Energy momentum tensor from a light front perspective

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September 19, 2022

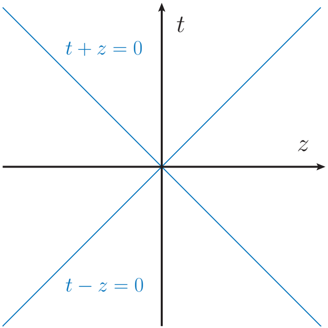
Introduction

- How are energy and momentum spatially distributed in hadrons?
  - Mass decomposition
  - Spin decomposition
  - Distribution of pressures, stresses, forces
  - **Energy-momentum tensor** (EMT)
- Hadrons are *manifestly relativistic*.
  - Their size is comparable to their Compton wavelength.
  - Their constituents move close to the speed of light.
  - **Light front coordinates** provide appropriate description.
- Based on work in:
  - AF & Gerald Miller, PRD**103**, 094023
  - AF & Gerald Miller, PRD**104**, 014024
  - AF & Gerald Miller, PRD**105**, 014003
  - AF & Wim Cosyn, arxiv:2207.10787
  - AF & Wim Cosyn, arxiv:2207.10788
  - AF & Gerald Miller, upcoming work

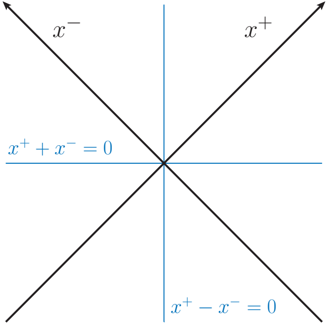
Light front coordinates

Light front coordinates are a different foliation of spacetime.

$$x^{\pm} = \frac{t \pm z}{\sqrt{2}} \qquad \mathbf{x}_\perp = (x, y) \qquad \tau = x^+ = \text{time}$$



Minkowski coordinates



Light front coordinates

Light front: Myths and Facts

- **Myth:** Light front coordinates are a reference frame.
- **Fact:** Light front coordinates can be employed in any reference frame.

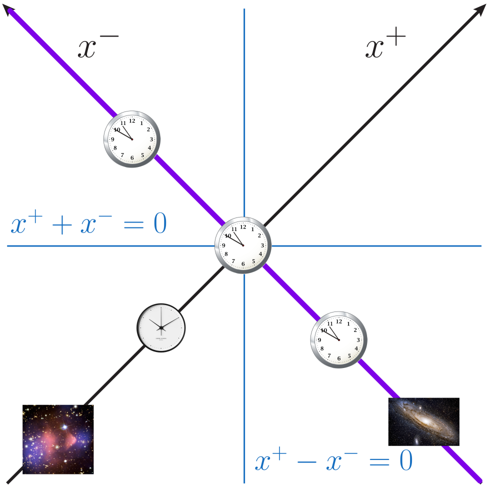
- **Myth:** Light front coordinates describe the perspective of light.
- **Fact:** Light front coordinates describe *our* perspective.
  - ...but only in the z direction!
  - Light has no perspective.

- **Myth:** Light front coordinates come from boosting to infinite momentum.
- **Fact:** Light front coordinates come from redefining:
  1. *Simultaneity*
  2. What we mean by *boosting*
  3. How we break the Poincaré group into generators (*Galilean subgroup*)

Light front & time synchronization

Light front *redefines simultaneity*

- Fixed $x^+ = \frac{t+z}{\sqrt{2}}$ means *simultaneous*
- Look in the $+\hat{z}$ direction...
  - Whatever you **see** *right now*,
    is **happening** *right now*.
  - *Only* true for $+\hat{z}$ direction though.
- Light front coordinates are what *we* see.
  - ...at least in one fixed direction.
  - (great for small systems—hadrons!)
  - *Not* what light "sees."



Terrell rotations

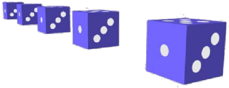
- Lorentz-boosted objects appear *rotated.*
  - **Terrell rotation**
  - Optical effect: contraction + delay
- **Light front transverse boost** *undoes* Terrell rotation:

$$B_x^{(\mathrm{LF})} = \frac{1}{\sqrt{2}}\Big(K_x + J_y\Big)$$

  - Combination of ordinary boost + rotation!
  - Leaves x^+ (time) invariant!
  - Changes p_z, but leaves p^+ invariant:

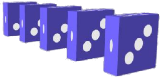
$$P^+ = \frac{E + p_z}{\sqrt{2}} = \frac{\sqrt{p_z^2 + \mathbf{p}_\perp^2 + M^2} + p_z}{\sqrt{2}}$$

- Dice images by Ute Kraus, `https://www.spacetimetravel.org/`

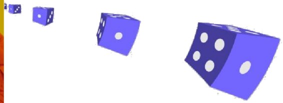


What happens upon boost?

Length contraction?



Terrell rotation



Galilean subgroup

- Poincaré group has a $(2+1)$D **Galilean subgroup**.
  - x^+ is time and $\mathbf{x}_\perp$ is space under this subgroup.
  - x^- can be integrated out.
  - $P^+ = \frac{1}{\sqrt{2}}(E_\mathbf{p} + p_z)$ is the central charge.
  - x^+ and P^+ are invariant under this subgroup!
- Basically, light front coordinates should give a **fully relativistic** 2D picture that looks like *non-relativistic* physics.
  - But with P^+ in place of M.

$$\frac{\mathrm{d}\mathbf{P}_\perp}{\mathrm{d}x^+} = P^+ \frac{\mathrm{d}^2\mathbf{x}_\perp}{\mathrm{d}{x^+}^2}$$

$$H = P^- = H_{\text{rest}} + \frac{\mathbf{P}_\perp^2}{2P^+}$$

$$\mathbf{v}_\perp = \frac{\mathbf{P}_\perp}{P^+}$$

The cost: lose one spatial dimension (2D densities).

Galilean group and densities

- Physical densities given by expectation values:

$$\rho_{\text{phys}}(x^+, \mathbf{x}_\perp) = \int \mathrm{d}x^- \, \langle \Psi | \hat{\rho}_{\text{op.}}(x) | \Psi \rangle$$

- Galilean subgroup means **barycentric position** and **impact parameter** dependence can be separated.
  - Allows wave packet dependence to be factored out.
  - Instant form coordinates **do not** have this luxury!
- **Simple densities:**

$$\rho_{\text{phys}}(x^+, \mathbf{x}_\perp) = \int \mathrm{d}^2\mathbf{R}_\perp \left| \Psi(\mathbf{R}_\perp, x^+) \right|^2 \rho_{\text{internal}}(\mathbf{x}_\perp - \mathbf{R}_\perp)$$

- **Compound densities:**

$$\rho_{\text{phys}}(x^+, \mathbf{x}_\perp) = \int \mathrm{d}^2\mathbf{R}_\perp \left\{ \mathcal{S}_\Psi^{(1)}(\mathbf{R}_\perp, x^+) \rho_{\text{int.}}^{(1)}(\mathbf{x}_\perp - \mathbf{R}_\perp) + \mathcal{S}_\Psi^{(2)}(\mathbf{R}_\perp, x^+) \rho_{\text{int.}}^{(2)}(\mathbf{x}_\perp - \mathbf{R}_\perp) \right\}$$

The energy-momentum tensor

- The **energy-momentum tensor** (EMT) is an operator characterizing the distribution and flow of energy and momentum.
  - Source of gravitation
  - Related to spacetime translation symmetry (Noether's theorems)
- Physical densities given by expectation values:

$$T^{\mu\nu}(x^+, \mathbf{x}_\perp) = \int \mathrm{d}x^- \, \langle \Psi | \hat{T}^{\mu\nu}(x) | \Psi \rangle$$

  - 2D density on transverse plane.
  - Every component is a simple or compound density.
  - Example: P^+ density is a **simple density**:

$$T^{++}(x^+, \mathbf{x}_\perp) = \int \mathrm{d}^2\mathbf{R}_\perp \left| \Psi(\mathbf{R}_\perp, x^+) \right|^2 T^{++}_{\text{intrinsic}}(\mathbf{x}_\perp - \mathbf{R}_\perp)$$

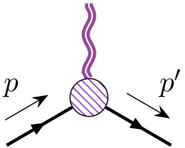
Form factors of the EMT

- EMT matrix elements give **gravitational form factors** (GFFs).
  - It's just a name.
  - EMT is the source of gravitation: $G^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi T^{\mu\nu}$
  - But we don't really use gravitation to measure them.
- Analogy to **electromagnetic form factors**.
- Spin-zero example:

$$\langle p' | \hat{J}^\mu(0) | p \rangle = 2P^\mu F(t)$$

$$\langle p' | \hat{T}^{\mu\nu}(0) | p \rangle = 2P^\mu P^\nu A(t) + \frac{1}{2}(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) D(t)$$

- $A(t)$ encodes momentum density (mass in NR limit)
- $D(t)$ encodes stress distributions (anisotropic pressures)



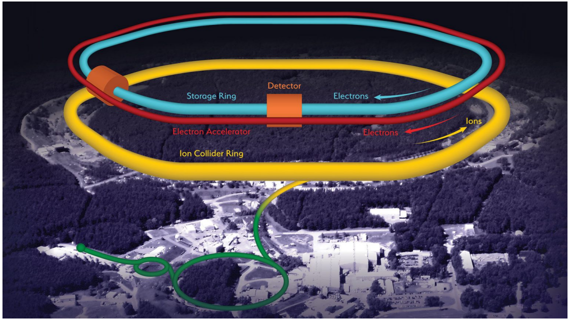
$$P = \frac{1}{2}(p + p')$$

$$\Delta = (p' - p)$$

$$t = \Delta^2$$

How to get the GFFs

- **Hard exclusive reactions** are used to measure GFFs—not gravity experiments.
  - Deeply virtual Compton scattering (DVCS) to probe quark structure.
  - Deeply virtual meson production (DVMP), *e.g.*, J/ψ or Υ to probe gluon structure.
  - ...and more!
- Measured at **Jefferson Lab** and the upcoming **Electron Ion Collider**.



Components of the EMT

$$T^{\mu\nu}(x) = \begin{bmatrix} T^{++}(x) & T^{+1}(x) & T^{+2}(x) & T^{+-}(x) \\ T^{1+}(x) & T^{11}(x) & T^{12}(x) & T^{1-}(x) \\ T^{2+}(x) & T^{21}(x) & T^{22}(x) & T^{2-}(x) \\ T^{-+}(x) & T^{-1}(x) & T^{-2}(x) & T^{--}(x) \end{bmatrix}$$

- **Momentum densities**
- **Energy density**
- **Stress tensor**
- ...and some other things

- **Angular momentum density** (z component) accessible too:

$$J_z(x) = x^1 T^{+2}(x) - x^2 T^{+1}(x)$$

...basically, from $\mathbf{x} \times \mathbf{p}$

Light front momentum densities

- **Physical** P^+ density is a simple density:

$$T^{++}_{\text{phys}}(\mathbf{x}_\perp) = P^+ \int \mathrm{d}^2\mathbf{R}_\perp \Big|\Psi(\mathbf{R}_\perp, x^+)\Big|^2 T^{++}_{\text{int.}}(\mathbf{x}_\perp - \mathbf{R}_\perp)$$

- Transverse momentum density involves *same internal density*:

$$\mathbf{T}^{+i}_{\text{phys}}(\mathbf{x}_\perp) = \int \mathrm{d}^2\mathbf{R}_\perp \Psi^*(\mathbf{R}_\perp, x^+) \frac{-i\overleftrightarrow{\nabla}^i_\perp}{2} \Psi(\mathbf{R}_\perp, x^+) T^{++}_{\text{int.}}(\mathbf{x}_\perp - \mathbf{R}_\perp)$$

  - But a different smearing function

- Internal density is simple Fourier transform:

$$T^{++}_{\text{int.}}(\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\mathbf{\Delta}_\perp}{(2\pi)^2} \frac{\langle p', \mathbf{s}_\perp | \hat{T}^{++}(0) | p, \mathbf{s}_\perp \rangle}{2(P^+)^2} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp}$$

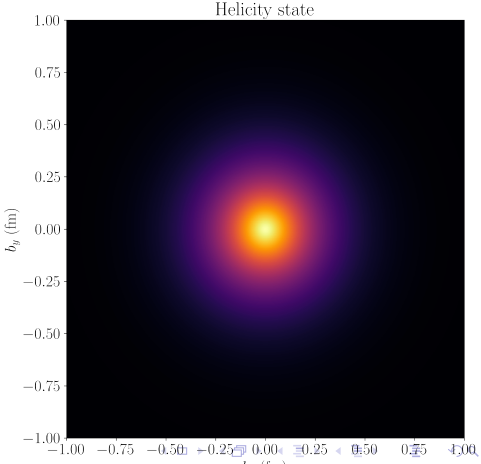
  - Internal structure has polarization dependence.

Light front momentum density

- P^+ density is a 2D Fourier transform:

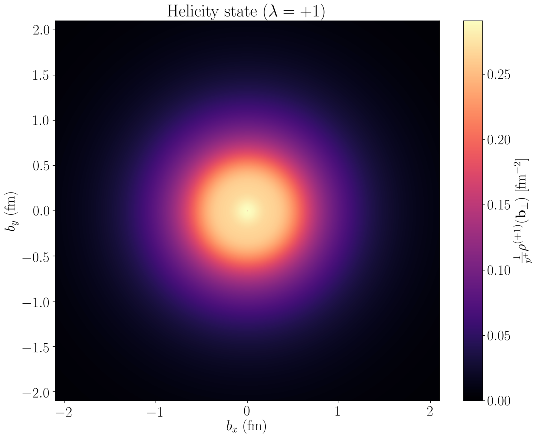
$$\rho^{(\mathrm{LF})}_{P^+}(\mathbf{b}_\perp, \mathbf{s}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \frac{\langle p', \mathbf{s}_\perp | T^{++}(0) | p, \mathbf{s}_\perp \rangle}{2(P^+)^2} e^{-i\boldsymbol{\Delta}_\perp \cdot \mathbf{b}_\perp}$$

  - Works for any polarization state.
  - Structure *relative to* center-of-P^+.
  - Boost invariance: $\mathbf{P}_\perp$ independent!
- Proton dipole model on right.
  - $f_2(1270)$ pole
- Also applicable to massless targets!
  - P^+, not M, is central charge.

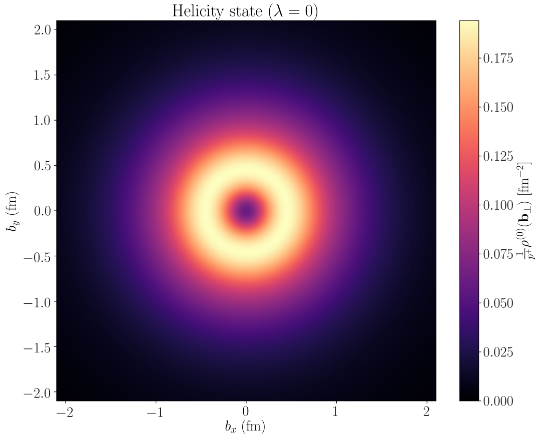


Spin-one targets

Helicity +1



Helicity 0

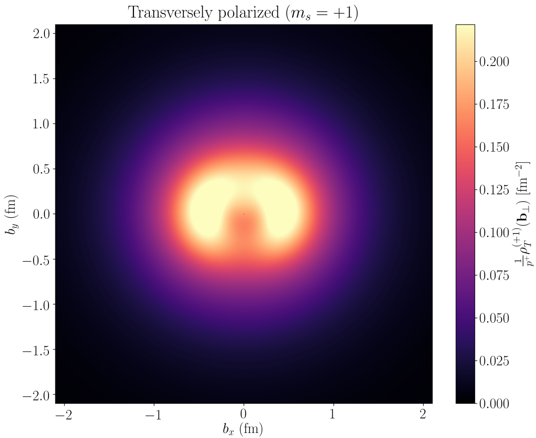


P^+ density depends on helicity for spin-one targets.

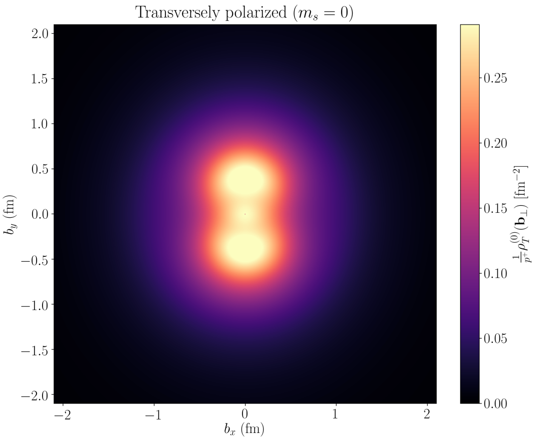
AF & Wim Cosyn, arxiv:2207.10787

Transverse polarization

Transverse, $m_s = +1$



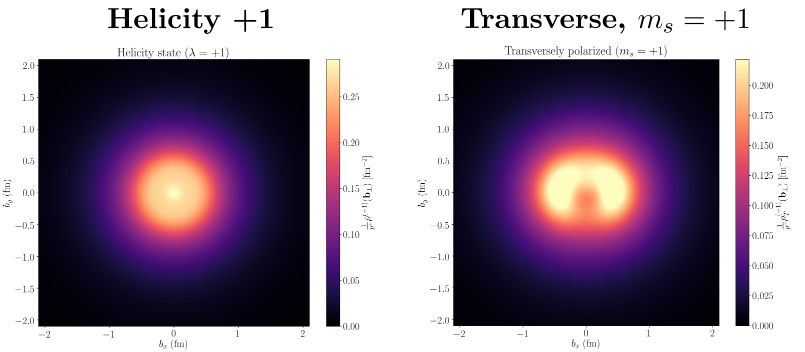
Transverse, $m_s = 0$

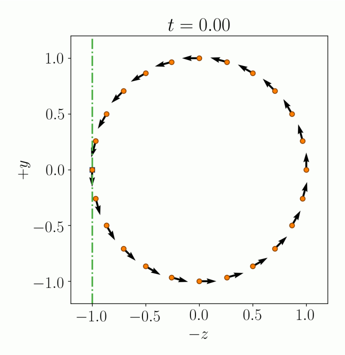


Transverse polarization contains helicity-flip contributions.

AF & Wim Cosyn, arxiv:2207.10787

Why angular modulations?

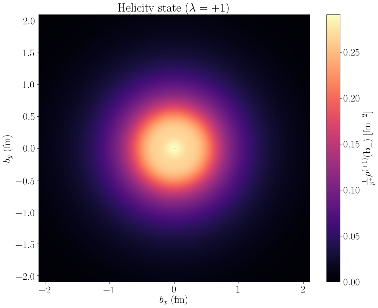




- Light takes a finite time to move across deuteron.
- Deuteron spins in the meantime.
- Greater density if spin is against light front.
- Distortion is what *we actually see* (even if stationary!).

Why angular modulations?

Helicity $+1$



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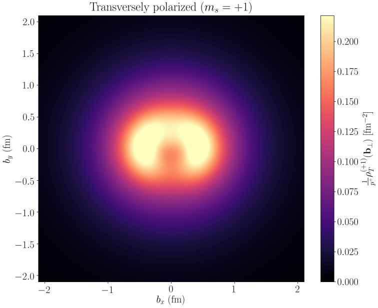

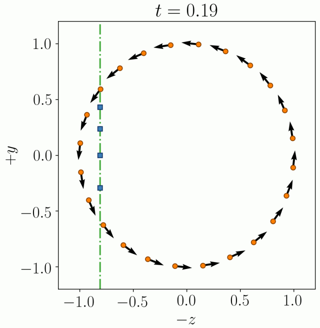

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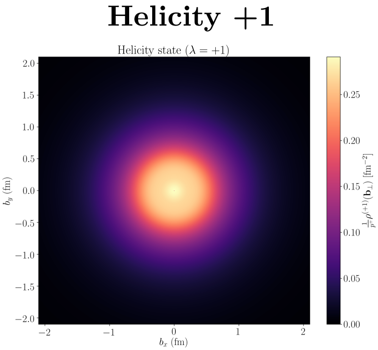

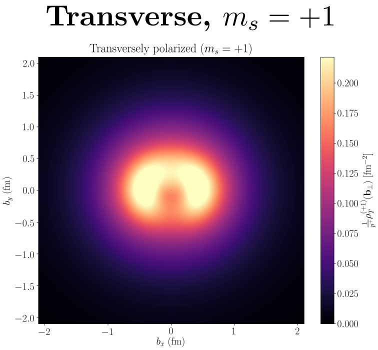

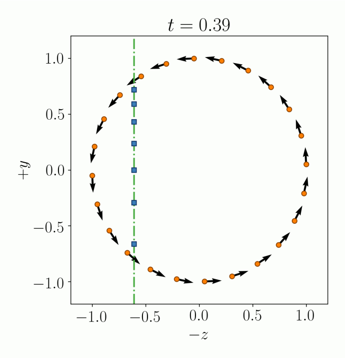

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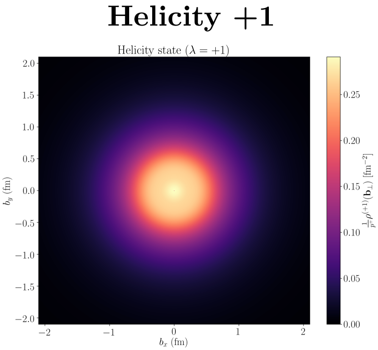

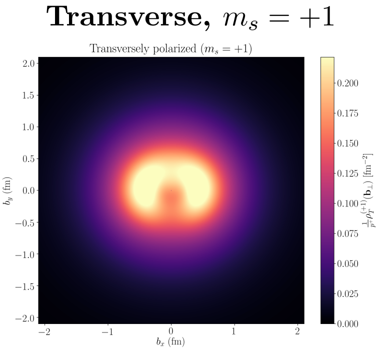

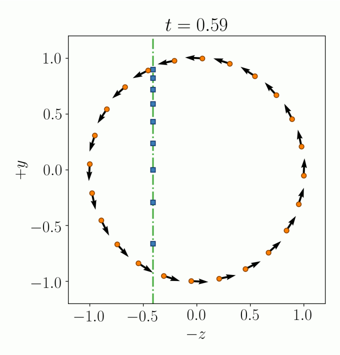

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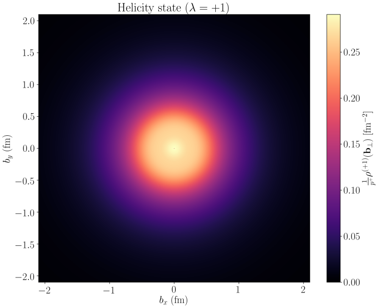
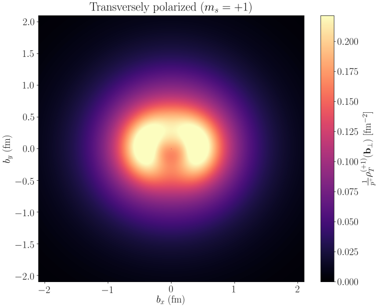
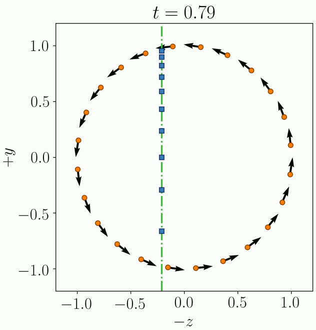
Why angular modulations?

Helicity +1



Transverse, $m_s = +1$





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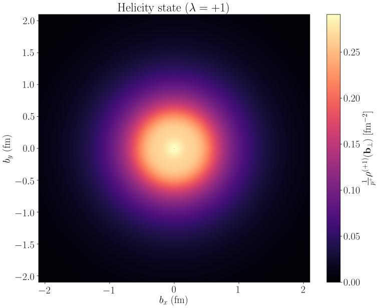

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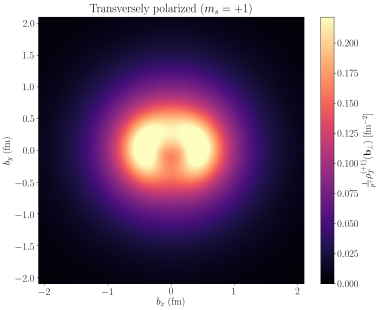

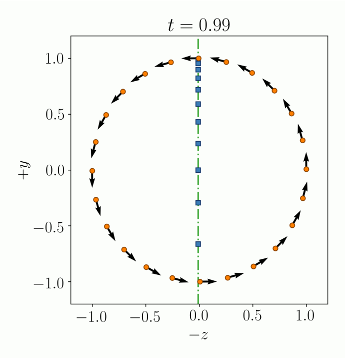

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Energy density

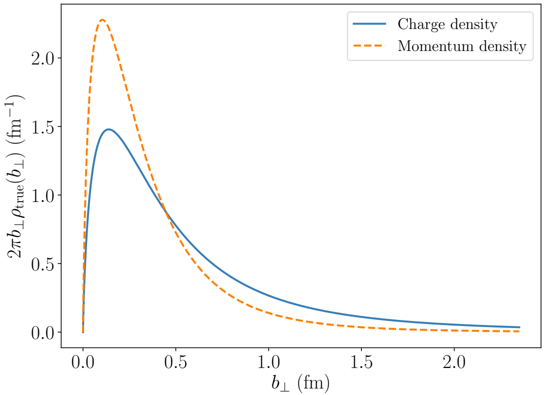
- **Light front energy density** is a *compound density*:

$$T^{+-}_{\text{phys}}(\mathbf{x}_\perp) = \frac{1}{2P^+}\int \mathrm{d}^2\mathbf{R}_\perp \Big\{ \Big|\Psi(\mathbf{R}_\perp, x^+)\Big|^2 T^{+-}_{\text{int.}}(\mathbf{x}_\perp - \mathbf{R}_\perp) \\ - \Psi^*(\mathbf{R}_\perp, x^+)\frac{\overleftrightarrow{\nabla}^2_\perp}{4}\Psi(\mathbf{R}_\perp, x^+) T^{++}_{\text{int.}}(\mathbf{x}_\perp - \mathbf{R}_\perp)\Big\}$$

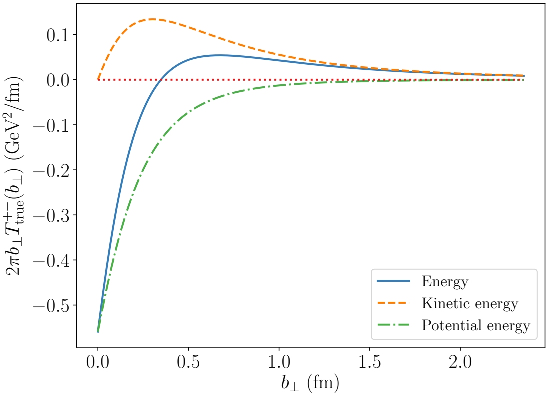
- First piece is **true intrinsic energy density**
  - Quark mass energy
  - Quark kinetic energy (relative to barycenter)
  - Potential energy & internal stresses
  - Literally the density of the $2P^+P^- - \mathbf{P}_\perp^2$ operator used by light front folks!
- Second piece is **barycentric kinetic energy**
  - It's literally just the $\mathbf{P}_\perp^2/(2P^+)$ density
  - Tells us nothing about internal dynamics
  - Galilean subgroup allows us to isolate it

Example densities from holographic model

Charge and momentum



Energy



Using soft wall holograhpic model of Brodsky & de Teramond, PRD77 (2008) 056007

Stress

- **Stress tensor** is also a *compound density*:

$$T^{ij}_{\text{phys}}(\mathbf{x}_\perp) = \frac{1}{P^+}\int \mathrm{d}^2\mathbf{R}_\perp \bigg\{ \Big|\Psi(\mathbf{R}_\perp, x^+)\Big|^2 T^{ij}_{\text{int.}}(\mathbf{x}_\perp - \mathbf{R}_\perp) - \Psi^*(\mathbf{R}_\perp, x^+) \frac{\overleftrightarrow{\nabla}^i_\perp \overleftrightarrow{\nabla}^j_\perp}{4} \Psi(\mathbf{R}_\perp, x^+) T^{++}_{\text{int.}}(\mathbf{x}_\perp - \mathbf{R}_\perp) \bigg\}$$

- First piece is **true intrinsic stress tensor**
  - Stresses seen by comoving observer
- Second piece is **stresses from hadron flow**
  - Includes motion of hadron
  - Includes wave function dispersion

Stress tensor and hadron flow

- Compound form of stress tensor mimics classical continuum mechanics
- In Galilean theory (eg **light front**):

$$T^{ij}(\mathbf{x},\mathbf{v},\nabla\mathbf{v}) = \mathbf{v}^i\mathbf{v}^j\rho(\mathbf{x}) + T^{ij}_{\text{pure}}(\mathbf{x},\nabla\mathbf{v})$$

  - $\mathbf{v}^i\mathbf{v}^j\rho(\mathbf{x})$ depends on wave packet.
- **Comoving stress tensor**:

$$S^{ij}(\mathbf{x},\nabla\mathbf{v}) = T^{ij}(\mathbf{x},\mathbf{v}=0,\nabla\mathbf{v})$$

  - Stresses seen by comoving obsever
  - True internal structure of hadron

Example: spin-zero with form factors

- Full stress tensor:

$$\frac{1}{2P^+}\langle p'|\hat{T}^{ij}(0)|p\rangle = P^+\frac{\mathbf{P}_\perp^i}{P^+}\frac{\mathbf{P}_\perp^j}{P^+}A(t) + \frac{1}{4P^+}\Big(\mathbf{\Delta}_\perp^i\mathbf{\Delta}_\perp^j - \mathbf{\Delta}_\perp^2\delta^{ij}\Big)D(t)$$

- **Hadron flow**:

$$V_{\mathrm{LF}}^{ij}(\mathbf{b}_\perp) = \left\langle \frac{\mathbf{P}_\perp^i}{P^+}\frac{\mathbf{P}_\perp^j}{P^+}\right\rangle P^+\int\frac{\mathrm{d}^2\mathbf{\Delta}_\perp}{(2\pi)^2}A(t)e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}$$

- **Pure stress tensor**:

$$S_{\mathrm{LF}}^{ij}(\mathbf{b}_\perp) = \frac{1}{4P^+}\int\frac{\mathrm{d}^2\mathbf{\Delta}_\perp}{(2\pi)^2}\Big(\mathbf{\Delta}_\perp^i\mathbf{\Delta}_\perp^j - \delta^{ij}\mathbf{\Delta}_\perp^2\Big)D(t)e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}$$

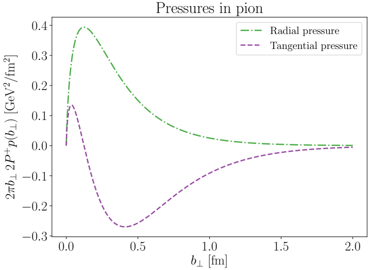
  - Only **D-term** appears in internal stresses.

Pion pressures and energy density

- Pion nearly massless—requires **light front** description.
- Densities by 2D Fourier transforms:

$$T^{ij}_{\text{pure}}(\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \frac{\boldsymbol{\Delta}^i_\perp \boldsymbol{\Delta}^j_\perp - \delta^{ij}\boldsymbol{\Delta}^2_\perp}{2} D(-\boldsymbol{\Delta}^2_\perp) e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp}$$

$$\mathcal{E}(\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \left(m_\pi^2 - \frac{\boldsymbol{\Delta}^2_\perp}{4}\right) A(-\boldsymbol{\Delta}^2_\perp) e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp} - \delta_{ij} T^{ij}_{\text{pure}}(\mathbf{b}_\perp)$$

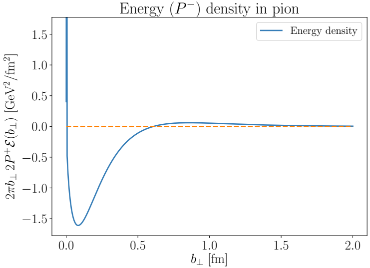


- Phenomenological form factors:

$$A(t) = \frac{1}{1 - t/m_{f_2}^2} \qquad m_{f_2} = 1270 \text{ MeV}$$

$$D(t) = \frac{-1}{(1 - t/m_{f_2}^2)(1 - t/m_\sigma^2)} \qquad m_\sigma = 630 \text{ MeV}$$

  - Forms inspired by Masjuan *et al* [PRD87 (2013) 014005]
  - Poles chosen to match Kumano's radii [PRD97 (2018) 014020]
  - Upcoming work (Freese & Miller) for more info!



Summary

- Relativistic densities can be given in **light front coordinates**
- The **energy momentum tensor** encodes momentum, energy & stress distributions.
- Several of these are **compound densities**.
- Energy density decompses into **barycentric kinetic energy** and **internal energy**.
- Stress tensor decomposes into **flow** and **intrinsic stresses**.
- **Galilean invariance** is needed to allow these decompositions.

Thank you for your time!