DYSON-SCHWINGER QUARK MODEL WITH DYNAMICAL MASS GENERATION IN MINKOWSKI SPACE

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In collaboration with Tobias Frederico, Wayne de Paula and Emanuel Ydrefors

Based on Phys. Rev. D 105, 114055 (2022)

Light Cone 2022 Online: Physics of Hadrons on the Light Front
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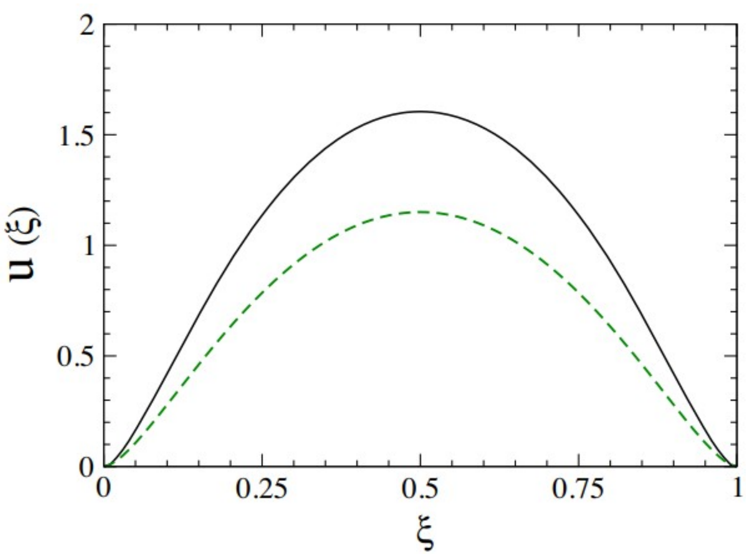
Universidade Federal de Santa Maria

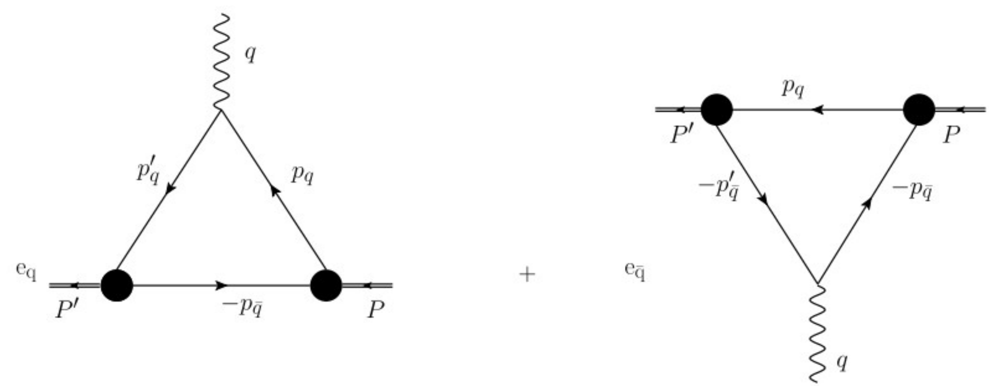
Outline

- Motivation
- DSE in Minkowski space
- Solving the rainbow-ladder DSE
- Numerical results
- Future perspectives and final remarks

Developments in Minkowski space

- Two-fermion homogeneous BSE. W. de Paula et al., PRD 94, 071901 (2016).
- Solution of the 3-body BSE. E.Ydrefors et al., PLB 791, 276 (2019).
- Pion electromagnetic form-factor. E. Ydrefors et al., PLB 820 136494 (2021).
- Proton image. Ydrefors and Frederico, PRD 104, 114012 (2021)
- ....





de Paula,Ydrefors,Nogueira,Frederico,Salmè, PRD 105, L071505 (2022).

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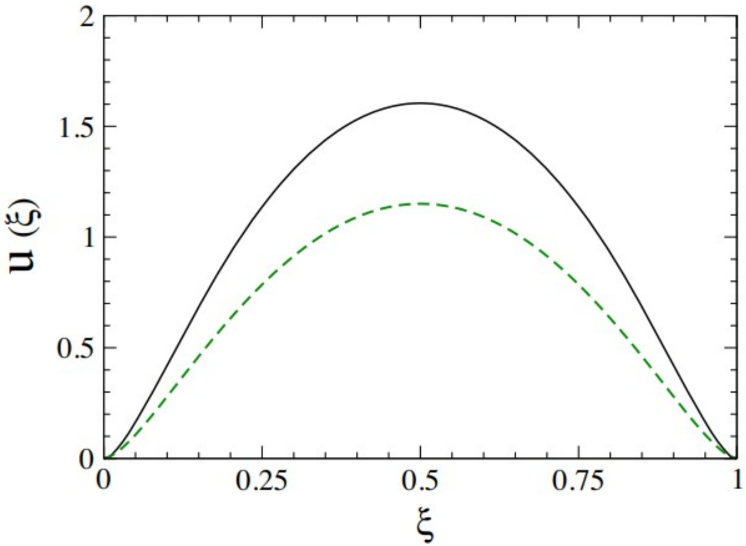
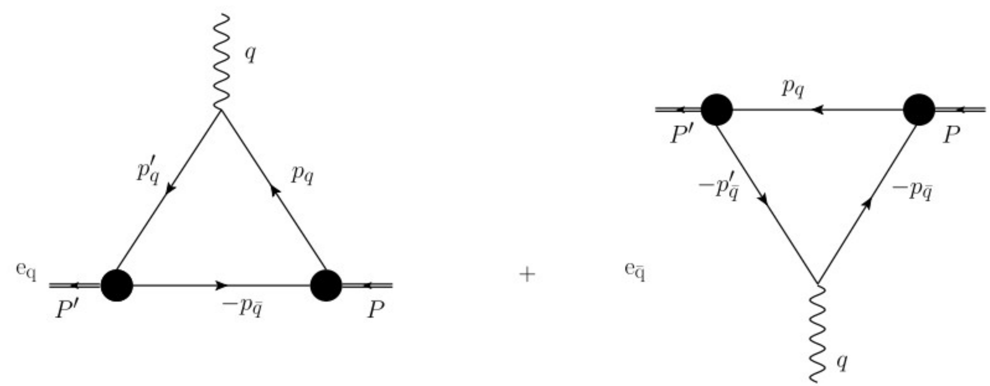
For more details check Wayne de Paula and Emanuel Ydrefors





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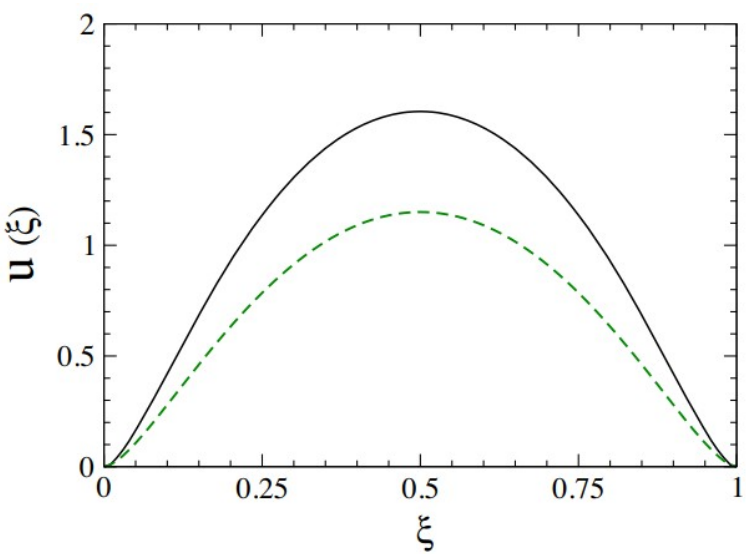

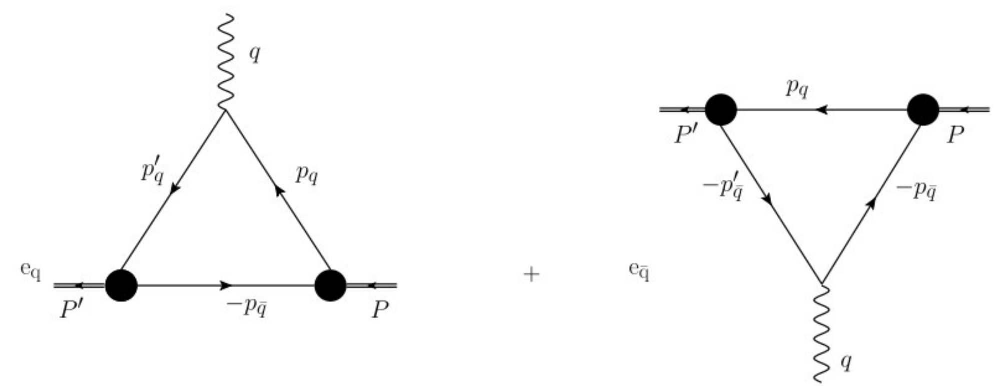

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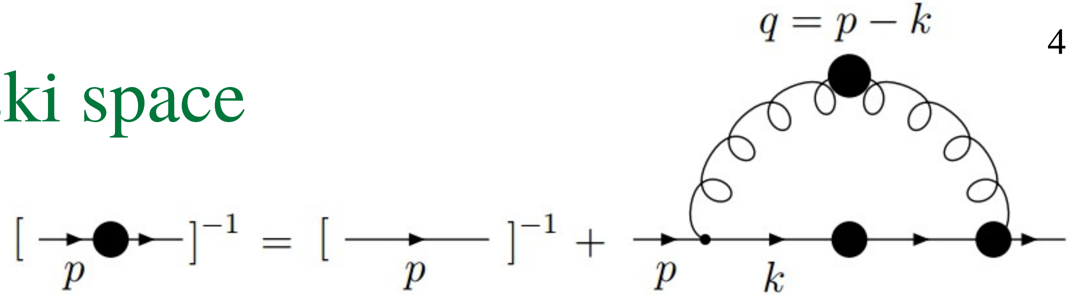
DSE in Minkowski space



- Dynamical observables defined in the light-front;
- Electromagnetic form-factors;
- Quark and gluon propagators to be used as inputs for the BS/Fadeev approach to hadronic physics;
- QCD at finite density: Lattice sign problem.

Main Tool: **Nakanishi Integral Representation**

- Generalization of the Källén-Lehmman integral representation of two point functions, for n-point functions. N. Nakanishi, Phys. Rev. 130, 1230 (1963) and Prog.Theor.Phys.Suppl. 43, 1 (1969).
- Wick rotation is the exact analytical continuation of the Minkowski space Nakanishi representation: Possibility of the exploration in the complex plane.

Comparison with Un-Wick rotated results: Bethe-Salpeter vertex

Un-Wick comparison with rotation in the ladder bosonic BSE

The Bethe-Salpeter approach to bound states: from Euclidean to Minkowski space

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[1]Instituto Tecnológico da Aeronáutica, DCTA, 12.228-900 São José dos Campos, SP, Brazil
[2]Università di Roma La Sapienza, INFN, Sezione di Roma, P.le A. Moro 5, 00187 Roma, Italy
[3]Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA

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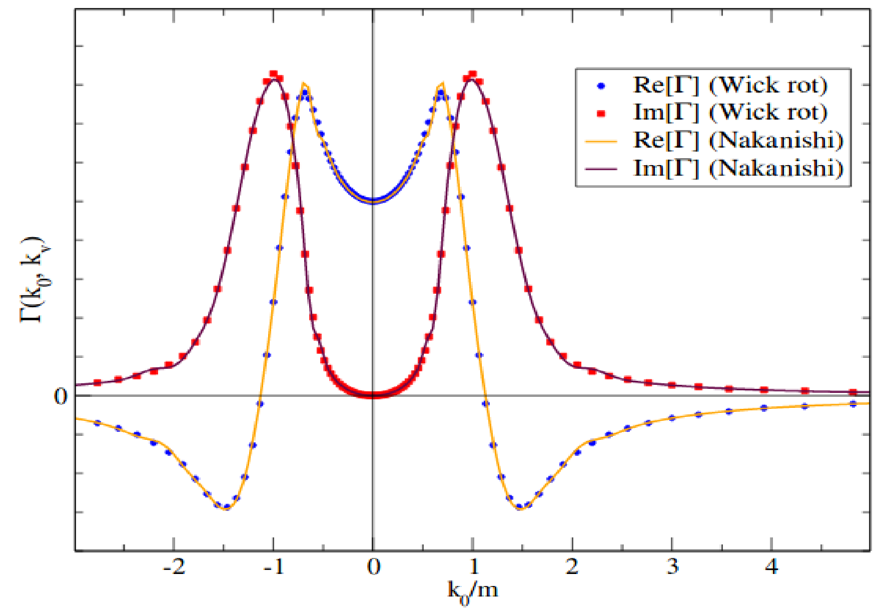
Abstract. The challenge to obtain from the Euclidean Bethe–Salpeter amplitude the amplitude in Minkowski is solved by resorting to un-Wick rotating the Euclidean homogeneous integral equation. The results obtained with this new practical method for the amputated Bethe–Salpeter amplitude for a two-boson bound state reveals a rich analytic structure of this amplitude, which can be traced back to the Minkowski space Bethe–Salpeter equation using the Nakanishi integral representation. The method can be extended to small rotation angles bringing the Euclidean solution closer to the Minkowski one and could allow in principle the extraction of the longitudinal parton density functions and momentum distribution amplitude, for example.



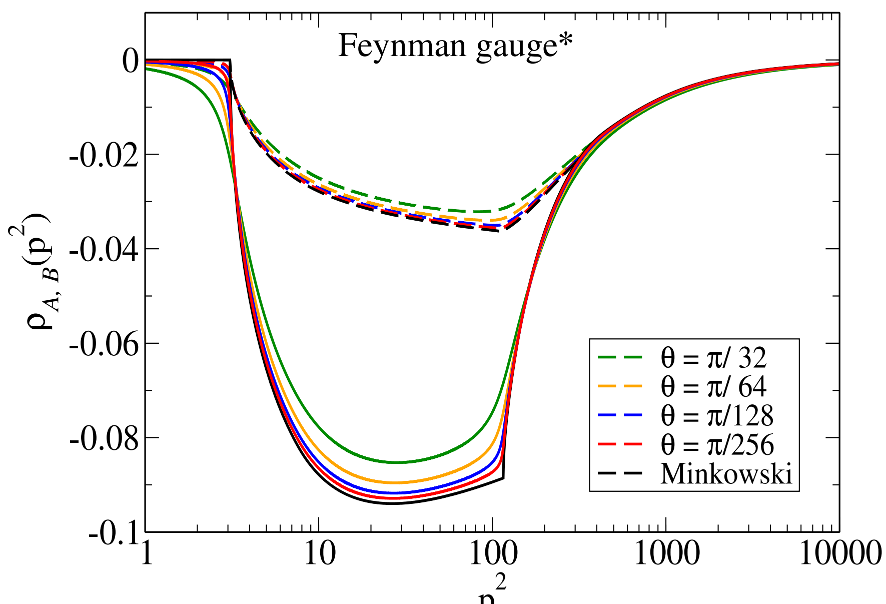
A. Castro et al., Journal of Physics: Conf. Series 1291 012006 (2019)

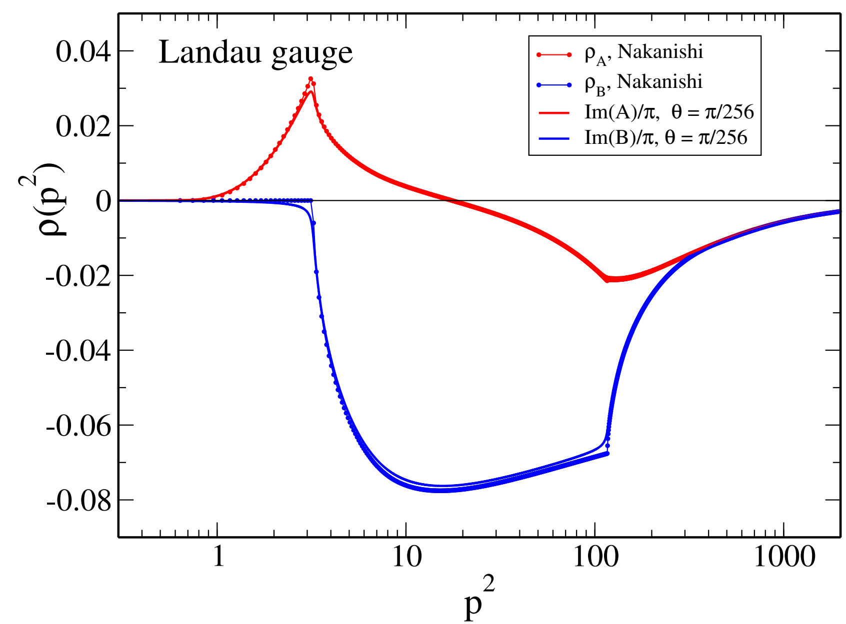
Comparison with Un-Wick rotated results:DSE in the Weak coupling limit

- From Euclidean space formulation, in increments of δ: $p_0 \Longrightarrow \exp(-i\delta)p_0$, $k_0 \Longrightarrow \exp(-i\delta)k_0$

- Minkowski space: δ = π/2, or in a more conveninent notation Θ = π/2 − δ.

$$\theta = 0, \begin{cases} p_0^2 = 0, & \vec{p}^2 > 0 \quad \text{spacelike region} \\ p_0^2 > 0, & \vec{p}^2 = 0 \quad \text{timelike region} \end{cases}$$





*S. Jia et al., Proceedings of HADRON-2019, arXiv:1912.00063, T. Frederico et al., Proceedings of NTSE-2018, arXiv:1905.00703.

Fermion Dyson Schwinger Equation (Rainbow-Ladder)

⟹ Dressed fermion propagator: $S_f\left(k\right) = \dfrac{1}{\not{k} - m_B + \not{k} A_f\left(k^2\right) - B_f\left(k^2\right) + i\epsilon}$

$$A_f(k^2) = \int_0^\infty ds \frac{\rho_A(s)}{k^2 - s + i\epsilon}\,, \qquad B_f(k^2) = \int_0^\infty ds \frac{\rho_B(s)}{k^2 - s + i\epsilon}$$

⟹ Integral representation of the fermion propagator:

$$S_f(k) = R\frac{\not{k} + \overline{m}_0}{k^2 - \overline{m}_0^2 + i\epsilon} + \not{k}\int_0^\infty ds \frac{\rho_v(s)}{k^2 - s + i\epsilon} + \int_0^\infty ds \frac{\rho_s(s)}{k^2 - s + i\epsilon}$$

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Vector and scalar Self-Energy densities

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Vector and scalar spectral densities

$$\not k A(k^2) - B(k^2) = ig^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2 - m_g^2 + i\epsilon}\left[g^{\mu\nu} - \frac{(1-\xi)\, q^\mu q^\nu}{q^2 - \xi m_g^2 + i\epsilon}\right]$$
$$- i\, g^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2 - \Lambda^2 + i\epsilon}\left[g^{\mu\nu} - \frac{(1-\xi) q^\mu q^\nu}{q^2 - \xi\Lambda^2 + i\epsilon}\right]$$

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Vector and scalar spectral densities

Gauge fixing

$$\not{k}A(k^2) - B(k^2) = ig^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2 - m_g^2 + i\epsilon}\left[g^{\mu\nu} - \frac{(1-\xi)\, q^\mu q^\nu}{q^2 - \xi m_g^2 + i\epsilon}\right]$$

$$- i\, g^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2 - \Lambda^2 + i\epsilon}\left[g^{\mu\nu} - \frac{(1-\xi) q^\mu q^\nu}{q^2 - \xi\Lambda^2 + i\epsilon}\right]$$

Pauli-Villars regulator

Fermion Dyson Schwinger Equation

- Parameters: $\alpha = \dfrac{g^2}{4\pi}$, Λ , m_g , $\overline{m}_0$.

- Spectral densities are obtained from the IR of the self-energy:

$$\rho_A(\gamma) = -\frac{1}{\pi}\mathrm{Im}\,[A(\gamma)]$$
$$\rho_B(\gamma) = -\frac{1}{\pi}\mathrm{Im}\,[B(\gamma)]$$

- Solutions of DSE obtained writing the trivial relation $S_f^{-1}S_f = 1$ in a suitable form:

$$\frac{R}{\gamma - \overline{m}_0^2 + \imath\epsilon} + \int_0^\infty ds \frac{\rho_v(s)}{\gamma - s + i\epsilon} = \frac{A(\gamma)}{\gamma A^2(\gamma) - B^2(\gamma) + i\epsilon}$$
$$\frac{R\,\overline{m}_0}{\gamma - \overline{m}_0^2 + i\epsilon} + \int_0^\infty ds \frac{\rho_s(s)}{\gamma - s + i\epsilon} = \frac{B(\gamma)}{\gamma A^2(\gamma) - B^2(\gamma) + i\epsilon}$$

Fermion DSE solution

$$\rho_A(\gamma) = R\mathcal{K}^{\xi}_{0A}(\gamma, \overline{m}_0^2, m_g^2) + \int_0^\infty ds\, \mathcal{K}^{\xi}_{A}(\gamma, s, m_g^2)\, \rho_v(s) - [m_g \to \Lambda]$$

$$\rho_B(\gamma) = R\,\overline{m}_0\, \mathcal{K}^{\xi}_{0B}(\gamma, \overline{m}_0^2, m_g^2) + \int_0^\infty ds\, \mathcal{K}^{\xi}_{B}(\gamma, s, m_g^2)\, \rho_s(s) - [m_g \to \Lambda]$$

- Driving term:

$$\mathcal{K}^{\xi}_{0A(0B)} = K_{A(B)} + m_g^{-2}\bar{K}^{\xi}_{A(B)}$$

- Kernel:

$$\mathcal{K}^{\xi}_{A}(\gamma, s, m_g^2) = K_A(\gamma, s, m_g^2)\,\Theta(s - (\overline{m}_0 + m_g)^2) + m_g^{-2}\bar{K}^{\xi}_{A}(\gamma, s, m_g^2)\,\Theta(s - (\overline{m}_0 + \sqrt{\xi} m_g)^2)$$

Connection Formulas

$$f_A(\gamma) = 1 + \int_0^\infty ds \frac{\rho_A(s)}{\gamma - s}$$

$$f_B(\gamma) = m_B + \int_0^\infty ds \frac{\rho_B(s)}{\gamma - s}$$

$$d(\gamma) = \left[\gamma f_A^2(\gamma) - \pi^2\,\gamma\,\rho_A^2(\gamma) - f_B^2(\gamma) + \pi^2\,\rho_B^2(\gamma)\right]^2 + 4\pi^2\left[\gamma\rho_A(\gamma)\, f_A(\gamma) - \rho_B(\gamma)\, f_B(\gamma)\right]^2$$

$$\rho_v(\gamma) = -2\frac{f_A(\gamma)}{d(\gamma)}\left[\gamma\rho_A(\gamma)\, f_A(\gamma) - \rho_B(\gamma)\, f_B(\gamma)\right] + \frac{\rho_A(\gamma)}{d(\gamma)}\left[\gamma\, f_A^2(\gamma) - \pi^2\gamma\rho_A^2(\gamma) - f_B^2(\gamma) + \pi^2\rho_B^2(\gamma)\right]$$

$$\rho_s(\gamma) = -2\frac{f_B(\gamma)}{d(\gamma)}\left[\gamma\rho_A(\gamma)\, f_A(\gamma) - \rho_B(\gamma)\, f_B(\gamma)\right] + \frac{\rho_B(\gamma)}{d(\gamma)}\left[\gamma\, f_A^2(\gamma) - \pi^2\gamma\rho_A^2(\gamma) - f_B^2(\gamma) + \pi^2\rho_B^2(\gamma)\right]$$

Feynman gauge kernel ($\xi = 1$):

$$K_A\left(\gamma, s, m_g^2\right) = -\frac{\alpha}{4\pi}\frac{\gamma - m_g^2 + s}{\gamma^2}\sqrt{(\gamma - m_g^2 + s)^2 - 4\gamma\, s}\;\Theta\left[\gamma - (m_g + \sqrt{s})^2\right]$$

$$K_B\left(\gamma, s, m_g^2\right) = -\frac{\alpha}{4\pi}\frac{4}{\gamma}\sqrt{(\gamma - m_g^2 + s)^2 - 4\gamma s}\;\Theta\left[\gamma - (m_g + \sqrt{s})^2\right]$$

Remaining arbitrary ξ-gauge contribution:

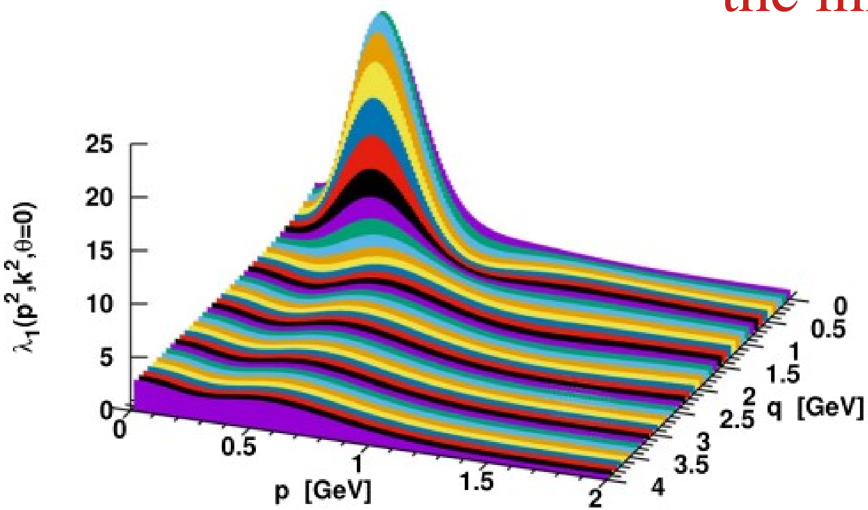
$$\bar{K}_A^{\xi}(\gamma, s, m_g^2) = -\frac{\alpha}{4\pi}\frac{\left(\gamma - s\right)^2 - m_g^2(\gamma + s)}{2\gamma^2}\sqrt{(\gamma - m_g^2 + s)^2 - 4\gamma s}$$
$$\times\,\Theta\left[\gamma - (m_g + \sqrt{s})^2\right] - [m_g^2 \to \xi m_g^2]$$

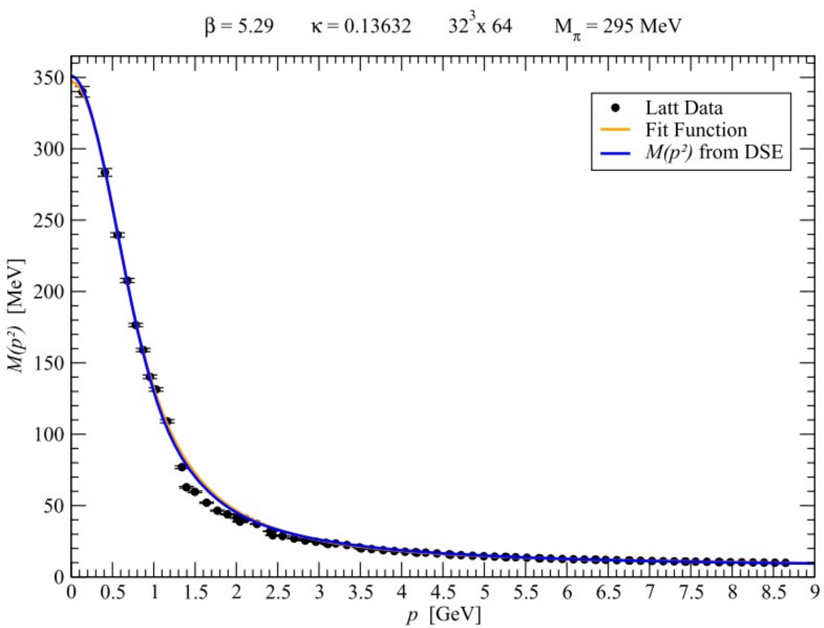
$$\bar{K}_B^{\xi}(\gamma, s, m_g^2) = \frac{\alpha\, m_g^2}{4\pi}\frac{\sqrt{(\gamma - m_g^2 + s)^2 - 4\gamma s}}{\gamma}\Theta\left[\gamma - (m_g + \sqrt{s})^2\right] - [m_g^2 \to \xi m_g^2]$$

Bare mass:

$$m_B = \overline{m}_0 + \overline{m}_0 \int_0^{\infty} ds \frac{\rho_A(s)}{\overline{m}_0^2 - s} - \int_0^{\infty} ds \frac{\rho_B(s)}{\overline{m}_0^2 - s}$$

DSE+Lattice QCD propagators: Enhancement of the quark-gluon vertex at the infrared region![†]





[†]O. Oliveira et al., EPJC 80:484 (2020).

- In our work: Pauli-Villars regulator can also be effectively associated with the form factor of the γ^μ component of the quark-gluon vertex:

$$\lambda_1(q^2) = \frac{m_g^2 - \Lambda^2}{q^2 - \Lambda + i\epsilon}$$

[†]Rojas et al., JHEP 10 (2013) 193; O. Oliveira et al., EPJC 79, 116 (2019).

Large coupling regime: Phenomenological model

Calibration of the model: Possibility to explore the chiral symmetry breaking region!

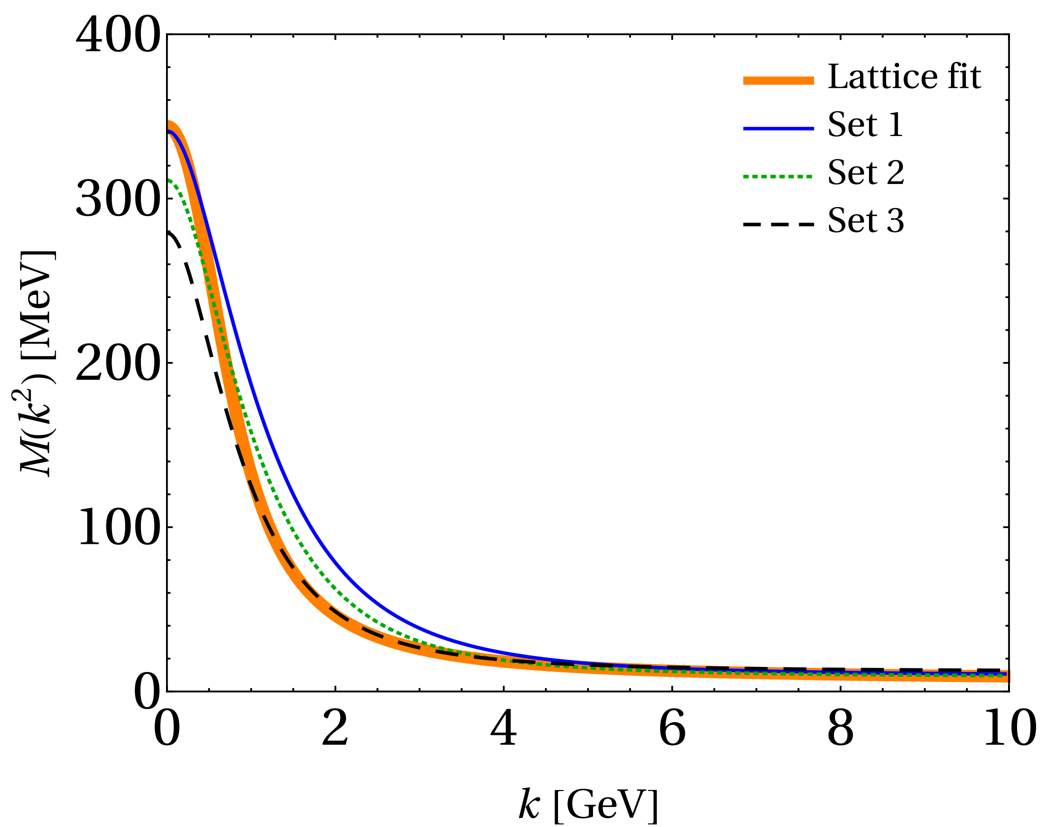
Set	$\bar{m}_0$ (GeV)	m_g (GeV)	Λ (GeV)	α
1	0.42	0.84	1.20	19.70
2	0.38	0.78	1.10	20.30
3	0.35	0.60	1.00	13.25

Set	(Outputs)	m_B (MeV)	R
1		9.06	2.22
2		8.53	2.09
3		12.25	2.64

Appropriate behavior in the infrared require a large enough Kernel

⇓

Λ cannot be large compared to m_σ, and as a consequence, α must increase!



DD, T. Frederico, W. de Paula, E. Ydrefors Phys. Rev. D 105, 114055 (2022).

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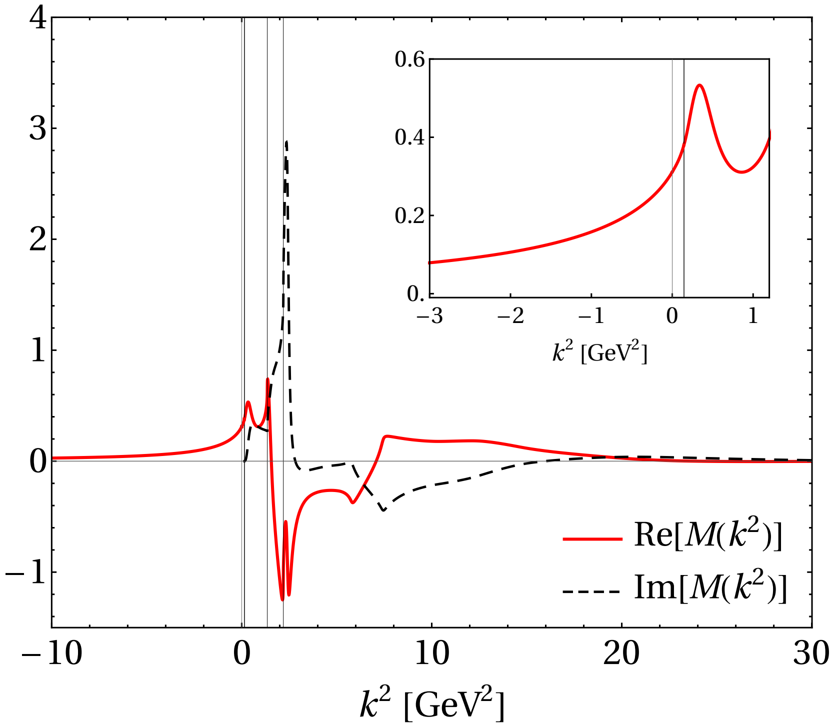

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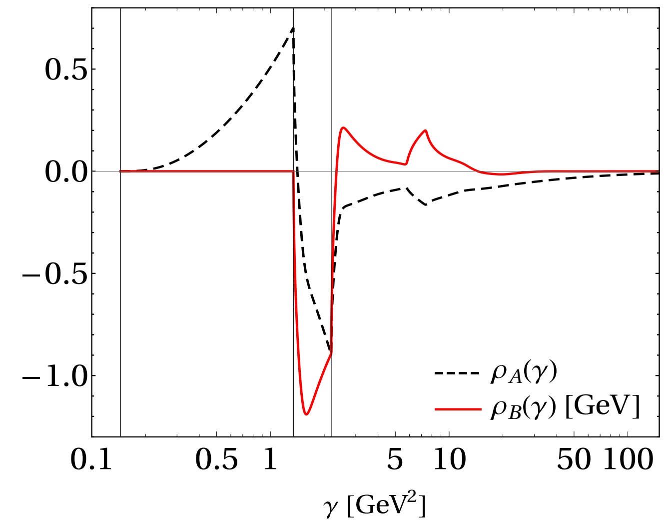

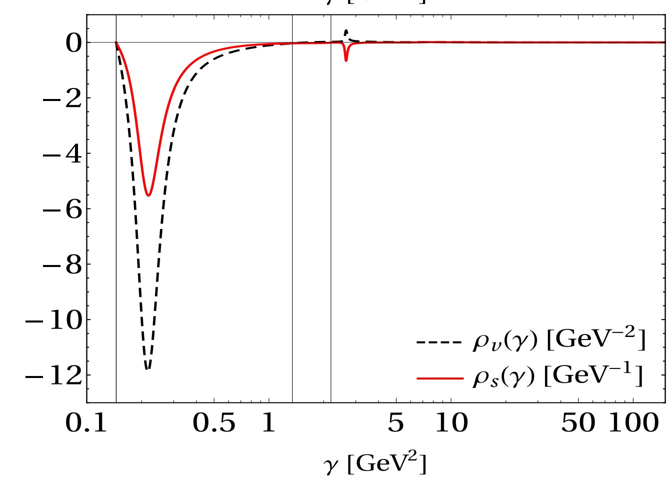

$\overline{m}_0^2$

$(\overline{m}_0 + m_g)^2$

$(\overline{m}_0 + \Lambda)^2$

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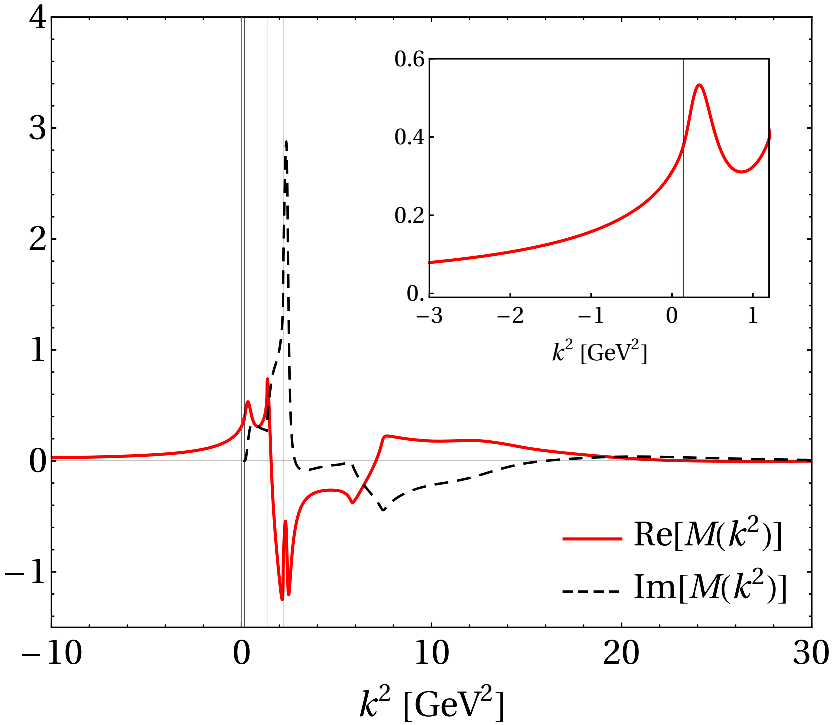
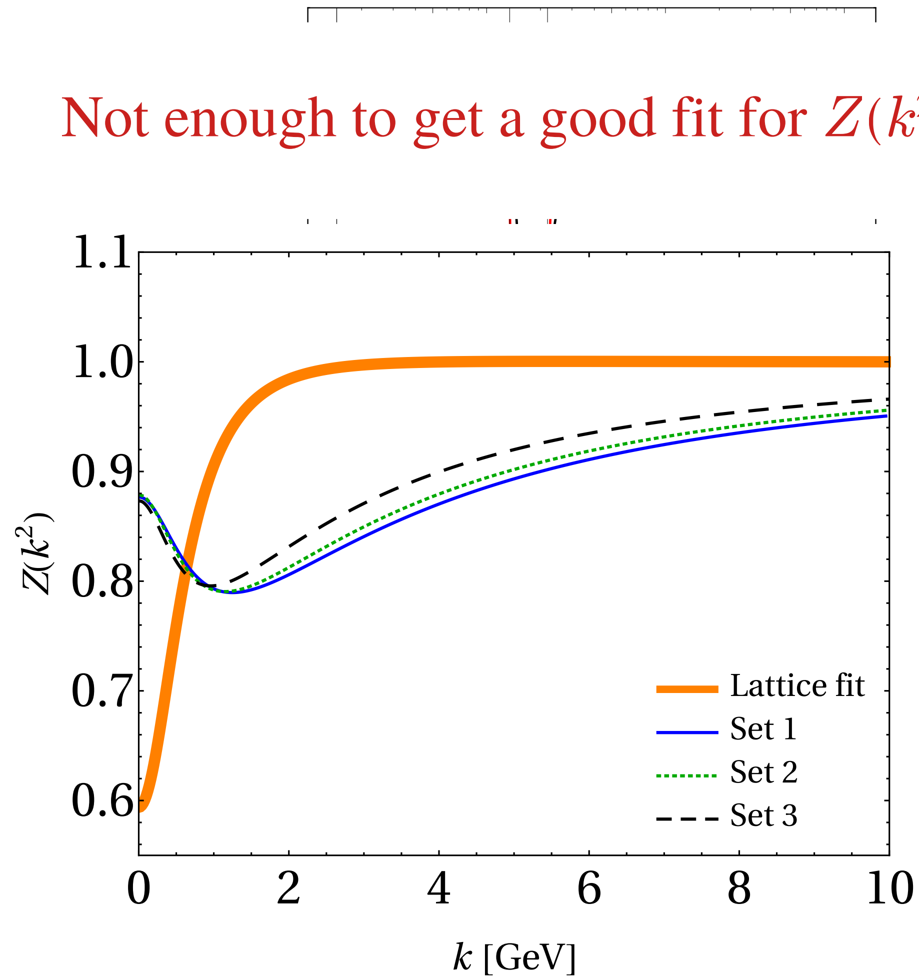


Not enough to get a good fit for $Z(k^2)$



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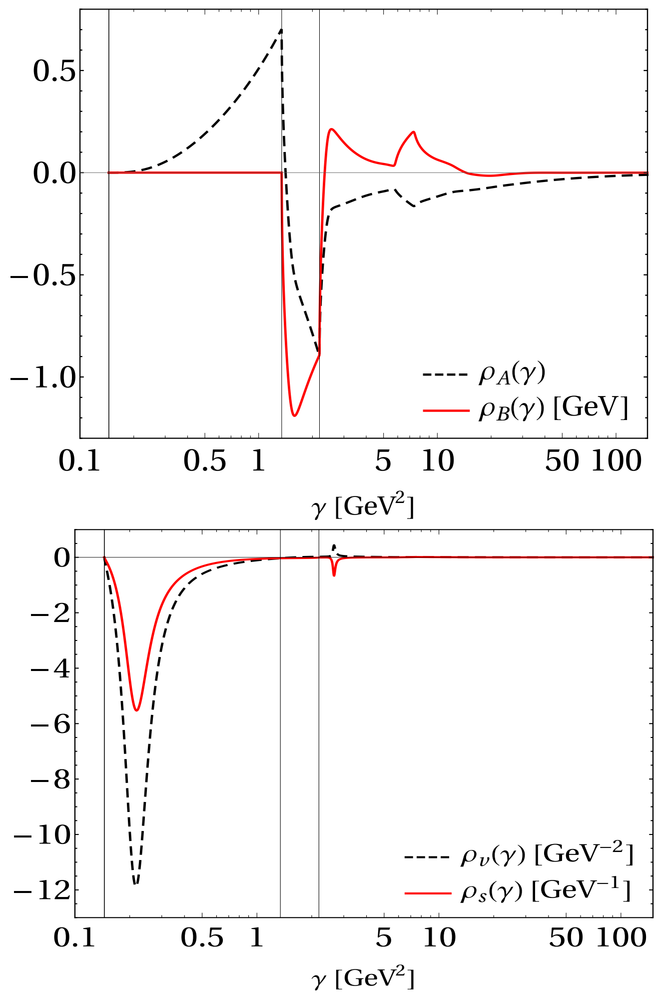
Recent developments and perspectives

Spectral densities evaluated by solving the DSE the method described previously as inputs for the pion Bethe-Salpeter equation.



PRD 105, 114055 (2022)

$$\Psi_\pi(k;p) = S_F(k_q)\Gamma_\pi(k;p)S_F(k_{\bar{q}})$$

$$S_F(k) = \frac{1}{A(k^2)\not{k} - B(k^2)}$$
$$= S_v(k^2)\not{k} + S_s(k^2)$$

$$S_v(k^2) = \frac{R}{k^2 - \overline{m}_0^2 + i\epsilon} + \int_0^\infty ds \frac{\rho_v(s)}{k^2 - s + \imath\epsilon}$$

$$S_s(k^2) = \frac{R\overline{m}_0}{k^2 - \overline{m}_0^2 + i\epsilon} + \int_0^\infty ds \frac{\rho_s(s)}{k^2 - s + i\epsilon}$$

$$\Gamma_\pi(k,p) = \gamma_5[\imath E_\pi(k,p) + F_\pi(k,p)$$
$$+ k^\mu p_\mu \, G_\pi(k,p) + \sigma_{\mu\nu} k^\mu p^\nu H_\pi(k,p)]$$

Recent developments and perspectives

- First approximation: Chiral limit! In this case, the pion quark-antiquark vertex is given by**

$$E_\pi(k) = -\frac{1}{f_\pi^0} B(k^2)\,, \qquad B(k^2) = \int_0^\infty ds \frac{\rho_B(s)}{k^2 - s + i\epsilon}$$

- Next steps: Calculation of observables, more rigorous study of chiral symmetry breaking, ingredients from LQCD...

BS amplitude normalization: $\mathrm{Tr}\left\{\int \frac{d^4k}{(2\pi)^4} \frac{\partial}{\partial p'^\mu}\left[S^{-1}\left(k - \frac{p'}{2}\right)\bar{\Psi}_\pi(k,p) S^{-1}\left(k + \frac{p'}{2}\right)\Psi_\pi(k,p)\right]_{p'=p}\right\} = -2ip_\mu$

Pion decay constant: $ip^\mu f_\pi = N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\gamma^\mu \gamma^5 \Psi_\pi(k,p)\right]$

Quark condensate: (Regularization??) $\langle \bar{q} q \rangle = -\mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} S_F(k)$

**C. S. Mello, et al., Phys. Lett. B, 766 86–93 (2017), L. Chang et al., PRL 110, 132001 (2013).

Final Remarks

- Possibility of calculation of dynamical observables in Minkowski space.
- The Integral Representation as a very important tool to solve DSE and BSE.
- Inclusion of more sophisticated ingredients, as quark-gluon vertex, Lattice QCD (self energy, vertex, ...) ⇒ more realistic theories!
- Wide range of applications: Form factors, parton distribution functions, analytic structure of pion, kaon, nucleon, Nakanishi weight functions ...

Thanks for your attention!