# Recent Progress on Radial Excitation of Hadron


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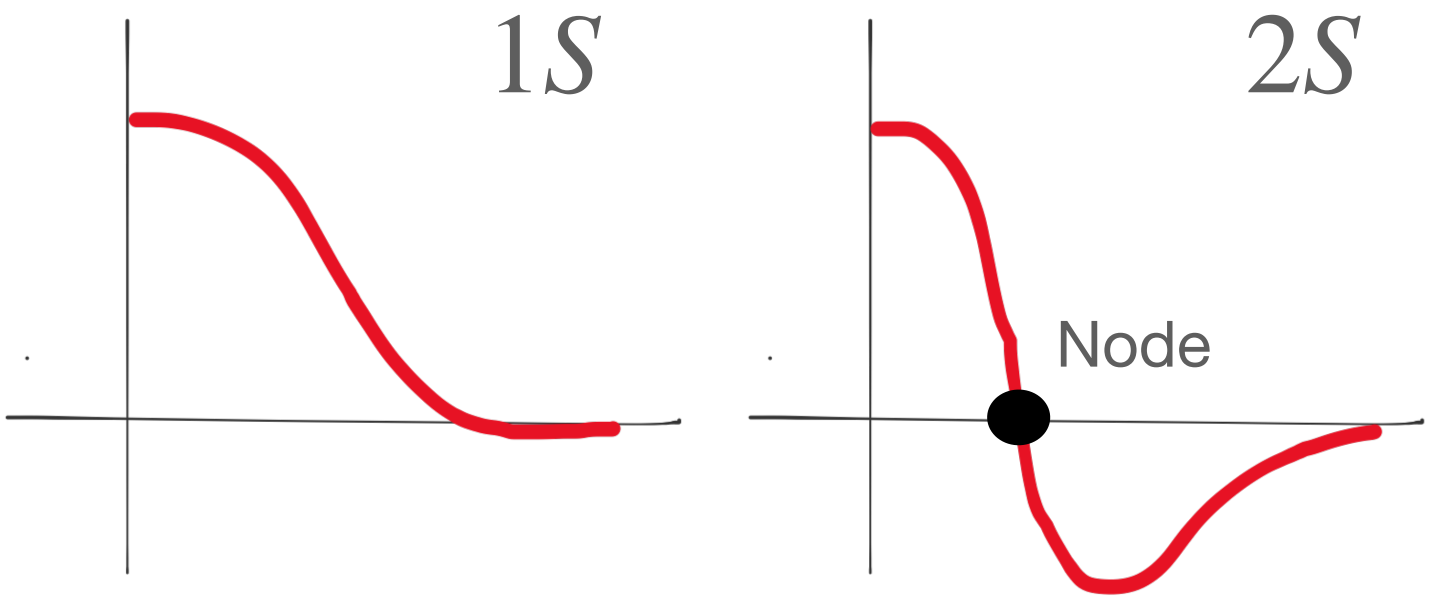
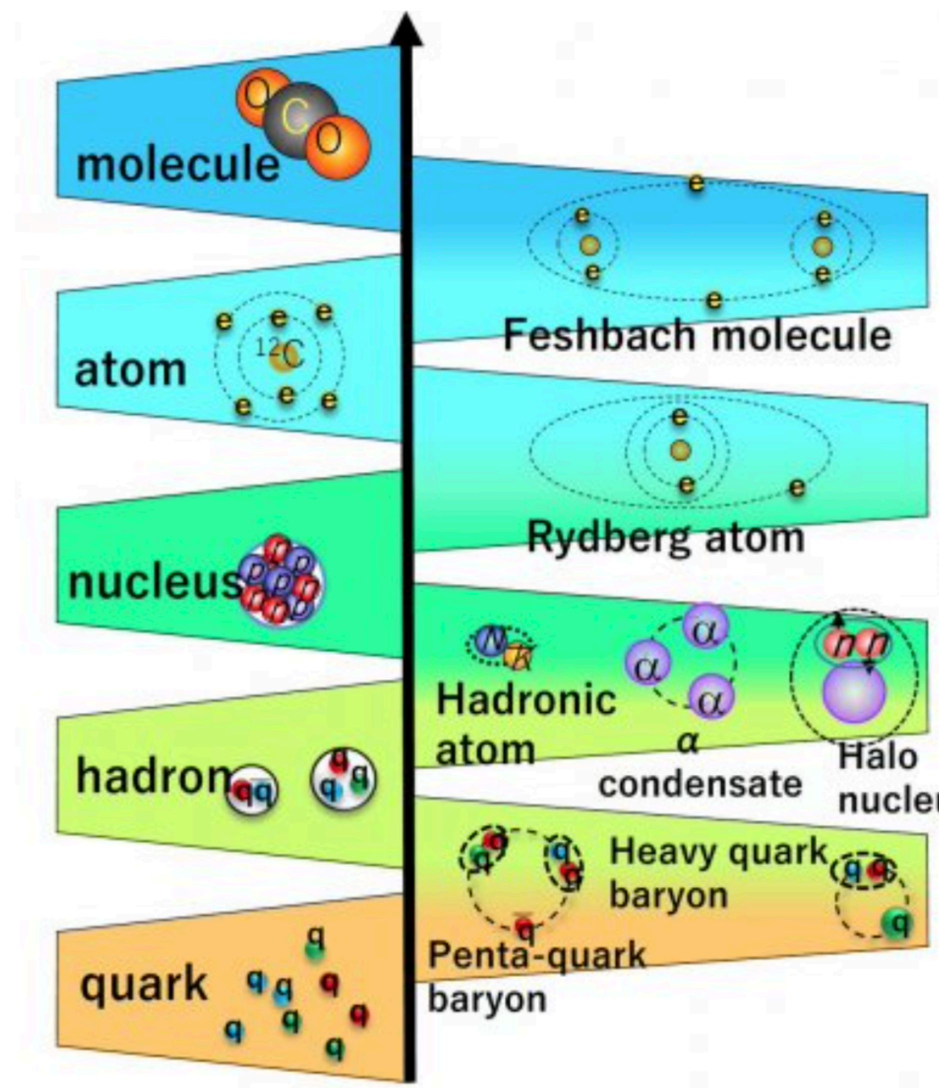
# Contents

- Radial excitation
- Three-body decays
  —> Dalitz plots
- Light-front quark model
  —> Mass spectra,
  —> Wave function,

# Radial excitation

# Radial excitation



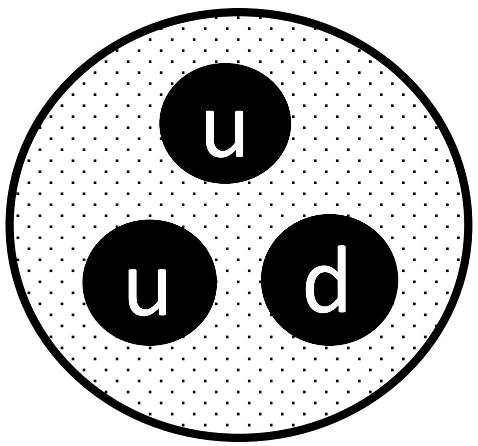




http://be.nucl.ap.titech.ac.jp/cluster/

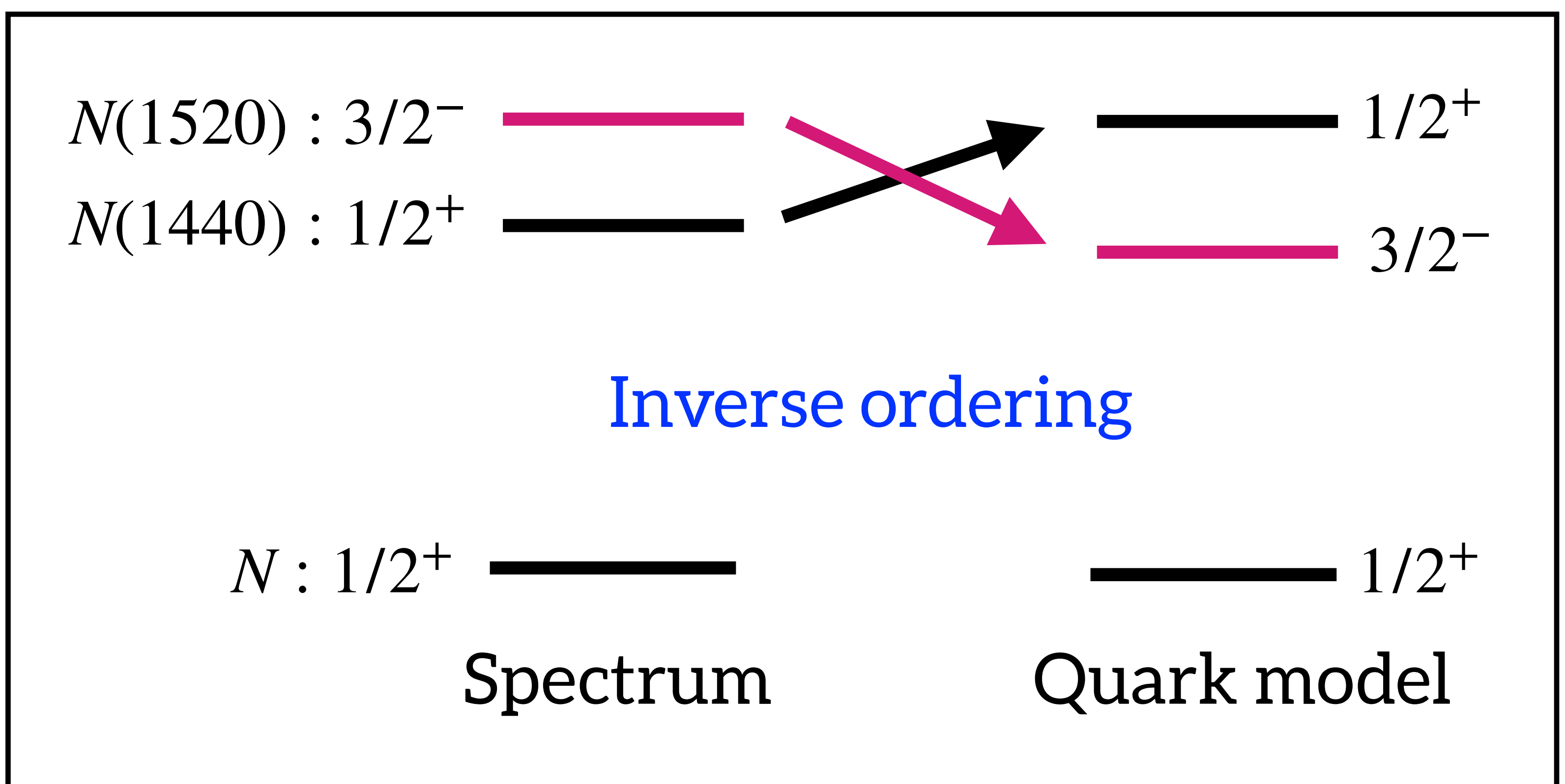
- **Radial excitation**
  —> a.k.a. Breathing mode,
  —> The same spin-parity with the g. s.
  —> Has "nodal" structure,

- **Hadron**: QCD
  —> E.g. Roper resonance
  —> Puzzle: mass, decay, etc.
  —> Similar behaviors

# Roper resonance




- In PDG –> $N(1440)$
- Discovered by **L. D. Roper** in 1963 (almost 60 years).
- A radial excitation of nucleon with $J^P = 1/2^+$.
- Incompatible with the quark model.
- Interpretation: quark core + meson cloud?



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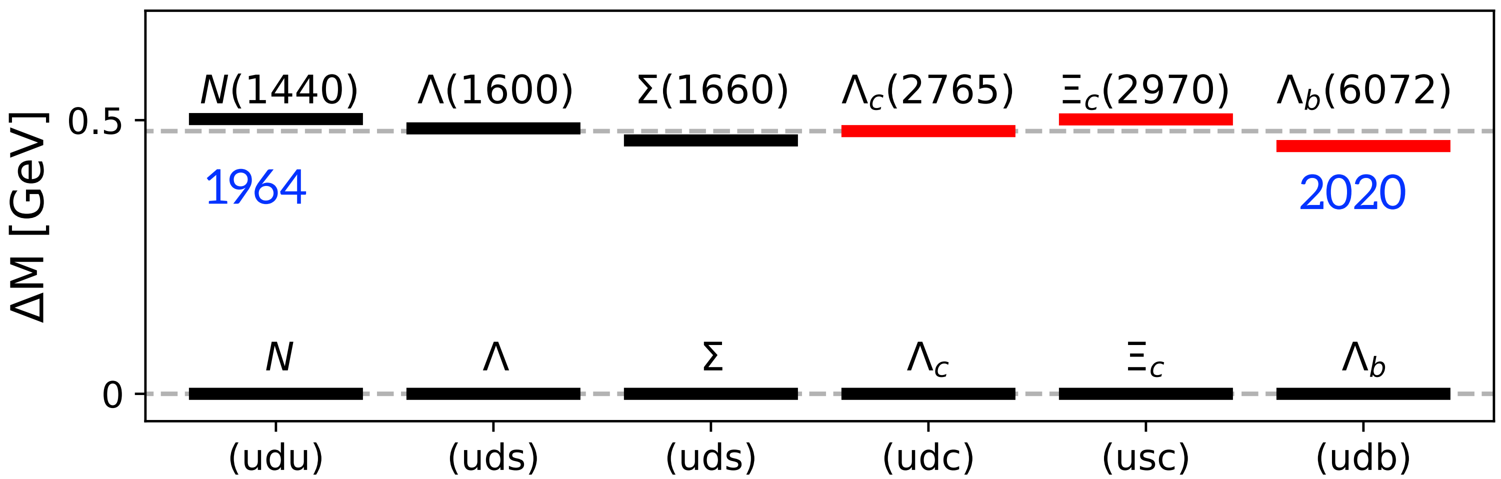
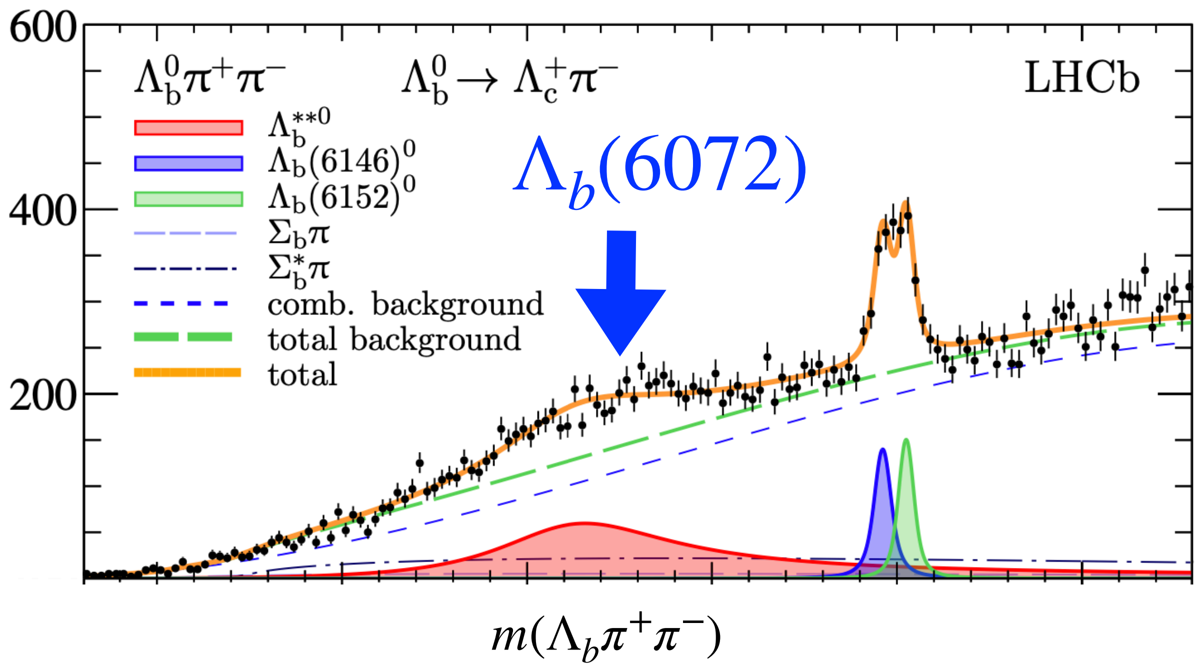
# 1S and 2S state baryons

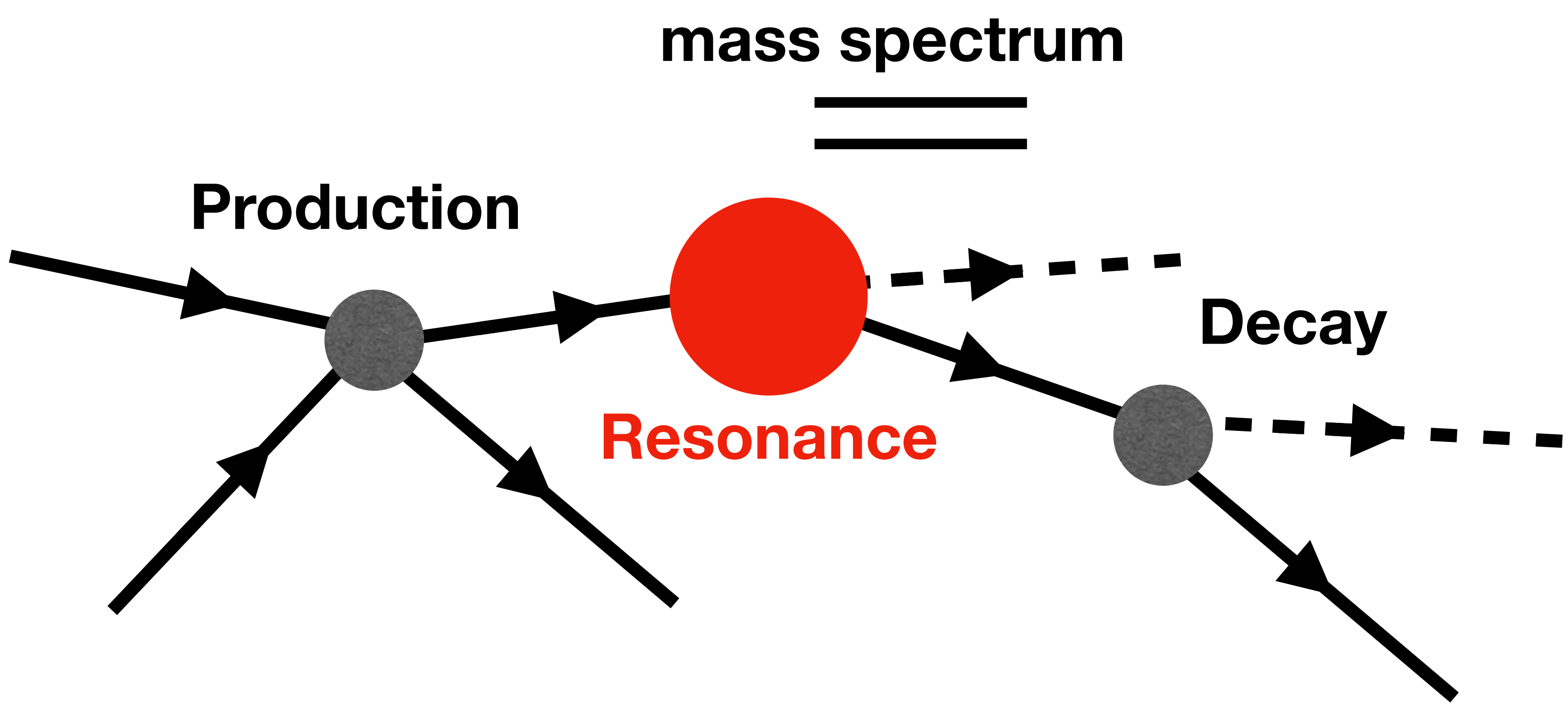



<> Similar mass gap
$\Delta M \sim 500$ MeV
Universal?
Accidental?




<> A broad resonance
($\Gamma = 72$ MeV).
Orthogonality?

# Methodology




**Internal structure:**

—> **Production,** Cross section, polarization observables, etc

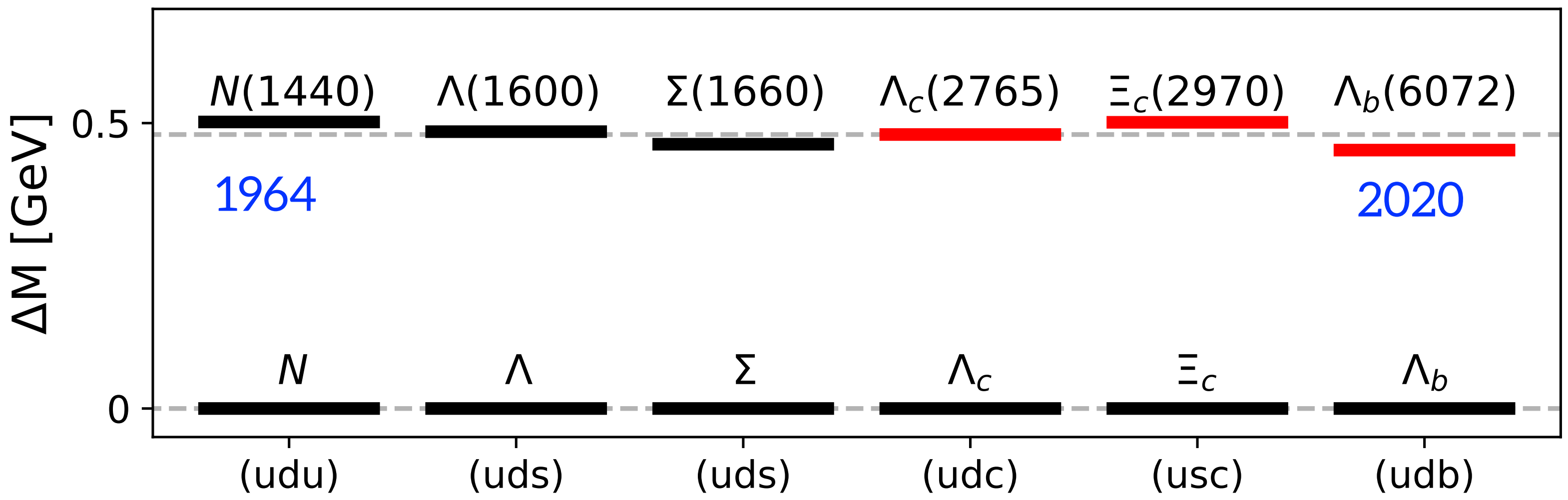
—> **Spectrum,** Mass, mass splitting, etc

—> **Decay,** Decay rates, branching fraction, etc

**Resonance:** mass, decay rate, spin, parity, isospin, etc

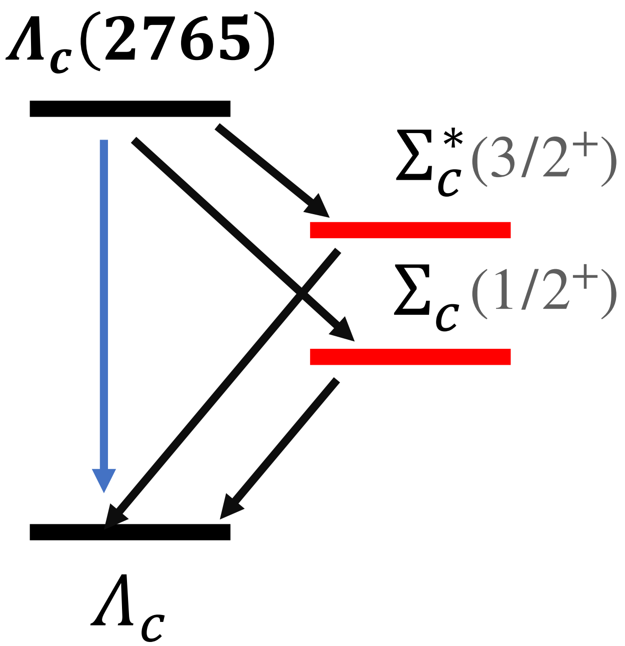
# Three-body decay

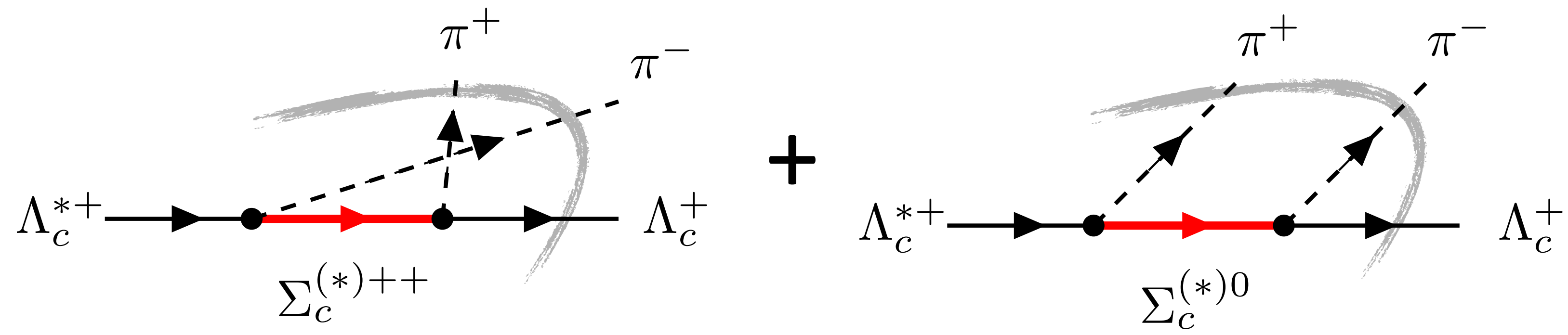
## Dalitz plot



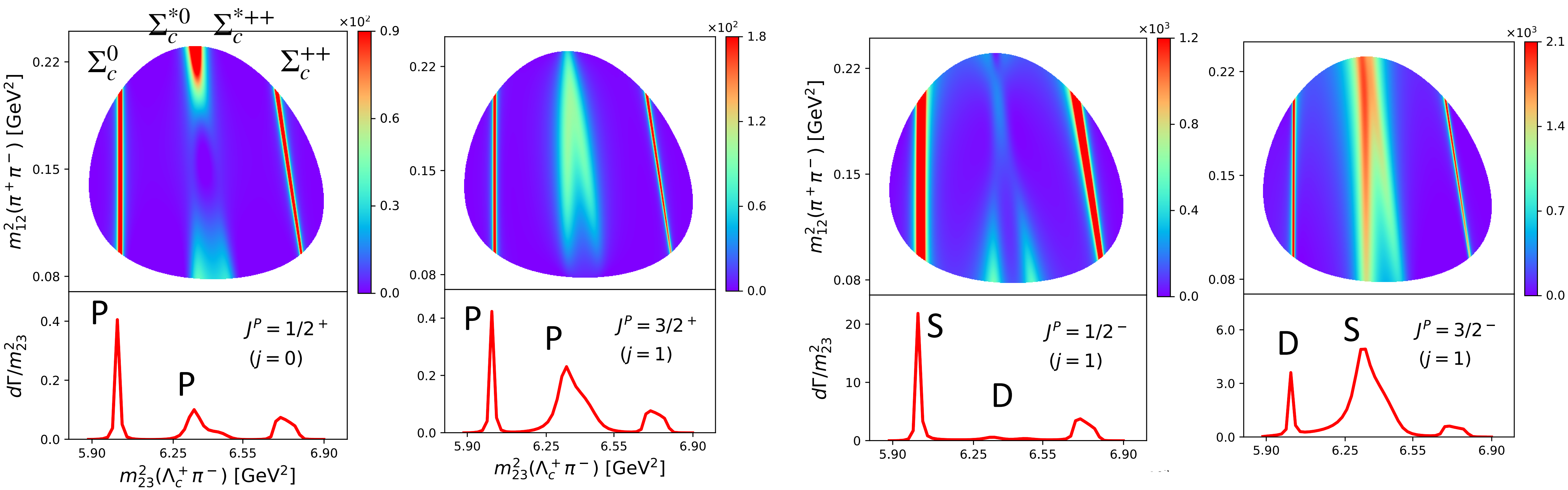
# Dalitz plot and Spin-parity

$\Lambda_c(2765)$




Sequential process going through $\Sigma_c^{(*)}$ resonances.




PRD101, 094023 (2020)



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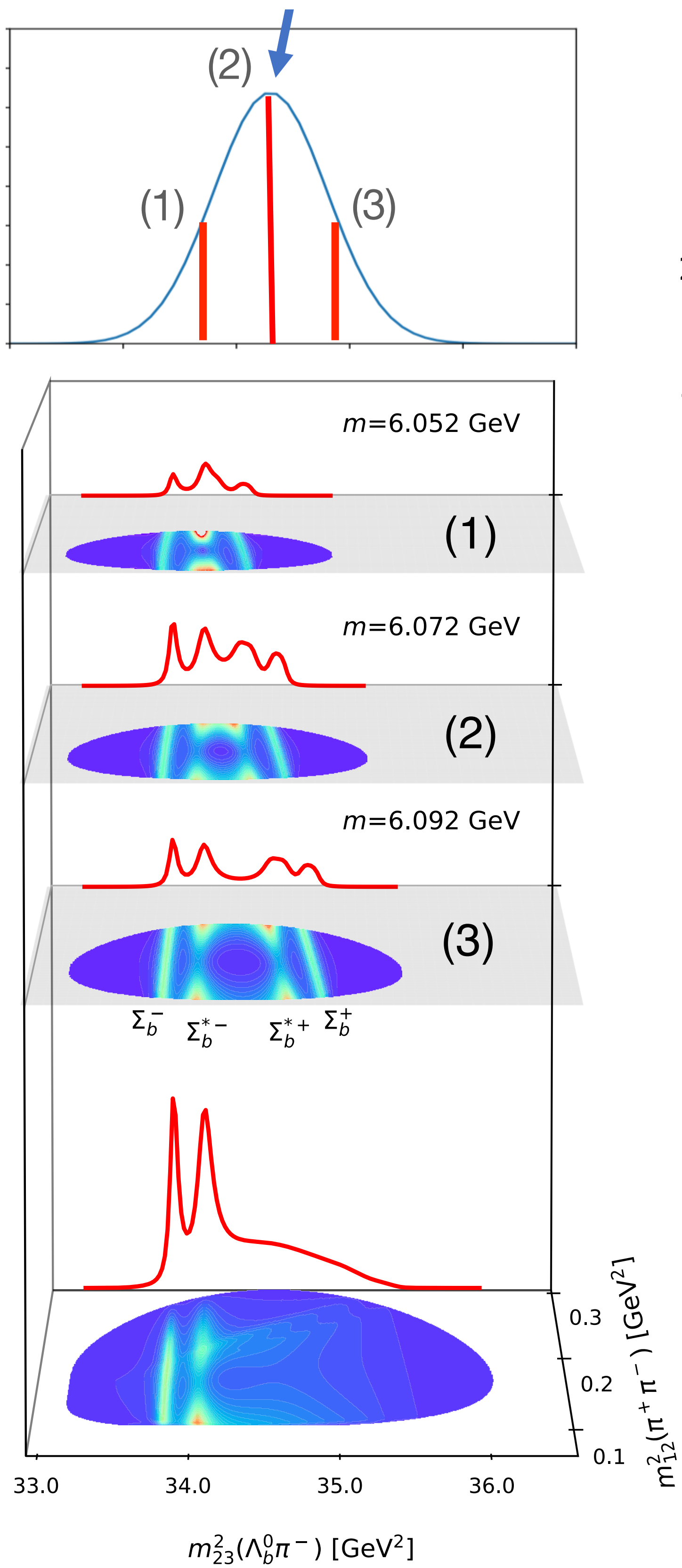
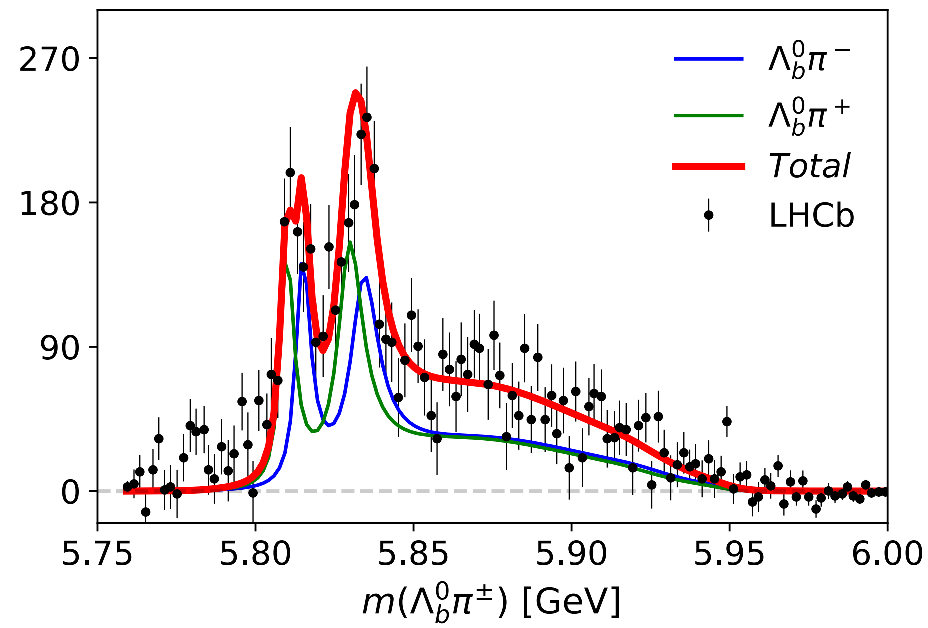
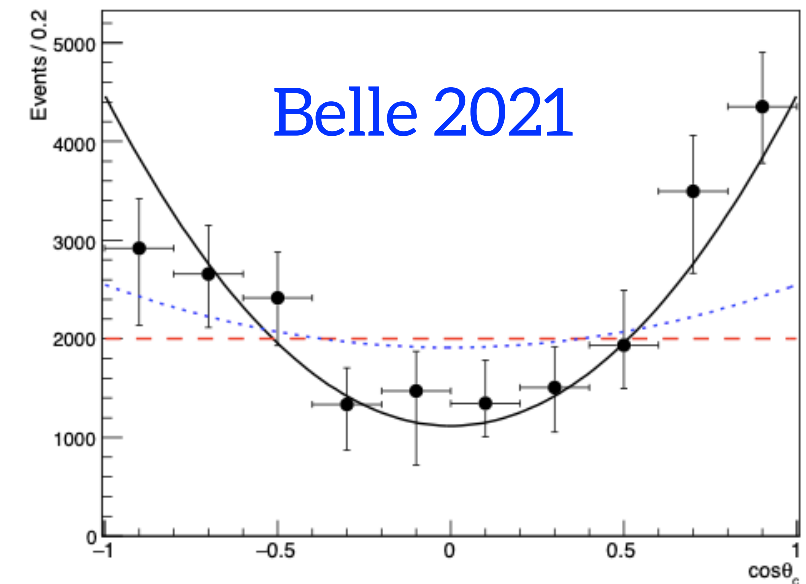
# Dalitz plot for a broad resonance




$\Lambda_b(6072)$



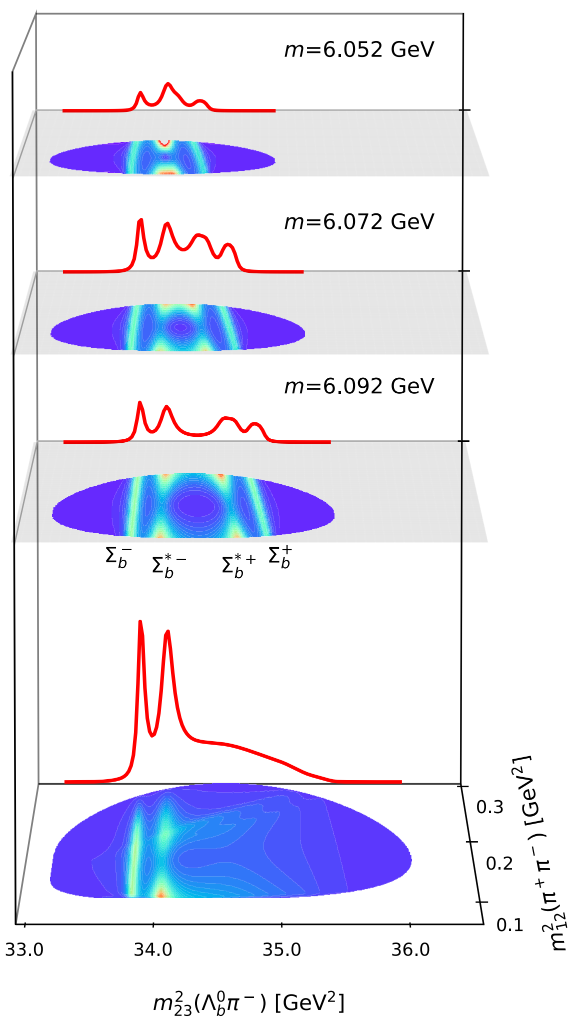

$\Xi_c(2970)$

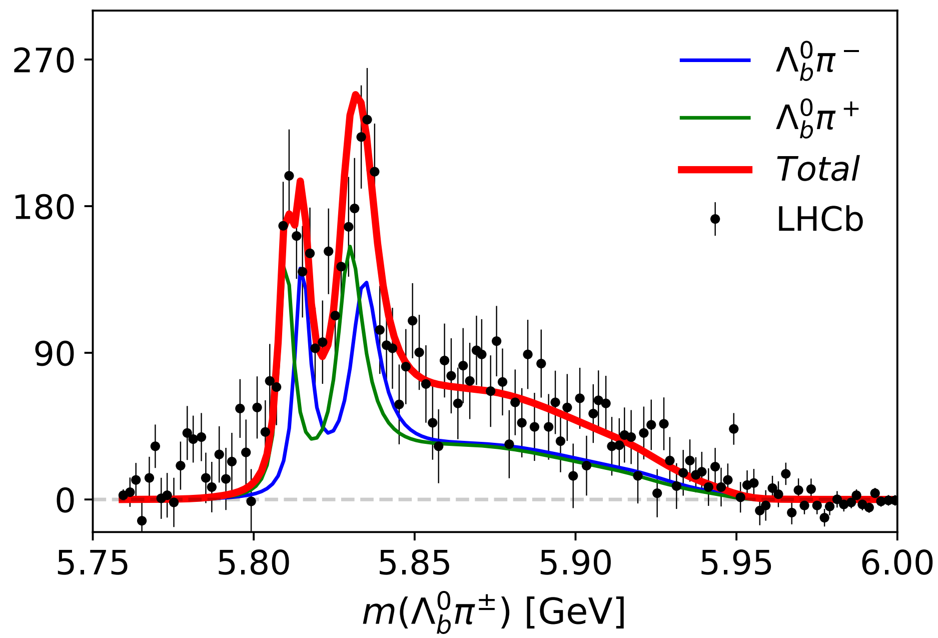


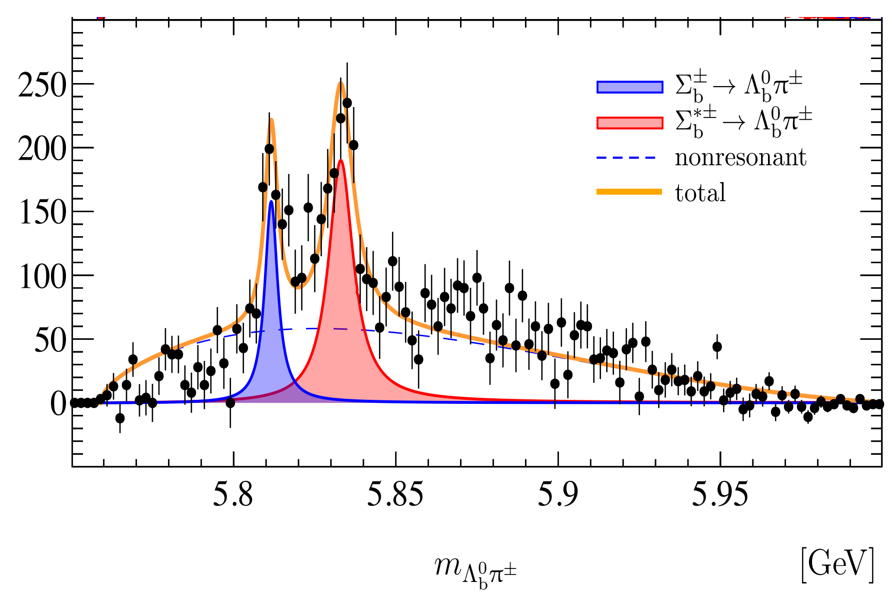

- They most likely belong to Roper's family.
  —> Invariant mass distribution
  —> Ratio of decay width
  —> Angular correlation

PRD101, 111502 (R) (2020)

# Comparison with LHCb analysis



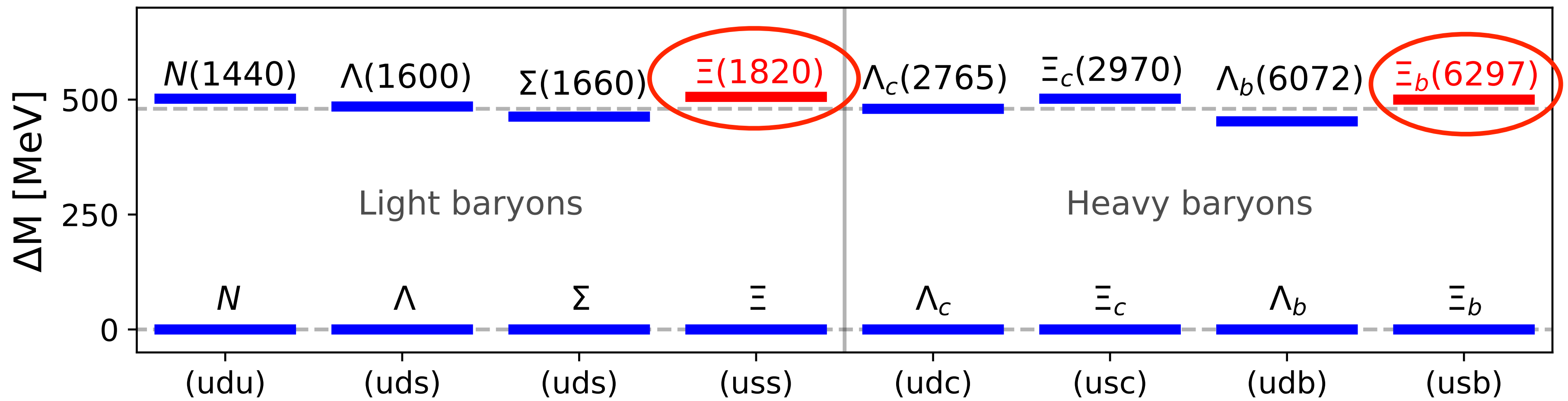

## Our analysis

- Background shape is from kinematical reaction.

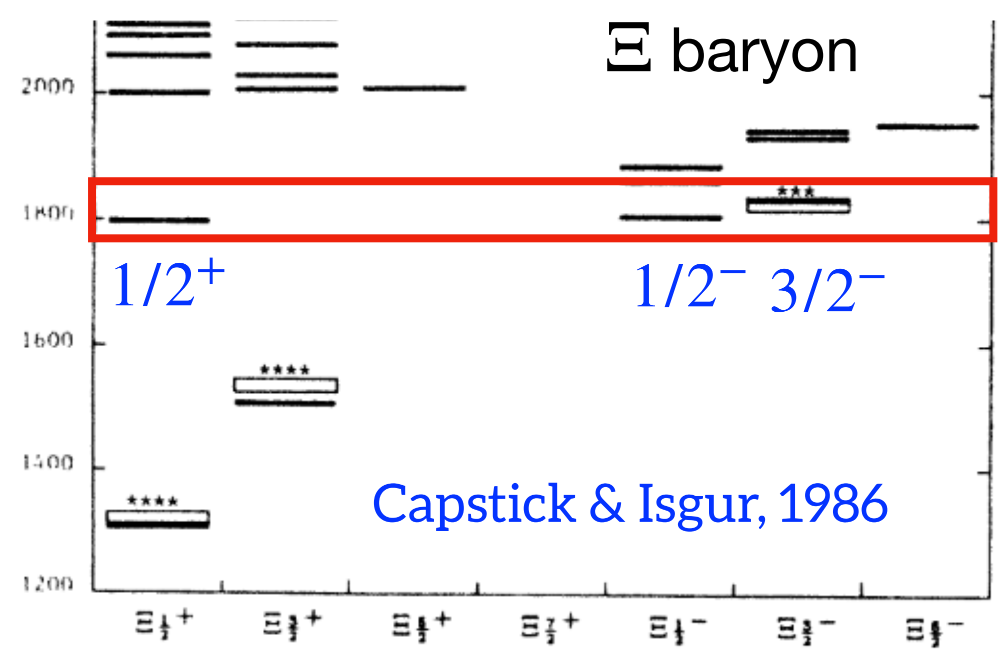
## LHCb

- Non-resonant contribution is relatively large.

# Missing Roper-like resonance




| | | |
|---|---|---|
| Ξ(1530) | 3/2+ | **** |
| Ξ(1620) | | * |
| Ξ(1690) | | *** |
| Ξ(1820) | 3/2− | *** |
| Ξ(1950) | | *** |

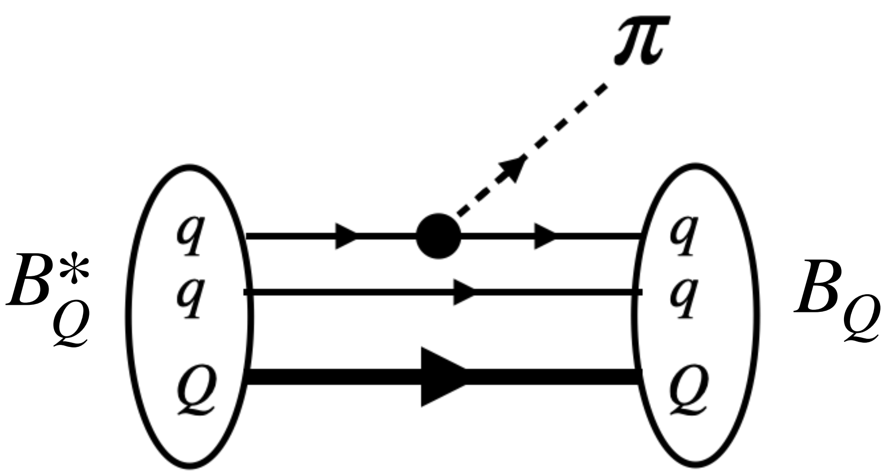


# Light-front quark model

Mass spectra & wave function

# Problem of NR Quark Model

- Strong decay of $\Lambda_b(6072)$




[1] Wave function
—> HO (gaussian)

[2] Quark-pion interaction

$$\mathcal{L}_{\pi qq} = \frac{g_A^q}{2f_\pi}\bar{q}\gamma^\mu\gamma_5\vec{\tau}q\cdot\partial_\mu\vec{\pi}$$

—-> Nonrelativistic expansion

$$\propto g(\sigma\cdot q - \frac{\omega}{2m}\sigma\cdot(p_i+p_f))$$

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<> $\Lambda_b(6072)$ —> $\Gamma$ ~ 5 MeV (narrow)
$\Gamma_{\rm exp} = 72$ MeV

- Orthogonality of w.f. ?
- Relativistic effect?

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# Relativistic correction

| Roper-like state | NR | NR + RC | Exp. |
|---|---|---|---|
| $\Lambda_b(6072) : 1/2^+, \lambda\lambda$ | 2 - 5 | 13 - 52 | 73 MeV |

- The overlap is orthogonal in the long-wavelength limit.

$$\left\langle \Sigma_b \middle| 1 \middle| \Lambda_b \right\rangle \propto q^2 \qquad \left\langle \Sigma_b \middle| p_i \middle| \Lambda_b \right\rangle \propto q \qquad \left\langle \Sigma_b \middle| p_i^2 \middle| \Lambda_b \right\rangle \propto a^2$$

negligible small large

- The leading order is somehow suppressed.
- The relativistic correction is essential.

**Light-front quark model**

PRD**103**, 094003 (2021)

# Light-front quark model

- Constituent quark model
- Light-front dynamics

⇨ Hadrons: $q\bar{q}$, $qqq$

[1] Trial wave function
 —> Gaussian (H.O. basis).

$$\phi_{1S}(x, \mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}} \sqrt{\frac{\partial k_z}{\partial x}} e^{-\vec{k}^2/2\beta^2},$$

$$\phi_{2S}(x, \mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\sqrt{6}\beta^{7/2}} \left(2\vec{k}^2 - 3\beta^2\right) \sqrt{\frac{\partial k_z}{\partial x}} e^{-\vec{k}^2/2\beta^2},$$

[2] Effective potentials
 —> Cornel potential, etc

[3] Variational Parameters $\beta$
 —> Fixed from mass spectra

$$M_{q\bar{q}} = \langle\Psi| \left[H_0 + V_{q\bar{q}}\right] |\Psi\rangle$$

$$\frac{\partial \langle\Psi| \left[H_0 + V_0\right] |\Psi\rangle}{\partial\beta} = 0$$

Problem!!
 —> Using pure HO basis
 —> Can't explain 2S decay constant.



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# Problem of decay constant

<> Decay constants of Upsilon (Exp)

—> f($\Upsilon(1S)$) = 689 MeV

—> f($\Upsilon(2S)$) = 497 MeV

$$\langle 0|\,\bar{q}\gamma^\mu\gamma_5 q\,|P\rangle = i f_P P^\mu,$$
$$\langle 0|\,\bar{q}\gamma^\mu q\,|V(P,\lambda)\rangle = f_V M \epsilon^\mu(\lambda),$$

$$f_P = \sqrt{6}\int \frac{dx\; d^2\mathbf{k}_\perp}{(2\pi)^3}\phi(x,\mathbf{k}_\perp)\frac{\mathcal{A}}{\sqrt{\mathcal{A}^2+\mathbf{k}_\perp^2}}$$

<> If we use pure 2S HO wave function

—> with the same $\beta$ parameters

—> always f($\Upsilon(2S)$) > f($\Upsilon(1S)$)

$$\phi_{2S}(x,\mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\sqrt{6}\beta^{7/2}}\left(2\vec{k}^2 - 3\beta^2\right)\sqrt{\frac{\partial k_z}{\partial x}}\,e^{-\vec{k}^2/2\beta^2},$$

<> To solve the problem:

—> Modify the wave function

—> Simply use different $\beta$ parameters

$$\mathrm{Exp}\left(-\frac{k^2}{2\beta}\right) \Rightarrow \mathrm{Exp}\left(-\frac{n^\delta}{2}\frac{k^2}{\beta}\right)$$

# Wave function of 1S and 2S states

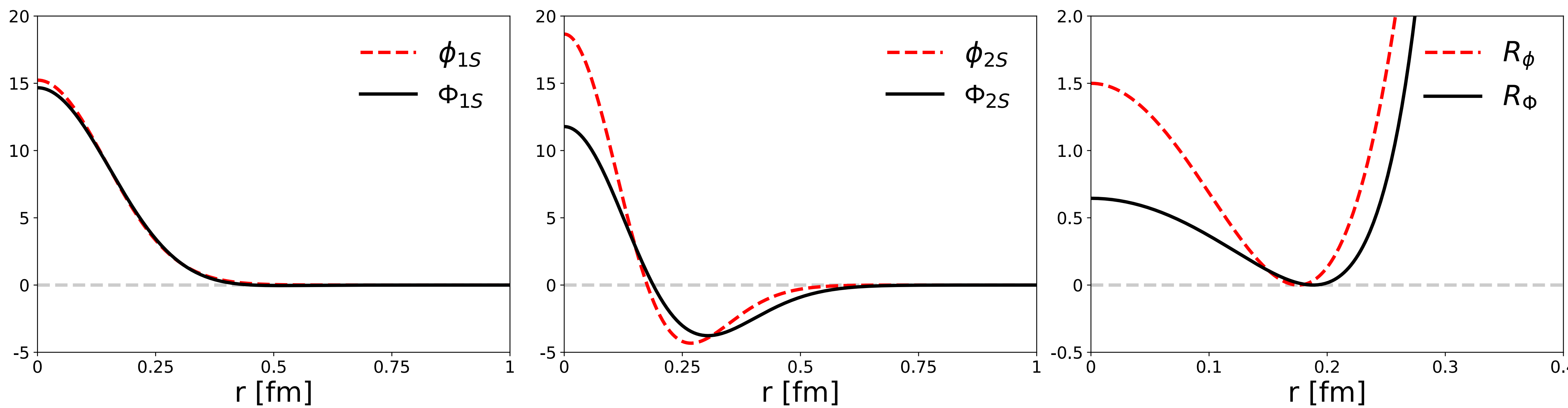
<> Minimal mixing

—> The same $\beta$ for 1S and 2S states

—> keep orthogonality

—> doesn't change 1S WF

$$\begin{pmatrix} \Phi_{1S} \\ \Phi_{2S} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_{1S} \\ \phi_{2S} \end{pmatrix},$$




<> Only need a small mixing

—> $\theta = 12^\circ$

—> $|\cos\theta|^2 = 95.7\,\%\,, |\sin\theta|^2 = 4.3\,\%\,,$

<> Huge impact to observable.

—> Mass spectra,

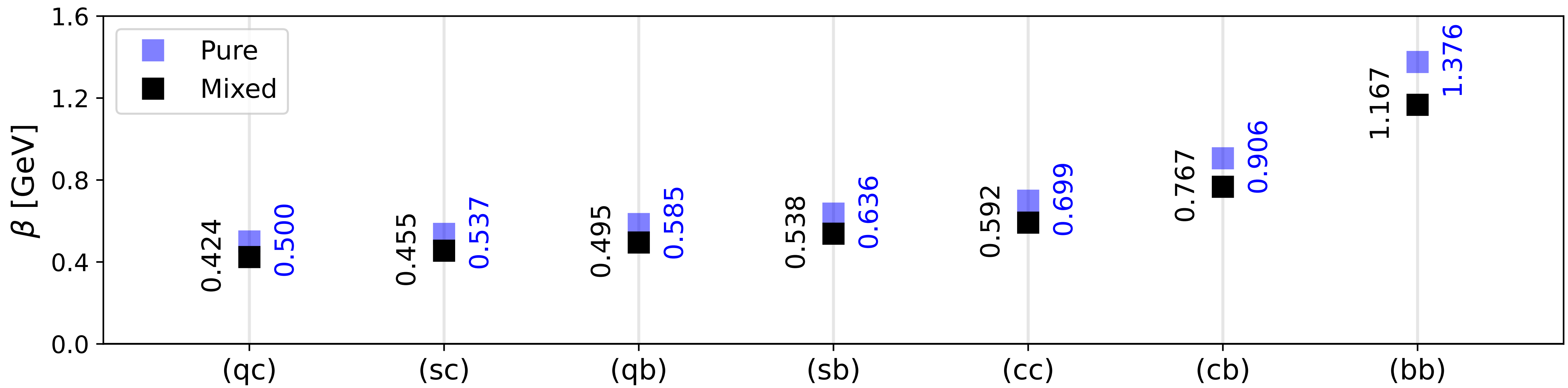
—> Decay constant,

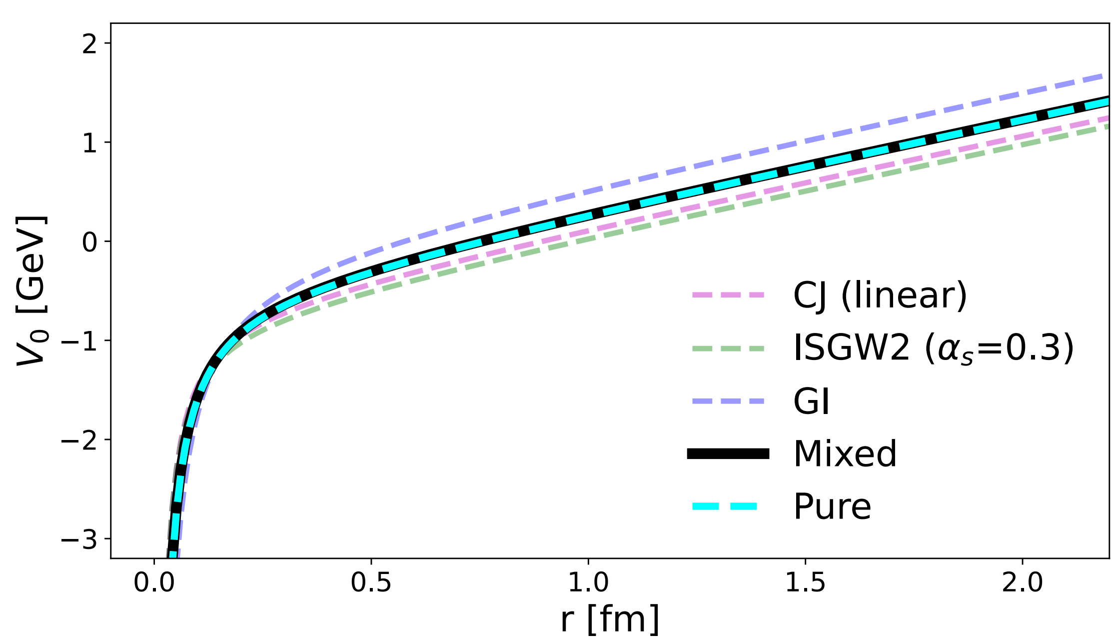
—> Charge radii, etc

# Model parameters

$$\phi_{1S}(x,\mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}e^{-\vec{k}^2/2\beta^2},$$

$$\phi_{2S}(x,\mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\sqrt{6}\beta^{7/2}}\left(2\vec{k}^2 - 3\beta^2\right)\sqrt{\frac{\partial k_z}{\partial x}}e^{-\vec{k}^2/2\beta^2},$$




<> In the mixed scenario:

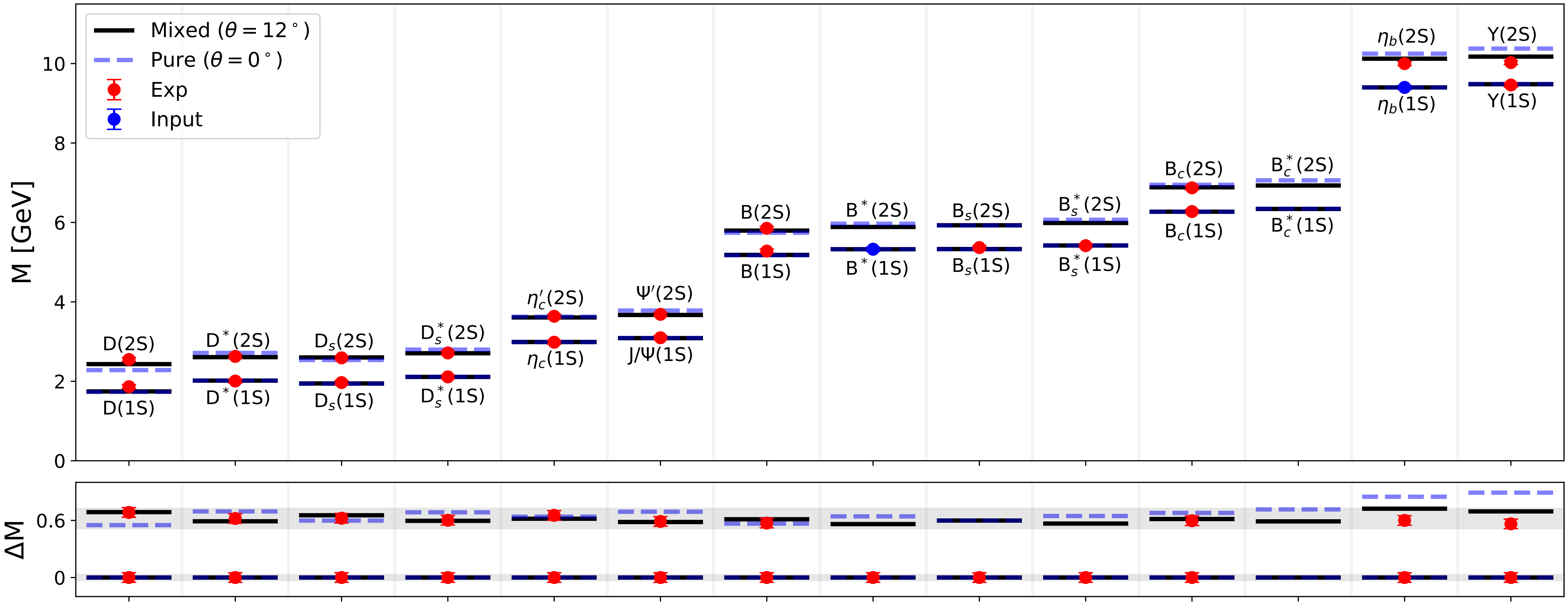
—> use the same quark mass.

—> $\beta$ systematically decrease.

—> Potential look the same.

$$V_{q\bar{q}} = a + br - \frac{4\alpha_s}{3r}.$$

# Mass spectra and gaps



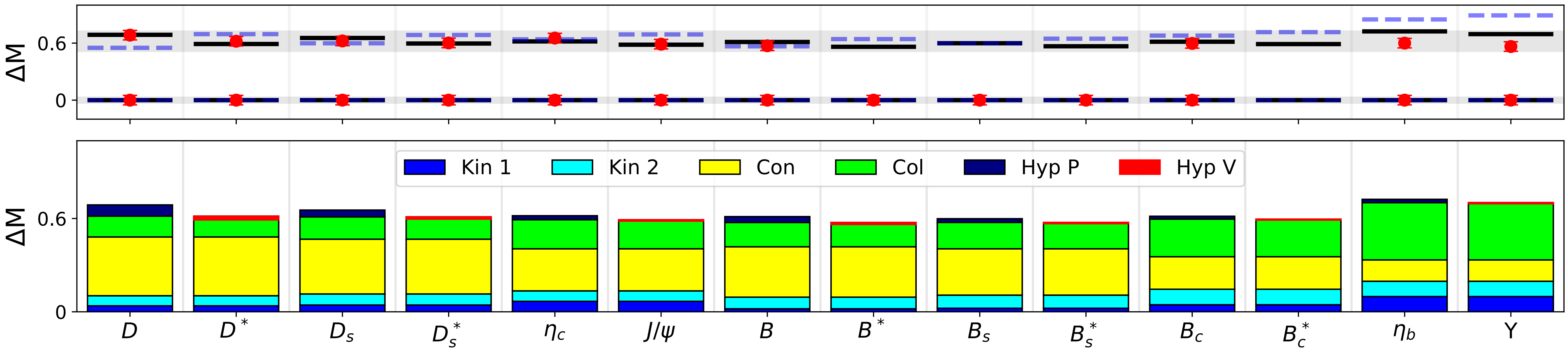

$$V_{q\bar{q}} = a + br - \frac{4\alpha_s}{3r} + \frac{2}{3}\frac{\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}}{m_q m_{\bar{q}}}\nabla^2 V_{\rm Coul}$$

$$M_{q\bar{q}} = \langle\Psi| \left[H_0 + V_{q\bar{q}}\right] |\Psi\rangle$$

<> Similar mass gap around 600 MeV

<> Mixed scenario
—> better agreement

# Mass spectra and gaps




<> Competing contribution:

—> Confinement int

$$\Delta M_{conf} \propto \frac{1}{\beta}$$

—> Coulomb int

$$\Delta M_{colmb} \propto \beta$$

<> Hyperfine int

—> Small but, very important

—> Mixing is needed

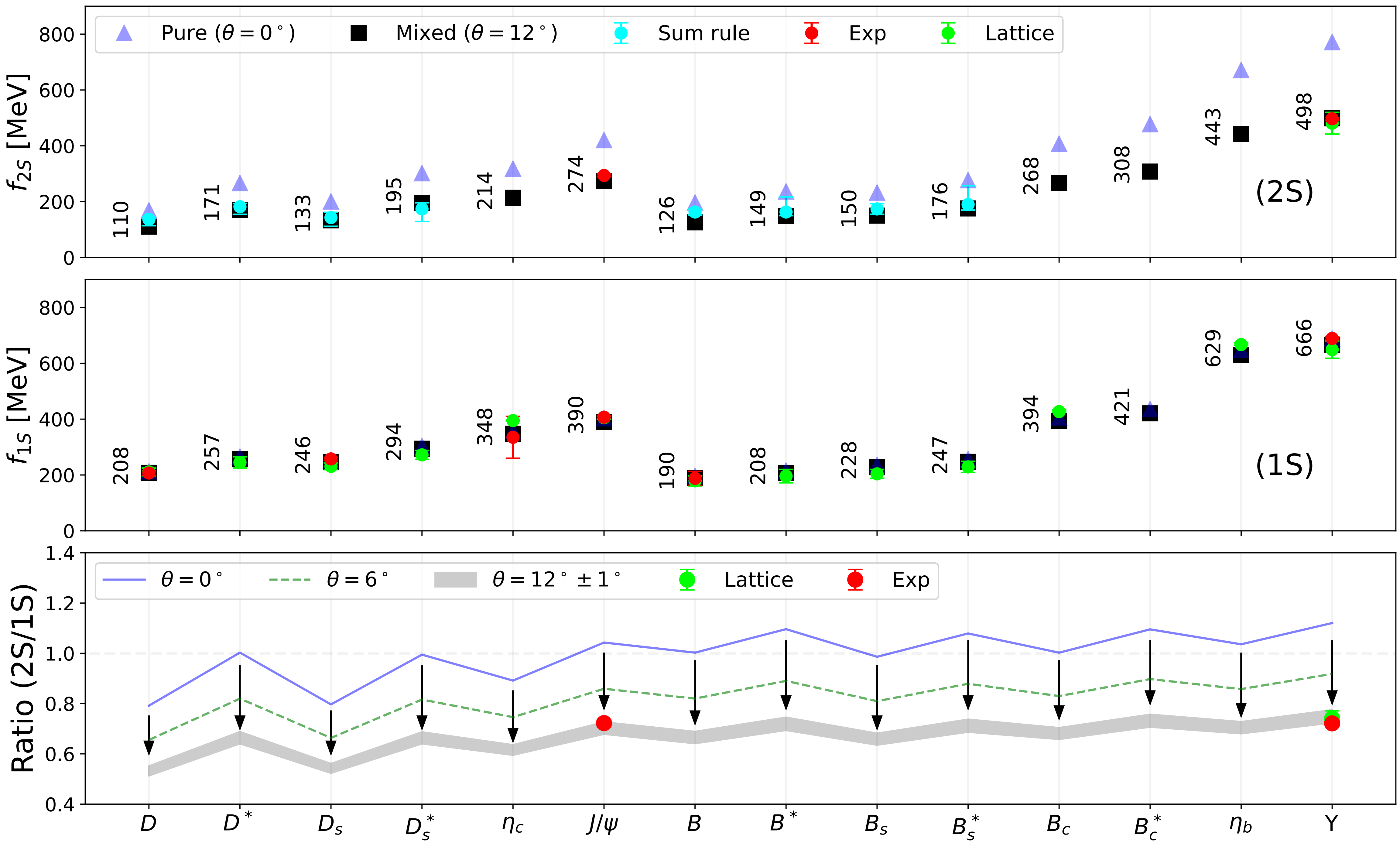
—> $\Delta M_P > \Delta M_V$

$$\Delta M_{hyp} \propto (S_q \cdot S_{\bar{q}})(\cos 2\theta - 2\sqrt{6}\sin 2\theta)$$

—> $\theta_c \approx 6^\circ$

# Decay constant

$$\langle 0| \bar{q}\gamma^{\mu}\gamma_5 q |P\rangle = i f_P P^{\mu},$$
$$\langle 0| \bar{q}\gamma^{\mu} q |V(P,\lambda)\rangle = f_V M \epsilon^{\mu}(\lambda),$$



# Decay constant with "bad current"

$$\langle 0|\, \bar{q}\gamma^\mu\gamma_5 q\, |P\rangle = i f_P P^\mu,$$

$$P^+ = P^0 + P^3$$

$$P^- = P^0 - P^3$$

$$P^- = \frac{M^2 + \mathbf{P}_\perp^2}{P^+}$$

$$P^2 = P^+P^- - \mathbf{P}_\perp^2 = M^2$$

$(\mu = +)$

$$f_p^+ = \frac{\sqrt{6}}{8\pi^3}\int dx d^2\mathbf{k}_\perp \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2+\mathbf{k}_\perp^2}}\mathcal{A}$$

$(\mu = -)$ —> prescription: $M \to M_0$

$$f_p^- = \frac{\sqrt{6}}{8\pi^3}\int dx d^2\mathbf{k}_\perp \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2+\mathbf{k}_\perp^2}}\frac{\mathcal{A}}{(M_0^2+\mathbf{P}_\perp^2)}\left[\frac{\mathbf{k}_\perp^2(\mathcal{A}'/\mathcal{A})+m_1m_2}{x(1-x)}+\mathbf{P}_\perp^2\right]$$

No difference: analytic proof

$$f_p^- - f_p^+ = \int dx d^2\mathbf{k}_\perp \frac{(m_2-m_1)M_0}{(\mathbf{P}_\perp^2+M_0^2)^2}(-2k_z)$$

$$P^- - P^+ = -2P^3$$

# A closer look at the problem

$$\langle 0|\, \bar{q}\gamma^{\mu}\gamma_5 q\, |P\rangle = i f_P P^{\mu},$$

$$P^- = \frac{M^2 + \mathbf{P}_\perp^2}{P^+}$$

## Historically

- Physics should be the same for different component.
- Not easy to get the equivalence.
- Replacement:
  —> $M \to M_0$ prescription
  —> Give the same result

## Fundamental

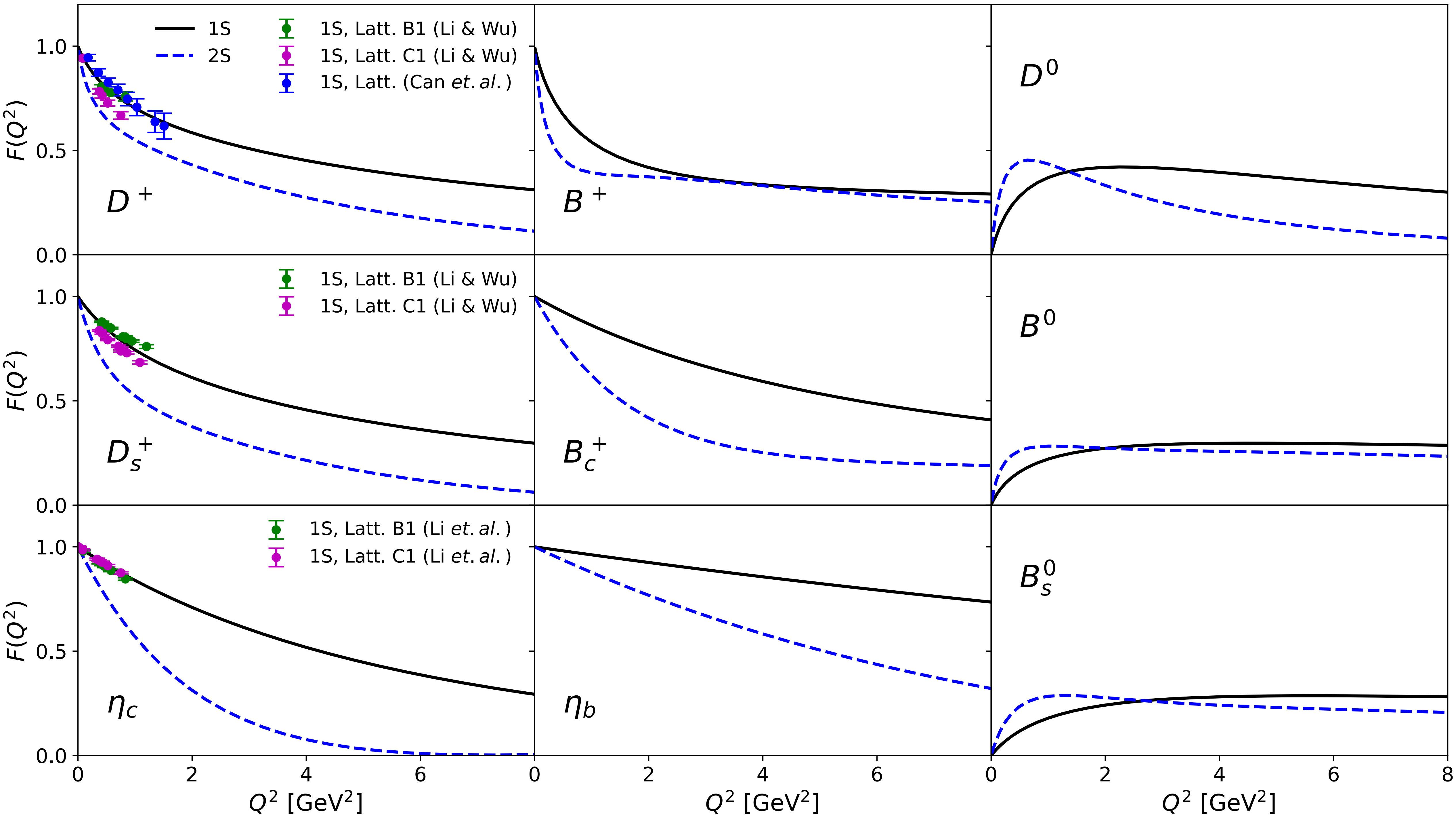
- Bakamjian-Thomas construction (1953)

$$|M(P)\rangle = \int d^3p_1 d^3p_2 \delta^3(P - p_1 - p_2)\ \psi(x, k_\perp)\, |p_1\rangle\ |p_2\rangle$$

$$P = p_1 + p_2$$

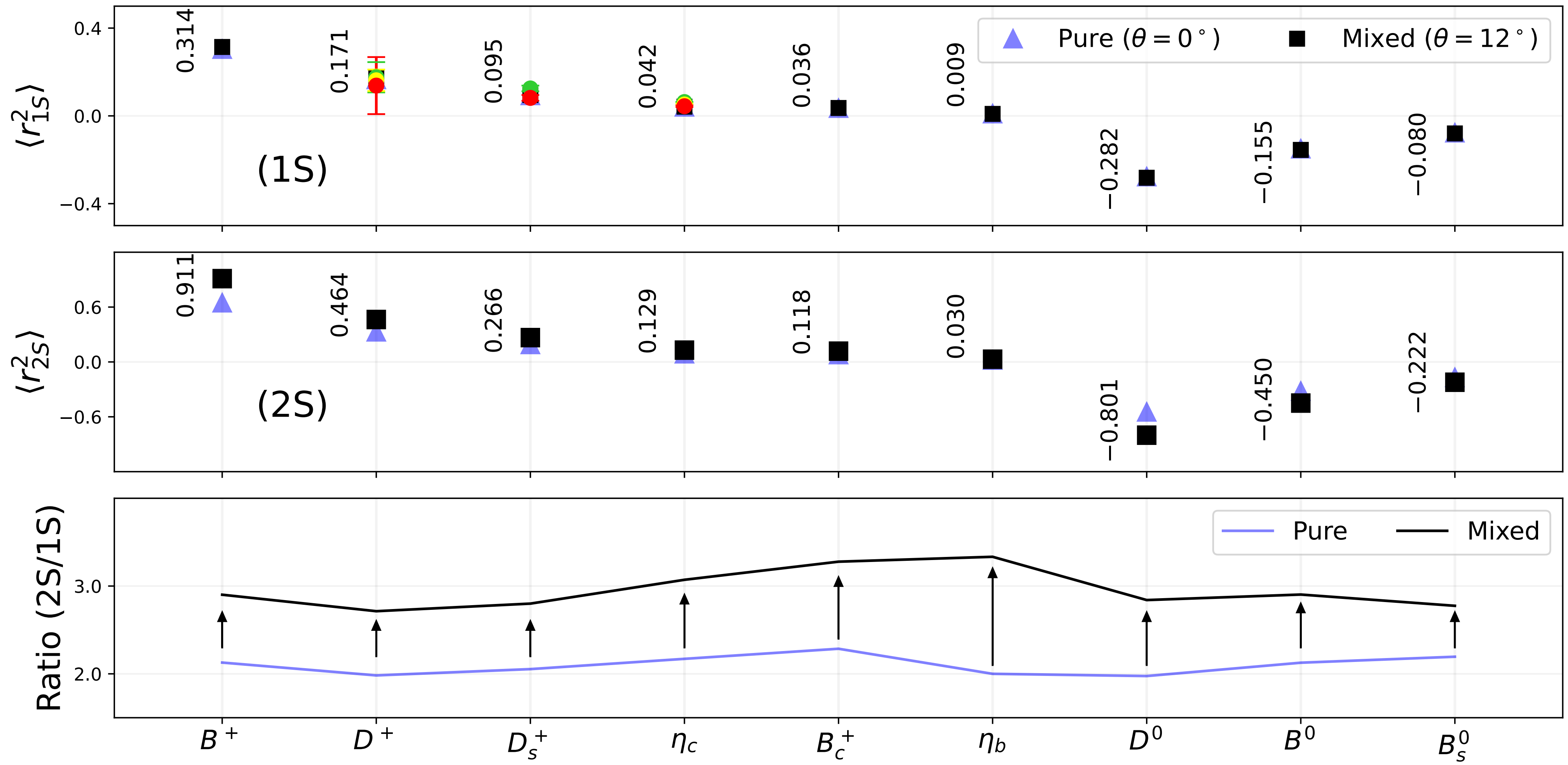
$$M \to M_0$$

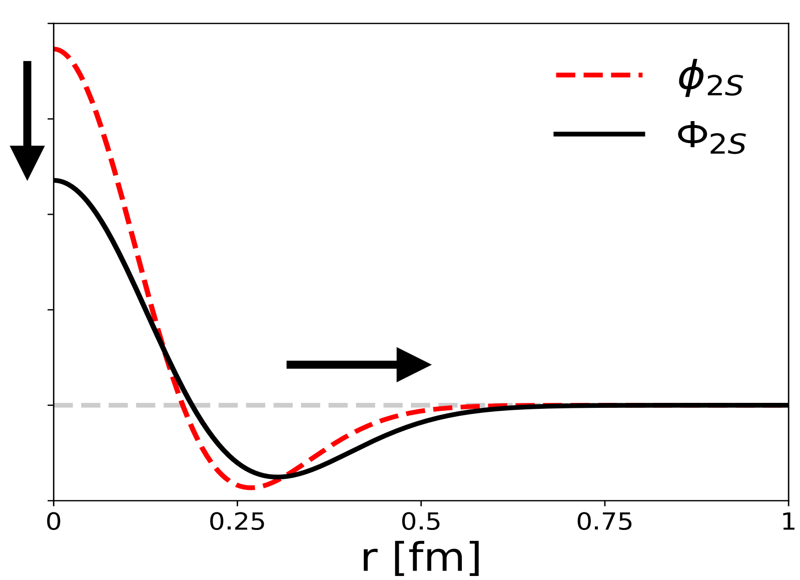
# E.m. Form factor



# Charge Radius

$$\langle r^2 \rangle = -6 \frac{dF(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$



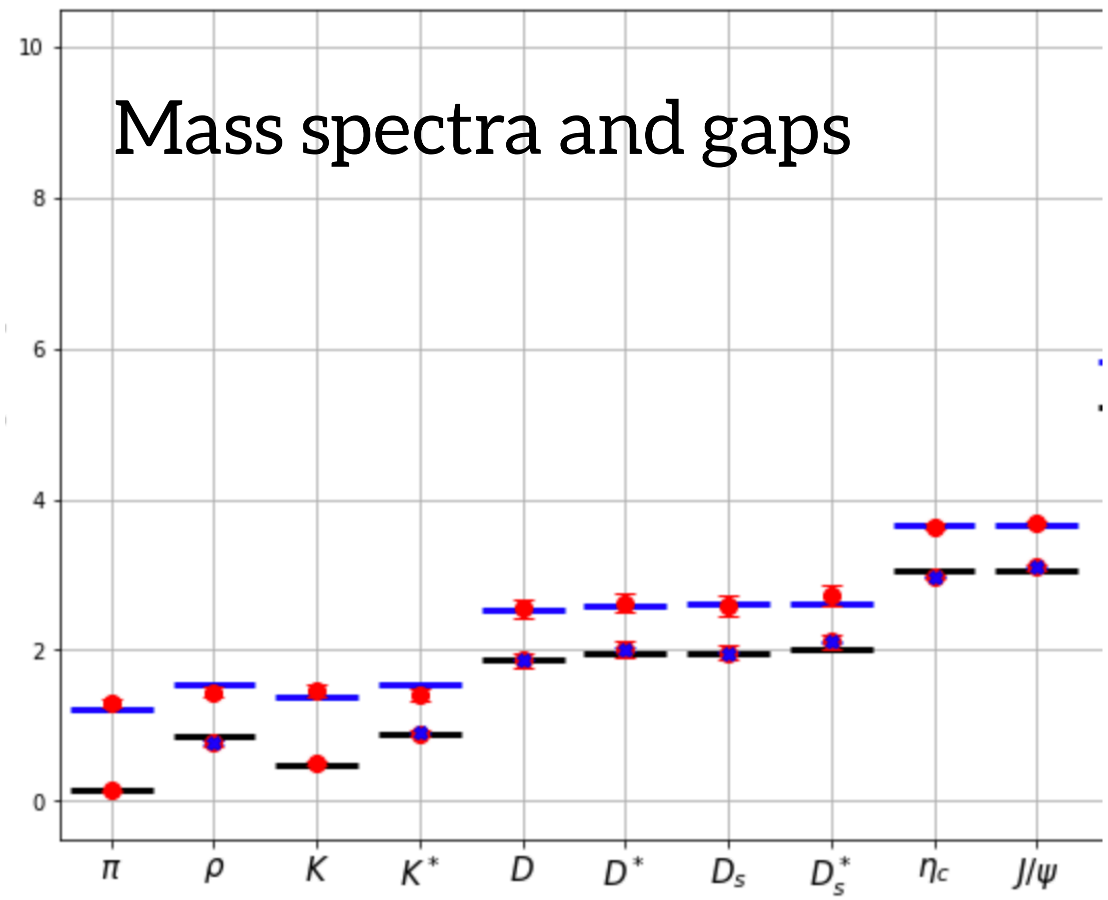



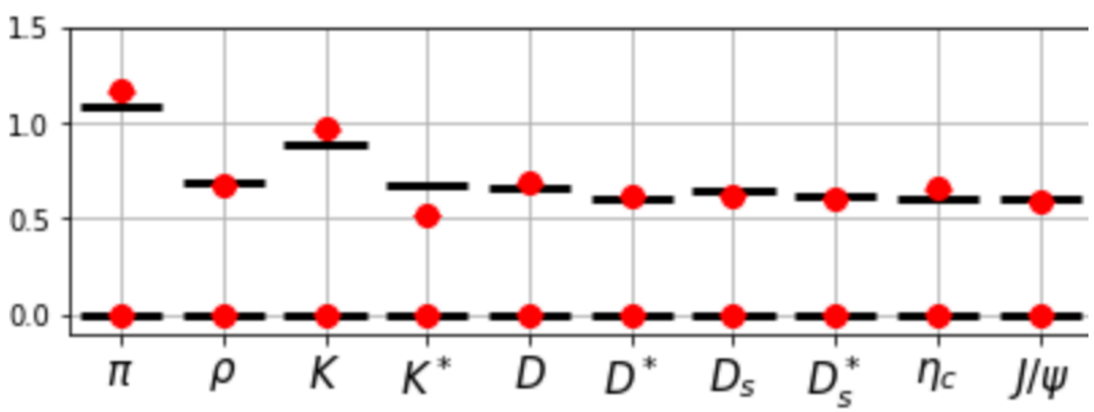

<> With mixing:

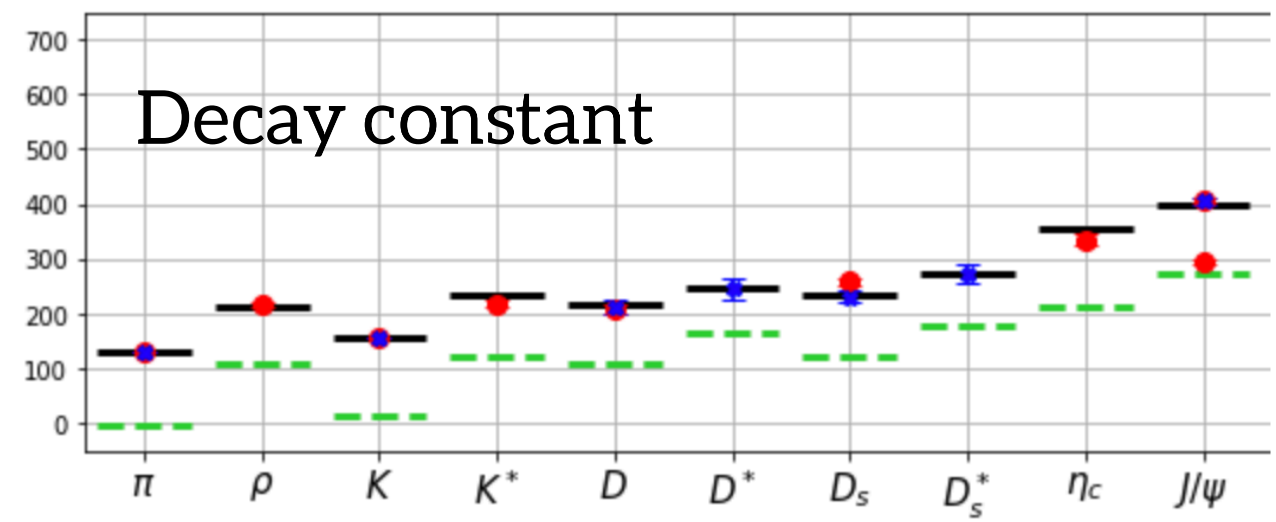
—> The 2S decay constants decrease

—> The 2S charge radii increase

# Light meson - Preliminary result




<> Mass gap
—> pion and kaon ~ 1 GeV

<> Decay constant
—> pion and kaon ~ few MeV

<> Connection to chiral symmetry?

# Summary

- Several discoveries of **radial excitations** of hadrons.
  —> need to understand their internal structure

- Similar mass gap: baryon & mesons
  —> Competing behavior: confinement vs coulomb
  —> Role of hyperfine int.

- Wave function of radial excitation:
  —> 2S HO needs some modification (mixing)
  —> Huge impact to the model prediction

- Future works of LFQM
  —> Various current component ($\mu = + , - , \perp$)
  —> Extension to baryon system
  —> Global analysis



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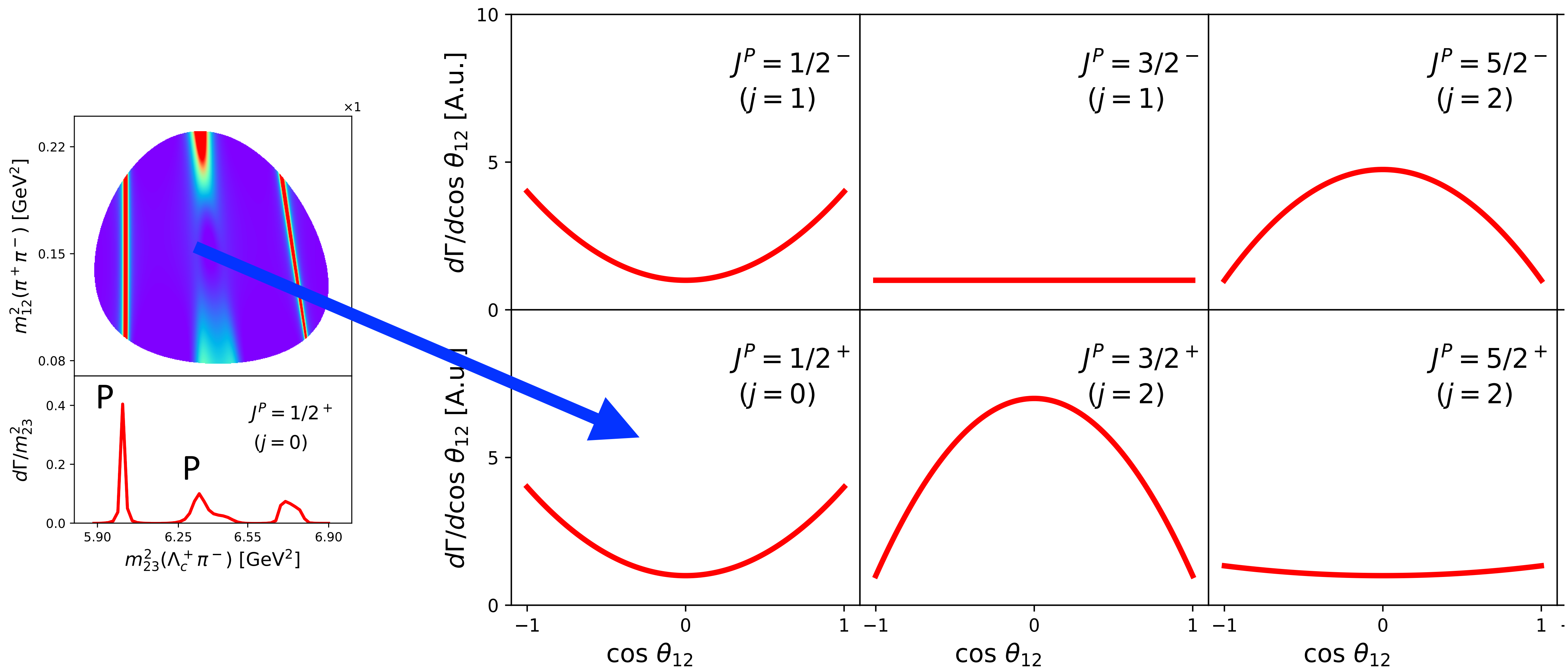
# Thank you very much

https://ajarifi.github.io

# Angular correlation: $\Sigma_c^*(3/2^+)$ band




$$W(\theta) \propto |A_{1/2}|^2\,(1 + 3\cos^2\theta) + |A_{3/2}|^2\,3\sin^2\theta$$

Dip　　　　　　　　Peak

$$\tilde{R} = \frac{|A_{3/2}|^2}{|A_{1/2}|^2}$$