The pressure inside the proton: a next-to-leading order analysis

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In collaboration with:

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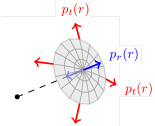
Hervé Moutarde and Pawel Sznajder

Introduction

- In relativistic hydrodynamics $\rightarrow$ pressure for a anisotropic fluid enters the description of the EMT θ:

$$\theta^{\mu\nu}(\mathbf{r}) = (\varepsilon + p_t)\frac{P^\mu P^\nu}{M^2} - p_t\eta^{\mu\nu} + (p_r - p_t)\frac{z^\mu z^\nu}{r^2}$$



Selcuk S. Bayin, Astrophys. J. 303, 101–110 (1986)
figure from C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

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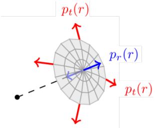

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- On can define isotropic pressure p and pressure anisotropy s:

$$p(\mathbf{r}) = \frac{p_r(\mathbf{r}) + 2p_t(\mathbf{r})}{3}$$
$$s(\mathbf{r}) = p_r(\mathbf{r}) - p_t(\mathbf{r})$$

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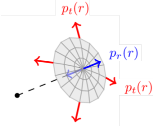

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Question

Can we obtain an analoguous definition within hadron physics?

Hadronic Energy-Momentum Tensor

- In QCD, the EMT is an operator given as:

$$T^{\mu\nu} = -G^{\mu\lambda}G^{\nu}_{\lambda} + \frac{1}{4}\eta^{\mu\nu}G^2 + \sum_a \bar{q}\gamma^{\mu}\overleftrightarrow{D}^{\nu}q$$

see e.g. A. Freese talk on Monday

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- In the case of spin 1/2 hadron one can parametrize the associated matrix element as:

$$\langle p_2|T^{\mu\nu}_a|p_1\rangle = \bar{u}(p_2)\Bigg\{\frac{P^{\mu}P^{\nu}}{M}A_a(t) + \frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^2}{M}C_a(t) + M\eta^{\mu\nu}\bar{C}_a(t)$$

$$+ \frac{P^{\{\mu}i\sigma^{\nu\}\rho}\Delta_{\rho}}{4M}[A_a(t) + B_a(t)] + \frac{P^{[\mu}i\sigma^{\nu]\rho}\Delta_{\rho}}{4M}D_a(t)\Bigg\}u(p_1)\,.$$

see e.g. C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

$$\varepsilon_a(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2}\left[B_a(t) - 4C_a(t)\right] \right\},$$

$$p_{r,a}(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ -\bar{C}_a(t) - \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{3/2}\, C_a(t)\right) \right\},$$

$$p_{t,a}(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ -\bar{C}_a(t) + \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}}{\mathrm{d}t}\left[t\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{3/2}\, C_a(t)\right)\right] \right\},$$

$$p_a(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ -\bar{C}_a(t) + \frac{2}{3}\frac{t}{M^2}\, C_a(t) \right\},$$

$$s_a(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ -\frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(t^{5/2}\, C_a(t)\right) \right\},$$

C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

Dictionnary

$$\varepsilon_a(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2}\left[B_a(t) - 4C_a(t)\right] \right\},$$

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C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

Experimental access?

Some coefficients are accessible through Generalised Parton Distributions

- Generalised Parton Distributions (GPDs):

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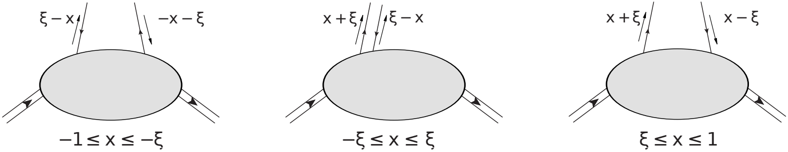
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,



- ★ x: average momentum fraction carried by the active parton
- ★ ξ: skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
- ★ t: the Mandelstam variable

- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,
  - are defined in terms of a non-local matrix element,

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5 u + \tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

- Generalised Parton Distributions (GPDs):
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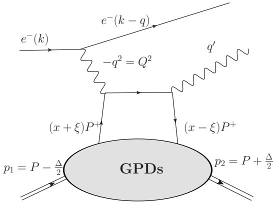
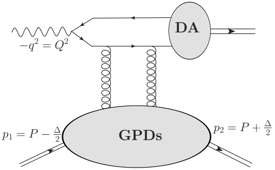
  - are related to PDF in the forward limit $H(x, \xi = 0, t = 0; \mu) = q(x; \mu)$
  - are universal, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$





- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x,\xi,t;\mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} A^q_{2j}(t;\mu) + mod(m,2)\xi^{m+1} C^q_{m+1}(t;\mu)$$

X. Ji, J.Phys.G 24 (1998) 1181-1205
A. Radyushkin, Phys.Lett.B 449 (1999) 81-88

Special case :

$$\int_{-1}^{1} \mathrm{d}x\ H^q(x,\xi,t;\mu) = F_1^q(t)$$

Lorentz Covariance

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left|H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2}E^q(x,\xi,t)\right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right)q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines
  $\rightarrow$ GPDs are continuous albeit non analytical at $x = \pm\xi$

J. Collins and A. Freund, PRD 59 074009 (1999)

Factorisation theorem

Properties

- Polynomiality Property:

  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Continuity at the crossover lines

  Factorisation theorem

- Scale evolution property
  $\rightarrow$ generalization of DGLAP and ERBL evolution equations

  D. Müller *et al.*, Fortschr. Phys. 42, 101 (1994)

  Renormalization

Properties

- Polynomiality Property:

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- Positivity property:

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  Factorisation theorem

- Scale evolution property

  Renormalization

Problem

- There is hardly any model fulfilling *a priori* all these constraints.
- Lattice QCD computations remain very challenging.

- In the limit $\xi \to 0$, one recovers a density interpretation:
  - 1D in momentum space (x)
  - 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)

Interpretation of GPDs I

2+1D structure of the nucleon

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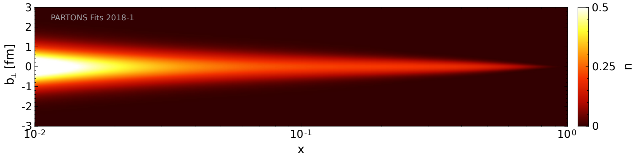

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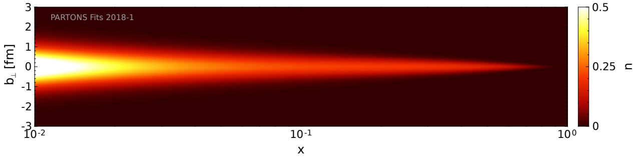

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- Correlation between x and $b_\perp \to$ going beyond PDF and FF.

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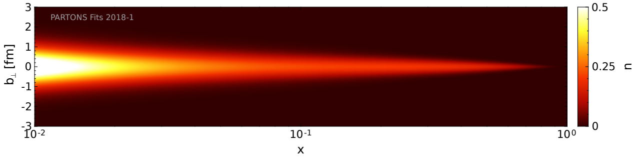

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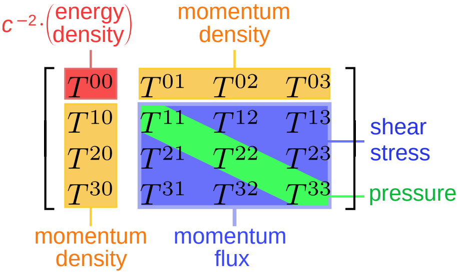
- Correlation between x and $b_\perp \to$ going beyond PDF and FF.
- Caveat: no experimental data at $\xi = 0$
  $\to$ extrapolations (and thus model-dependence) are necessary

Connection to the Energy-Momentum Tensor



How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé *et al.*, PLB 776 (2018) 38-47,
M. Polyakov and P. Schweitzer, IJMPA 33 (2018) 26, 1830025
C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

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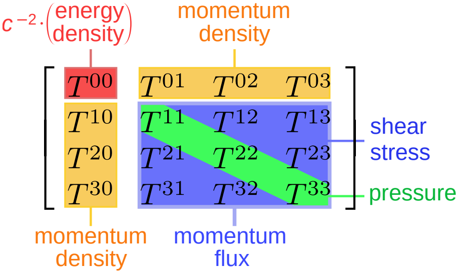

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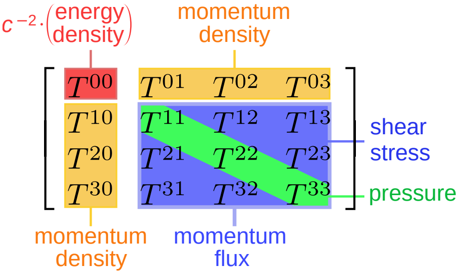
$$\left. + M g^{\mu\nu} \bar{C}_{q,g}(t;\mu) + \frac{P^{\{\mu} i\sigma^{\nu\}\Delta}}{2M} B_{q,g}(t;\mu) + \frac{P^{[\mu} i\sigma^{\nu]\Delta}}{2M} D_{q,g}(t;\mu) \right] u$$



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$$\int_{-1}^{1} \mathrm{d}x\, x\, H_q(x,\xi,t;\mu) = A_q(t;\mu) + (2\xi)^2 C_q(t;\mu)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E_q(x,\xi,t;\mu) = B_q(t;\mu) - (2\xi)^2 C_q(t;\mu)$$

- Ji sum rule
- Fluid mechanics analogy

X. Ji, PRL 78, 610-613 (1997)
M.V. Polyakov PLB 555, 57-62 (2003)

Polynomiality and EMT

- The polynomiality property yields

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x,\xi,t;\mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} (2\xi)^{2j} A^q_{2j}(t;\mu) + mod(m,2)(2\xi)^{m+1} C^q_{m+1}(t;\mu^2)$$

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- We can define the D-term D as a generating function:

$$\int_{-1}^{1} \mathrm{d}\alpha \alpha^m D^q(\alpha,t,\mu^2) = 2^{m+1} C^q_{m+1}(t;\mu^2)$$

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- Thus in the case $m = 1$

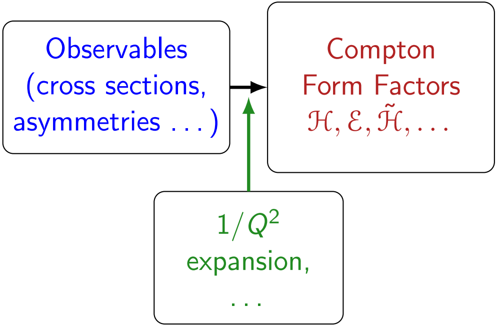
$$\int_{-1}^{1} \mathrm{d}\alpha\, \alpha D^q(\alpha,t,\mu^2) = 4C^q(t;\mu^2)$$

and the connection to the pressure and pressure anistropy is contained in the D-term.

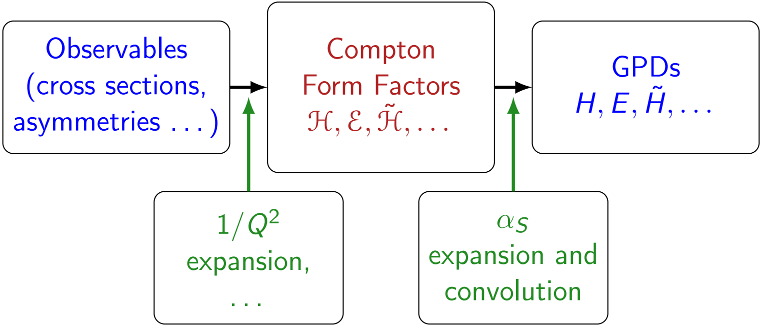
Experimental access to the nucleon pressure

Experimental connection to GPDs

Observables
(cross sections,
asymmetries . . .)


Observables
(cross sections,
asymmetries ...)
Compton
Form Factors
$\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \ldots$
$1/Q^2$
expansion,
...

Experimental connection to GPDs



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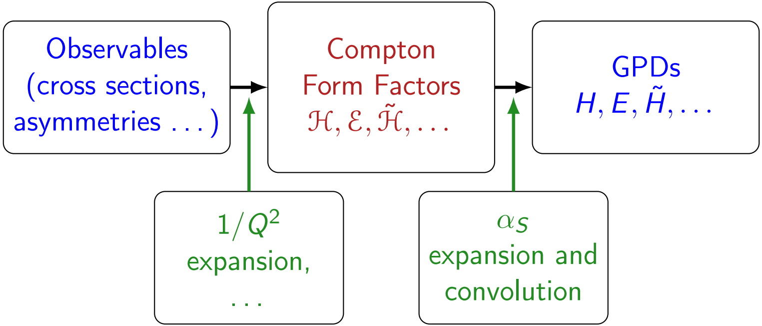
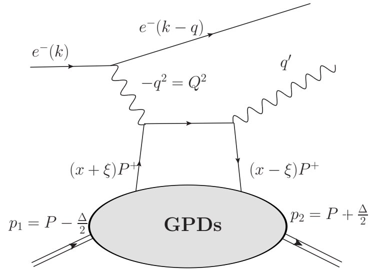
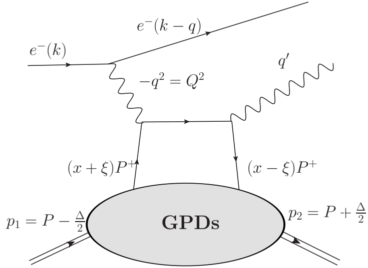


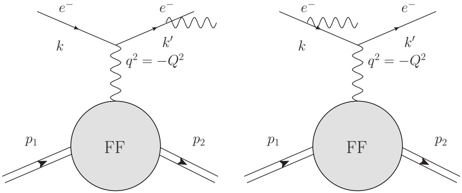
- Multiple extraction procedure:
  - CFF are extracted from data
  - GPDs are extracted from CFF
  - EMT FF are computed from GPDs



- Best studied experimental process connected to GPDs
  → Data taken at Hermes, Compass, JLab 6, JLab 12

Deep Virtual Compton Scattering

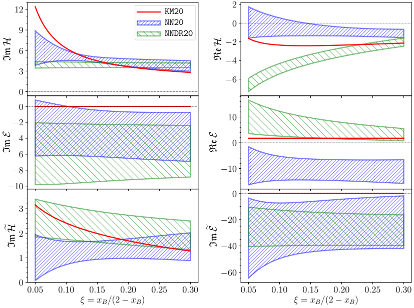




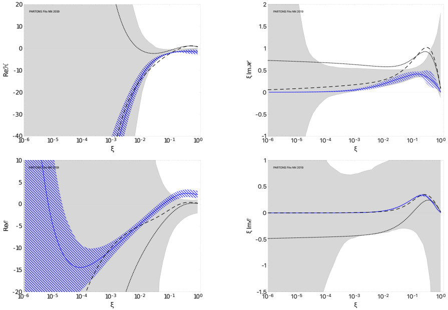
- Best studied experimental process connected to GPDs
  $\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler (BH) process
  - Blessing: Interference term boosted w.r.t. pure DVCS one
  - Curse: access to the angular modulation of the pure DVCS part difficult

M. Defurne *et al.*, Nature Commun. 8 (2017) 1, 1408

Recent CFF extractions



M. Cuič *et al.*, PRL 125, (2020), 232005



H. Moutarde *et al.*, EPJC 79, (2019), 614

- Recent effort on bias reduction in CFF extraction (ANN)

  additional ongoing studies, J. Grigsby *et al.*, PRD 104 (2021) 016001
- Studies of ANN architecture to fulfil GPDs properties (dispersion relation,polynomiality,. . .)
- Recent efforts on propagation of uncertainties (allowing impact studies for JLAB12, EIC and EicC)

  see *e.g.* H. Dutrieux *et al.*, EPJA 57 8 250 (2021)

Introducing shadow GPDs

CFF Definition

$$\underbrace{\mathcal{H}(\xi, t, Q^2)}_{\text{Observable}} = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \underbrace{T\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)}_{\text{Perturbative DVCS kernel}} H(x, \xi, t, \mu^2)$$

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Shadow GPD definition

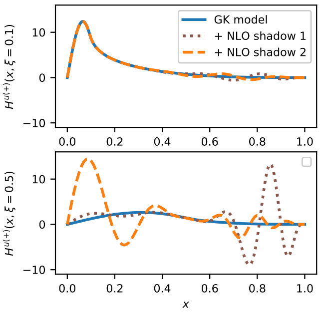
We define shadow GPD $H^{(n)}$ of order n such that when T is expanded in powers of α_s up to n one has:

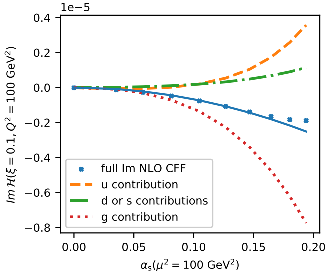
$$0 = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^{(n)}\left(\frac{x}{\xi}, \frac{Q^2}{\mu_0^2}, \alpha_s(\mu_0^2)\right) H^{(n)}(x, \xi, t, \mu_0^2) \quad \text{invisible in DVCS}$$

$$0 = H^{(n)}(x, 0, 0) \quad \text{invisible in DIS}$$

A part of the GPD functional space is invisible to DVCS and DIS combined

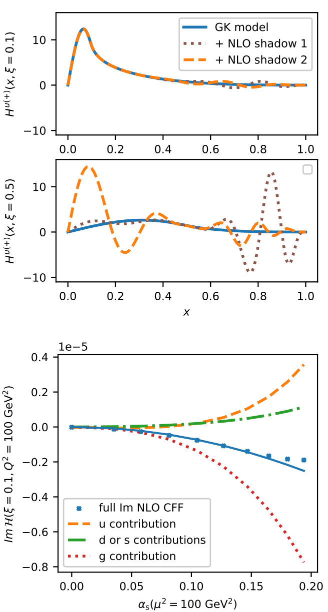
The DVCS deconvolution problem II





- NLO analysis of shadow GPDs:
  - Cancelling the line $x=\xi$ is necessary but **no longer** sufficient
  - Additional conditions brought by NLO corrections reduce the size of the "shadow space"...
  - ... but do not reduce it to 0 $\rightarrow$ NLO shadow GPDs

H. Dutrieux *et al.*, PRD 103 114019 (2021)



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A. Freund PLB 472, 412 (2000)

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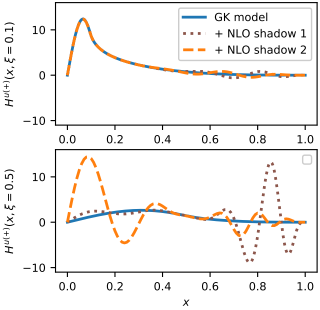
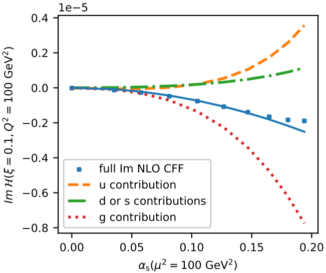
H. Dutrieux *et al.*, PRD 103 114019 (2021)





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H. Dutrieux *et al.*, PRD 103 114019 (2021)

Theoretical uncertainties promoted to main source of GPDs uncertainties

PARTONS and Gepard

Integrated softwares as a mandatory step for phenomenology

PARTONS	Gepard
partons.cea.fr	gepard.phy.hr
B. Berthou *et al.*, EPJC 78 (2018) 478	K. Kumericki, EPJ Web Conf. 112 (2016) 01012

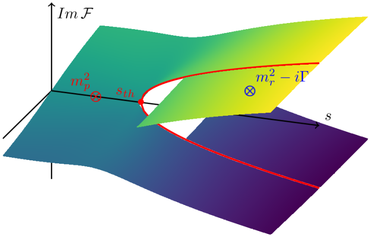
- Similarities : NLO computations, BM formalism, ANN, ...
- Differences : models, evolution, ...

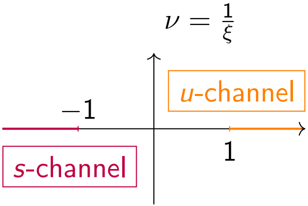
Physics impact

These integrated softwares are the mandatory path toward reliable multichannel analyses.

Dispersion relation:
bypassing GPDs extraction

- DVCS amplitude obeys dispersion relations coming from:
  - The mathematical property of the amplitude (Unitarity + Reflexion principle)
  - The analysis of the singularities in the complex plane (poles and cuts)





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$$\Re(\mathcal{F}(\xi, t, Q^2)) = \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \Im(\mathcal{F}(\xi', t, Q^2)) \left[\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right] + \mathcal{S}(t, Q^2)$$

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  - $\Re\mathcal{F}$ and $\Im\mathcal{F}$ are measurable
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- In principle, we can extract $\mathcal{S}(t,Q^2)$

- At leading order, the subtraction constant is related the "D-term":

$$\mathcal{S}(t, Q^2) \underset{\mathrm{LO}}{=} 2\int_{-1}^{1} \mathrm{d}z \frac{D(z, t, Q^2)}{1 - z}$$

I. Anikin and O. Teryaev, PRD 76 056007

The DVCS subtraction constant

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M. Diehl and D. Ivanov, EPJC 52 (2007) 919-932

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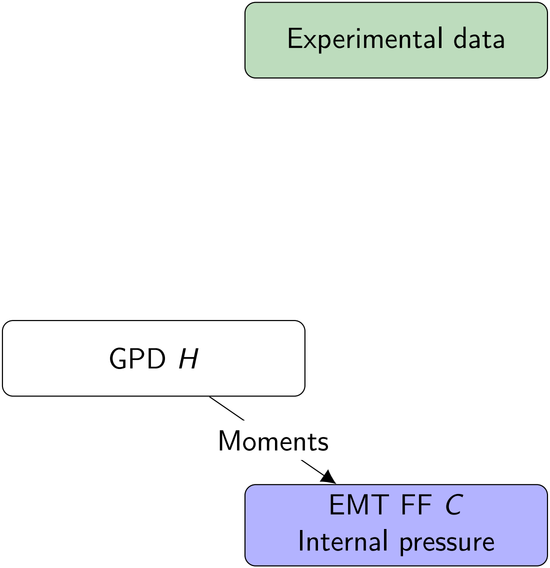
Throught $D(z, t, Q^2)$ we can access the EMT FF $C(t)$!

Two paths to the EMT

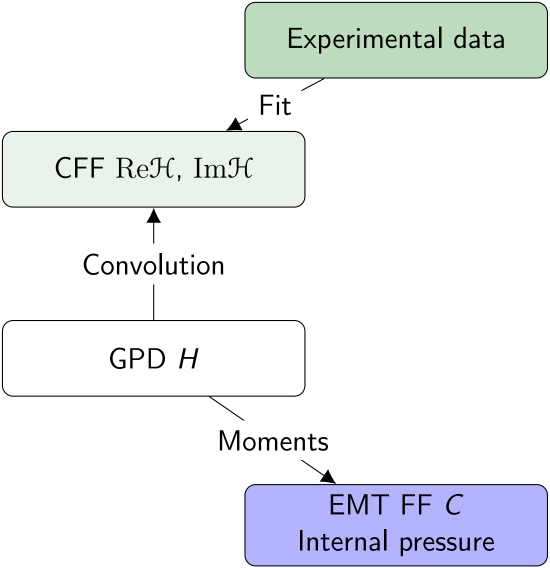
Experimental data

EMT FF C
Internal pressure

Two paths to the EMT



Two paths to the EMT



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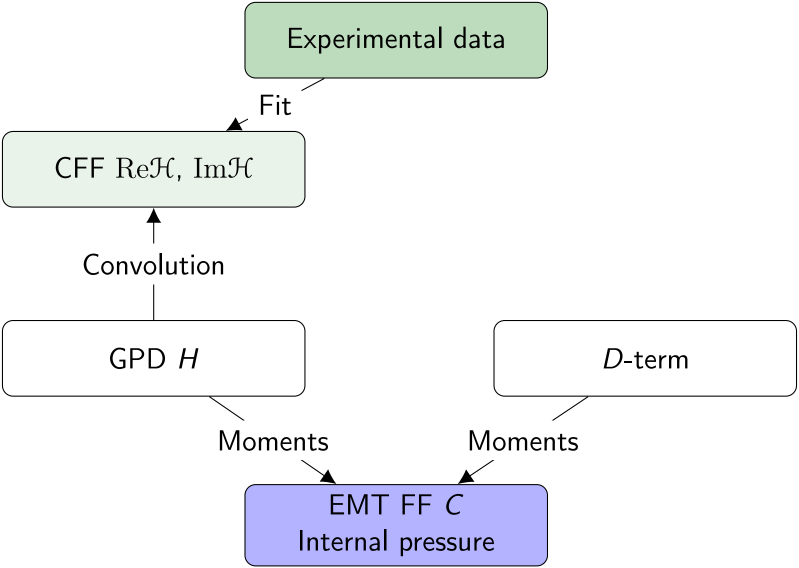

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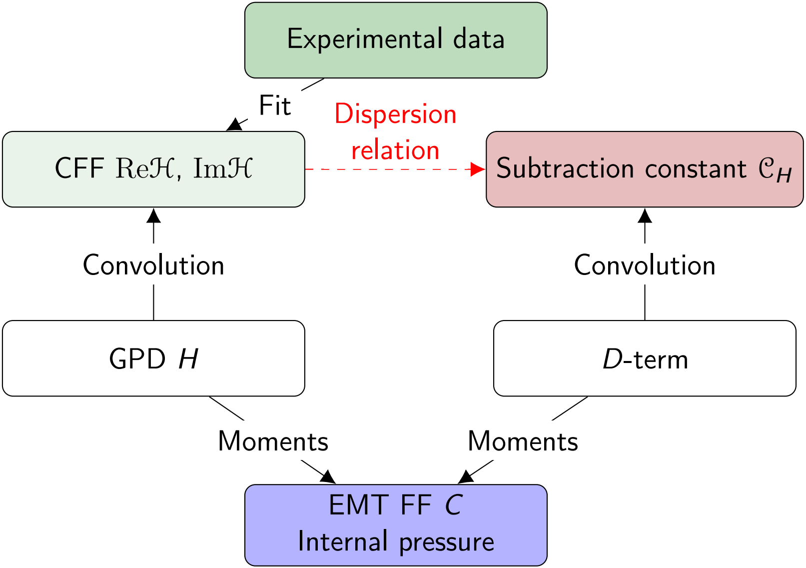

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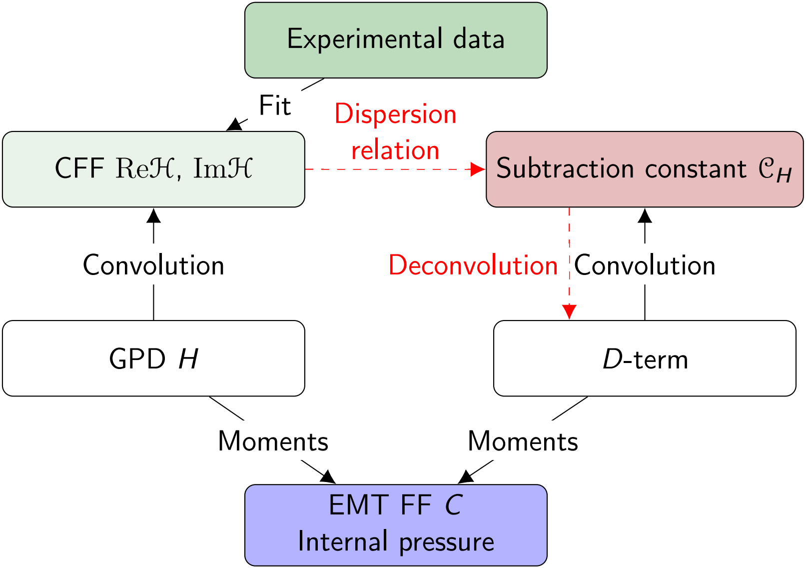

Deconvolution of $C(t)$ at LO

- Decomposition of the D-term on a Gegenbauer Polynomial basis (diagonalisation of the LO ERBL equations)

$$D^q(z,t,\mu^2) = (1-z^2)\sum_{\substack{j\\ \text{odd}}} d_j^q(t,\mu^2) G_j^{3/2}(z)$$

$$D^g(z,t,\mu^2) = \frac{3}{2}(1-z^2)^2\sum_{\substack{j\\ \text{odd}}} d_j^g(t,\mu^2) G_{j-1}^{5/2}(z)$$

- The subtraction constant becomes:

$$\mathcal{S}(t,Q^2) \underset{\text{LO}}{=} 4\sum_q e_q^2 \sum_{\substack{j\\ \text{odd}}} d_j^q(t,Q^2) \quad \text{with} \quad d_1^q(t,Q^2) = 5C^q(t,Q^2)$$

- Several comments are in order:
  - only quark contributes directly to $\mathcal{S}$ at LO
  - we need to rely on the Q^2 dependence to disantangle d_1 from the d_j
  - at fixed Q^2, one can easily build "shadow D-term" such as $d_1 = -d_3$

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- But at fixed j, quarks and gluons mix:

$$\begin{pmatrix} d_j^g(\mu^2) \\ d_j^{q_1}(\mu^2) \\ \dots \\ d_j^{q_n}(\mu^2) \end{pmatrix} = \begin{pmatrix} E_j^{gg}(\mu^2,\mu_0^2) & \dots & E_j^{gq_n}(\mu^2,\mu_0^2) \\ E_j^{q_1 g}(\mu^2,\mu_0^2) & \dots & E_j^{q_1 q_n}(\mu^2,\mu_0^2) \\ \dots & \dots & \dots \\ E_j^{q_n g}(\mu^2,\mu_0^2) & \dots & E_j^{q_n q_n}(\mu^2,\mu_0^2) \end{pmatrix} \begin{pmatrix} d_j^g(\mu_0^2) \\ d_j^{q_1}(\mu_0^2) \\ \dots \\ d_j^{q_n}(\mu_0^2) \end{pmatrix}$$

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At leading order, gluons play an indirect role through evolution equations

Deconvolution of $C(t)$ at NLO

- A NLO gluons starts to play a direct role in the expression of the subtraction constant:

$$\mathcal{S}(t,Q^2) = \sum_q 4e_q^2 \left(1+\alpha_s T^q_{1,1}, 1+\alpha_s T^q_{3,1}, \ldots\right) \begin{pmatrix} d_1^q \\ d_3^q \\ \ldots \end{pmatrix} + \alpha_s \left(T^g_{1,1}, T^g_{3,1}, \ldots\right) \begin{pmatrix} d_1^g \\ d_3^g \\ \ldots \end{pmatrix}$$

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In practice what happens?

Experimental extraction of the Subtraction Constant

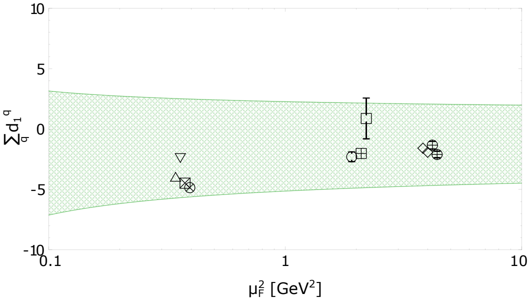

figure from H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4

- Green band → extraction of the subtraction constant using a bias-reduced technique (ANN) is compatible with zero H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4
- Other ANN study yields a similar result K. Kumericki, Nature 570 (2019) 7759, E1-E2
- One should really pay attention to systematic uncertainties

D-term Expansion and Shadow D-term

$$\mathcal{S} = \int_{-1}^{1} \mathrm{d}\alpha \frac{D^{uds}(\alpha, t)}{1-\alpha} = 2d_1^{uds}(t) + 2 \sum_{\text{n odd}>1} d_n^{uds}(t)$$

- Fitting scenario with d_1 only:
  $d_1(\mu_F^2) = -0.5 \pm 1.2$

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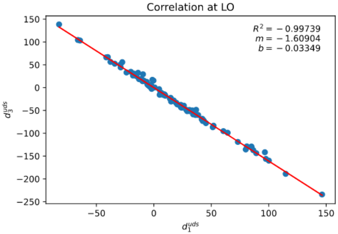

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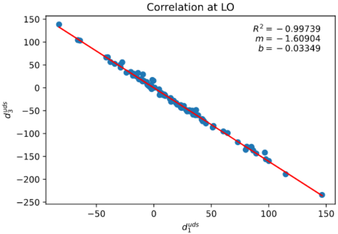

Shadow D-term

We extract shadow D-term yielding vanishing contribution to the subtraction constant. The range in Q^2 is too small.

Gluon at LO

Pure evolution generation

Assuming $d_1^g(\mu_0^2 = 0.1\mathrm{GeV}^2) = 0$
$\rightarrow d_1^g(2\mathrm{GeV}^2) = -0.5 \pm 1.2$

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Free gluon parameter

Fitting $d_1^g(2\mathrm{GeV}^2)$ as a free parameter
$\rightarrow d_1^g(2\mathrm{GeV}^2) = 51 \pm 111$

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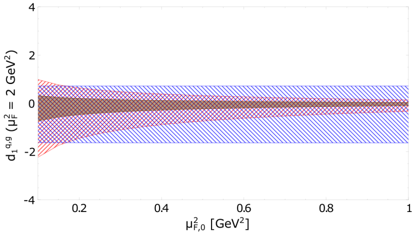
Evolution operator $\Gamma^{qg} << \Gamma^{qq}$ and gluons hardly impact the quark sector



figures from H. Dutrieux, Ph.D. Thesis

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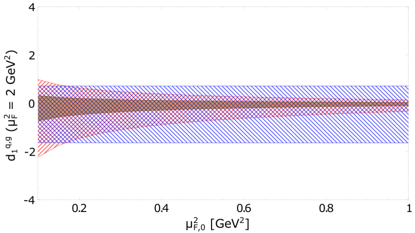

figures from H. Dutrieux, Ph.D. Thesis

In practice, $\mathcal{S}$ is insensitive to gluon through evolution on the range of Q^2 of data currently accessible.

Impact of NLO corrections

$$\mathcal{S} = \int_{-1}^{1} \mathrm{d}\alpha \, \Re T(\alpha) \, D(\alpha, t) = F(d_i^q, d_i^g)$$

- First fit strategy: only d_1^{uds} is a free parameter

Parameters	LO + evolution fit
$d_1^{uds}(2\mathrm{GeV}^2)$	-0.5 ± 1.2
$d_1^g(2\mathrm{GeV}^2)$	-0.6 ± 1.6

Parameters	NLO fit
$d_1^{uds}(2\mathrm{GeV}^2)$	-0.5 ± 1.4
$d_1^g(2\mathrm{GeV}^2)$	-0.7 ± 1.9

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$d_1^g(2\mathrm{GeV}^2)$	-0.6 ± 1.6

Parameters	NLO fit
$d_1^{uds}(2\mathrm{GeV}^2)$	-0.5 ± 1.4
$d_1^g(2\mathrm{GeV}^2)$	-0.7 ± 1.9

- Second fit strategy: d_1^{uds} and d_1^g as free parameters

Parameters	LO + evolution fit
$d_1^{uds}(2\mathrm{GeV}^2)$	-0.7 ± 1.2
$d_1^g(2\mathrm{GeV}^2)$	51 ± 111

Parameters	NLO fit
$d_1^{uds}(2\mathrm{GeV}^2)$	0.4 ± 2.8
$d_1^g(2\mathrm{GeV}^2)$	5.3 ± 19

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Impact of NLO corrections

$$\mathcal{S} = \int_{-1}^{1} \mathrm{d}\alpha \, \Re T(\alpha) \, D(\alpha, t) = F(d_i^q, d_i^g)$$

- First fit strategy: only d_1^{uds} is a free parameter

Parameters	LO + evolution fit
$d_1^{uds}(2\mathrm{GeV}^2)$	-0.5 ± 1.2
$d_1^g(2\mathrm{GeV}^2)$	-0.6 ± 1.6

Parameters	NLO fit
$d_1^{uds}(2\mathrm{GeV}^2)$	-0.5 ± 1.4
$d_1^g(2\mathrm{GeV}^2)$	-0.7 ± 1.9

- Second fit strategy: d_1^{uds} and d_1^g as free parameters

Parameters	LO + evolution fit
$d_1^{uds}(2\mathrm{GeV}^2)$	-0.7 ± 1.2
$d_1^g(2\mathrm{GeV}^2)$	51 ± 111

Parameters	NLO fit
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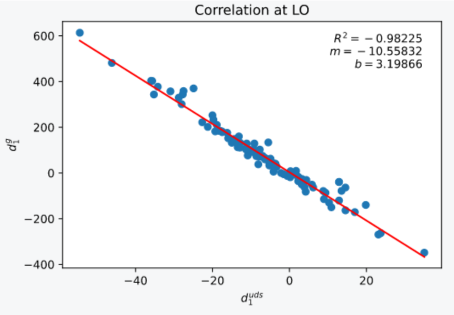
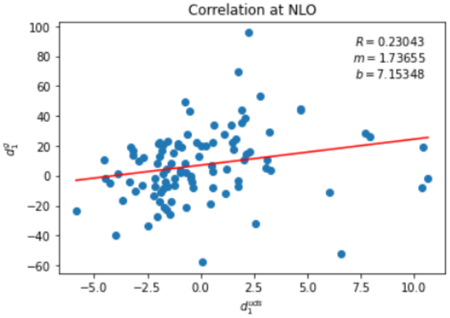
NLO impact

The direct sensitivity of $\mathcal{S}$ to gluons triggers a significant improvement on the knowledge on gluon contributions to the EMT (almost a factor 6).

Impact of EIC

$$\mathcal{S} = \int_{-1}^{1} \mathrm{d}\alpha\, \Re T(\alpha)\, D(\alpha, t) = F(d_i^q, d_i^g)$$



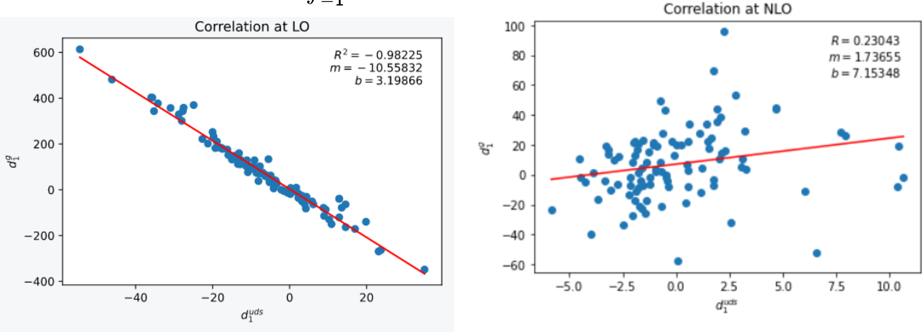


figures from H. Dutrieux *et al.*, in preparation

Data-driven extractions of (d_1^q, d_1^g) possible at EIC

Impact of EIC

$$\mathcal{S} = \int_{-1}^{1} \mathrm{d}\alpha\, \Re T(\alpha)\, D(\alpha, t) = F(d_i^q, d_i^g)$$



figures from H. Dutrieux *et al.*, in preparation

Data-driven extractions of (d_1^q, d_1^g) possible at EIC

However $(d_1^q, d_1^g, d_3^q, d_3^g)$ remains tainted by shadow D-terms

Conclusions

Summary

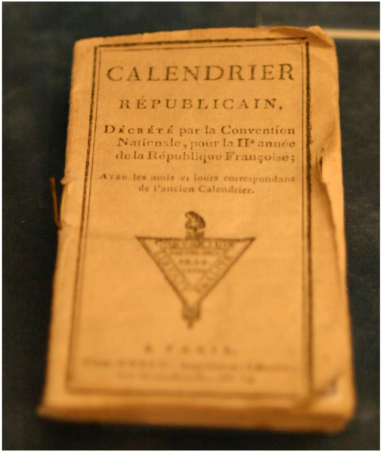
- Hydrodynamics analogy allows us to define pressure profiles based on the EMT of the nucleon
- In principle, extracting such component from data is possible
- In practice, the bad conditioning of the deconvolution problem makes a **reliable** and non vanishing extraction **out of reach** with current data.

Perspectives

- Impact studies with EIC kinematics should be performed
- The question of multichannel analysis should be asked
- Ab-initio computations may provide insights in the next decade

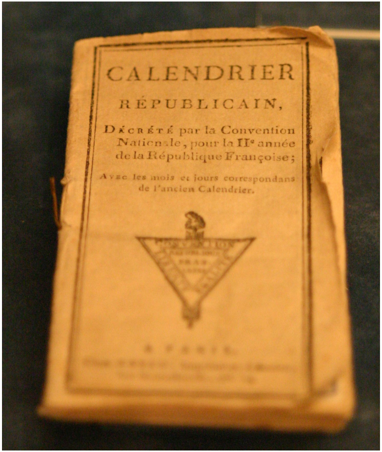
In the perspective of EIC and EicC, a lot of work remains to be done to exploit the forthcoming data.

Today is new year eve !



- Today is the last day of republican year CCXXX
- Tomorrow will be 1$^{\rm st}$ Vendémiaire CCXXXI

Today is new year eve !



- Today is the last day of republican year CCXXX
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Happy new republican year to everybody !

Thank you for your attention

Additional materials