Confinement, chiral symmetry breaking, holography and the 3D image of the pion

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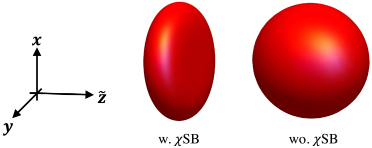
Light Cone 2022 Online: Physics of Hadrons on the Light Front

September 19-24, 2022 @ Zoom

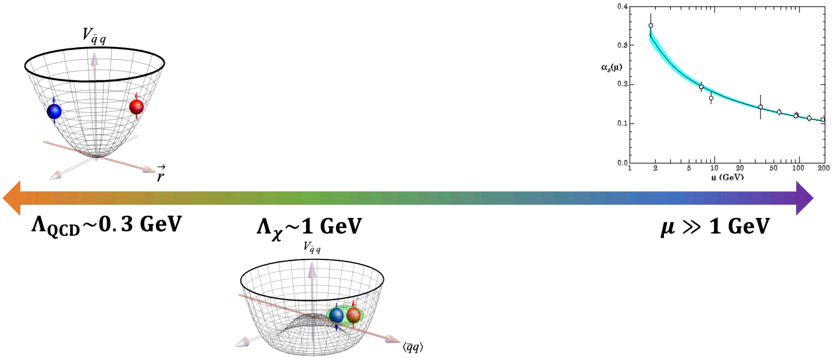
Contents

Based on:
YL, Maris, Vary, arXiv:2203.14447 [hep-th];
YL, Vary, PLB (2022), arXiv:2103.09993 [hep-ph];
YL, Vary, PRD (2022), arXiv: 2202.05581 [hep-ph]

Pion light-front amplitude in 3D



Non-perturbative QCD

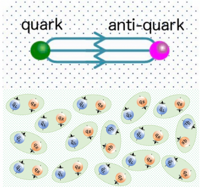


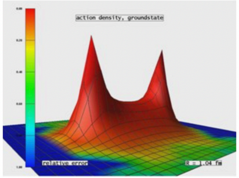
- Confinement
- Chiral symmetry breaking

$r_{q\bar{q}} \lesssim \Lambda_{\mathrm{QCD}}^{-1}$

$r_\pi \to \infty$

QCD vacuum





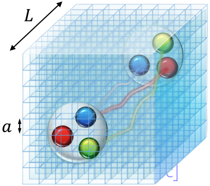
QCD Lite $\longleftarrow$ $q\bar{q}$ $s\bar{s}$ $c\bar{c}$ $b\bar{b}$ $\longrightarrow$ **Yang-Mills world**

- Confinement in QCD: various possibilities [Greensite:2016pfc]

  flux tube, dual superconductor, central vortices, Gribov horizon, ...

- Chiral symmetry breaking: [Faber:2017alm]

``Berlin wall'': $$C = K\left(\frac{M_\rho}{M_\pi}\right)^{2\sim 6} a^{-(6\sim 7)} L^5,$$



Light-front QCD

Is there any simplified description of the pion on the light front?

[Brodsky et. al., Nature Rev. Phys. (2022)]

- Same physics: (non-local) chiral condensate on the LF

[Wu:2003vn; Beane:2013oia, Beane:2015ufo, Burkardt:1996pa]

$$\frac{m_q}{2}\langle 0|\bar{q}\frac{\gamma^+}{i\overleftrightarrow{\partial}^+}q|0\rangle_{\text{LF}} = \langle 0|\bar{q}q|0\rangle_{\text{IF}}$$

- In-hadron condensate: χSB encoded in hadronic wave functions

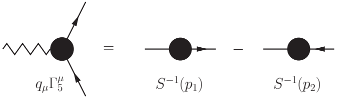
[Brodsky:2010xf; Casher:1974xd]

$$Q_5|0\rangle_{\text{LF}} = 0$$

- DSE/BSE: Axial-vector Ward-Takahashi identity [Maris:1997hd]

$$f_\pi E_\pi(k,0) = B_q(k^2), \cdots$$

Tantalizing implication to dynamical mass generation



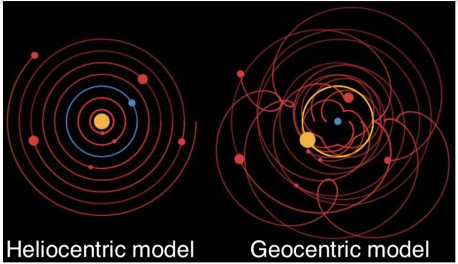
nature reviews physics

Artificial dynamical effects in quantum field theory

Stanley J. Brodsky, Alexandre Deur and Craig D. Roberts

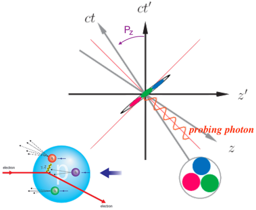
Abstract | In Newtonian mechanics, studying a system in a non-Galilean reference frame can lead to inertial pseudoforces appearing, such as the centrifugal force that seems to arise in dynamics analysed in a rotating frame. Likewise, artificial

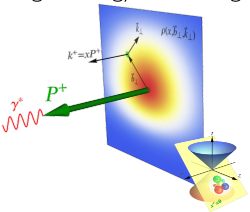
Today, no approach to QFT is perfect, but LF quantization possesses many merits. It often provides the simplest known description of nature. As always in physics, one can formulate a problem using any framework one desires. But if the wrong approach is chosen, the costs are high[65].



Light-front physics

Light-front physics underlines hadron structures measured in high-energy scattering experiments

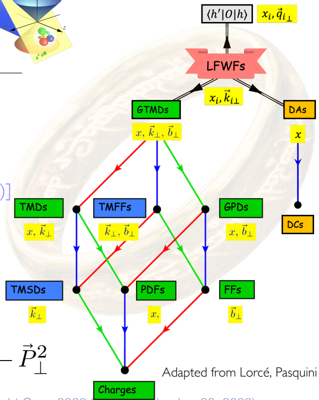




Access to light front amplitudes:

- OPE + moments reconstruction [e.g. Chang:2013pq]
- Nakanishi representation & LF projection of BSA [dePaula:2022pcb; de Paula's Talk (T)]
- un-Wick rotation [Eichmann:2021vnj, Frederico:2019noo; Ferreira's Talk (W)]
- IMF and large momentum effective theory [Ji:2013dva, Ma:2021yqx; Ma's Talk (W)]
- Schrödinger-Einstein equation [review: Hiller:2016itl]

$$H_{\rm LFQCD}|\psi_h\rangle = M_h^2|\psi_h\rangle, \qquad H_{\rm LFQCD} = P^+P^-_{\rm LFQCD} - \vec{P}_\perp^2$$



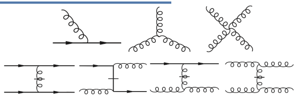
Adapted from Lorcé, Pasquini

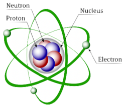
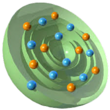
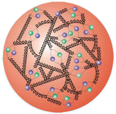
Semiclassical first approximation to QCD

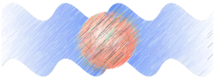
[Review: Brodsky:2014yha]

Light-front QCD in
light cone gauge $A^+ = 0$
[Tomboulis:1973jn, Casher:1976ae]

$$H_{\mathrm{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^{\,2} + m_i^2}{x_i} + U_i + \sum_{ij} V_{ij}^{(\mathrm{QCD})} - \delta_{ij} U_i$$



Atoms	Nuclei	Hadrons
		
Coulomb interaction	NN, NNN interactions	QCD interactions
↓	↓	↓
Bohr Model $\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$	Shell Model $\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$	?



zitterbewegung
[Miller:2018ybm; Li:2022ldb]

Light-front Schrödinger wave equation (LFSWE)
[Brodsky:2014yha, Chabysheva:2012fe]

$$\left(\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + U\right)\psi(x, \vec{k}_\perp) = M^2 \psi(x, \vec{k}_\perp)$$

factorization ansatz $\Downarrow$ $\psi = \varphi(\vec{\zeta}_\perp)\chi(x)$

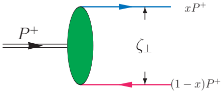
[Li:2021jqb, deTeramond:2021yyi, Ahmady:2021lsh, Ahmady:2021yzh, Li:2022izo, Lyubovitskij:2022rod, Ahmady:2022dfv]

$$\left[-\nabla_{\zeta\perp}^2 + U_\perp(\vec{\zeta}_\perp)\right]\varphi(\vec{\zeta}_\perp) = M_\perp^2 \varphi(\vec{\zeta}_\perp), \quad \left[\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel(\tilde{z})\right]\chi(x) = M_\parallel^2 \chi(x)$$

Light-front holography

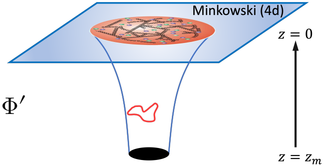
[Review: Brodsky:2014yha; cf. Erlich:2005qh, Karch:2006pv]

LFH is a unique correspondence between LFQCD_{3+1} and string motion in AdS_5/QCD [Brodsky:2003px]



- LF wave equation
- $\zeta_\perp = \sqrt{x(1-x)}r_\perp$, conjugate to off-shell energy ε

semiclassical LFQCD	$\leftrightarrow$	AdS/QCD
$\zeta_\perp$	$\leftrightarrow$	z,
ϕ_{Jm}	$\leftrightarrow$	ϕ_J
U_J	$\leftrightarrow$	$\frac{1}{4}\Phi'^2 + \frac{1}{2}\Phi'' + \frac{2J-3}{2z}\Phi'$
$m^2 - (J-2)^2$	$\leftrightarrow$	$(\mu R)^2$



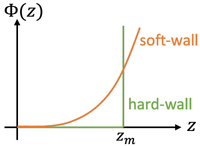
- Based on gravity/gauge duality
- $z \to 0$: CFT
- $z \to \infty$: low-energy QCD pheno.
- Dilaton $\Phi(z)$ to break conf. sym.
- $z^{-1} \sim \mu_R$, RG scale

$$\Big[-\frac{1}{\zeta_\perp}\frac{\mathrm{d}}{\mathrm{d}\zeta_\perp}\Big(\zeta_\perp\frac{\mathrm{d}}{\mathrm{d}\zeta_\perp}\Big) + \frac{m^2}{\zeta_\perp^2} + U_J(\zeta_\perp)\Big]\varphi_{Jm}(\zeta_\perp) = M^2\varphi_{Jm}(\zeta_\perp)$$

$$\updownarrow$$

$$\Big[-\frac{z^{3-2J}}{e^{\Phi(z)}}\frac{\mathrm{d}}{\mathrm{d}z}\Big(\frac{e^{\Phi(z)}}{z^{3-2J}}\frac{\mathrm{d}}{\mathrm{d}z}\Big) + \frac{\mu^2R^2}{z^2}\Big]\varphi_J(z) = M^2\varphi_J(z)$$

where $\varphi_J(z) = \left(\frac{R}{z}\right)^{J-\frac{3}{2}} e^{-\frac{1}{2}\Phi(z)}\phi_J(z)$, $\varphi = \phi/\sqrt{\zeta_\perp}$



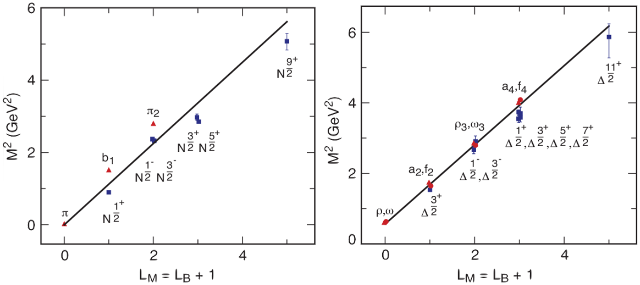
Confinement in AdS/QCD

[Review: Brodsky:2014yha]

Breaking of the conformal symmetry by the dilaton field $\Phi(z)$ at IR $z \to \infty$ [Erlich:2005qh]

- Regge trajectory: $\Phi(z \to \infty) \sim z^2$ [Karch:2006pv]
- Soft-wall AdS/QCD: $\Phi(z) = \lambda z^2$
  - *Effective LF confining potential:* $U(z) = \lambda^2 z^2 + 2(J-1)\lambda$
  - *Superconformal symmetry: pion as a massless susy singlet* [Brodsky:2013ar; see de Téramond's Talk (R)]

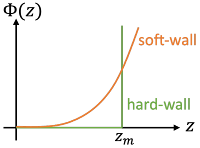
[cf. deAlfaro:1976vlx, Fubini:1984hf; Miyazawa:1966mfa, Miyazawa:1968zz]



What about chiral symmetry breaking?

Chiral symmetry breaking in soft-wall AdS/QCD

[Erlich:2005qh, Karch:2006pv]



PRL **95,** 261602 (2005)

QCD and a Holographic Model of Hadrons

Joshua Erlich,[1] Emanuel Katz,[2] Dam T. Son,[3] and Mikhail A. Stephanov[4]

Hard-wall model: with chiral symmetry breaking, but no Regge trajectory

PHYSICAL REVIEW D **74,** 015005 (2006)

Linear confinement and AdS/QCD

Andreas Karch,[1,*] Emanuel Katz,[2,†] Dam T. Son,[3,‡] and Mikhail A. Stephanov[4,§]

Soft-wall model: with Regge trajectory, but no chiral symmetry breaking

tion between bulk and boundary theories [12,19]. The action at quadratic order in the fields and derivatives reads

$$I = \int d^5x e^{-\Phi(z)} \sqrt{g} \Big\{ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \Big\}$$

asymptotics $e^{z^2} \to \infty$ and $\exp\{-(3/4)z^{-2}\} \to 1$. Since the equation is linear, selecting one of the solutions in the IR (the $X < \infty$ one, of course) gives Σ simply proportional to M. This is not what one wants in a theory with spontaneous symmetry breaking such as QCD. It is clear that one has to consider higher order terms in the potential $U(X, \ldots)$ for X and all other scalar condensates. Such a potential would

Various implementations of χSB in soft-wall AdS/QCD:

[Gherghetta:2009ac, Zuo:2009dz, Sui:2009xe, Sui:2010ay, Iatrakis:2010zf, Iatrakis:2010jb, Vega:2010ne, Jarvinen '12; Afonin:2012jn, Li:2012ay, Li:2013oda, Cui:2013xva, Chelabi:2015cwn, Fang:2016nfj, Ballon-Bayona:2021ibm, Rinaldi:2022dyh, ...]

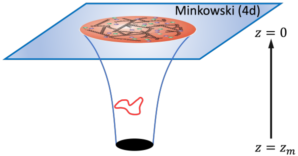
Chiral symmetry breaking in soft-wall AdS/QCD [Gherghetta:2009ac, Kapusta:2010mf]

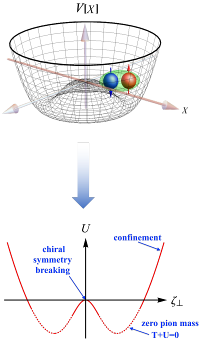
Scalar field X dual to $\bar{q}q$, with a non-vanishing VEV: $\langle X\rangle = \frac{1}{2}\chi(z)$

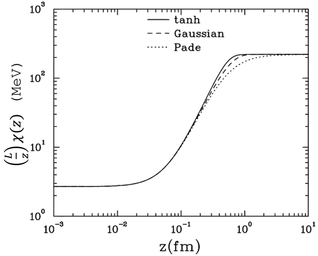
$$S = -\int \mathrm{d}^5x\sqrt{-g}e^{-\Phi(z)}\mathrm{Tr}\Big\{\big|DX\big|^2 - V[X] + \frac{1}{4g_5^2}F^2\Big\},$$

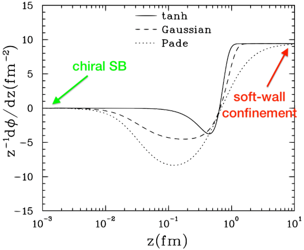
Higgs potential: $V[X] = -m_X^2\big|X\big|^2 + \kappa\big|X\big|^4$, $m_X^2 = -3$

- At the CFT boundary ($z\to 0$): $\chi(z) \sim m_q z + \Sigma z^3$ [Klebanov:1999tb]
- (Non-linear) Eq. of Motion $\Rightarrow$ $\Phi(z)\sim z^6 \Rightarrow U(z) \sim -z^4$ at $z\to 0$
- χSB & confinement dictate the UV & IR behaviors of $U(\zeta_\perp)$, respectively







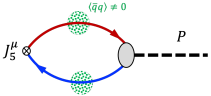


Model independent constraints from light-front dynamics?

Partially conserved axial-vector current (PCAC)

$$\partial_\mu J_5^\mu = 2im_q\bar{q}\gamma_5 q \quad \Rightarrow \quad \langle 0|\partial_\mu J_5^\mu - 2im_q\bar{q}\gamma_5 q|P(p)\rangle = 0.$$

where, $J_5^\mu(x) = \bar{q}(x)\gamma^\mu\gamma_5 q(x)$ is the axial-vector current; $q(x)$ is the quark field operator.



$$J_5^\mu = i\partial^\mu\phi_\pi + J_{5,N}^\mu$$

$$q(x) = \sum_s \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2p^+}\Big\{b_s(p)u_s(p)e^{ip\cdot x} + d_s^\dagger(p)v_s(p)e^{-ip\cdot x}\Big\}\Big|_{x^+=0}$$

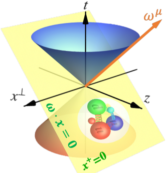
$$|P(p)\rangle = \sum_{s,\bar{s}} \int \frac{\mathrm{d}x}{2x(1-x)} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3}\psi_{s\bar{s}/P}(x,\vec{k}_\perp)\frac{1}{\sqrt{N_C}}\sum_i b_{si}^\dagger(p_1)d_{\bar{s}i}^\dagger(p_2)|0\rangle + \text{high Fock sectors}$$

where $x = p_1^+/p^+$, and $\vec{k}_\perp = \vec{p}_{1\perp} - x\vec{p}_\perp$.

The *most general covariant structure* of the pion valence LFWF: covariant LFD [Carbonell:1998rj]

$$\psi_{s\bar{s}/P}(x,\vec{k}_\perp) = \bar{u}_s(p_1)\Big[\gamma_5\,\phi_1(x,k_\perp) + \hat{f}_\chi\frac{\gamma_5\not{\omega}}{\omega\cdot p}\phi_2(x,k_\perp)\Big]v_{\bar{s}}(p_2),$$

$$V\cdot\omega = V^+$$



- ω-terms: rotational & conformal invariance: $L_{\text{int}}^{\mu\nu} = i\omega^{[\mu}\partial/\partial\omega_{\nu]}$, $\omega^\mu \to \lambda\omega^\mu$
- Chiral broken phase: $f_\pi \neq 0 \;\Rightarrow\; \hat{f}_\chi \neq 0$

$$\frac{f_P}{2\sqrt{2N_C}} = \int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\psi_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp).$$

Chiral sum rule

[Li:2022ytx]

In the chiral limit $m_q \to 0$,

$$\int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \frac{k_\perp^2}{x(1-x)} \psi^{(0)}_{\uparrow\downarrow-\downarrow\uparrow/P}(x, \vec{k}_\perp) = 0$$

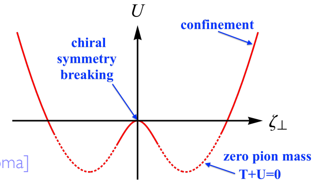
- Exact relation - no Fock sector truncation
- If $\hat{f}_\chi = 0$, this sum rule is automatically satisfied. However, $f_\pi = 0 \;\to$ chiral symmetric phase (not QCD)
- Gell-Mann-Oakes-Renner relation: $f_P^{(0)2} M_P^2 = 2 m_q g_P^{(0)} + O(m_q^2)$, where $g_P = \langle 0 | j_5 | P(p) \rangle$
- Additional sum rules from further light-front current algebra [Beane:2013oia, Beane:2015ufo]

Application to light-front Schrödinger wave equation:

$$\boxed{f_P \nabla_\perp^2 \varphi_P(\zeta_\perp = 0) = 0}$$

Recall: $\left[-\nabla_\perp^2 + U(\zeta_\perp)\right] \varphi_P(\zeta_\perp) = M_P^2 \varphi_P(\zeta_\perp)$,

$\Rightarrow \quad U(\zeta_\perp = 0) = 0 \qquad (\text{cf. } U_{\text{Higgs}} \sim -\zeta_\perp^4 \to 0)$



N.B. for excited pions, the decay constants $f_{\pi^n} = 0$. [Maris:1997hd, Ballon-Bayona:2014oma]

Analytic model

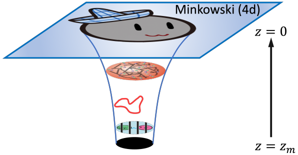
[Li:2022ytx]

Let's build a chiral pion:

$$\Big[-\nabla_\perp^2 + U(\zeta_\perp)\Big]\varphi_P(\zeta_\perp) = M_P^2\varphi_P(\zeta_\perp), \qquad \nabla_\perp^2\varphi_\pi(\zeta_\perp=0)=0, \quad U(\zeta_\perp)\to\begin{cases}\zeta_\perp^2 & \zeta_\perp\to\infty\\ -\zeta_\perp^4 & \zeta_\perp\to 0\end{cases}$$

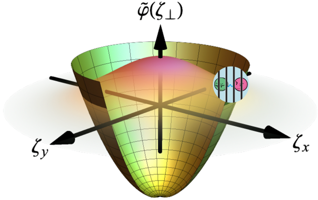
We propose the following pion wave function based on light-front holography:

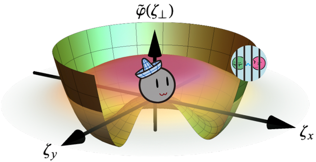
$$\varphi_\pi(\zeta_\perp) = \underbrace{\Big(1+\frac{1}{2}\zeta_\perp^2+\frac{c}{8}\zeta_\perp^4\Big)}_{\text{chiral symmetry breaking}}\overbrace{e^{-\frac{\zeta_\perp^2}{2}}}^{\text{confinement}}$$



Given the pion wave function, the potential is ($c=1$),

$$U(\zeta_\perp) = \frac{\nabla_\perp^2\varphi_\pi(\zeta_\perp)}{\varphi_\pi(\zeta_\perp)} = \frac{\zeta_\perp^4(\zeta_\perp^2-6)}{\zeta_\perp^4+4\zeta_\perp^2+8}$$





3D image of the pion

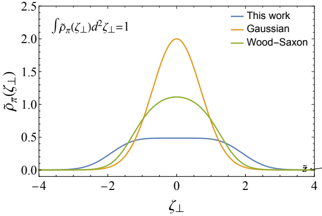
[Li:2022ytx]

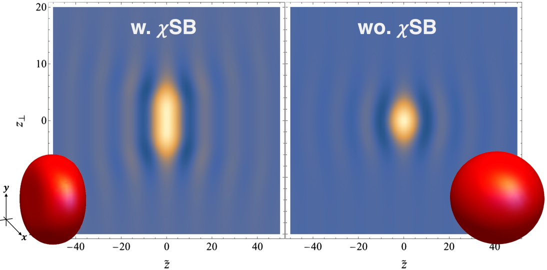
$$\widetilde{\psi}_\pi(\vec{z}_\perp, \tilde{z}) = \int_0^1 \frac{\mathrm{d}x}{2\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} e^{ix\tilde{z} - i\vec{k}_\perp\cdot\vec{z}_\perp} \psi_\pi(x, \vec{k}_\perp),$$

$$= \langle 0 | \bar{q}(-\tfrac{1}{2}z) \frac{\not{\omega}\gamma_5}{\omega\cdot p} q(+\tfrac{1}{2}z) | P(p) \rangle_{z^+=0}$$

where $\tilde{z} = p\cdot z$ is the Ioffe time of Miller and Brodsky. [Miller:2019ysh]

- Infinitely long in the longitudinal direction, $r_\pi \to \infty$ [Weller:2021wog, Li:2022izo]
- Plateau in the transverse direction up to $z_\perp \sim \kappa_{\text{conf}}^{-1}$: uniform within

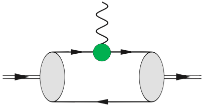




Pion electromagnetic form factor & PDF

[In preparation]

$$F_\pi(q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \rho_\pi(\zeta_\perp) V_{\mathrm{SW}}(q^2, \zeta_\perp) = \int \mathrm{d}x\, H_\pi(x, t = q^2)$$

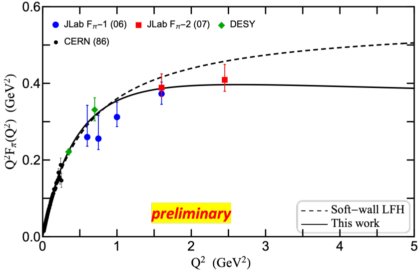


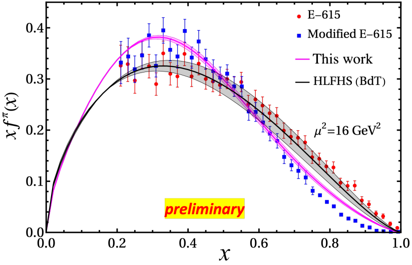
- Soft-wall AdS/QCD: only leading-twist contribution $\sim Q^{-2}$

$$\rho_\pi(\zeta_\perp) = N \exp(-\zeta_\perp^2) \;\Rightarrow\; F_\pi(q^2) = F_{\tau=2}(q^2)$$

- χSB modifies the large-Q^2 behavior: incorporating high-twist contributions

$$\rho_\pi(\zeta_\perp) = [1 + \frac{1}{2}\zeta_\perp^2 + \frac{c}{8}\zeta_\perp^4]^2 \exp(-\zeta_\perp^2) \;\Rightarrow\; F_\pi(q^2) = N\Big\{\underbrace{F_{\tau=2}(q^2)}_{[q\bar{q}]} + \underbrace{F_{\tau=3}(q^2)}_{[q\bar{q}g]} + c_1 \underbrace{F_{\tau=4}(q^2)}_{[q\bar{q}q\bar{q}]} + \cdots\Big\}$$





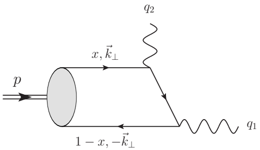
Pion transition form factor

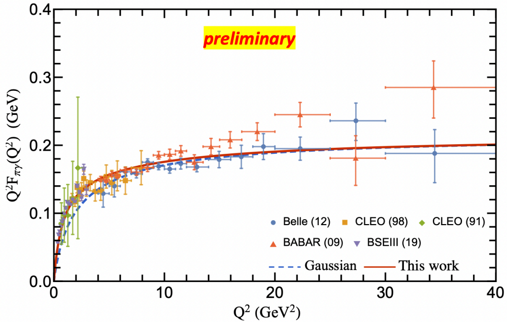
[In preparation]

Singly-virtual two-photon transition form factor:

$$F_{\pi\gamma}(Q^2) = e_f^2 2\sqrt{2N_C} \int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \frac{\psi_\pi(x, \vec{k}_\perp)}{k_\perp^2 + x(1-x)Q^2}$$

- VMD: $Q^2 \to Q^2 + M_\rho^2$
- pQCD normalization: $Q^2 F_{\pi\gamma}(Q^2) \to 2f_\pi$, $Q^2 \to \infty$





Dynamical mass generation

$$\left[-\nabla_\perp^2 + U(\zeta_\perp)\right]\varphi_P(\zeta_\perp) = M_P^2\varphi_P(\zeta_\perp),$$

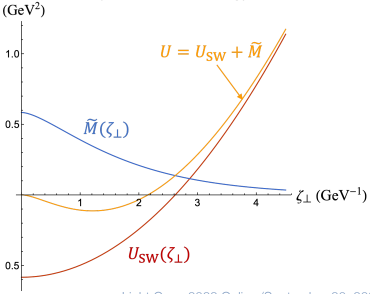
- Dynamically generated mass: $m_q \to m_q + M(k^2)$ with $M(0) \sim 300\,\text{MeV}$
- In LFH, an effective quark mass $m_{\{u,d\}} \sim$ 46 MeV is adopted
- In our model, $m_q \equiv 0$, viz. $M(k^2)$ is absorbed in the effective potential $U(\zeta_\perp)$
- DMG turns the quadratic potential to a sombrero: χSB is closely related to quark self-energy [cf. Maris:1997hd]

Consider a mass function inspired by the Lattice data:

[Mello:2017mor]

$$M(k^2) = \frac{M(0)}{1 - k^2/\Lambda^2} \overset{\text{F.T.}}{\Rightarrow} \widetilde{M}(\zeta_\perp) = M_0^2 \Lambda \zeta_\perp K_1(\Lambda\zeta_\perp)$$

$$k^2 = -\frac{k_\perp^2}{x(1-x)} = \nabla_{\zeta\perp}^2$$

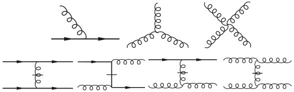


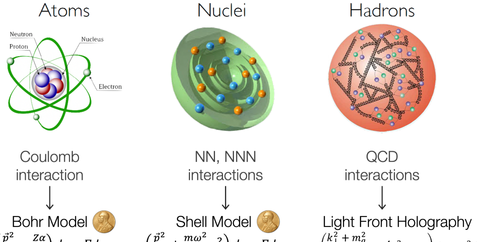
Implication to *ab initio* LFQCD

[Review: Brodsky:1997de, cf. Wilson:1994fk]

Light-front QCD in light cone gauge $A^+ = 0$

$$H_{\text{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^{\,2} + m_i^2}{x_i} + U_i + \sum_{ij} V_{ij}^{(\text{QCD})} - \delta_{ij} U_i$$

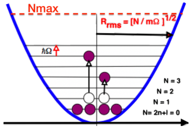




$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$

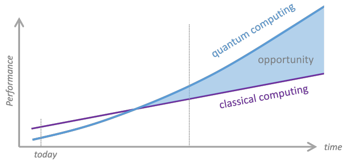
$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$

$\left(\frac{k_\perp^2 + m_q^2}{x(1-x)} + \kappa^4 \zeta_\perp^2 + U_{||}\right)\psi = M^2\psi$



[Review: Barrett:2013nh (NCSM)]

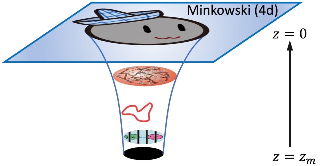
- Preserving all kinematical symmetries under the truncation is important: BLFQ & CLFD [Vary:2009gt, Carbonell:1998rj]
- Realistic QQ interactions: phenomenological → EFT → RG + LFQCD (e.g. RGPEP) [Serafin's talk (T)]
- Many-body dynamics: dynamical sea & gluons [Mondal's talk (T)]
- Exponential wall $\dim = N^{dN}$: QMBT + HPC/QC [Kreshchuk's talk (F)]

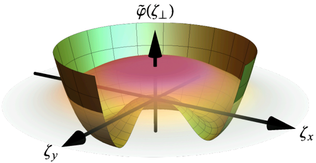


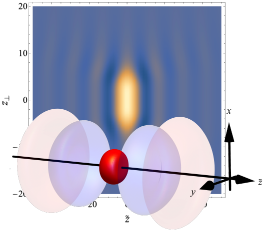
Summary

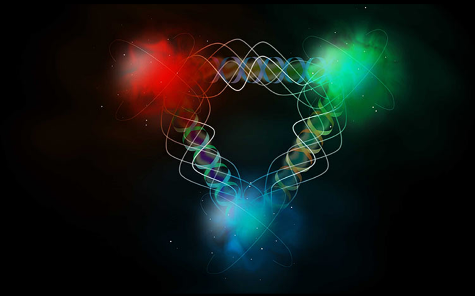
[YL, Maris, Vary, arXiv:2203.14447 [hep-th]]

- The pion is the key to understand confinement and chiral symmetry breaking in QCD
- Light-front wave functions provide the direct access to the parton structure of the pion
- Obtained an exact sum rule for the valence sector wave function based on the most general covariant structure and PCAC
- This chiral sum rule is consistent with chiral symmetry breaking in soft-wall AdS/QCD and leads to a remarkable feature of the pion structure









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