account !

- Macroscopic locality ($\equiv$ cluster separability (relevant in nuclear physics)): i.e. observables associated to different space-time regions must commute in the limit of large spacelike separation (i.e. causally disconnected), rather than for arbitrary (microscopic-locality) spacelike separations. In this way, when a system is separated into disjoint subsystems by a sufficiently large spacelike separation, then the subsystems behave as independent systems.

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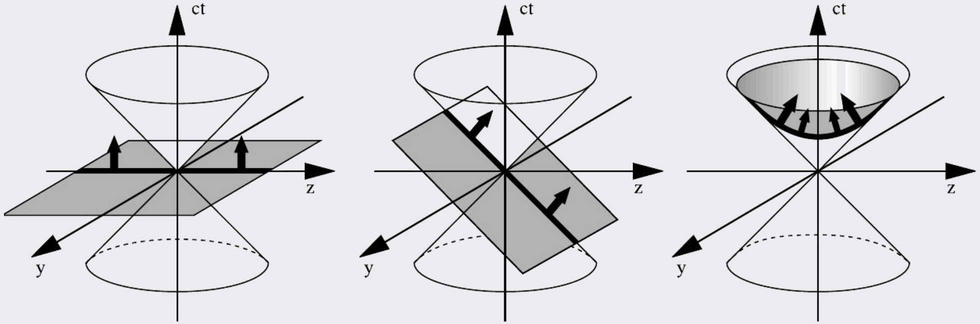
4 Forms of relativistic Dynamics

P.A.M. Dirac, 1949



The instant form

$$\tilde{x}^0 = ct$$
$$\tilde{x}^1 = x$$
$$\tilde{x}^2 = y$$
$$\tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The front form

$$\tilde{x}^0 = ct+z$$
$$\tilde{x}^1 = x$$
$$\tilde{x}^2 = y$$
$$\tilde{x}^3 = ct-z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

The point form

$$\tilde{x}^0 = \tau \quad , \quad ct = \tau \cosh\omega$$
$$\tilde{x}^1 = \omega \quad , \quad x = \tau \sinh\omega \sin\theta \cos\phi$$
$$\tilde{x}^2 = \theta \quad , \quad y = \tau \sinh\omega \sin\theta \sin\phi$$
$$\tilde{x}^3 = \phi \quad , \quad x = \tau \sinh\omega \cos\theta$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2 \sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2 \sinh^2\omega \sin^2\theta \end{pmatrix}$$

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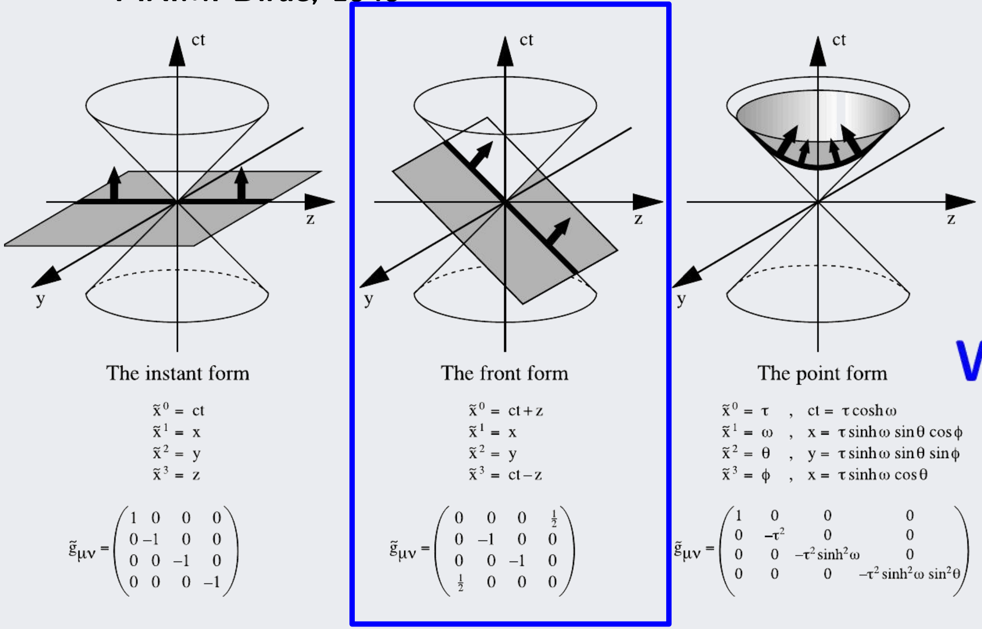

We choose the Front Form!

Fig. from **Brodsky, Pauli, Pinsky Phys.Rept. 301 (1998) 299-486**

4 Forms of relativistic Dynamics - The front form

The **Light-Front framework** has several advantages:

- 7 Kinematical generators: i) three LF boosts (In instant form they are dynamical!), ii) $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$, iii) Rotation around the z-axis.
- The LF boosts have a subgroup structure : trivial Separation of intrinsic and global motion, as in the NR case. important to correctly treat the boost between initial and final states !
- $P^+ \geq 0 \longrightarrow$ meaningful Fock expansion, once massless constituents are absent
- No square root in the dynamical operator P^-, propagating the state in the LF-time.
- The infinite-momentum frame (IMF) description of DIS is easily included.

Drawback: the transverse LF-rotations are dynamical

But within the Bakamjian-Thomas (BT) construction of the generators in an interacting theory, one can construct an intrinsic angular momentum fully kinematical!

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★ The Mass Operator, developed within a non relativistic framework, is fully acceptable for a BT construction of the Poincaré generators★

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5 Bakamjian-Thomas construction and LFHD

- Bakamjian and Thomas **(PR 92 (1953) 1300)** proposed an explicit construction of 10 Poincaré generators in presence of **interactions**.

  The key ingredient is the mass operator:

  i) only the mass operator M contains the **interaction**

  ii) it generates the dependence of the 3 dynamical generators (P^- and LF transverse rotations $\vec{F}_\perp$) upon the interaction

- The mass operator is given by the sum of M_0 with an interaction V, or . The interaction, U or V, must commute with all the kinematical generators and with the non-interacting angular momentum, as in the NR case.

- In the Few-body case, one can easily embed the NR phenomenology:

  i) the mass equation for, e.g. the ^{2}H: $[M_0^2(12) + U]\,|\psi_D\rangle = \left[4m^2 + 4k^2 + U\right]\,|\psi_D\rangle = M_D^2\,|\psi_D\rangle = [2m - B_D]^2\,|\psi_D\rangle$ becomes a Schr. eq.

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  i) the mass equation for, e.g. the ^{2}H: $[M_0^2(12) + U]\,|\psi_D\rangle = \left[4m^2 + 4k^2 + U\right]\,|\psi_D\rangle = M_D^2\,|\psi_D\rangle = [2m - B_D]^2\,|\psi_D\rangle$
  becomes a Schr. eq.
  $\left[4m^2 + 4k^2 + 4m\ V^{NR}\right]\,|\psi_D\rangle = \left[4m^2 - 4mB_D\right]|\psi_D\rangle$ where $U \equiv 4mV^{NR}$ neglecting $(B_D/2m)^2$

  ii) The eigensolutions of the mass equation for the continuum are identical to the solutions of the Lippmann-Schwinger equation.

5 Bakamjian-Thomas construction and LFHD

- Bakamjian and Thomas **(PR 92 (1953) 1300)** proposed an explicit construction of 10 Poincaré generators in presence of **interactions**.

  The key ingredient is the mass operator:

  i) only the mass operator M contains the **interaction**
  ii) it generates the dependence of the 3 dynamical generators (P^- and LF transverse rotations $\vec{F}_\perp$) upon the interaction

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6 The BT Mass operator for an A=3 system

- In the three-body case, the mass operator is: $M_{BT}(123) = M_0(123) + \underbrace{V^{BT}_{12,3} + V^{BT}_{23,1} + V^{BT}_{31,2}}_{\text{2-body forces}} + \underbrace{V^{BT}_{123}}_{\text{3-body force}}$

$$\underbrace{M_0^2(123) = \sum_{i=1}^{3} \sqrt{m^2 + \boxed{k_i^2}}}_{\textbf{free mass operator}}$$

momenta in the intrinsic reference frame $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$

- The commutation rules impose to V^{BT} invariance for translations and rotations as well as independence on the total momentum, as it occurs for V^{NR}.

- One can assume $M_{BT}(123) \sim M^{NR}$

- Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

The eigenfuntions of M^{NR} do not fulfill the cluster separability, but we take care of macroscopic locality in the spectral function.

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$$|\mathbf{k}; \frac{1}{2}, \sigma\rangle_c = \sum_{\sigma'} \underbrace{D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))}_{\text{Wigner rotation for the } \mathcal{J}=1/2 \text{ case}} \, |\tilde{\mathbf{k}}; \frac{1}{2}, \sigma'\rangle_{LF}$$

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- $R_M(\tilde{\mathbf{k}})$ is the Melosh rotation connecting the intrinsic LF and canonical frames, reached through different boosts from a given frame where the particle is moving

$$D^{1/2}[R_M(\tilde{\mathbf{k}})]_{\sigma'\sigma} = \chi^\dagger_{\sigma'} \frac{m + k^+ - \imath \boldsymbol{\sigma} \cdot (\hat{z} \times \mathbf{k}_\perp)}{\sqrt{(m + k^+)^2 + |\mathbf{k}_\perp|^2}} \chi_\sigma = {}_{LF}\langle \tilde{\mathbf{k}}; s\sigma' | \mathbf{k}; s\sigma\rangle_c$$

$\longrightarrow$ two-dimensional spinor

N.B. If $|\mathbf{k}_\perp| << k^+, m \longrightarrow D_{\sigma'\sigma} \simeq I_{\sigma'\sigma}$

8

The spin-dependent LF spectral function

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

The Spectral Function: probability distribution to find inside a bound system a particle with a given $\tilde{\kappa}$ when the rest of the system has energy ϵ , with a polarization vector S:

$$\mathcal{P}^{\tau}_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S) \quad = \quad \rho(\epsilon)\sum_{JJ_z\alpha}\sum_{Tt} {}_{LF}\langle tT;\alpha,\epsilon;JJ_z;\tau\sigma',\tilde{\boldsymbol{\kappa}}|\Psi_{\mathcal{M}};ST_z\rangle\,\langle ST_z;\Psi_{\mathcal{M}}|\tilde{\boldsymbol{\kappa}},\sigma\tau;JJ_z;\epsilon,\alpha;Tt\rangle_{LF}$$

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- $|\Psi_{\mathcal{M}};ST_z\rangle = \sum_m \boxed{|\Psi_m;S_zT_z\rangle}\, D^{\mathcal{J}}_{m,\mathcal{M}}(\boxed{\alpha,\beta,\gamma}) \longrightarrow$ Euler angles of rotation from the z-axis to the **polarization vector S**

  $\hookrightarrow$ three-body bound eigenstate of $M_{BT}(123) \sim M^{NR}$

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- $|\tilde{\boldsymbol{\kappa}},\sigma\tau;JJ_z;\epsilon,\alpha;T\tau\rangle_{LF}$ tensor product of a plane wave for particle 1 with LF momentum $\tilde{\kappa}$ in the intrinsic reference frame of the $[1+(23)]$ cluster times the fully interacting state of the (23) pair of energy eigenvalue ϵ. It has eigenvalue:

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$$\mathcal{M}_0(1,23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} + E_S \quad E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2} \qquad M_S = 2\sqrt{m^2 + m\epsilon}$$

and fulfills the macroscopic locality **(Keister, Polyzou, Adv. N. P. 20, 225 (1991)).**

$$\tilde{\boldsymbol{\kappa}} = (\kappa^+ = \xi \, \mathcal{M}_0(1,23), \mathbf{k}_\perp = \boldsymbol{\kappa}_\perp)$$

8 The spin-dependent LF spectral function

The LF overlaps for ^{3}He SF in terms of the IF ones are

$$\langle T\tau;\alpha,\epsilon;JJ_z;\tau_1\sigma,\tilde{\boldsymbol{\kappa}}\,|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{LF} = \sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}\sum_{\sigma'_1} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'_1}\sqrt{(2\pi)^3\;2E(\mathbf{k})}\sqrt{\frac{\kappa^+E_{23}}{k^+E_S}}\sum_{\sigma''_2,\sigma''_3}\sum_{\sigma'_2,\sigma'_3}$$

$${}_{IF}\langle T,\tau;\alpha,\epsilon;JJ_z|\mathbf{k}_{23},\sigma''_2,\sigma''_3;\tau_2,\tau_3\rangle\ \langle\sigma'_3,\sigma'_2,\sigma'_1;\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{IF}$$

$$\mathcal{D}_{\sigma''_2,\sigma'_2}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\;\mathcal{D}_{\sigma''_3,\sigma'_3}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)$$

effect of boosts in the Jacobians and in the transformations:

$$\mathcal{D}_{\sigma''_i,\sigma'_i}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i) = \sum_{\sigma_i} D^{\frac{1}{2}}[\mathcal{R}^{\dagger}_M(\pm\tilde{\mathbf{k}}_{23})]_{\sigma''_i\sigma_i}\; D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_i)]_{\sigma_i\sigma'_i}$$

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★ Through the Bakamjian-Thomas construction, one is allowed to approximate the momentum space wave functions for the 2- and 3-body systems

$$\langle\quad\ldots\quad|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{IF} = \langle\quad\ldots\quad|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{NR}$$

preserving the Poincaré covariance but using the successful NR phenomenology

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

9 Reference frames

CAUTION !

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- The intrinsic LF frame of the cluster, $(1;23)$, where $\tilde{P} = (\mathcal{M}_0(1,23), \vec{0}_\perp)$, with

$$\kappa^+(1;23) = \xi \, \mathcal{M}_0(1,23) \text{ and } \mathcal{M}_0(1,23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} \; + \; \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2}$$

$$M_S = 2\sqrt{m^2 + m\epsilon}$$

while $\mathbf{p}_\perp(lab) = \mathbf{k}_\perp(123) = \boldsymbol{\kappa}_\perp(1,23)$

10 LF spectral function decomposition

The LF spin-dependent spectral function (SF), for a nucleus with polarization **S**, can be macroscopically decomposed in terms of the available vectors:

- the unit vector $\hat{n}$, ⊥ to the hyperplane $n^\mu x_\mu = 0$. Our choice is $n^\mu \equiv \{1, 0, 0, 1\} \Rightarrow \hat{n} \equiv \hat{z}$
- the polarization vector **S**
- the transverse (wrt the $\hat{z}$ axis) momentum component of the constituent, i.e. $\mathbf{k}_\perp(123) = \mathbf{p}_\perp(Lab) = \boldsymbol{\kappa}_\perp(1;23)$

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The scalar functions $\mathcal{B}^{\mathcal{T}}_{i,\mathcal{M}}(\dots)$ depend, for $\mathcal{J} = 1/2$, $|\mathbf{k}_{\perp}|$, x, ϵ

$x = \kappa^{+}(1;23)/\mathcal{M}_0(1;23)$

11 LF spectral function and momentum distribution

By integrating the LF SF on κ^-, equivalent to the integration on the ϵ ≡ internal energy of the spectator system, one straightforwardly gets the LF spin-dependent momentum distribution

$$\mathcal{N}^{\tau}_{\sigma'\sigma}(x,\mathbf{k}_\perp;\mathcal{M},\mathbf{S}) = \frac{1}{2}\left\{b_{0,\mathcal{M}}(\dots) + \boldsymbol{\sigma}\cdot\mathbf{f}_{\mathcal{M}}(x,\mathbf{k}_\perp;\mathbf{S})\right\}_{\sigma'\sigma}$$

$$\int d\epsilon\ \mathcal{F}^{\tau}_{\mathcal{M}}(x,\mathbf{k}_\perp;\epsilon,\mathbf{S})$$

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The decomposition is useful to get:

an explicit interplay between transverse momentum component and spin dofs

relations between Transverse-momentum distributions (TMDs) in the *valence sector*

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$$b_{0,\mathcal{M}}(\dots) \longrightarrow \int d\epsilon \;\mathcal{B}^{\tau}_{i,\mathcal{M}}(\dots)$$

$$\mathbf{f}_{\mathcal{M}}(x, \mathbf{k}_{\perp}; \mathbf{S}) \longrightarrow \int d\epsilon \;\mathcal{F}^{\tau}_{\mathcal{M}}(x, \mathbf{k}_{\perp}; \epsilon, \mathbf{S})$$

The decomposition is useful to get:

an explicit interplay between transverse momentum component and spin dofs

relations between Transverse-momentum distributions (TMDs) in the *valence sector*

12 LF spectral function and LC Correlator

The fermion correlator in terms of the LF coordinates is **[e.g., Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002)]**

isospin

p = fermion momentum

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \frac{1}{2}\int d\xi^- d\xi^+ d\boldsymbol{\xi}_T\; e^{i\,p\xi}\,\underbrace{\langle P,S,A|}_{\text{parent system (nucleus, nucleon..)}}\bar{\psi}^{\tau}_{\beta}(0)\psi^{\tau}_{\alpha}(\xi)|A,S,P\rangle$$

The particle contribution to the correlator in valence approximation, i.e. the result obtained if the antifermion contributions are disregarded, is related to the LF SF:

$$\Phi^{\tau p}(p,P,S) = \frac{(\not{p}_{on} + m)}{2m}\,\Phi^{\tau}(p,P,S)\,\frac{(\not{p}_{on} + m)}{2m} = \frac{2\pi\,(P^+)^2}{(p^+)^2\,4m}\,\frac{E_S}{\mathcal{M}_0[1,(23)]}\sum_{\sigma\sigma'}\left\{\,u_\alpha(\tilde{\mathbf{p}},\sigma')\,\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\,\bar{u}_\beta(\tilde{\mathbf{p}},\sigma)\,\right\}$$

In deriving this expression it naturally appears the momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the cluster [1,(23)], where particle 1 is free and the (23) pair is **fully interacting**.

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The correlator function (related to the LF spectral function) at leading twist can be decomposed:

$$\Phi(p,P,S) = \frac{1}{2}\,\not{P}\,A_1 + \frac{1}{2}\,\gamma_5\,\not{P}\left[A_2\,S_z + \frac{1}{M}\,\widetilde{A}_1\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\right] + \frac{1}{2}\,\not{P}\,\gamma_5\left[A_3\,\not{S}_\perp + \widetilde{A}_2\,\frac{S_z}{\underset{\hookrightarrow}{M}}\,\not{p}_\perp + \frac{1}{M^2}\,\widetilde{A}_3\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\,\not{p}_\perp\right]$$

mass of the system

The functions A_j, $\widetilde{A}_j$ $(j = 1, 2, 3)$ can be obtained by proper traces of $\Phi(p,P,S)$ and Γ matrices.

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The 6 TMDs can be obtained:

$$f(x,\mathbf{p}_\perp^2) = \mathcal{O}[A_1] \qquad \Delta f(x,|\mathbf{p}_\perp|^2)=\mathcal{O}[A_2] \qquad g_{1T}(x,|\mathbf{p}_\perp|^2)=\mathcal{O}\left[\widetilde{A}_1\right]$$

$$\Delta'_T f(x,|\mathbf{p}_\perp|^2) = \mathcal{O}\left[A_3+\frac{|\mathbf{p}_\perp|^2}{2M^2}\widetilde{A}_3\right] \qquad h_{1L}^\perp(x,|\mathbf{p}_\perp|^2)=\mathcal{O}\left[\widetilde{A}_2\right] \qquad h_{1T}^\perp(x,|\mathbf{p}_\perp|^2)=\mathcal{O}\left[\widetilde{A}_3\right]$$

with

$$\mathcal{O}[A_j]=\frac{1}{2}\int\frac{dp^+dp^-}{(2\pi)^4}\,\delta[p^+-xP^+]\,P^+\,[A_j]$$

14 TMDs and LF spectral function

$$\text{TMDs} \xrightarrow{\text{obtained from}} \text{Tr LC correlator} \xrightarrow{\text{obtained from}} \text{Tr Spectral function}$$

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$$D = \frac{(P^+)^2}{p^+} \frac{\pi}{m} \frac{E_S}{\mathcal{M}_0[1,(23)]}$$

$$\boxed{\begin{aligned}
\mathrm{Tr}(\gamma^+\Phi^{\mathrm{P}}) &= D\ Tr\left[\hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right] \\
\mathrm{Tr}(\gamma^+\gamma_5\ \Phi^{\mathrm{P}}) &= D\ Tr\left[\sigma_z\ \hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right] \\
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The integration $\frac{1}{2} \int \frac{dp^+ dp^-}{(2\pi)^4}\ \delta[p^+ - xP^+]\ P^+$ of Tr of SF

$$\Downarrow$$

$$f(x, \mathbf{p}_\perp^2) = b_0 \quad \Delta f(x, |\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + b_{5,\mathcal{M}} \qquad g_{1T}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{4,\mathcal{M}}$$

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$\delta[p^+ - xP^+]\ P^+$ of Tr of SF

$\Downarrow$

There is a one-to-one correspondence between the $\epsilon-$ integral of proper components of the SF (the functions $b_{i,\mathcal{M}}$) and the TMDs of ³He: the latter can be accurately obtained from the wave function!

Pace, Salmè, Scopetta et al, Phys.Rev.C 104 (2021) 6, 065204

$$f(x, \mathbf{p}_\perp^2) = b_0 \quad \Delta f(x, |\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + b_{5,\mathcal{M}} \quad g_{1T}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{4,\mathcal{M}}$$

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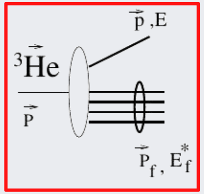
15 TMDs and ^{3}He LF spectral function

The procedure works for any three-body J $= 1/2$ system (in valence approx!)

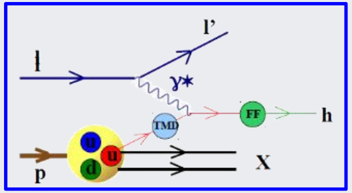
^{3}He



- p, p, n
- $(e, e'p)$ reactions
- p detection
- PW Impulse Approximation
- spin-dep response functions
- light-cone momentum distributions
- norms, effective polarizations

CORRESPONDENCE

Proton



- u_v, u_v, d_v
- SIDIS
- no q_v detection, fragmentation...
- leading twist
- TMDs
- PDFs
- charges (axial, tensor...)

- the ^{3}He TMDs could be obtained from spin asymmetries in $^3\vec{He}(\vec{e}, e'p)$ experiments: in progress!
- We show our calculation for the TMDs of He using Av18 + UIX wfs (A. Kievsky, M. Viviani et al.) Thus testing LFRHD and of the importance of Relativity in nuclear structure.

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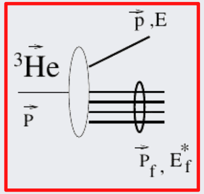

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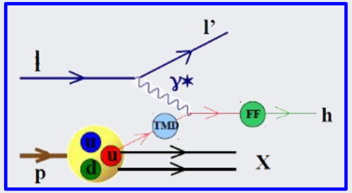

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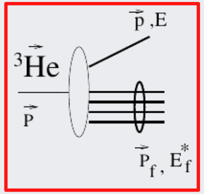

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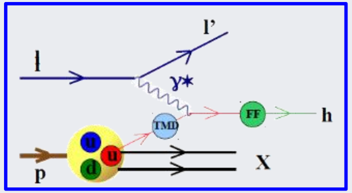

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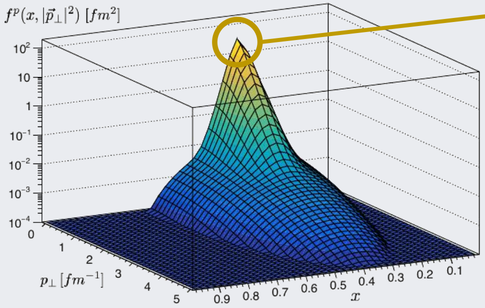
16 ^{3}He TMDs

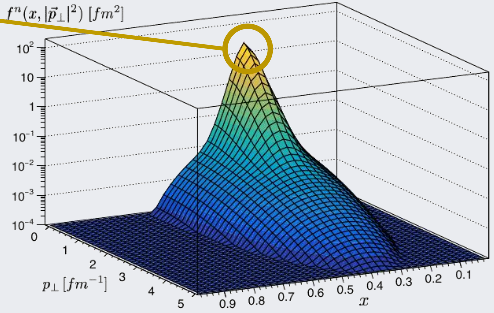
Numerical results **A. Del Dotto, E. Pace, G. Perna, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)**

PROTON



peak around x=1/3

NEUTRON



$f^\tau(x, |\mathbf{p}_\perp|^2)$, unpolarized TMD in an unpolarized ^{3}He.

16

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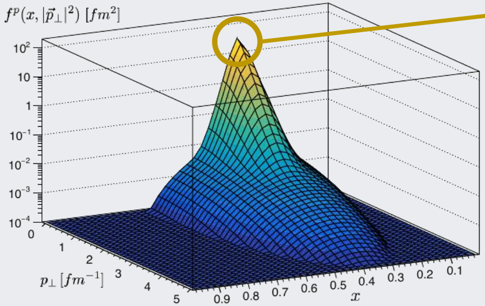

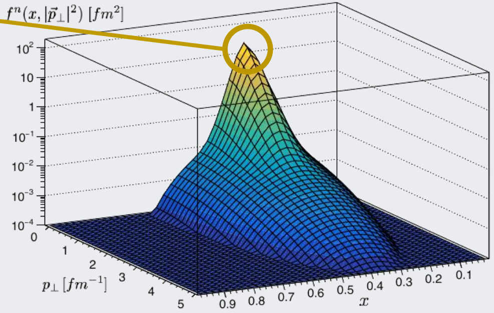

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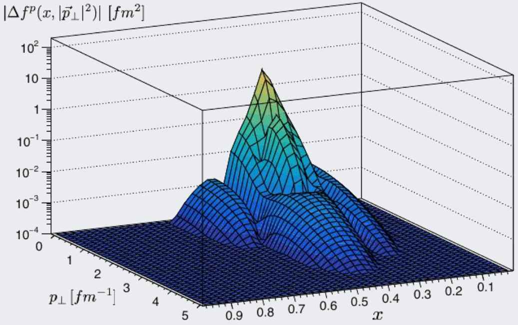

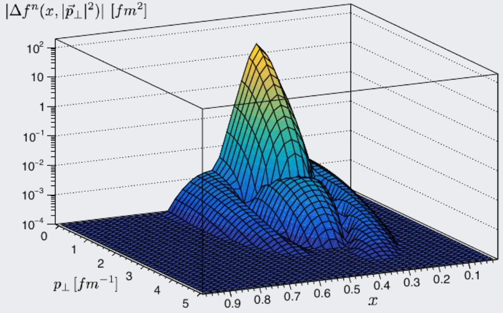

Absolute value of the nucleon longitudinal-polarization distribution $\Delta f^\tau(x, |\mathbf{p}_\perp|^2)$ in a longitudinally (wrt virtual photon axes) polarized ^{3}He.

16 ^{3}He TMDs

Numerical results **A. Del Dotto, E. Pace, G. Perna, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)**

PROTON

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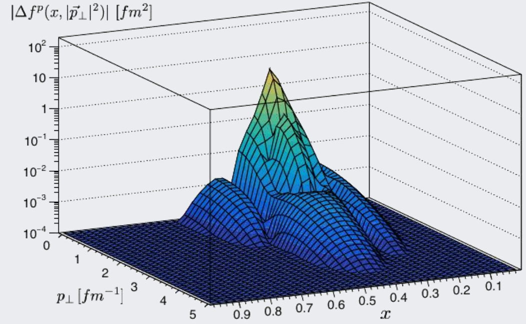

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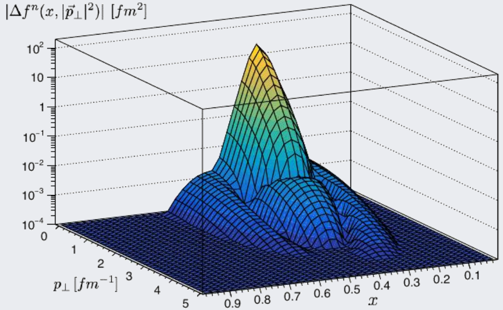
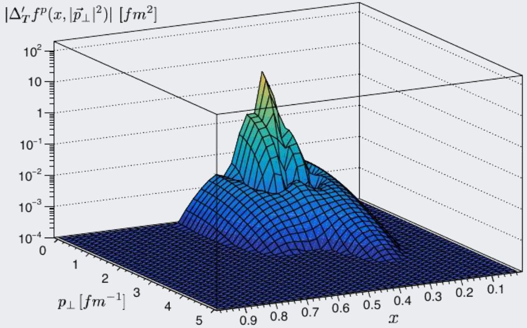
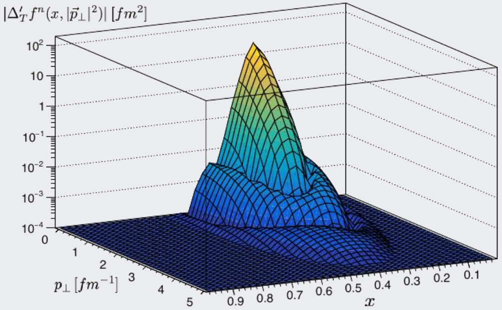




Absolute value of the nucleon transverse-polarization distribution, $\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$in a ^{3}He transversely polarized in the same direction of the nucleon polarization.



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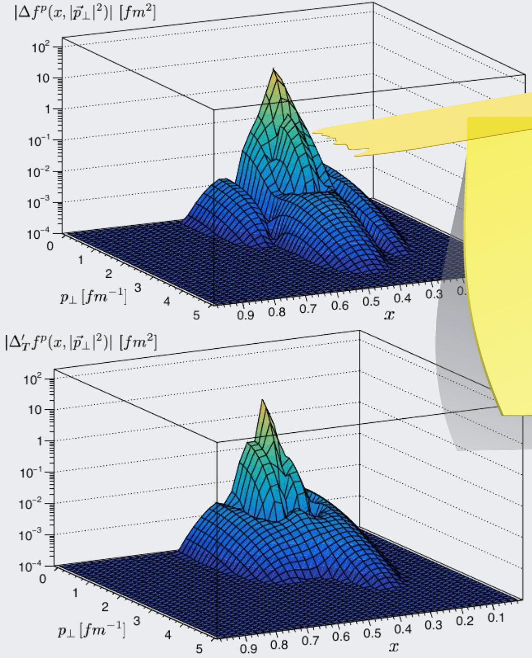

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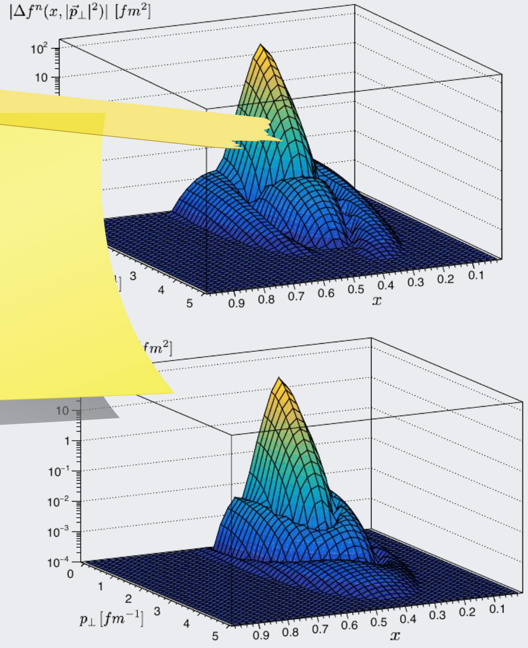

solute value of the

(Small) Difference wrt the 2 lines due to relativistic effects

Absolute value of the nucleon transverse-polarization distribution, $\Delta'_T f^\tau(x,|\mathbf{p}_\perp|^2)$ in a ³He transversely polarized in the same direction of the nucleon polarization.

16 ^{3}He TMDs

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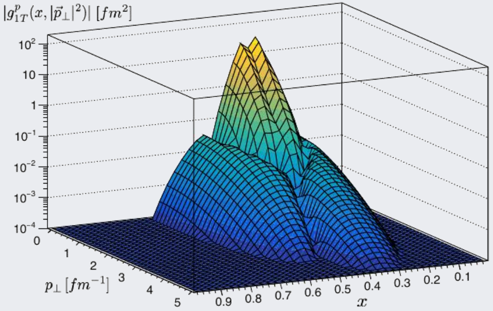
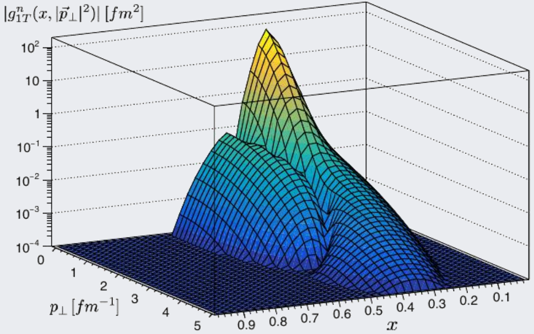
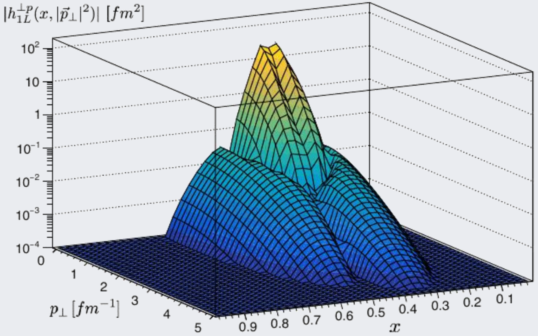
PROTON

NEUTRON

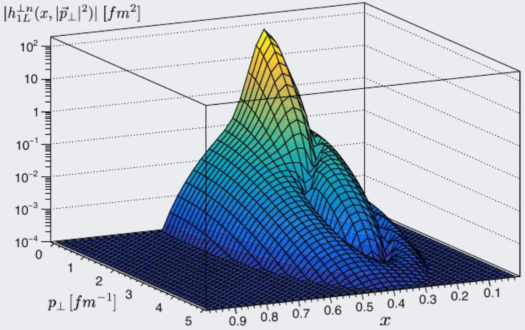


Absolute value of the nucleon longitudinal-polarization distribution, $g^\tau_{1T}(x, |\mathbf{p}_\perp|^2)$, in a transversely polarized ^{3}He.





Absolute value of the nucleon transverse-polarization distribution, $h^{\perp\tau}_{1L}(x, |\mathbf{p}_\perp|^2)$ in a longitudinally polarized ^{3}He.



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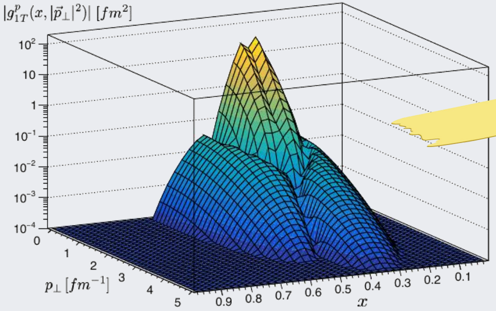
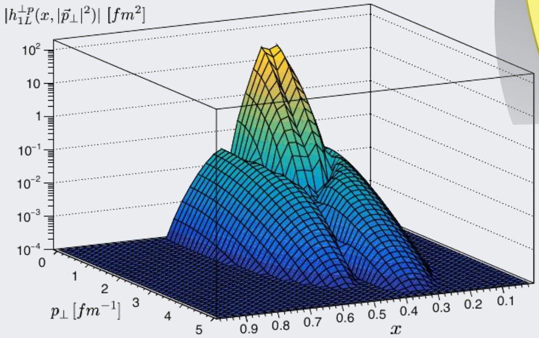
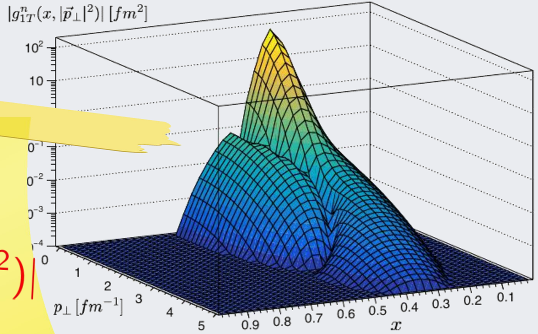
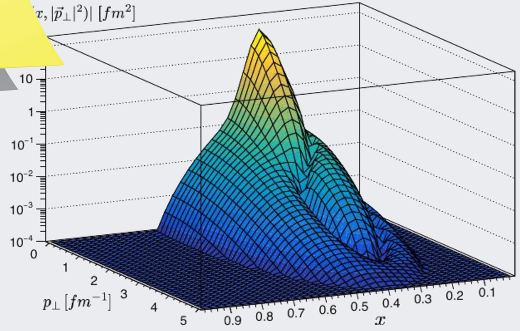
PROTON

NEUTRON









ute value of the

$$|h_{1L}^{\perp\tau}(x,|\mathbf{p}_\perp|^2)| \sim |g_{1T}^{\tau}(x,|\mathbf{p}_\perp|^2)|$$

Absolute value of the nucleon transverse-polarization distribution, $h_{1L}^{\perp\tau}(x,|\mathbf{p}_\perp|^2)$ in a longitudinally polarized ^{3}He.

17 Relations among nuclear TMDs

We remark that from the general principles implemented in the SF, TMDs receive contributions from both $L = 0$ and $L = 2$ orbital angular momenta.

Some approximated relations among TMDs are:

$$\Delta f(x, |\mathbf{p}_\perp|^2) = \Delta'_T f(x, |\mathbf{p}_\perp|^2) \; + \; \frac{|\mathbf{p}_\perp|^2}{2M^2} \, h^\perp_{1T}(x, |\mathbf{p}_\perp|^2)$$

$$g_{1T}(x, |\mathbf{p}_\perp|^2) = \; - \, h^\perp_{1L}(x, |\mathbf{p}_\perp|^2)$$

$$(g_{1T})^2 \; + \; 2 \, \Delta'_T f \;\; h^\perp_{1T} = 0$$

R. Jacob et al. [NPA 626, 937 (1997)]
B. Pasquini et al. [PRD 78, 034025 (2008)]

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- first relation recovered retaining only the L = 0 contribution. Taking into account both L = 0, 2, the difference between the lhs and rhs is small for the neutron, not negligible for the proton;

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$$\Delta f(x,|\mathbf{p}_\perp|^2) = \Delta'_T f(x,|\mathbf{p}_\perp|^2) \; + \; \frac{|\mathbf{p}_\perp|^2}{2M^2} \, h_{1T}^\perp(x,|\mathbf{p}_\perp|^2)$$

$$g_{1T}(x,|\mathbf{p}_\perp|^2) = \; - \; h_{1L}^\perp(x,|\mathbf{p}_\perp|^2)$$

$$(g_{1T})^2 \; + \; 2\,\Delta'_T f \;\; h_{1T}^\perp = 0$$

R. Jacob et al. [NPA 626, 937 (1997)]
B. Pasquini et al. [PRD 78, 034025 (2008)]

- first relation recovered retaining only the L = 0 contribution. Taking into account both L = 0, 2, the difference between the lhs and rhs is small for the neutron, not negligible for the proton;
- the second relation holds in modulus, since if the L = 0 component, tiny for those TMDs, is retained the minus sign works, while the dominant L = 2 contribution leads to a plus sign.

17 Relations among nuclear TMDs

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Measurable effect!
Importance of 2-3 interactions

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- the second relation holds in mo , tiny for those TMDs, is retained the minus sign works, n leads to a plus sign.
- The third relation does not hold, even if the L = 2 contribution is vanishing. Noteworthy, the integration on k23 , imposed by **Macro-locality**, spoils the relation!

18 LC momentum distributions

From the normalization of the Spectral Function one has

$$\boxed{f_\tau^A(\xi) = \int d\mathbf{k}_\perp \; n^\tau(\xi,\mathbf{k}_\perp)} \longrightarrow \int_0^1 d\xi \; f_\tau^A(\xi) \;=\; 1$$

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LC momentum distributions

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We can define the essential **sum rules** that must be satisfied:

$$\underbrace{N_A = \int d\xi \; \left[Z f_p^A(\xi) + (A - Z) f_n^A(\xi)\right] = 1}_{\text{Baryon number sum rule}} \qquad \underbrace{MSR = \int d\xi \; \xi \; \left[Z f_p^A(\xi) + (A - Z) f_n^A(\xi)\right] = 1}_{\text{Momentum sum rule}}$$

Within the LFHD we are able to fulfill **both sum rules at the same time!**

E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485

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Not possible within the IF! (**Frankfurt & Strikman; Miller;....80's**)

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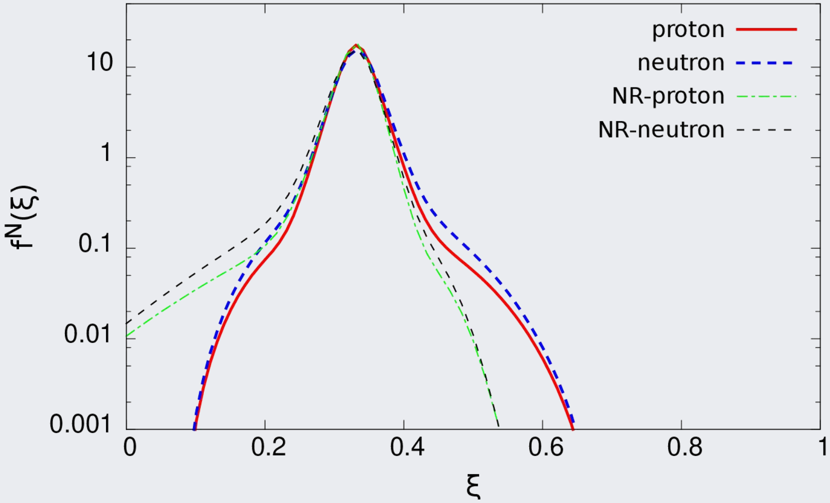

unpolarized distribution

E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485

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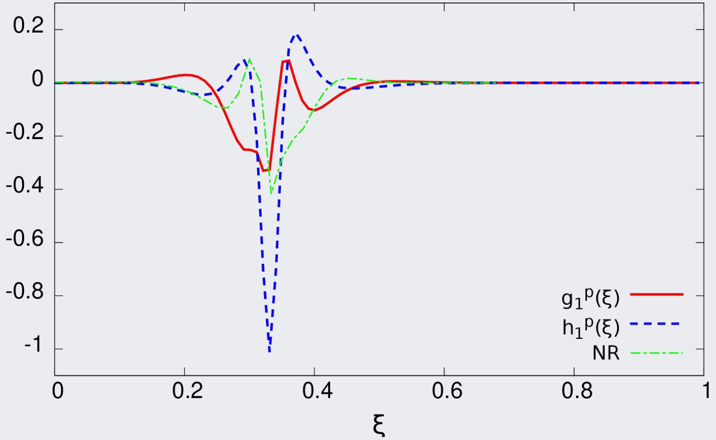
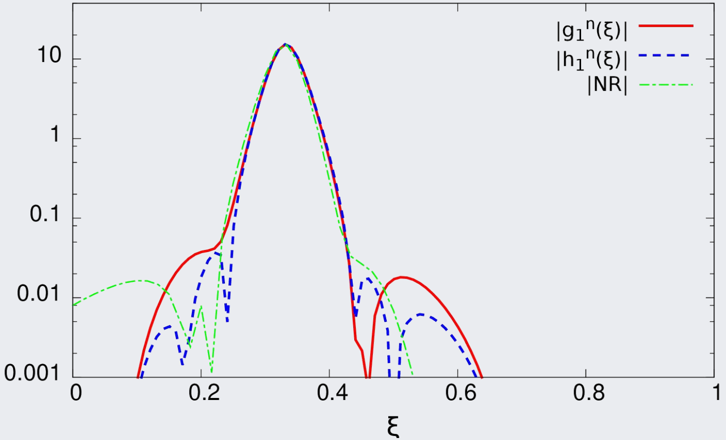
LC momentum distributions



PROTON



NEUTRON

E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485

$g_1^n(\xi)$ longitudinal-polarization distribution

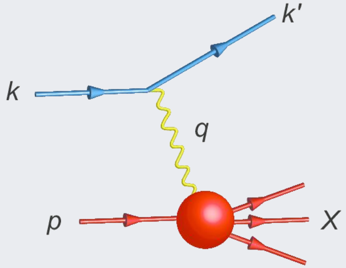
$h_1^n(\xi)$ transverse-polarization distribution

- They would be the same in a NR framework;
- Crucial for the extraction of the neutron information from DIS and SIDIS off ^{3}He.
  Work in progress to LF update our NR results ⟶ important for JLab12, EIC

The EMC effect

Almost 40 years ago, the European Muon Collaboration (EMC) measured (in DIS processes)

$$R(x) = F_2^{^{56}Fe}(x)/F_2^{^{2}H}(x)$$



Expected result: $\boldsymbol{R(x) = 1}$ up to corrections of the Fermi motion

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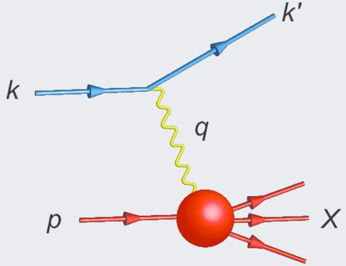

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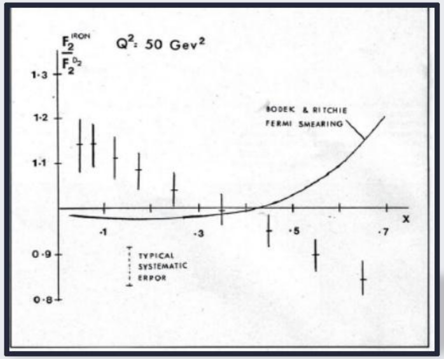
Aubert et al. Phys.Lett. B123 (1983) 275
1488 citations (inSPIRE)



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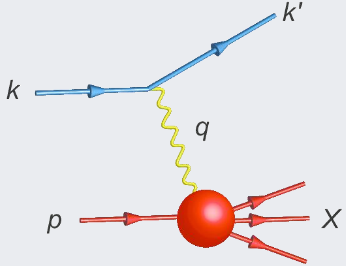

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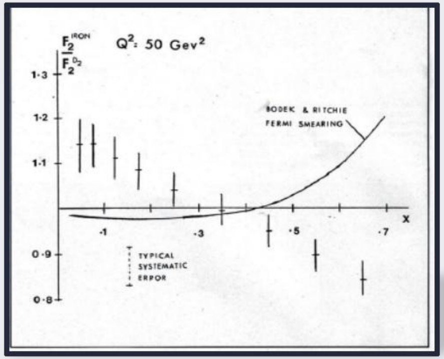
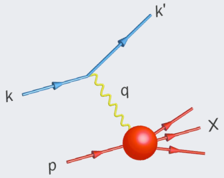


Naive parton model interpretation:
"Valence quarks, in the bound nucleon, are in average slower that in the free nucleon"

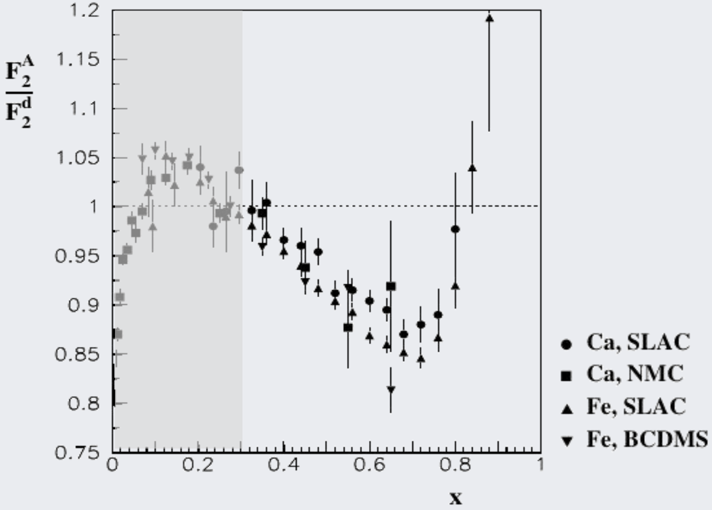
Is the bound proton bigger than the free one???

The EMC effect in details

We remind that for DIS off nuclei: $0 \leq x = \dfrac{Q^2}{2M\nu} \leq \dfrac{M_A}{M} \simeq A$



- x ⩽ 0.3 “Shadowing region”: coherence effects, the photon interacts with partons belonging to different nucleons



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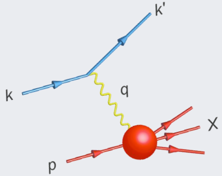

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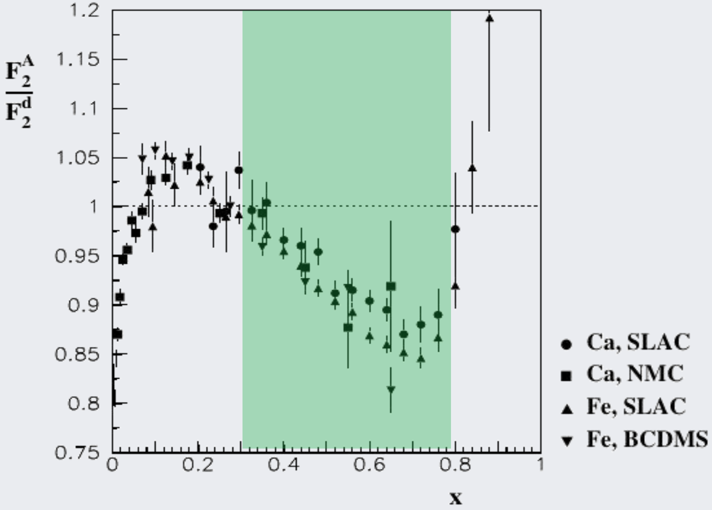

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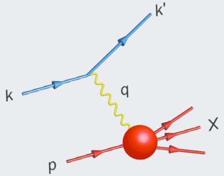

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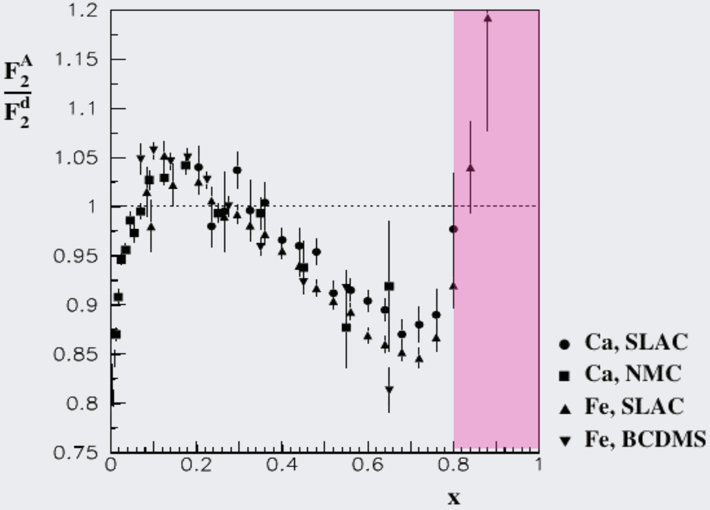

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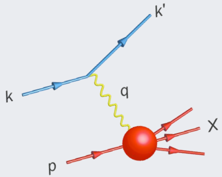

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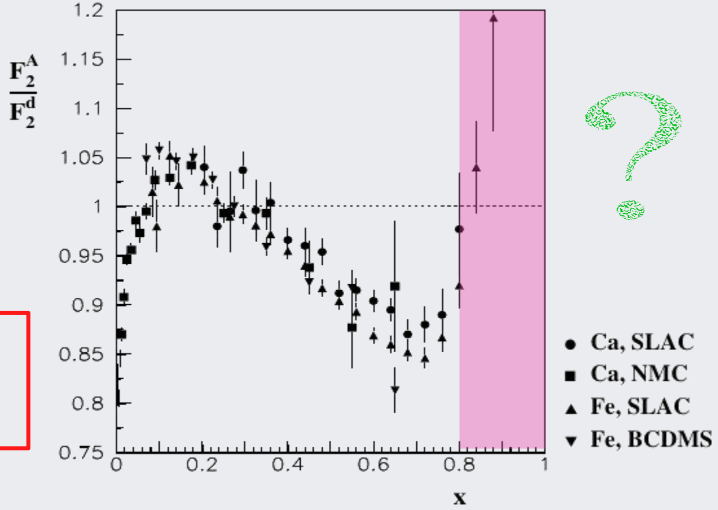
main features: universal behaviour independent on Q^2 ; weakly dependent on A; scales with the density $\rho \rightarrow$ global property? Or due to correlations...Local...



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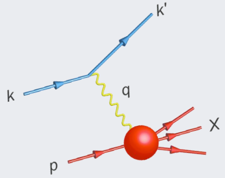

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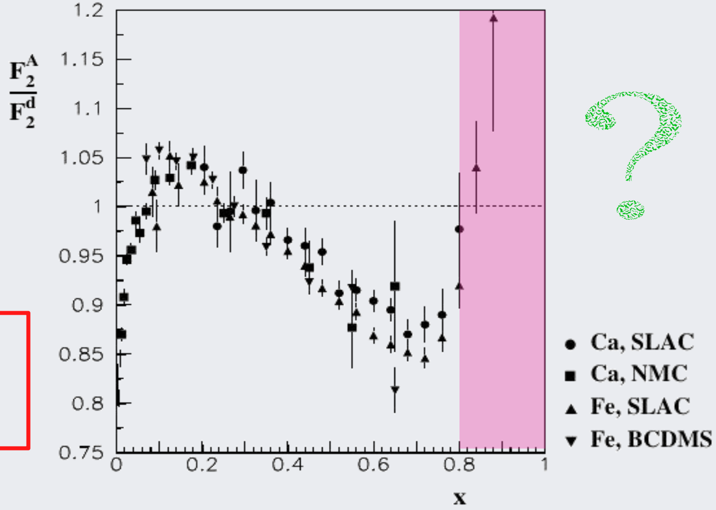

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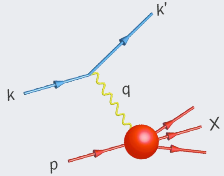
Explanation (exotic) advocated: confinement radius bigger for bound nucleons, quarks in bags with 6, 9,..., 3A quark, pion cloud effects... Alone or mixed with conventional ones...

19

The EMC effect explanations and perspective?



Situation: **basically not understood. Very unsatisfactory**. We need to know the reaction mechanism of hard processes off nuclei and the degrees of freedom which are involved:

- the knowledge of nuclear PDFs is crucial for the analysis of heavy ions collisions;
- neutron parton structure measured with nuclear targets; several QCD sum rules involve the neutron information (Bjorken SR, for example): importance of Nuclear Physics for QCD

Inclusive measurements cannot distinguish between models
One has probably to go beyond (not treated here...) **(R. Dupré and S.S., EPJA 52 (2016) 159)**

- Hard Exclusive Processes (GPDs)
- SIDIS (TMDs)

Status of "Conventional" calculations for light nuclei:

- **IF (NR) Calculations**: qualitative agreement but no fulfillment of both particle and MSR... Not under control
- **LF Calculations**: in heavy systems, mean field approaches do not find an EMC effect in the valence region **(Miller and Smith, PRC C 65 (2002) 015211)**; For light nuclei, no realistic calculations available (approximate attempt in **Oelfke, Sauer and Coester NPA 518 (1990) 593**)

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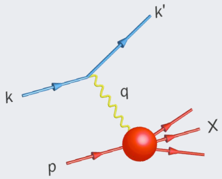

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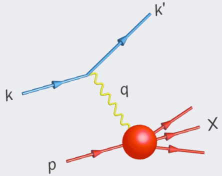

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The nuclear structure function F_2

The hadronic tensor is found to be (**Pace, M.R., Salmè and S. Scopetta, Phys. Scri. 2020**)

$$W_A^{\mu\nu}(P_A, T_{Az}) = \sum_N \sum_\sigma \sum\!\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \; d\kappa^+}{(2\pi)^3 \; 2 \; \kappa^+} \, \frac{1}{\xi} \, \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \boxed{w_{N,\sigma}^{\mu\nu}(p,q)} \rightarrow$$

hadronic tensor of the bound nucleon

In the Bjorken limit the nuclear structure function can be obtained from the hadronic tensor:

$$F_2^A(x) = \sum_N \sum_\sigma \sum\!\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \; d\kappa^+}{(2\pi)^3 \; 2 \; \kappa^+} \, \frac{1}{\xi} \; \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \; (-x) \; g_{\mu\nu} \;\; w_{N,\sigma}^{\mu\nu}(p,q) = \sum_N \sum\!\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \; d\kappa^+}{(2\pi)^3 \; 2 \; \kappa^+} \; \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \; \frac{P_A^+}{p^+} \; \frac{x}{z} \boxed{F_2^N(z)}$$

$x = \frac{Q^2}{2P_A \cdot q}$ Bjorken variable $\xi = \frac{\kappa^+}{\mathcal{M}_0(1,23)} \neq x$ $z = \frac{Q^2}{2p \cdot q}$

nucleon structure function

$F_2^N(z) = -z \; g_{\mu\nu} \; \sum_\sigma \; w_{N,\sigma}^{\mu\nu}(p,q)$

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nucleon structure function

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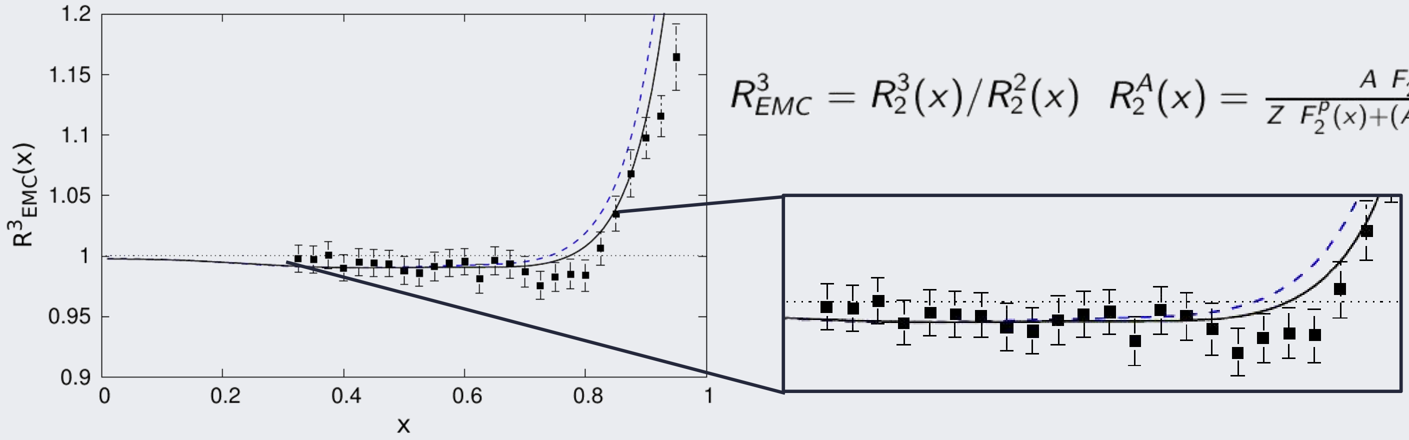
One should notice that: $\int d\epsilon \int d\kappa^+ \neq \int d\kappa^+ \int d\epsilon$ but in the BJ limit $\int d\epsilon \int d\kappa^+ = \int d\kappa^+ \int d\epsilon$

therefore, **F_2 and the EMC effect can be evaluated from TMDs and the LC momentum distribution directly!**

The ³He EMC effect within the LFHD

The numerical calculations **E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485**



$$R^3_{EMC} = R^3_2(x)/R^2_2(x) \quad R^A_2(x) = \frac{A \ F^A_2(x)}{Z \ F^p_2(x){+}(A{-}Z) \ F^n_2(x)}$$

- **Solid line**: with Av18 description of ³He, **Dashed line**: including three-body forces (U-IX) with "SMC" nucleon structure functions (**Adeva et al PLB 412, 414 (1997)**).
- **Full squares**: data from J. Seely et al., PRL. 103, 202301 (2009) reanalyzed by S. A. Kulagin and R. Petti, PRC 82, 054614 (2010)

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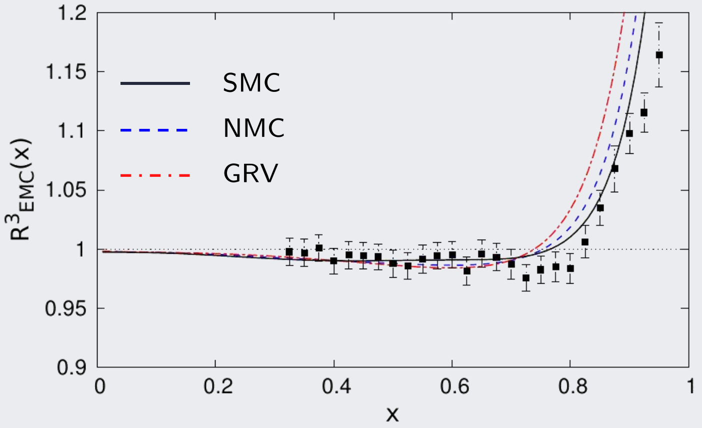

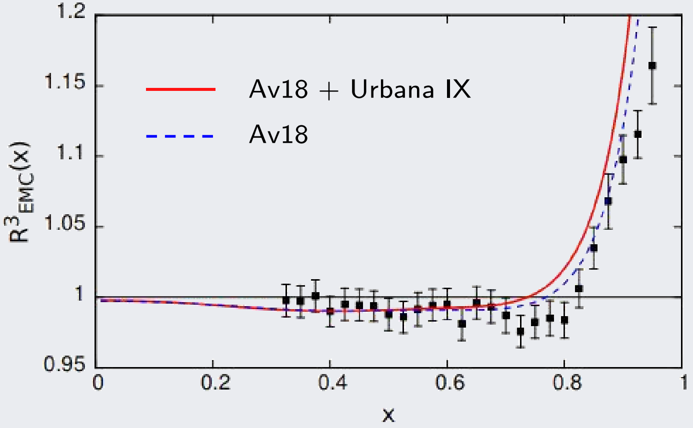

$F_2^n(x)$ extracted from the MARATHON data
[MARATHON, PRL 128,132003 (2022)]

$F_2^p(x)$ SMC

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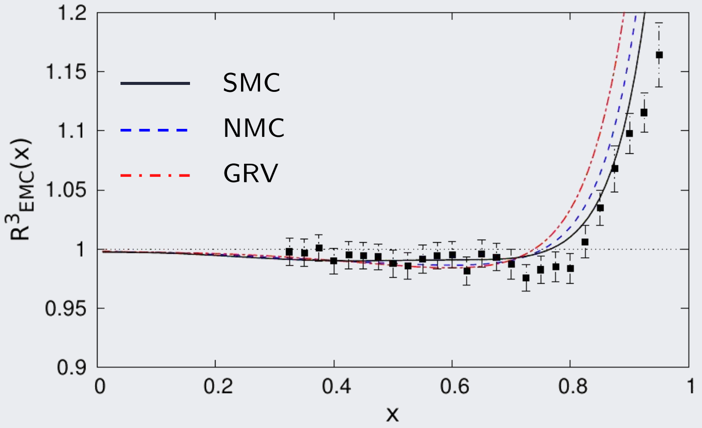

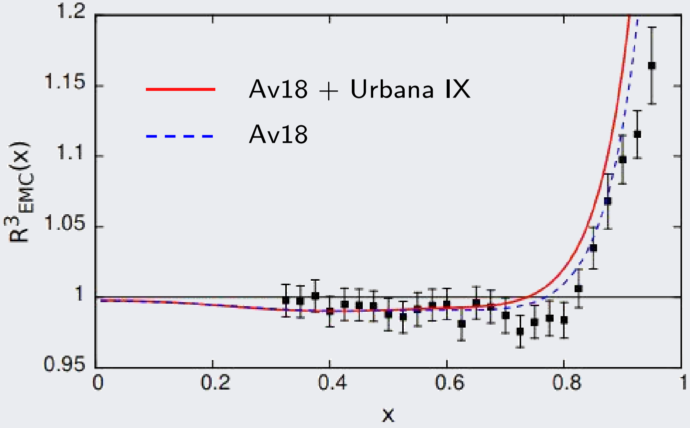

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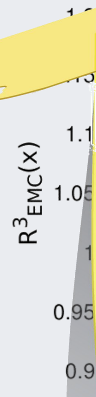
$F_2^p(x)$ SMC

Conclusions: **small but solid effect**; essential the extension to ^{4}He (which presents a bigger effect)

The ^{3}He EMC effect within the LFHD

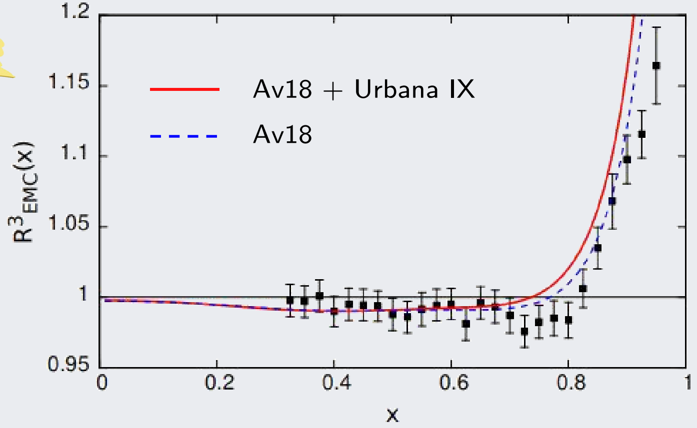
The numerical calculations **E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485**



ATTENTION PLEASE

We calculated the valence contribution to the EMC effect within an appoach:

i) able to include relativistic effects
ii) fulfill number and momentum sum rules at the same time!
iii) including conventional nuclear effects



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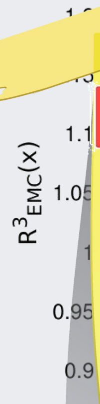
$F_2^p(x)$ SMC

Conclusions: **small but solid effect**; essential the extension to ^{4}He (which presents a bigger effect)

20 The ^{3}He EMC effect within the LFHD

The numerical calculations **E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485**



ATTENTION PLEASE

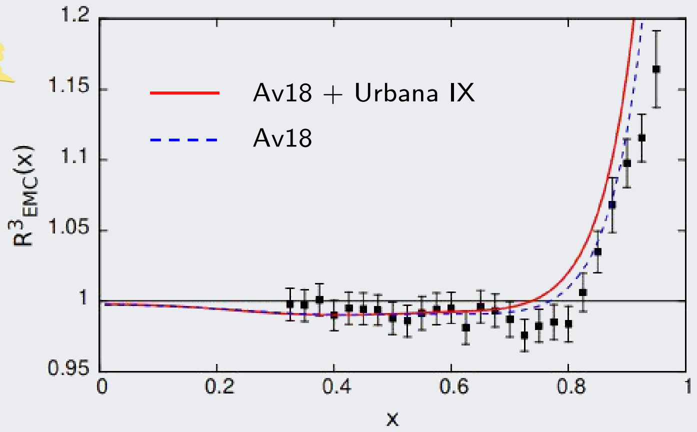
We calculated the valence contribution to the EMC effect within an appoach:

i) able to include relativistic effects
ii) fulfill number and momentum sum rules at the same time!
iii) including conventional nuclear effects

We are not excluding the existence of effects beyond the conventional ones!

↓

We need to test the approach with heavier nuclei



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CONCLUSIONS

✓ **A Poincaré covariant description of nuclei, based on the light-front Hamiltonian dynamics, has been proposed**. The Bakamjian-Thomas construction of the Poincaré generators allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré covariant framework.
N.B. Normalization and momentum sum rule are both automatically fulfilled.

✓ The Spectral Function is related to the valence contribution to the **correlator** introduced for a QFT description of SiDIS reactions involving the nucleon, applied for the first time to ^{3}He.

✓ General principles fulfilled by the LF Spectral function entail **relations among T-even twist-2 (and also twist-3) valence TMDs**, with interesting angular momentum dependence.

✓ Encouraging calculation of ^{3}He EMC, shedding light on the role of a reliable description of the nucleus. Also the LC spin-dependent momentum distributions are available, for both longitudinal and transverse polarizations of the nucleon. Crucial extension to ^{4}He!

Analyses of A(e,e',p)X reactions, with polarized initial and final states, for accessing nuclear TMD's in ^{3}He are in progress

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Backup Slides: effective polarizations

Effective polarizations

Key role in the extraction of neutron polarized structure functions and neutron Collins and Sivers single spin asymmetries, from the corresponding quantities measured for ^{3}He

Effective longitudinal polarization (axial charge for the nucleon)

$$p^{\tau}_{||} = \int_0^1 dx \int d\mathbf{p}_\perp \; \Delta f^{\tau}(x, |\mathbf{p}_\perp|^2)$$

Effective transverse polarization (tensor charge for the nucleon)

$$p^{\tau}_{\perp} = \int_0^1 dx \int d\mathbf{p}_\perp \; \Delta'_T f^{\tau}(x, |\mathbf{p}_\perp|^2)$$

Effective polarizations	proton	neutron
LF longitudinal polarization	-0.02299	0.87261
LF transverse polarization	-0.02446	0.87314
non relativistic polarization	-0.02118	0.89337

- The difference between the LF polarizations and the non relativistic results are up to 2% in the neutron case (larger for the proton ones, but it has an overall small contribution), and should be ascribed to the intrinsic coordinates, implementing the Macro-locality, and not to the Melosh rotations involving the spins.
- N.B. Within a NR framework: $p^{\tau}_{||}(NR) = p^{\tau}_{\perp}(NR)$

Backup Slides: effective polarizations

The BT Mass operator for A=3 nuclei - II

The NR mass operator is written as

$$M^{NR} = 3m + \sum_{i=1,3} \frac{k_i^2}{2m} + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR}$$

and must obey to the commutation rules proper of the Galilean group, leading to translational invariance and independence of total 3-momentum.
Those properties are analogous to the ones in the BT construction. This allows us to consider the standard non-relativistic mass operator as a sensible BT mass operator, and embed it in a Poincaré covariant approach.

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

The 2-body phase-shifts contain the relativistic dynamics, and the Lippmann-Schwinger equation, like the Schrödinger one, has a suitable structure for the BT construction. Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

The eigenfuntions of M^{NR} do not fulfill the cluster separability, but we take care of Macro-locality in the spectral function.

Diagrams and infographics