Tetraquark bound and resonant states

Atsushi Hosaka
Research **C**enter for **N**uclear **P**hysics, Osaka University
Advanced **S**cience **R**esearch **C**enter, **J**apan **A**tomic **E**nergy **A**gency

1. Exotics — previous century

Beyond the standard: $qq\bar{q}\bar{q}$ mesons, $qqqq\bar{q}$ baryons and **more**

Some histories
20th and this centuries

20th centuries

A SCHEMATIC MODEL OF BARYONS AND MESONS

Phys. Lett. 8, 214 (1964)

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the represen-

Molecular Charmonium: A New Spectroscopy?*

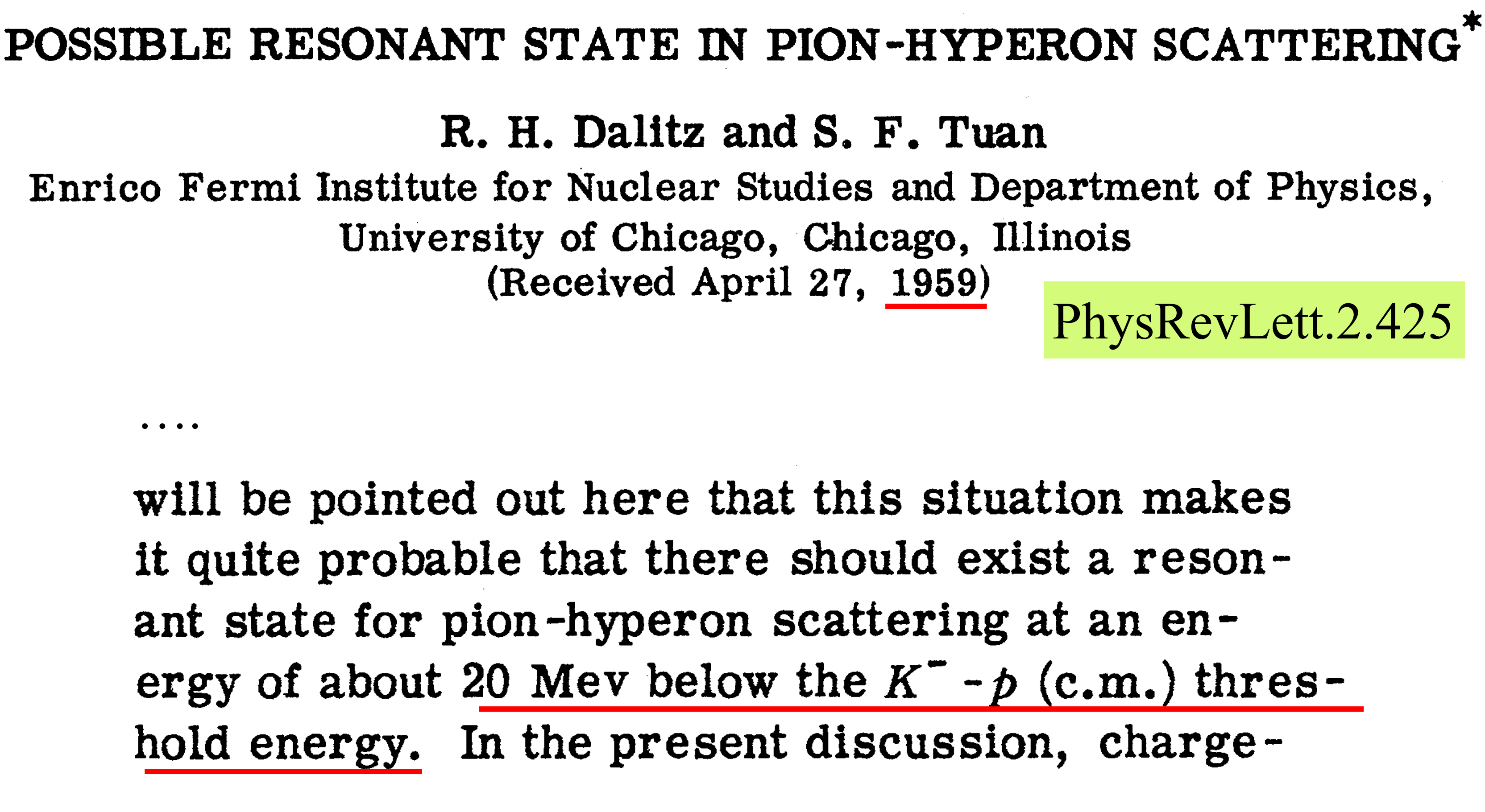
Phys. Lett. 38, 317 (1977)

A. De Rújula, Howard Georgi,† and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 23 November 1976)

Recent data compel us to interpret several peaks in the cross section of e^-e^+ annihilation into hadrons as being due to the production of four-quark molecules, i.e., resonances between two charmed mesons. A rich spectroscopy of such states is predicted and may be studied in e^-e^+ annihilation.

$\bar{K}N$ molecule — $\Lambda(1405)$


POSSIBLE RESONANT STATE IN PION-HYPERON SCATTERING*

R. H. Dalitz and S. F. Tuan

Enrico Fermi Institute for Nuclear Studies and Department of Physics,
University of Chicago, Chicago, Illinois

(Received April 27, 1959)

....

will be pointed out here that this situation makes it quite probable that there should exist a resonant state for pion-hyperon scattering at an energy of about 20 Mev below the $K^- - p$ (c.m.) threshold energy. In the present discussion, charge-

....

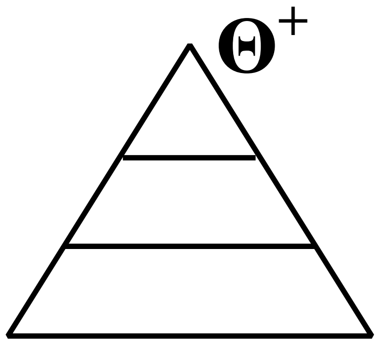
PhysRevLett.2.425

This is being confirmed….

This century Θ^+

Theiry came first….

D. Diakonov in Osaka 2012



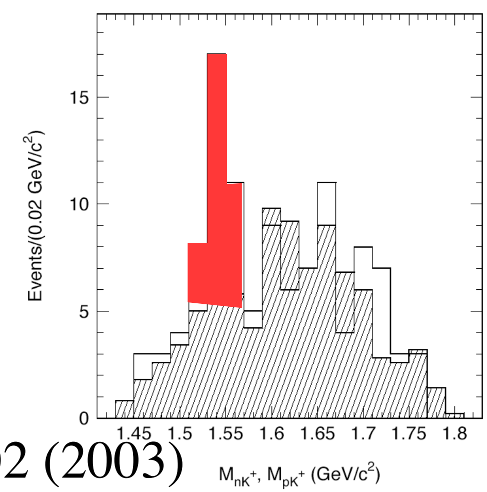
$uudd\bar{s}$

Prediction
by the chiral Solitons

Z.Phys. A359 (1997) 305-314

T. Nakano

PRL91, 012002 (2003)



LEPS@SPring-8

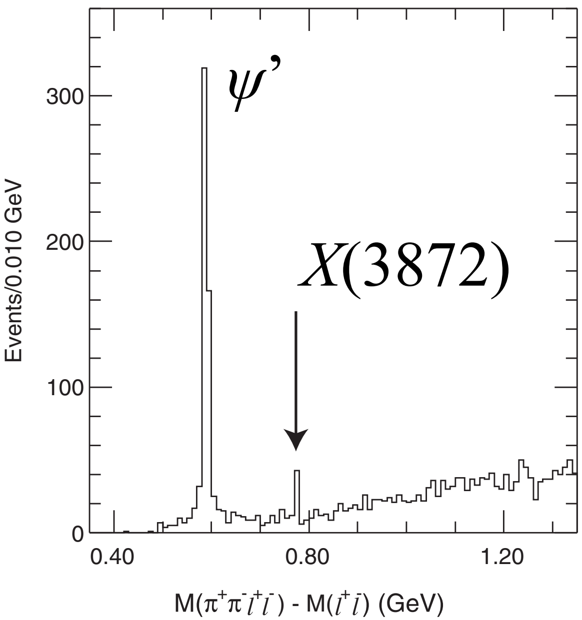
Further study is on going at LEPS

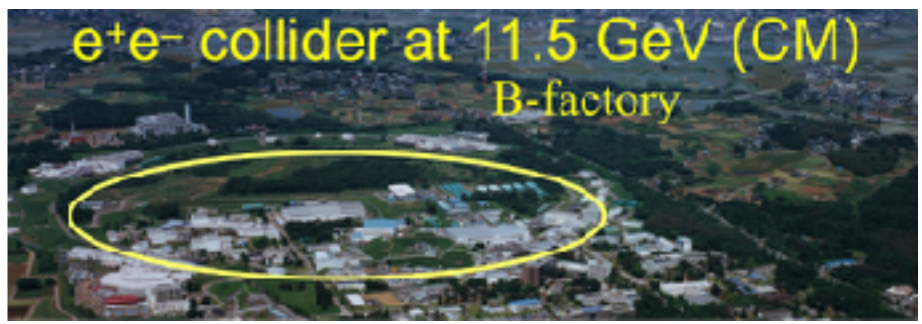
$X(3872)$

Pentaquark ***Tetraquark***

Experiment came first….

Belle@KEK, PRL91, 262001 (2003)
and further confirmed at Fermi Lab, SLAC, LHC, BEP, …





$\boxed{u\bar{u}c\bar{c},\ d\bar{d}c\bar{c}}$

Heavy and light quarks

Many other findings have are following

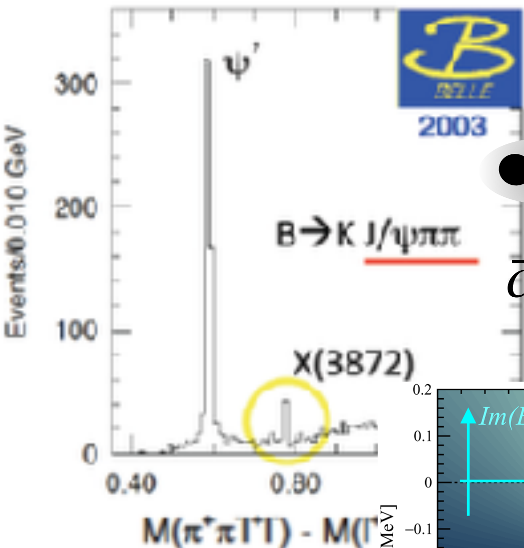
The existence confirmed
but its nature note yet settled

LHC’s continuous reports

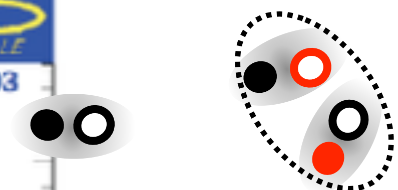
LHCb, PRL122 (2019) 222001

X(3872)

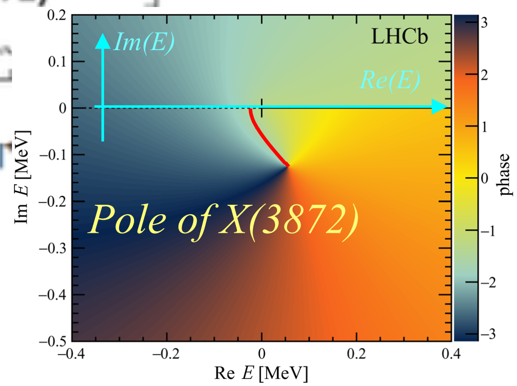
Belle, PRL91 (2003) 262001



Hadronic molecule?



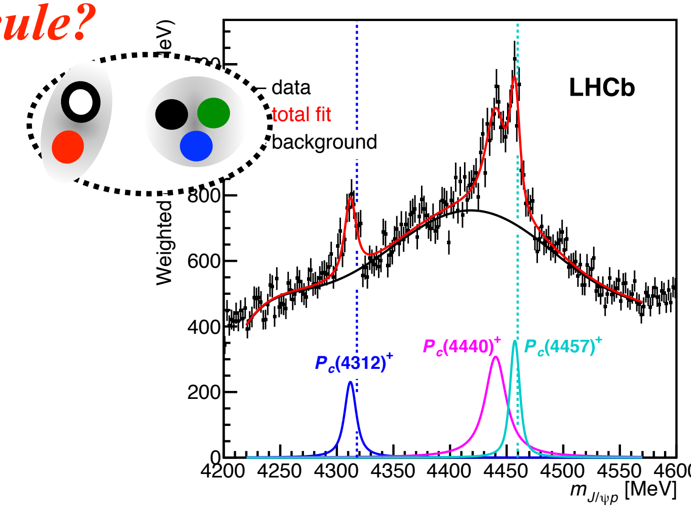
$\bar{c}c$ or D^{*}–$\bar{D}$



LHCb, PRD102 092005 (2020)

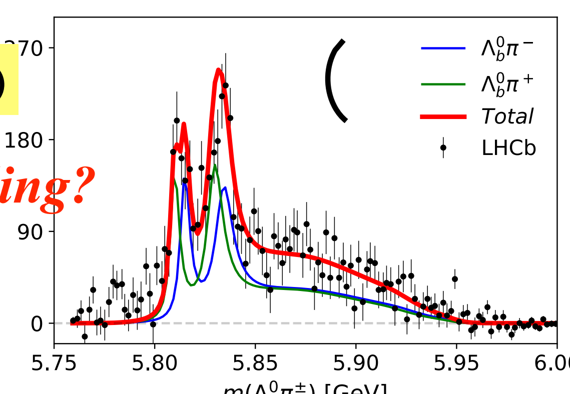
Pc(4310, 4460, 4520)

$\bar{c}cuud \to (\bar{c}u)(cdd)$



$\Lambda_b(6072)$

Roper’s sibling?



And more . . .

LHCb, JHEP 06 (2020) 136

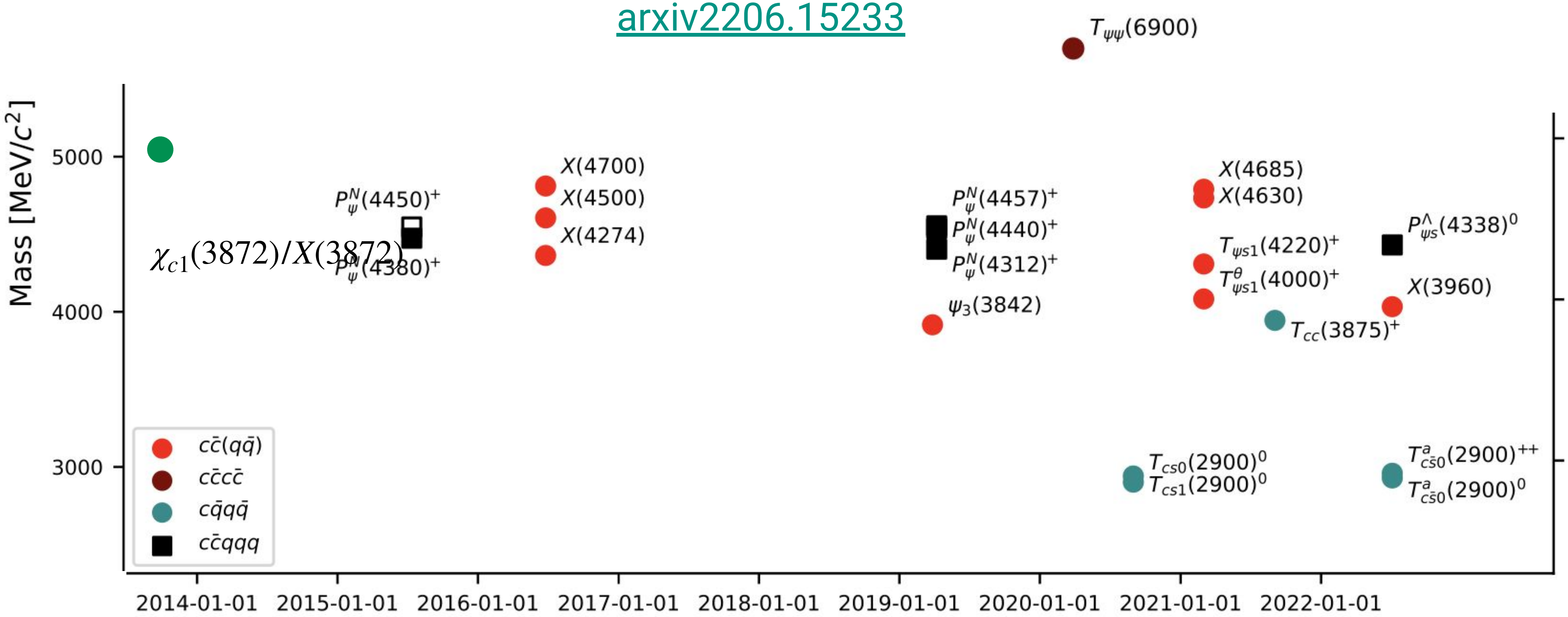
Arifi et al., PRD 101

Latest status

CERN-LHC Seminar on Tuesday 5 July, https://indico.cern.ch/event/1176505/

59 hadrons have been observed by LHCb

More than 15 states are exotics:
⇒ New naming scheme
arxiv2206.15233



What’s the next?

Marek Karliner: Questions to be answered

1. Do they exist?
2. If they do, which ones?
3. What is their internal structure?
4. How best to look for them?

Marek Karliner, QNP proceedings, 2018@Tsukuba
https://journals.jps.jp/doi/book/10.7566/QNP2018

Marek Karliner

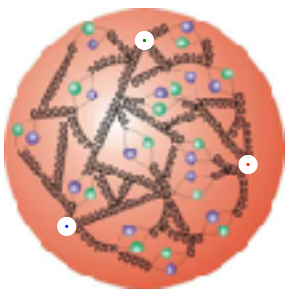
Studying heavy (exotic) hadrons is somewhat similar to investigating the social life of various quarks:

(a) Who with whom?
(b) For how long?
(c) A short episode? or
(d) “Till Death Us Do Part”?

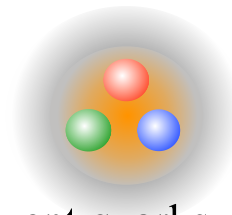
These are for exotics, but then …

Furthermore?

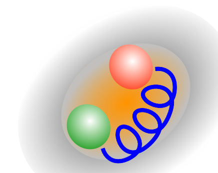
- Bare quarks and gluons ==> **Effective degrees of freedom** for hadrons



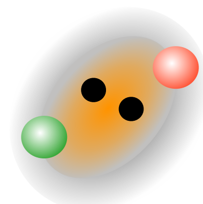
http://ppssh.phys.sci.kobeu.
ac.jp/~yamazaki/lectures/07/modernphys-yamazaki07.pdf



Constituent quarks



Constituent gluons



Quark model

Question of
Effective degrees of freedom ⇄ the non-trivial QCD vacuum

Nontrivial QCD vacuum

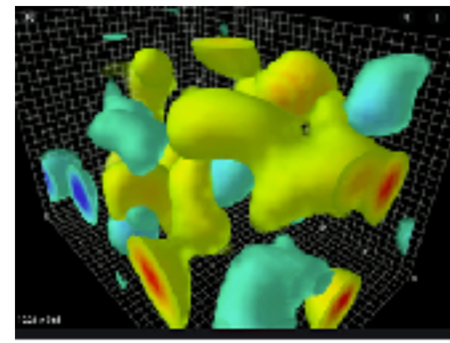
Uniqueness of QCD as a many-body problem → Non-trivial dynamics

QCD vacuum is not empty ~ Instantons are created and annihilated

- Extended (topological) object of gluons, of size ~ 0.2 fm
- QCD vacuum is topologically nontrivial
- Chiral symmetry is broken spontaneously $m \neq 0$

$$\langle \bar{q}q \rangle \sim \int \frac{d^4k}{i(2\pi)^4} \mathrm{tr} \frac{1}{m - \not{k}} \sim \int_{\infty}^{\infty} d\lambda \nu(\lambda) \frac{\mu}{\lambda^2 + \mu^2} |_{\mu \to 0}$$

Banks-Casher, NPB169(1989)193
D. Diakonov, PPNP51(2003)173
Fukaya et al, PRL104.122002 (2010), PRD.83.074501 (2011)

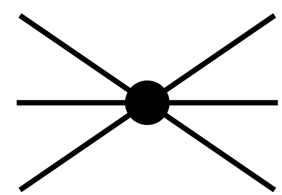


Snapshot of topological densities fluctuating in the vacuum
Derek Leinweber, 2003, 2004
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/index.html

- Instanton Induced Interaction (III) with $U_A(1)$ breaking

Kobayashi-Maskawa_PTP44(1970)1422
G. 't Hooft, PRL37.8 (1976), PRD14, 3432 (1976)

$$\mathscr{L}_{III} = g_D \left(\det[\bar{q}_i (1 - \gamma_5) q_j] + h.c. \right)$$

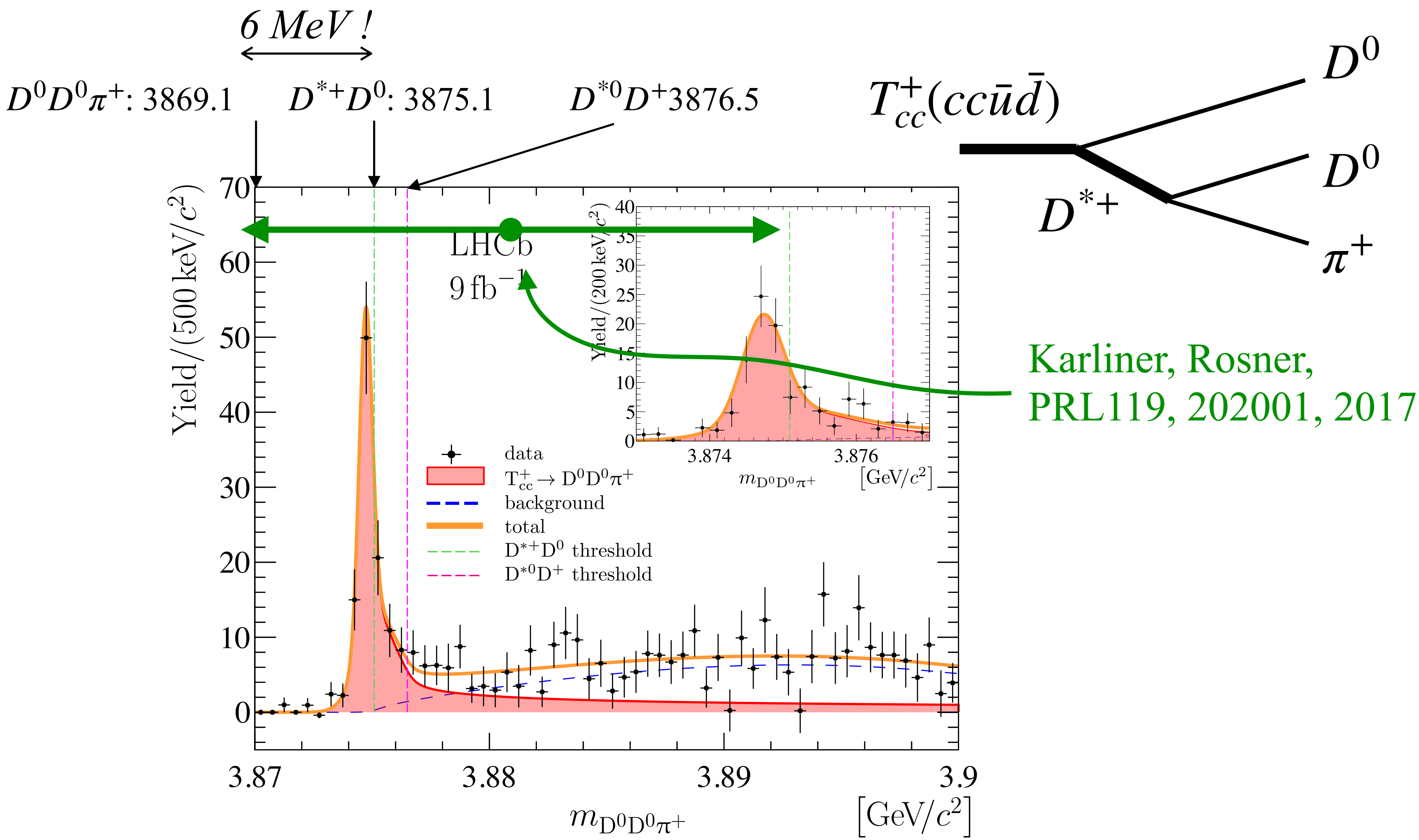


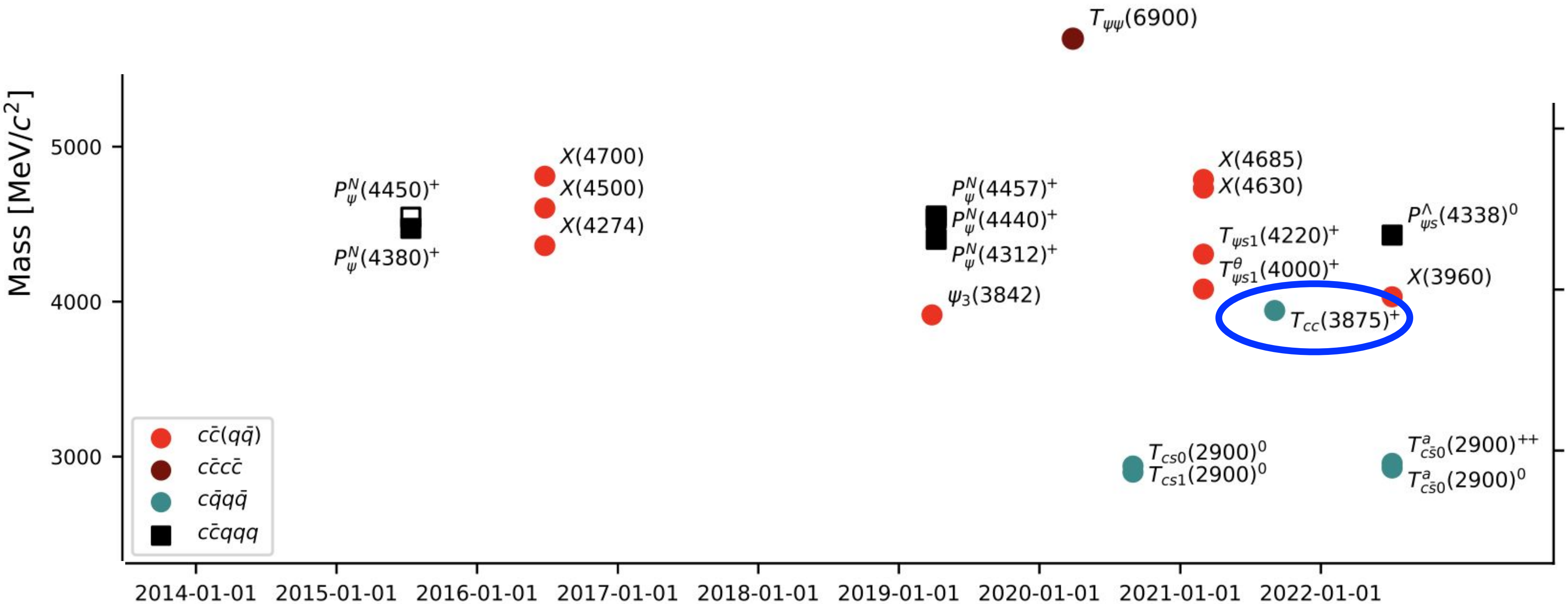
Systematic study: Hatsuda-Kunihiro: Phys. Repts. 247 (1994) 221-367

2. Tetraquarks $QQ'\bar{q}\bar{q}'$

LHCb

Nature Commun. 13 (2022) 1, 3351, arXiv: 2109.01056




$T_{\psi\psi}(6900)$
Mass [MeV/c^2]
5000
4000
3000
$P_\psi^N(4450)^+$
$P_\psi^N(4380)^+$
$X(4700)$
$X(4500)$
$X(4274)$
$P_\psi^N(4457)^+$
$P_\psi^N(4440)^+$
$P_\psi^N(4312)^+$
$\psi_3(3842)$
$X(4685)$
$X(4630)$
$T_{\psi s1}(4220)^+$
$T_{\psi s1}^\theta(4000)^+$
$P_{\psi s}^\Lambda(4338)^0$
$X(3960)$
$T_{cc}(3875)^+$
$T_{cs0}(2900)^0$
$T_{cs1}(2900)^0$
$T_{c\bar{s}0}^a(2900)^{++}$
$T_{c\bar{s}0}^a(2900)^0$
$c\bar{c}(q\bar{q})$
$c\bar{c}c\bar{c}$
$c\bar{q}q\bar{q}$
$c\bar{c}qqq$
2014-01-01
2015-01-01
2016-01-01
2017-01-01
2018-01-01
2019-01-01
2020-01-01
2021-01-01
2022-01-01

Why T_{cc} is interesting

- Toward answering “Who with whom?
- Check many theoretical models
  Quarks, Diquarks, triquarks, molecules, hybrid, …
- Are they bound or resonant states?
- The role of heavy vs light quarks

— why not clear evidence to find exotics only with light q’s

$$\text{Light} << \Lambda_{QCD} << m_Q$$

Interplay of light and heavy scales of QCD

Discuss in terms of the standard quark model by precisely solving four-body system

Q, $\bar{Q}$, and q, $\bar{q}$: two distinct scales

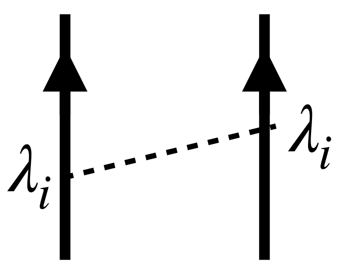
$q, \bar{q}$ $\qquad$ Λ_{QCD} $\qquad$ $Q, \bar{Q}$

Non-perturbative $\qquad$ Perturbative ~ Color-Coulomb

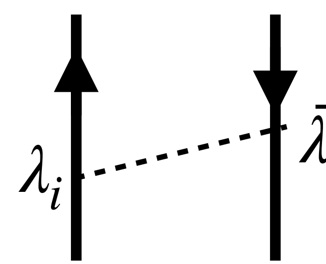
Color Coulomb
for $Q, \bar{Q}$ sector

$$\boxed{H = \frac{p^2}{2M_Q} - \frac{\alpha}{r}}$$

$QQ: \; 3 \times 3 = \bar{3} + 6$ $\qquad$ $Q\bar{Q}: 3 \times \bar{3} = 1 + 8$



$$-\frac{1}{2}\frac{\alpha}{r}$$

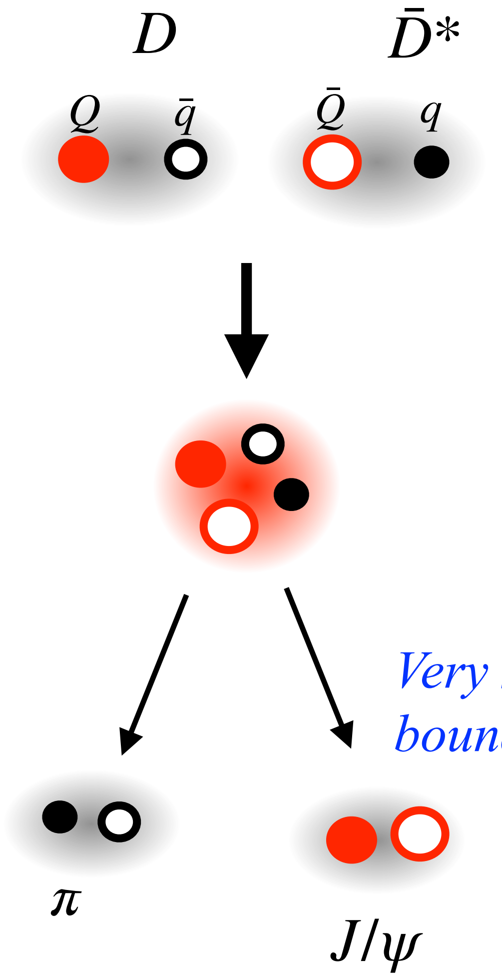


$$-1\frac{\alpha}{r}$$

$$\boxed{E_B = \frac{1}{2}\alpha^2 M_Q \gg \Lambda_{QCD}}$$

Stability

$\boxed{Q\bar{Q}q\bar{q}}$



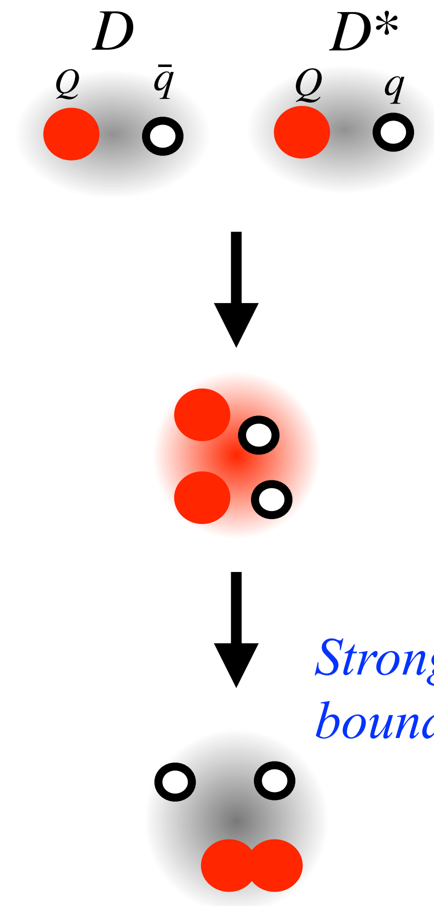
$E_B \sim \Lambda_{QCD}$

Very strongly bound $Q\bar{Q}$ $-1\dfrac{\alpha}{r}$

$E_B \sim \alpha M_Q$

Decay into ordinary mesons

$\boxed{QQ\bar{q}\bar{q}}$



Strongly bound QQ $-\dfrac{1}{2}\dfrac{\alpha}{r}$

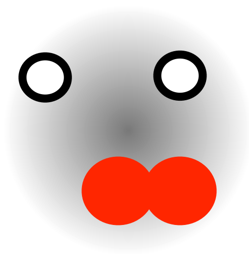
Stays as stable T_{QQ}

We expect:

- $Q\bar{Q}q\bar{q} \to [Q\bar{Q}] + [q\bar{q}], \quad Q\bar{Q}Q\bar{Q} \to [Q\bar{Q}] + [Q\bar{Q}]$
  => form moleculars near thresholds (with suitable force)
  => decay into mesons

- $QQ\bar{q}\bar{q} \to [QQ][\bar{q}\bar{q}]$
  => stay as stable tetraquark

Expected J^P



- Orbitally in S-state
- QQ must has $j^P = 1^+$ due to Pauli principle
- $\bar{q}\bar{q}$ is a good diquark $S = I = 0$

$$\boxed{\text{The lowest } T_{QQ} \text{ has } j^P = 1^+, I = 0}$$

Quark model — 4-body calculation

Meng et al, PLB814 (2021) 136095

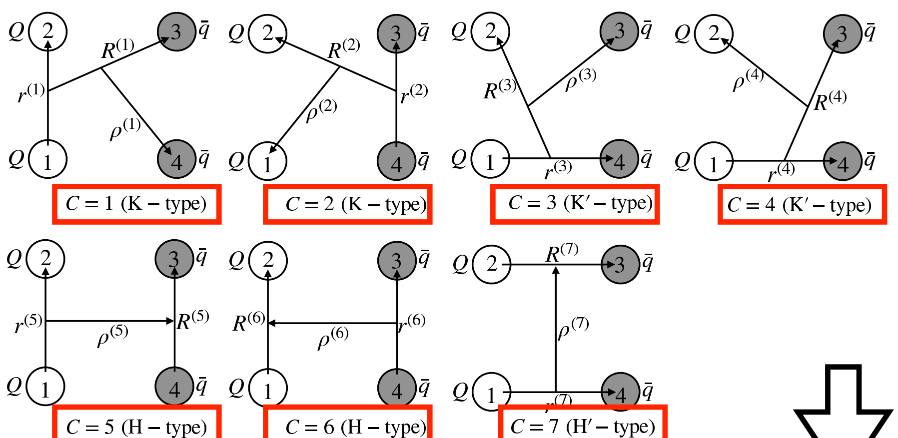
Gauss expansion method ~Hiyama et al, Prog. Part. Nucl. Phys. 51 (2003) 223

Hamiltonian

$$H = \sum_{i}^{4} \left(m_i + \frac{\boldsymbol{p}_i^{\,2}}{2m_i} \right) - T_G \qquad V_{ij}(\boldsymbol{r}) = -\frac{\kappa}{r} + \lambda r^p - \Lambda$$

$$-\frac{3}{16} \sum_{i<j=1}^{4} \sum_{a}^{8} \left((\lambda_i^a \cdot \lambda_j^a) V_{ij}(\boldsymbol{r}_{ij}) \right) \qquad + \frac{2\pi\kappa'}{3m_i m_j} \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

Ansatz Expand WF by different combinations of coordinates



$$\Psi_{I,JM} = \sum_{C} \xi_1^{(C)} \sum_{\gamma} B_\gamma^{(C)} \eta_I^{(C)} \Big[\Big[\big[[\chi_{\frac{1}{2}} \chi_{\frac{1}{2}}]_s \chi_{\frac{1}{2}} \big]_\Sigma \chi_{\frac{1}{2}} \Big]_K \times \big[[\phi_{n\ell}^{(C)}(\mathbf{r}_C) \psi_{NL}^{(C)}(\mathbf{R}_C)]_\Lambda \phi_{\nu\lambda}^{\prime(C)}(\rho_C) \big]_G \Big]_{JM}, \quad (3)$$

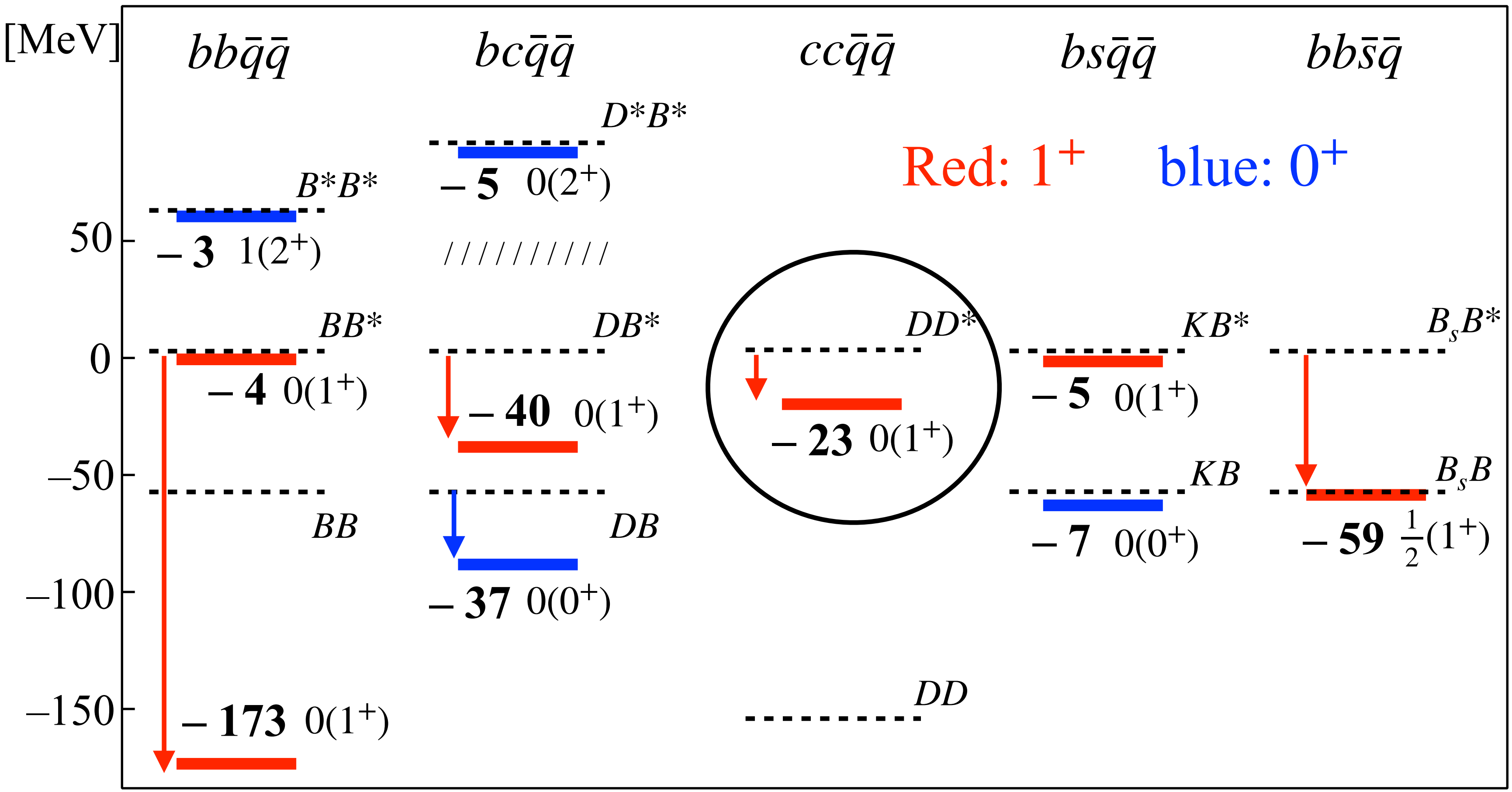
Diagonalize

Comparison with threshold energies important

=> Consistency check with meson masses ~ accuracy of the model/method

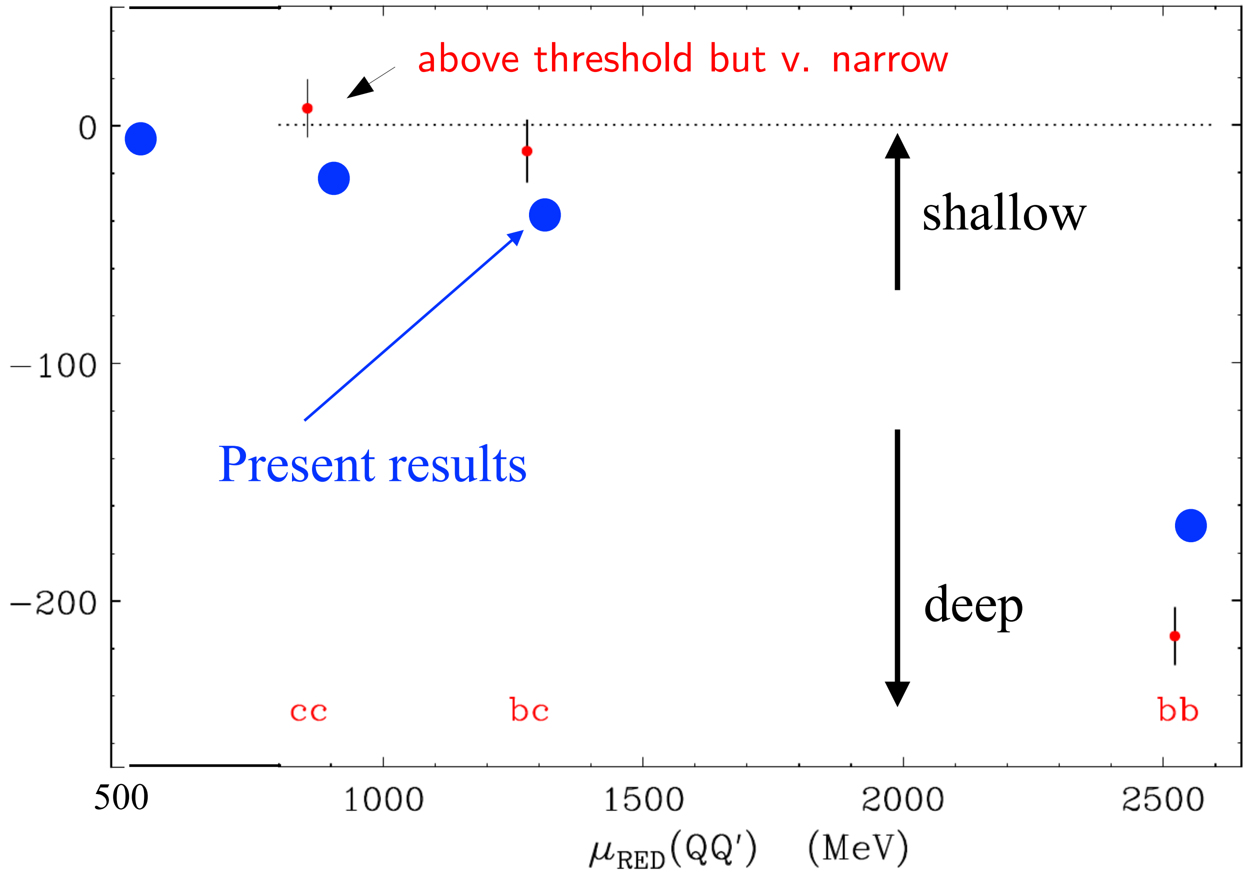
Parameters		Masses (MeV)		
			Cal	Exp
$m_{u,d}$ (GeV)	0.277	$\eta_b(0^-)$	9375	9399
m_s (GeV)	0.593	$\Upsilon(1^-)$	9433	9460
m_c (GeV)	1.826	$\eta_c(0^-)$	2984	2984
m_b (GeV)	5.195	$J/\psi(1^-)$	3102	3097
p	2/3	$B^-(0^-)$	5281	5279
κ	0.4222	$B^{*-}(1^-)$	5336	5325
κ'	1.7925	$B_s(0^-)$	5348	5367
λ (GeV$^{5/3}$)	0.3798	$B_s^*(1^-)$	5410	5415
Λ (GeV)	1.1313	$D^-(0^-)$	1870	1870
A (GeV^{B-1})	1.5296	$D^{*-}(1^-)$	2018	2010
B	0.3263			

Results — bound states



Arrows indicate the energy gain (binding energy) from the relevant thresholds

M. Karliner:
Proc. 8th Int. Conf. Quarks and Nuclear Physics (QNP2018)
JPS Conf. Proc. 26, 011005 (2019) https://doi.org/10.7566/JPSCP.26.011005
Blue dots are added by AH from our results



Comparison with lattice results

	$I(J^P)$	This work	[27]	[28]	[29]	[30]	[31]
$bb\bar{q}\bar{q}$	$0(1^+)$	-173	-189 ± 13	-143 ± 34	$-$	-186 ± 15	-128 ± 26
$bc\bar{q}\bar{q}$	$0(1^+)$	-40	$-$	$-$	13 ± 3	$-$	$-$
$cc\bar{q}\bar{q}$	$0(1^+)$	-23	$-$	-23 ± 11	$-$	$-$	$-$
$bs\bar{q}\bar{q}$	$0(1^+)$	-5	$-$	$-$	16 ± 2	$-$	$-$
$bb\bar{s}\bar{q}$	$\frac{1}{2}(1^+)$	-59	-98 ± 10	-87 ± 32	$-$	$-$	$-$
$bb\bar{q}\bar{q}$	$1(0^+)$	N	$-$	-5 ± 18	$-$	$-$	$-$
$bc\bar{q}\bar{q}$	$0(0^+)$	-37	$-$	$-$	17 ± 3	$-$	$-$
$cc\bar{q}\bar{q}$	$1(0^+)$	N	$-$	26 ± 11	$-$	$-$	$-$
$bs\bar{q}\bar{q}$	$0(0^+)$	-7	$-$	$-$	18 ± 2	$-$	$-$

[27] A. Francis, R.J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. 118,(2017) 142001
$m_\pi = 164,\ 299,\ 415\ MeV$

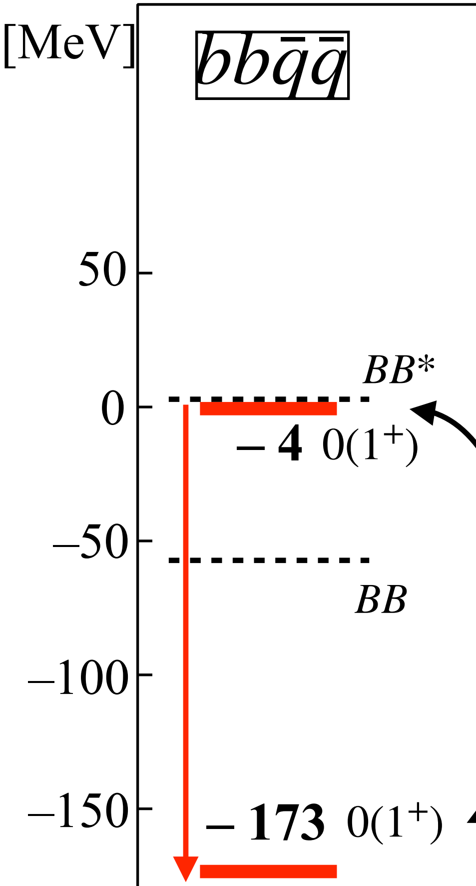
[28] P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D 99 (2019) 034507,
$m_\pi = 153 - 689\ MeV$

[29] R. Hudspith, B. Colquhoun, A. Francis, R. Lewis, K. Maltman, Phys Rev D.102.114506 (2020).
$m_\pi = 164,\ 299,\ 415\ MeV$

[30] P. Mohanta, S. Basak, Phys Rev D.102. 094516 (2020)

[31] L. Leskovec, S. Meinel, M. Pflaumer, M. Wagner, Phys. Rev. D 100 (1) (2019)

Results — bound states



There are two states of $J^P = 1^+$
near thresholds BB^* and B^*B^*

One deep bound state

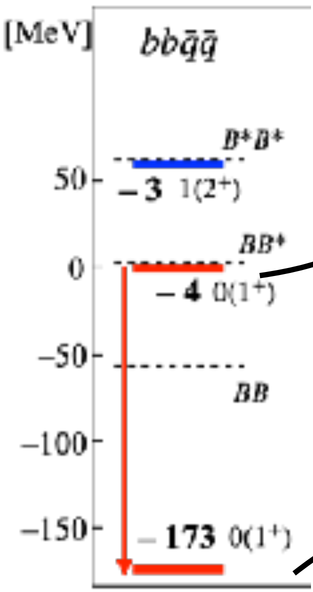
With "tightly bound" *bb* heavy-diquark)
Compact structure

Another is shallow

With "radially excited" *bb* heavy-diquark
Molecular-like

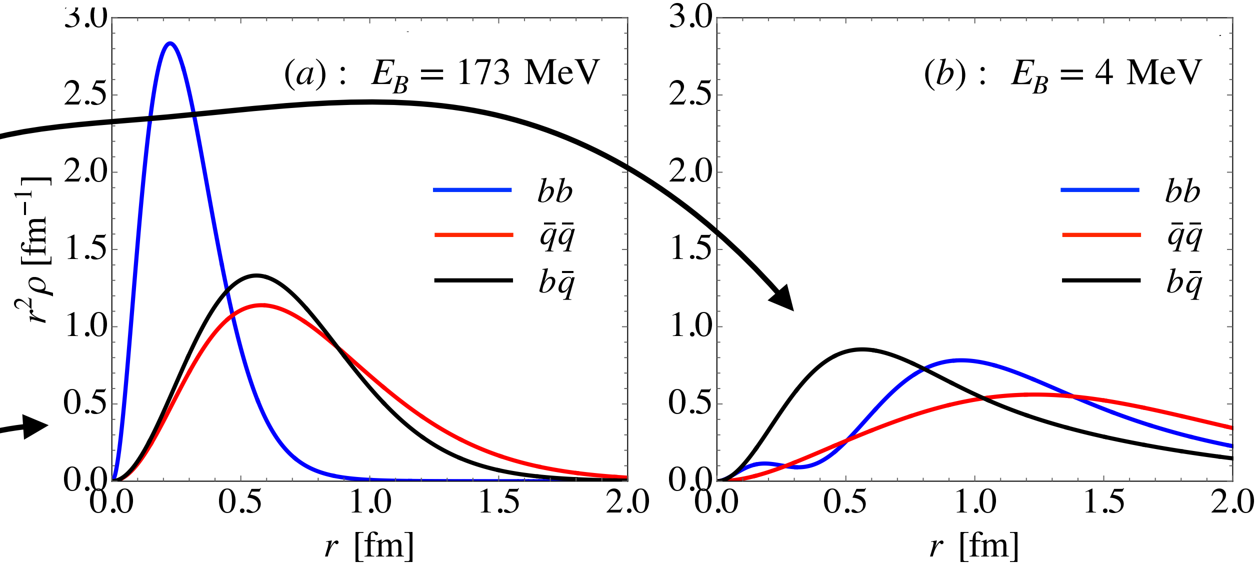
Arrows indicate the energy gain (binding energy) from the relevant thresholds

Results — bound states



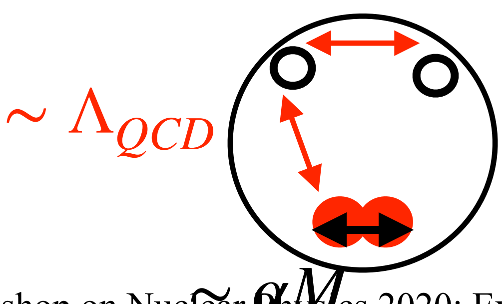
$\boxed{R_{qq'}}$

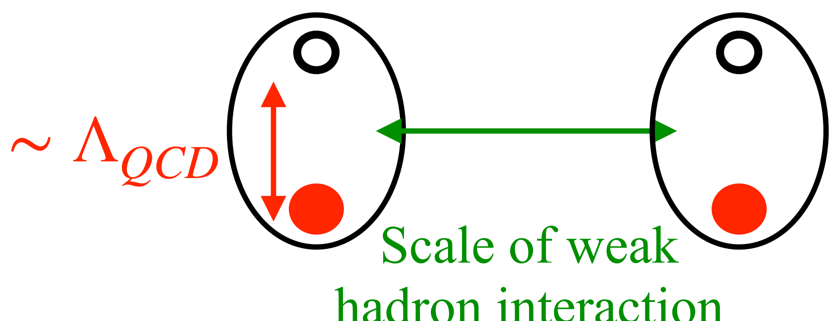
Deeply bound **Shallow bound**



$$R_{qq'} \equiv \left(\int r^2 \rho_{qq'}(r)\, r^2 dr \Big/ \int \rho_{qq'}(r)\, r^2 dr \right)^{1/2} \qquad \rho_{qq'}(r_{qq'}) = \int d\hat{\boldsymbol{r}}_{qq'} d\boldsymbol{x}_1 d\boldsymbol{x}_2 \, |\Psi_{JM}(\boldsymbol{r}_{qq'}, \boldsymbol{x}_1, \boldsymbol{x}_2)|^2$$

Singly heavy baryon like **Molecular**

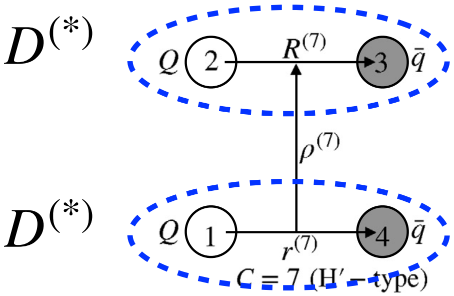




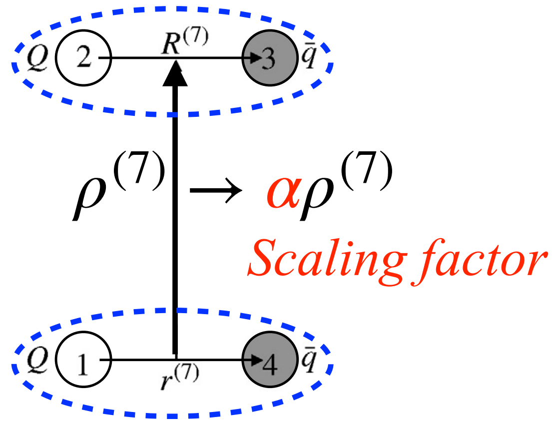
Results — Resonant states

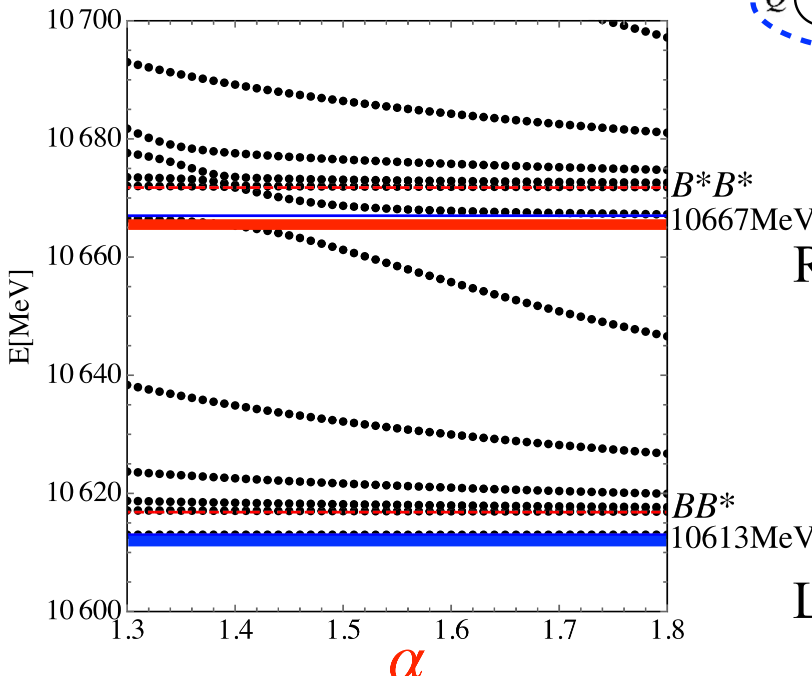
Meng et al, PLB824 (2022) 136800

Scaling method



They can be scattering states





Resonances

Position:

Sequence of horizontal lines that repel each other.

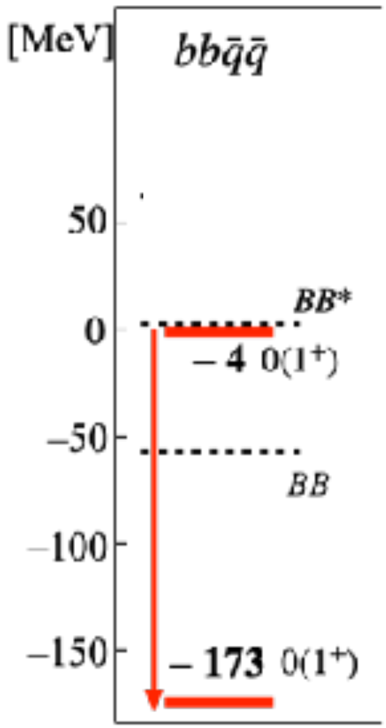
Width:

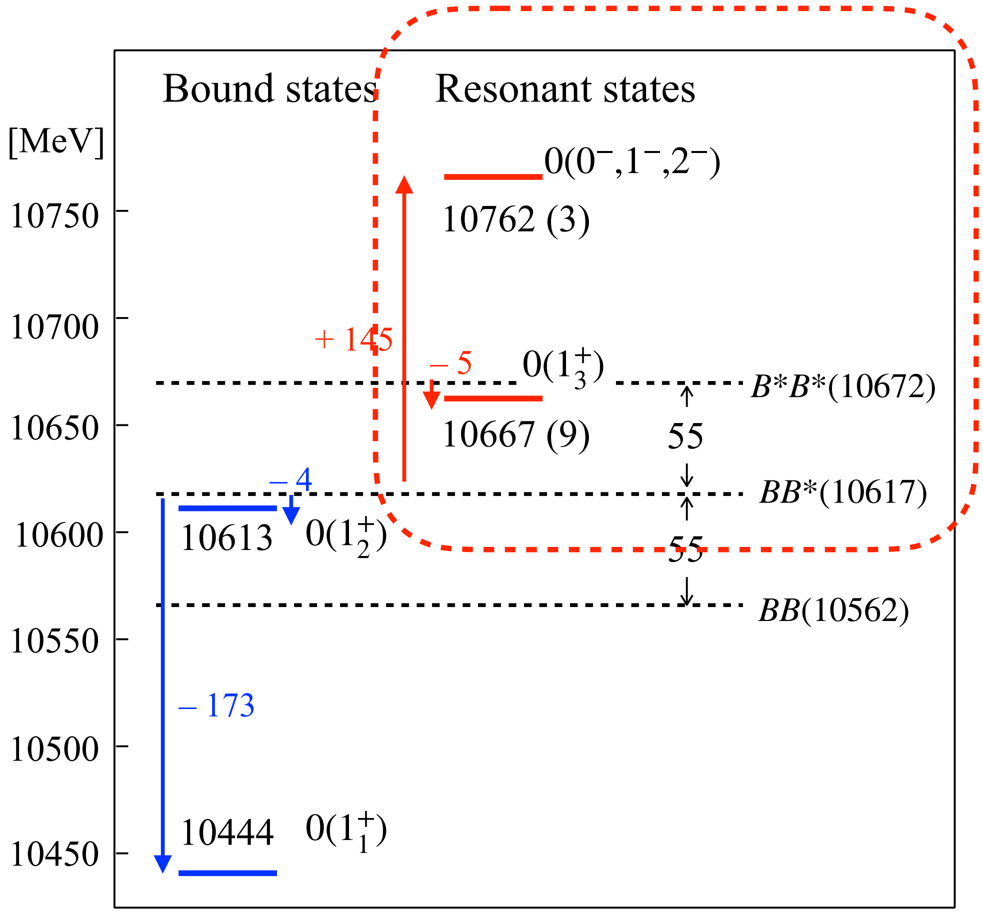
Distance of repulsion

Resonant state

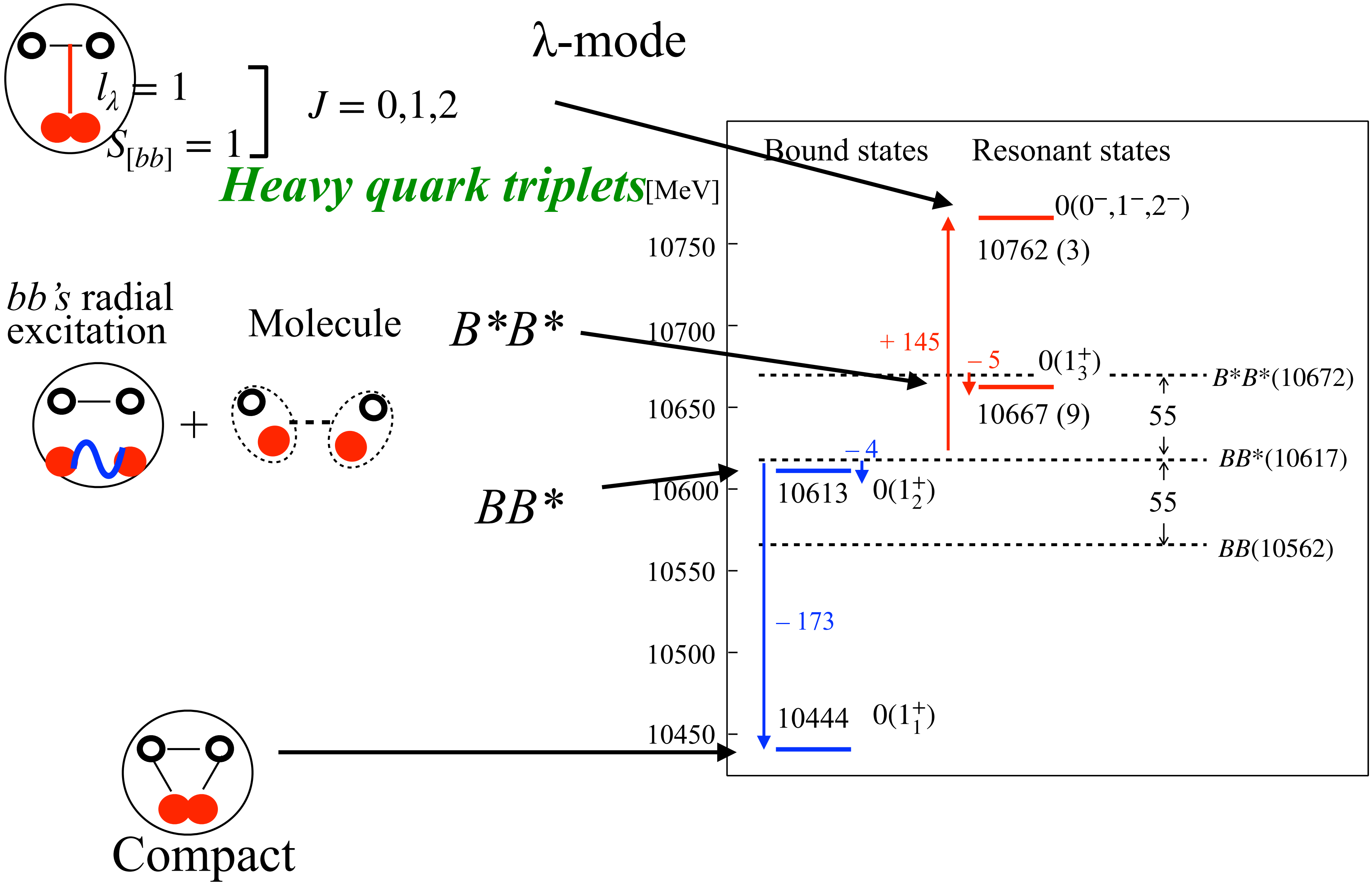
Loosely bound state

Results — Resonant states





Results — Resonant states



Summary for T_{QQ}

- Stable tetraquarks exist for $QQ\bar{q}\bar{q}$ (Heavy + light)
- Different configurations are formed depending on their energies

- The most stable one looks like a $[QQ]\bar{q}\bar{q} \sim \bar{Q}\bar{q}\bar{q}$ ($\sim Qqq$)
- Shallow ones are like molecule
- No stable all heavy $QQ\bar{Q}\bar{Q}$ ($> Q\bar{Q} + Q\bar{Q}$)

- There are also resonances;
  Negative parity ones ($L = 1$) may form heavy quark triplet,
  $J = L + S = 0,1,2$

Future

- Decays, inclusion of pion exchange interaction