Emergent superconformal structure in holographic light-front QCD

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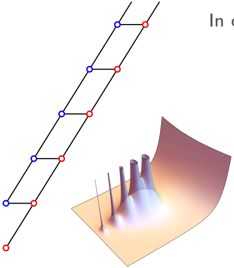

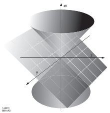

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- Integer spin
- Half-integer spin

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- Hadron form factors and quark distribution functions
- Gravitational form factors and gluon distribution functions

Emergent QCD properties

A basic understanding of fundamental features of hadron physics from first principles QCD has remained elusive

Other important aspects of the strong interactions which were manifest in dual models, before QCD, are also not explicit properties of the QCD Lagrangian

Emergent properties of QCD:

1. Mechanism of color confinement
2. The origin of the hadron mass scale
3. Chiral symmetry breaking and confinement
4. A massless pion *vs.* a massive proton in the chiral limit
5. The pattern of hadronic excitations
6. How does Regge theory emerge from QCD at large distances?

...

Present goal: To understand how emerging QCD properties can be incorporated in an effective computational framework of hadron structure

Critical role of the number of space-time dimensions

We start with the $SU(N)_C$ Lagrangian of QCD

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \tfrac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu}$$

where $D_\mu = \partial_\mu - igT^a A^a_\mu$ and $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f_{abc}A^b_\mu A^c_\nu$

Dimensional analysis from the QCD action

$$S = \int d^d x\, \mathcal{L}$$

in d-dimensional space-time gives

$$[\psi] \sim M^{(d-1)/2}$$
$$[A] \sim M^{(d-2)/2}$$
$$[g] \sim M^{(4-d)/2}$$

QCD$(1+1)$ $[g] \sim M$: Can be solved for any number of constituents and colors (DLCQ), but no emerging phenomena

QCD$(3+1)$ $[g] \sim 1$: Complex hadronic phenomena which should, in principle, emerge from a simple Lagrangian in the nonperturbative domain

Holographic light-front QCD (HLFQCD)

HLFQCD originates from the precise mapping of the AdS and LF expressions of FFs for an arbitrary number of partons [S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

Nonperturbative analytic approach follows from a semiclassical approximation to light-front QCD and its holographic embedding in AdS space: It leads to relativistic wave equations similar to the Schrödinger equation in atomic physics

Underlying superconformal algebraic structure introduces a mass scale and fix the effective confinement potential: It is not SUSY QCD

The zero mass eigenmode is identified with the pion and is massless in the chiral limit

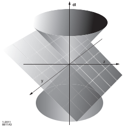
The new framework leads to relations between the Regge trajectories of mesons, baryons, and tetraquarks

Holographic QCD also incorporates features of the Veneziano model as emerging properties

Further extensions incorporate the exclusive-inclusive connection in QCD and provide nontrivial relations between hadron form factors and quark and gluon distributions

Semiclassical approximation to light-front QCD

Light-front (LF) quantization uses the null plane $x^+ = x^0 + x^3 = 0$ tangent to the light cone as the initial surface (Dirac 1949), thus without reference to a specific Lorentz frame



Evolution in LF time x^+ is given by the Hamiltonian equation

$$i \frac{\partial}{\partial x^+} |\psi\rangle = P^- |\psi\rangle, \qquad P^- |\psi\rangle = \frac{\mathbf{P}_\perp^2 + M^2}{P^+} |\psi\rangle,$$

for a hadron with 4-momentum $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, where P^- is a dynamical generator and P^+ and $\mathbf{P}_\perp$ are kinematical

Hadron mass spectra from LF invariant Hamiltonian $\;P^2 = P_\mu P^\mu = P^+ P^- - \mathbf{P}_\perp^2$

$$P^2 |\psi(P)\rangle = M^2 |\psi(P)\rangle, \qquad |\psi\rangle = \sum_n \psi_n |n\rangle$$

Simple structure of the LF vacuum allows for a quantum-mechanical probabilistic interpretation of hadronic states in terms of invariant LF wave functions, $\psi_n = \langle n|\psi\rangle$, written in terms of the quark and gluon degrees of freedom in the Fock expansion

QCD(1+1)

A semiclassical LF Schrödinger equation derived from first principles QCD

G. 't Hooft (1974), K. Hornbostel, S. J. Brodsky and H. C. Pauli (1990)

Properties of QCD$(1+1)$

- Gluons are not dynamical, no self-couplings of gluons
- quarks have chirality but no spin
- Coupling g has dimensions of mass
- Confining field theory for any coupling

Express the hadron 2-momentum generator $P = (P^+, P^-)$, $P^\pm = P^0 \pm P^3$, in the $A^+ = 0$ gauge in terms of the fields $\psi_\pm \equiv \psi_{R,L}$ and A^-

(1+1) Hamiltonian P^- is given in terms of ψ_+, only dynamical variable, from inverting A^- using the LF constraint equations

$$P^- = \tfrac{1}{2} \int_{-\infty}^{\infty} dx^- \left(\psi_+^\dagger \frac{m^2}{i\partial^+} \psi_+ + g^2 j^{+a} \frac{1}{(i\partial^+)^2} j^{+a} \right)$$

where $j^{+a} = \psi_+^\dagger T^a \psi_+$

Interaction term

$$V = -g^2 \int dx^- dy^- j^{+a}(x^-) \left| x^- - y^- \right| j^{+a}(y^-)$$

See also: B. Ma and C. R. Ji, PRD **104**, 036004 (2021)

Mass spectrum computed from LF eigenvalue equation

$$P^+ P^- |\chi(P^+)\rangle = M_\pi^2 |\chi(P^+)\rangle$$

For the $q\bar{q}$ valence state (or large N_C) it leads to the 't Hooft equation (1974)

$$\boxed{\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)\chi(x) + \frac{g^2 N_C}{\pi} P \int_0^1 dx' \, \frac{\chi(x) - \chi(x')}{(x-x')^2} = M_\pi^2 \, \chi(x)}$$

where x is the longitudinal momentum fraction of the $q\bar{q}$ state

Cancellation of singularities at $x = \epsilon$ and $x = 1 - \epsilon$ for the approximate solution

$$\chi(x) \sim x^{\beta_q} (1-x)^{\beta_{\bar{q}}}$$

leads for $m_q^2 / \pi g^2 N_C \ll 1$ to $\beta_q = \left(3 m_q^2 / \pi g^2 N_C\right)^{1/2}$ and

$$M_\pi^2 = g \sqrt{\frac{\pi N_C}{3}} \, (m_q + m_{\bar{q}}) + \mathcal{O}\left((m_q + m_{\bar{q}})^2\right)$$

QCD(1+1): Both, the value of the CSB "condensate"

$$\langle \psi \psi \rangle = g \, f_\pi^2 \sqrt{\frac{\pi N_C}{3}}$$

and the strength of linear confinement depend on the value of the coupling g in the QCD Lagrangian, and are not emerging properties

QCD(1+ 3)

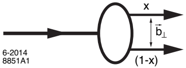
GdT and S. J. Brodsky, PRL **102**, 081601 (2009)

(3+1) Hamiltonian P^- written in terms of the dynamical field ψ_+

$$P^- = \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \bar{\psi}_+^\dagger \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+}\psi_+ + \text{ interactions}$$

We factor out the longitudinal $X(x)$ and orbital $e^{iL\theta}$ dependence from the LFWF ψ

$$\psi(x,\zeta,\varphi) = e^{iL\theta}X(x)\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$



where $\zeta^2 = x(1-x)b_\perp^2$ is the invariant transverse separation between two quarks and L their relative LF orbital angular momentum

Mass spectrum from LF invariant Hamiltonian equation $P_\mu P^\mu|\psi(P)\rangle = M^2|\psi(P)\rangle$

Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple and we find

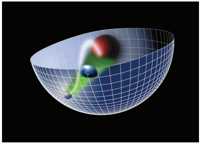
$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)}$$

- Effective potential U includes all interactions, including from higher Fock states
- Critical value $L = 0$ corresponds to the lowest possible stable solution
- Relativistic and frame-independent semiclassical WE
- It has identical structure of AdS WE provided that $z = \zeta$

Higher-spin wave equations in AdS

GdT, H. G. Dosch and S. J. Brodsky, PRD **87**, 075005 (2013)

$S = \int d^d dx\, dz\, e^{\varphi(z)} \mathcal{L}$



$ds^2 = \frac{R^2}{z^2}(dx_\mu^2 - dz^2)$

Integer spin

We start with the AdS$_{d+1}$ action for a tensor-J field $\Phi_{N_1 \dots N_J}$ with a dilaton φ to modify the IR region of AdS space

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z\right) + \frac{(\mu\, R)^2}{z^2}\right] \Phi_J(z) = M^2 \Phi_J(z)}$$

Upon the substitution $\Phi_J(z) = z^{(d-1)/2-J} e^{-\varphi(z)/2}\, \phi_J(z)$ we find for $d = 4$ the semiclassical QCD light-front wave equation (z is the fifth dimension of AdS space)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right) \phi(\zeta) = M^2 \phi(\zeta)$$

where $\zeta^2 = z^2 = x(1-x) b_\perp^2$ and

$$U(\zeta, J) = \frac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2\zeta}\varphi'(\zeta)$$

the effective LF confinement potential with AdS mass-radius $(\mu R)^2 = -(2-J)^2 + L^2$

Half-integer spin

We start with the Rarita-Schwinger action in AdS for a spinor-J field $\Psi_{N_1 \dots N_{J-1/2}}$ with potential V (No dynamical dilaton for fermions)

Upon the substitution $\Psi_J^{\pm}(z) = z^{(d-1)/2-J}\psi_J^{\pm}(z)u^{\pm}$ we find for the chiral components $\psi^{\pm}$

$$
\begin{aligned}
-\frac{d}{dz}\psi_- - \frac{\nu+\frac{1}{2}}{z}\psi_- - V(z)\psi_- &= M\psi_+ \\
\frac{d}{dz}\psi_+ - \frac{\nu+\frac{1}{2}}{z}\psi_+ - V(z)\psi_+ &= M\psi_-
\end{aligned}
$$

with $|\mu R| = \nu + 1/2$ and equal probability $\int dz\ \psi_+(z)^2 = \int dz\ \psi_-^2(z)$

System of linear Eqs. is equivalent to second order Eqs. ($z \to \zeta$):

$$
\boxed{
\begin{aligned}
\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U^+(\zeta)\right)\psi_+ &= M^2\psi_+ \\
\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + U^-(\zeta)\right)\psi_- &= M^2\psi_-
\end{aligned}
}
$$

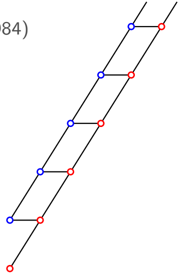
the semiclassical LF WE with ψ_+ and ψ_- corresponding to LF orbital L and $L+1$ with

$$
U^{\pm}(\zeta) = V^2(\zeta) \pm V'(\zeta) + \frac{1+2L}{\zeta}V(\zeta), \qquad L = \nu,
$$

a J-independent potential in agreement with the observed degeneracy in the baryon spectrum

Superconformal QM: Emergence of a mass scale and confinement

de Alfaro, Fubini and Furlan, (1976), Witten (1981) and Fubini and Rabinovici (1984)



Embedding LF physics in AdS leads to important insights on the nonperturbative structure of bound state equations for any spin, but the effective confinement potential is not determined

The potential $V(\zeta)$ in the baryon equations plays the role of the superpotential in SUSY QM

SUSY QM is based on a graded Lie algebra consisting of two anticommuting supercharges Q and $Q^\dagger$ which commute with the Hamiltonian H

$$H = \tfrac{1}{2}\{Q, Q^\dagger\}, \quad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \quad [Q, H] = [Q^\dagger, H] = 0$$

If $|E\rangle$ is an eigenstate with energy E, $H|E\rangle = E|E\rangle$, then $Q^\dagger|E\rangle$ is degenerate with the state $|E\rangle$ for $E \neq 0$, but for $E = 0$ we have $Q^\dagger|E = 0\rangle = 0$: zero mode has no SUSY partner

Key result for the supermultiplet structure and the pattern of the hadron spectrum

Following F&R we consider the scale-deformed supercharge operator $R_\lambda = Q + \lambda S$, where S is the generator of special conformal transformations.

The generator R_λ gives rise to a new scale-dependent Hamiltonian G which also closes under the graded algebra

$$G = \tfrac{1}{2}\{R_\lambda, R_\lambda^\dagger\}, \quad \{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0, \quad [R_\lambda, G] = [R_\lambda^\dagger, H] = 0$$

The new supercharge R_λ has the matrix representation

$$R_\lambda = \begin{pmatrix} 0 & r_\lambda \\ 0 & 0 \end{pmatrix}, \qquad R_\lambda^\dagger = \begin{pmatrix} 0 & 0 \\ r_\lambda^\dagger & 0 \end{pmatrix}$$

with

$$r_\lambda = -\partial_x + \frac{f}{x} + \lambda x, \qquad r_\lambda^\dagger = \partial_x + \frac{f}{x} + \lambda x$$

The parameter f is dimensionless and λ has the dimension $[M^2]$

A mass scale is introduced in the Hamiltonian without leaving the conformal group !

The Hamiltonian equation $G|E\rangle = E|E\rangle$ leads to the wave equations:

$$\boxed{\begin{aligned} \left(-\frac{d^2}{dx^2} + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} + \lambda^2 x^2 + 2\lambda f - \lambda\right)\phi_1 &= E\,\phi_1 \\ \left(-\frac{d^2}{dx^2} + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} + \lambda^2 x^2 + 2\lambda f + \lambda\right)\phi_2 &= E\,\phi_2 \end{aligned}}$$

thus to harmonic confinement, Regge trajectories, and a massless pion !

Light-front mapping and baryons

GdT, H. G. Dosch, S. J. Brodsky, PRD **91**, 045040 (2015)

Upon the substitution in the superconformal equations

$$x \mapsto \zeta, \qquad E \mapsto M^2, \qquad f \mapsto L + \tfrac{1}{2}$$
$$\phi_1 \mapsto \psi_-, \qquad \phi_2 \mapsto \psi_+$$

we recover the LF/AdS nucleon bound-state equations

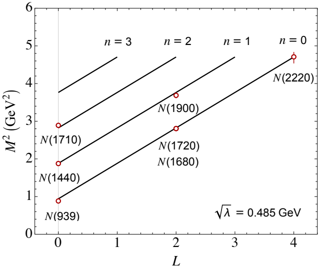
$$\boxed{\begin{aligned}
&\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L+1)\right)\psi_+ = M^2\psi_+ \\
&\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda L\right)\psi_- = M^2\psi_-
\end{aligned}}$$

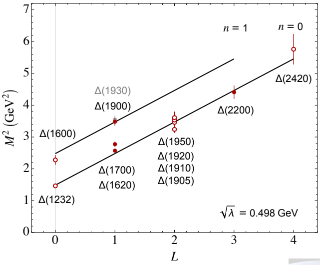
Eigenvalues

$$M^2 = 4\lambda(n + L + 1)$$

Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2)$$
$$\psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^2/2} L_n^{L+1}(\lambda\zeta^2)$$





See also: Abidin and Carlson (2009) and Gutsche, Lyubovitskij, Schmidt and Vega (2012)

Superconformal meson-baryon-tetraquark symmetry

H. G. Dosch, GdT, S. J. Brodsky, PRD **91**, 085016 (2015)

Upon the substitution in the superconformal equations

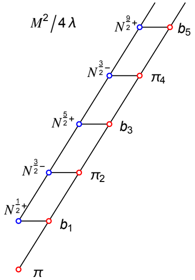
$$x \mapsto \zeta, \quad E \mapsto M^2,$$

$$\lambda \mapsto \lambda_B = \lambda_M, \quad f \mapsto L_M - \tfrac{1}{2} = L_B + \tfrac{1}{2}$$

$$\phi_1 \mapsto \phi_M, \quad \phi_2 \mapsto \phi_B$$

we find the LF meson/baryon bound-state equations

$$\boxed{\begin{aligned} &\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M(L_M - 1)\right)\phi_M = M^2\,\phi_M \\ &\left(-\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_N + 1)\right)\phi_B = M^2\,\phi_B \end{aligned}}$$



$$\Phi = \begin{pmatrix} \phi_M & \phi_B^- \\ \phi_B^+ & \phi_T \end{pmatrix}$$

Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $\boxed{L_M = L_B + 1}$

L_M is the LF angular momentum between the quark and antiquark in the meson and L_B between the active quark and spectator diquark cluster in the baryon

Full hadron 4-plet: meson-baryon-tetraquark

S. J. Brodsky, GdT, H. G. Dosch and C. Lorce, PLB **759**, 171 (2016)

Special role of the pion as a unique state of zero energy $\quad Q^\dagger |M, L\rangle = |B, L-1\rangle, \quad Q^\dagger |M, L=0\rangle = 0$

Zero mode invariant under deformations of scale λ

Spin-dependent Hamiltonian

$$H = \{Q_\lambda^\dagger, Q_\lambda\} + 2\lambda\, s$$

s internal spin of the meson, or the spin of the diquark cluster of the baryon partner

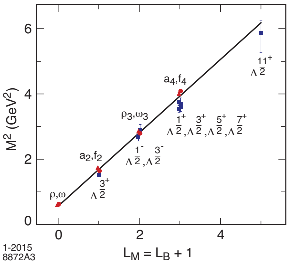
Supersymmetric 4-plet

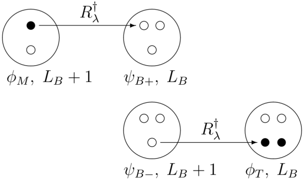
$$\begin{aligned} M_M^2 &= 4\lambda\,(n + L_M) + 2\lambda S \\ M_B^2 &= 4\lambda\,(n + L_B + 1) + 2\lambda S \\ M_T^2 &= 4\lambda\,(n + L_T + 1) + 2\lambda S \end{aligned}$$

Quark masses and CSB from longitudinal dynamics

$$\left(-\,\sigma^2 \partial_x \left(x(1-x)\,\partial_x\right) + \frac{m_q^2}{x} + \frac{m_{\bar q}^2}{1-x}\right)\chi(x) = \Delta M^2\,\chi(x)$$

$$M_\pi^2 = \Delta M^2 = \sigma(m_q + m_{\bar q}) + \mathcal{O}\big((m_q + m_{\bar q})^2\big)$$

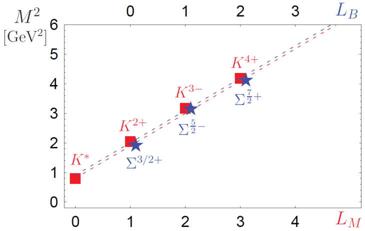




$\bar{\mathbf{3}} \to \mathbf{3} \otimes \mathbf{3} \qquad \mathbf{3} \to \bar{\mathbf{3}} \otimes \bar{\mathbf{3}}$

Recent work: Li and Vary (2021), GdT and Brodsky (2021), Ahmady *et al.* (2021), Shuryak and Zahed (2021), Weller and Miller (2021), Lyubovitskij and Schmidt (2022), Rinaldi *et al.* (2022)

HLFQCD predictions for the K^* and Σ^* trajectories with $\sqrt{\lambda} = 0.51$ GeV and quark mass corrections (H. G. Dosch, GdT and S. J. Brodsky (2015)



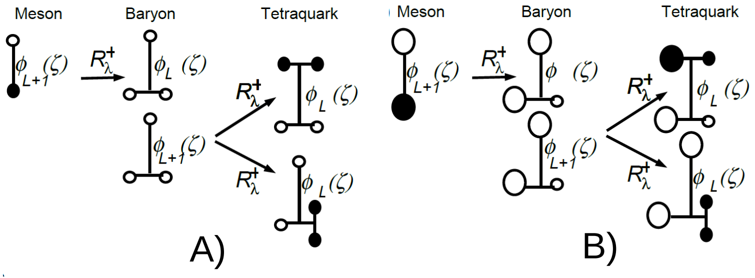
Predictions for double-heavy tetraquarks

quark content	J^P	predicted Mass [MeV]	strong decay	threshold [MeV]
$cq\overline{cq}$	0^+	3660	$\eta_c\pi\pi$	3270
$cc\overline{qq}^{(!)}$	1^+	3870	D^*D	3880
$bq\overline{bq}$	0^+	10020	$\eta_b\pi\pi$	9680
$bb\overline{qq}^{(!)}$	1^+	10230	B^*B	10800
$bc\overline{qq}^{(!)}$	0^+	6810	BD	7150

Predicted double heavy tetraquarks in HLFQCD. Exotics stable under strong interactions (!)
(H. G. Dosch, S. J. Brodsky, GdT, M. Nielsen and L. Zou (2020)

The doubly charmed stable boson T_{cc} with mass 3875 MeV was observed at LHCb a year later !

It completes the 4-plet with constituents $c\bar{c}$, ccq, $cc\bar{q}\bar{q}$:



A) Light hadrons (Tetraquark mixes with conventional hadrons)

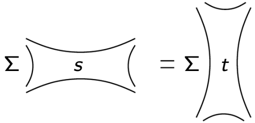
Example: 2^{++}, $\frac{3}{2}^{+}$, 1^{+} 4-plet $\boxed{f_2(1270),\ \ \Delta(1232),\ \ a_1(1260)}$

B) Double-heavy hadrons (Tetraquarks do not mix with conventional hadrons)

Example: 2^{++}, $\frac{3}{2}^{+}$, 1^{+} 4-plet $\boxed{\chi_{c2}(3565),\ \ \Xi_{cc}(3770),\ \ T_{cc}(3875)}$

From: H. G. Dosch, S. J. Brodsky, GdT, M. Nielsen and L. Zou (2020)

Veneziano amplitudes and holographic QCD



Veneziano 4-point amplitude amplitude (1968)

$$A(s,t) \sim B(1-\alpha(s), 1-\alpha(t))$$

where $\alpha(t) = \alpha_0 + \alpha' t$ is the linear Regge trajectory

Sum of poles in the direct s *or* crossed t-channels: Accounts for duality in strong interactions (Dolen-Horn-Schmid (1967))

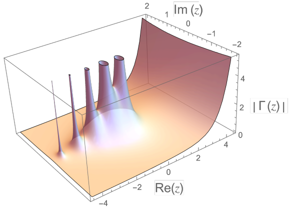
For fixed t and large s the result $A(s,t) \sim s^{\alpha(t)-1}$ is found

Scattering is exponentially suppressed with increasing t: It cannot produce collisions at large angles (soft scattering)

Veneziano model can be extended to N-particle amplitudes and/or external currents

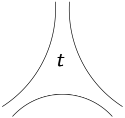
Pole structure and high-energy Regge behavior at tree level (nonperturbative mathematical structure)

Features of Regge theory and the Veneziano model as emerging properties of holographic QCD



Hadron form factors

Form Factor as a 3-point Veneziano amplitude



Ademollo and Del Giudice (1969), Landshoff and Polkinghorne (1970)

GdT, Liu, Sufian, Dosch, Brodsky and Deur (HLFHS 2018)

s-channel dependence is replaced by a fixed pole allowed by unitarity

$$F(t) \sim B(\gamma, 1 - \alpha(t)) \to Q^{-2\gamma} \quad \text{for large} \quad Q^2 = -t > 0$$

Compare with Brodsky-Farrar (B-F) hard counting rules (1973) for large Q^2

$$F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}$$

where the twist τ is the number of constituents in a given Fock component: $\gamma = \tau - 1$

The B-F asymptotic counting rules allow us to incorporate the underlying (hard) constituent pointlike structure of hadrons into the (soft) Veneziano amplitudes with external currents

$$\boxed{F_\tau(t) = \frac{1}{N_\tau} B(\tau - 1, 1 - \alpha(t))}$$

$\alpha(t)$ is the Regge trajectory of the VM which couples to the quarks in the hadron

For $\tau = N$, the number of constituents, the FF is an $N-1$ product of poles

$$F_\tau(Q^2) = \frac{1}{\left(1+\frac{Q^2}{M^2_{n=0}}\right)\left(1+\frac{Q^2}{M^2_{n=1}}\right)\cdots\left(1+\frac{Q^2}{M^2_{n=\tau-2}}\right)}$$

located at

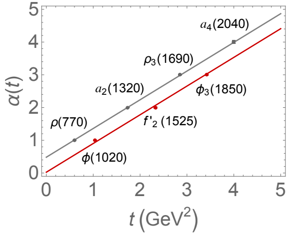
$$-Q^2 = M_n^2 = \frac{1}{\alpha'}\big(n+1-\alpha(0)\big)$$

It generates the radial spectrum of the exchanged t-channel vector mesons

For the ρ trajectory $\alpha_0 = 1/2$ and $\alpha' = 1/4\lambda$, thus

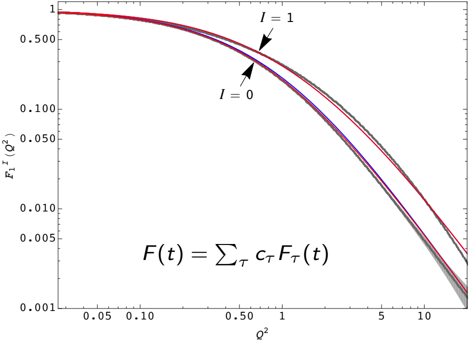
$$\boxed{M_n^2 = 4\lambda\left(n+\tfrac{1}{2}\right)}$$

which exactly matches the HLFQCD result for the ρ VM and its radial excitations for $n = 0, 1, 2, \ldots, \tau - 2$ (The "Regge daughter trajectories")



Note: The holographic SWM model, however, leads to leads to $M_n^2 = 4\lambda(n+1)$, thus to linear Regge trajectories with the same slope, but with the wrong intercept $\alpha_0 = 0$
Karch, Katz, Son, and Stephanov (2006)

Nucleon isospin form factors $F^{I=0,1}(t) = F_p(t) \pm F_n(t)$



HLFHS (2020): —— Valence contribution only

HLFHS (2020): —— Including $u\bar{u}$ and $d\bar{d}$

Ye et al. (2018): —— z-expansion data analysis

Quark distribution functions

GdT, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, A. Deur (HLFHS) PRL **120**, 182001 (2018)
T. Liu, R. S. Sufian, GdT, H. G. Dosch, S. J. Brodsky, A. Deur (HLFHS) PRL **124**, 082003 (2020)

Using the integral representation of the Beta function the FF is expressed in a reparametrization invariant form

$$F(t)_\tau = \frac{1}{N_\tau} \int_0^1 dx\, w'(x) w(x)^{-\alpha(t)} \left[1 - w(x)\right]^{\tau - 2}$$

with $w(0) = 0, \quad w(1) = 1, \quad w'(x) \geq 0$

Flavor FF is given in terms of the valence GPD $H^q_\tau(x, \xi = 0, t)$ at zero skewness

$$F^q_\tau(t) = \int_0^1 dx\, H^q_\tau(x, t) = \int_0^1 dx\, q_\tau(x) \exp[t f(x)]$$

with the profile function $f(x)$ and PDF $q(x)$ determined by $w(x)$

$$f(x) = \frac{1}{4\lambda} \log\left(\frac{1}{w(x)}\right)$$
$$q_\tau(x) = \frac{1}{N_\tau} [1 - w(x)]^{\tau - 2} w(x)^{-\alpha(0)} w'(x)$$

At $x \sim 0$, $w(x) \sim x$ from Regge behavior, and $w'(1) = 0$ to recover the counting rules at $x \to 1$, $q_\tau(x) \sim (1 - x)^{2\tau - 3}$ (inclusive-exclusive connection)

If $w(x)$ fixed by nucleon PDFs: $w(x) = x^{1-x}\, e^{-a(1-x)^2}$, the pion PDF is a prediction

Effective LFWFs

Effective LFWFs which incorporate the correct pole structure follow from

$$\begin{aligned} H_\tau^q(x,t) &= \int_0^1 dx\, q_\tau(x) \exp[tf(x)] \\ &= 2\pi \int_0^\infty db\, b\, J_0\big(bQ(1-x)\big) |\psi_{\rm eff}(x,b)|^2 \end{aligned}$$

We find

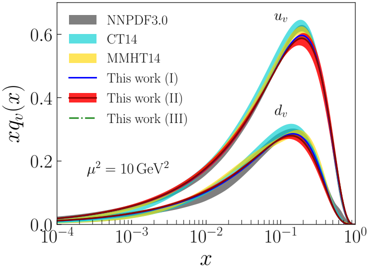
$$\psi_{\rm eff}^\tau(x,\mathbf{b}_\perp) = \frac{1}{2\sqrt{\pi}} \sqrt{\frac{q_\tau(x)}{f(x)}} (1-x) \exp\left[-\frac{(1-x)^2}{8f(x)}\, \mathbf{b}_\perp^2\right]$$

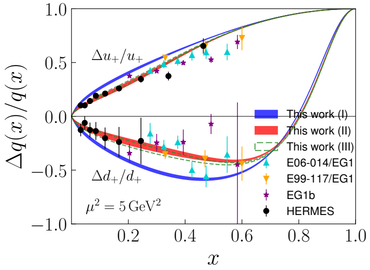
in the transverse impact space representation and

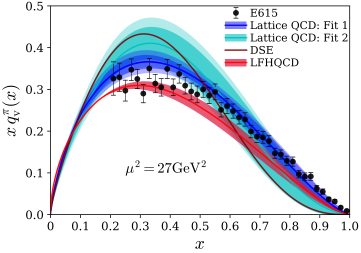
$$\psi_{\rm eff}^\tau(x,\mathbf{k}_\perp) = 8\pi \frac{\sqrt{q_\tau(x) f(x)}}{1-x} \exp\left[-\frac{2f(x)}{(1-x)^2}\, \mathbf{k}_\perp^2\right]$$

in transverse momentum space

Unpolarized and polarized GPDs and PDFs   HLFHS Collaboration (2018, 2020)







Separation of chiralities from the axial current
Coefficients c_τ are fixed from the vector current

Regge trajectory from HLFQCD

$$\alpha_A(t) = \frac{t}{4\lambda}$$

$$\lim_{x\to 1} \frac{\Delta q(x)}{q(x)} = 1, \quad \lim_{x\to 0} \frac{\Delta q(x)}{q(x)} = 0$$

DGLAP NNLO evolution from initial scale $\mu \simeq 1$ GeV from soft-hard matching in α_s

Infrared behavior of the strong coupling

A. Deur, S. J. Brodsky and GdT (2010, 2015, 2016, 2017)

Initial DGLAP evolution scale from IR-UV matching of QCD coupling

IR behavior of strong coupling in HLFQCD for the HLFQCD dilaton $e^{\lambda z^2}$

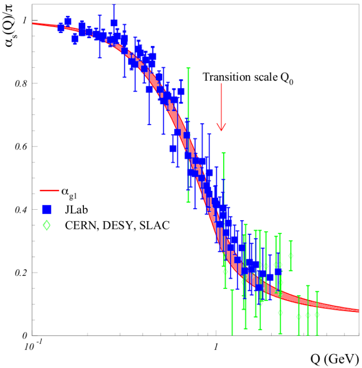
$$\alpha_s^{IR}(Q^2) \sim \int_0^\infty z dz J_0(zQ) e^{-\lambda z^2}$$
$$\sim e^{-Q^2/4\lambda},$$

with $\alpha_{g1}^{IR}(Q^2) = \pi$

Λ_{QCD} and transition scale Q_0 from matching perturbative and nonperturbative regimes for $\sqrt{\lambda} = 0.534 \pm 0.05$ GeV

Transition scale: $Q_0^2 \simeq 1$ GeV^2

Connection between proton mass, $M_p^2 = 4\lambda$, the ρ mass, $M_\rho^2 = 2\lambda$, and the perturbative QCD scale Λ_{QCD} in any RS !



Comparison of QCD strong coupling data from Bjorken sum with holographic and pQCD predictions matched at the IR-UV transition scale Q_0 (red)

Similar behavior of the IR coupling was obtained from the DSE

D. Binosi *et al.* (2017) and Z. F. Cui, et al. Chin. Phys. C **44**, 083102 (2020)

Intrinsic charm-anticharm asymmetry in the proton

Sufian, T. Liu, Alexandru, Brodsky, GdT, Dosch, Draper, K. F. Liu and Y. B. Yang (2020)

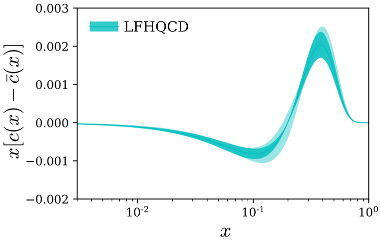
Intrinsic charm in the proton introduced by Brodsky, Hoyer, Peterson and Sakai (1980)

Charm FF normalization computed with with three gauge ensembles in LGTH (one at the physical pion mass) and charm distribution from HLFQCD

Intrinsic charm asymmetry $c(x) - \bar{c}(x)$,

$$c(x) - \bar{c}(x) = \sum_{\tau} c_\tau \left(q_\tau(x) - q_{\tau+1}(x) \right)$$

with $\int_0^1 dx [c(x) - \bar{c}(x)] = 0$



J/Ψ Regge trajectory

$$\alpha(t)_{J/\Psi} = -2.066 + \frac{t}{4\lambda_c}, \quad \lambda_c = 0.874 \ \mathrm{GeV}^2$$

from HLFQCD and HQET

Nielsen, Brodsky, GdT, Dosch, Navarra and Zou (2018)

Gravitational form factors and gluon distribution functions

GdT, H. G. Dosch, T. Liu, R. S. Sufian, S. J. Brodsky, A. Deur (HLFHS) PRD **104**, 114005 (2021)

Spin-2 gluon gravitational FF $A(t)$ from the coupling of the metric fluctuations induced by the spin-two Pomeron with the energy momentum tensor in AdS

$$\int d^4x\, dz \sqrt{g}\, h_{MN} T^{MN}$$

$$\boxed{A^g_\tau(t) \sim B(\tau - 1, 2 - \alpha_P(t))}$$

with Pomeron Regge trajectory

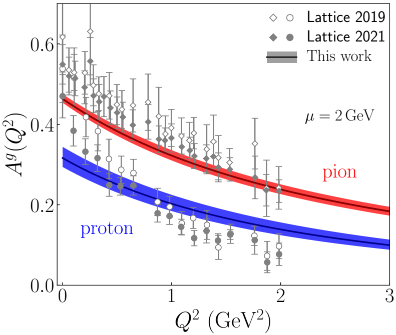
$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t$$

where $\alpha_P(0) \simeq 1.08$ and $\alpha' = 0.25\ \mathrm{GeV}^{-2}$

Radial spectrum from t-channel poles in the 2^{++} trajectory

$$-Q^2 = M_n^2 = \frac{1}{\alpha'}\left(n + 2 - \alpha(0)\right)$$

with $M_0 \simeq 1.92$ GeV



Lattice data from Shanahan *et al.* (2018) and Pefkou *et al.* (2021)

Intrinsic gluon distribution in the proton and pion

Gluon GPD $H^g_\tau(x,t) = g_\tau(x) e^{tf(x)}$

$$f(x) = \alpha'_P \log\left(\frac{1}{w(x)}\right),$$

$$g_\tau(x) = \frac{1}{N_\tau}\frac{w'(x)}{x}[1-w(x)]^{\tau-2} w(x)^{1-\alpha_P(0)}$$

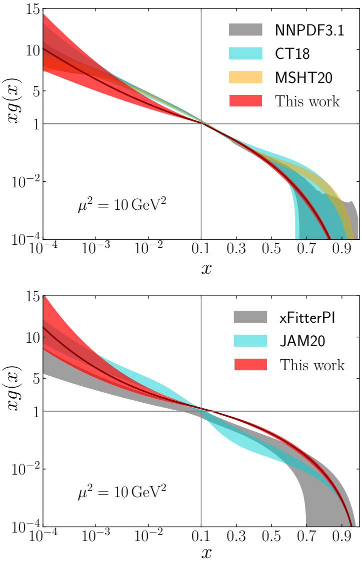
Normalization of $A^g(0)$ determined from the sum rule:

$$\sum_q \langle x\rangle_q + \sum_{\bar q}\langle x\rangle_{\bar q} + \langle x\rangle_g = 1$$

Basic parameters fixed in quark sector:
No adjustable parameters

Single Pomeron (HLFHS 2022))

Hard Pomeron from the evolution of the nonperturbative gluon distribution function



Gluon PDF: Proton upper figure and pion lower figure

Thank you !