
Chiral anomaly and the pion properties in the light-front quark model

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Based on [HMC, C.R. Ji, PRD 102, 036005\(2020\)](#)

Outline

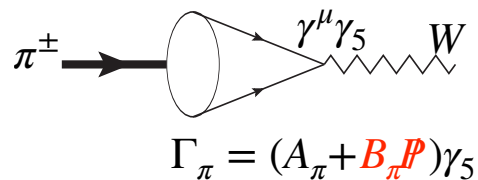
1. Motivation

2. Model Description: Light-Front Quark Model(LFQM)

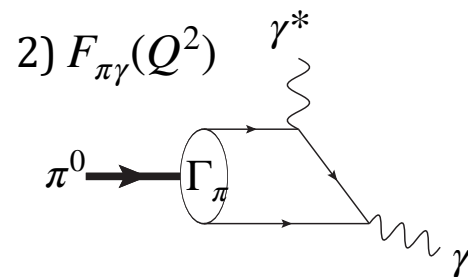
- **Role of axial-vector coupling** in the chiral limit ($M_\pi, m_Q \rightarrow 0$)

3. Application

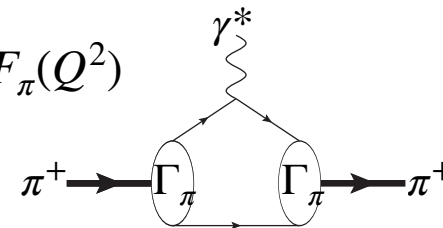
1) f_π



2) $F_{\pi\gamma}(Q^2)$



3) $F_\pi(Q^2)$

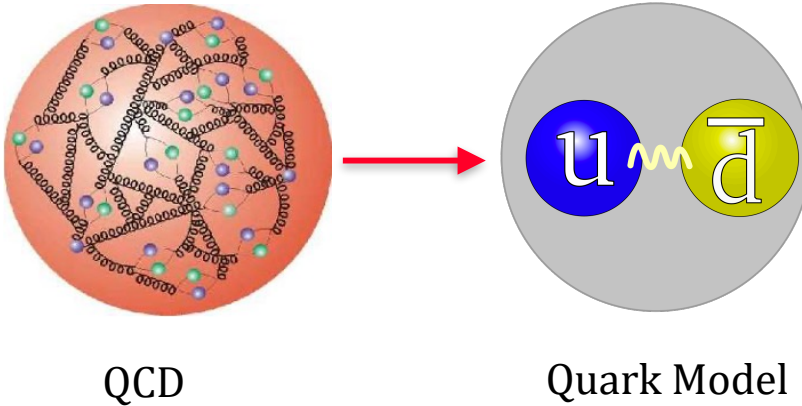


4. Numerical Results

5. Conclusion

1. Motivation

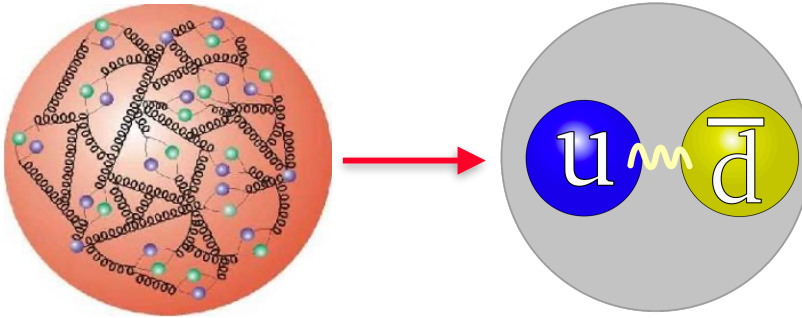
$$|\pi\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \dots \equiv \Psi_{Q\bar{Q}}^\pi|Q\bar{Q}\rangle$$



- In our previous LFQM for the pion, we investigated
 1. **Effect of the mass evolution on $F_\pi(Q^2)$ in a LFBS model** [Kissinger, HMC, C. Ji, PRD**63**, 113005(01)]
[HMC, C. Ji, PRD**74**, 093010(06); **77**, 113004(08)]
 2. **Constraint of conformal symmetry for pion form factor both in space- and time-like regions**
→ confirmed the AdS/CFT correspondence [Brodsky and de Teramond, PLB**582**,211(04), PRL**96**, 201601(06)]
 3. **Consistency of the LFQM with chiral symmetry** [HMC, C. Ji, PRD**91**, 014018(15)]
→ complication of LF zero mode and its resolution
 4. **$\pi^0 \rightarrow \gamma\gamma^*$ both in space- and time-like regions** [HMC, H. Ryu, C. Ji, PRD**96**, 056008(17)]
→ developed a new direct method to explore the timelike region

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QCD

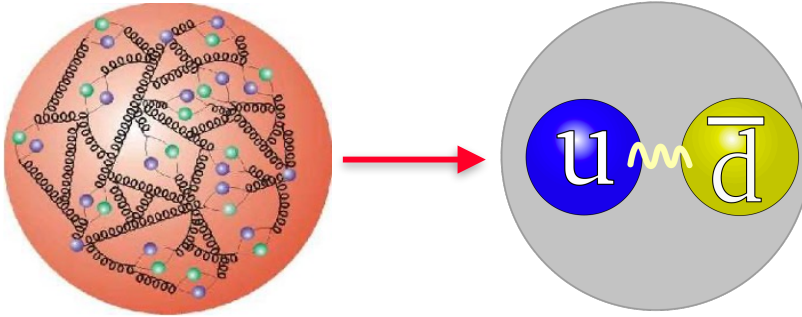
Quark Model

Hard to explain the
 $\Gamma_\pi = \gamma_5$ Chiral limit phenomena...

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QCD

Quark Model

Higher Fock states
(Non-trivial QCD vacuum)



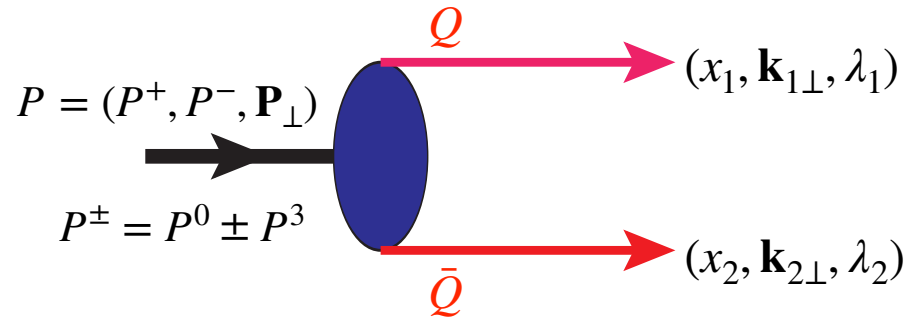
Constituent Q and \bar{Q}

$$\Gamma_\pi = (A_\pi + B_\pi \not{P})\gamma_5$$

- The main purpose of this work is to explore “the correlation between the nontrivial QCD vacuum effect and the constituent quark mass” through the analysis of $f_\pi, F_{\pi\gamma}(Q^2), F_\pi(Q^2)$ with an emphasis on the role of axial-vector coupling in the exact chiral limit.

2. Model Description

$$|\pi\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \dots \equiv \Psi_{Q\bar{Q}}^\pi|Q\bar{Q}\rangle$$



$$x_i = \frac{k_i^+}{P^+} \quad \sum_{i=1}^2 x_i = 1, \quad \sum_{i=1}^2 \mathbf{k}_{i\perp} = 0$$

$$\Psi_{Q\bar{Q}}^\pi \equiv \Psi_\pi(x_i, \mathbf{k}_{i\perp}, \lambda_i) = \underbrace{\phi_R(x_i, \mathbf{k}_{i\perp})}_{\text{Radial w.f.}} \underbrace{\chi(x_i, \mathbf{k}_{i\perp}, \lambda_i)}_{\text{Spin-orbit w.f.}}$$

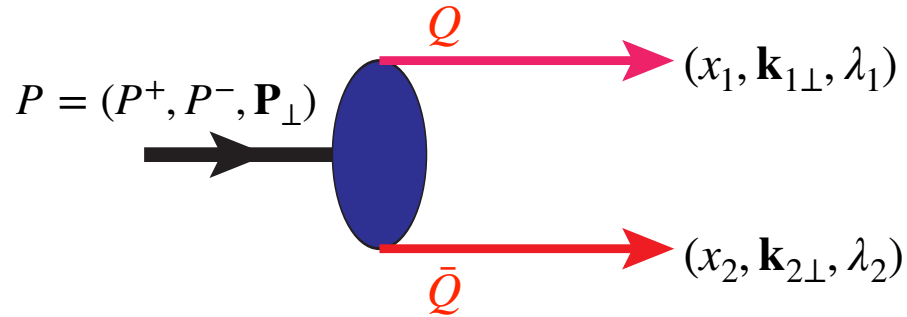
Normalization: $\langle \Psi_{Q\bar{Q}}^\pi | \Psi_{Q\bar{Q}}^\pi \rangle = P_{Q\bar{Q}}$

Radial part:

$$\phi_R(x, \mathbf{k}_\perp) = \sqrt{P_{Q\bar{Q}}} \frac{4\pi^{3/4}}{\beta^{3/2}} \sqrt{\frac{\partial k_z}{\partial x}} e^{-\frac{\bar{k}^2}{2\beta^2}},$$

$$\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |\phi_R(x, \mathbf{k}_\perp)|^2 = P_{Q\bar{Q}}.$$

2. Model Description



$$M_0^2 = \frac{m^2 + \mathbf{k}_{1\perp}^2}{x_1} + \frac{m^2 + \mathbf{k}_{2\perp}^2}{x_2}$$

Spin-orbit part: $\chi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) = \mathcal{N} \bar{u}_{\lambda_1}(k_1) \Gamma_\pi v_{\lambda_2}(k_2) \rightarrow \langle \chi_{\lambda_1\lambda_2} | \chi_{\lambda_1\lambda_2} \rangle = 1$

$$\Gamma_\pi = (M_\pi + B_\pi \mathbf{P}) \gamma_5$$

$$\chi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) = \begin{pmatrix} \chi_{\uparrow\uparrow} & \chi_{\uparrow\downarrow} \\ \chi_{\downarrow\uparrow} & \chi_{\downarrow\downarrow} \end{pmatrix} \propto \begin{pmatrix} -k^L \mathcal{M} & m \mathcal{M} + x(1-x) B_\pi \epsilon_B \\ -m \mathcal{M} - x(1-x) B_\pi \epsilon_B & -k^R \mathcal{M} \end{pmatrix}$$

$$\mathcal{M} = M_\pi + 2B_\pi m, \epsilon_B = M_\pi^2 - M_0^2, \quad k^{R(L)} = k_x \pm ik_y$$

$$\chi_{\lambda_1 \lambda_2}(x, \mathbf{k}_\perp) = \mathcal{N} \bar{u}_{\lambda_1}(k_1) (M_\pi + B_\pi \not{P}) \gamma_5 v_{\lambda_2}(k_2)$$

Chiral limit
($M_\pi, m \rightarrow 0$)

Zero-binding limit
($M_\pi \rightarrow M_0$ or $\epsilon_B = 0$)

$$\Gamma_\pi \propto \gamma_5$$

$$\chi_{\lambda_1 \lambda_2}^{\text{chiral}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \text{sgn}(-B_\pi)$$

$$\chi_{\lambda_1 \lambda_2}^{(M_0)} = \frac{1}{\sqrt{2(\mathbf{k}_\perp^2 + m^2)}} \begin{pmatrix} -k^L & m \\ -m & -k^R \end{pmatrix}$$

Invariant mass scheme

$$\text{sgn}(-B_\pi) = -\text{sgn}(B_\pi)$$

$$\text{sgn}(B_\pi) = \begin{cases} 1 & \text{for } B_\pi > 0 \\ -1 & \text{for } B_\pi < 0 \\ 0 & \text{for } B_\pi = 0 \end{cases}$$

Chiral limit
($m \rightarrow 0$)

$$\chi_{\uparrow\downarrow}^{(M_0)} = \chi_{\downarrow\uparrow}^{(M_0)} = 0$$

$$\chi_{\lambda_1 \lambda_2}^{\text{chiral}} = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$$

Axial vector coupling (i.e. $B_\pi < 0$) is absolutely required to have non-zero chiral limit!

3. Application

(1) Pion decay constant

$$\langle 0 | \bar{q} \gamma^\mu (1 - \gamma_5) q | \pi(P) \rangle = i f_\pi P^\mu$$



Using $\mu = +$

$$f_\pi = 2\sqrt{2N_c} \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_\pi(x, \mathbf{k}_\perp),$$

$$\psi_\pi(x, \mathbf{k}_\perp) = \frac{1}{\sqrt{2}} (\chi_{\uparrow\downarrow} - \chi_{\downarrow\uparrow}) \phi_R(x, \mathbf{k}_\perp)$$

Twist-2 pion Distribution amplitude(DA):

$$\phi_\pi(x) = \int^{Q^2} \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_\pi(x, \mathbf{k}_\perp),$$

$$\int_0^1 dx \phi_\pi(x) = \frac{f_\pi}{2\sqrt{2N_c}}.$$

In the chiral limit, we obtain

$$f_\pi^{\text{chiral}} = \sqrt{P_{Q\bar{Q}}} \frac{\sqrt{3}\beta}{2^{3/4}\pi^{1/4}} \Gamma\left(\frac{5}{4}\right),$$

$$\phi_\pi^{\text{chiral}}(x) = \frac{2\sqrt{2}f_\pi^{\text{chiral}}}{\sqrt{3}\pi} \sqrt{x(1-x)} = \phi_\pi^{\text{AdS/CFT}}(x)$$

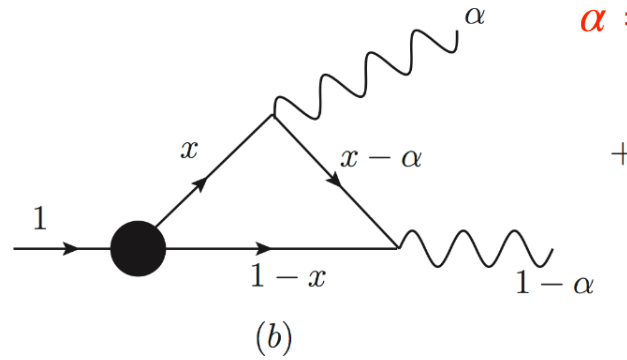
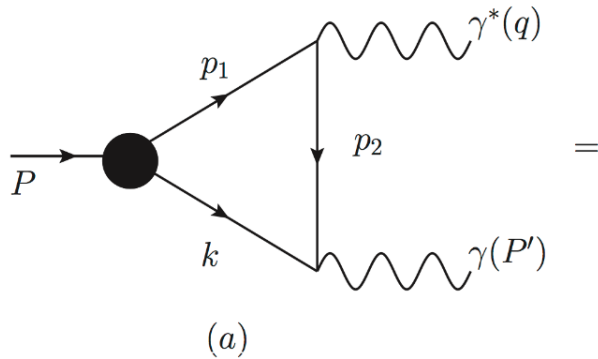
[Brodsky, de Teramond 06,08]

3. Application

(2) $\pi^0 \rightarrow \gamma\gamma^*$ transition form factor (TFF)

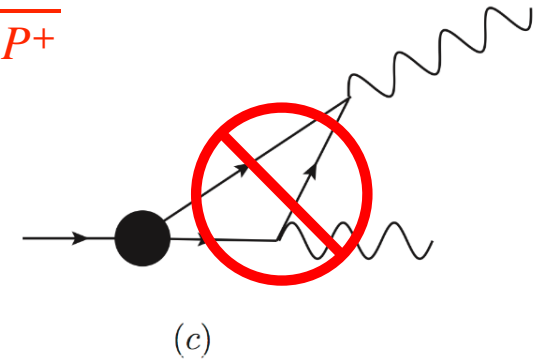
$$\langle \gamma(P - q) | J_{\text{em}}^\mu | \pi^0(P) \rangle = ie^2 F_{\pi\gamma}(Q^2) \epsilon^{\mu\nu\rho\sigma} P_\nu \epsilon_\rho q_\sigma,$$

$$q^2 = q^+ q^- - \mathbf{q}_\perp^2$$



$$(\alpha < x < 1)$$

$$\alpha = \frac{q^+}{P^+}$$



$$(0 < x < \alpha)$$

(i) For $q^+ = 0$ frame ($\alpha = 0$): **spacelike region**, i.e. $q^2 = -\mathbf{q}_\perp^2 = -Q^2 < 0$

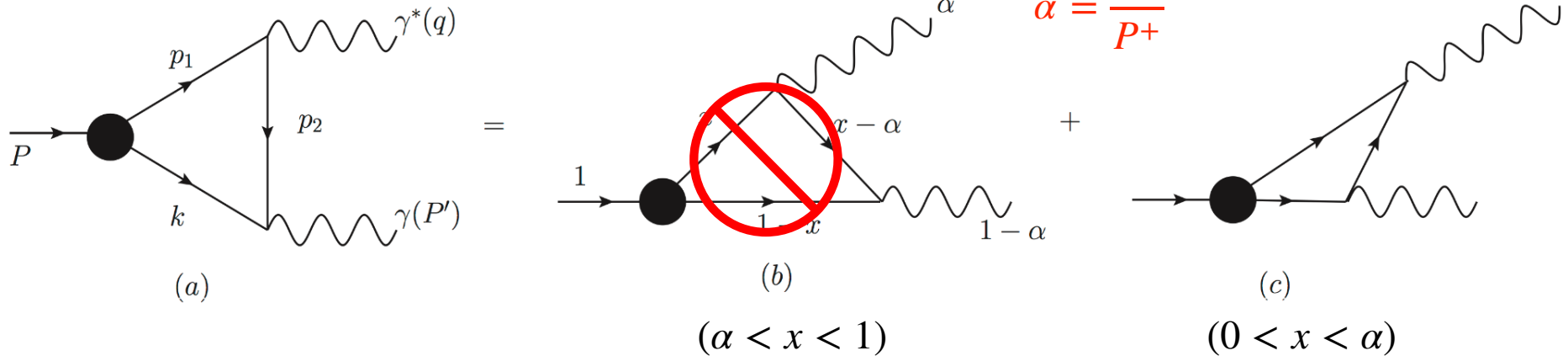
$$F_{\pi\gamma}(q^2) = \frac{e_u^2 - e_d^2}{\sqrt{2}} \frac{\sqrt{2N_c}}{4\pi^3} \int_0^1 (1-x) dx \int d^2\mathbf{k}_\perp \frac{\psi_\pi(x, \mathbf{k}_\perp)}{\mathbf{k}'_\perp{}^2 + m^2}, \quad \mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$$

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$$q^2 = q^+ q^- - \mathbf{q}_\perp^2$$



(ii) For $q^+ = P^+$ with $\mathbf{q}_\perp = 0$ frame ($\alpha = 1$): **timelike region**, i.e. $q^2 = q^+ q^- > 0$

$$F_{\pi\gamma}(q^2) = \frac{e_u^2 - e_d^2}{\sqrt{2}} \frac{\sqrt{2N_c}}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \frac{\psi_\pi(x, \mathbf{k}_\perp)}{M_0^2 - q^2}$$

HMC, Ryu, Ji, PRD 96, 056008(17)

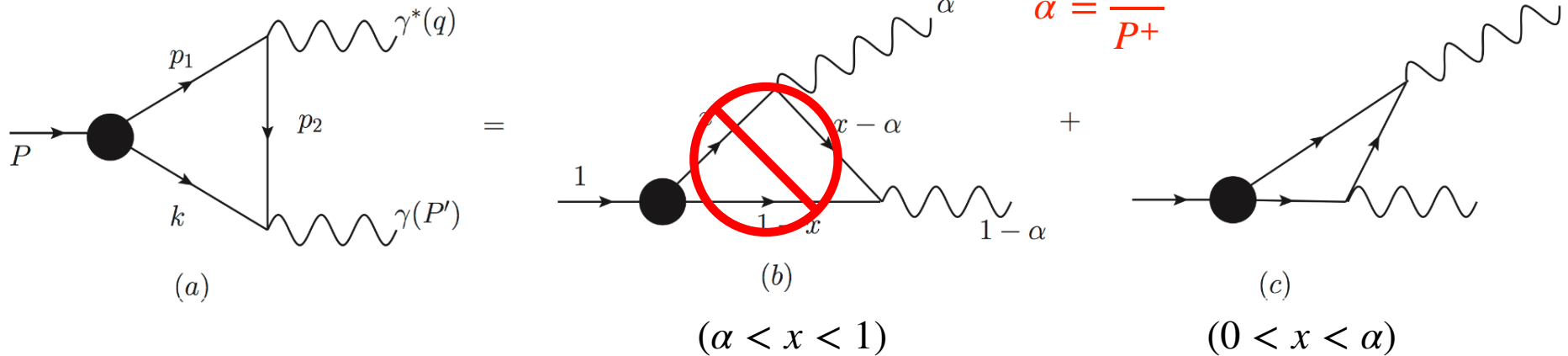
→ Imaginary part starts
at $q_{\text{th}}^2 = 4m^2$

3. Application

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$$Q^2 F_{\pi\gamma}(Q^2) \rightarrow \text{constant as } Q^2 \rightarrow \infty$$

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- TFF at $Q^2 = 0$

$$\Gamma_{\pi^0 \rightarrow \gamma\gamma} = \frac{\pi}{4} \alpha_{\text{em}}^2 M_\pi^3 |F_{\pi\gamma}(0)|^2$$

$$F_{\pi\gamma}^{\text{exp}}(0) = 0.272(3) \text{ GeV}^{-1}$$

Adler-Bell-Jackiw(ABJ) anomaly (or chiral anomaly):

$$F_{\pi\gamma}^{\text{ABJ}}(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi^{\text{Exp}}} \simeq 0.276 \text{ GeV}^{-1}$$

[Adler 69, Bell & Jackie 69]

To fit both

$$(F_{\pi\gamma}^{\text{Exp}}(0), f_\pi^{\text{Exp}})$$

$P_{Q\bar{Q}} < 0.1$ is required!

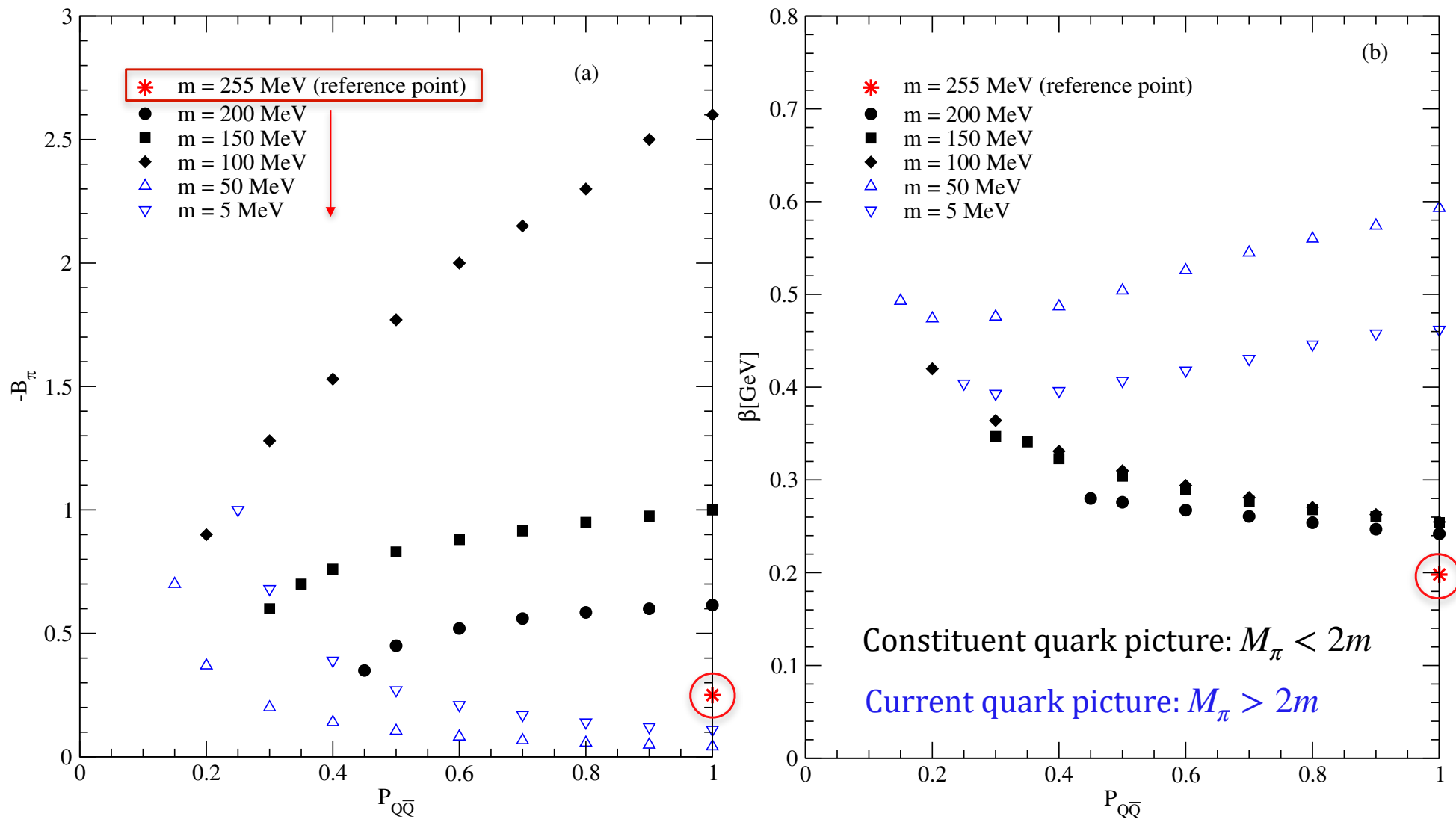
Significant higher Fock-states
contribute in the chiral limit !

$P_{Q\bar{Q}}$ increases as m increases!

$$F_{\pi\gamma}^{\text{chiral}}(0) = \frac{\sqrt{\frac{\pi^3}{32}} [\Gamma(\frac{1}{4})]^2 P_{Q\bar{Q}}}{2\sqrt{2}\pi^2 f_\pi^{\text{chiral}}}$$

4. Numerical Results

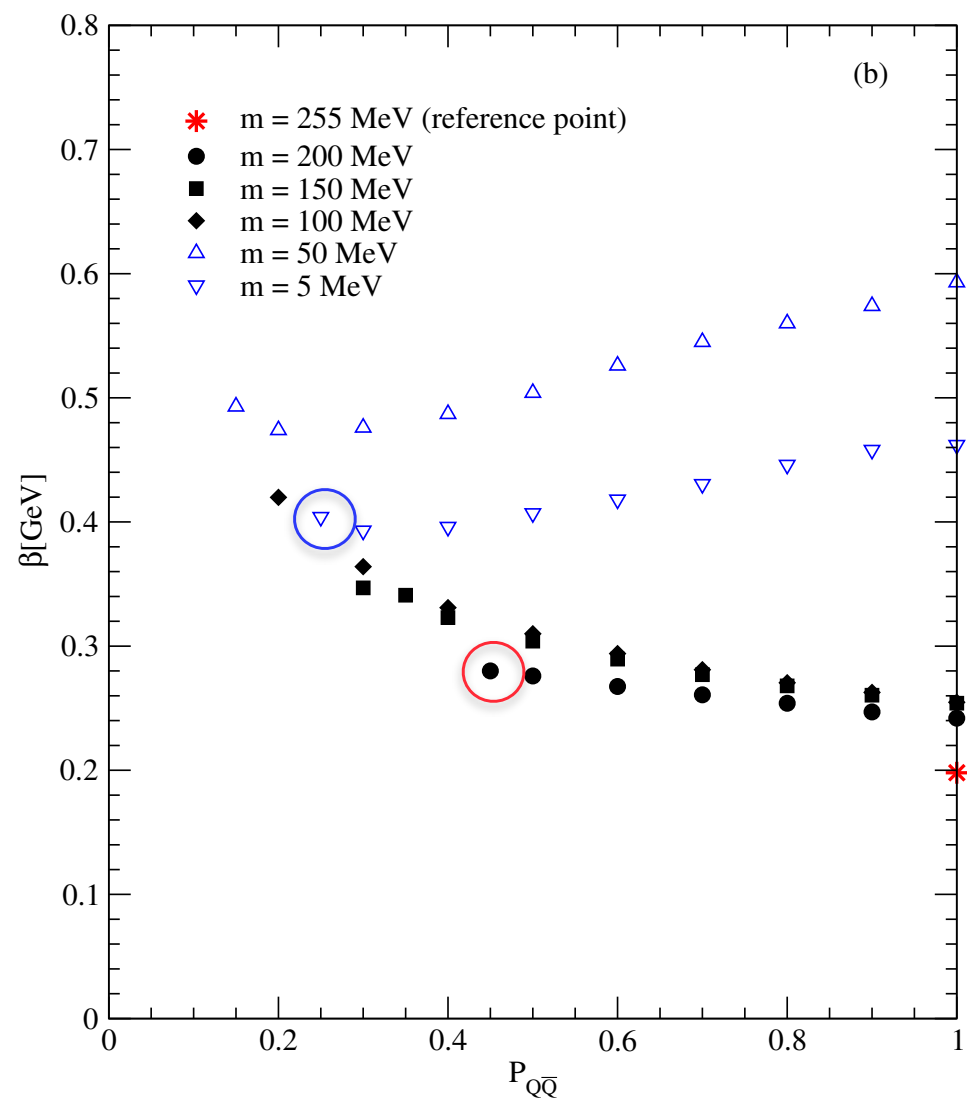
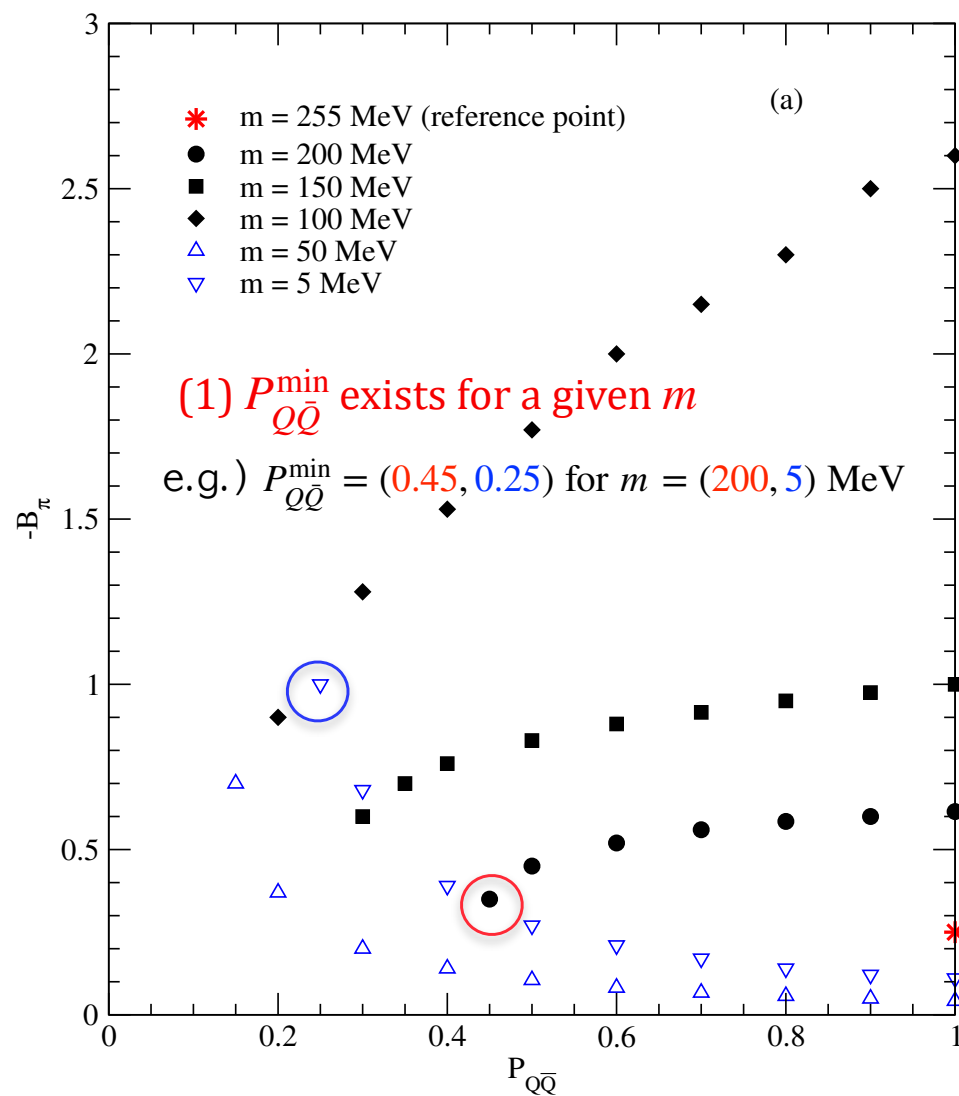
- Possible solution sets for $(-B_\pi$ vs $P_{Q\bar{Q}}$) and $(\beta$ vs $P_{Q\bar{Q}})$ satisfying both f_π^{Exp} and $F_{\pi\gamma}^{\text{Exp}}(0)$.



c.f.) $(M_\pi, m) = (135, 5)$ MeV satisfies the GMOR relation $M_\pi^2 f_\pi^2 = -2(m_q + m_{\bar{q}})\langle q\bar{q} \rangle$

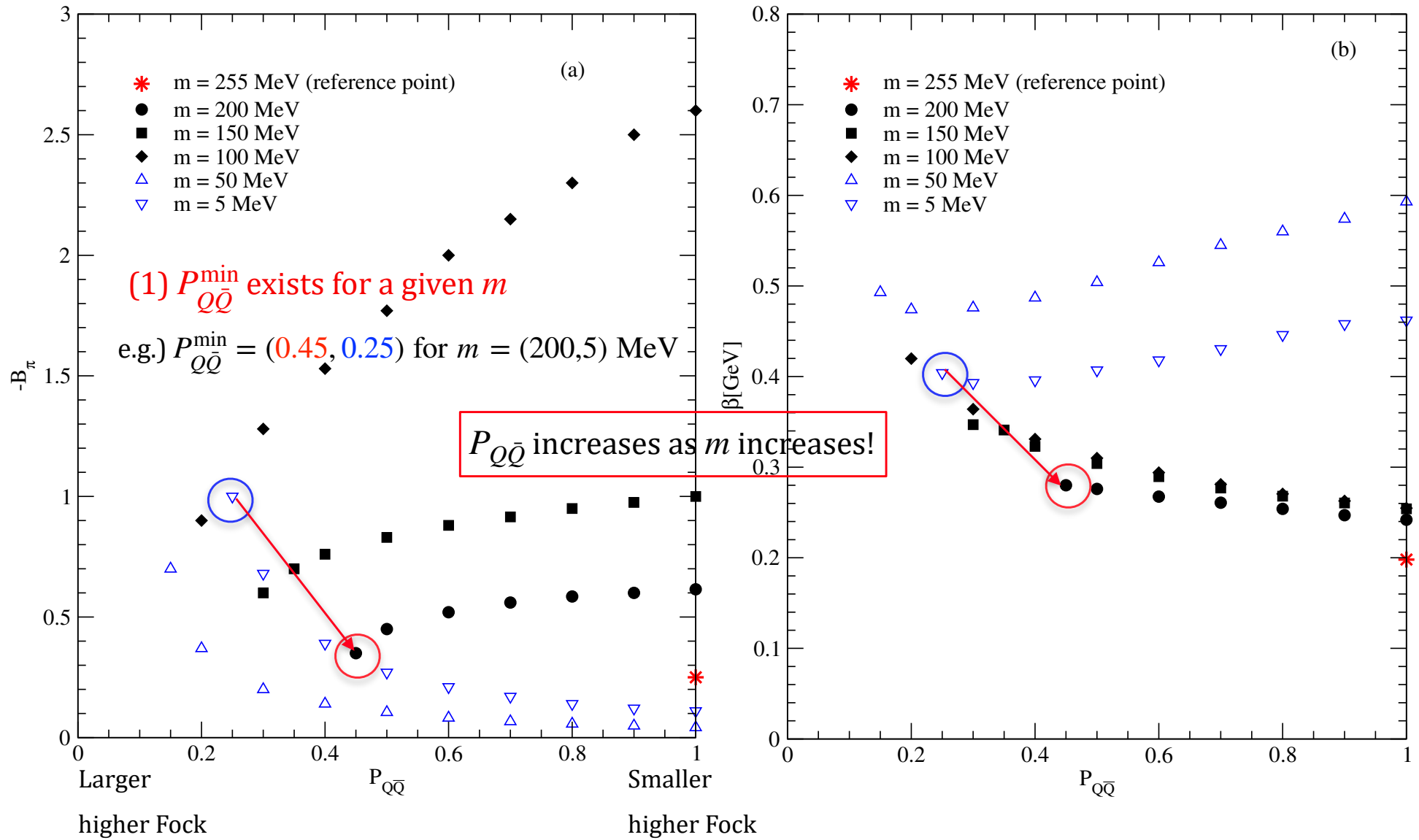
4. Numerical Results

- Our main findings for the model parameters:



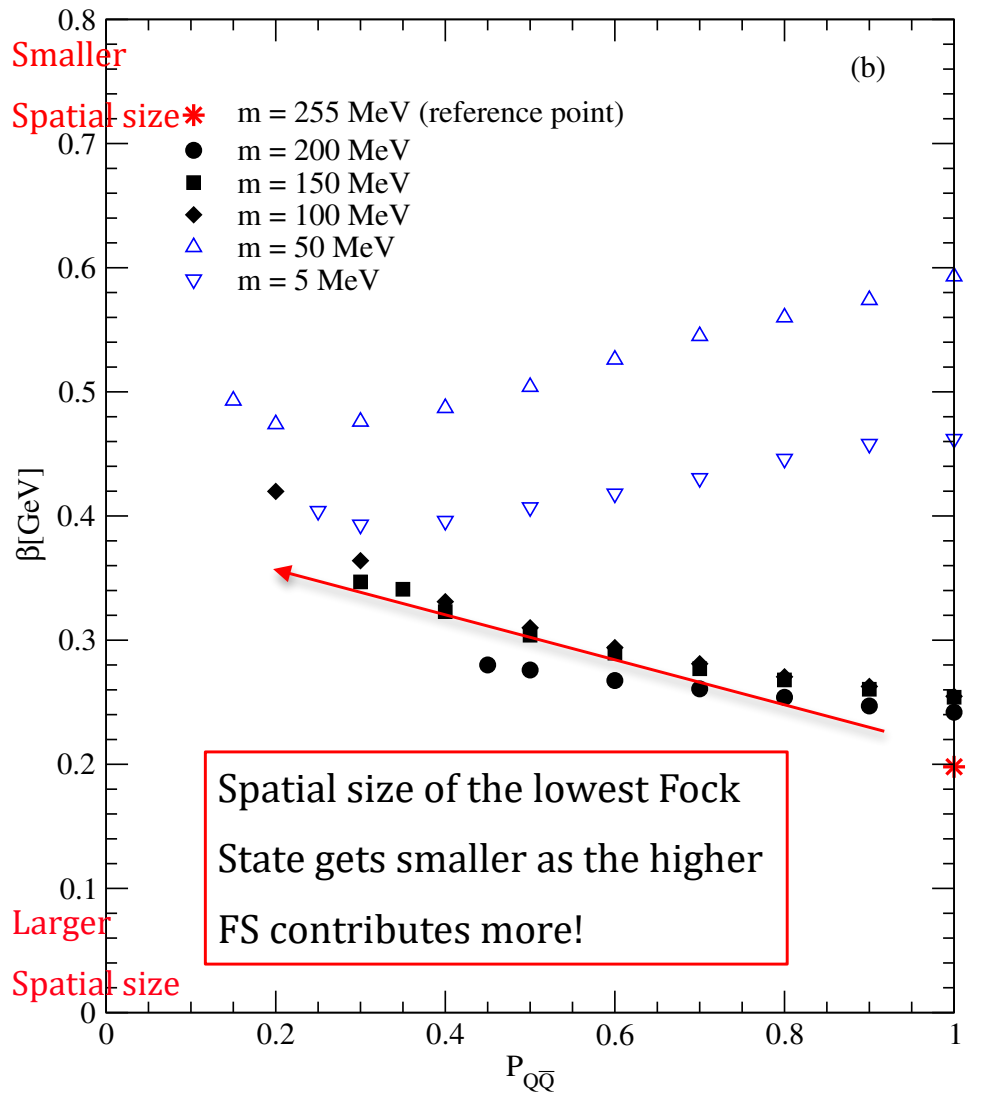
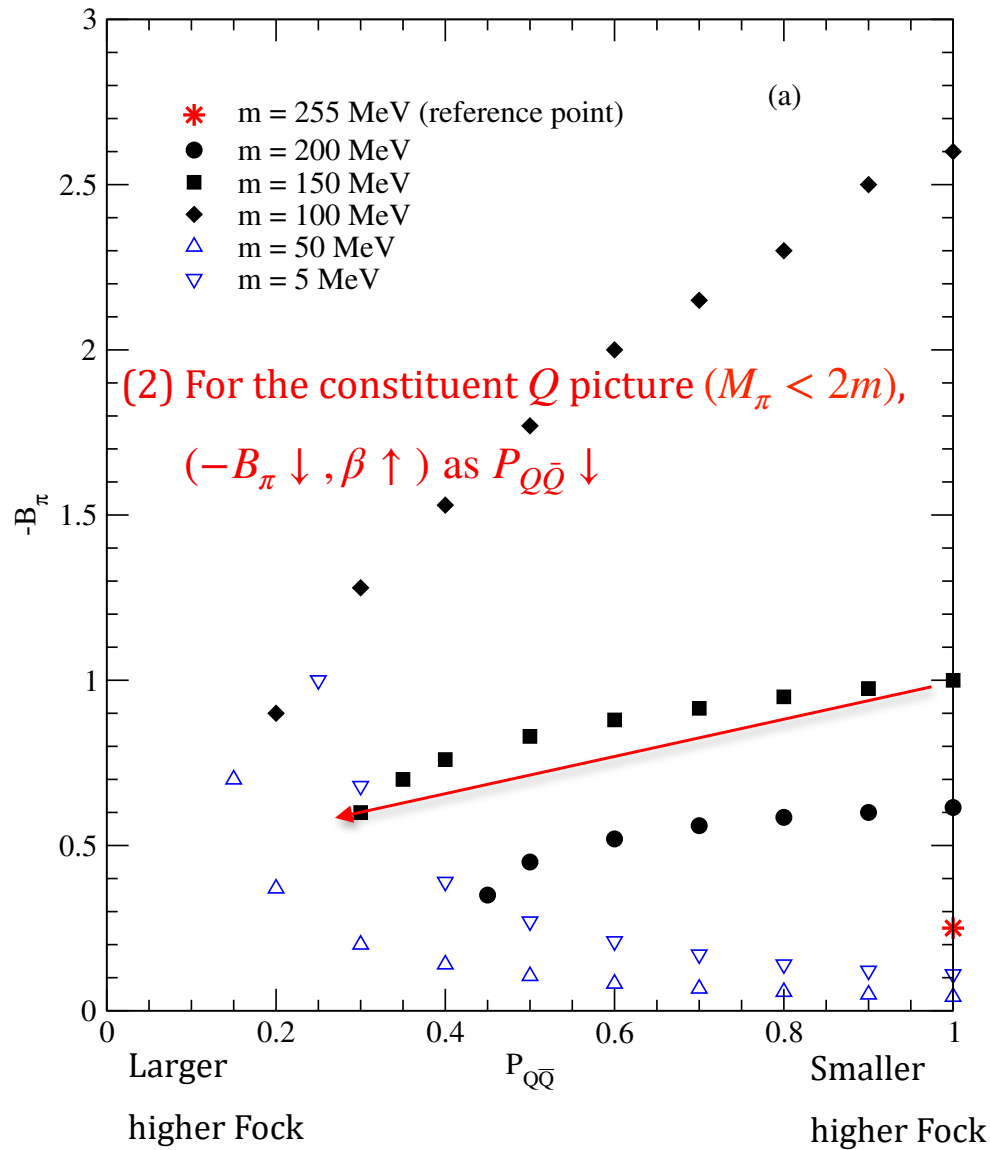
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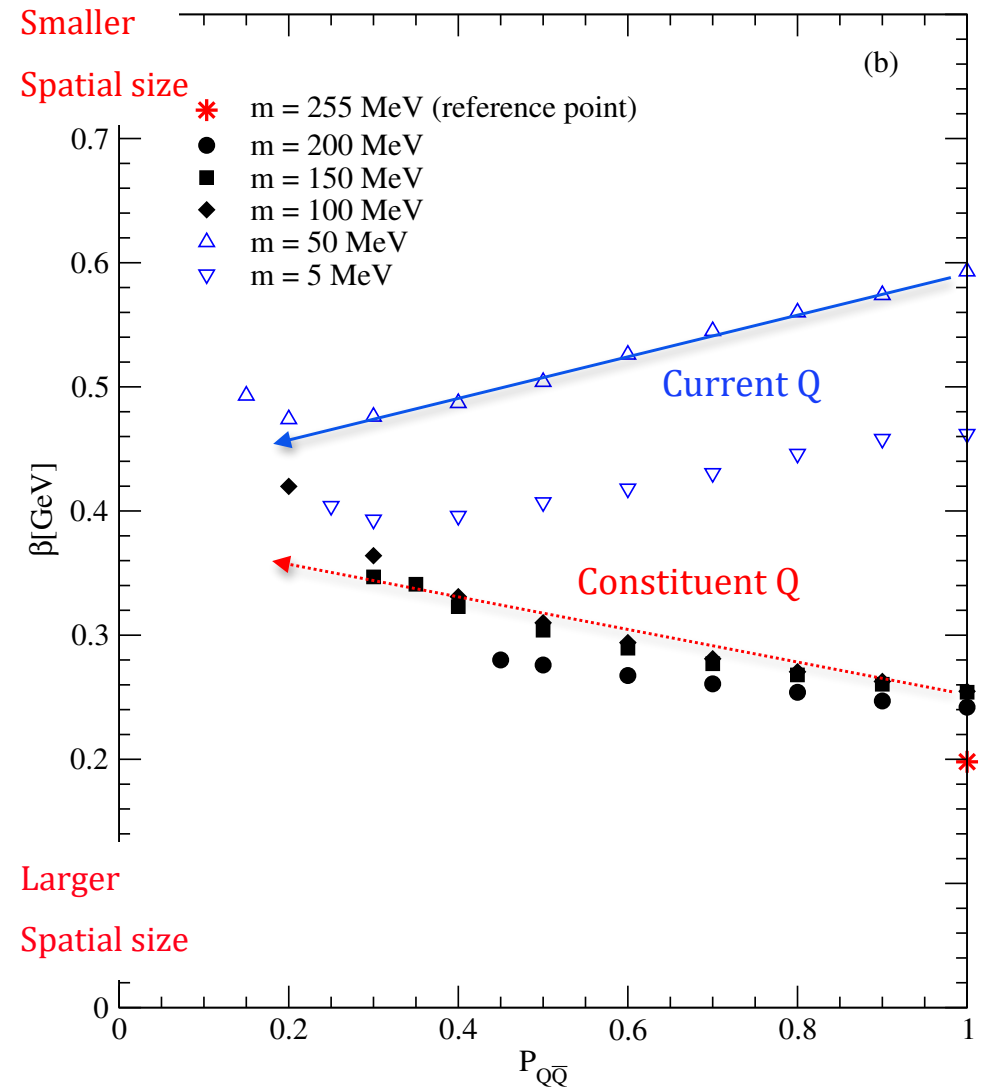
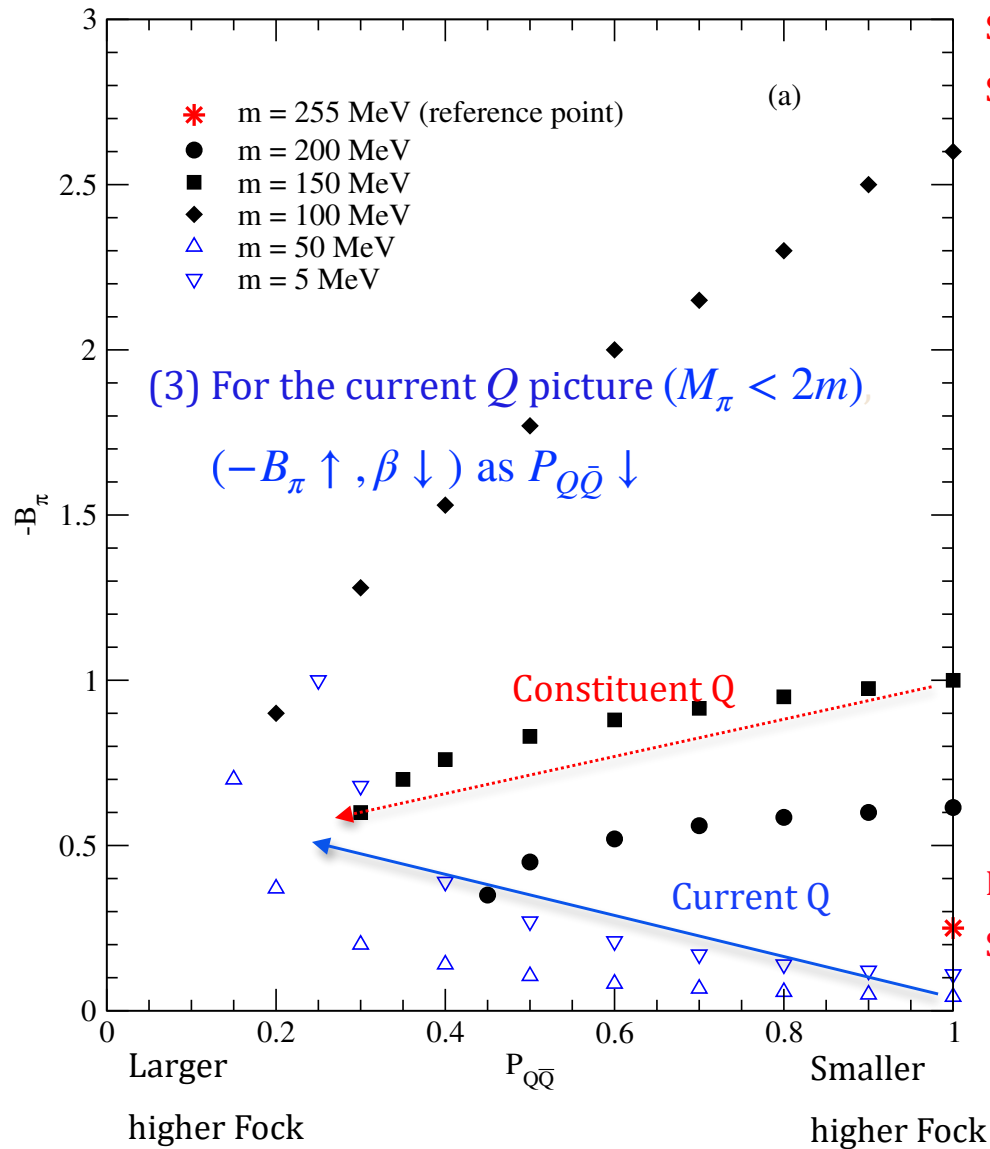
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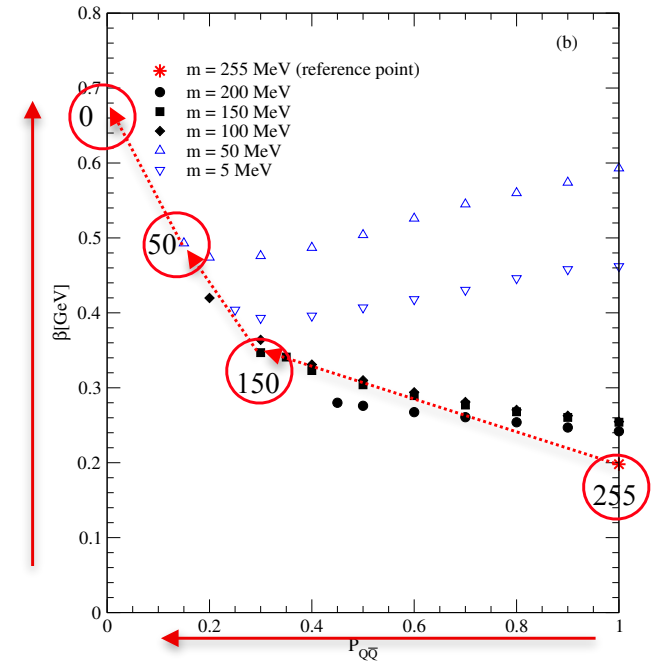
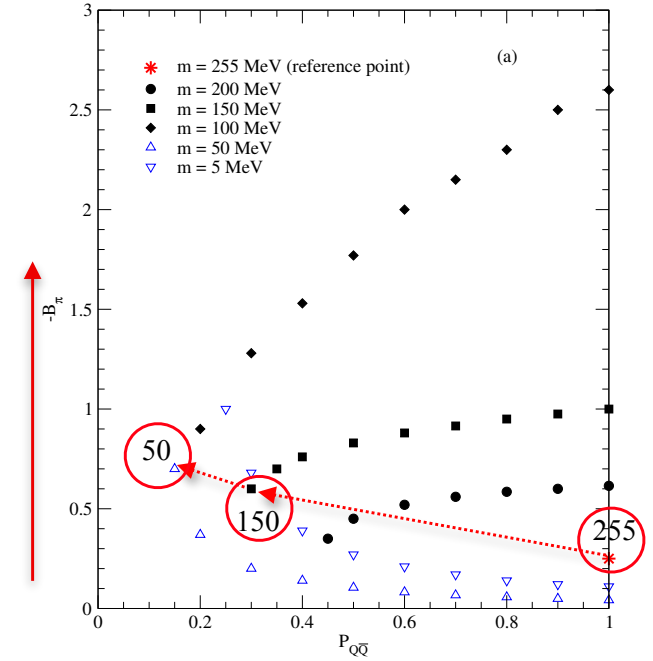
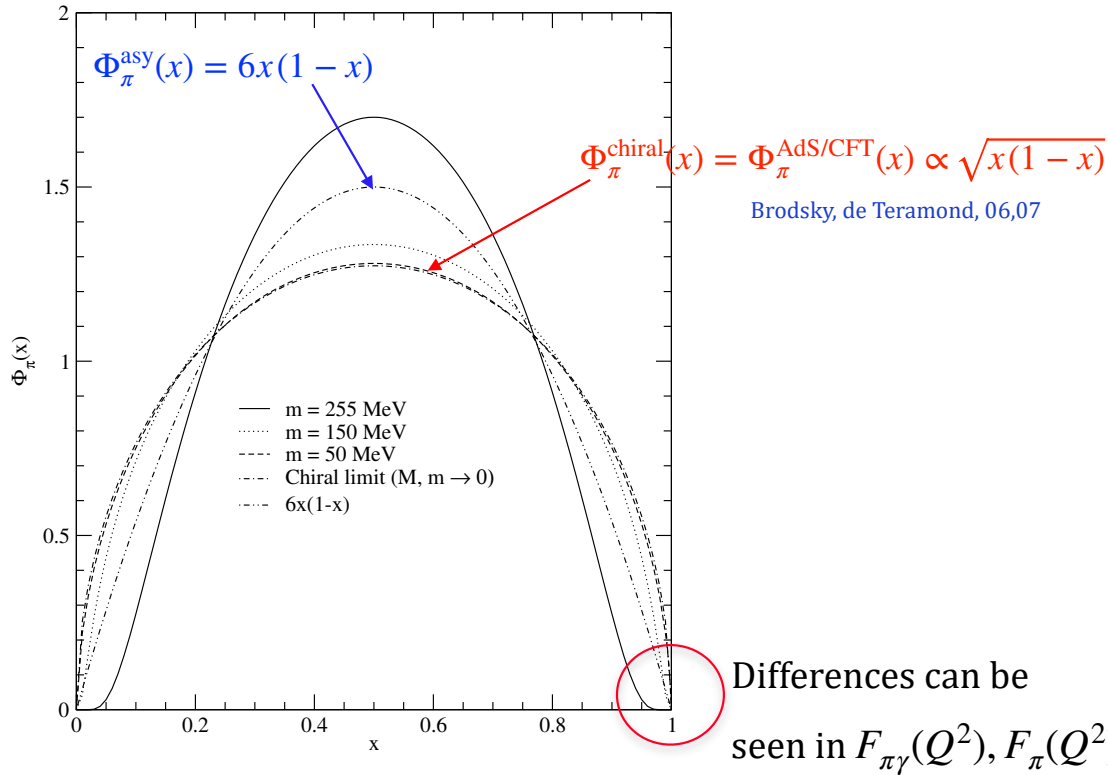


Larger
Spatial size

Simulate Mass Variation Effect

TABLE I: Model parameters (B_π, β) depending on the variation of (M_π, m) and $P_{Q\bar{Q}}$. We denote (M_π, m, β, f_π) in unit of MeV.

(M_π, m)	$P_{Q\bar{Q}}$	B_π	β	f_π^{Th}	$F_{\pi\gamma}^{\text{Th}}(0)$ [GeV^{-1}]
(135,255)	1	-0.25	198.0	130.4	0.271
(135,150)	0.3	-0.60	346.9	130.6	0.272
(135,50)	0.15	-0.7	493.0	130.7	0.271
(0,0)	0.078	< 0	668.5	130.9	0.276
Exp. [44]	—	—	—	130.2(1.7)	0.272(3)



- Simulation of the **quark mass variation effect** on Q^2 evolution of $F_{\pi\gamma}(Q^2)$ and $F_\pi(Q^2)$

As a first attempt to estimate the quark mass variation effect, we use **the mixing between m_{ref} and $m (< m_{\text{ref}})$ via**

$$\langle \Psi_{m'}^\pi | \Psi_m^\pi \rangle = \delta_{m'm} \sqrt{P_{m'} P_m} = \delta_{m'm} P_m \quad m_{\text{ref}} = m = 255 \text{ MeV and } P_{Q\bar{Q}} = P_{m_{\text{ref}}} = 1$$

e.g.) For the mixing between m_{ref} and $m = 150 \text{ MeV}$:

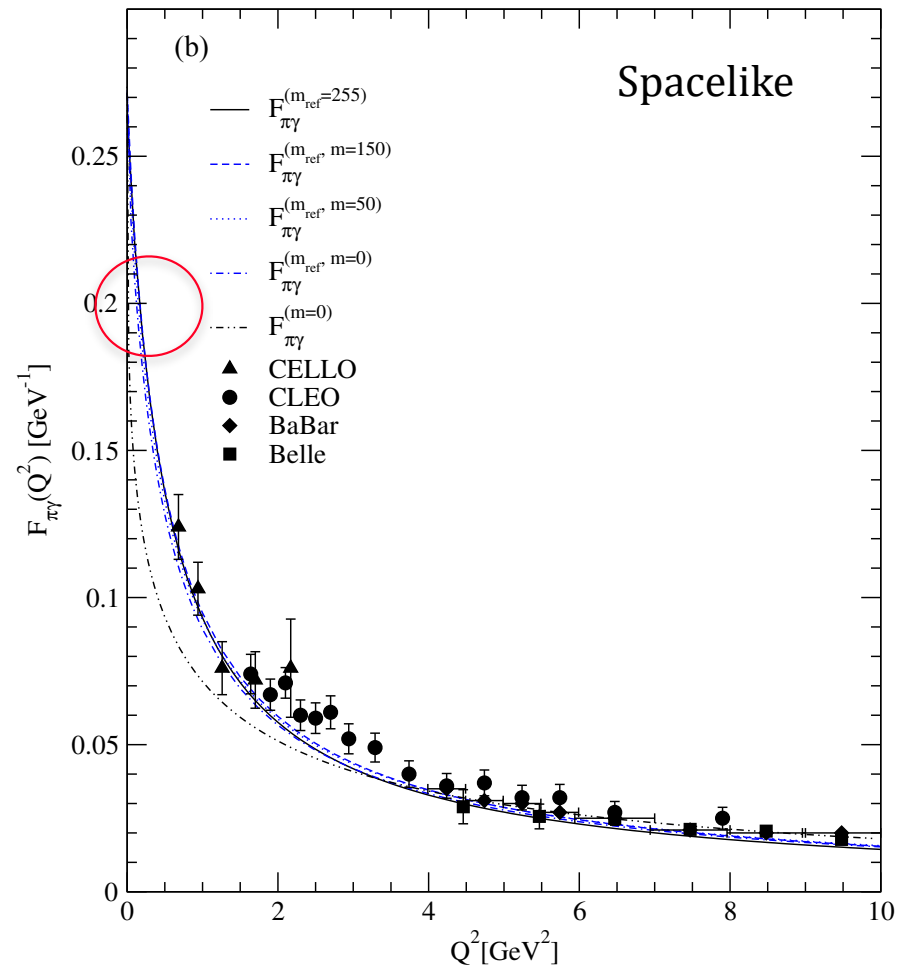
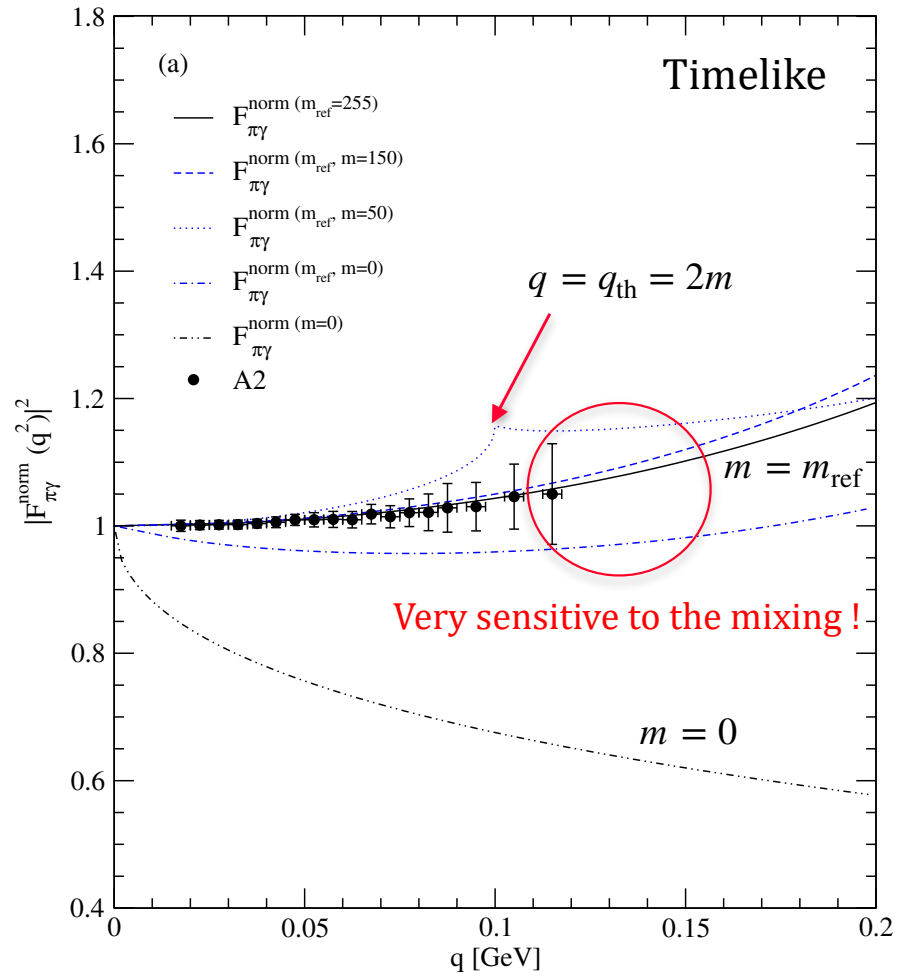
$$F_{\pi\gamma}^{(m_{\text{ref}},m)}(Q^2) = c_1 F_{\pi\gamma}^{(m_{\text{ref}})}(Q^2) + c_2 F_{\pi\gamma}^{(m)}(Q^2), \quad \text{with } F_{\pi\gamma}^{(m_{\text{ref}},m)}(0) = F_{\pi\gamma}^{\text{Exp}}(0).$$

$$F_\pi^{(m_{\text{ref}},m)}(Q^2) = d_1 F_\pi^{(m_{\text{ref}})}(Q^2) + d_2 F_\pi^{(m)}(Q^2), \quad \text{with } F_\pi^{(m_{\text{ref}},m)}(0) = 1.$$

$$c_1 = \frac{\sqrt{1 - \tilde{P}_m}}{\sqrt{1 - \tilde{P}_m} + \sqrt{\tilde{P}_m}}, \quad c_2 = 1 - c_1 \quad d_1 = 1 - \tilde{P}_m, \quad d_2 = \tilde{P}_m$$

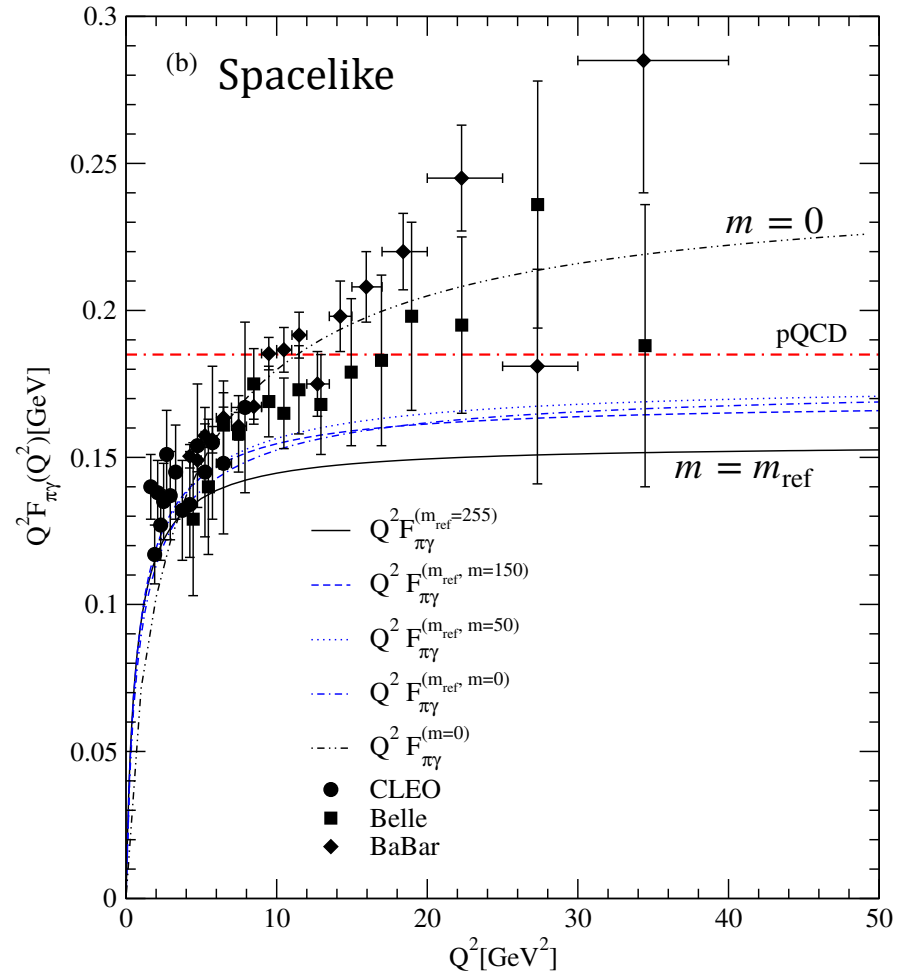
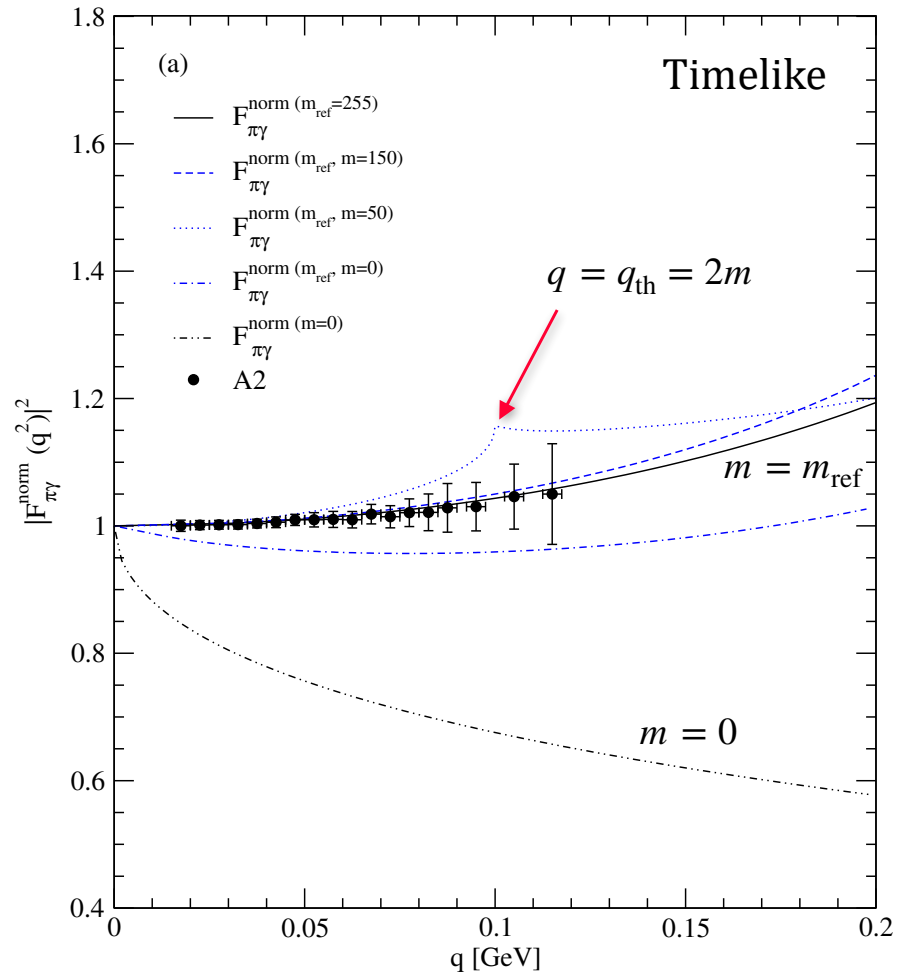
$$\tilde{P}_m = \frac{P_m}{(P_{m_{\text{ref}}} + P_m)} = \frac{0.3}{1.3} \approx 0.23 \quad \text{: renormalized probability}$$

- Quark mass variation effect on $F_{\pi\gamma}(Q^2)$

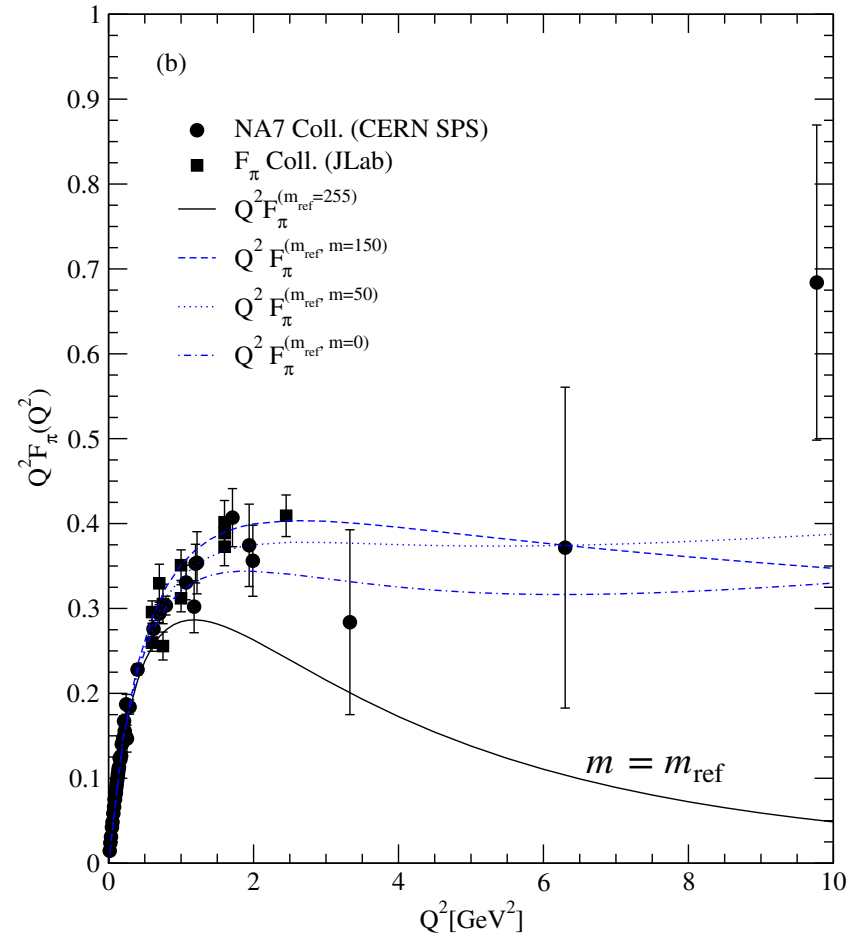
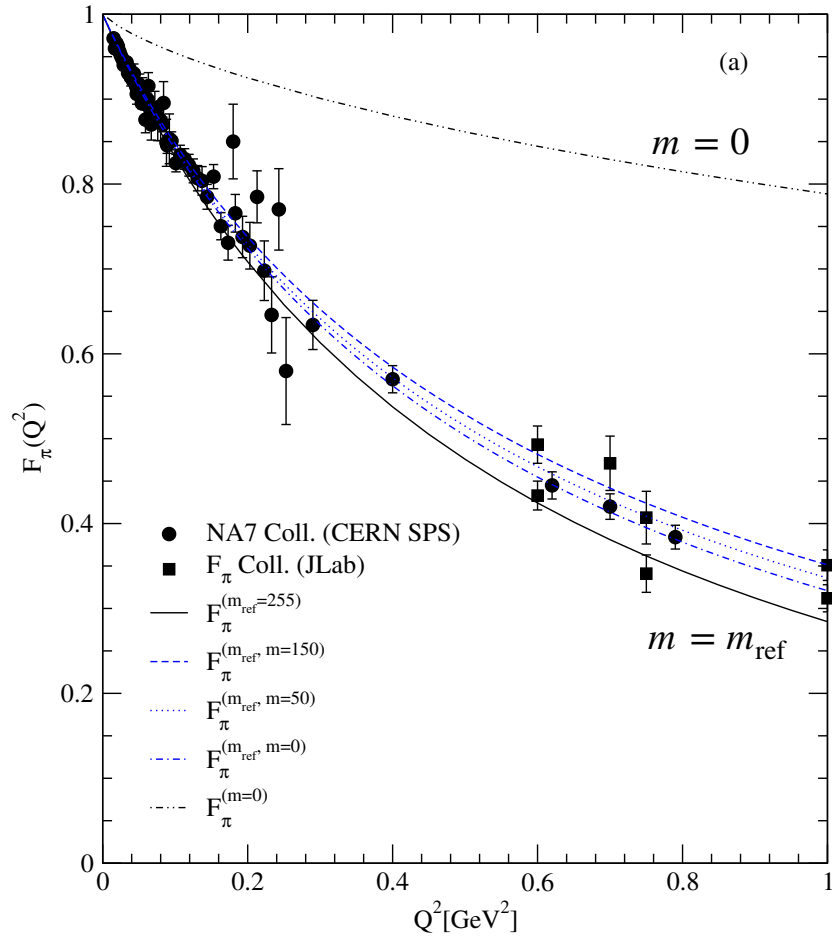


- In constraining the model parameters, **the analysis in timelike region is important.**

- Quark mass variation effect on $F_{\pi\gamma}(Q^2)$



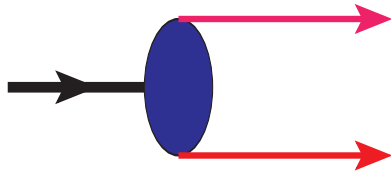
- Quark mass variation effect on $F_\pi(Q^2)$



Shows the necessity of the
quark mass evolution effect

5. Conclusion

- We explored the link between the chiral sym. of QCD and the numerical results of the LFQM analyzing $f_\pi, F_{\pi\gamma}(Q^2), F_\pi(Q^2)$.



$$\Gamma_\pi = (M_\pi + B_\pi \not{P}) \gamma_5$$

- Axial-vector coupling with $B_\pi < 0$ is essential to describe the correct chiral limit expression in the LFQM.
- Our chiral limit results for f_π and $\phi_\pi(x)$ are exactly the same as AdS/CFT predictions.
- In constraining the model parameters, we found that **the chiral anomaly plays a critical role and the analysis of $F_{\pi\gamma}(q^2)$ in timelike region is important.**
- Our results indicate that **the constituent quark picture is very effective** in describing both $F_{\pi\gamma}(Q^2), F_\pi(Q^2)$ **in the low energy regime,** but **the quark mass evolution seems inevitable as Q^2 grows.**