The QCD running coupling

A. Deur

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Work done in collaboration with:

- S. J. Brodsky and G. de Téramond (HLFQCD),
- V. Burkert, J-P Chen and W. Korsch (experimental).

The strong coupling α_s

α_s is the most important quantity of QCD.

Well understood at high energy where it is small: $\alpha_s \sim 0.1 \Rightarrow$ pQCD.

Very active research to understand α_s at low energy where it is large: $\alpha_s \sim 1$.

Outline:

- α_s in pQCD.
- Extension to non-perturbative QCD.
- Experimental determination of the coupling (perturbative + non-perturbative domains).
- HLFQCD calculation. Comparison with data and other predictions.
- Application: determination of the hadron spectrum using α_s and HLFQCD.

The strong coupling α_s at short distances (large Q^2)

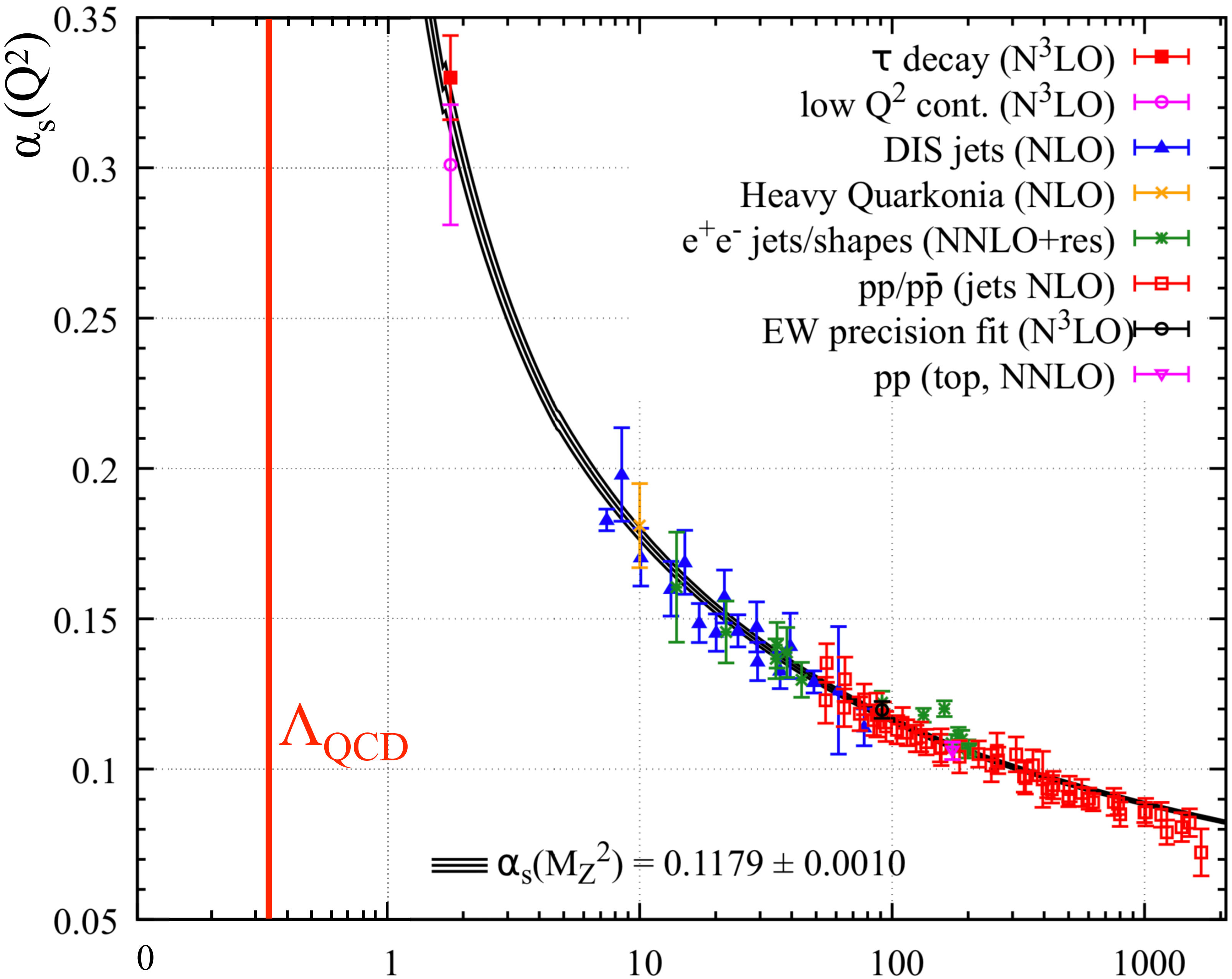
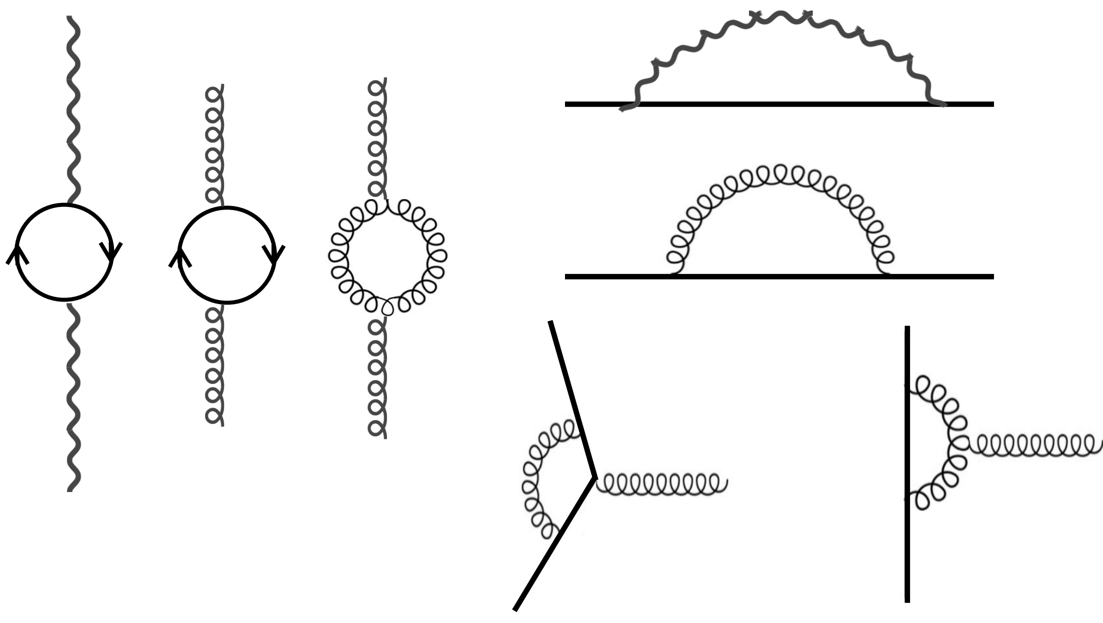


Figure adapted from Particle Data Group, 2020.

α_s is not constant due to loops in gluon propagator, fermion self-energy, and vertex corrections:



At first order, propagator $\propto 1/Q^2$ force $\propto 1/r^2$.

$\Rightarrow \alpha_s(Q^2)$: Quantum effects that make the force to depart from $1/r^2$ are folded into the coupling constant.

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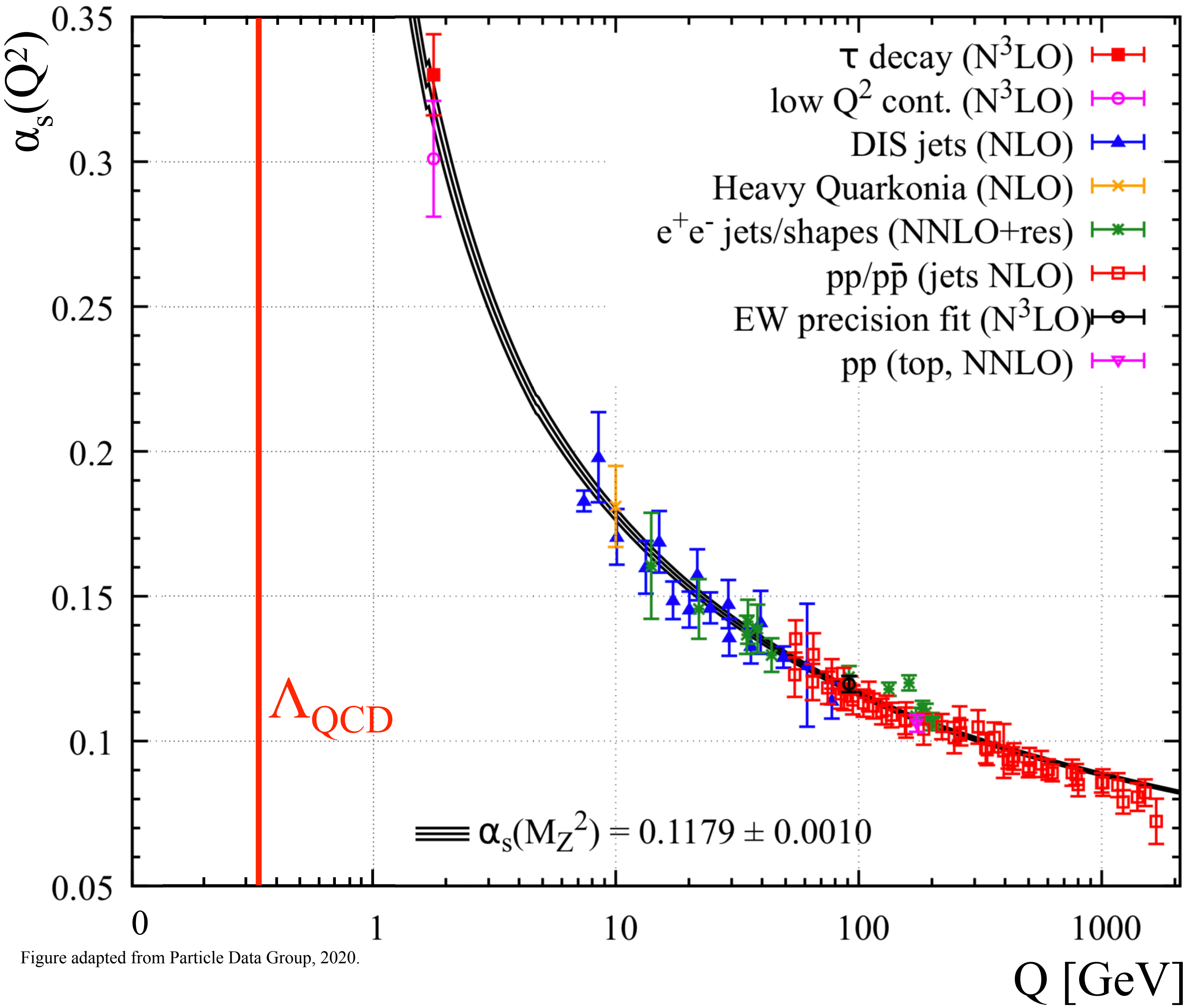

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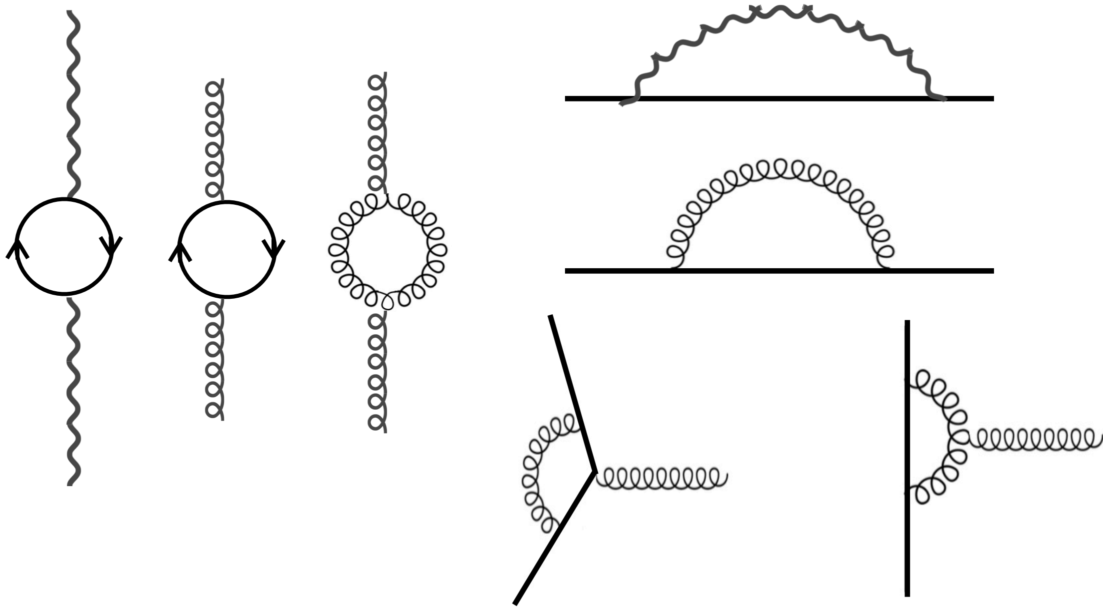

α_s becomes small at large Q^2: Asymptotic freedom.

pQCD applicable $\Rightarrow \alpha_s(Q^2)$ is well defined there.

Large Q^2 determinations agree well with pQCD expectation of Q^2-evolution.

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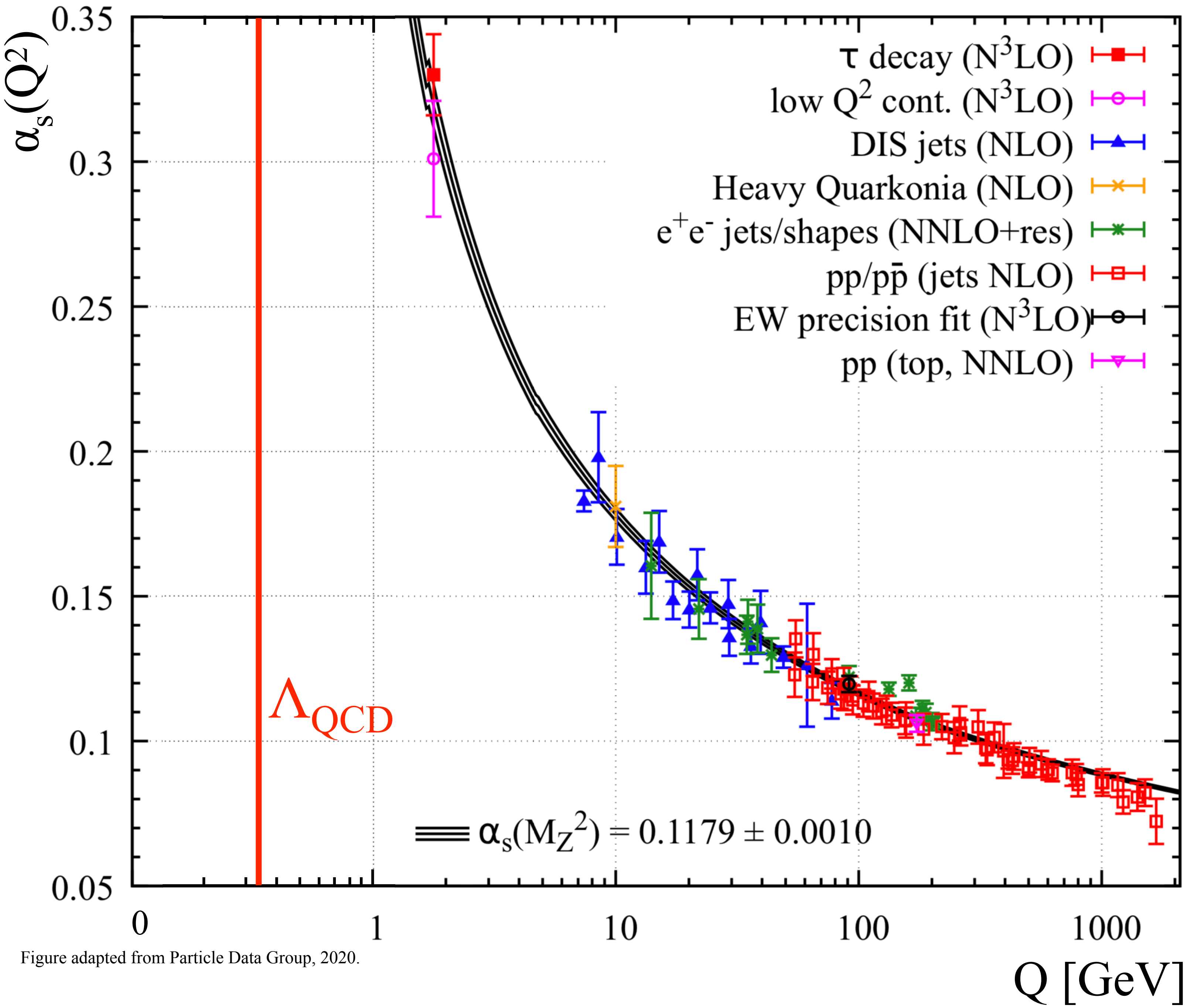

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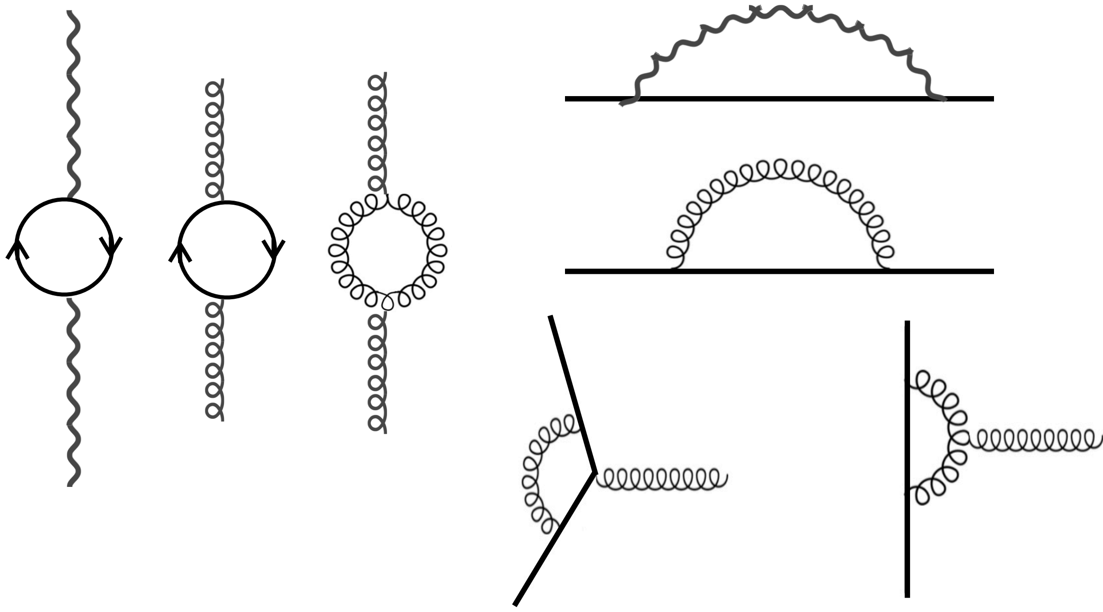

Loops in perturbative expansion ⇒ renormalization needed.

Here, α_s is in $\overline{MS}$ scheme (most common choice. α_s is gauge-independent in this scheme)

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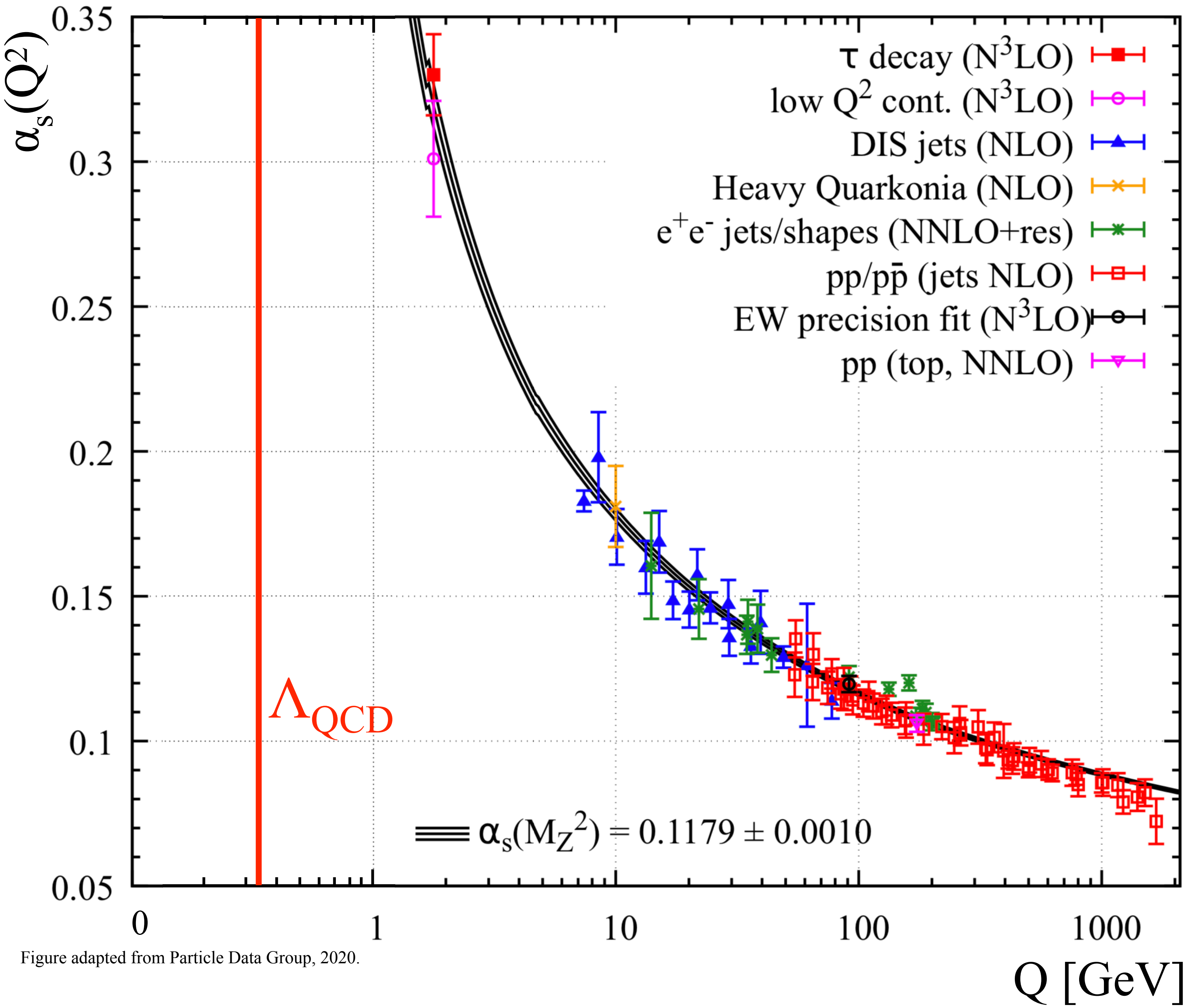

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$\alpha_s(Q^2)=f(Q^2/\Lambda^2_{QCD}) \Rightarrow$ needs non-perturbative methods (e.g., lattice QCD) or data for an absolute determination of $\alpha_s(Q^2)$, even in pQCD domain.

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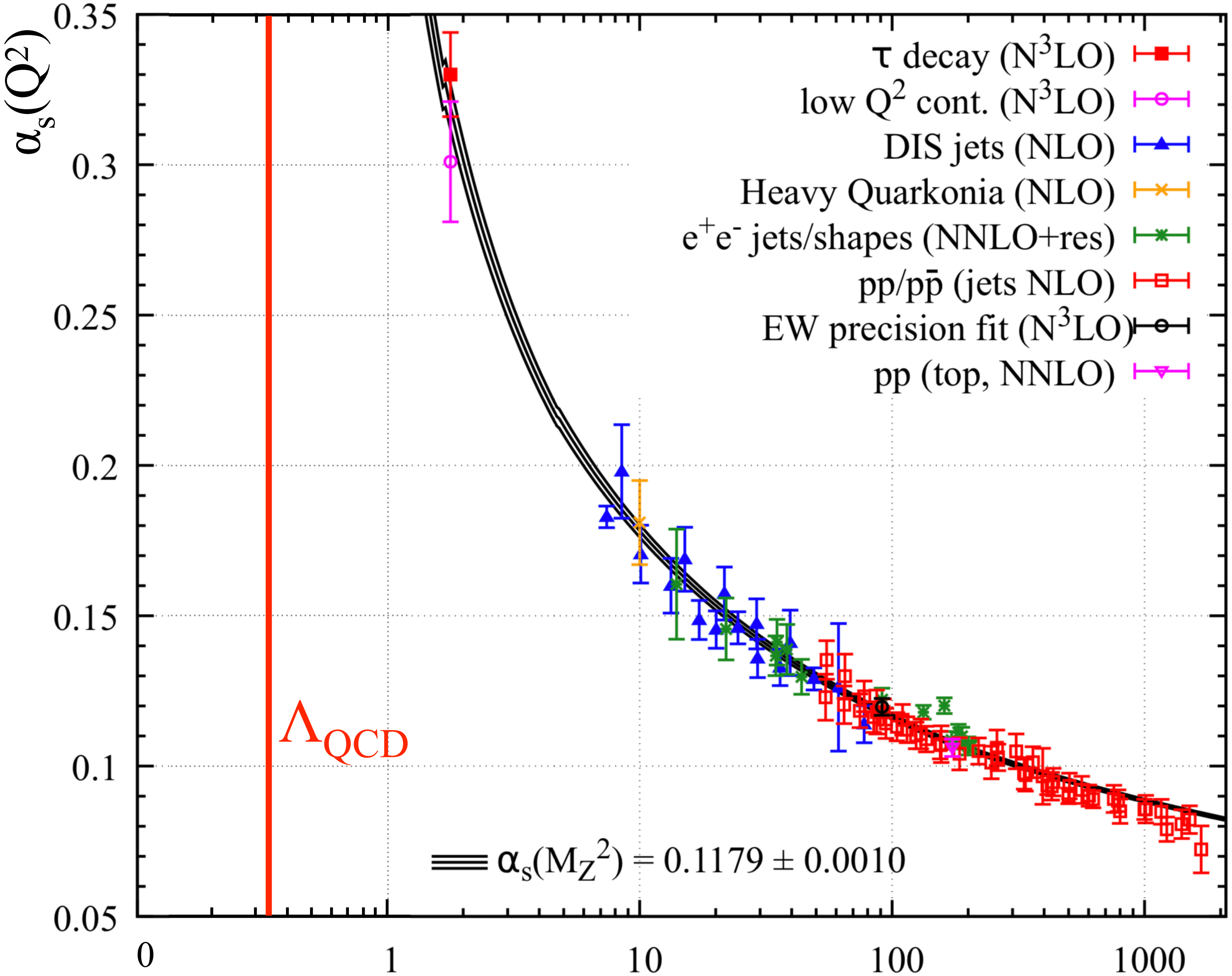

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Ex: Bjorken sum rule:

$$\int (g^p_1 - g^n_1)dx = \frac{1}{6} g_A\left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right)$$

$g^p_1(Q^2,x)$, $g^n_1(Q^2,x)$: proton & neutron longitudinal spin structure functions.

x: Bjorken-x

g_A: nucleon axial charge (well known).

After scheme-independent LO term, pQCD series is expressed in a particular renormalization scheme. $\Rightarrow$ α_s also in that scheme.

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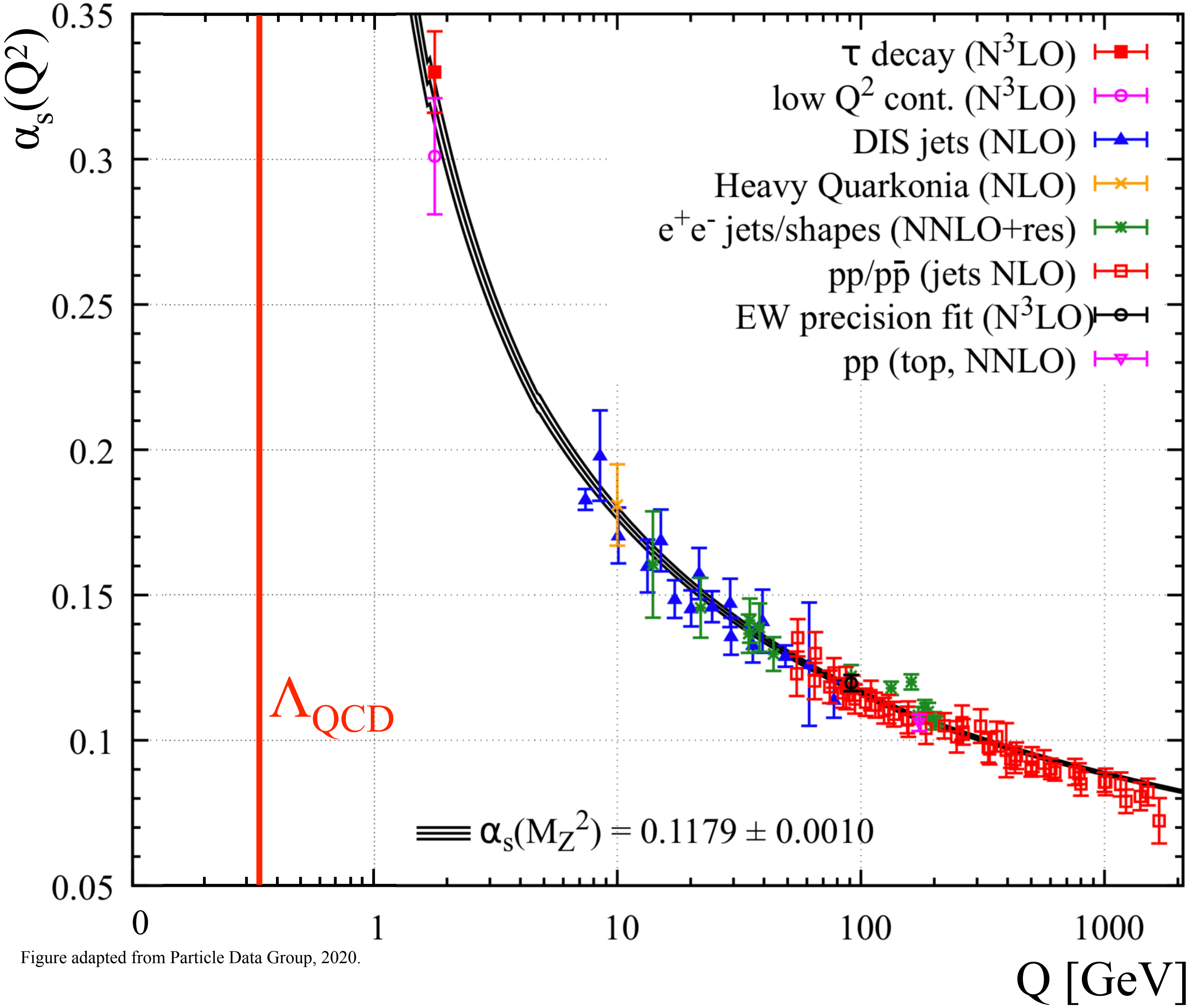

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At $Q^2 \lesssim 1 GeV^2$, pQCD cannot be used to define α_s: if pQCD is trusted, $\alpha_s \to \infty$ when $Q \to \Lambda_{QCD}$.

Contradicts the perturbative hypothesis: inconsistency (leads to unphysical results).

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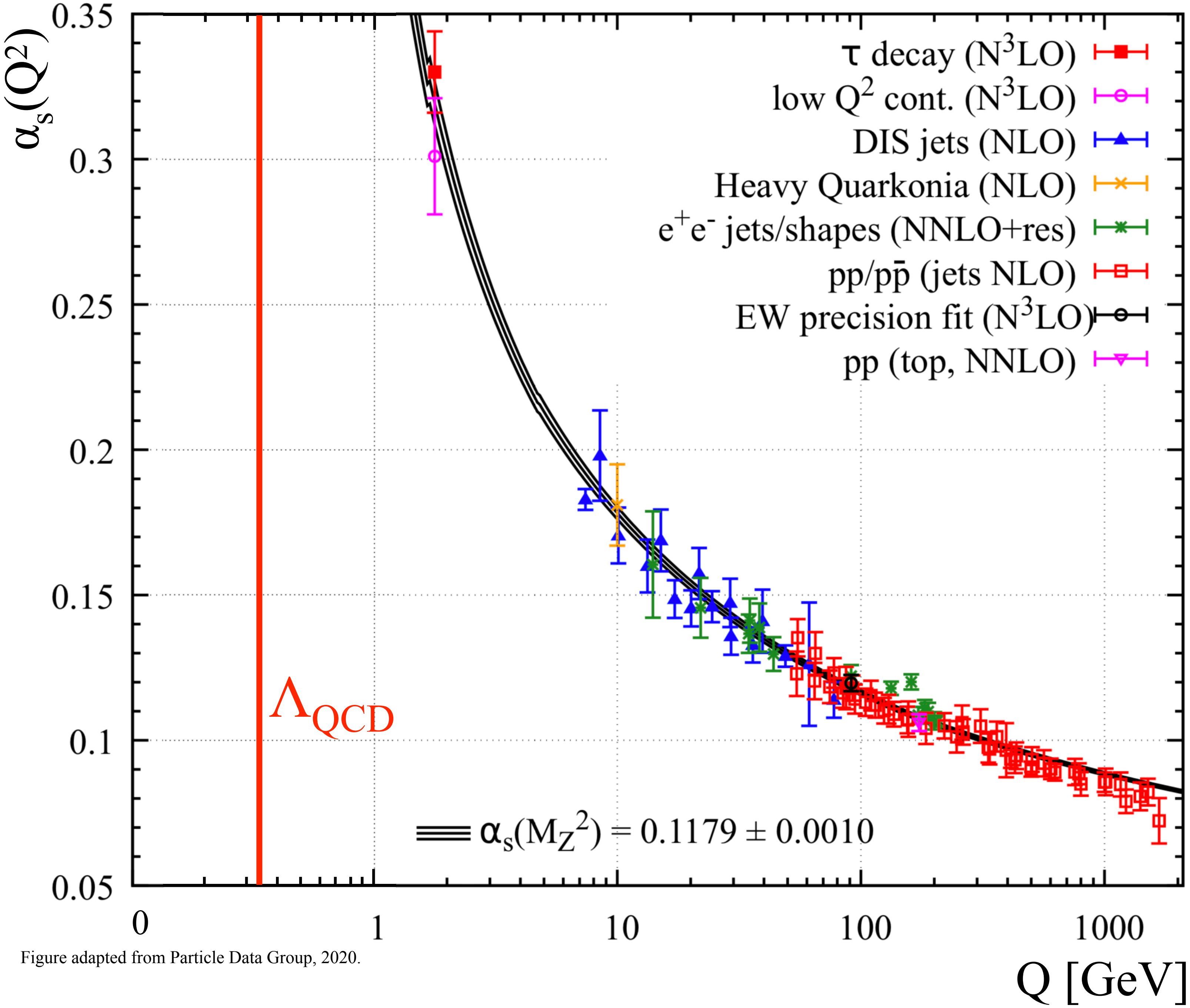

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Definition and computation of α_s at low Q^2?

α_s at long distance (low Q^2)

Prescription: Define an effective coupling from an observable's perturbative series truncated to first order in α_s. G. Grunberg, PLB B95 70 (1980); PRD 29 2315 (1984); PRD 40 680(1989).

This definition is in analogy to α, QED's coupling definition (Gell-Mann Low coupling).

Proposed for pQCD. Can be extended to non-perturbative QCD.

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Nucleon axial charge.

pQCD corrections (gluon bremsstrahlung)

Higher Twists: $1/Q^{2n}$ corrections.
Non-perturbative quantities. Express correlations between parton distributions and confinement forces.

$$\Rightarrow \boxed{\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A \left(1 - \frac{\alpha_{g1}}{\pi}\right)}$$

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This means that additional short distance effects, and long distance confinement force and parton distribution correlations are now folded into the definition of α_s.

Analogous to original coupling constant becoming an effective coupling when short distance quantum loops are folded into its definition.

α_s at long distance (low Q^2)

The effective coupling is back to be an observable.

⇒ • Extractable at any Q^2;
- Free of divergence;
- Renormalization scheme independent.

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But it is

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⇒There is *a priori* a different α_s for each different process.

However these α_s can be related (Commensurate Scale Relations).

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Such definition of α_s using a particular process is equivalent to a particular choice of renormalization scheme.
(process dependence) ⇔ (scheme dependence)

$\alpha_{g1} = \alpha_s$ in the "g_1 scheme".
Relations between g_1 scheme and other schemes are known in pQCD domain, e.g.
$\Lambda_{g_1} = 2.70\Lambda_{\overline{MS}} = 1.48\Lambda_{MOM} = 1.92\Lambda_V = 0.84\Lambda_\tau$.

α_s at long distance (low Q^2)

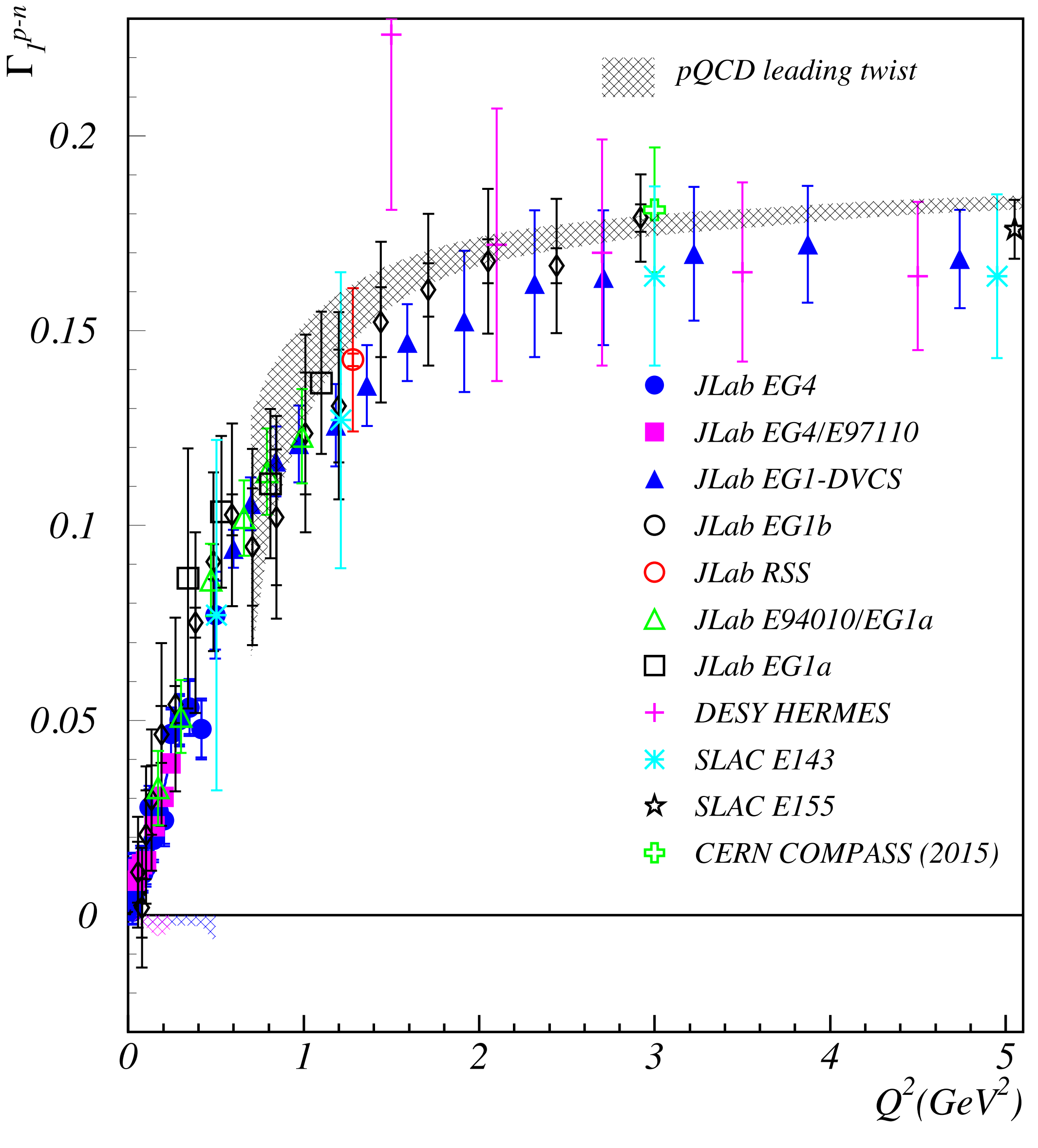
Advantages of extracting α_s from the Bjorken Sum Rule:

- Bjorken sum rule: simple perturbative series.
- Data exist at low, intermediate, and high Q^2.
- Rigorous Sum Rules dictate the behavior of α_{g1} in the unmeasured $Q^2 \rightarrow 0$ and $Q^2 \rightarrow \infty$ regions.

$\Rightarrow$ We can obtain α_{g1} at **any** Q^2.

α_{g1} from the Bjorken Sum data

Bjorken sum $\Gamma_1^{p\text{-}n}$ measurements



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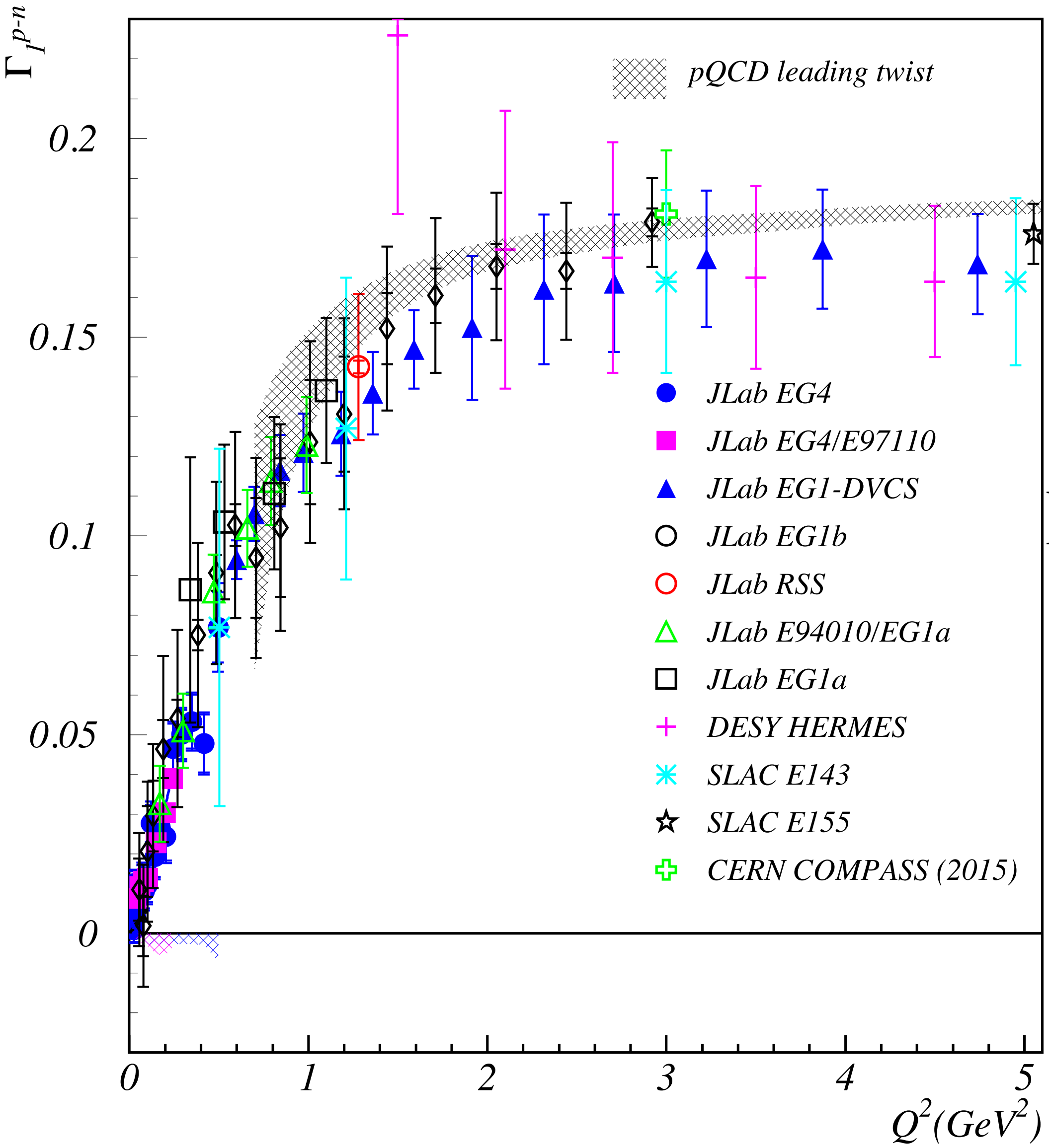
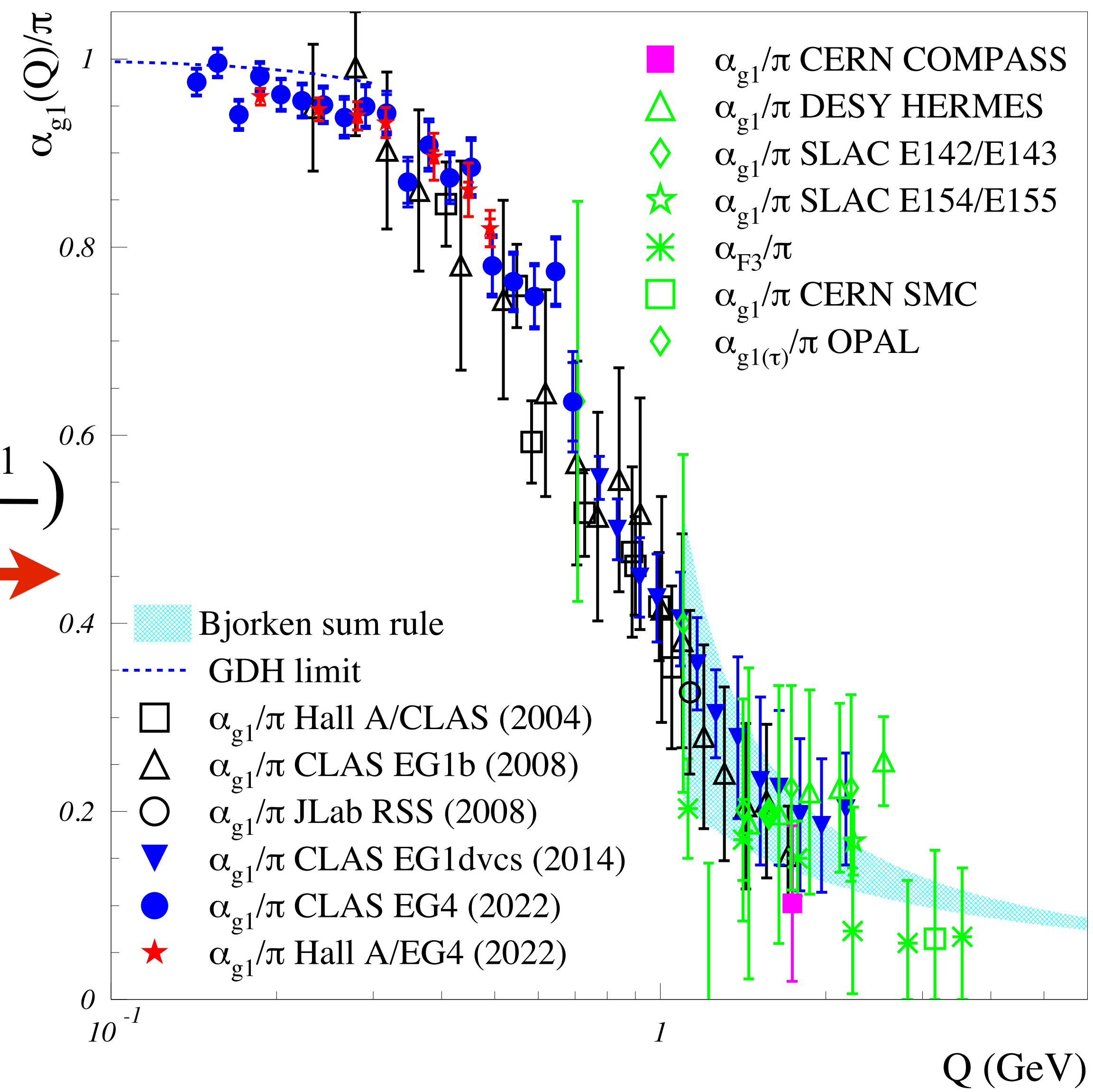


$$\Gamma_1^{p\text{-}n} \triangleq \frac{1}{6} g_A \left(1 - \frac{\alpha_{g1}}{\pi}\right)$$



Low Q^2 limit

At $Q^2 = 0$, a sum rule related to the Bjorken sum rule exists: the Gerasimov-Drell-Hearn (GDH) sum rule:

At $Q^2 = 0$, GDH sum rule:

$$\Gamma_1 = \frac{-\kappa^2 Q^2}{8M^2}$$

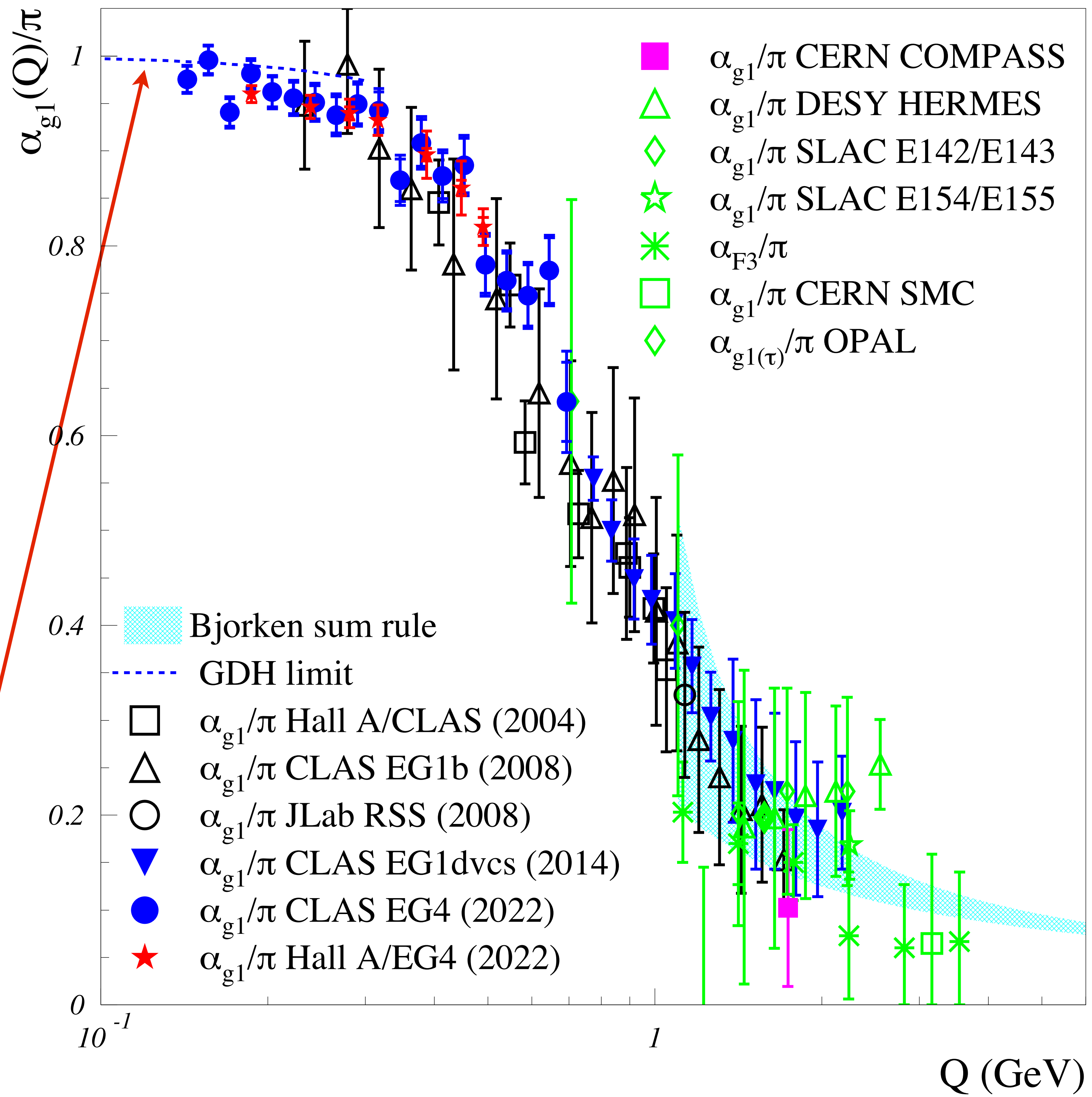
anomalous magnetic moment (κ)

Nucleon mass (M)

$\Rightarrow$ $Q^2 = 0$ constraints:

$$\Rightarrow \begin{cases} \alpha_{g1} = \pi \\ \dfrac{d\alpha_{g1}}{dQ^2} = \dfrac{3\pi}{4g_A}\left(\dfrac{\kappa_n^2}{M_n^2} - \dfrac{\kappa_p^2}{M_p^2}\right) \end{cases}$$

$Q^2=0$



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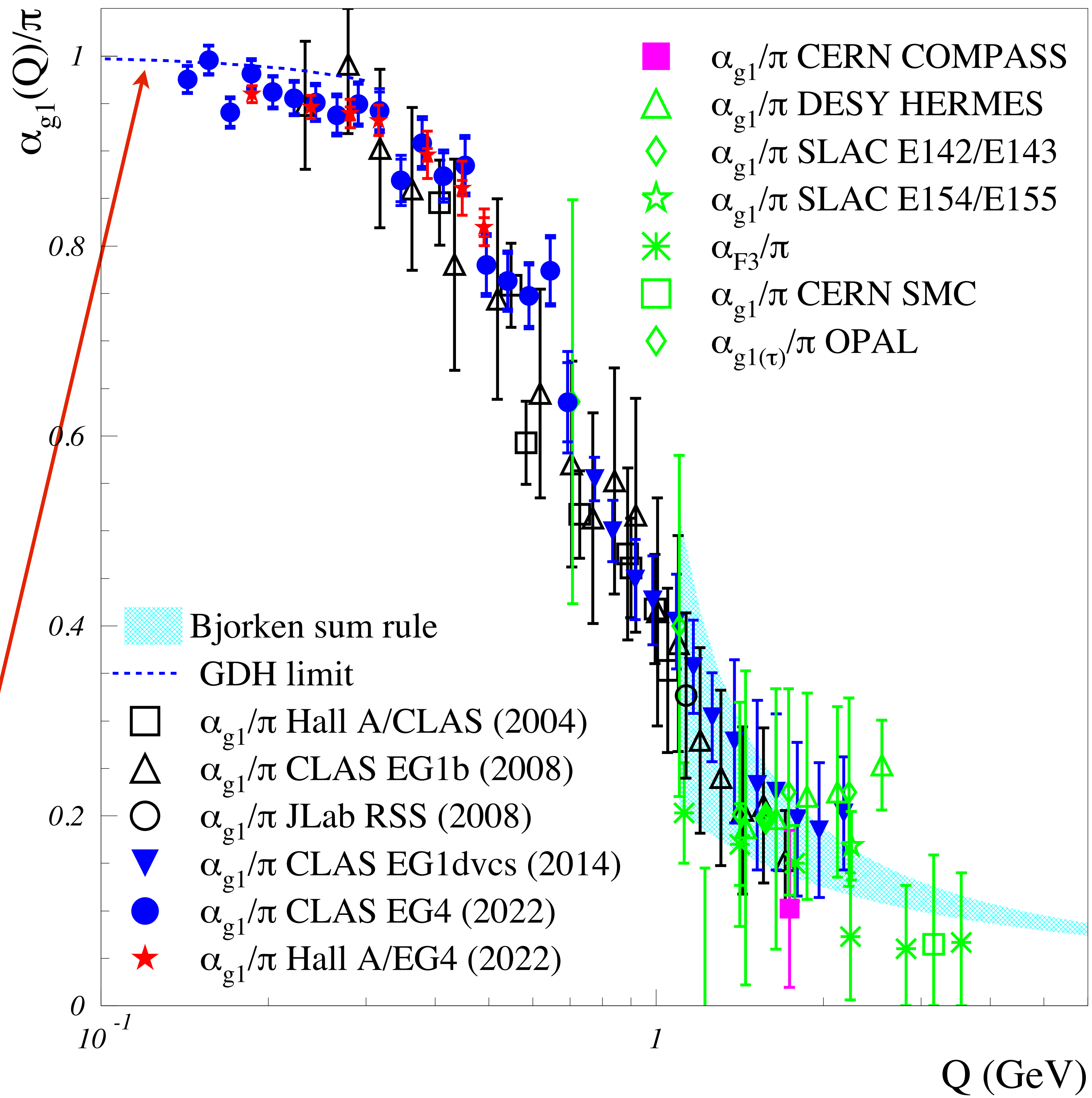
κ_n^2: 3.66, κ_p^2: 3.20



First experimental evidence of nearly *conformal behavior* (i.e. no Q^2-dependence) of QCD at low Q^2.

Large Q^2 limit

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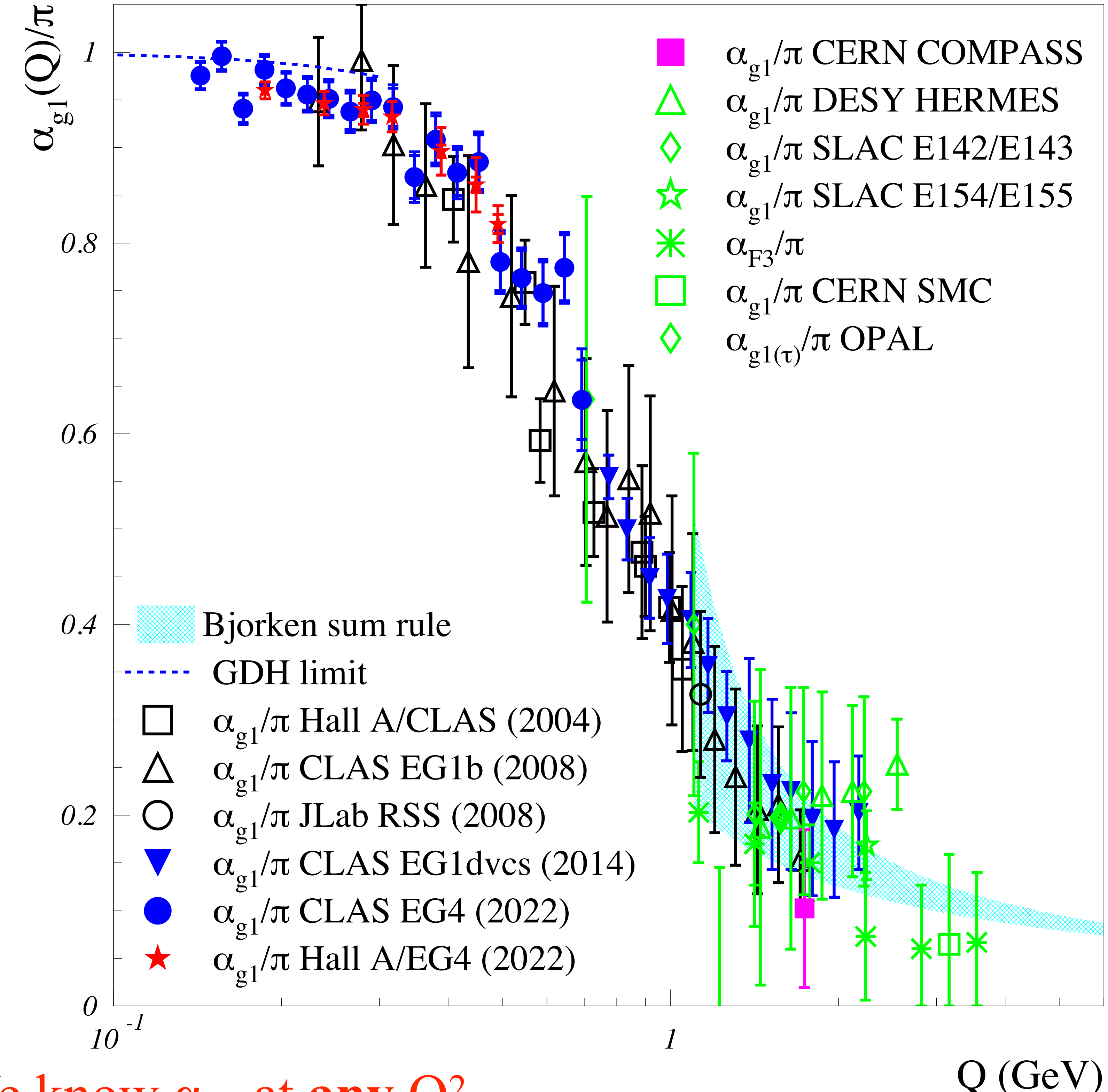
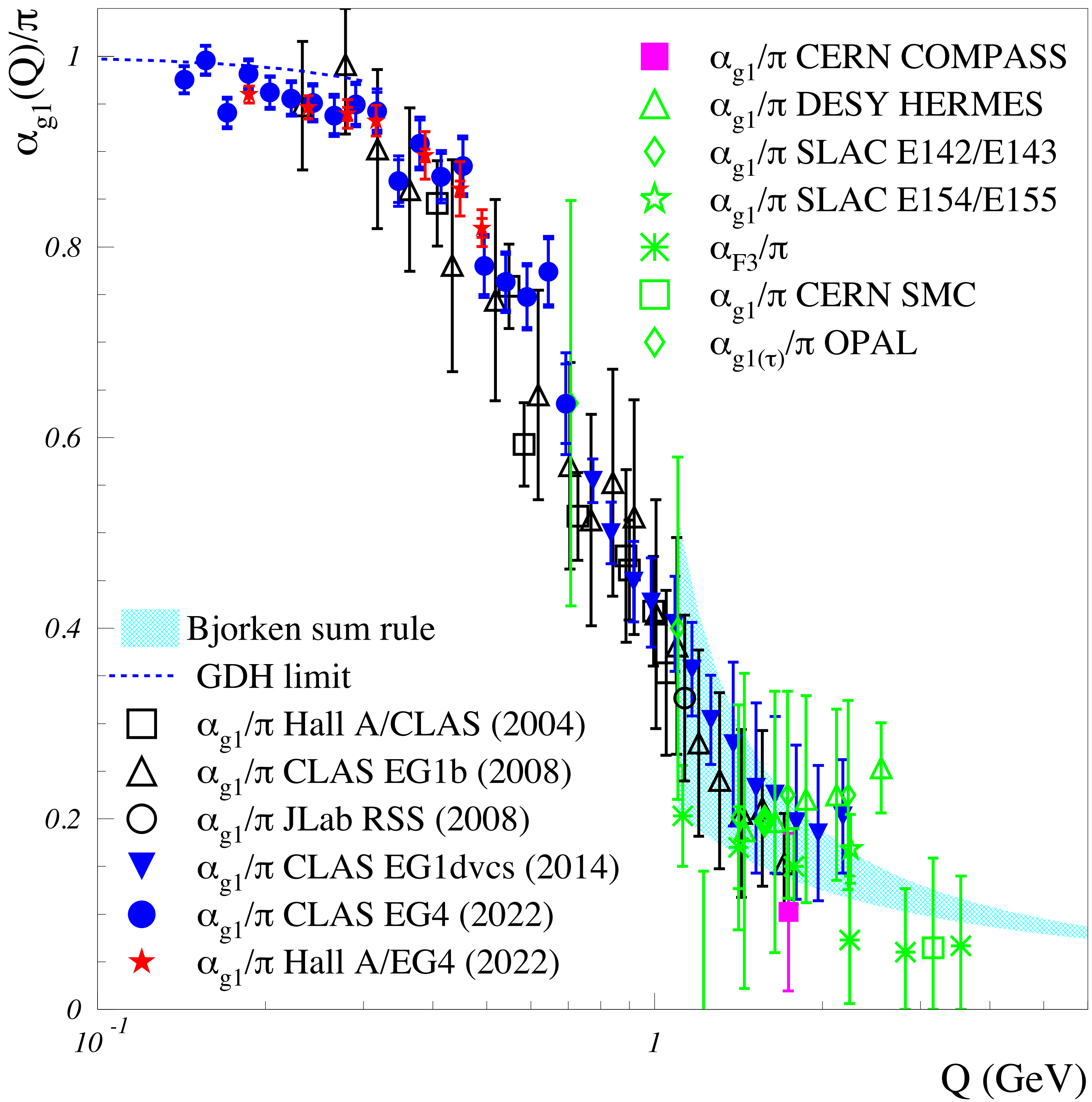


⇒ We know α_{g1} at **any** Q^2.

Low Q^2 limit

QCD is conformal
at low Q^2.



⇒ One can use AdS/CFT correspondence to study non-perturbative QCD.

The Light-Front holography approximation (HLFQCD)

Review: Brodsky, de Teramond, Dosch, Erlich, Phys. Rep. 05 (2015) 001. arXiv:1407.8131

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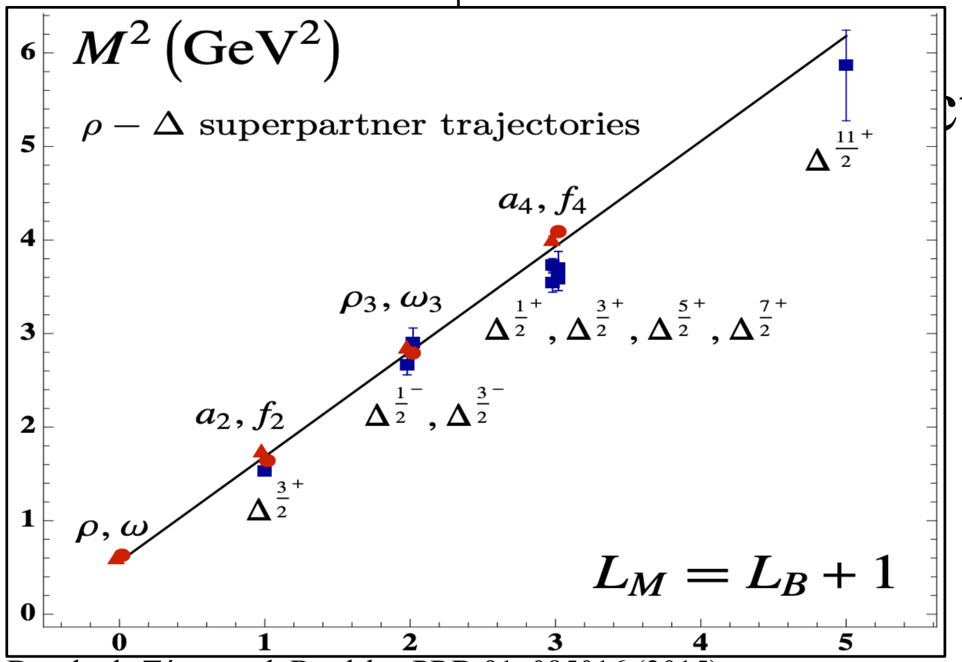

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Harmonic oscillator on light front ⇒ in AdS space, $ds^2 \to \exp(\kappa^2 z^2) ds^2$

z is the 5th dimension of AdS space. z^2 is the scale at which the hadron is probed, i.e. $1/Q^2$.

κ is the universal scale factor of HLFQCD.

α_s from HLFQCD

Perturbative QCD:

pQCD effective coupling $\alpha_s(Q^2)$: small distance QCD effects are folded into the definition of the coupling constant α_s.

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Non-perturbative QCD:

Likewise for $\alpha_{g1}(Q^2)$ at long distance, confinement forces and PDF correlations are folded into the definition of $\alpha_s(Q^2)$.

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General Relativity Action: $S \propto \int d^4x \sqrt{g} \frac{1}{G_N} R$, with R the Ricci scalar and $g = \det(g_{\mu\nu})$

AdS Action: $S \propto \int d^5x \sqrt{g} \frac{1}{g^2_5} F^2$, with F the gauge field and g_5 the coupling

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General Relativity Action: $S \propto \int d^4x \sqrt{g} \frac{1}{G_N} R$, with R the Ricci scalar and $g=\det(g_{\mu\nu})$

AdS Action: $S \propto \int d^5x \sqrt{g} \frac{1}{g^2_5} F^2$, with F the gauge field and g_5 the coupling

Deformed AdS Action: $S \propto \int d^5x \sqrt{g}\, e^{\kappa^2 z^2} \frac{1}{g^2_5} F^2$

Confinement potential

α_s from HLFQCD

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Effective coupling at large distance

Transforming to momentum space:

$$\alpha_s^{HLF}(Q^2)=\alpha_s^{HLF}(Q^2=0)e^{(-Q^2/4\kappa^2)}$$

Brodsky, de Téramond, Deur.
Phys. Rev. D 81, 096010 (2010)

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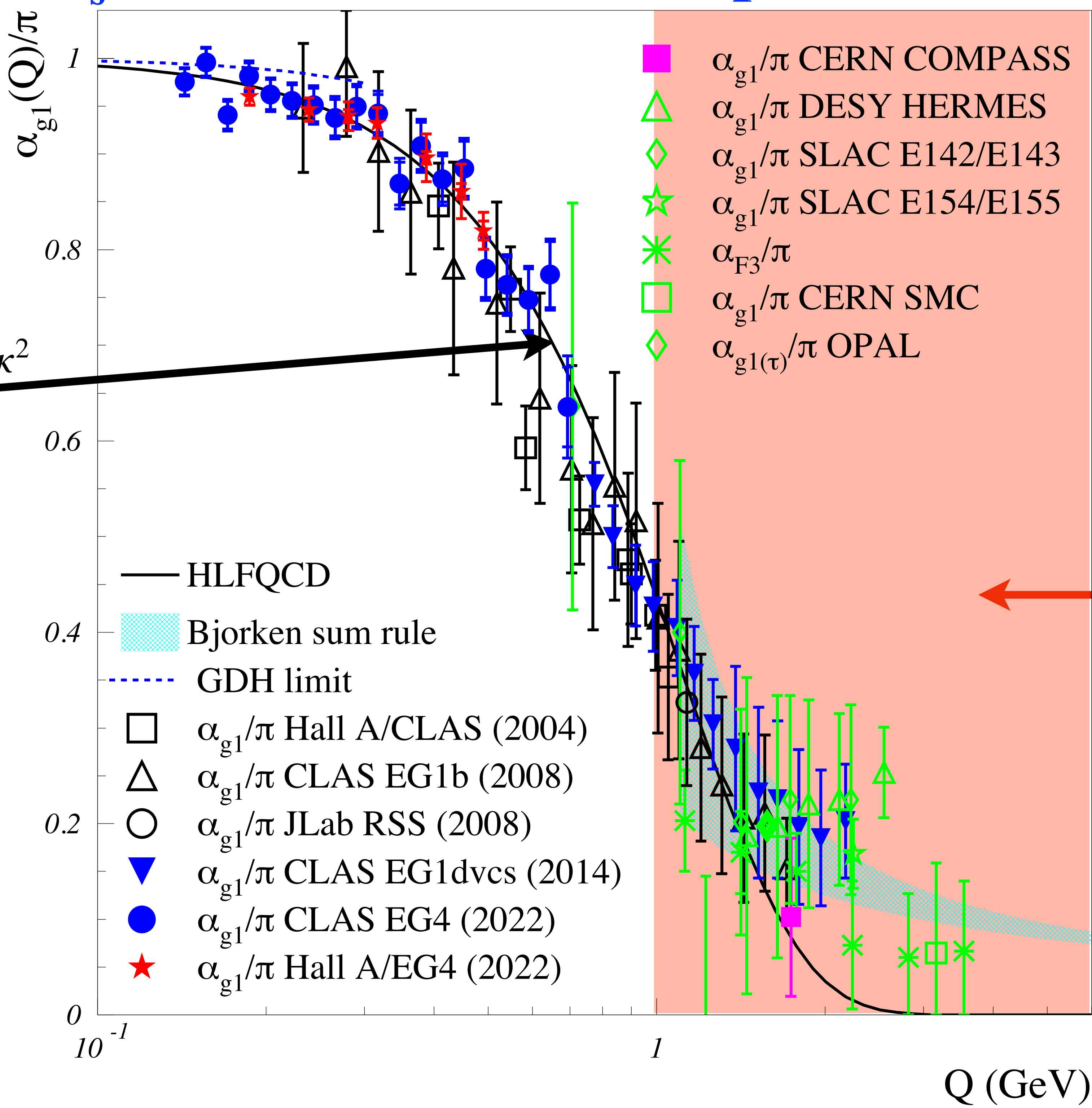
$$\alpha_s^{HLF}(Q^2) = \alpha_s^{HLF}(Q^2=0)\, e^{(-Q^2/4\kappa^2)}$$

$\alpha_s^{HLF}(0) \equiv \pi$: $\alpha_s^{HLF}(Q^2)$ in the g_1 scheme.

α_s and HLFQCD: Comparison with data

$$\frac{\alpha_s^{HLF}(Q^2)}{\pi} = e^{-Q^2/4\kappa^2}$$

$$\kappa = M_\rho/\sqrt{2}$$

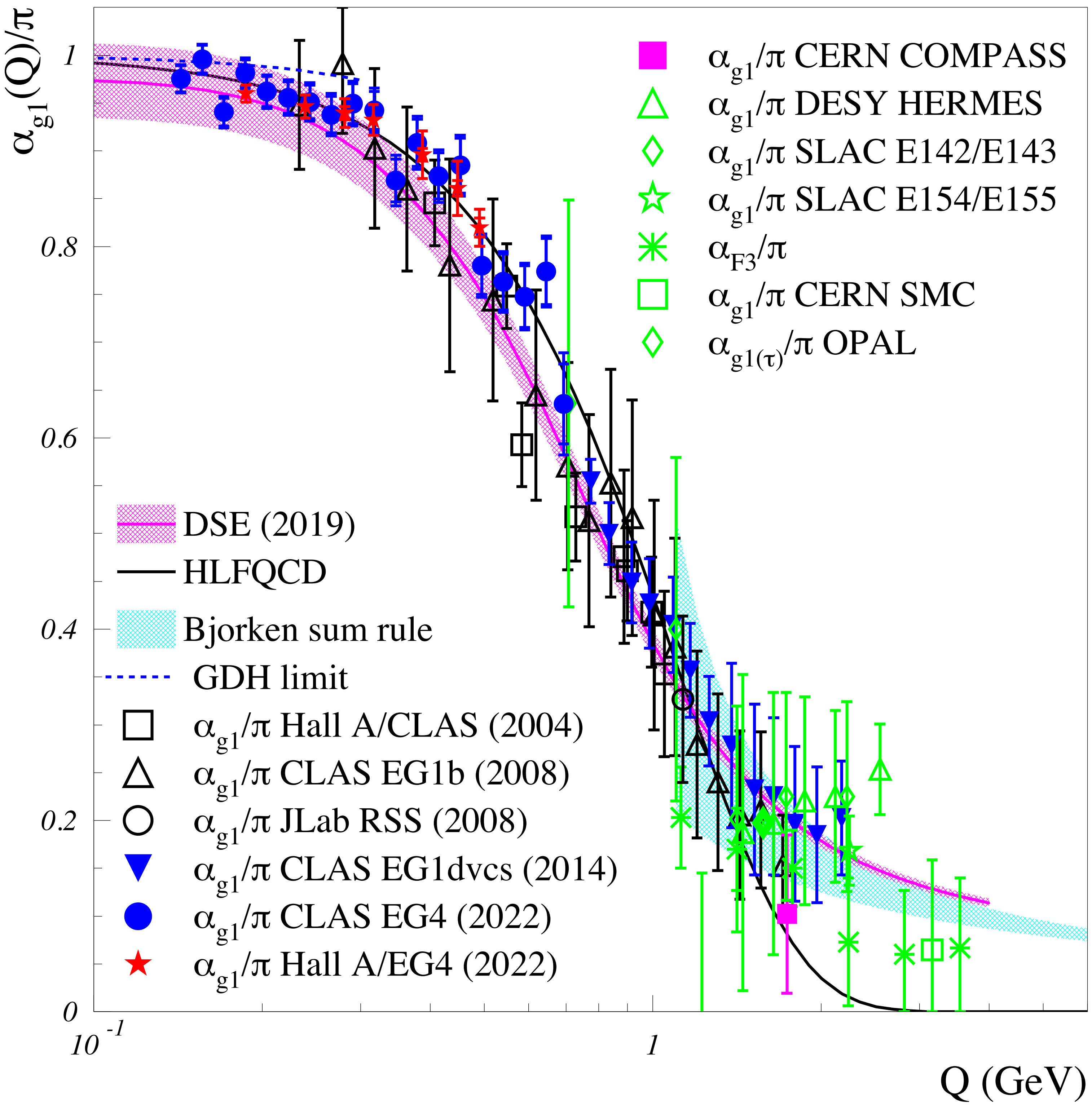


HLFQCD not applicable

⇒ Prediction for α_s at long distances. No free parameters ($\kappa=M_\rho/\sqrt{2}$). Agrees very well with the α_s extracted from JLab's Bjorken sum data.

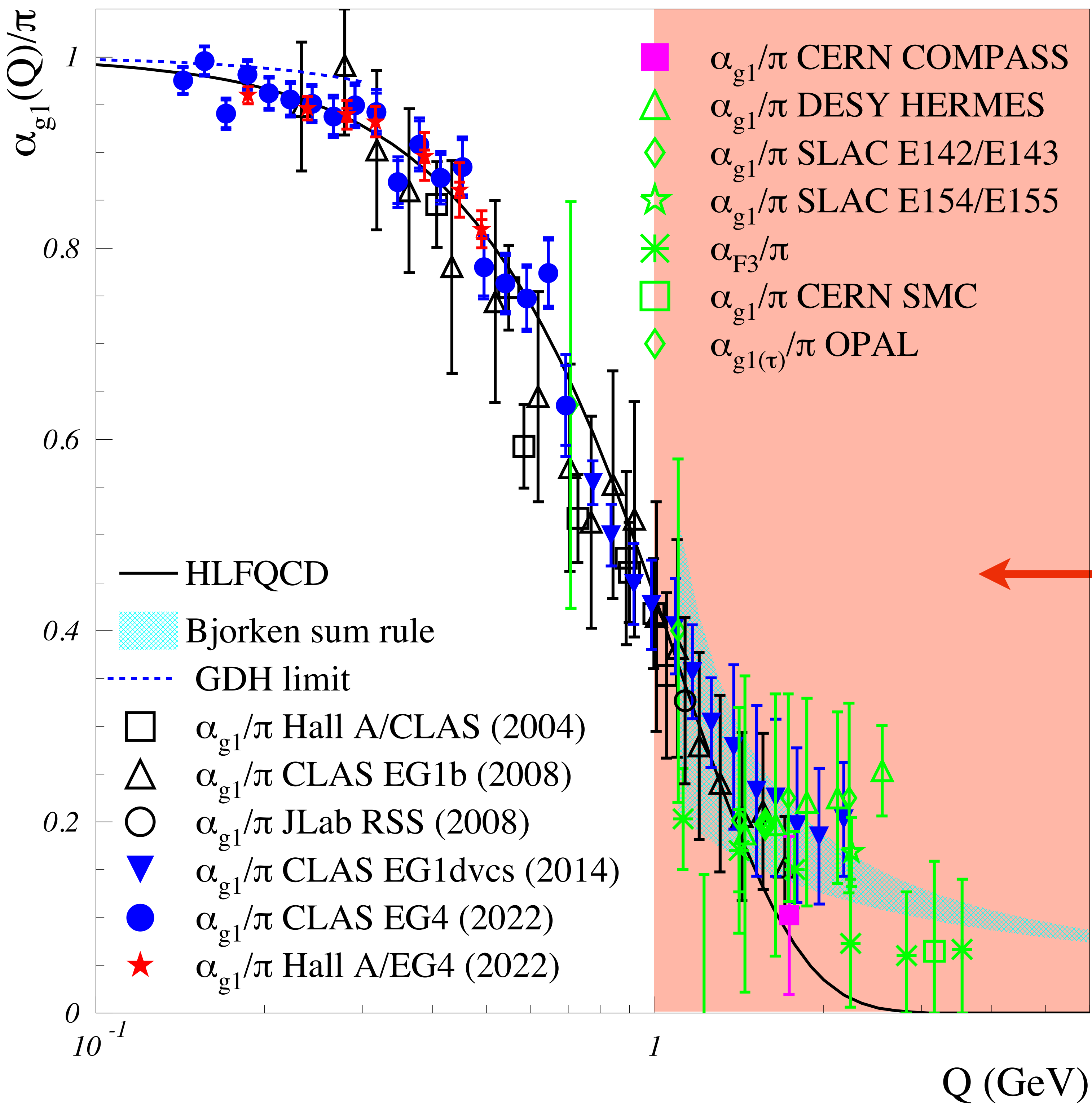
Comparison with process-independent α_s from SDE calculation

Cui et al. Chin.Phys.C 44 8, 083102 (2020)



⇒ **SDE, HLFQCD and data agree very well.**

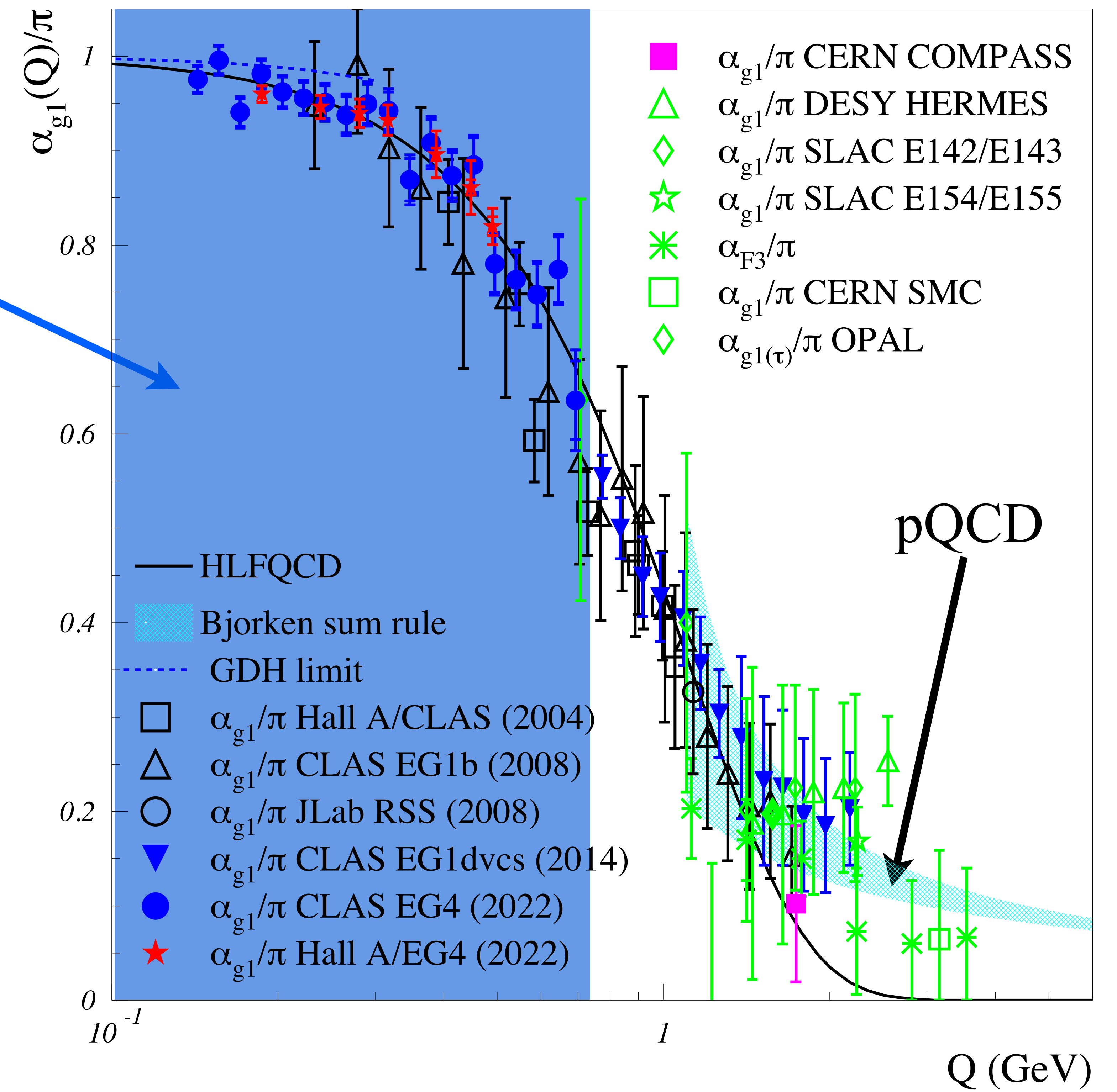
Predictions of the hadronic mass spectrum



HLFQCD not applicable

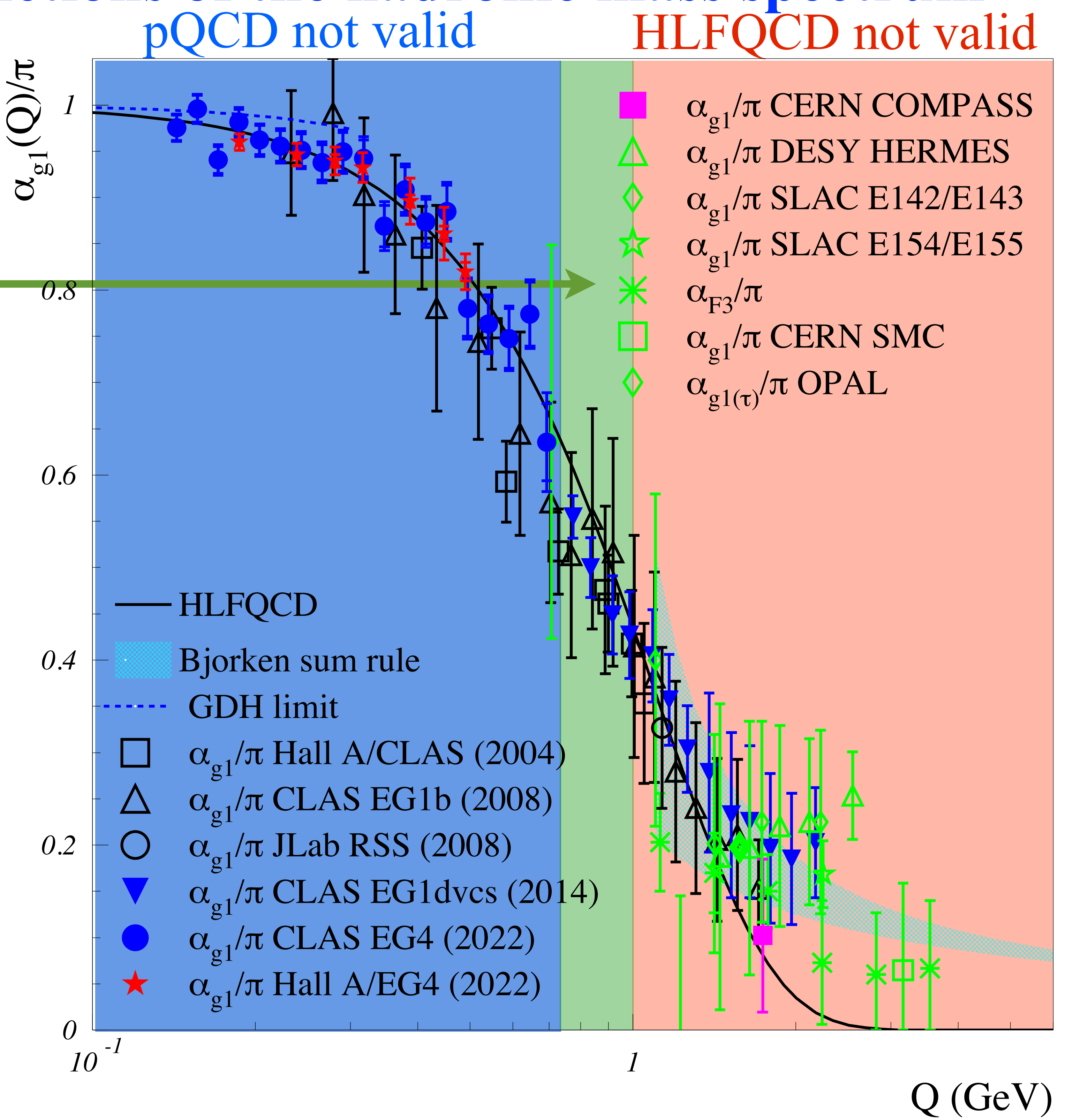
Predictions of the hadronic mass spectrum

pQCD not valid



Predictions of the hadronic mass spectrum

pQCD and HLFQCD both provide a good description of α_{g1} (i.e. the Bjorken sum)



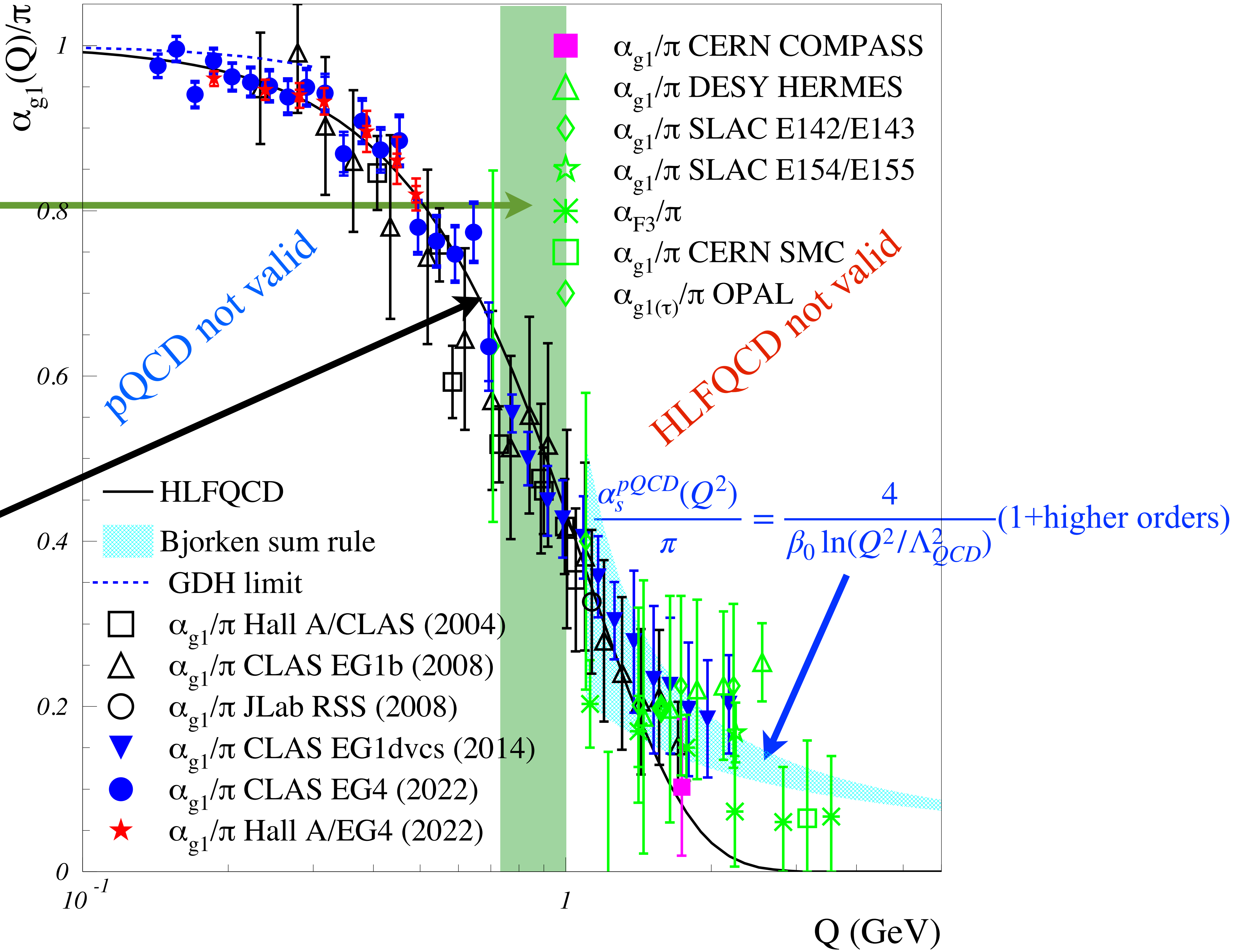
Match HLFQCD and pQCD expressions of α_{g1} and its β-function:

$\Rightarrow$ Relate **hadronic masses** to fundamental QCD parameter Λ_{QCD}.

Connecting κ to Λ_{QCD}

pQCD and HLFQCD both provide a good description of α_{g1} (i.e. the Bjorken sum)

$$\frac{\alpha_s^{HLF}(Q^2)}{\pi} = e^{-Q^2/4\kappa^2}$$



$\kappa = \Lambda_{QCD}\, e^{(a+1)}(a/2)^{1/2}$ At LO

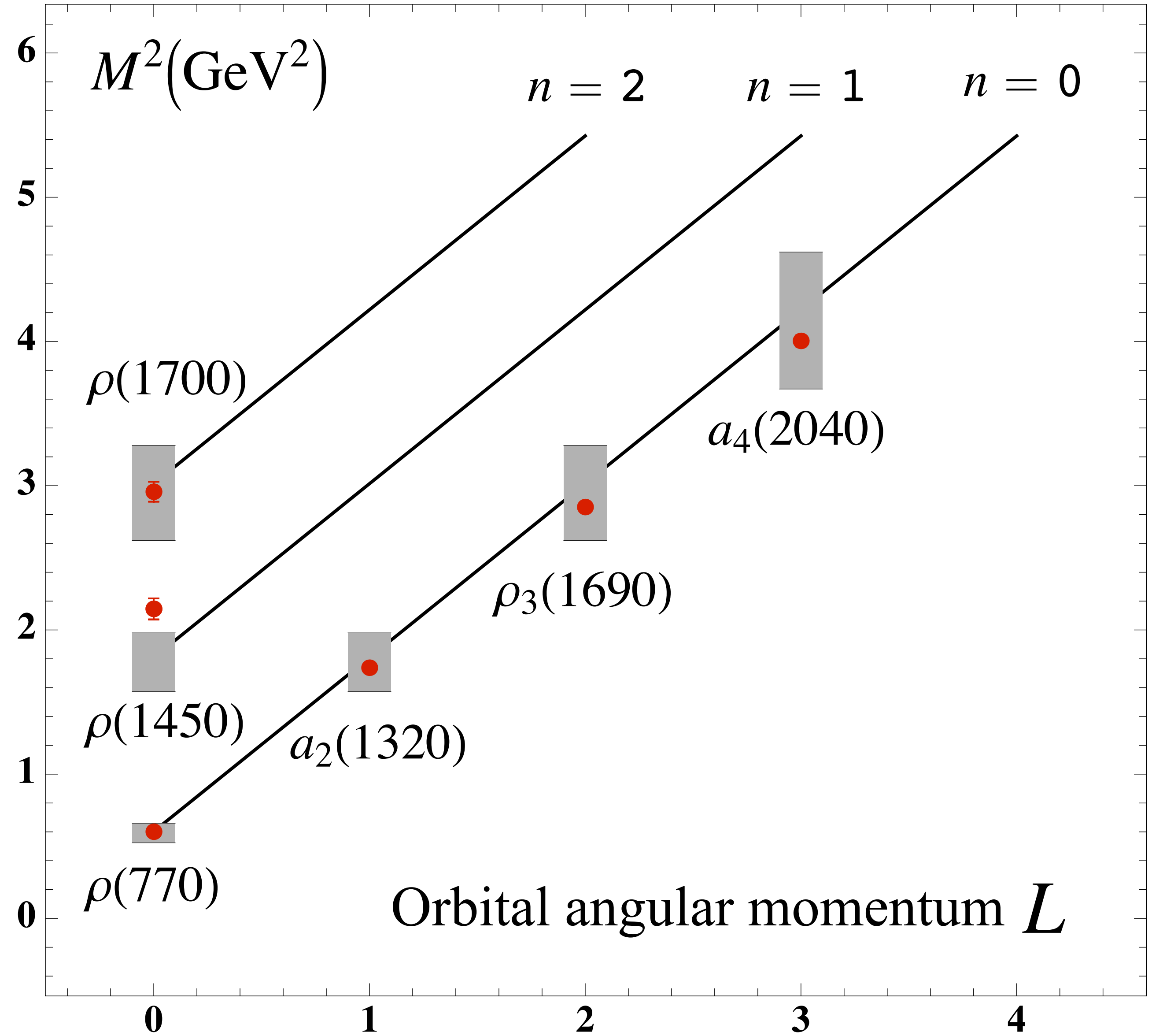
$a=4(\sqrt{\ln(2)^2-1+\beta_0/4}-\ln(2))/\beta_0$

$\kappa = 1.607\Lambda_{\overline{MS}}$ At N³LO

Deur, Brodsky, de Téramond, PLB 750, 528 (2015)

Predictions of the hadronic mass spectrum

$\kappa = 1.607\Lambda_{\overline{MS}}$
$\kappa = M_\rho/\sqrt{2}$



: HLFQCD predictions with Λ_{QCD} from Part. Data Group as only input.
: Slopes predicted by HLFQCD.
: Measurements.

Baryon spectrum obtained from hadronic supersymmetry or from proton mass.

Brodsky, de Téramond, Dosch and Lorcé, Int. J. Mod. Phys. A 31, 1630029 (2016)

For hadrons with heavy valence quarks, heavy mass quarks are also needed as input.

Analytic determination of hadron spectrum with Λ_{QCD} as only input (+heavy quark mass if needed)

Summary

- α_s is the fundamental parameter of QCD.
- Reason for the running of the (effective) coupling, including in non-pert. domain.
- Bjorken Sum Rule is advantageous to define an effective coupling α_{g1}.
- Data and sum rules allow us to know α_{g1} at all Q^2.
- α_{g1} ~constant at low Q^2 ⇒ Application of AdS/CFT on LF to non-perturbative QCD.
- α_s obtained with HLFQCD.
  - Its form is **imposed** by respecting QCD's basic (approximate) symmetries: either conformal symmetry of QCD Lagrangian (mass scale emerging in QCD's Action: dAFF mechanism), or chiral symmetry (massless pion). Or requiring hadronic supersymmetry.
  - No free parameters (uses only one parameter, κ, known from very different phenomenology).
  - Remarkable agreement with α_{g1} data and recent SDE calculation.
- Analytic determination of hadron mass spectrum with Λ_{QCD} as the only input.