New results on gluon and heavy quark content in the nucleon

Valery Lyubovitskij

Eberhard Karls Universität Tübingen, Germany and Universidad Técnica Federico Santa María/CCTVal, Chile

In Collaboration with

Ivan Schmidt (UTFSM/CCTVal, Chile) and Stan Brodsky (SLAC, USA)

Phys Rev D 105 114032 (2022); 104, 014001 (2021); 103, 094017 (2021); 102, 034011 (2020); 2209.00403 [hep-ph]

Light Cone 2022 Online: Physics of Hadrons on the Light Front

Plan

I. Gluon TMDs in LF QCD

- Using LFWFs for $g + 3q$ Fock component in nucleon we derive gluon TMDs
- TMDs — factorized product of two LFWFs and gluonic matrix encoding information about both T-even and T-odd TMDs
- TDMs obey Mulders-Rodrigues inequalities, small-x and large-x behavior
- New sum rules (SRs) involving TMDs

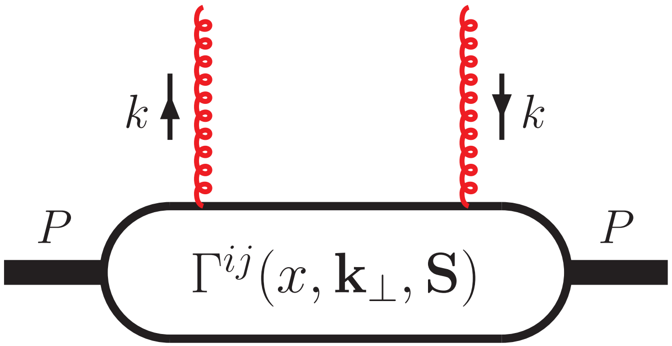
II. Heavy quark contribution to nucleon properties

- LFWFs for the Fock state containing heavy quark-antiquark component
- LF decomposition and results for
heavy quark-antiquark symmetry/asymmetry $Q(x) \pm \bar{Q}(x)$,
EM form factors of nucleons (Q^2 dependence, magnetic moments, radii)
- Novel asymmetry due to interference of Fock states producing $Q\bar{Q}$

III. Summary

Gluon TMDs in QCD

- Mulders and Rodrigues, PRD63, 094021 (2001):
  Expansion of gluon correlator $\Gamma^{ij}(x, \mathbf{k}_\perp, \mathbf{S})$ in QCD



- i, j – gluon polarization indices; nucleon spin $S^\mu = (0, \mathbf{S})$ and $\mathbf{S} = (S_L, \mathbf{S}_T)$
- $S_L = \cos\theta$ and $\mathbf{S}_T = (\cos\phi\sin\theta, \sin\phi\sin\theta)$
- θ and ϕ – polar and azimuthal angles (orientation of the spin-vector $\mathbf{S}$)
- $k_\perp^\mu = (0, 0, \mathbf{k}_\perp) = \sqrt{\mathbf{k}_\perp^2}\,(0, 0, \cos\phi_k, \sin\phi_k)$ with $k_\perp^2 = -\mathbf{k}_\perp^2$,
  ϕ_k – azimuthal angle (orientation of $\mathbf{k}_\perp$ in the T-plane).

Gluon TMDs in QCD

- $U(2)$ group acting in 2D transverse space:
  3 symmetric g_T^{ij}, η_T^{ij}, ξ_T^{ij} and 1 antisymmetric ϵ_T^{ij} transverse tensors

$$
\begin{aligned}
g_T^{ij} &= -\delta^{ij} = \mathrm{diag}(-1,-1) \\
\eta_T^{ij} &= \tau_3^{ij}\ \cos 2\phi_k + \tau_1^{ij}\ \sin 2\phi_k = \begin{pmatrix} \cos 2\phi_k & \sin 2\phi_k \\ \sin 2\phi_k & -\cos 2\phi_k \end{pmatrix} \\
\xi_T^{ij} &= -\tau_3^{ij}\ \sin 2\phi_k + \tau_1^{ij}\ \cos 2\phi_k = \begin{pmatrix} -\sin 2\phi_k & \cos 2\phi_k \\ \cos 2\phi_k & \sin 2\phi_k \end{pmatrix} \\
\epsilon_T^{ij} &= i\tau_2^{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
\end{aligned}
$$

- In Mulders-Rodrigues 5 tensors for the classification of gluon TMD, including

$$
\omega_T^{ij} = 2\left[\xi_T^{ij}\, \mathbf{S}_T \mathbf{k}_\perp + \eta_T^{ij}\ e^{\mathbf{S}_T \mathbf{k}_\perp}\right] = 2\,|\mathbf{S}_T|\,|\mathbf{k}_\perp| \begin{pmatrix} -\sin\delta & \cos\delta \\ \cos\delta & \sin\delta \end{pmatrix}, \qquad \delta = \phi + \phi_k
$$

Gluon TMDs in QCD

- Exclusion of ω_T^{ij} from expansion of gluon correlator has several advantages:

  (i) Reduction the number of tensors ($5 \to 4$) involved in the expansion

  (ii) $\omega_T^{\mu\nu}$ involves transverse spin, while other 4 tensors ($g_T^{\mu\nu}$, $\eta_T^{\mu\nu}$, $\xi_T^{\mu\nu}$, $\epsilon_T^{\mu\nu}$) are manifestly independent on S_T

  (iii) Substitution of $\omega_T^{\mu\nu}$ (linear combination of $\eta_T^{\mu\nu}$ and $\xi_T^{\mu\nu}$) gives separation of T-odd transversity TMDs with L-polarized gluons in T-polarized nucleon:

  - Symmetric transversity TMD $h_{1T}^{+g}(x, \mathbf{k}_\perp^2)$ standing at structure $\xi_T^{\mu\nu}\, \mathbf{S}_T \mathbf{k}_\perp$ symmetric under $\mathbf{S}_T \leftrightarrow \mathbf{k}_\perp$ interchange

  - Antisymmetric transversity TMD $h_{1T}^{-g}(x, \mathbf{k}_\perp^2)$ standing at $\eta_T^{\mu\nu}\, e^{\mathbf{S}_T \mathbf{k}_\perp}$ structure antisymmetric under $\mathbf{S}_T \leftrightarrow \mathbf{k}_\perp$ interchange

Gluon TMDs in QCD

- Mulders-Rodrigues: Gluon correlator $\Gamma^{ij}(x, \mathbf{k}_\perp, \mathbf{S}) = \sum_{P=U,L,T} \Gamma_P^{ij}(x, \mathbf{k}_\perp, \mathbf{S})$

- Γ_U^{ij} (U-polarized nucleon): U-polarized $f_1^g(x, \mathbf{k}_\perp^2)$ and L-polarized $h_1^{\perp g}(x, \mathbf{k}_\perp^2)$

$$\Gamma_U^{ij} = -g_T^{ij}\, f_1^g + \eta_T^{ij}\, h_1^{(1)\perp g}$$

- Γ_L^{ij} (L-polarized nucleon): C-polarized $g_{1L}^g(x, \mathbf{k}_\perp^2)$ and L-polarized $h_{1L}^{\perp g}(x, \mathbf{k}_\perp^2)$

$$\Gamma_L^{ij} = -i\epsilon_T^{ij}\, S_L\, g_{1L}^g + \xi_T^{ij}\, S_L\, h_{1L}^{(1)\perp g}$$

- Γ_T^{ij} (T-polarized nucleon): U-polarized $f_{1T}^{\perp g}(x, \mathbf{k}_\perp^2)$, C-polarized $g_{1T}^g(x, \mathbf{k}_\perp^2)$ and two L-polarized $h_{1T}^{+g}(x, \mathbf{k}_\perp^2)$ and $h_{1T}^{-g}(x, \mathbf{k}_\perp^2)$

$$\Gamma_T^{ij} = -g_T^{ij}\, \frac{e^{\mathbf{S}_T \mathbf{k}_\perp}}{M_N}\, f_{1T}^{\perp g} - i\epsilon_T^{ij}\, \frac{\mathbf{S}_T \mathbf{k}_\perp}{M_N}\, g_{1T}^g + \xi_T^{ij}\, \frac{\mathbf{S}_T \mathbf{k}_\perp}{M_N}\, h_{1T}^{+g} + \eta_T^{ij}\, \frac{e^{\mathbf{S}_T \mathbf{k}_\perp}}{M_N}\, h_{1T}^{-g}$$

$$\mathrm{TMD}^{(1/2)}(x, \mathbf{k}_\perp^2) = \frac{|\mathbf{k}_\perp|}{M_N}\, \mathrm{TMD}(x, \mathbf{k}_\perp^2)\,, \qquad \mathrm{TMD}^{(n)}(x, \mathbf{k}_\perp^2) = \left[\frac{\mathbf{k}_\perp^2}{2M_N^2}\right]^n \mathrm{TMD}(x, \mathbf{k}_\perp^2)$$

Gluon TMDs in QCD

- Our decomposition

$$\Gamma_T^{ij} = -g_T^{ij}\, \frac{e^{\mathbf{S}_T \mathbf{k}_\perp}}{M_N}\, f_{1T}^{\perp g} - i\epsilon_T^{ij}\, \frac{\mathbf{S}_T \mathbf{k}_\perp}{M_N}\, g_{1T}^{g} + \xi_T^{ij}\, \frac{\mathbf{S}_T \mathbf{k}_\perp}{M_N}\, h_{1T}^{+g} + \eta_T^{ij}\, \frac{e^{\mathbf{S}_T \mathbf{k}_\perp}}{M_N}\, h_{1T}^{-g}$$

- Comparison with Mulders-Rodrigues (MR): our L-polarized gluon TMDs $h_{1T}^{\pm g}$ are related to the corresponding MR TMDs – linearity ΔH_T and pretzelosity $\Delta H_T^\perp$

$$h_{1T}^{\pm g}(x, \mathbf{k}_\perp^2) = -\frac{1}{2}\left[\Delta H_T(x, \mathbf{k}_\perp^2) \pm \frac{\mathbf{k}_\perp^2}{2M_N^2}\, \Delta H_T^\perp(x, \mathbf{k}_\perp^2)\right]$$

- Comparison with Boer et al: analogous sets $(h_{1T}^{g},\, h_{1T}^{\perp g})$ and $(h_1,\, h_{1T},\, h_{1T}^\perp)$ used in PRL116, 122001 (2016) and JHEP10, 13 (2016)

$$\begin{aligned} h_{1T}^{+g}(x, \mathbf{k}_\perp^2) &= h_{1T}^{g}(x, \mathbf{k}_\perp^2) - h_{1T}^{\perp g}(x, \mathbf{k}_\perp^2) = \frac{1}{2}\left[h_1(x, \mathbf{k}_\perp^2) + \frac{\mathbf{k}_\perp^2}{2M_N^2} h_{1T}^\perp(x, \mathbf{k}_\perp^2)\right] \\ h_{1T}^{-g}(x, \mathbf{k}_\perp^2) &= h_{1T}^{g}(x, \mathbf{k}_\perp^2) = \frac{1}{2}\left[h_1(x, \mathbf{k}_\perp^2) - \frac{\mathbf{k}_\perp^2}{2M_N^2} h_{1T}^\perp(x, \mathbf{k}_\perp^2)\right] = \frac{1}{2} h_{1T}(x, \mathbf{k}_\perp^2) \end{aligned}$$

Gluon TMDs in QCD

- Full gluon correlation tensor in the more compact form:

$$
\begin{aligned}
\Gamma^{ij}(x,\mathbf{k}_\perp,\mathbf{S}) &= -g_T^{ij}\,F_1^g(x,\mathbf{k}_\perp^2;\mathbf{S}_T) - i\epsilon_T^{ij}\,\mathbf{S}\,\mathbf{G}_1^g(x,\mathbf{k}_\perp^2) \\
&+ \eta_T^{ij}\,H_1^{(\eta)g}(x,\mathbf{k}_\perp^2;\mathbf{S}_T) + \xi_T^{ij}\,\mathbf{S}\,\mathbf{H}_1^{(\xi)g}(x,\mathbf{k}_\perp^2)
\end{aligned}
$$

where

$$
\begin{aligned}
F_1^g(x,\mathbf{k}_\perp^2;\mathbf{S}_T) &= f_1^g(x,\mathbf{k}_\perp^2) + \frac{e^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\,f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) && U - \text{polarized gluon} \\
\mathbf{G}_1^g(x,\mathbf{k}_\perp^2) &= \left(g_{1L}^g(x,\mathbf{k}_\perp^2), \frac{\mathbf{k}_\perp}{M_N} g_{1T}^g(x,\mathbf{k}_\perp^2)\right) && C - \text{polarized gluon} \\
H_1^{(\eta)g}(x,\mathbf{k}_\perp^2;\mathbf{S}_T) &= h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) + \frac{e^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\,h_{1T}^{-g}(x,\mathbf{k}_\perp^2) && L - \text{polarized gluon} \\
\mathbf{H}_1^{(\xi)g}(x,\mathbf{k}_\perp^2) &= \left(h_{1L}^{(1)\perp g}(x,\mathbf{k}_\perp^2), \frac{\mathbf{k}_\perp}{M_N}\,h_{1T}^{+g}(x,\mathbf{k}_\perp^2)\right) && L - \text{polarized gluon}
\end{aligned}
$$

and $\mathbf{S} = (S_L, \mathbf{S}_T)$

Gluon TMDs in QCD

- Following Mulders-Rodrigues expand the gluon tensor in the nucleon spin basis

$$\Gamma^{ii'}(x, \mathbf{k}_\perp, \mathbf{S}) = \sum_{\Lambda,\Lambda'} \rho_{\Lambda'\Lambda}(\mathbf{S})\, \Gamma^{ii'}_{\Lambda\Lambda'}(x, \mathbf{k}_\perp)\,, \qquad \rho(\mathbf{S}) = \frac{1}{2}\Big(\mathbf{1} + \mathbf{S}\,\sigma\Big)$$

- $4 \otimes 4$ matrix $\Gamma^{ii'}_{\Lambda\Lambda'}$ in the gluon-nucleon circular basis

$$\begin{pmatrix} F_1^+ & F_{1T}^+ & H_1^- & \Delta H_{1T} \\ \Big(F_{1T}^+\Big)^\dagger & F_1^- & H_{1T} & H_1^+ \\ \Big(H_1^-\Big)^\dagger & \Big(H_{1T}\Big)^\dagger & F_1^- & F_{1T}^- \\ \Big(\Delta H_{1T}\Big)^\dagger & \Big(H_1^+\Big)^\dagger & \Big(F_{1T}^-\Big)^\dagger & F_1^+ \end{pmatrix}$$

$$F_1^\pm = f_1^g \pm g_{1L}^g\,, \quad F_{1T}^\pm = \pm\frac{|\mathbf{k}_\perp|}{M_N}\, e^{-i\phi_k}\left[g_{1T}^g \pm i f_{1T}^{\perp g}\right], \quad H_1^\pm = -e^{-2i\phi_k}\left[h_1^{(1)\perp g} \pm i h_{1L}^{(1)\perp g}\right],$$

$$H_{1T} = \frac{i|\mathbf{k}_\perp|}{M_N}\, e^{-i\phi_k}\left[h_{1T}^{+g} + h_{1T}^{-g}\right], \quad \Delta H_{1T} = \frac{i|\mathbf{k}_\perp|}{M_N}\, e^{-3i\phi_k}\left[h_{1T}^{+g} - h_{1T}^{-g}\right]$$

Gluon TMDs in QCD

- Small-x behavior Boer et al, PRL116, 122001 (2016) and JHEP10, 13 (2016)
- U-polarized tensor

$$\frac{x}{2}\Gamma_U^{ij}(x,\mathbf{k}_\perp) \overset{x\to 0}{\longrightarrow} \frac{\mathbf{k}_\perp^i\,\mathbf{k}_\perp^j}{2M_N^2}\,e_U(\mathbf{k}_\perp^2) = \frac{1}{2}\left[-g_T^{ij}\,\frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_U(\mathbf{k}_\perp^2) + \eta_T^{ij}\,\frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_U(\mathbf{k}_\perp^2)\right]$$

- Leading to the identity

$$\lim_{x\to 0}\,x f_1^g(x,\mathbf{k}_\perp^2) = \lim_{x\to 0}\,x h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) = \frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_U(\mathbf{k}_\perp^2) = e_U^{(1)}(\mathbf{k}_\perp^2)$$

$e_U(\mathbf{k}_\perp^2)$ – scalar function defining U-part of g Wilson loop LF correlator at small x

- L-polarized tensor

$$\frac{x}{2}\Gamma_L^{ij}(x,\mathbf{k}_\perp) = 0$$

- Leading to the vanishing of the corresponding TMDs:

$$\lim_{x\to 0}\,x g_{1L}^g(x,\mathbf{k}_\perp^2) = 0\,, \qquad \lim_{x\to 0}\,x h_{1L}^{\perp g}(x,\mathbf{k}_\perp^2) = 0$$

Gluon TMDs in QCD

- T-polarized tensor

$$\frac{x}{2}\Gamma_T^{ij}(x,\mathbf{k}_\perp) \quad \xrightarrow{x\to 0} \quad \frac{\mathbf{k}_\perp^i\mathbf{k}_\perp^j}{2M_N^2}\,\frac{\epsilon_T^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\,e_T(\mathbf{k}_\perp^2) = \frac{\epsilon_T^{\mathbf{S}_T\mathbf{k}_\perp}}{2M_N}\left[-g_T^{ij}\,\frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_T(\mathbf{k}_\perp^2)\right.$$

$$\left. + \quad \eta_T^{ij}\,\frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_T(\mathbf{k}_\perp^2)\right],$$

$e_T(\mathbf{k}_\perp^2)$ – scalar function defining T-part of g Wilson loop LF correlator at small x

- It follows

$$\begin{aligned}
\lim_{x\to 0} x f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) &= \lim_{x\to 0} x h_{1T}^{-g}(x,\mathbf{k}_\perp^2) = \lim_{x\to 0} x h_1(x,\mathbf{k}_\perp^2) \\
&= -\frac{\mathbf{k}_\perp^2}{2M_N^2}\lim_{x\to 0} x h_{1T}^{\perp}(x,\mathbf{k}_\perp^2) = \frac{1}{2}\lim_{x\to 0} x h_{1T}(x,\mathbf{k}_\perp^2) \\
&= \frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_T(\mathbf{k}_\perp^2) = e_T^{(1)}(\mathbf{k}_\perp^2)
\end{aligned}$$

and

$$\lim_{x\to 0} x h_{1T}^{+g}(x,\mathbf{k}_\perp^2) = 0$$

Gluon TMDs in LF QCD

- Following Brodsky-Hwang-Ma-Schmidt, NPB593, 311 (2001)
- We derive LFWFs $\psi^{\lambda_N}_{\lambda_g;\lambda_X}(x, \mathbf{k}_\perp)$ for bound state of gluon g and 3q spectator X with helicities $\lambda_N = \uparrow, \downarrow$, $\lambda_g = \pm 1$, $\lambda_X = \pm\frac{1}{2}$:

$$\begin{aligned}
\psi^{\uparrow}_{+1+\frac{1}{2}}(x, \mathbf{k}_\perp) &= -\Big[\psi^{\downarrow}_{-1-\frac{1}{2}}(x, \mathbf{k}_\perp)\Big]^\dagger = \frac{k^1 - ik^2}{\kappa}\,\varphi^{(2)}(x, \mathbf{k}_\perp)\,, \\
\psi^{\uparrow}_{+1-\frac{1}{2}}(x, \mathbf{k}_\perp) &= +\Big[\psi^{\downarrow}_{-1+\frac{1}{2}}(x, \mathbf{k}_\perp)\Big]^\dagger = \varphi^{(1)}(x, \mathbf{k}_\perp^2)\,, \\
\psi^{\uparrow}_{-1+\frac{1}{2}}(x, \mathbf{k}_\perp) &= -\Big[\psi^{\downarrow}_{+1-\frac{1}{2}}(x, \mathbf{k}_\perp)\Big]^\dagger = -\frac{k^1 + ik^2}{\kappa}\,(1-x)\,\varphi^{(2)}(x, \mathbf{k}_\perp^2)\,, \\
\psi^{\uparrow}_{-1-\frac{1}{2}}(x, \mathbf{k}_\perp) &= \psi^{\downarrow}_{+1+\frac{1}{2}}(x, \mathbf{k}_\perp) = 0\,,
\end{aligned}$$

$\varphi^{(1,2)}(x, \mathbf{k}_\perp)$ are expressed through the gluon PDF functions $G^\pm(x)$ as

$$\varphi^{(1)} = \frac{4\pi}{\kappa}\sqrt{G^+(x) - \frac{G^-(x)}{(1-x)^2}}\,\exp\Big[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\Big]\,, \quad \varphi^{(2)} = \frac{4\pi}{\kappa}\,\frac{\sqrt{G^-(x)}}{1-x}\,\exp\Big[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\Big]$$

$\kappa \sim 350 - 500$ MeV – dilaton scale parameter; x-dependent scale $\kappa(x) = \kappa D(x)$; $D(x)$ is fixed from scaling arguments

Gluon TMDs in LF QCD

- $G^+ = G_{g\uparrow/N\uparrow}$ and $G^- = G_{g\downarrow/N\uparrow}$ – helicity-aligned and antialigned gluon PDFs.
- Gluon unpolarized $G = G^+ + G^-$ and polarized $\Delta G = G^+ - G^-$ PDFs.
- G and ΔG are expressed in terms of derived LFWFs $\psi^{\lambda_N}_{\lambda_g;\lambda_X}(x, \mathbf{k}_\perp)$

$$\left(\begin{array}{r} G(x) \\ \Delta G(x) \end{array} \right) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[|\psi^{+}_{+1+\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 + |\psi^{+}_{+1-\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 \pm |\psi^{+}_{-1+\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 \right]$$

- First calculation in QCD – Brodsky, Schmidt, PLB234, 144 (1990):

$$G^+(x) = N_g \, (1-x)^4 \, (1+4x)/x, \quad G^-(x) = N_g \, (1-x)^6/x$$

$N_g = 0.8967$ fixed from 1st moment $\langle x_g \rangle = \int_0^1 dx \, x \, G(x) = (10/21) \, N_g$

- Lattice result: $\langle x_g \rangle = 0.427$ Alexandrou et al, PRD101, 094513 (2020).
- We are not strict to any explicit form of gluon PDFs and one can use results of world data analysis obey very important model-independent constraints:

Gluon TMDs in LF QCD

- Gluon correlator $\Gamma_{\lambda\lambda';\Lambda\Lambda'}(x, \mathbf{k}_\perp)$ in LF QCD reads:

$$\Gamma_{\lambda\lambda';\Lambda\Lambda'}(x, \mathbf{k}_\perp) = \sum_{i=1}^{8} \Gamma^{(i)}_{\lambda,\lambda';\Lambda\Lambda'}(x, \mathbf{k}_\perp)$$

$$\Gamma^{(i)}_{\lambda,\lambda';\Lambda\Lambda'}(x, \mathbf{k}_\perp) = \psi^{\dagger\Lambda_1}_{\lambda_1\lambda_X}(x, \mathbf{k}_\perp) \, \frac{G^{\lambda_1\lambda_2\Lambda_1\Lambda_2\lambda_X\lambda'_X;(i)}_{\lambda,\lambda';\Lambda\Lambda'}(x, \mathbf{k}_\perp)}{32\pi^3} \, \psi^{\Lambda_2}_{\lambda_2\lambda'_X}(x, \mathbf{k}_\perp)$$

where $G^{(i)}$ are interaction kernels including both T-even and T-odd structures

- T-odd TMDs contain loop functions $R_{\rm TMD}(x, \mathbf{k}^2_\perp)$ encoding $g - 3q$ rescattering

- Factorization

$$\psi^\dagger(x, \mathbf{k}_\perp) \int d^2\mathbf{k}'_\perp F_{\rm TMD}(x, \mathbf{k}_\perp, \mathbf{k}'_\perp) \, \psi(x, \mathbf{k}'_\perp) = \psi^\dagger(x, \mathbf{k}_\perp) \, R_{\rm TMD}(x, \mathbf{k}^2_\perp) \, \psi(x, \mathbf{k}_\perp)$$

Gluon TMDs in LF QCD

- Tensorial structures

$$
\begin{aligned}
G^{(1)} &= \delta_{\lambda\lambda'}\,\delta_{\Lambda\Lambda'}\,\delta_{\lambda_1\lambda_2}\,\delta_{\Lambda_2\Lambda_1}\,\delta_{\lambda_X\lambda'_X} \\
G^{(2)} &= \tau^3_{\lambda\lambda'}\,\sigma^3_{\Lambda\Lambda'}\,\tau^3_{\lambda_1\lambda_2}\,\sigma^3_{\Lambda_2\Lambda_1}\,\delta_{\lambda_X\lambda'_X} \\
G^{(3)} &= \tau^3_{\lambda\lambda'}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda\Lambda'}}{M_N}\,\tau^3_{\lambda_1\lambda_2}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda_2\Lambda'}}{M_N}\,\delta_{\lambda_X\lambda'_X} \\
G^{(4)} &= \eta_{\lambda\lambda'}\,\delta_{\Lambda\Lambda'}\,\eta_{\lambda_1\lambda_2}\,\delta_{\Lambda_2\Lambda_1}\,\delta_{\lambda_X\lambda'_X} \\
G^{(5)} &= \xi_{\lambda\lambda'}\,\sigma^3_{\Lambda\Lambda'}\,(\tau^3\xi)_{\lambda_1\lambda_2}\,\sigma^3_{\Lambda_2\Lambda_1}\,\tau^3_{\lambda_X\lambda'_X}\,iR_{h^g_{1L}}(x,\mathbf{k}^2_\perp) \\
G^{(6)} &= \xi_{\lambda\lambda'}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda\Lambda'}}{M_N}\,(\xi\tau^3)_{\lambda_1\lambda_2}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda_2\Lambda_1}}{M_N}\,\tau^3_{\lambda_X\lambda'_X}\,iR_{h^{+g}_{1T}}(x,\mathbf{k}^2_\perp) \\
G^{(7)} &= \eta_{\lambda\lambda'}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp})_{\Lambda\Lambda'}}{M_N}\eta_{\lambda_1\lambda_2}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp}\sigma^3)_{\Lambda_2\Lambda_1}}{M_N}\tau^3_{\lambda_X\lambda'_X}\,iR_{h^{-g}_{1T}}(x,\mathbf{k}^2_\perp) \\
G^{(8)} &= \delta_{\lambda\lambda'}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp})_{\Lambda\Lambda'}}{M_N}\delta_{\lambda_1\lambda_2}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp}\sigma^3)_{\Lambda_2\Lambda_1}}{M_N}\tau^3_{\lambda_X\lambda'_X}\,iR_{f^{\perp g}_{1T}}(x,\mathbf{k}^2_\perp)
\end{aligned}
$$

Gluon TMDs in LF QCD

- Tensorial structures generate eight leading-twist gluon TMDs

$$
\begin{aligned}
\delta_{\lambda\lambda'}\,\delta_{\Lambda\Lambda'}\,f_1^g(x,\mathbf{k}_\perp^2) &= \Gamma^{(1)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)\\
\tau^3_{\lambda\lambda'}\,\sigma^3_{\Lambda\Lambda'}\,g^g_{1L}(x,\mathbf{k}_\perp^2) &= \Gamma^{(2)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)\\
\tau^3_{\lambda\lambda'}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda\Lambda'}}{M_N}\,g^g_{1T}(x,\mathbf{k}_\perp^2) &= \Gamma^{(3)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)\\
\eta_{\lambda\lambda'}\,\delta_{\Lambda\Lambda'}\,h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) &= \Gamma^{(4)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)\\
\xi_{\lambda\lambda'}\,\sigma^3_{\Lambda\Lambda'}\,h_{1L}^{(1)\perp g}(x,\mathbf{k}_\perp^2) &= \Gamma^{(5)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)\\
\xi_{\lambda\lambda'}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda\Lambda'}}{M_N}\,h^{+g}_{1T}(x,\mathbf{k}_\perp^2) &= \Gamma^{(6)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)\\
\eta_{\lambda\lambda'}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp})_{\Lambda\Lambda'}}{M_N}\,h^{-g}_{1T}(x,\mathbf{k}_\perp^2) &= \Gamma^{(7)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)\\
\delta_{\lambda\lambda'}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp})_{\Lambda\Lambda'}}{M_N}\,f^g_{1T}(x,\mathbf{k}_\perp^2) &= \Gamma^{(8)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)
\end{aligned}
$$

Gluon TMDs in LF QCD

- Analytical expressions for the gluon TMDs in terms of LFWFs are:
- T-even TMDs

$$
\begin{aligned}
f_1^g(x,\mathbf{k}_\perp^2) &= \frac{1}{16\pi^3}\left[\left(\varphi^{(1)}(x,\mathbf{k}_\perp^2)\right)^2 + \frac{\mathbf{k}_\perp^2}{M_N^2}\left[1+(1-x)^2\right]\left(\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right)^2\right] \\
g_{1L}^g(x,\mathbf{k}_\perp^2) &= \frac{1}{16\pi^3}\left[\left(\varphi^{(1)}(x,\mathbf{k}_\perp^2)\right)^2 + \frac{\mathbf{k}_\perp^2}{M_N^2}\left[1-(1-x)^2\right]\left(\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right)^2\right] \\
g_{1T}^g(x,\mathbf{k}_\perp^2) &= \frac{1}{8\pi^3}\,\varphi^{(1)}(x,\mathbf{k}_\perp^2)\,\varphi^{(2)}(x,\mathbf{k}_\perp^2)\,(1-x) \\
h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) &= \frac{1}{8\pi^3}\,\frac{\mathbf{k}_\perp^2}{M_N^2}\left[\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right]^2(1-x)
\end{aligned}
$$

- T-odd TMDs

$$
\begin{aligned}
h_{1L}^{(1)\perp g}(x,\mathbf{k}_\perp^2) &= \frac{1}{8\pi^3}\,\frac{\mathbf{k}_\perp^2}{M_N^2}\left[\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right]^2(1-x)\,R_{h_{1L}^{(1)\perp g}}(x,\mathbf{k}_\perp^2) \\
h_{1T}^{\pm g}(x,\mathbf{k}_\perp^2) &= \frac{1}{8\pi^3}\,\varphi^{(1)}(x,\mathbf{k}_\perp^2)\,\varphi^{(2)}(x,\mathbf{k}_\perp^2)\,R_{h_{1T}^{\pm g}}(x,\mathbf{k}_\perp^2) \\
f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) &= \frac{1}{8\pi^3}\,\varphi^{(1)}(x,\mathbf{k}_\perp^2)\,\varphi^{(2)}(x,\mathbf{k}_\perp^2)\,(1-x)\,R_{f_{1T}^{\perp g}}(x,\mathbf{k}_\perp^2)
\end{aligned}
$$

Gluon TMDs in LF QCD

- T-odd in terms of T-even without referring to specific choice of $\varphi^{(1,2)}(x, \mathbf{k}_\perp^2)$

$$
\begin{aligned}
h_{1L}^{(1)\perp g}(x, \mathbf{k}_\perp^2) &= \frac{f_1^g(x, \mathbf{k}_\perp^2) - g_{1L}^g(x, \mathbf{k}_\perp^2)}{1-x} \, R_{h_{1L}^{(1)\perp g}}(x, \mathbf{k}_\perp^2) \\
&= h_1^{(1)\perp g}(x, \mathbf{k}_\perp^2) \, R_{h_{1L}^{(1)\perp g}}(x, \mathbf{k}_\perp^2)
\end{aligned}
$$

$$
h_{1T}^{\pm g}(x, \mathbf{k}_\perp^2) = \frac{g_{1T}^g(x, \mathbf{k}_\perp^2)}{1-x} \, R_{h_{1T}^{\pm g}}(x, \mathbf{k}_\perp^2)
$$

$$
f_{1T}^{\perp g}(x, \mathbf{k}_\perp^2) = g_{1T}^g(x, \mathbf{k}_\perp^2) \, R_{f_{1T}^{\perp g}}(x, \mathbf{k}_\perp^2)
$$

Gluon TMDs in LF QCD

- Next, using our parametrization for the LFWFs one gets

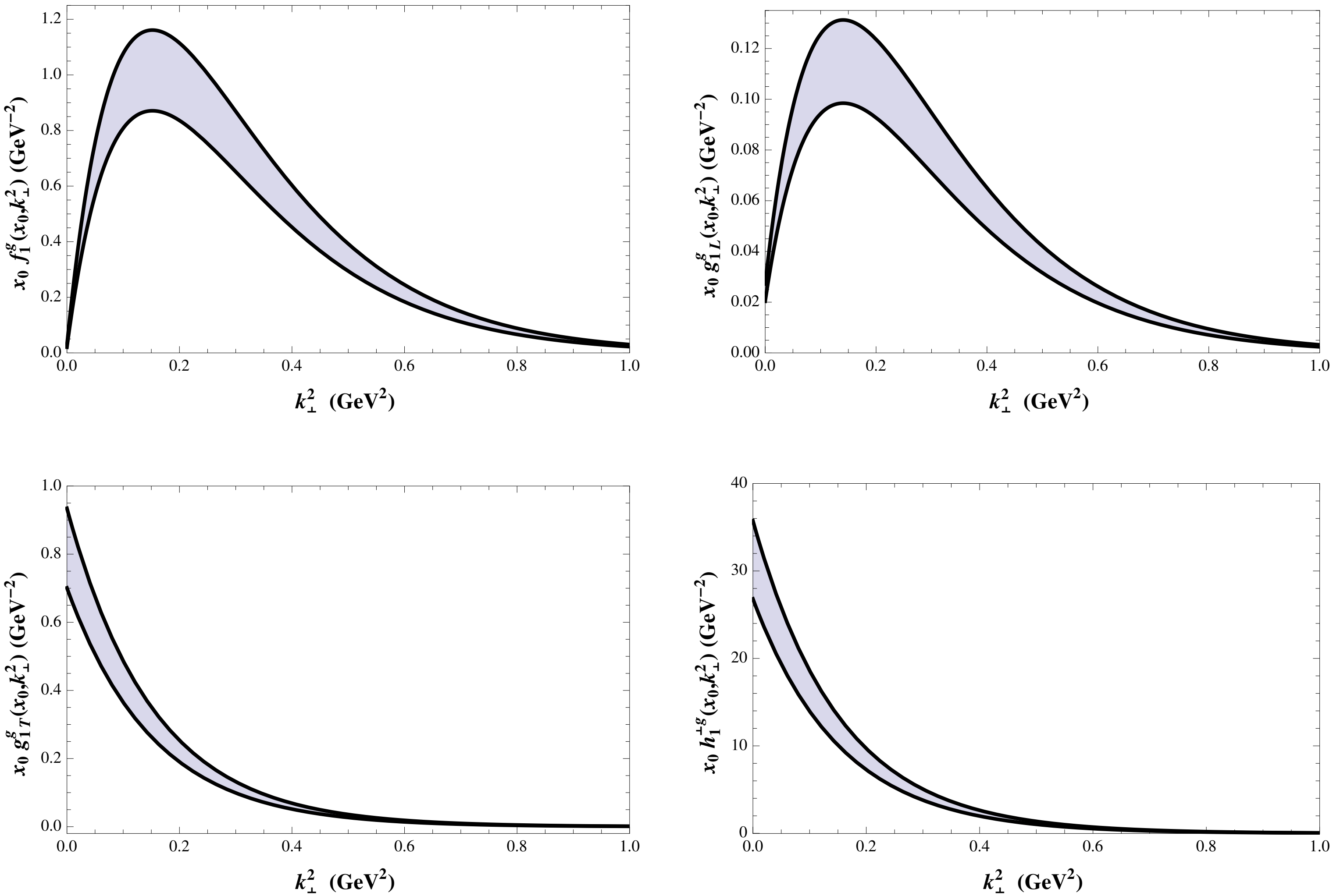
$$
\begin{aligned}
f_1^g(x,\mathbf{k}_\perp^2) &= \frac{1}{\pi\kappa^2}\left[G(x)+G^-(x)\,\alpha_+(x)\left(\frac{\mathbf{k}_\perp^2}{\kappa^2}-1\right)\right]\,\exp\left[-\frac{\mathbf{k}_\perp^2}{\kappa^2}\right] \\
g_{1L}^g(x,\mathbf{k}_\perp^2) &= \frac{1}{\pi\kappa^2}\left[\Delta G(x)+G^-(x)\,\alpha_-(x)\left(\frac{\mathbf{k}_\perp^2}{\kappa^2}-1\right)\right]\,\exp\left[-\frac{\mathbf{k}_\perp^2}{\kappa^2}\right] \\
g_{1T}^g(x,\mathbf{k}_\perp^2) &= \frac{M_N}{\pi\kappa^3}\sqrt{G^2(x)-\Delta G^2(x)}\;\beta(x)\,\exp\left[-\frac{\mathbf{k}_\perp^2}{\kappa^2}\right] \\
h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) &= \frac{\mathbf{k}_\perp^2}{\pi\kappa^4}\,\frac{G(x)-\Delta G(x)}{1-x}\;\exp\left[-\frac{\mathbf{k}_\perp^2}{\kappa^2}\right]
\end{aligned}
$$

where

$$
\alpha_\pm(x)=\frac{1\pm(1-x)^2}{(1-x)^2}\,,\quad \beta(x)=\sqrt{1-\frac{G^-(x)}{G^+(x)(1-x)^2}}
$$

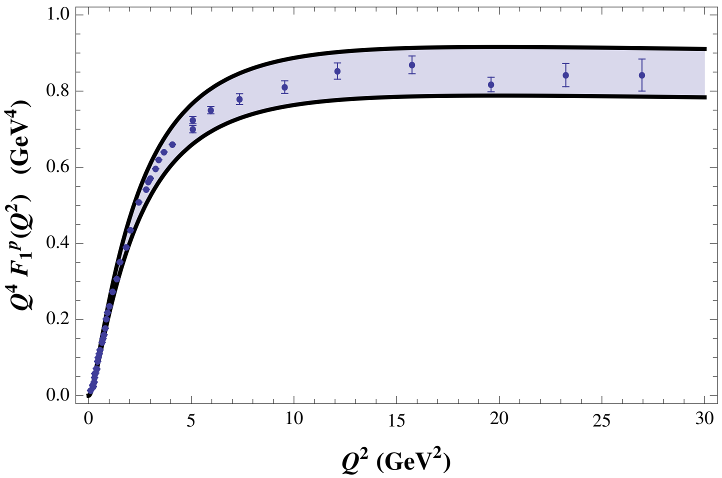
Gluon TMDs: selected results

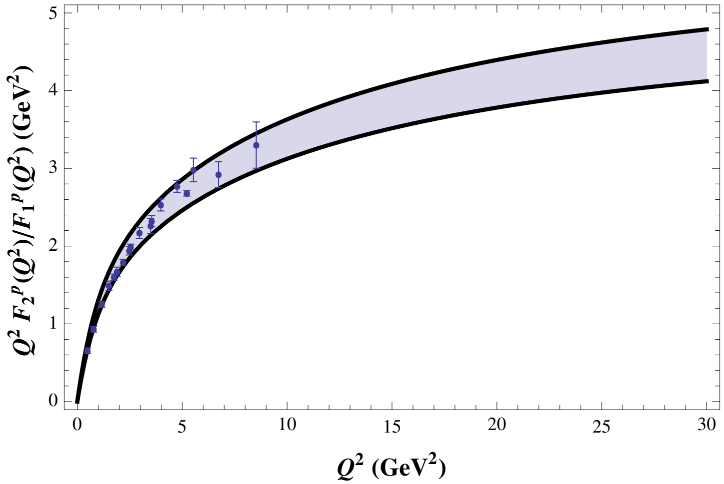
- T-even $x\,\mathrm{TMD}(x, \mathbf{k}_\perp^2)$ at $x = 0.1$ and for $\kappa = 380 \pm 30$ MeV
- Very good agreement with Pavia group, Bacchetta et al, EPJC80, 733 (2020)

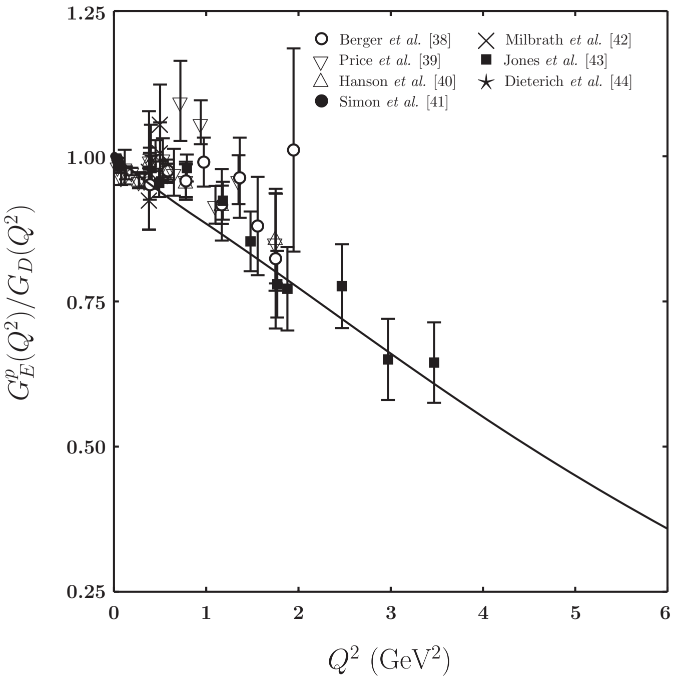


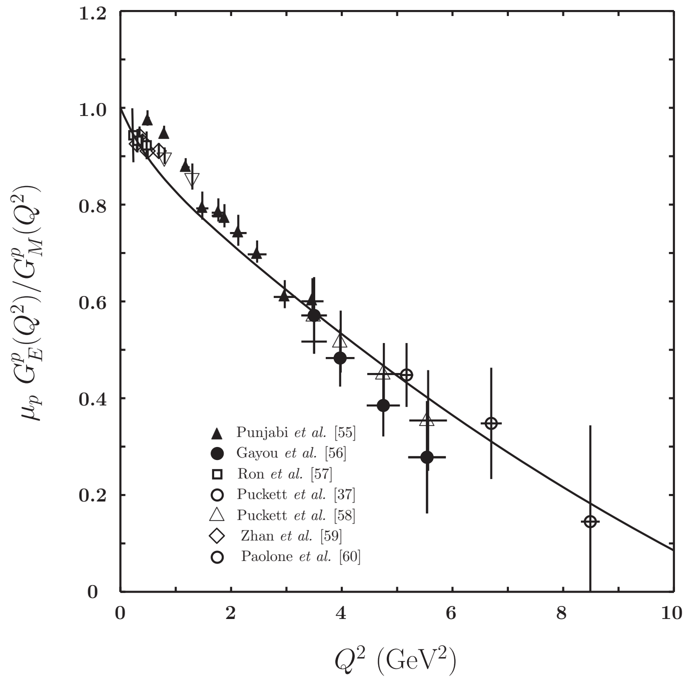
Nucleon EM: selected results

- $Q^4 F_1^p(Q^2)$, ratio $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$, ratio of Sachs FF









Sum Rules for TMDs

- Two sum rules for T-even TMDs without referring to explicit form of $\varphi^{(1,2)}(x, \mathbf{k}_\perp^2)$

$$\begin{aligned}\left[f_1^g(x,\mathbf{k}_\perp^2)\right]^2 &= \left[g_{1L}^g(x,\mathbf{k}_\perp^2)\right]^2 + \left[g_{1T}^{(1/2)g}(x,\mathbf{k}_\perp^2)\right]^2 + \left[h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2)\right]^2 \\ f_1^g(x,\mathbf{k}_\perp^2) - g_{1L}^g(x,\mathbf{k}_\perp) &= (1-x)\, h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2)\end{aligned}$$

- Square of the unpolarized TMD = Sum of the squares of three polarized TMDs
- These two SR are derived at α_s^0
- Consistent with Mulders-Rodrigues positivity bounds

$$\sqrt{\left[g_{1L}^g(x,\mathbf{k}_\perp^2)\right]^2 + \left[g_{1T}^{(1/2)g}(x,\mathbf{k}_\perp^2)\right]^2} \le f_1^g(x,\mathbf{k}_\perp^2)$$

$$\sqrt{\left[g_{1L}^g(x,\mathbf{k}_\perp^2)\right]^2 + \left[h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2)\right]^2} \le f_1^g(x,\mathbf{k}_\perp^2)$$

$$\sqrt{\left[g_{1T}^{(1/2)g}(x,\mathbf{k}_\perp^2)\right]^2 + \left[h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2)\right]^2} \le f_1^g(x,\mathbf{k}_\perp^2)$$

Sum Rules for TMDs

- Based on the SR derived for T-even gluon TMDs, we make a conjecture that there should two additional SRs involving T-odd gluon TMDs, valid at α_s and α_s^2
- We conjecture that the derived SRs are consequence of the condition

$$\det\left[\Gamma_{\lambda\lambda';\Lambda\Lambda'}\right] = 0$$

  signaling that the gluon TMDs are not independent and are related via SRs

- 3 SRs at orders $\mathcal{O}(1)$, $\mathcal{O}(\alpha_s)$, and $\mathcal{O}(\alpha_s^2)$
- From $\det\left[\Gamma_{\lambda\lambda';\Lambda\Lambda'}\right] = 0$ follows the condition

$$\left[R_0 + 2R_1 + R_2\right]\left[R_0 - 2R_1 + R_2\right] = 0$$

- R_0, R_1, R_2 are the combinations of TMDs at orders $\mathcal{O}(1)$, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2)$

Sum Rules for TMDs

- 3 Sum Rules

$$
\begin{aligned}
R_0 &= \left[g_{1L}^{g}\right]^2 + \left[g_{1T}^{(1/2)g}\right]^2 + \left[h_1^{(1)\perp g}\right]^2 - \left[f_1^{g}\right]^2 = 0 \\
R_1 &= f_1^{g}\, h_{1T}^{(1/2)-g} + g_{1L}^{g}\, h_{1T}^{(1/2)+g} - g_{1T}^{(1/2)g}\, h_{1L}^{(1)\perp g} - h_1^{(1)\perp g}\, f_{1T}^{(1/2)\perp g} = 0 \\
R_2 &= \left[f_{1T}^{(1/2)\perp g}\right]^2 + \left[h_{1L}^{(1)\perp g}\right]^2 + \left[h_{1T}^{(1/2)+g}\right]^2 - \left[h_{1T}^{(1/2)-g}\right]^2 = 0
\end{aligned}
$$

- 1st SR $R_0 = 0$ is exactly our SR involving four T-even TMDs
- 2nd SR $R_1 = 0$ couples T-even and T-odd TMDs
- 3rd SR $R_2 = 0$ involves only T-odd TMDs.

Small-x behavior of TMDs

- Another amazing result: SRs $R_i = 0$ at $x \to 0$ reduce to QCD results (Boer et al)
- In particular, at small x we get:

$$f_1^g = h_1^{(1)\perp g}\,, \quad h_{1T}^{(1/2)-g} = f_{1T}^{(1/2)\perp g}$$

$$g_{1L}^g = g_{1T}^g = h_{1T}^{(1/2)+g} = 0\,.$$

- Droping vanishing TMDs, the SRs $R_i = 0$ are simplified at small x as read

$$\begin{aligned} R_0 &= \left[h_1^{(1)\perp g}\right]^2 - \left[f_1^g\right]^2 = 0 \\ R_1 &= f_1^g\, h_{1T}^{(1/2)-g} - h_1^{(1)\perp g}\, f_{1T}^{(1/2)\perp g} = 0 \\ R_2 &= \left[f_{1T}^{(1/2)\perp g}\right]^2 - \left|h_{1T}^{(1/2)-g}\right]^2 = 0 \end{aligned}$$

- It means that at small x LF QCD is consistent with QCD

Large-x behavior of TMDs

- Large-x scaling: $f_1^g \sim g_{1L}^g \sim (1-x)^4, \quad g_{1T}^{(1/2)g} \sim h_1^{(1)\perp g} \sim (1-x)^5$
- Scaling of $f_{1T}^{(1/2)\perp g}$ and $h_{1L}^{(1)\perp g}$ is similar to $g_{1T}^{(1/2)g}$, up to corresponding loop factors $R_{h_{1L}^{(1)\perp g}}\Big|_{x\to 1}$ and $R_{f_{1T}^{\perp g}}\Big|_{x\to 1}$, expected to be constants or power of $(1-x)$
- Scaling of $h_{1T}^{(1/2)\pm g}$ is similar to f_1^g up to corresponding loop factor $R_{h_{1T}^{\pm g}}\Big|_{x\to 1}$, expected to be constant or power of $(1-x)$
- At large x SRs are simplified to

$$
\begin{aligned}
R_0 &= \left[g_{1L}^g\right]^2 - \left[f_1^g\right]^2 = 0 \\
R_1 &= f_1^g \left[h_{1T}^{(1/2)-g} + h_{1T}^{(1/2)+g}\right] = 0 \\
R_2 &= \left[h_{1T}^{(1/2)+g}\right]^2 - \left[h_{1T}^{(1/2)-g}\right]^2 = 0
\end{aligned}
$$

- From $R_1 = 0$ and $R_2 = 0$ follows $h_{1T}^{(1/2)+g} = -h_{1T}^{(1/2)-g}$

HQ contribution to nucleon properties in LF QCD

- Two nonperturbative Fock states $|uud + g\rangle$ (via $g \to Q\bar{Q}$ splitting) and $|uud + Q\bar{Q}\rangle$ produce intrinsic heavy content

(a) (b)

- Derive LFWFs effectively describing the heavy quark-antiquark content in the nucleon without specifying mechanism
- Later we consider interference of $|uud + Q\bar{Q}\rangle$ and $|uud + g\rangle$

HQ contribution to nucleon properties in LF QCD

- LFWFs for heavy struck quark distribution $\psi^{\lambda_N}_{Q;\lambda_Q}(x,\mathbf{k}_\perp)$:

$$
\begin{aligned}
\psi^{\uparrow}_{Q;+\frac{1}{2}}(x,\mathbf{k}_\perp) &= \varphi(x,\mathbf{k}_\perp) \\
\psi^{\uparrow}_{Q;-\frac{1}{2}}(x,\mathbf{k}_\perp) &= -\frac{k^1+ik^2}{\kappa}\,\varphi(x,\mathbf{k}_\perp) \\
\psi^{\downarrow}_{Q;+\frac{1}{2}}(x,\mathbf{k}_\perp) &= -\Big[\psi^{\uparrow}_{Q;-\frac{1}{2}}(x,\mathbf{k}_\perp)\Big]^\dagger = \frac{k^1-ik^2}{\kappa}\,\varphi(x,\mathbf{k}_\perp) \\
\psi^{\downarrow}_{Q;-\frac{1}{2}}(x,\mathbf{k}_\perp) &= \Big[\psi^{\uparrow}_{Q;\frac{1}{2}}(x,\mathbf{k}_\perp)\Big]^\dagger = \varphi(x,\mathbf{k}_\perp) \\
\varphi(x,\mathbf{k}_\perp) &= \frac{2\pi\sqrt{2}}{\kappa}\,\sqrt{Q_{\rm in}(x)}\,\exp\Big[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\Big]
\end{aligned}
$$

$Q_{\rm in}(x)$ is intrinsic heavy quark PDF expressed in terms of LFWFs as

$$
\begin{aligned}
Q_{\rm in}(x) &= \int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\Big[|\psi^{\uparrow}_{Q;+\frac{1}{2}}(x,\mathbf{k}_\perp)|^2+|\psi^{\uparrow}_{Q;-\frac{1}{2}}(x,\mathbf{k}_\perp)|^2\Big] \\
&= \int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\Big[|\psi^{\downarrow}_{Q;+\frac{1}{2}}(x,\mathbf{k}_\perp)|^2+|\psi^{\downarrow}_{Q;-\frac{1}{2}}(x,\mathbf{k}_\perp)|^2\Big]
\end{aligned}
$$

HQ contribution to nucleon properties in LF QCD

- QCD prediction: Brodsky et al, PRD23, 2745 (1981), PLB93, 451 (1980)

$$Q_{\rm in}(x) = N_{Q_{\rm in}}\, x^2 \left[(1-x)(1+10x+x^2) + 6x(1+x)\log(x)\right] \sim (1-x)^5$$

  where $N_{Q_{\rm in}}$ is the normalization constant fixed from data.

- From data $N_{c_{\rm in}} = 6$ corresponding 1% intrinsic charm contribution to proton PDF.
- In the case of bottom distribution $N_{b_{\rm in}} = 6\, m_c^2/m_b^2$
- Following Sufian et al, PLB808, 135633 (2020) we suppose that antiquark PDF $\bar{Q}_{\rm in}(x)$ has at least $(1-x)$ falloff at large x in comparison with $Q_{\rm in}(x)$; i.e., $\bar{Q}_{\rm in}(x) \sim (1-x)^6$
- To construct LFWF with struck heavy antiquark we replace heavy quark PDF by heavy antiquark PDF

$$Q_{\rm in}(x) \to \bar{Q}_{\rm in}(x) = N\,(1-x)\,Q_{\rm in}(x) \sim (1-x)^6$$

HQ contribution to nucleon properties in LF QCD

- Following PLB808, 135633 (2020), we introduce the asymmetric heavy-anti-heavy quark distribution function $Q_{\rm asym}(x) = Q_{\rm in}(x) - \bar{Q}_{\rm in}(x)$.

- Normalization constant $N = 7/5$ in $\bar{Q}_{\rm in}(x)$ is fixed using the condition that zero moments of the $Q_{\rm in}(x)$ and $\bar{Q}_{\rm in}(x)$ PDFs should be equal
or zero moment of the asymmetry PDF $Q_{\rm asym}(x)$ should vanish

- Finally, LFWFs with a heavy struck antiquark $\psi^{\lambda_N}_{\bar{Q};\lambda_{\bar{Q}}}(x, \mathbf{k}_\perp)$ are related to the corresponding LFWFs with heavy struck quark as:

$$\psi^{\lambda_N}_{\bar{Q};\lambda_{\bar{Q}}}(x, \mathbf{k}_\perp) = \sqrt{\frac{7}{5}(1-x)}\, \psi^{\lambda_N}_{Q;\lambda_Q}(x, \mathbf{k}_\perp)\,.$$

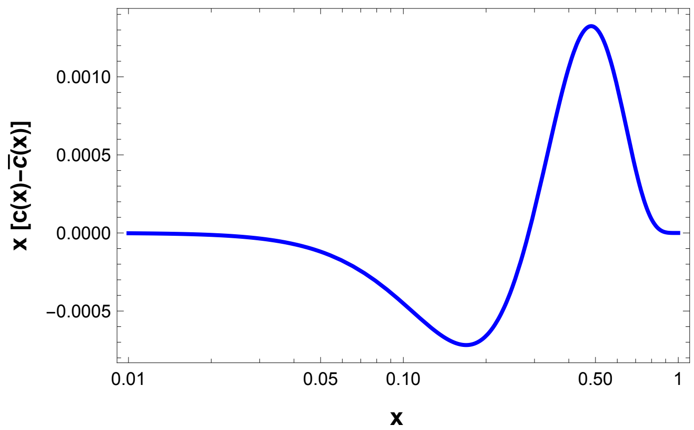
- Another quantity of interest is the sum of the heavy quark and antiquark PDFs

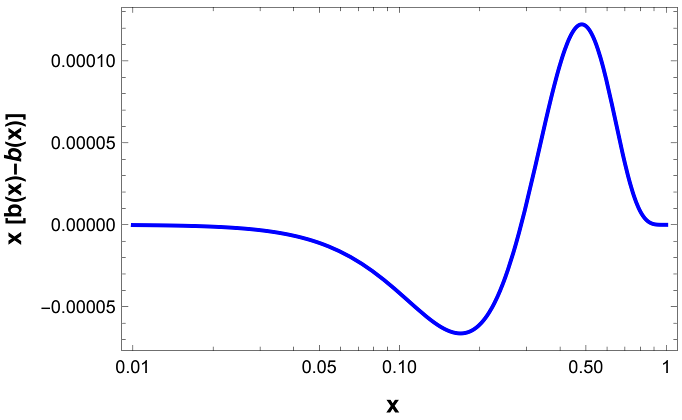
$$Q^+(x) = Q_{\rm in}(x) + \bar{Q}_{\rm in}(x)\,,$$

which was recently extracted by Ball et al (NNPDF Coll), Nature 608, 483 (2022)

HQ contribution to nucleon properties in LF QCD

- Results for $Q_{\rm asym}(x)$ are in very good agreement with the predictions of PLB808





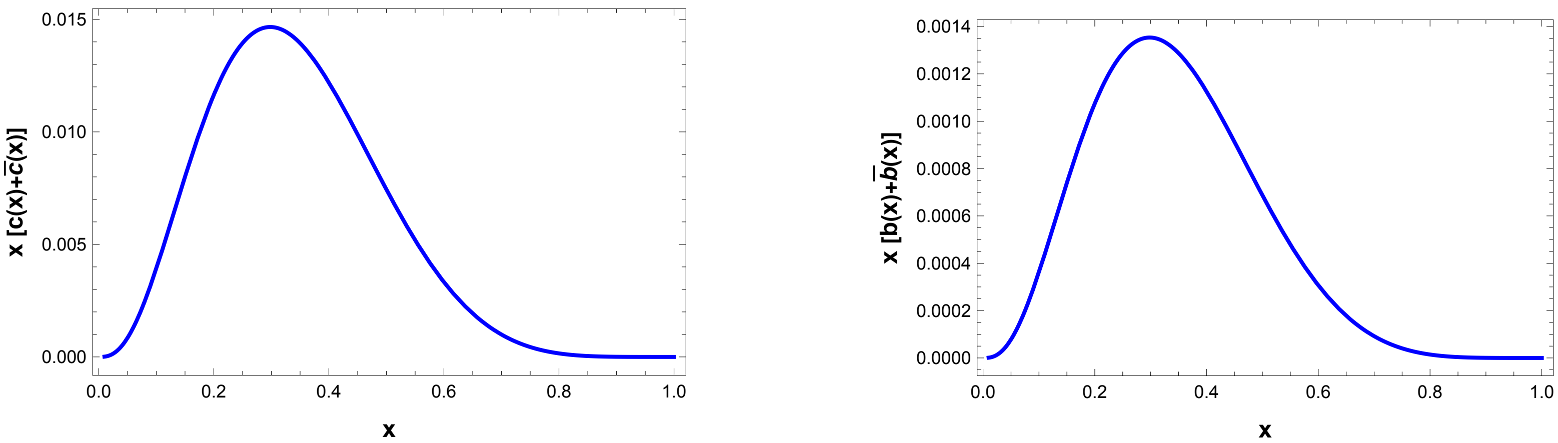
- 1st moment of the asymmetry $\langle x\rangle_{c-\bar{c}} = \int\limits_0^1 dx\, x\, c_{\rm asym}(x) = \dfrac{1}{3500} \simeq 0.00029$ is in order of magnitude agreement with prediction of PLB808: $\langle x\rangle_{c-\bar{c}} = 0.00047(15)$

- $\langle x\rangle_{b-\bar{b}}$ is suppressed as expected by a factor $(m_c/m_b)^2$

- Using $m_c = 1.27$ GeV and $m_b = 4.18$ GeV from PDG gives $\langle x\rangle_{b-\bar{b}} = 0.000026$

HQ contribution to nucleon properties in LF QCD

- $Q^+(x)$ in very good agreement with NNPDF Collaboration



- Difference: at $x = 0$ and small x our results are consistent with the physical meaning of PDFs: they are distributions, and must be positively defined quantities (both for heavy quark and antiquark), while the NNPDF results show negative distributions, due to uncertainties of the empirical extraction)

HQ contribution to nucleon properties in LF QCD

- Drell-Yan-West and Brodsky-Drell formulas for Dirac and Pauli EM FF

$$F_1^Q(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \sum_{\lambda_Q=\pm\frac{1}{2}} \psi^{\dagger\uparrow}_{Q;\lambda_Q}(x,\mathbf{k}'_\perp)\, \psi^{\uparrow}_{Q;\lambda_Q}(x,\mathbf{k}_\perp)$$

$$F_2^Q(Q^2) = -\frac{2M_N}{q^1 - iq^2} \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \sum_{\lambda_Q=\pm\frac{1}{2}} \psi^{\dagger\uparrow}_{Q;\lambda_Q}(x,\mathbf{k}'_\perp)\, \psi^{\downarrow}_{Q;\lambda_Q}(x,\mathbf{k}_\perp)$$

for Q and by analogy for $\bar{Q}$. Here $\mathbf{k}'_\perp = \mathbf{k}_\perp + \mathbf{q}_\perp(1-x)$.

- Heavy $Q-\bar{Q}$ form factors $F_i^{Q-\bar{Q}}(Q^2) = F_i^Q(Q^2) - F_i^{\bar{Q}}(Q^2)\,, \quad i = 1,2$

- $\mu^{Q-\bar{Q}}$ is directly related he 1st moment of the asymmetry distribution $\langle x \rangle_{Q-\bar{Q}}$

$$\mu^{Q-\bar{Q}} = F_2^{Q-\bar{Q}}(0) = \frac{M_N}{\kappa} \int_0^1 dx\, (1-x)\, Q_{\rm asym}(x) = -\frac{M_N}{\kappa}\, \langle x \rangle_{Q-\bar{Q}}$$

HQ contribution to nucleon properties in LF QCD

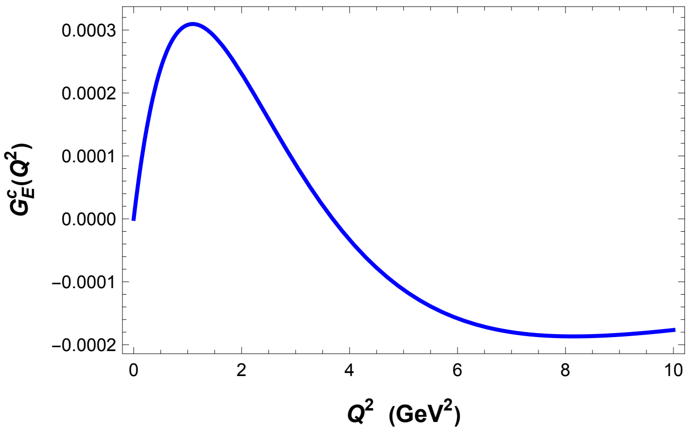
- Magnetic moment and radii of proton

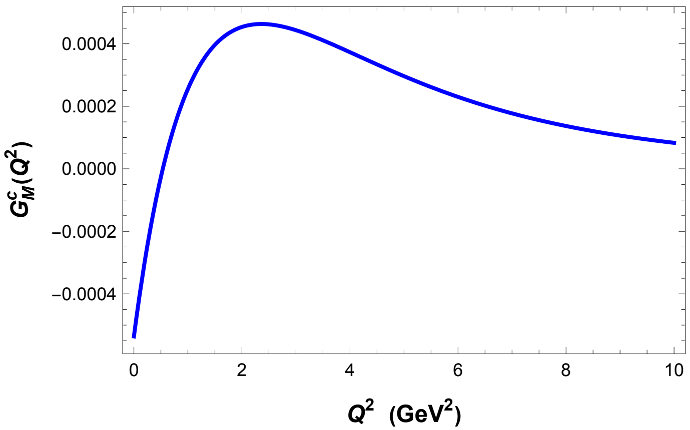
$$\mu^{c-\bar{c}} = -5.36 \times 10^{-4}\,, \qquad \mu^{b-\bar{b}} = -4.94 \times 10^{-5}\,,$$
$$\langle r_E^2 \rangle^{c-\bar{c}} = -1.70 \times 10^{-4}\ \mathrm{fm}^2\,, \qquad \langle r_M^2 \rangle^{c-\bar{c}} = -1.12 \times 10^{-4}\ \mathrm{fm}^2\,,$$
$$\langle r_E^2 \rangle^{b-\bar{b}} = -1.57 \times 10^{-5}\ \mathrm{fm}^2\,, \qquad \langle r_M^2 \rangle^{b-\bar{b}} = -1.03 \times 10^{-5}\ \mathrm{fm}^2\,.$$

- Scaling: $\mu^{b-\bar{b}}/\mu^{c-\bar{c}} = \langle r^2_{E,M}\rangle^{b-\bar{b}}/\langle r^2_{E,M}\rangle^{c-\bar{c}} \sim (m_c/m_b)^2 \sim 0.1$

- Results are of the same sign as in PLB808 but suppressed by 1 order of magnitude

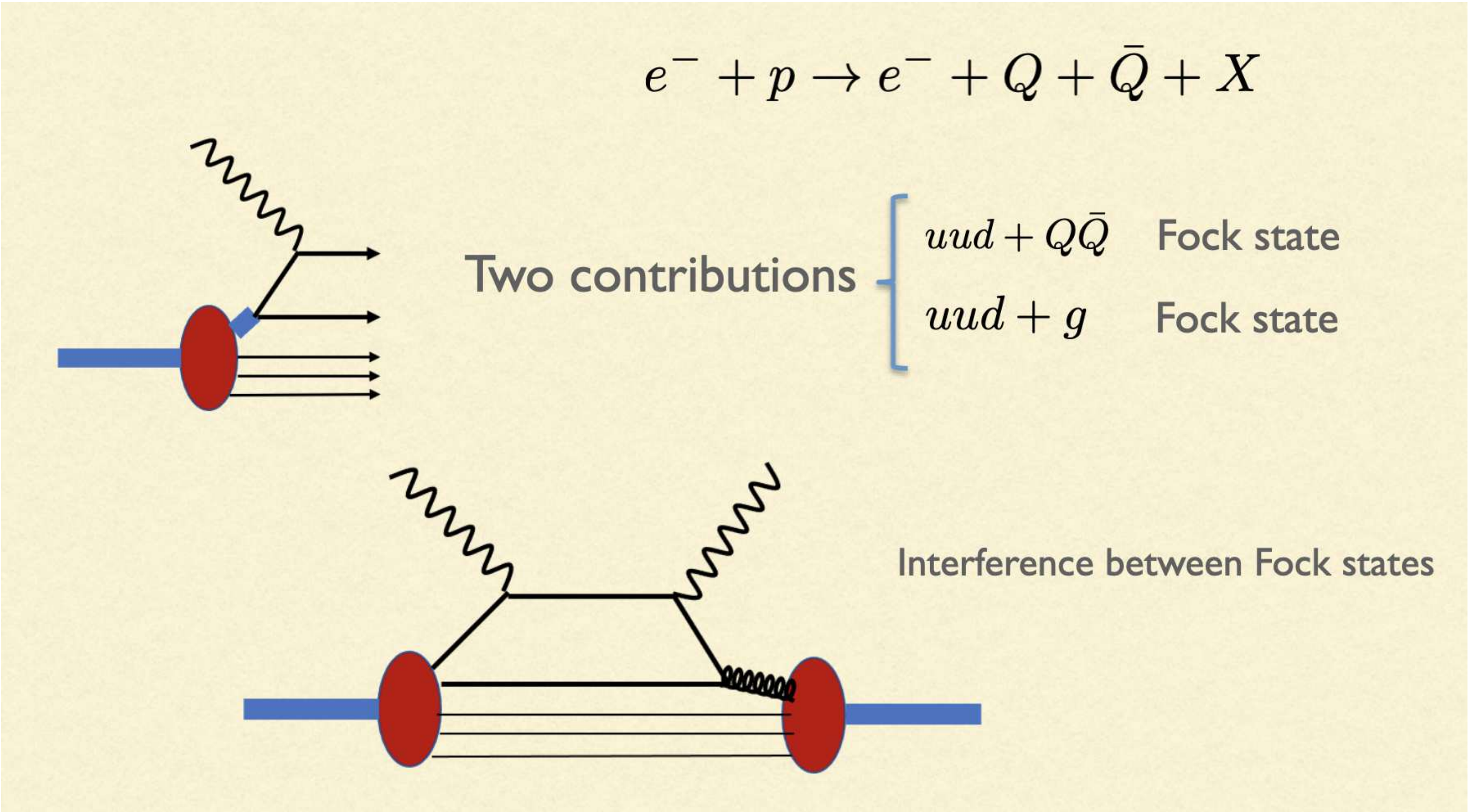
- Sachs FF





Novel asymmetry due to interence of Fock states

- Novel asymmetry in QCD due the interference of two Fock states producing the heavy quark pair in proton electroproduction.



- Asymmetry is given by the imaginary part of the diagram shown in Figure

Novel asymmetry due to interence of Fock states

- Total amplitude $M = M_I + M_{II}$

$$M_I(e^- + p \to e^- + p) = \psi^{\lambda_N}_{Q;\lambda_q}(x, \mathbf{k}_\perp)\, \mathcal{M}(e^- + Q \to e^- + Q)$$

is generated by the $|uud + Q\bar{Q}\rangle$ Fock state, while the amplitude

$$\begin{aligned} M_{II}(e^- + p \to e^- + p) &= \alpha_s\, C_F\, x^2\, (1-x)\, P_{\lambda_g}(x, \mathbf{k}_\perp) \\ &\times \psi^{\lambda_N}_{\lambda_g;\lambda_X}(x, \mathbf{k}_\perp)\, \mathcal{M}(e^- + Q \to e^- + Q) \end{aligned}$$

is generated by the $|uud + g\rangle$ Fock state.

- Differential cross section for the electroproduction of heavy quarks is given by

$$\frac{d^2\sigma_{e-p}}{dx dQ^2} = \frac{1}{8\pi s^2} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |M(x, \mathbf{k}_\perp)|^2 = \frac{2\pi\alpha^2}{xQ^4}\, F_2^Q(x)\, (1 + (1-y)^2)$$

Here $T(x, \mathbf{k}_\perp) = \frac{1}{4} \sum_{\text{pol}} |M(x, \mathbf{k}_\perp)|^2$ – square of matrix element,
$F_2^Q(x) = 2\, e_Q^2\, x\, Q_{\text{full}}(x)$ – predictied heavy quark structure function including contributions of both $|uud + g\rangle$ and $|uud + Q\bar{Q}\rangle$ Fock states and their interference

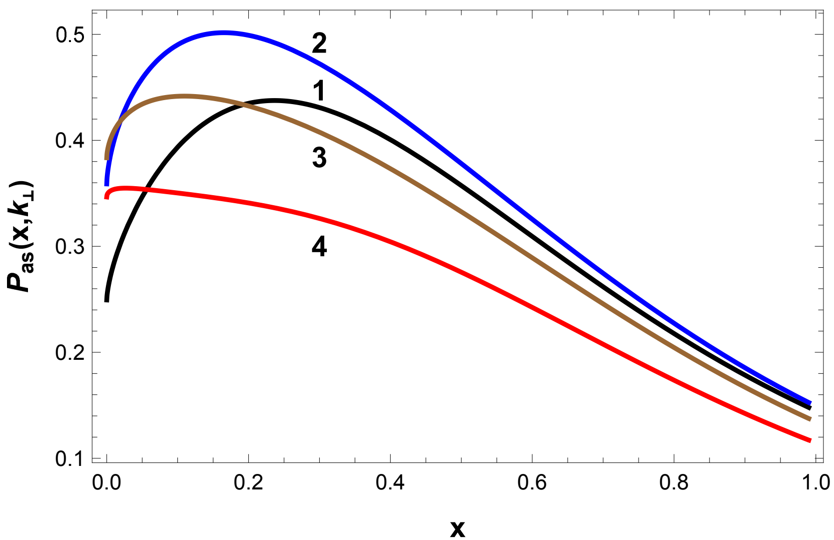
Novel asymmetry due to interence of Fock states

- Asymmetry $\mathcal{P}$ in terms of $T(x,\mathbf{k}_\perp)$

$$\mathcal{P}_{\rm as}(x,\mathbf{k}_\perp) = \frac{T(x,\mathbf{k}_\perp) - T(x,-\mathbf{k}_\perp)}{T(x,\mathbf{k}_\perp) + T(x,-\mathbf{k}_\perp)}$$

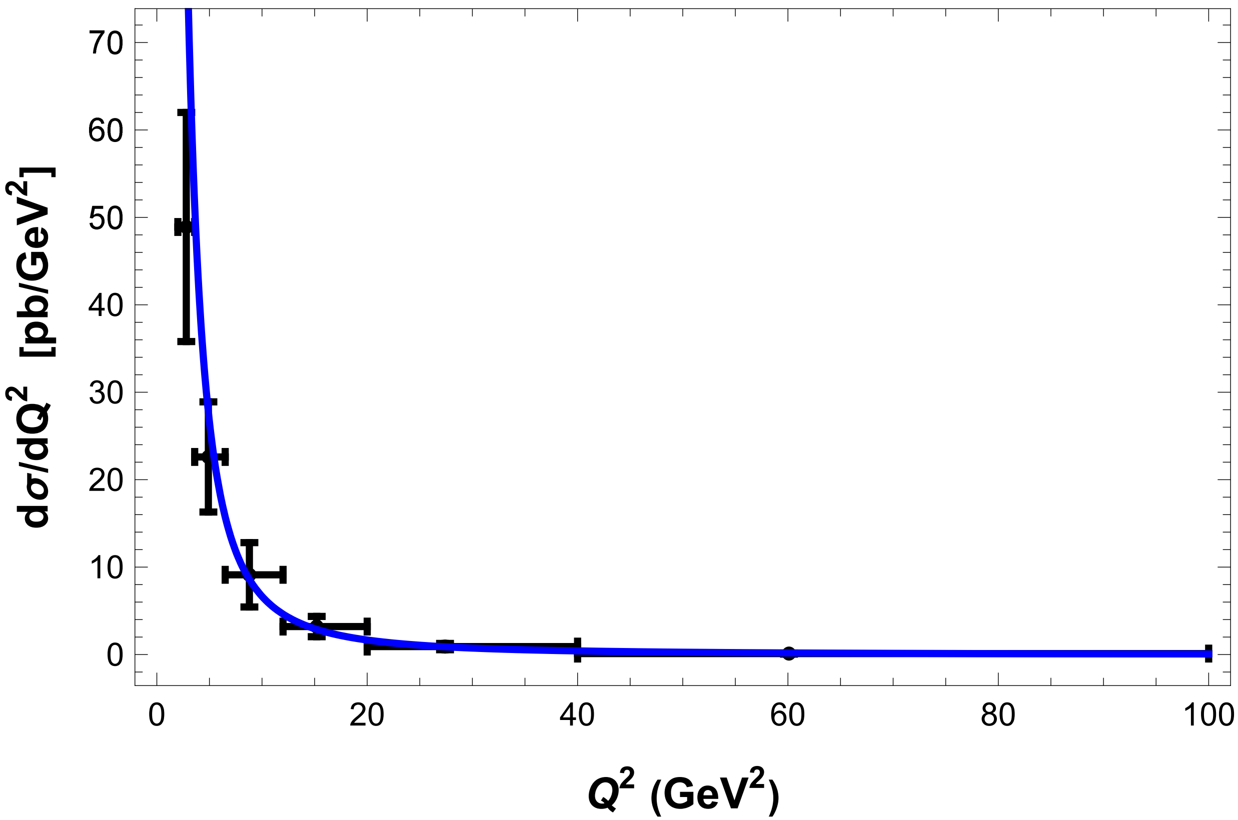
  is independent on the square of the elastic $e^- + Q \to e^- + Q$ amplitude

- Encodes asymmetry with respect to the inverse of the transverse momentum $\mathbf{k}_\perp$
- $\mathcal{P}_{\rm as}(x,\mathbf{k}_\perp)$ as function of x at $|\mathbf{k}_\perp| = 0.5, 0.75, 1, 1.25$ GeV
- Curves increase in magnitude for $|\mathbf{k}_\perp| < 0.75 - 0.8$ GeV and then decrease



Novel asymmetry due to interence of Fock states

- Comparison of $d\sigma_{e-p}/dQ^2$ at $y = 1$ with data from H1 Collaboration/DESY



Summary

- New decomposition of gluon correlator producing TMDs at leading twist
- Clear interpretation of 2 transversity T-odd TMDs with L-polarization of gluons symmetric and antisymmetric under permutation of nucleon S_T and gluon q_T
- Gluon TMDs in LF QCD using LFWFs for $g + 3q$ Fock component in nucleon
- TDMs obey Mulders-Rodrigues inequalities, small-x and large-x behavior
- New Sum Rules involving TMDs
- LFWFs for the Fock state containing heavy quark-antiquark component
- LF decomposition and numerical results for heavy quark-antiquark symmetry/asymmetry $Q(x) \pm \bar{Q}(x)$, EM form factors of nucleons
- Novel asymmetry due to interference of Fock states producing $Q\bar{Q}$