Electromagnetic interaction of baryon resonances in the timelike region studied via the reaction

$$\pi N \to N e^+ e^-$$

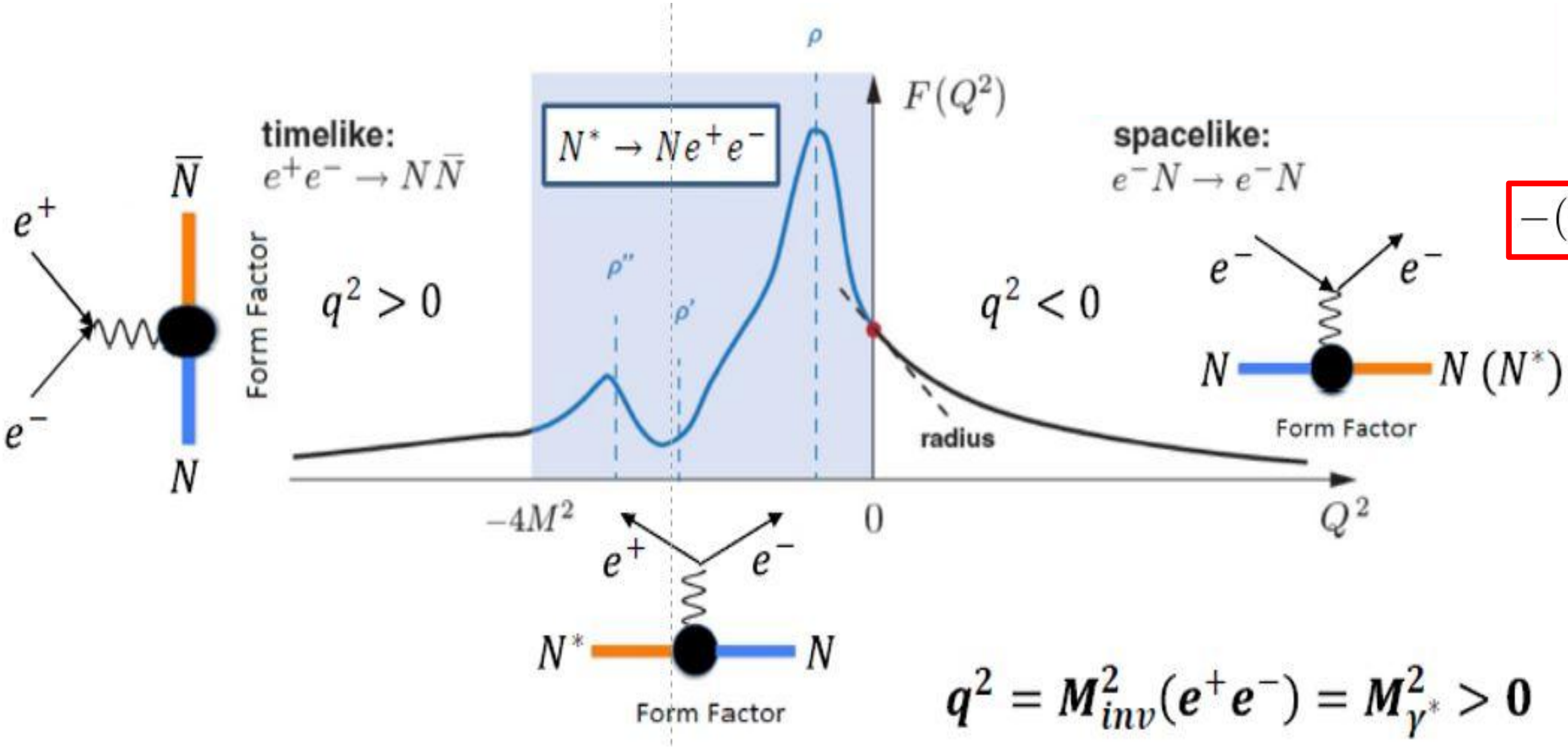
Miklós Zétényi
Wigner RCP

APCTP Workshop on Nuclear Physics
11-16 July 2022

Outline

- Introduction
- $\pi N \rightarrow N e^{+} e^{-}$ in helicity formalism
  - Polarization & anisotropy from symmerties
- Effective Lagrangian model
- Comparison to new results from HADES

EM transition form factors of baryon resonances



$$q^2 = M^2_{inv}(e^+e^-) = M^2_{\gamma^*} > 0$$

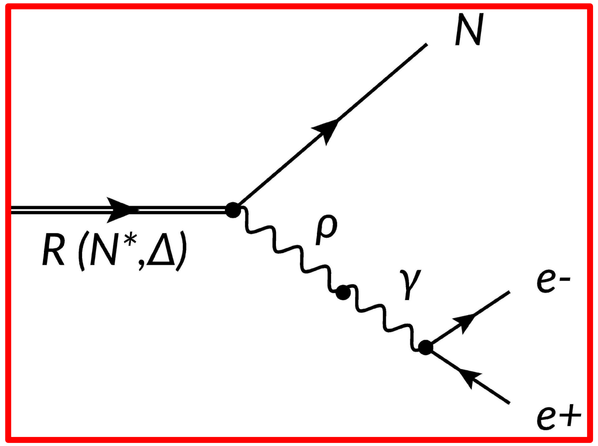
Domains of $Q^2 = -q^2$

$Q^2 < -(m_R + m_N)^2$: e+e- annihilation

$-(m_R - m_N)^2 < Q^2 < 0$: N* Dalitz-decay

$Q^2 = 0$: photoproduction

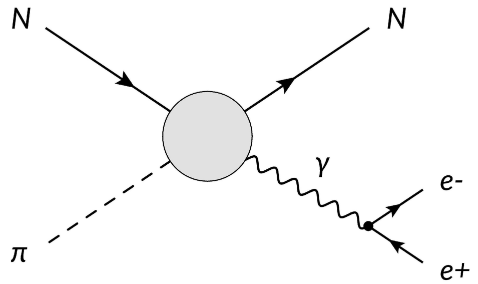
$0 < Q^2$: electroproduction



Δ(1232) Dalitz-decay measured by HADES in p+p

J. Adamczewski-Musch et al. (HADES Collaboration) Phys. Rev. C **95**, 065205 (2017)

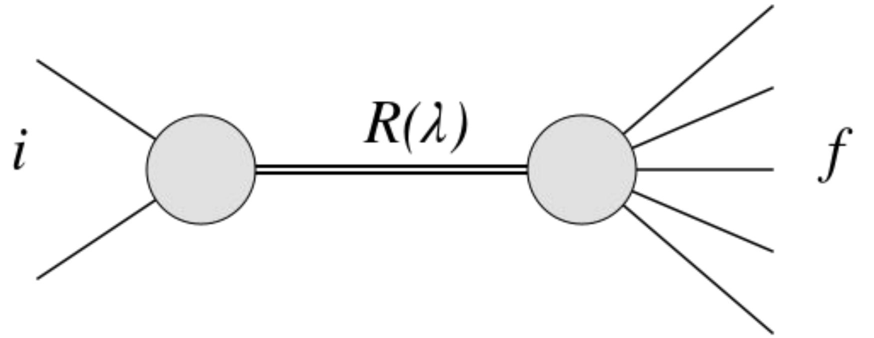
HADES @GSI:
pion-induced reactions @ $\sqrt{s_{\pi N}}$ = 1.49 GeV
(future: 1.7 GeV)



Resonance polarization - density matrices

Creation and decay of a resonance:

$$\mathcal{M}_{fi} = \sum_{\lambda} \left\langle f \left| T_{R\to f} \right| R(\lambda) \right\rangle \left\langle R(\lambda) \left| T_{i\to R} \right| i \right\rangle$$



Squared amplitude (weighted average over initial polarization):

$$\sum_i P_i |\mathcal{M}_{fi}|^2 = \sum_{\lambda,\lambda'} \rho^{\text{cre}}_{\lambda\lambda'} \rho^{\text{dec}}_{\lambda'\lambda}$$

c.f. $\langle \mathcal{O} \rangle = \mathrm{Tr}\,(\rho\, \mathcal{O})$

$\rho \sim \rho^{\text{cre}}_{\lambda\lambda'}$ polarization state of the resonance

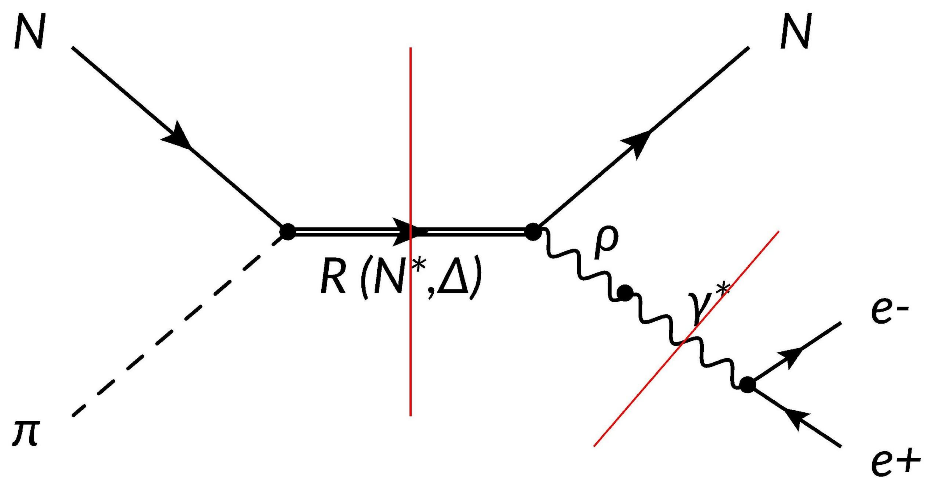
Polarization density matrices:

$\mathcal{O} \sim \rho^{\text{dec}}_{\lambda'\lambda}$ measured quantity $\longrightarrow$ angular distribution of decay products

$$\rho^{\text{cre}}_{\lambda\lambda'} = \sum_i P_i \left\langle R(\lambda) \left| T_{i\to R} \right| i \right\rangle \left\langle i \left| T^{\dagger}_{i\to R} \right| R(\lambda') \right\rangle$$

$$\rho^{\text{dec}}_{\lambda'\lambda} = \left\langle R(\lambda') \left| T^{\dagger}_{R\to f} \right| f \right\rangle \left\langle f \left| T_{R\to f} \right| R(\lambda) \right\rangle$$

The process $\pi N \to N e^+ e^-$



Important contribution:
baryon resonance in the *s*-channel

3 steps - 2 intermediate resonances

- baryon resonance *R* (*N** or Δ)
- vector particle (ϱ or γ*)

each of the 3 steps is a 1→2 or 2→1 process

$\pi + N \to R$ preparation of resonance state

$R \to N + \gamma^*$ E.M. transition of resonance

$\gamma^* \to e^+ e^-$ "analyser"

1⟶2 processes

Wigner-Eckart theorem for transition amplitude $R \longrightarrow 1+2$:

$$\left\langle \mathbf{p} s_1 \lambda_1; -\mathbf{p} s_2 \lambda_2 \middle| T_{R\to 1+2} \middle| \mathbf{p}_R = 0,\, JM \right\rangle = \sqrt{\frac{2J+1}{4\pi}} F^J_{\lambda_1\lambda_2} D^J_{M,\lambda_1-\lambda_2}(\Omega)^*$$

Wigner matrix:
angular distribution - from symmetry

Helicity amplitude:
from dynamics

Restrictions on helicity amplitudes:

from Wigner matrix $D^J_{M,\lambda_1-\lambda_2}(\Omega)$: $-J \le \lambda_1 - \lambda_2 \le J$

from parity conservation: $F^J_{\lambda_1\lambda_2} = \eta_R \eta_1 \eta_2 (-1)^{J-s_1-s_2} F^J_{-\lambda_1,-\lambda_2}$

Helicity formalism

1⟶2 processes

Wigner matrix:
angular distribution - from symmetry

Wigner-Eckart theorem for transition amplitude *R* ⟶1+2:

$$\left\langle \mathbf{p} s_1 \lambda_1; -\mathbf{p} s_2 \lambda_2 \middle| T_{R\to 1+2} \middle| \mathbf{p}_R = 0,\ JM \right\rangle = \sqrt{\frac{2J+1}{4\pi}} F^J_{\lambda_1\lambda_2} D^J_{M,\lambda_1-\lambda_2}(\Omega)^*$$

Helicity amplitude:
from dynamics

Restrictions on helicity amplitudes:

from Wigner matrix $D^J_{M,\lambda_1-\lambda_2}(\Omega)$: $-J \le \lambda_1 - \lambda_2 \le J$

from parity conservation: $F^J_{\lambda_1\lambda_2} = \eta_R \eta_1 \eta_2 (-1)^{J-s_1-s_2} F^J_{-\lambda_1,-\lambda_2}$

$\pi + N \to R$:

Quantization axis = beam axis: $D^J_{\lambda_R,\lambda_N}(\theta=0,\phi=0) = \delta_{\lambda_R,\lambda_N}$ $\longrightarrow$ $\lambda_R = \lambda_N$

The nucleon state is a mixture of +½ and -½ helicities ⟶ so is the resonance *R*

⟶ for $J \ge 3/2$ resonances the $\lambda_R \ge 3/2$ states are missing ⟶ *R* has nontrivial polarization

Helicity formalism

1⟶2 processes

Wigner-Eckart theorem for transition amplitude $R \longrightarrow 1+2$:

Wigner matrix:
angular distribution - from symmetry

$$\left\langle \mathbf{p} s_1 \lambda_1; -\mathbf{p} s_2 \lambda_2 \,\middle|\, T_{R\to 1+2} \,\middle|\, \mathbf{p}_R = 0,\, JM \right\rangle = \sqrt{\frac{2J+1}{4\pi}} F^J_{\lambda_1\lambda_2} D^J_{M,\lambda_1-\lambda_2}(\Omega)^*$$

Helicity amplitude:
from dynamics

Restrictions on helicity amplitudes:

from Wigner matrix $D^J_{M,\lambda_1-\lambda_2}(\Omega)$: $-J \le \lambda_1 - \lambda_2 \le J$

from parity conservation: $F^J_{\lambda_1\lambda_2} = \eta_R \eta_1 \eta_2 (-1)^{J-s_1-s_2} F^J_{-\lambda_1,-\lambda_2}$

$\gamma^* \to e^+ e^-$:

Two independent helicity amplitudes:

$$A_0 \equiv F^1_{\frac{1}{2},\frac{1}{2}} = F^1_{-\frac{1}{2},-\frac{1}{2}}$$

$$A_1 \equiv F^1_{\frac{1}{2},-\frac{1}{2}} = F^1_{-\frac{1}{2},\frac{1}{2}}$$

They satisfy the QED relation:

$$\frac{A_0}{A_1} = -\sqrt{2}\frac{m_e}{m_{\gamma^*}}$$

Helicity formalism

$\gamma^* \to e^+ e^-$ - decay density matrix:

$$\rho^{\text{dec}}_{\lambda',\lambda} \propto \begin{pmatrix} 1+\cos^2\theta_e+\alpha & -\sqrt{2}\cos\theta_e\sin\theta_e e^{-i\phi_e} & \sin^2\theta_e e^{-2i\phi_e} \\ -\sqrt{2}\cos\theta_e\sin\theta_e e^{i\phi_e} & 2(1-\cos^2\theta_e)+\alpha & \sqrt{2}\cos\theta_e\sin\theta_e e^{-i\phi_e} \\ \sin^2\theta_e e^{2i\phi_e} & \sqrt{2}\cos\theta_e\sin\theta_e e^{i\phi_e} & 1+\cos^2\theta_e+\alpha \end{pmatrix} \qquad \alpha = \frac{2m_e^2}{|\mathbf{k}_e|^2}$$

Angular distribution of the lepton pair:

$$\sum_{\lambda,\lambda'} \rho^{\text{cre}}_{\lambda\lambda'}\rho^{\text{dec}}_{\lambda'\lambda} \propto (1+\cos^2\theta_e)\left(\rho^{\text{cre}}_{1,1}+\rho^{\text{cre}}_{-1,-1}\right)+2\sin^2\theta_e\,\rho^{\text{cre}}_{0,0}$$
$$+\sqrt{2}\sin 2\theta_e\left[\cos\phi_e(\text{Re}\rho^{\text{cre}}_{1,0}-\text{Re}\rho^{\text{cre}}_{-1,0})\right.$$
$$\left.+\sin\phi_e(\text{Im}\rho^{\text{cre}}_{1,0}+\text{Im}\rho^{\text{cre}}_{-1,0})\right]$$
$$+2\sin^2\theta_e\left(\cos 2\phi_e\text{Re}\rho^{\text{cre}}_{1,-1}+\sin 2\phi_e\text{Im}\rho^{\text{cre}}_{1,-1}\right)$$

depends on γ^* polarization ($\rho^{\text{cre}}_{\lambda\lambda'}$)

↓

we can obtain $\rho^{\text{cre}}_{\lambda\lambda'}$ via fitting the experimental angular distribution

Helicity formalism

1⟶2 processes

Wigner-Eckart theorem for transition amplitude $R \longrightarrow 1+2$:

Wigner matrix:
angular distribution - from symmetry

$$\left\langle \mathbf{p} s_1 \lambda_1; -\mathbf{p} s_2 \lambda_2 \middle| T_{R\to 1+2} \middle| \mathbf{p}_R = 0,\ JM \right\rangle = \sqrt{\frac{2J+1}{4\pi}} F^J_{\lambda_1\lambda_2} D^J_{M,\lambda_1-\lambda_2}(\Omega)^*$$

Helicity amplitude:
from dynamics

Restrictions on helicity amplitudes:

from Wigner matrix $D^J_{M,\lambda_1-\lambda_2}(\Omega)$: $-J \le \lambda_1 - \lambda_2 \le J$

from parity conservation: $F^J_{\lambda_1\lambda_2} = \eta_R \eta_1 \eta_2 (-1)^{J-s_1-s_2} F^J_{-\lambda_1,-\lambda_2}$

$R \to N + \gamma^*$:

3 independent helicity amplitudes for $J \ge 3/2$ resonances:

$$A_{1/2} \equiv F^J_{1,\frac{1}{2}} = \pm F^J_{-1,-\frac{1}{2}}$$

$$S_{1/2} \equiv F^J_{0,\frac{1}{2}} = \pm F^J_{0,-\frac{1}{2}}$$

$$A_{3/2} \equiv F^J_{-1,\frac{1}{2}} = \pm F^J_{1,-\frac{1}{2}}$$

$S_{1/2} \to 0$ when $m_{\gamma^*} \to 0$

(real photons are transverse)

Helicity formalism

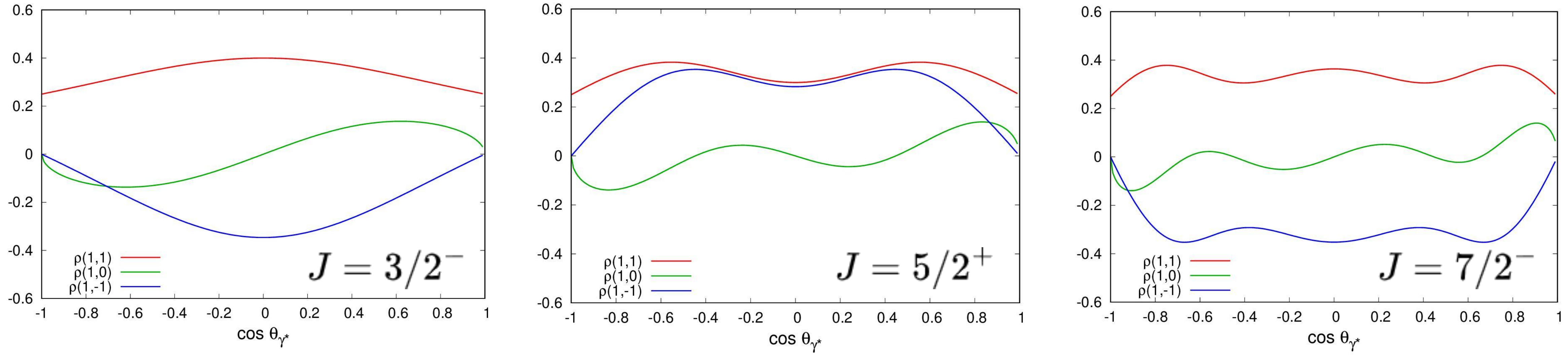
$R \to N + \gamma^*$ - creation density matrix:

$$\rho^{\text{cre}}_{\lambda\lambda'} \propto \sum_{\lambda_R,\lambda_N} F^J_{\lambda\lambda_N} F^{J*}_{\lambda'\lambda_N} P_{\lambda_R} D^J_{\lambda_R,\lambda-\lambda_N}(\Omega)^* D^J_{\lambda_R,\lambda'-\lambda_N}(\Omega)$$

Unpolarized $J = 3/2^-$ baryon resonance [e.g. *N*(1520)]:

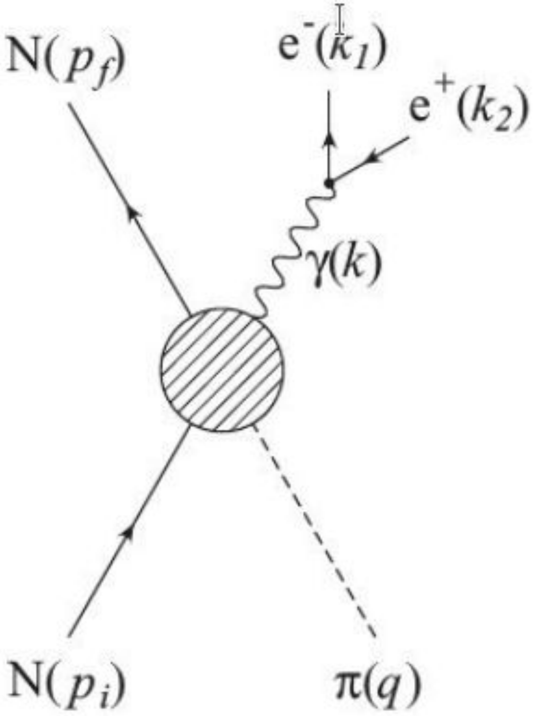
$$\rho^{\text{cre},3/2}_{\lambda,\lambda'} = \mathcal{N} \begin{pmatrix} A^2_{1/2} + 3\left(A^2_{1/2}\cos^2\theta + A^2_{3/2}\sin^2\theta\right) & -\sqrt{3}A_{3/2}S_{1/2}\sin 2\theta & 2\sqrt{3}A_{1/2}A_{3/2}\sin^2\theta \\ -\sqrt{3}A_{3/2}S_{1/2}\sin 2\theta & S^2_{1/2}(5+3\cos 2\theta) & \sqrt{3}A_{3/2}S_{1/2}\sin 2\theta \\ 2\sqrt{3}A_{1/2}A_{3/2}\sin^2\theta & \sqrt{3}A_{3/2}S_{1/2}\sin 2\theta & A^2_{1/2} + 3\left(A^2_{1/2}\cos^2\theta + A^2_{3/2}\sin^2\theta\right) \end{pmatrix}$$

E.g. if $A_{3/2} = A_{1/2} = S_{1/2}$:



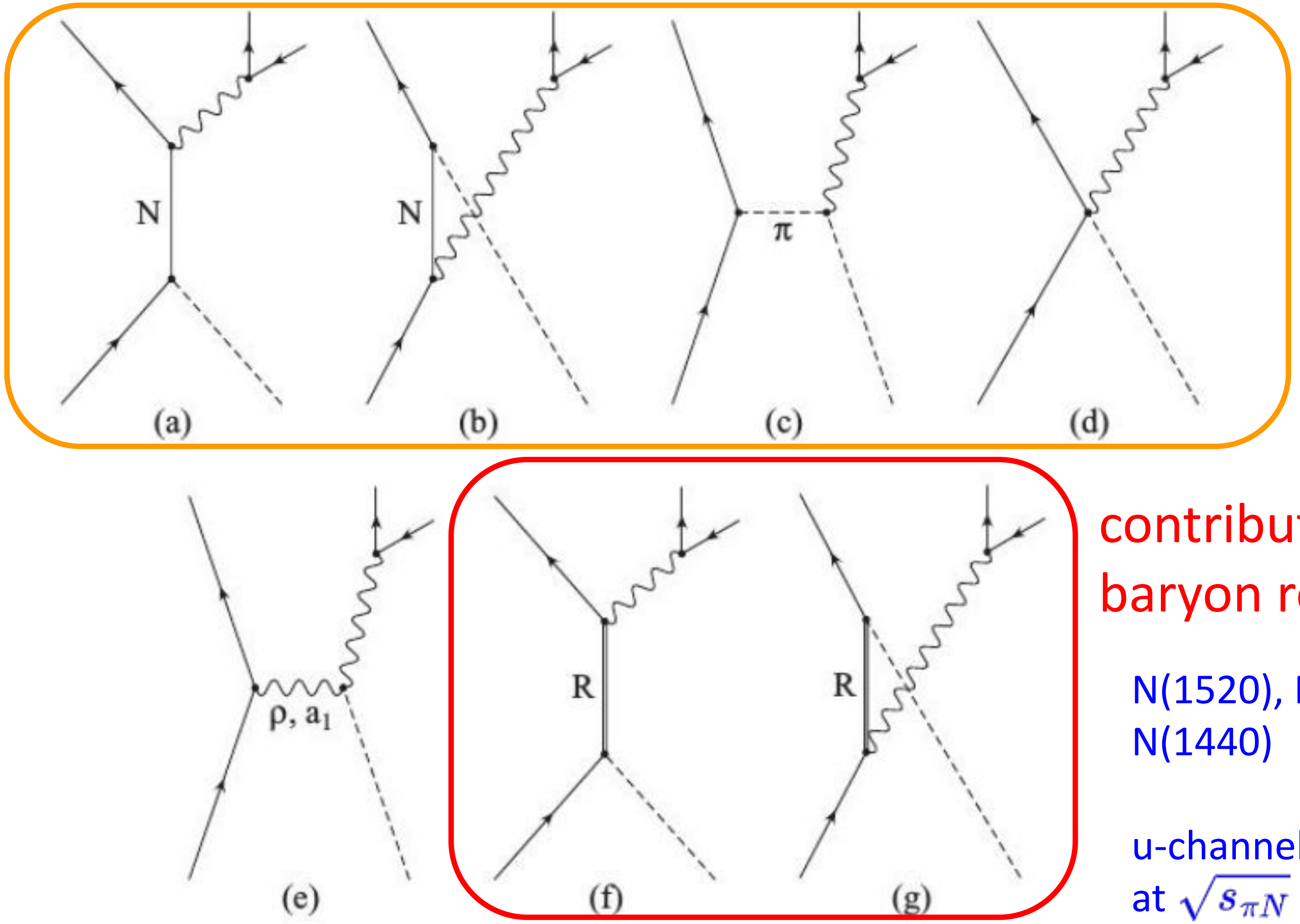
Effective Lagrangian model

All contributions to the process $\pi N \rightarrow N e^+ e^-$:



=

Born terms



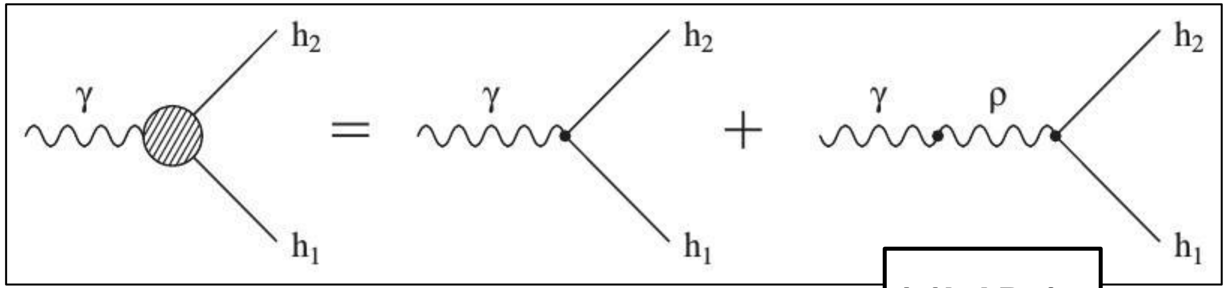
contributions of baryon resonances

N(1520), N(1535), N(1440)

u-channel unimportant at $\sqrt{s_{\pi N}}$ = 1.5 GeV

Effective Lagrangian model

Vector Meson Dominance



VMD1

Electromagnetic interaction of hadrons - two versions:

direct photon-hadron coupling

$-\frac{ieq^2}{g_\rho}$

γ and ρ couplings can be determined independently

$$\mathcal{L}_{\mathrm{VMD1}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - g_{\rho\pi\pi}\rho_\mu J^\mu - eA_\mu J^\mu - \frac{e}{2g_\rho}F_{\mu\nu}\rho^{\mu\nu}$$

hadron-rho interaction

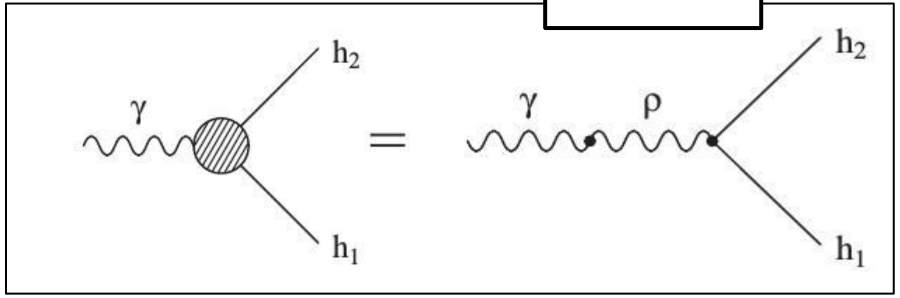
$$\mathcal{L}_{\mathrm{VMD2}} = -\frac{1}{4}(F'_{\mu\nu})^2 - \frac{1}{4}(\rho'_{\mu\nu})^2 + \frac{1}{2}m_\rho^2(\rho'_\mu)^2 - g_{\rho\pi\pi}\rho'_\mu J^\mu - \frac{e' m_\rho^2}{g_\rho}\rho'_\mu A'^\mu + \frac{1}{2}\left(\frac{e'}{g_\rho}\right)^2 m_\rho^2 (A'_\mu)^2$$

$-\frac{iem_\rho^2}{g_\rho}$

hadron current: J_μ

VMD2

rho field-strength tensor: $\vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g\vec{\rho}_\mu \times \vec{\rho}_\nu$



Effective Lagrangian model

Consistent interaction scheme for $J \geq 3/2$ resonances:

$$\mathcal{L}^{(1)}_{R_{3/2}N\rho} = \frac{ig_1}{4m_N^2}\bar{\Psi}^{\mu}_{R}\vec{\tau}\Gamma\gamma^{\nu}\psi_N \cdot \vec{\rho}_{\nu\mu} + \text{h.c.}$$

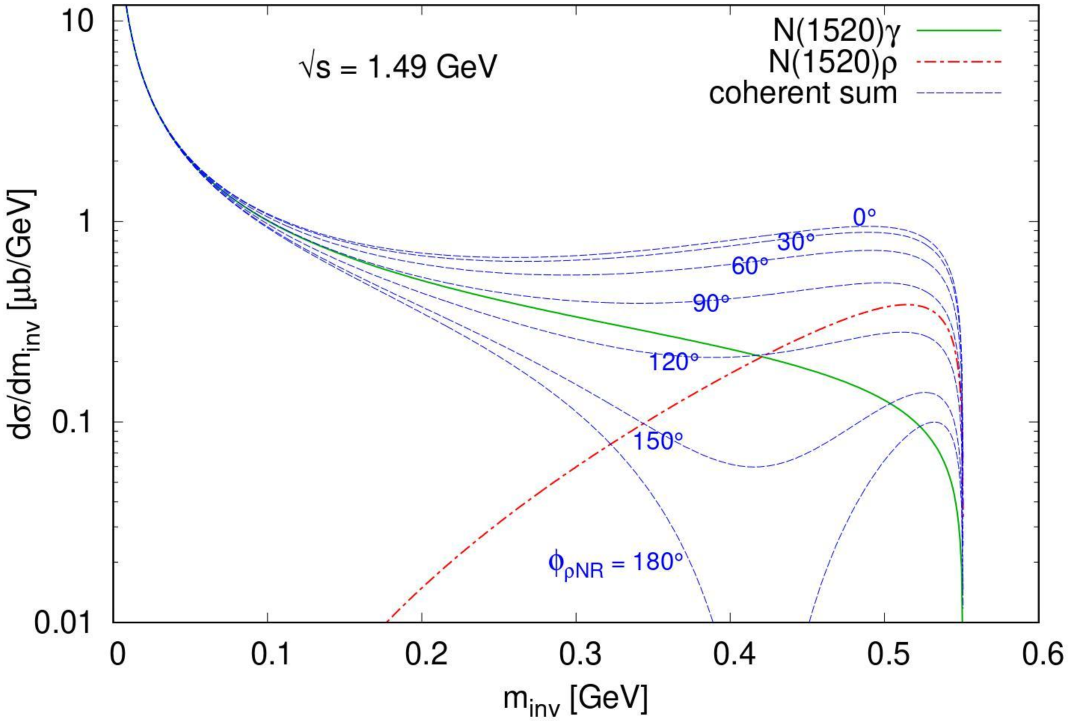
$$\mathcal{L}^{(2)}_{R_{3/2}N\rho} = -\frac{g_2}{8m_N^3}\bar{\Psi}^{\mu}_{R}\vec{\tau}\Gamma\partial^{\nu}\psi_N \cdot \vec{\rho}_{\nu\mu} + \text{h.c.}$$

$$\mathcal{L}^{(3)}_{R_{3/2}N\rho} = -\frac{g_3}{8m_N^3}\bar{\Psi}^{\mu}_{R}\vec{\tau}\Gamma\psi_N\partial^{\nu} \cdot \vec{\rho}_{\nu\mu} + \text{h.c.},$$

$$\Psi^{\mu}_{R} = i\gamma_{\nu}(\partial^{\mu}\psi^{\nu}_{R} - \partial^{\nu}\psi^{\mu}_{R})$$

[V. Pascalutsa, Phys. Rev. D 58 (1998) 096002;
T. Vrancx, et al. Phys. Rev. C 84 (2011) 045201]

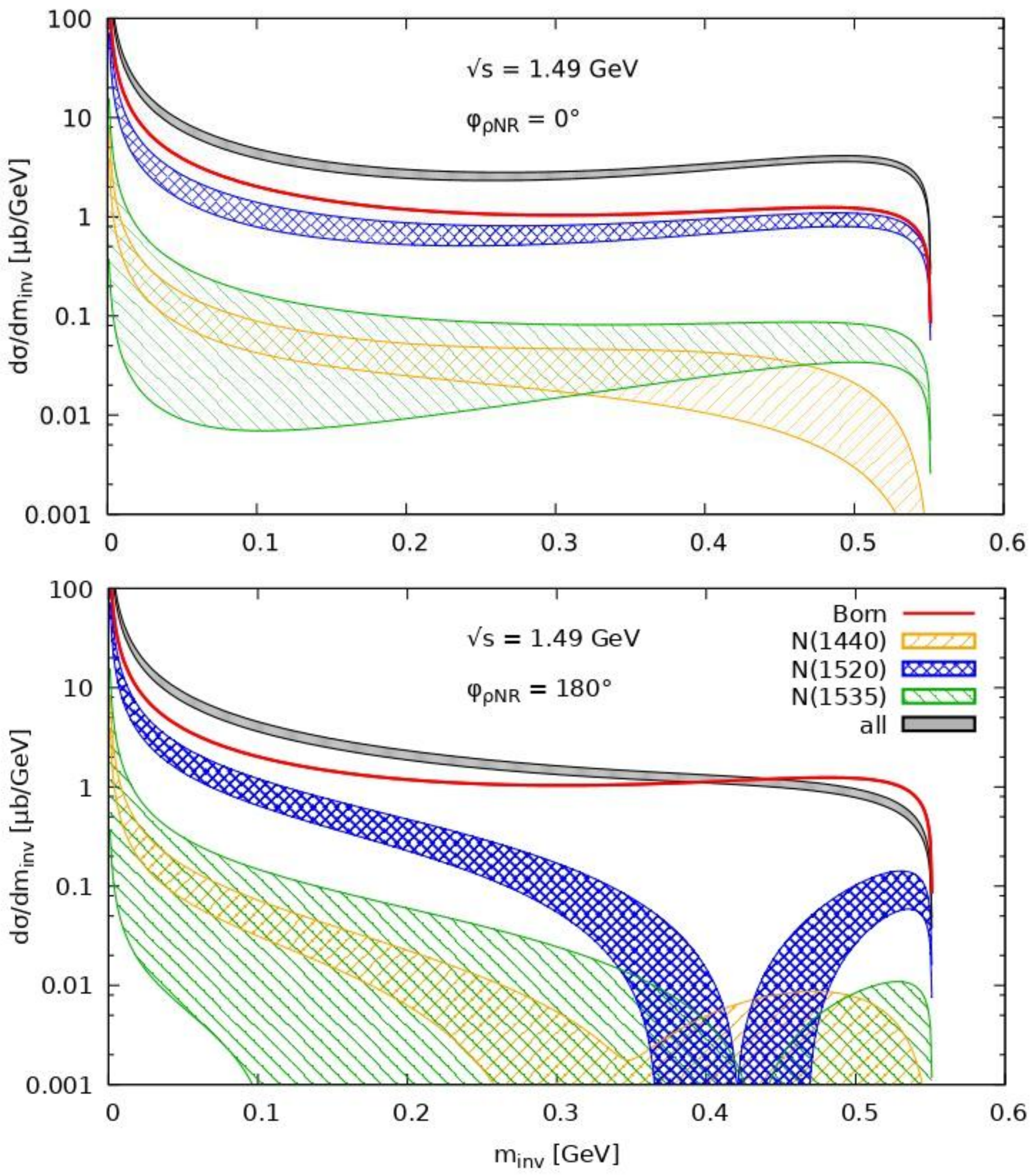
- coupling constants from $R \rightarrow N\rho$ branching ratios (HADES, $\pi N \rightarrow N\pi\pi$ [Phys. Rev. C 102 (2020) 024001])
- analogous Lagrangians for $RN\gamma$ coupling
- VMD1: relative phase of ρ and direct γ contribution is not fixed $\longrightarrow$ various interference patterns
- ρ contribution is important although we are below threshold



Effective Lagrangian model

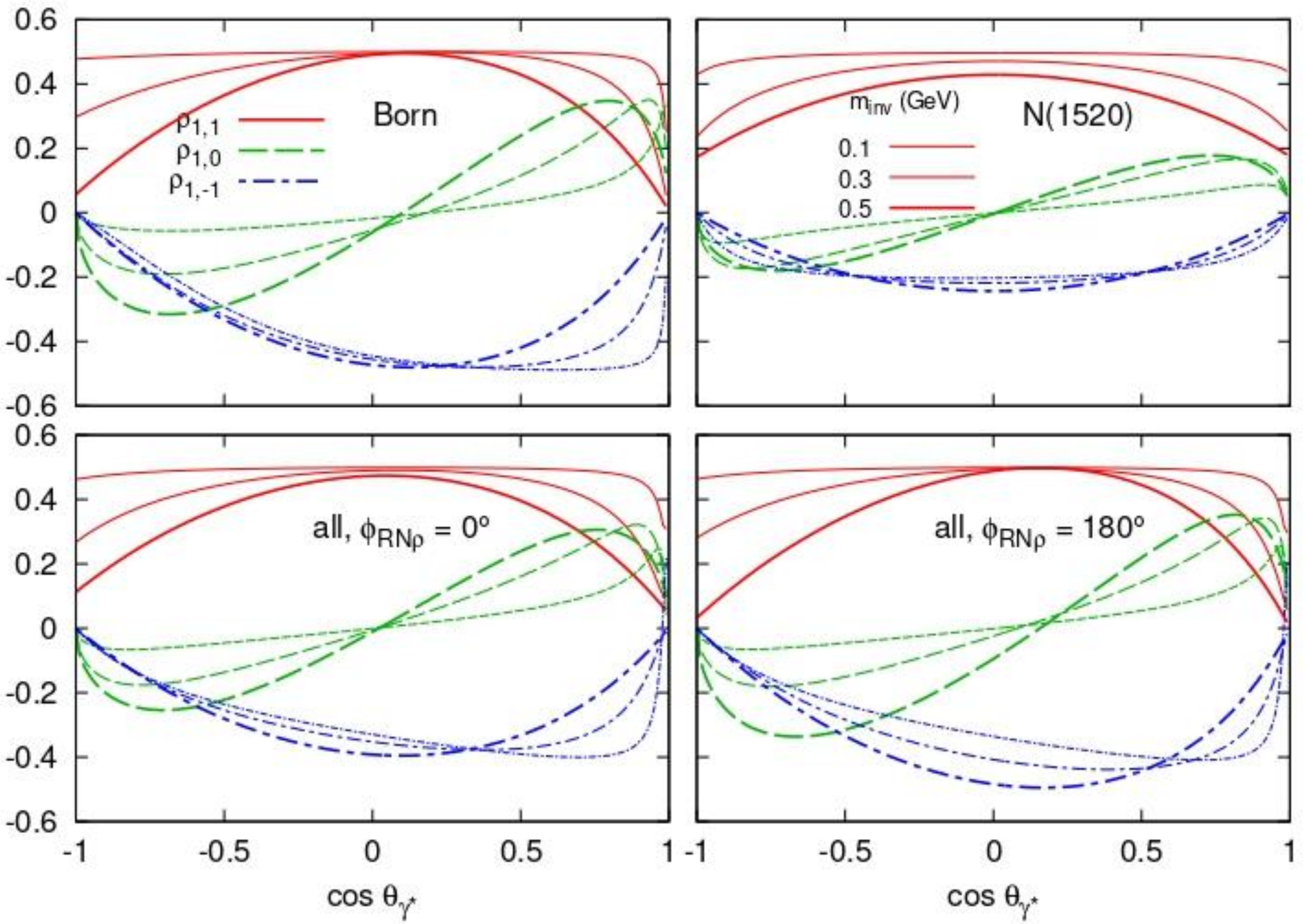
Differential cross section of e^+e^- production

- dominant sources are Born and N(1520)
- error bands: uncertainty of resonance widths and branching ratios
- relative phase of ρ and direct γ has a strong influence



Effective Lagrangian model

Density matrix elements



- Born and N(1520) contributions have similar shapes
- longitudinal polarization disappears as $m_{\rm inv} \to 0$

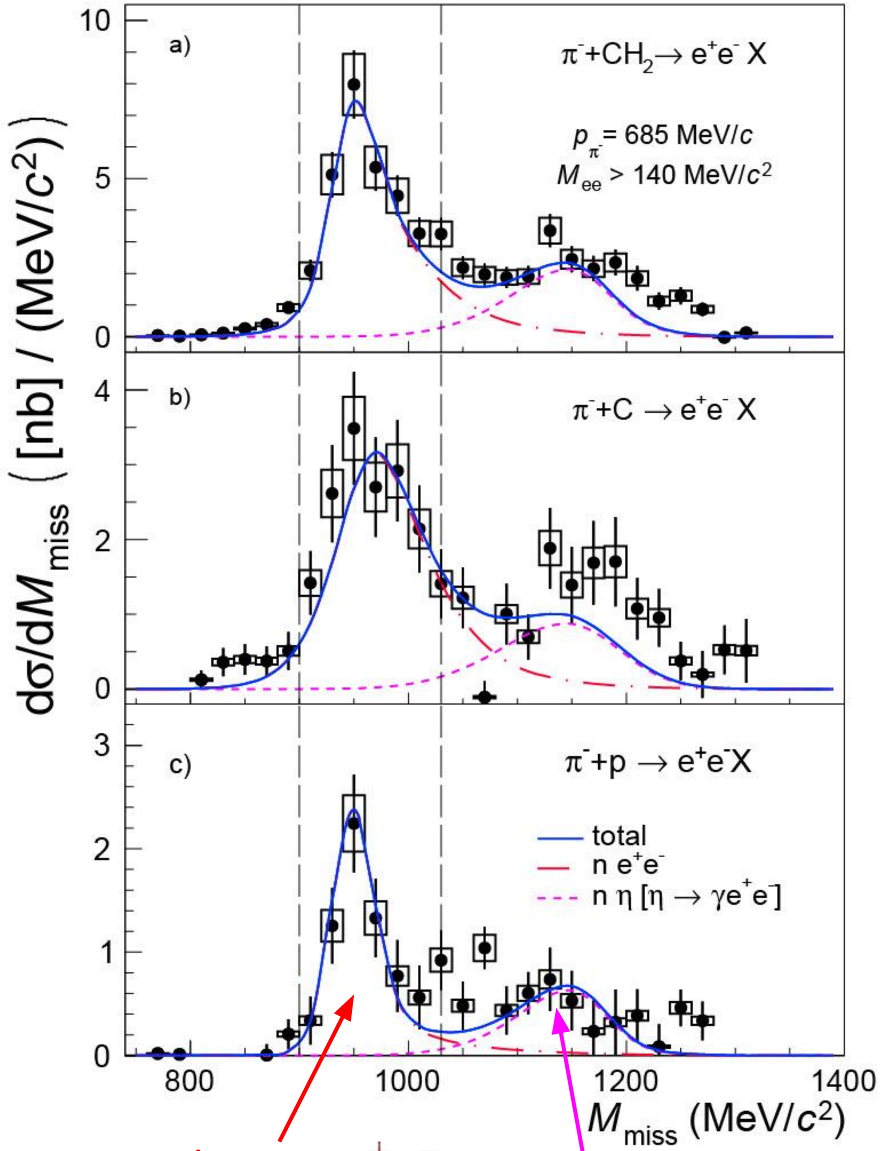
HADES results

Quasifree $\pi^- p \to n e^+ e^-$ by HADES

[R. Abou Yassine et al., e-Print: 2205.15914 [nucl-ex] (2022)]

- π^- beam: p_π = 0.658 GeV ($\sqrt{s_{\pi N}}$ = 1.49 GeV)
- targets: polyethylene (CH_2), carbon (C)
- missing mass cut: select exclusive $n e^+ e^-$
- difference of CH_2 and C $\longrightarrow$ free $\pi^- p \to n e^+ e^-$
- low statistics for carbon target
  $\longrightarrow$ quasifree $\pi^- p \to n e^+ e^-$:
  π^- + CH_2 & effective proton number

Missing mass of $e^+ e^- X$



exclusive $n e^+ e^-$

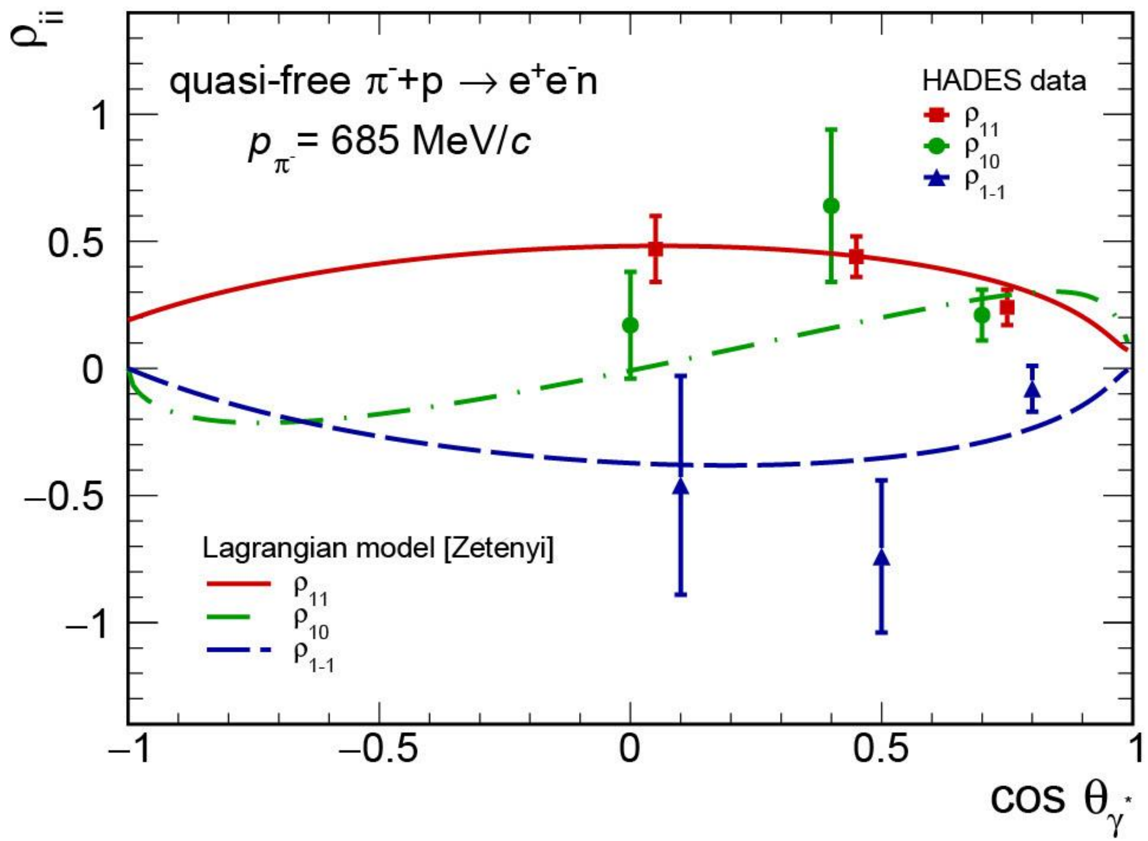
η Dalitz

Quasifree $\pi^- p \to n e^+ e^-$ by HADES

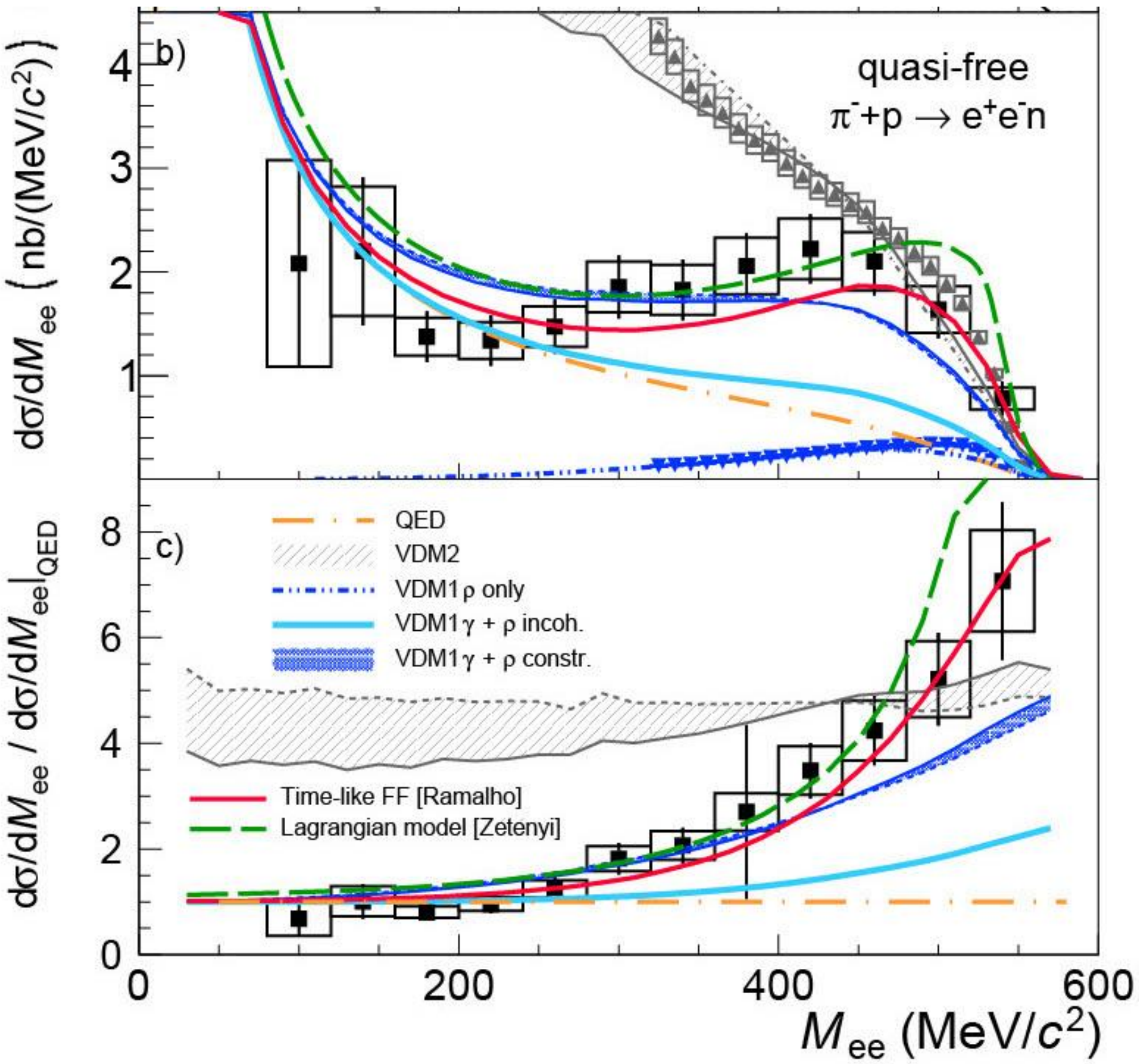
Polarization density matrix elements:

- extracted from experiment via fitting
  $d^4\sigma/d\cos\theta_{\gamma^*}\, dM_{e^+e^-}\, d\cos\theta_e\, d\phi_e$
  for $M_{e^+e^-} > 300\,\mathrm{MeV}/c^2$ and 3 bins in $\cos\theta_{\gamma^*}$

- ~ consistent with effective Lagr. model (dominance of N(1520) and Born terms)



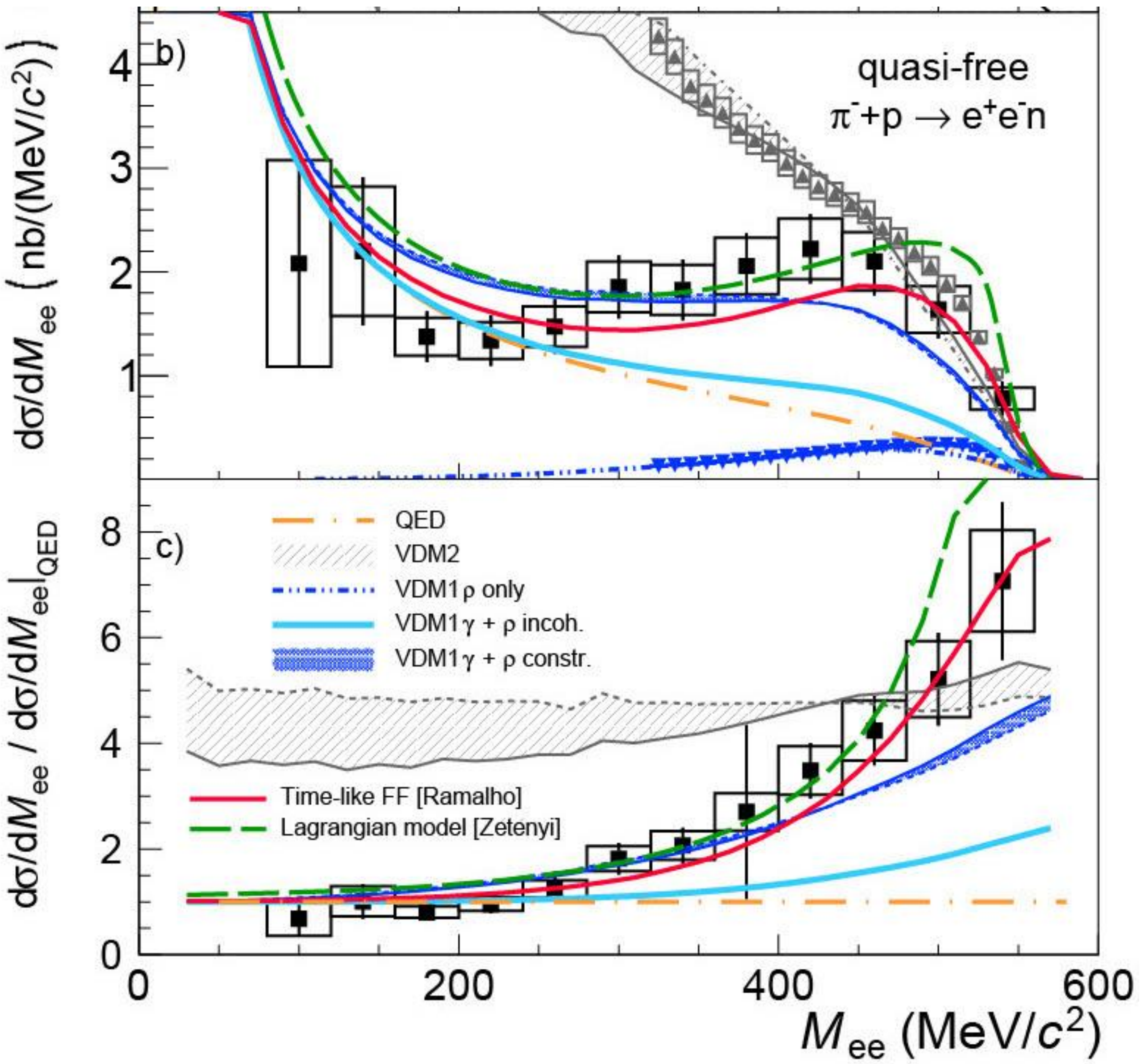
Quasifree $\pi^- p \to n e^+ e^-$ by HADES



Differential cross section vs. models

- large excess compared to QED [pointlike N(1520) and N(1535)] for M_{e+e-} > 200 MeV
- ▲ and ▼: reconstructed from $\pi^- p \to n\rho^0$ [known from PWA of $\pi^- p \to n\pi^+\pi^-$ (Bonn-Gatchina, HADES data)], via
  - VMD1: $\Gamma(M_{e^+e^-}) = \Gamma_0 \frac{M_{e^+e^-}}{M_0}$
  - VMD2: $\Gamma(M_{e^+e^-}) = \Gamma_0 \left(\frac{M_0}{M_{e^+e^-}}\right)^3$

Quasifree $\pi^- p \to n e^+ e^-$ by HADES



Differential cross section vs. models

- VMD2: overestimates for low inv. mass
- VMD1: constructive $\gamma - \rho$ interference gives the best description of the data
- Covariant Spectator Quark model
  [G. Ramalho, T. Pena, Phys. Rev. D **95**, 014003 (2017)]
- our effective Lagrangian model with $\phi_{\rho NR} = 90^\circ$
  ($\sim$ incoherent sum)

[M. Z., D. Nitt, M. Buballa, T. Galatyuk, Phys. Rev. C **104**, 015201 (2021)]

Summary

- Quasifree $\pi^- p \to n e^+ e^-$ measured at $\sqrt{s} =$ 1.49 GeV by HADES
- Related to $\pi^- p \to n \pi^+ \pi^-$ by vector meson dominance (VMD)
- Spin density matrix elements (= polarization state) of γ^* accessible in the experiment
- Differential cross section confronted with various models
    - timelike e.m. transition of the relevant baryon resonances
    - various versions of VMD
- Plans for higher energies

Thank You!

R. Abou Yassine et al. (HADES Collaboration & M.Z.), e-Print: 2205.15914 [nucl-ex] (2022)
M. Z., D. Nitt, M. Buballa, T. Galatyuk, Phys. Rev. C **104**, 015201 (2021)
E. Speranza, M. Z., B. Friman, Phys. Lett. B **764**, 282 (2017)
M. Z., Gy. Wolf, Phys. Rev. C **86**, 065209 (2012)
B. Zhang, M. Z., in preparation

Collaborators, Acknowledgements

Deniz Nitt, Enrico Speranza, Baiyang Zhang
Michael Buballa, Bengt Friman

Members of HADES:
Piotr Salabura, Beatrice Ramstein, Tetyana Galatyuk