Photo- and Electroproduction with JBW

Michael Döring for Jülich-Bonn-Washington (**JBW**) collaboration

With D. Rönchen, M. Mai *et al.*

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JÜLICH
FORSCHUNGSZENTRUM

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[several slides by
D. Rönchen and M. Mai]

Degrees of freedom: Quarks or hadrons, or both?

e.g.: Review by [Thiel, Afzal, Wunderlich, arXiv:2202.05055]

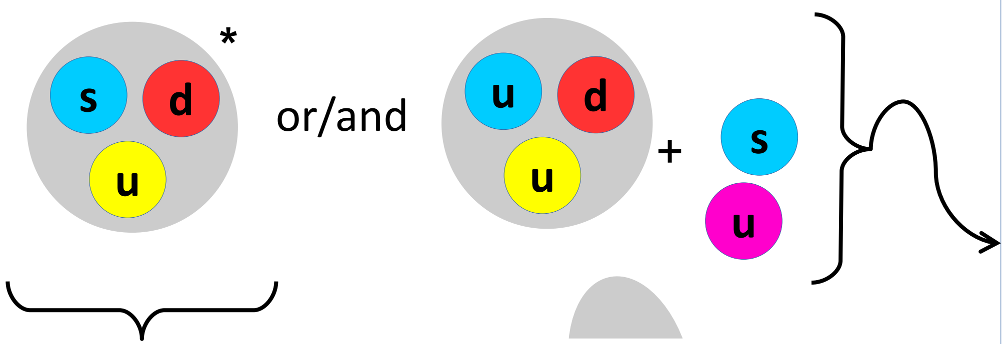
QCD at low energies → *mass generation* & *confinement*

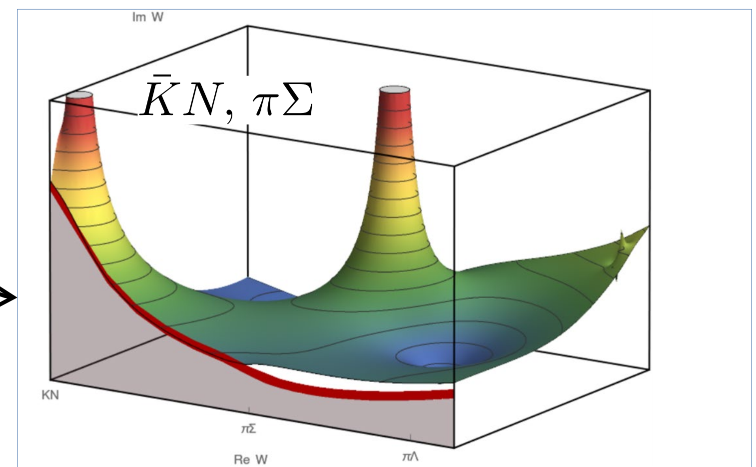
Non-perturbative dynamics → rich spectrum of excited states

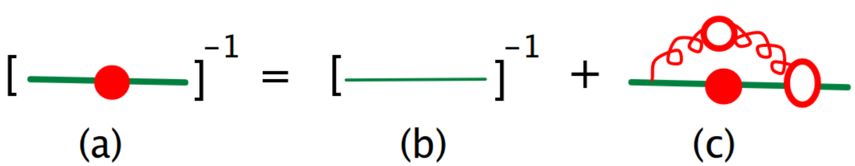
How many states are there? → missing resonance problem)

What are they? → 2-quark/3-quark, hadron molecules, …

$\Lambda(1405)$

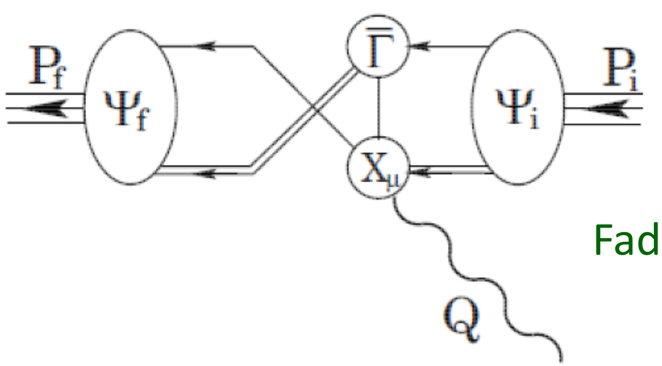






Hadronic molecule

Review of the (1405), M. Mai, 2010.00056 [nucl-th]

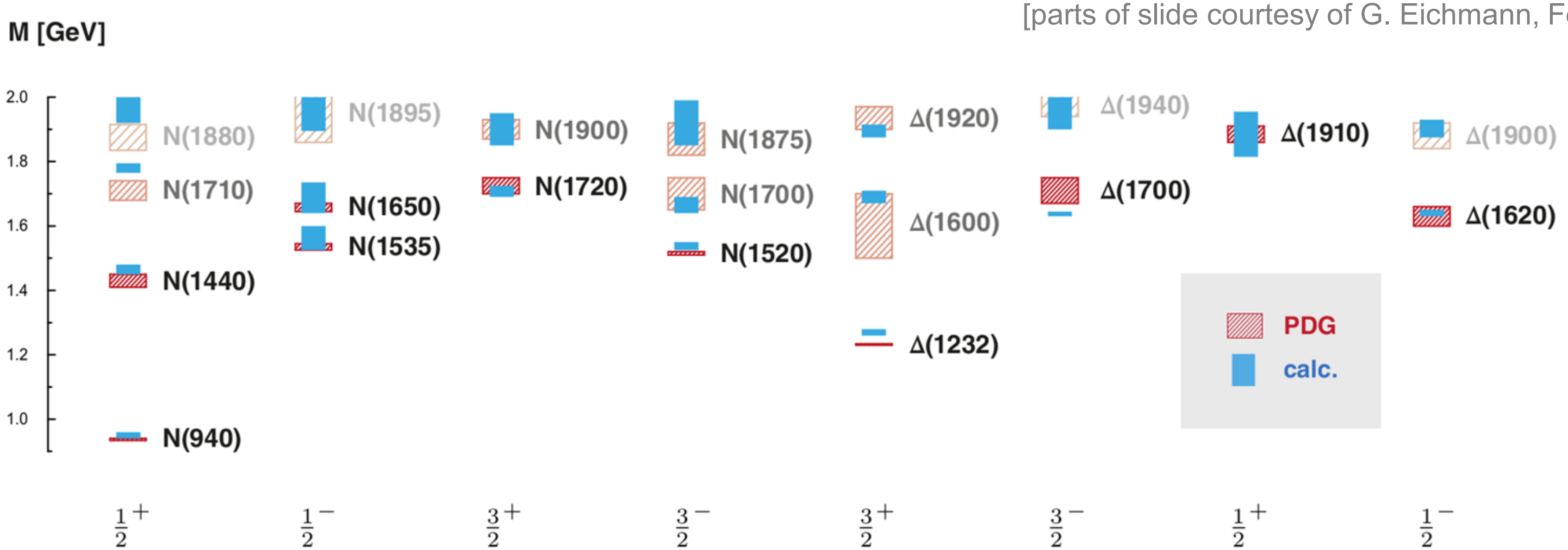


Faddeev Eq. / DSE (Binosi, Cloet, Chang, Roberts,…);

Results in dynamical quark picture

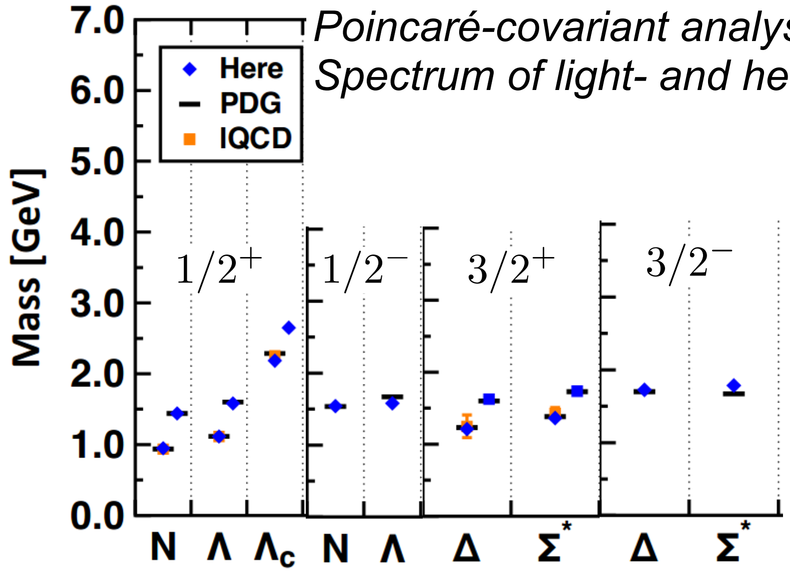
Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

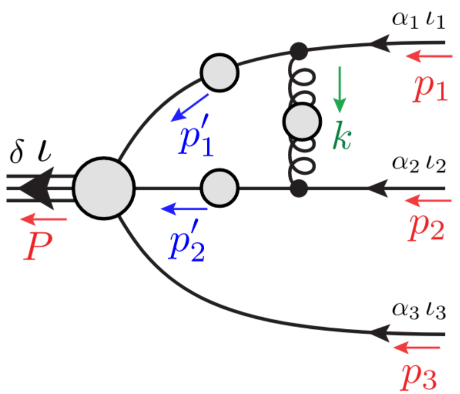
[parts of slide courtesy of G. Eichmann, Few Body 2018]



Poincaré-covariant analysis of heavy-quark baryons, Qin, Roberts, Schmidt, PRD (2018)

Spectrum of light- and heavy-baryons, Qin, Roberts, Schmidt, Few Body Syst. 60 (2019)

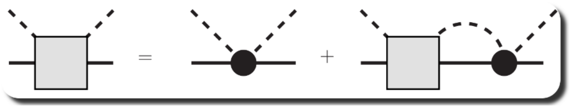




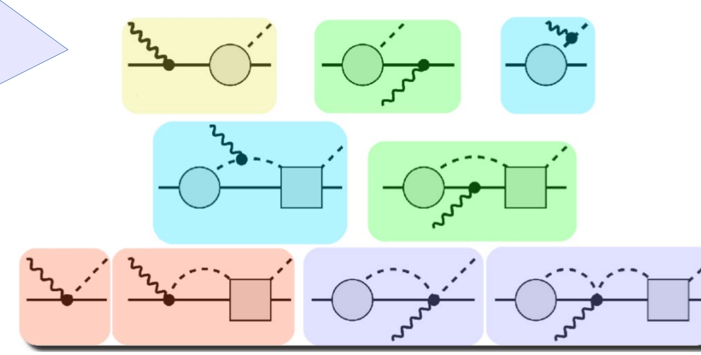
Using ONLY meson-baryon degrees of freedom (no explicit quark dynamics):

Manifestly gauge invariant approach based on full BSE solution

[Ruic, M. Mai, U.-G. Meissner PLB 704 (2011)]



Gauge invariance

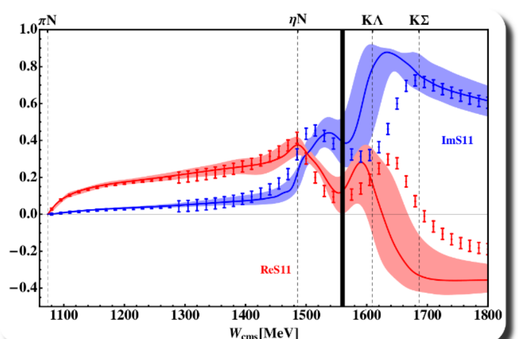


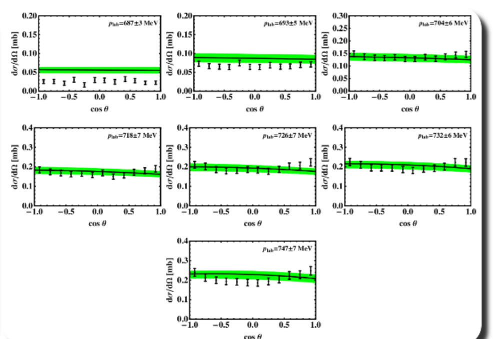
Fit

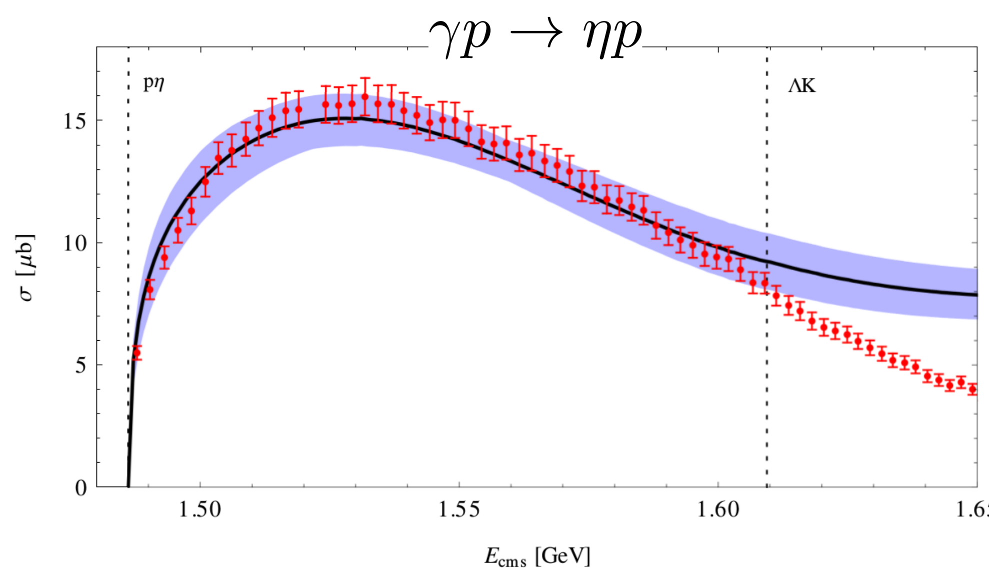
- Exact unitary meson-baryon scattering amplitude T with parameters, fixed to reproduce:
  - πN-partial wave S_{11} and S_{31} for $\sqrt{s} < 1560$ MeV

    Arndt et al. (2012)
  - $\pi^- p \to \eta n$ differential cross sections

    Prakhov et al. (2005)

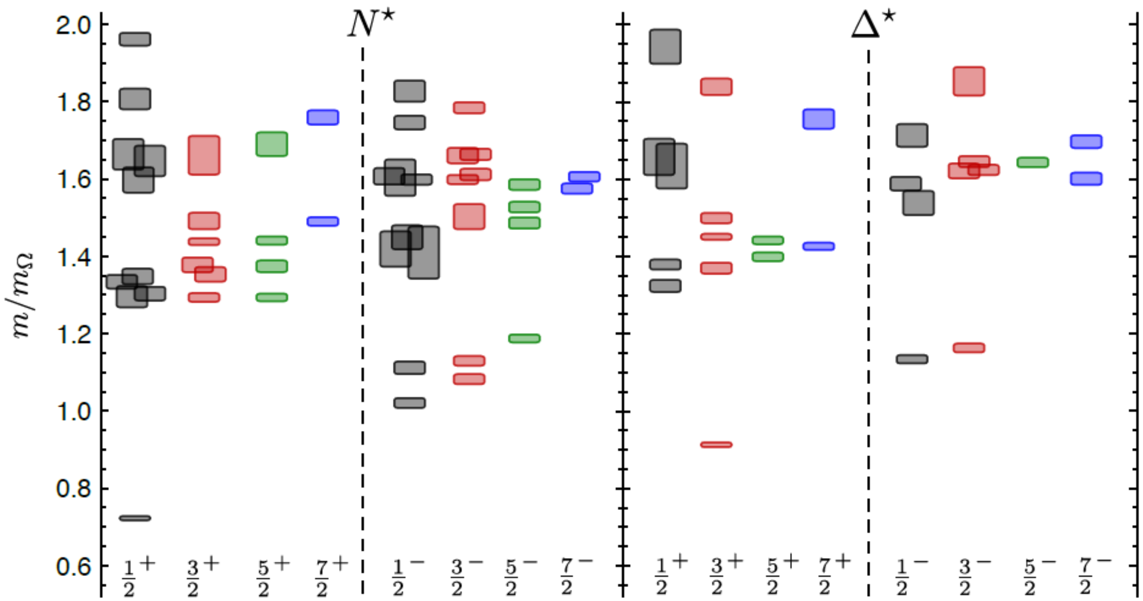




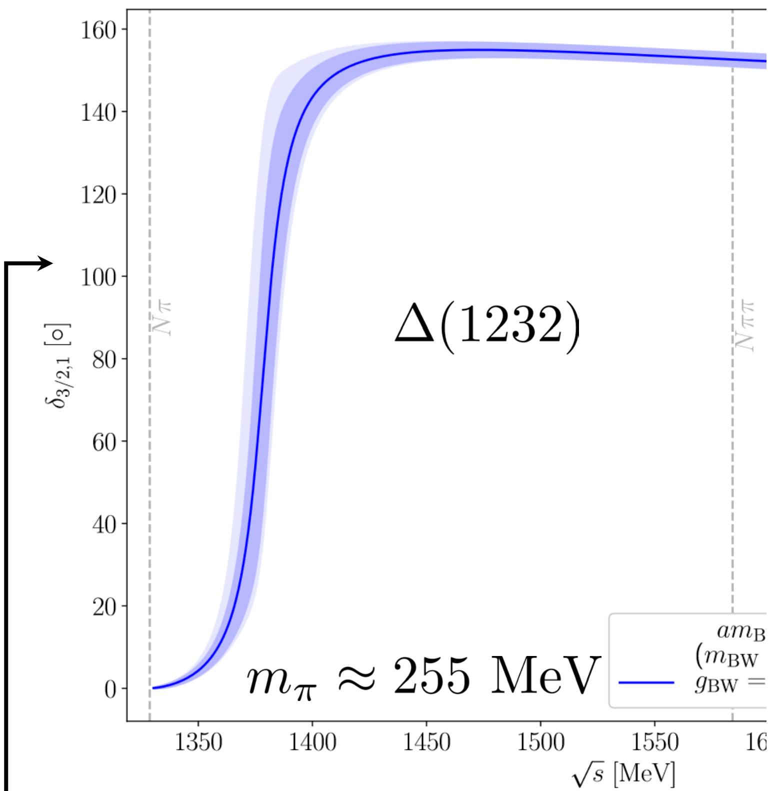


→ Making the "Missing resonance problem" worse ?!

Lattice QCD for excited baryons



$m_\pi = 396$ MeV [Edwards *et al.*, Phys.Rev. D84 (2011)]



[G. Silvi et. al., arXiv: 2101.00689]

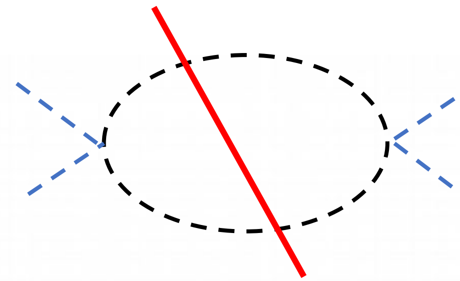
- Pioneering spectroscopic calculations
- Information on existence, width & properties of resonances requires
  - Meson-baryon interpolating operators
  - Detailed finite-volume analysis

How about $\pi\pi N$?
Roper resonance?

Analyticity of the scattering amplitude in different approaches

Cuts and Unitarity

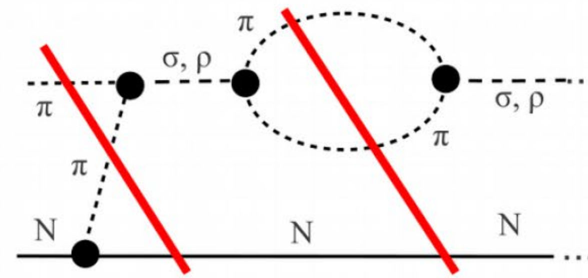
$$S = \mathbb{1} + iT$$



2-body

Unitarity: $SS^\dagger = 1 \Leftrightarrow -i(T - T^\dagger) = T\,T^\dagger$

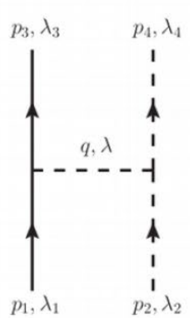
- 3-body unitarity:
  discontinuities from t-channel exchanges
  $\rightarrow$ Meson exchange from requirements of the S-matrix



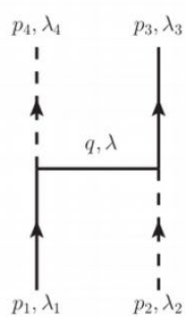
3-body

Other cuts

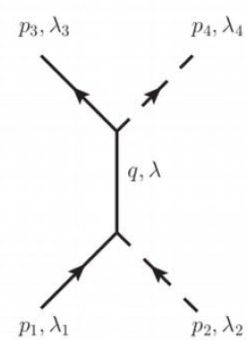
- to approximate left-hand cut $\rightarrow$ Baryon u-channel exchange
- σ, ρ exchanges from crossing plus analytic continuation.



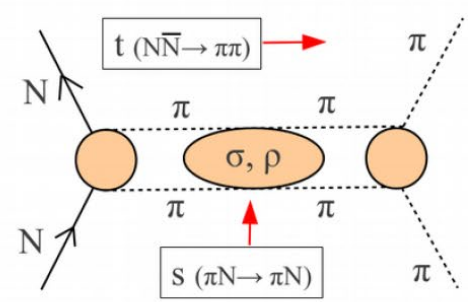
$\vec{q} = \vec{p}_1 - \vec{p}_3$



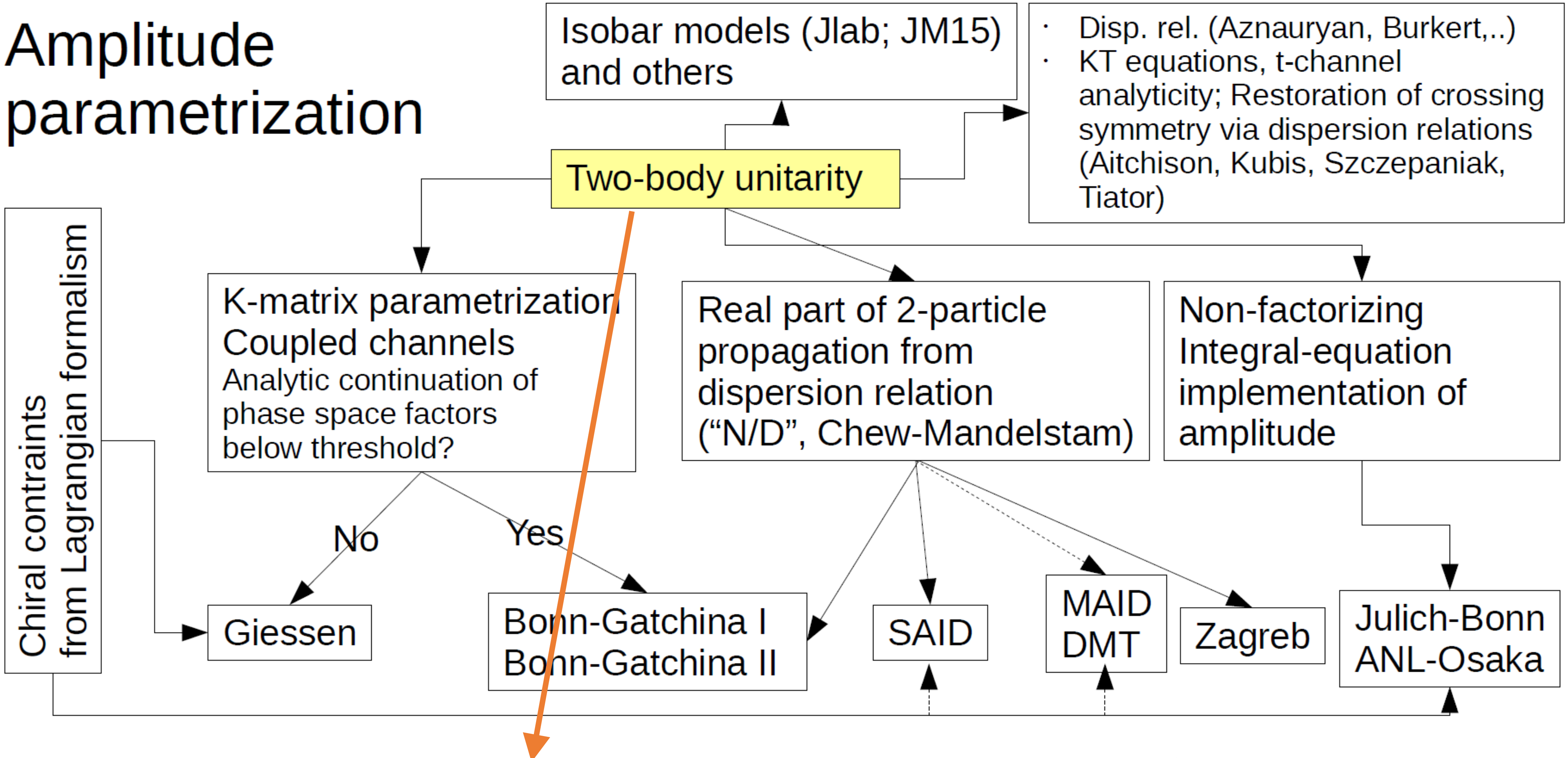
$\vec{q} = \vec{q}_1 - \vec{p}_4$

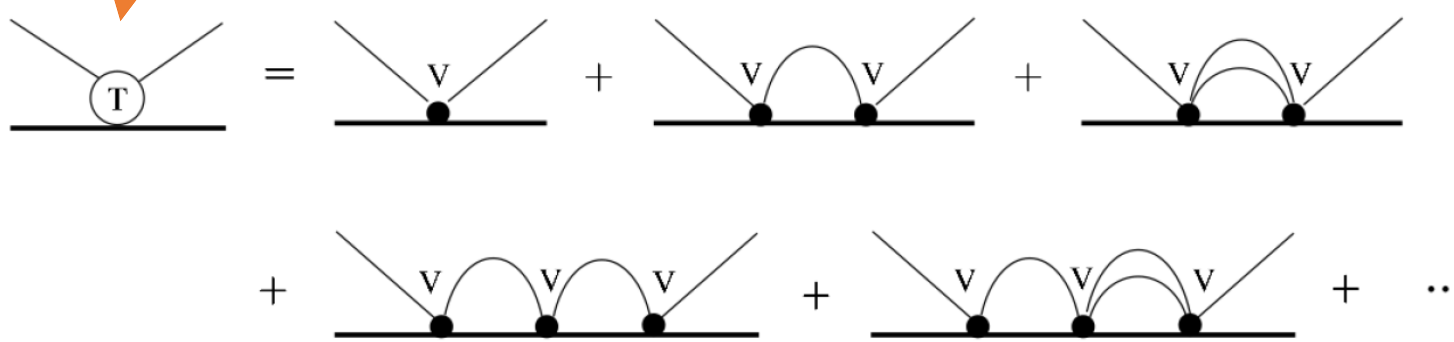


$\vec{q} = \vec{p}_1 + \vec{p}_2 = 0$



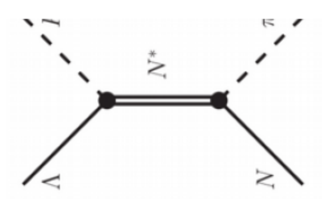
Amplitude parametrization





$$T = V + VGT,$$

Genuine Resonance:

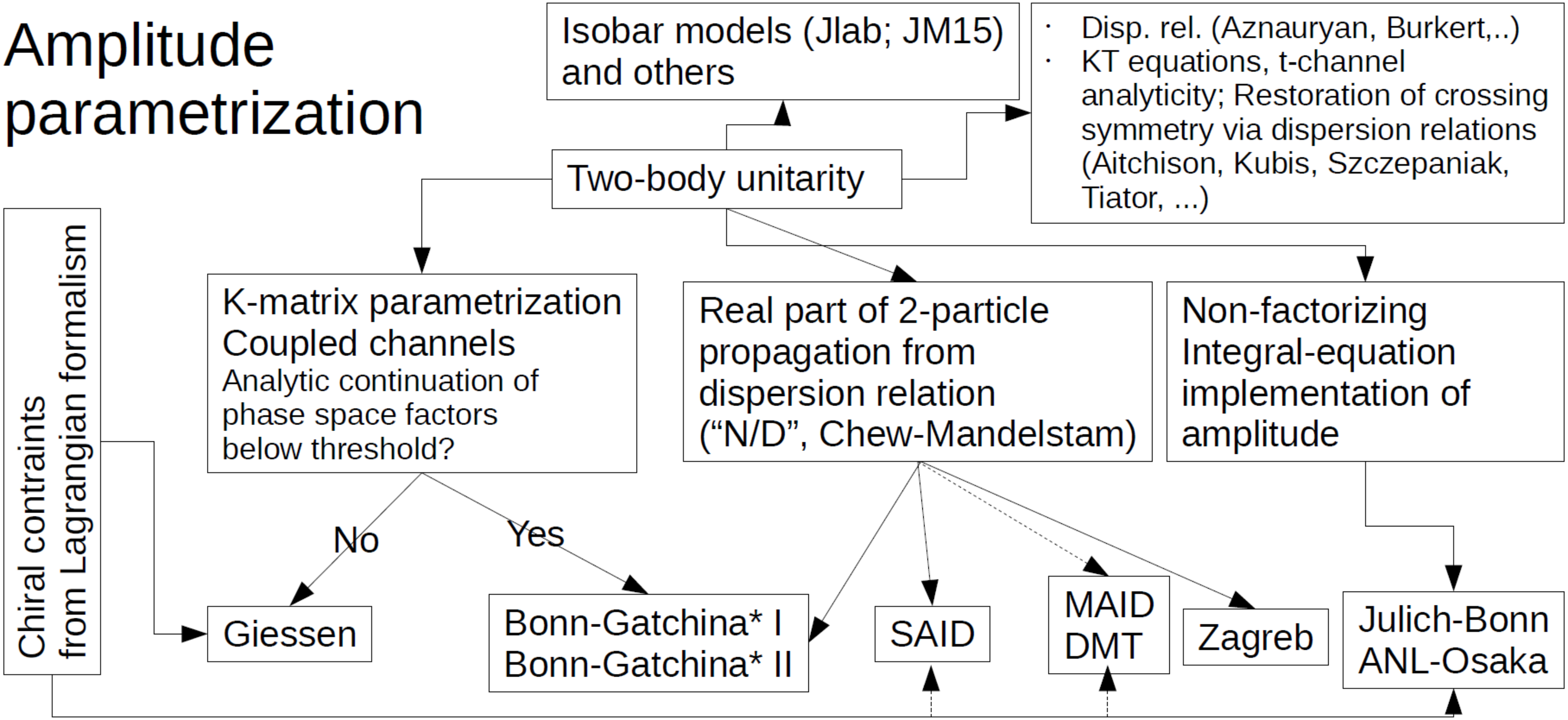


Unitarity loop G:

- Re G → 0: K-matrix
- V point-like: SAID

  Integral equation: Julich-Bonn, ANL-Osaka

Amplitude parametrization



Explicit resonance Terms?	Yes	Yes	No	(Yes)	Yes	Yes/No
Analyticity (math.)	No	Yes	Yes	Yes	Yes	Yes
Analyticity (disp.)	No	No/Yes	Yes			Yes
Effective $\pi\pi N$?			Yes			
Three-body unitarity?			No			Yes

* largest set of analyzed reactions

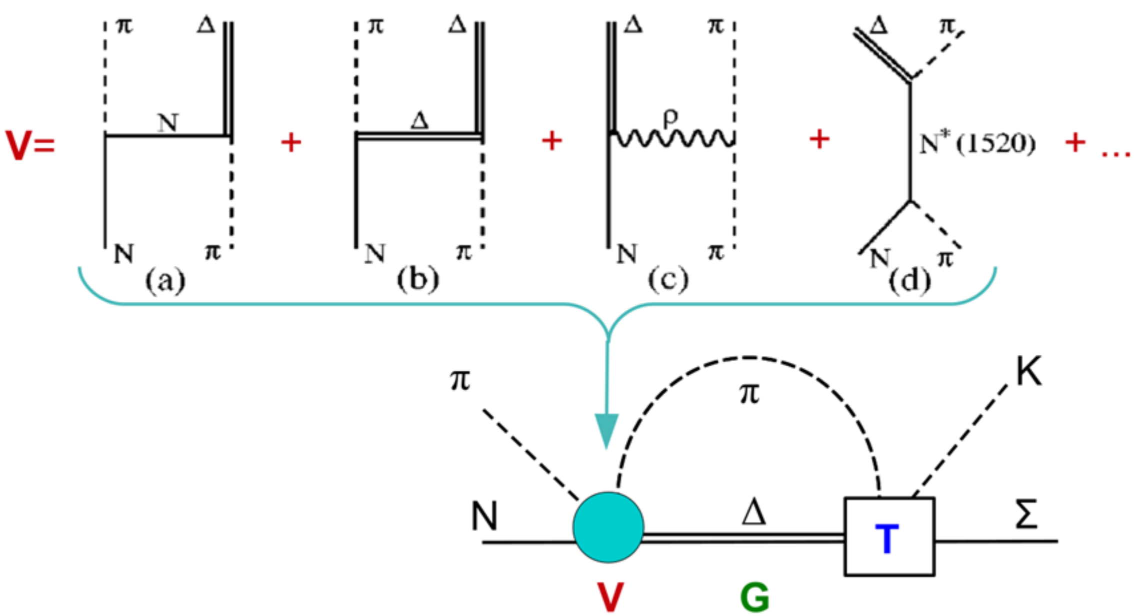
The Julich-Bonn Dynamical Coupled-Channel Approach

e.g. EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon}\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$



- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P

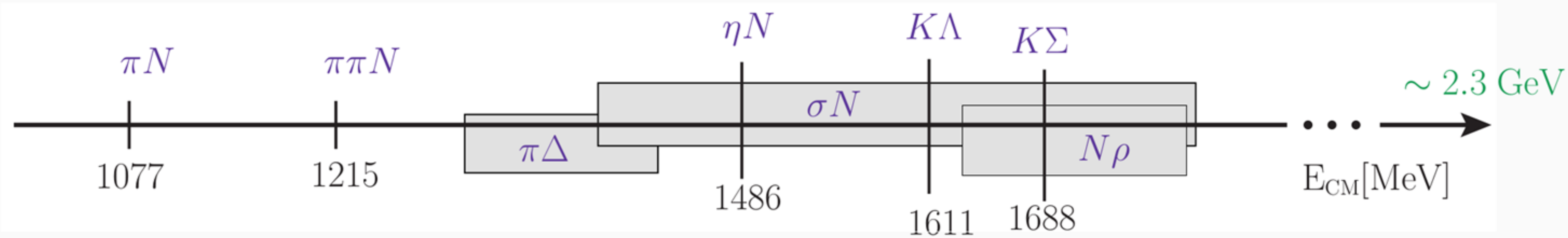
  genuine resonance states
- t- and u-channel: T^{NP}

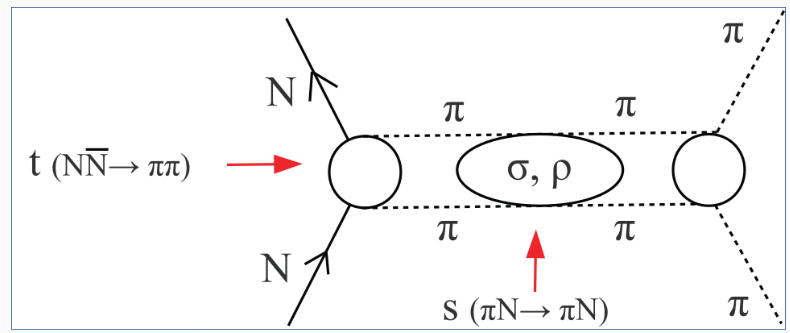
  dynamical generation of poles

  partial waves strongly correlated

JBW: Channels and Analytic Structure

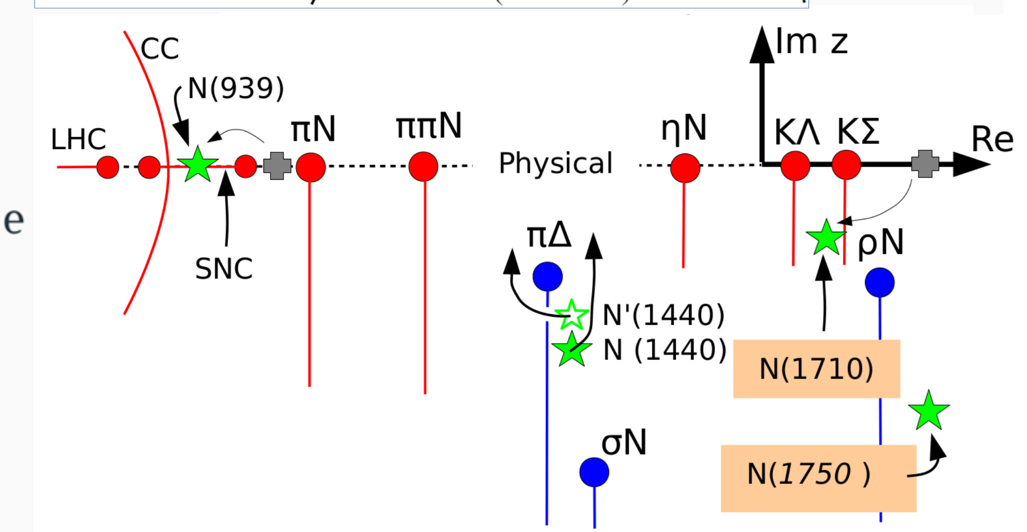
Channels included:





- (2-body) unitarity and analyticity respected
- 3-body $\pi\pi N$ channel:
  - parameterized effectively as $\pi\Delta$, σN, ρN
  - $\pi N/\pi\pi$ subsystems fit the respective phase shifts

  ↳ branch points move into complex plane



Single-meson photoproduction with JBW

A boundary condition for electroproduction analysis

e.g.: D. Ronchen et al., EPJA (2018), arXiv: 1801.10458

JBW: Photoproduction Data base

- $\pi N \to X$: $>$ **7,000** data points ($\pi N \to \pi N$: GW-SAID WI08 (ED solution))
- $\gamma N \to X$:

New JPARC data on pion-induced reactions coming!

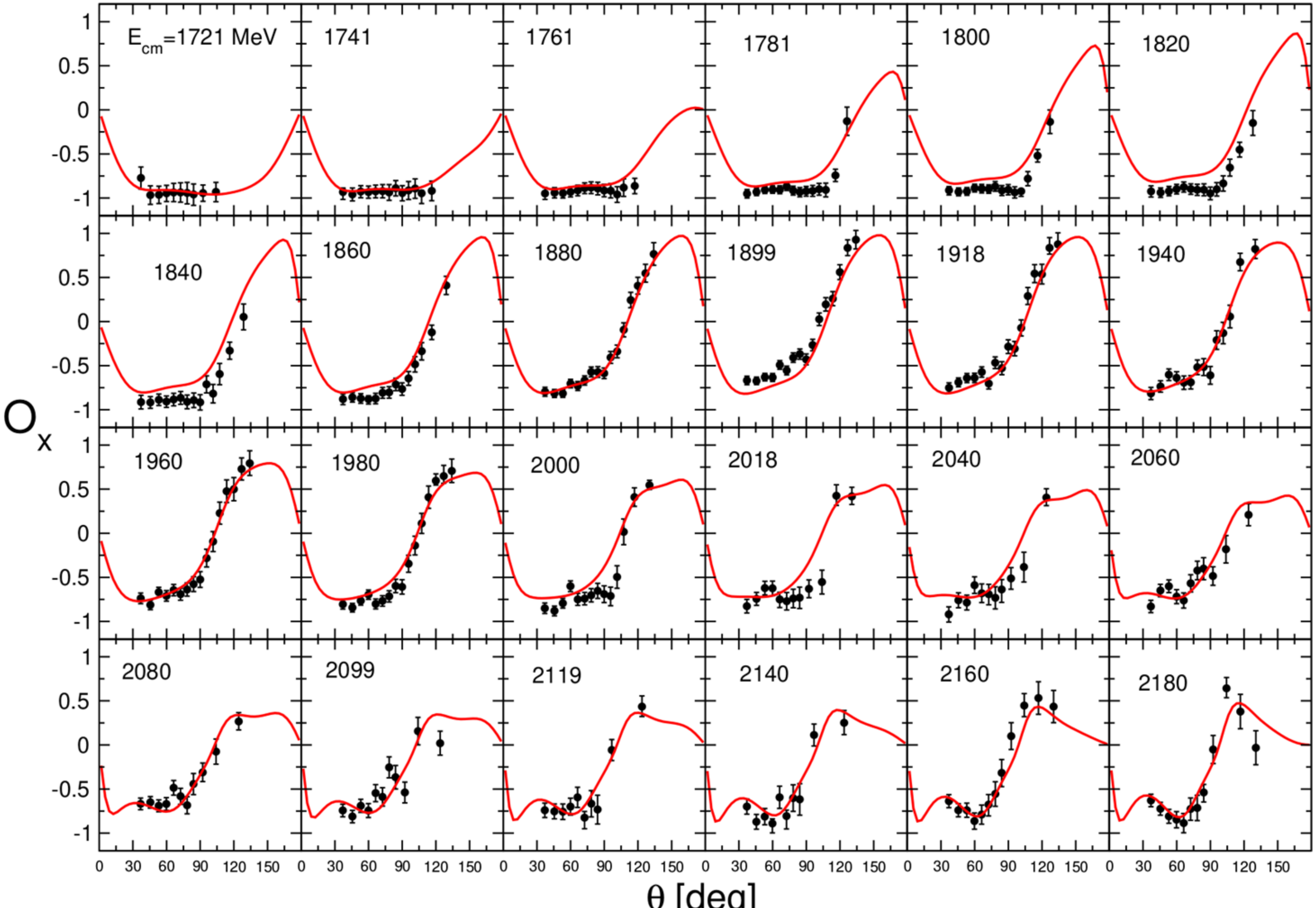
Reaction	Observables (# data points)	p./channel
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140), G (393), H (225), E (467), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	25,542
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231), G (86), H (128), E (903)	9,748
$\gamma p \to \eta p$	$d\sigma/d\Omega$ (9112), Σ (403), P (7), T (144), F (144), E (129)	9,939
$\boldsymbol{\gamma p \to K^+ \Lambda}$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383), $C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\boldsymbol{\gamma p \to K^+ \Sigma^0}$	$d\sigma/d\Omega$ (4271), P (422), Σ (280), T (127), $C_{x',z'}$ (188), $O_{x,z}$ (254)	5,542
$\boldsymbol{\gamma p \to K^0 \Sigma^+}$	$d\sigma/d\Omega$ (242), P (78)	320
	in total	**57,027**

A new web interface [https://jbw.phys.gwu.edu/]

Selected Fit Results (I)

- $\gamma p \to K^+ \Lambda$:

http://collaborations.fz-juelich.de/ikp/meson-baryon/main



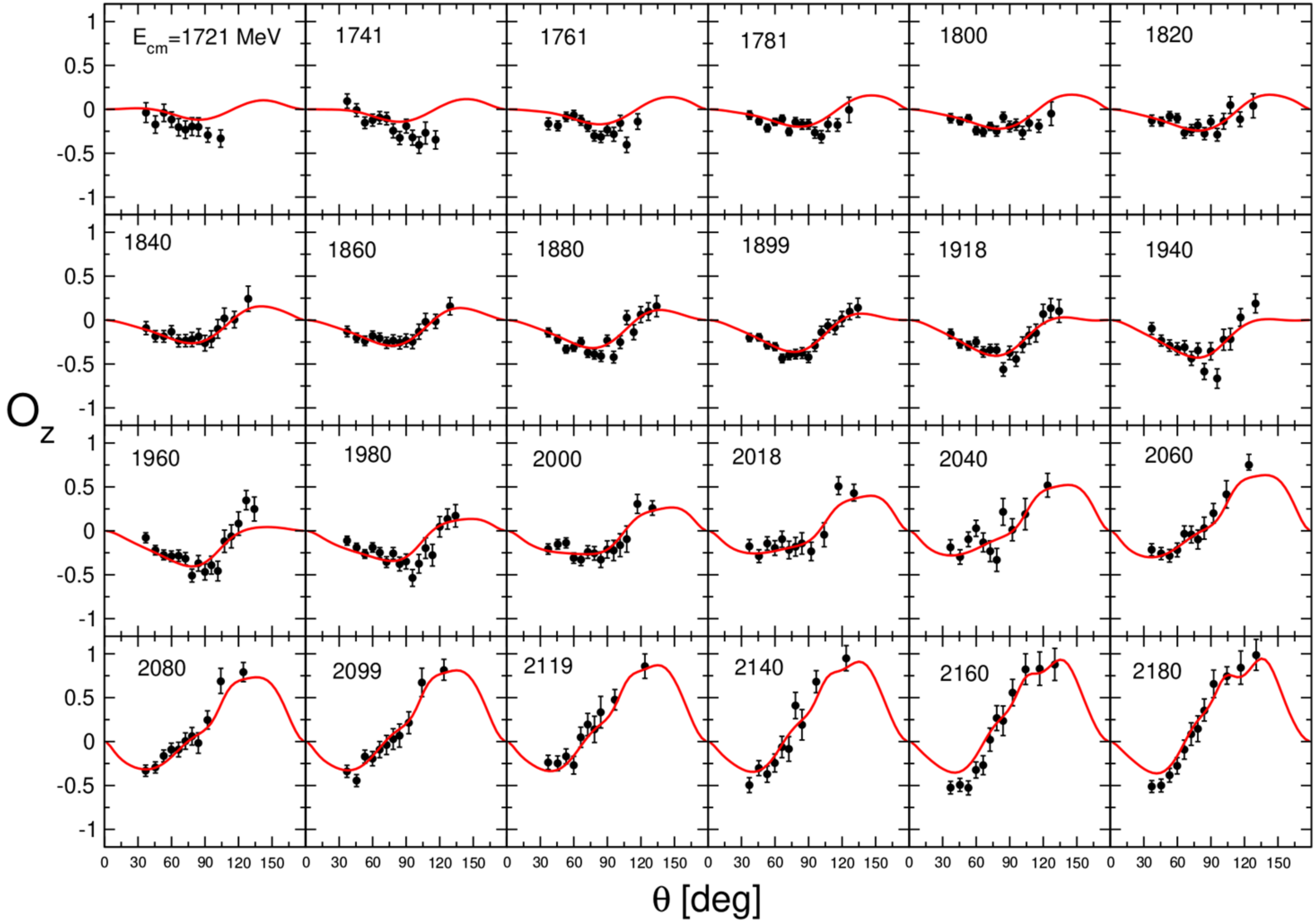
[D. Roenchen, M. D., U.-G. Meißner, EPJ A 54, 110 (2018)]

data: Paterson (CLAS) PRC 93, 065201 (2016), red line: fit JüBo2019

Selected Fit Results (II)

- $\gamma p \to K^+ \Lambda$:

http://collaborations.fz-juelich.de/ikp/meson-baryon/main



[D. Roenchen, M. D., U.-G. Meißner, EPJ A 54, 110 (2018)]

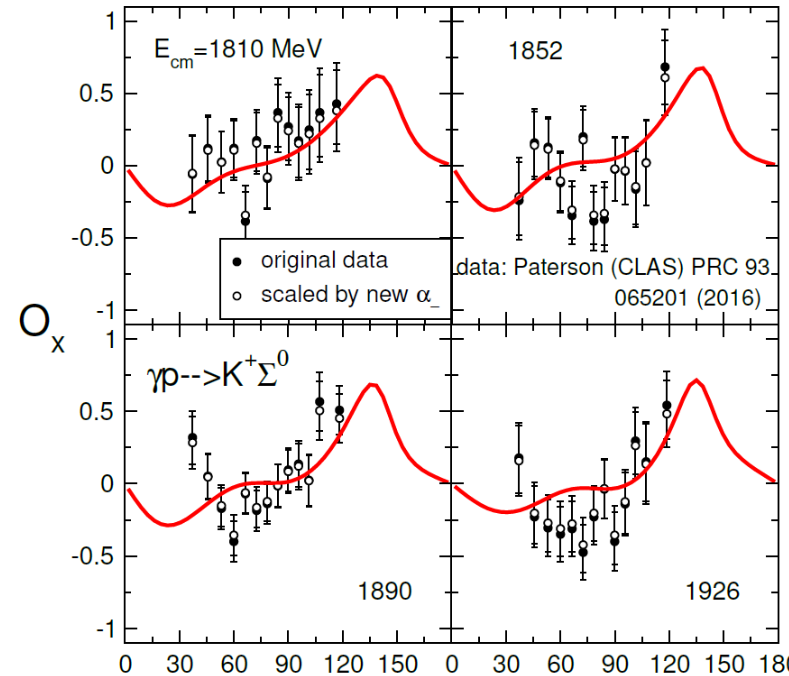
data: Paterson (CLAS) PRC 93, 065201 (2016), red line: fit JüBo2019

Extension to $K\Sigma$ photoproduction

[D. Roenchen et al., preliminary]

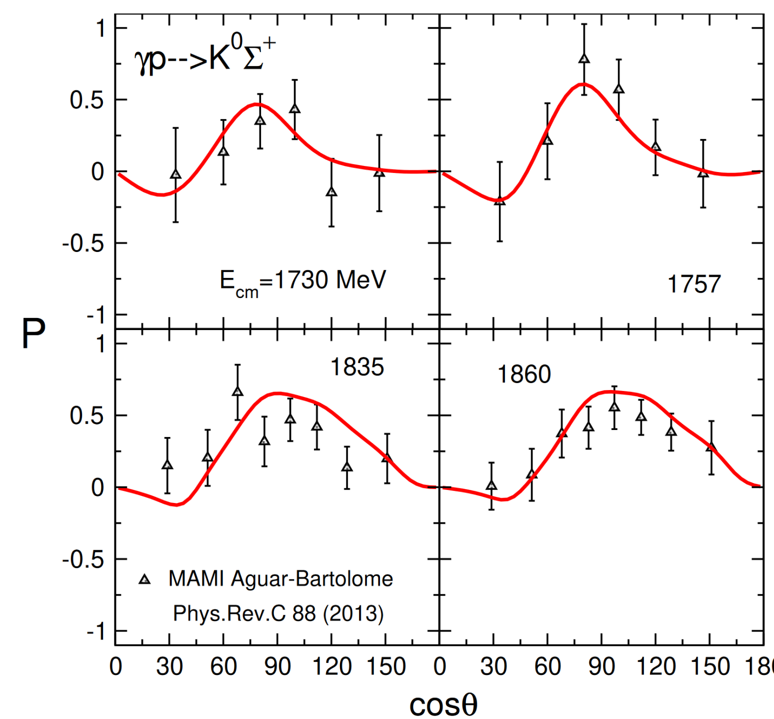
Simultaneous analysis of $\pi N \to \pi N, \eta N, K\Lambda, K\Sigma$ and $\gamma p \to \pi N, \eta N, K\Lambda, K\Sigma$

- $>$ 67,000 data points in total
  - $\gamma p \to K^+\Sigma^0$: $d\sigma/d\Omega$, P, Σ, T, $C_{x',z'}$, $O_{x,z}$ = 5,652
  - $\gamma p \to K^0\Sigma^+$: $d\sigma/d\Omega$, P = 448
- polarizations scaled by new Λ decay constant α_- (Ireland PRL 123 (2019), 182301), if applicable
- χ^2 minimization with MINUIT on JURECA [Jülich Supercomputing Centre, JURECA: JLSRF 2, A62 (2016)]



Resonance analysis:

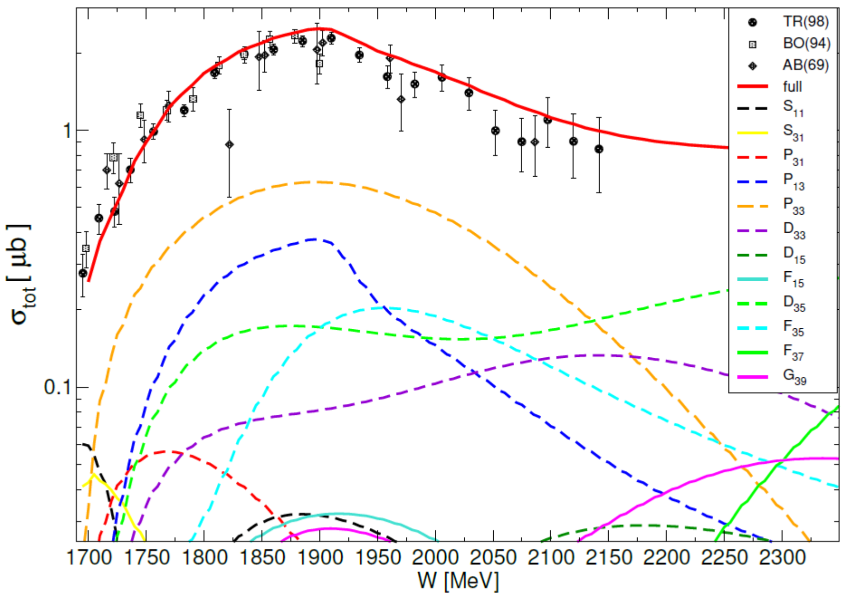
- all 4-star N and Δ states up to $J = 9/2$ are seen (exception: $N(1895)1/2^-$) + some states rated less than 4 stars
- no additional s-channel diagram, but indications for new dyn. gen. poles

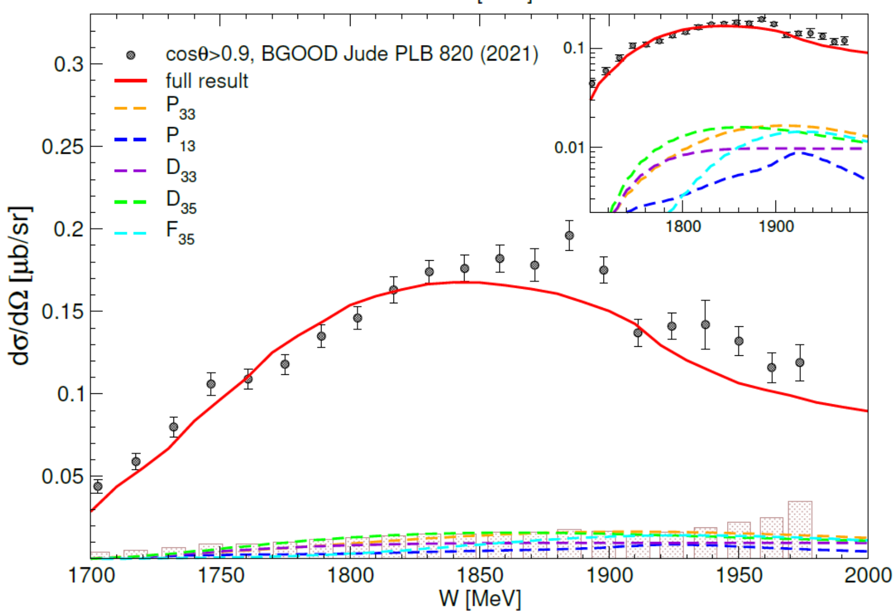


Resonances in $K\Sigma$ photoproduction

[D. Roenchen et al., preliminary]

$$\gamma p \to K^+ \Sigma^0$$





- dominant partial waves: $I = 3/2$

Exception: P_{13} partial wave ($I = 1/2$):

$N(1720)\ 3/2^+$ $\ast\ast\ast\ast$	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
2022	1726	185	5.9	82
2017	1689(4)	191(3)	0.6(0.4)	26(58)
PDG 2021	1675 ± 15	250^{+150}_{-100}	—	—

$N(1900)\ 3/2^+$ $\ast\ast\ast\ast$	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
2022	1905	93	1.3	-40
2017	1923(2)	217(23)	10(7)	$-34(74)$
PDG 2021	1920 ± 20	150 ± 50	4 ± 2	110 ± 30

- drop in cross section ("cusp-like structure") due to $N(1900)3/2^+$

Pion and eta Electroproduction

A first step towards a coupled-channel photo- and electroproduction analysis

M. Mai et al., 2104.07312 [nucl-th], 2111.04774 (PRC)

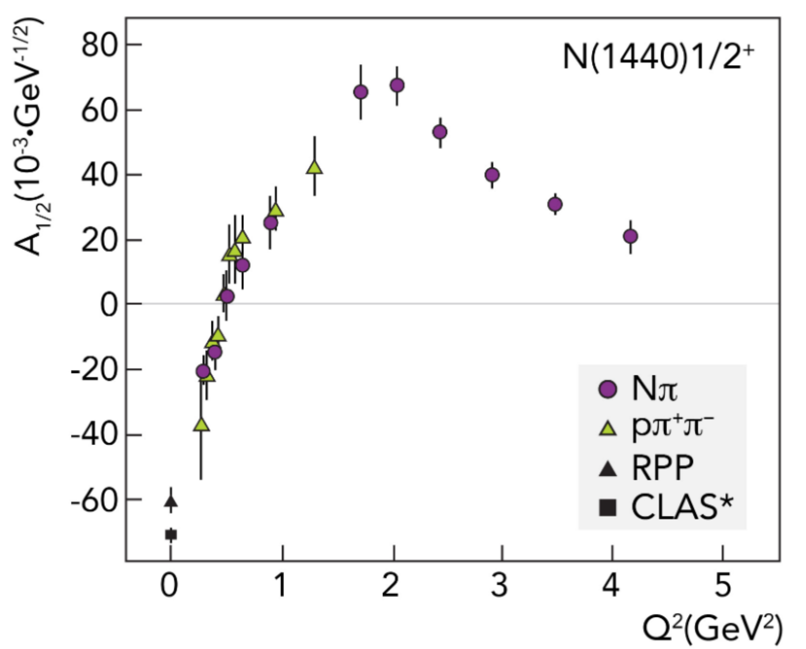
Single-meson electroproduction to reveal resonance structure

- **ANL-Osaka** PRC **80**, 025207 (2009), Few-Body Syst. 59, 24 (2018),...
- **Aznauryan, Burkert, Mokeev** et al., PRC **80**, 055203 (2009), Int. J. Mod. Phys. E22, 1330015 (2013),..
- **EtaMAID2018**, EPJA 54 (2018), 210
- **MAID2007**, EPJA 34 (2007) 69
- **SAID**, PiN Newsletter 16, 150 (2002)
- **Gent group** Phys. Rev. C 89, 065202 (2014),...

Highlights:
- Simultaneous description of pion photo- and electroproduction (MAID)
- Consistent extraction of the Roper form factor from single and double pion electroproduction
- New resonance in electroproduction claimed

  Mokeev et al., PLB (2020) 2004.13531 [nucl-ex]

Burkert, Roberts, 1710.02549



Needed: Coupled-channel electroproduction analysis

Take advantage of multi-channel approach
$\rightarrow$ analyze simultaneously final states $\pi N,\ \eta N,\ K\Lambda$

~10^5 pion electroproduction data; $\eta N,\ K\Lambda$:

Reaction	Observable	Q^2 [GeV]	W [GeV]	Ref.
$ep \rightarrow e'p'\eta$	$\sigma_U,\ \sigma_{LT},\ \sigma_{TT}$	$1.6 - 4.6$	$2.0 - 3.0$	[132]
	$\sigma_U,\ \sigma_{LT},\ \sigma_{TT}$	$0.13 - 3.3$	$1.5 - 2.3$	[137]
	$d\sigma/d\Omega$	$0.25 - 1.5$	$1.5 - 1.86$	[138]
$ep \rightarrow e'K^+\Lambda$	P_N^0	$0.8 - 3.2$	$1.6 - 2.7$	[139]
	$\sigma_U,\ \sigma_{LT},\ \sigma_{TT},\ \sigma_{LT'}$	$1.4 - 3.9$	$1.6 - 2.6$	[140]
	$P'_x,\ P'_z$	$0.7 - 5.4$	$1.6 - 2.6$	[141]
	$\sigma_T,\ \sigma_L,\ \sigma_{LT},\ \sigma_{TT}$	$0.5 - 2.8$	$1.6 - 2.4$	[142]
	$P'_x,\ P'_z$	$0.3 - 1.5$	$1.6 - 2.15$	[143]

Table 1: Overview of ηp and $K^+\Lambda$ electroproduction data measured at CLAS for different photon virtualities Q^2 and total energy W. Based on material provided by courtesy of D. Carman (JLab) and I. Strakovsky (GW).

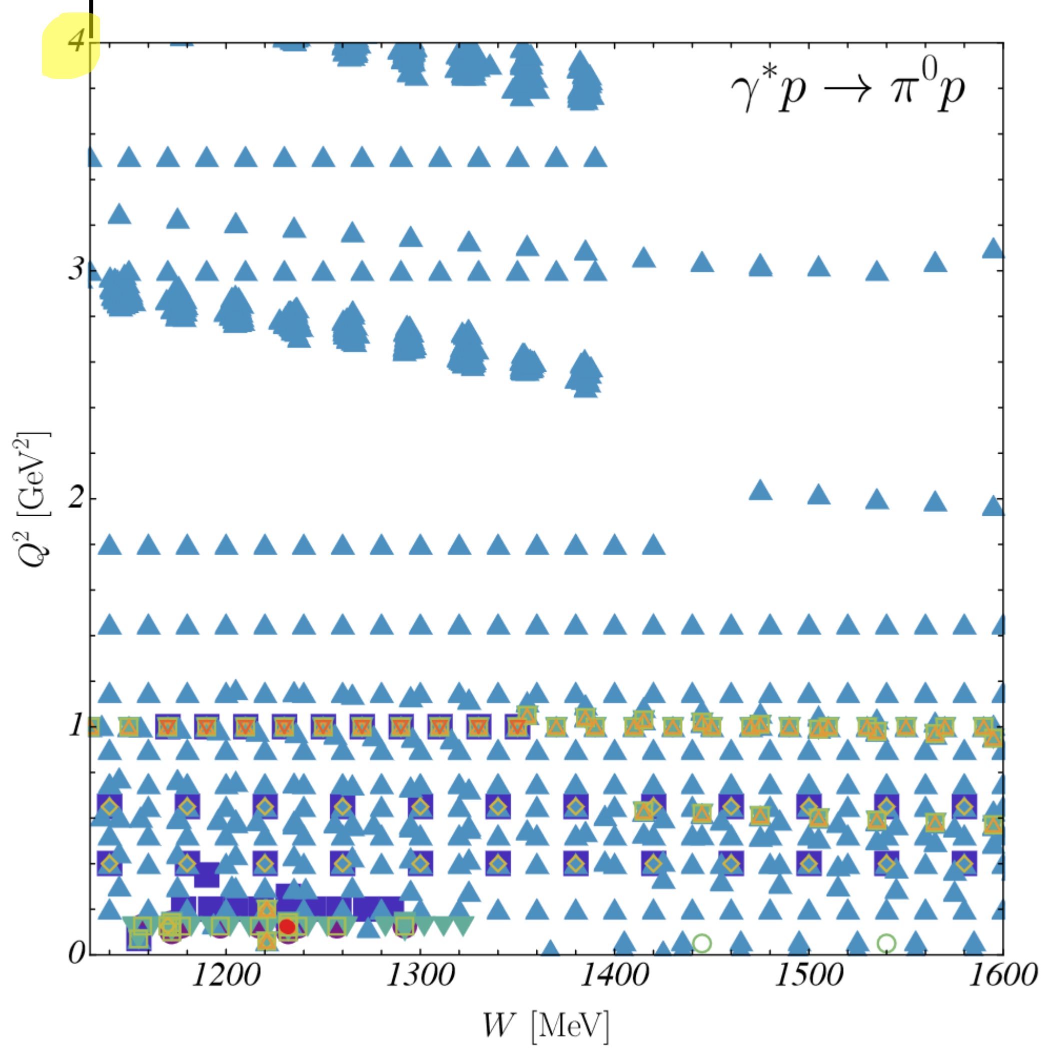
- Many of these (and similar) data await analysis.
- Many more data to emerge at Jlab ($Q^2 = 5 - 12\ \mathrm{Gev}^2$)

  e.g.: Carman, Joo, Mokeev, Few Body Syst. 61, 29 (2020)

- Approved Jlab experiments to study
  - Higher-lying nucleon resonances
  - Hybrid baryons
  - Transition regime between nonperturbative and perturbative regions

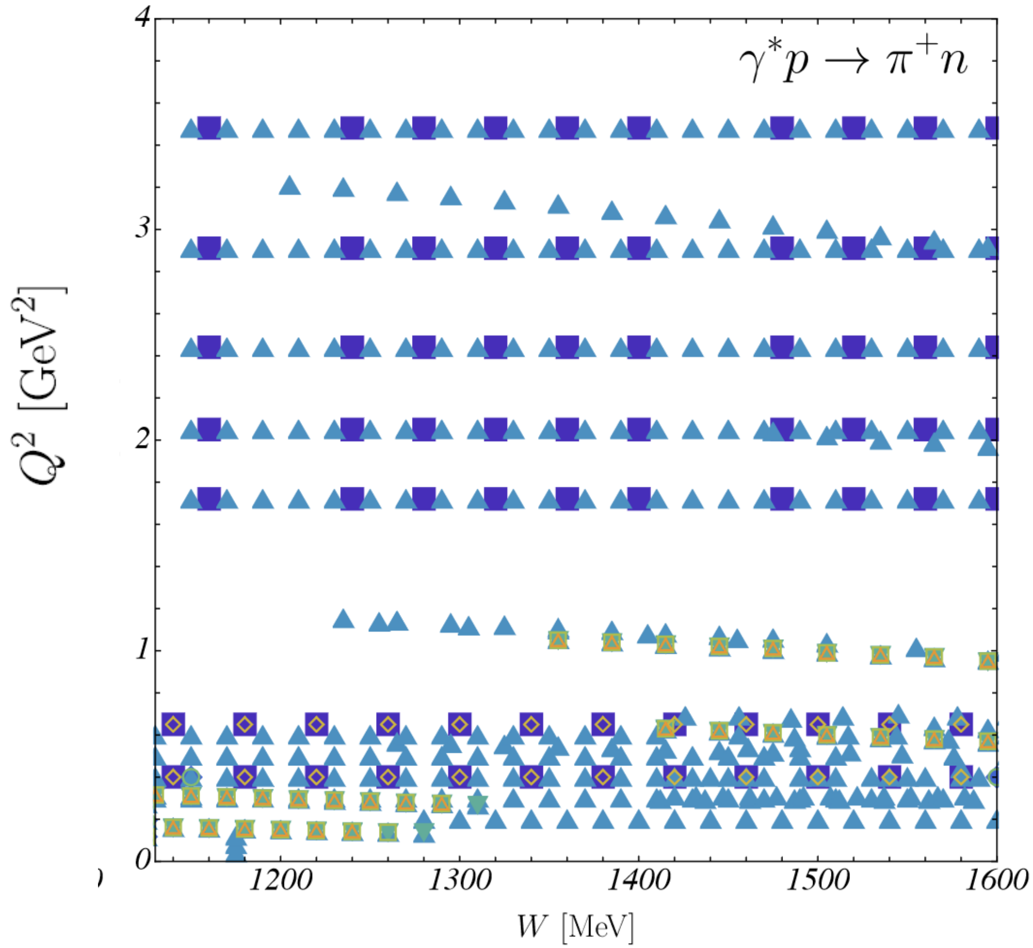
Pion Electroproduction – data base

Extended Q^2 up to 6 GeV^2 to check dependence of results



Type	N_{data}
ρ_{LT}	45
$\rho_{LT'}$	2644
σ_L	–
$d\sigma/d\Omega$	39942
$\sigma_T + \epsilon\sigma_L$	318
σ_T	10
σ_{LT}	312
$\sigma_{LT'}$	198
σ_{TT}	266
K_{D1}	1527
P_Y	2

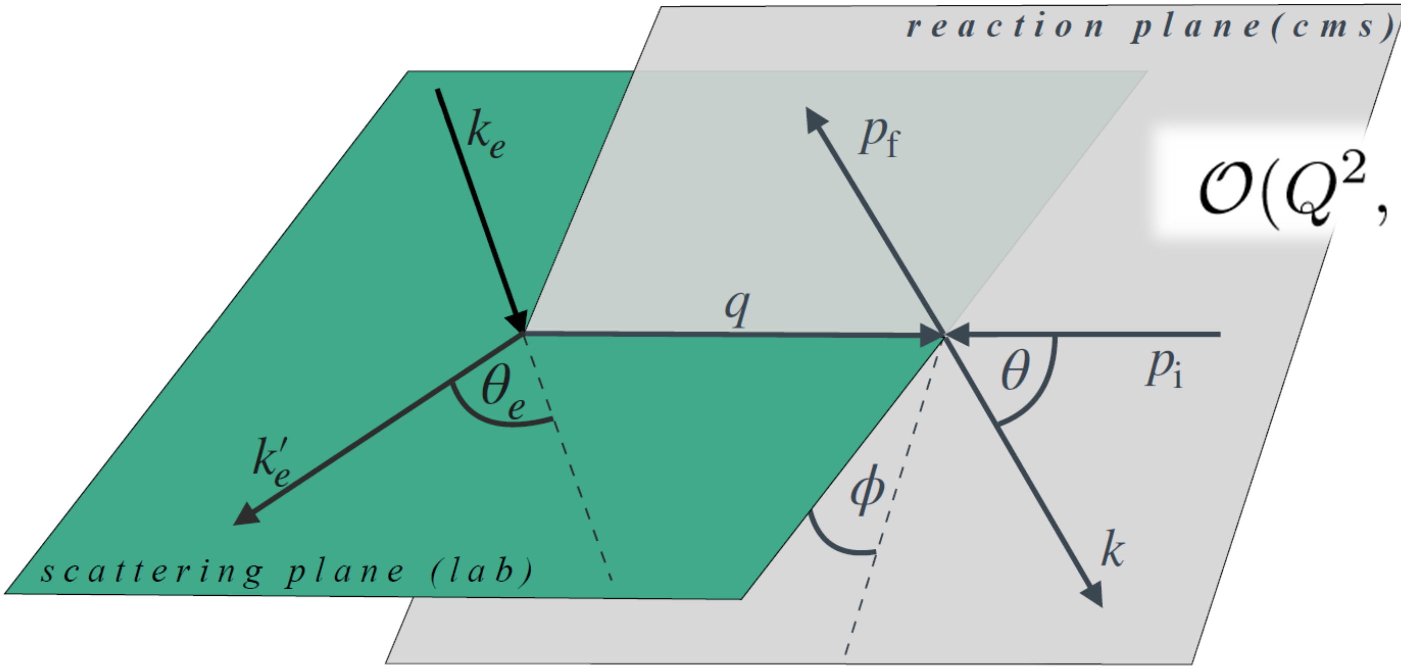
Pion Electroproduction – data base



	Type	N_{data}
●	ρ_{LT}	–
■	$\rho_{LT'}$	4354
◆	σ_L	2
▲	$d\sigma/d\Omega$	32813
▼	$\sigma_T + \epsilon\sigma_L$	144
○	σ_T	2
□	σ_{LT}	106
◇	$\sigma_{LT'}$	192
△	σ_{TT}	91
▽	K_{D1}	–
●	P_Y	–

- Data base grown over decades with recent input mostly by CLAS, MAMI.
- Far from complete: Kinematic gaps & consistency issues. Need to combine information from different (W, Q^2) regions
- Need to combine information from simultaneous analysis of different final states ($\pi N/\eta N/K Y/\pi \pi N$,….) to extract resonance helicity couplings

Kinematics



$$\mathcal{O}(Q^2, W, \phi, \theta, \epsilon)$$

$$\frac{d\sigma}{d\Omega_f dE_f d\Omega} = \left(\frac{\alpha}{2\pi^2}\frac{E_f}{E_i}\frac{q_L}{Q^2}\frac{1}{1-\epsilon}\right)\frac{d\sigma^v}{d\Omega},$$

(Un)polarized differential cross section:

$$\frac{d\sigma^v}{d\Omega} = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi + \epsilon\sigma_{TT}\cos 2\phi + h\sqrt{2\epsilon(1-\epsilon)}\sigma_{LT'}\sin\phi$$

$$\epsilon = 1 + 2\frac{q_L^2}{Q^2}\tan^2\frac{\theta_e}{2}$$

Polarized Observables

- CLAS: Structure functions $\sigma_{LT'}$

  K. Joo et al. [CLAS], Phys. Rev. C 68 (2003),

  K. Joo et al. [CLAS], Phys. Rev. C 70 (2004).

- Jlab-Hall A for $K_{1D} = \{K_{1D}^X | X = A, B, .., T\}$

  J. J. Kelly, Phys. Rev. Lett. 95 (2005).

- Response functions (R) ⇔ Kelly notation (RL, RT, …) ⇐ Helicity amplitudes *H* ⇔ CGNL amplitude. For example:

$$\sigma_T = \frac{k}{q_\gamma} R_T^{00}\,, \quad \sigma_L = \frac{k}{q_\gamma}\frac{Q^2}{\omega^2} R_L^{00}\,, \quad \sigma_{TT} = \frac{k}{q_\gamma} R_{TT}^{00}$$

$$\sigma_{LT} = \frac{k}{q_\gamma}\frac{\sqrt{Q^2}}{\omega} R_{LT}^{00}\,, \quad \sigma_{LT'} = \frac{k}{q_\gamma}\frac{\sqrt{Q^2}}{\omega} R_{LT'}^{00}\,.$$

$$P_Y = -\sqrt{2\epsilon(1+\epsilon)}\,\frac{\omega}{\sqrt{Q^2}}\,\frac{R_{LT}^{0Y}}{R_T^{00} + \epsilon\omega^2/Q^2 R_L^{00}}$$

$$\rho_{LT} = \sqrt{2\epsilon(1+\epsilon)}\,\frac{R_{LT}^{00}}{R_T^{00} + \epsilon(R_L^{00} + R_{TT}^{00})}\,,$$

$$\rho_{LT'} = \sqrt{2\epsilon(1-\epsilon)}\,\sin\phi\,\frac{\sigma_{LT'}}{d\sigma^v/d\Omega}\,,$$

Parameterization

- Photoproduction solution as constraint
- Constraints from (Pseudo)-threshold:

			$k \to 0$	$q \to 0$
$E_{\ell+}$	and $L_{\ell+}$	for $\ell \geq 0$	k^ℓ	q^ℓ
$L_{\ell-}$		for $\ell = 1$	k	q
$M_{\ell+}$	and $M_{\ell-}$	for $\ell \geq 1$	k^ℓ	q^ℓ
$E_{\ell-}$	and $L_{\ell-}$	for $\ell \geq 2$	k^ℓ	$q^{\ell-2}$

$$q = \frac{\sqrt{\lambda(W^2, m^2, -Q^2)}}{2W}$$

$$k = \frac{\sqrt{\lambda(W^2, m^2, M^2)}}{2W}$$

- Siegert's theorem at pseudo-threshold:

$$\frac{E_{l_+}}{L_{l_+}} \to 1, \qquad \frac{E_{l_-}}{L_{l_-}} \to \frac{-l}{l-1}$$

Amaldi, Fubini, Furlan,
Springer Tracts Mod. Phys. 83, 1 (1979)
Tiator, Few-body Systems 57, 1087 (2016)

- Watson's theorem, multi-channel unitarity

$$M_{\mu\gamma^*}(q, W, Q^2) = V_{\mu\gamma^*}(q, W, Q^2) + \sum_\kappa \int dp p^2 T_{\mu\kappa}(q, p, W) G_\kappa(p, W) V_{\nu\gamma^*}(p, W, Q^2)$$

$$V_{\mu\gamma^*}(p, W, Q^2) = \alpha^{NP}_{\mu\gamma^*}(p, W, Q^2) + \sum_i \frac{\gamma^a_{\mu;i}(p)\gamma^c_{\gamma^*;i}(W, Q^2)}{W - m^b_i}$$

Parameterization (2)

- Up to *D*-waves included (photoproduction part includes up to *J=9/2*)
- Energy range up to $W \approx 1.6$ allows to include ηN electro-production without much extra effort, but *KY* electroproduction requires additional work
- Final state interaction given by JuBo/JBW model such that pole positions and hadronic branching ratios (pole residues) are universal as required by reaction dynamics
- Q^2-dependence: Several analytic forms tested; settled for:

$$\tilde{F}(Q^2) = \tilde{F}_D(Q^2)\, e^{-\beta_0 Q^2/m^2}\, P^N(Q^2/m^2)$$

where P^N: Polynomial

$$\tilde{F}_D(Q^2) = \frac{1}{(1+Q^2/b^2)^2}\,\frac{1+e^{-Q_r^2/Q_w^2}}{1+e^{(Q^2-Q_r^2)/Q_w^2}}$$

- Other analytic forms?

Ramalho et al., arXiv:1909.00013

Results (1): Fit Strategies

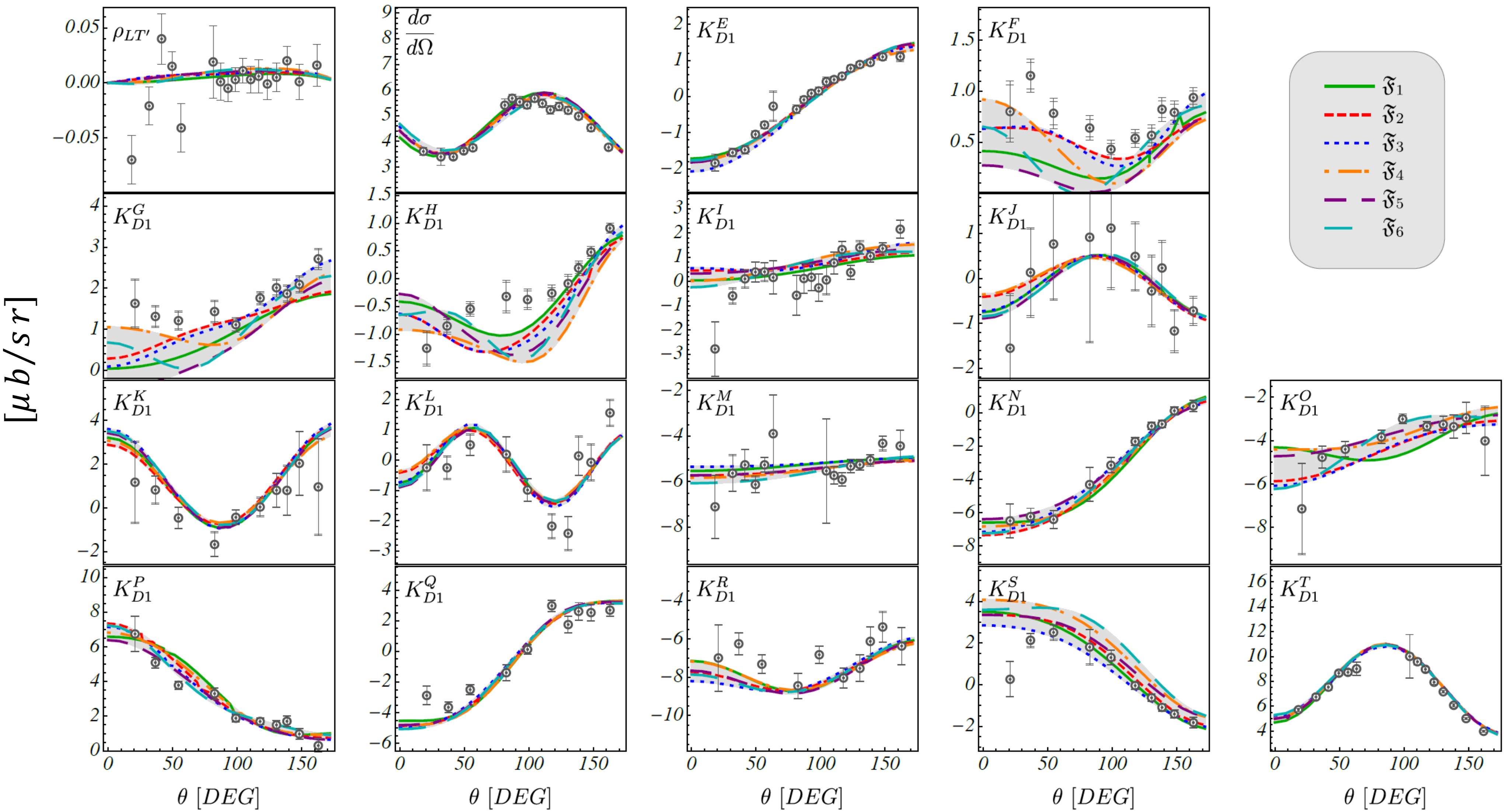
- Six different fit strategies:
    - Avoid fitting structure function if corresponding cross sections can be fitted (respect data correlations)
    - Sequential S → S+P → S+P+D waves;
    - Subsets of data until full data set reached
    - Simultaneous fit all parameters (209) set to zero without any (!) guidance
    - Extend data range from $0 < Q^2 < 4$ Gev² to $0 < Q^2 < 6$ Gev² to check for stability

Fit	σ_L		$d\sigma/d\Omega$		$\sigma_T + \epsilon\sigma_L$		σ_T		σ_{LT}		$\sigma_{LT'}$		σ_{TT}		K_{D1}		P_Y		ρ_{LT}		$\rho_{LT'}$		χ^2_{dof}
	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	π^0p	π^+n	
$\mathfrak{F}_1$	–	9	65355	53229	870	418	87	88	1212	133	862	762	4400	251	4493	–	234	–	525	–	3300	10294	1.77
$\mathfrak{F}_2$	–	4	69472	55889	1081	619	65	78	1780	150	1225	822	4274	237	4518	–	325	–	590	–	3545	10629	1.69
$\mathfrak{F}_3$	–	8	66981	54979	568	388	84	95	1863	181	1201	437	3934	339	4296	–	686	–	687	–	3556	9377	1.81
$\mathfrak{F}_4$	–	22	63113	52616	562	378	153	107	1270	146	1198	1015	4385	218	5929	–	699	–	604	–	3548	11028	1.78
$\mathfrak{F}_5$	–	20	65724	53340	536	528	125	81	1507	219	1075	756	4134	230	5236	–	692	–	554	–	3580	11254	1.81
$\mathfrak{F}_6$	–	18	71982	58434	1075	501	29	68	1353	135	1600	1810	3935	291	5364	–	421	–	587	–	3932	11475	1.78

χ^2

Results (2): Kelly data

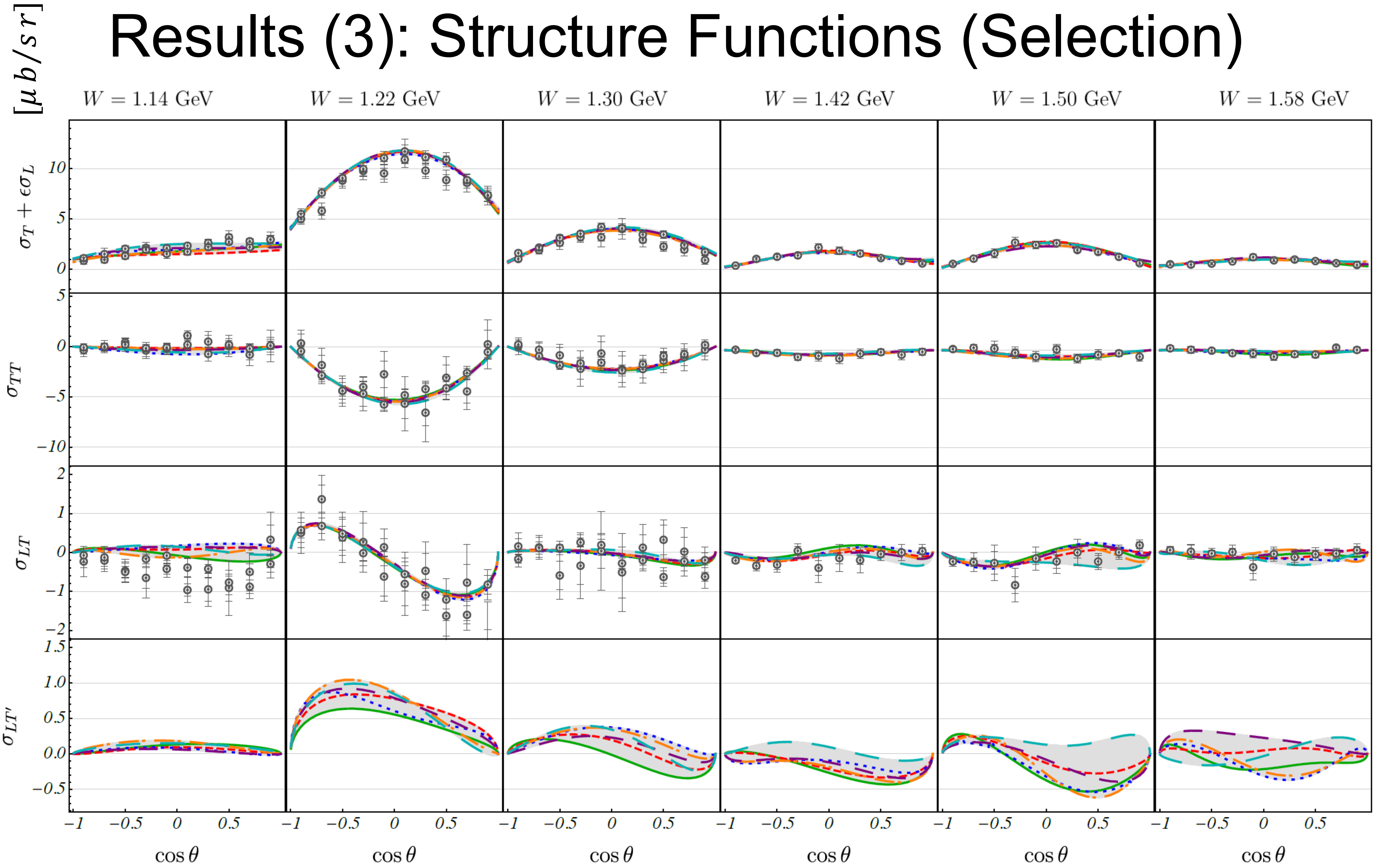
The closest to a complete data set there is



$\pi^0 p$, Q^2=1 GeV^2, W=1.23 GeV, ϕ=15^0

J. J. Kelly, [Phys. Rev. Lett. 95 (2005)](#).

Results (3): Structure Functions (Selection)

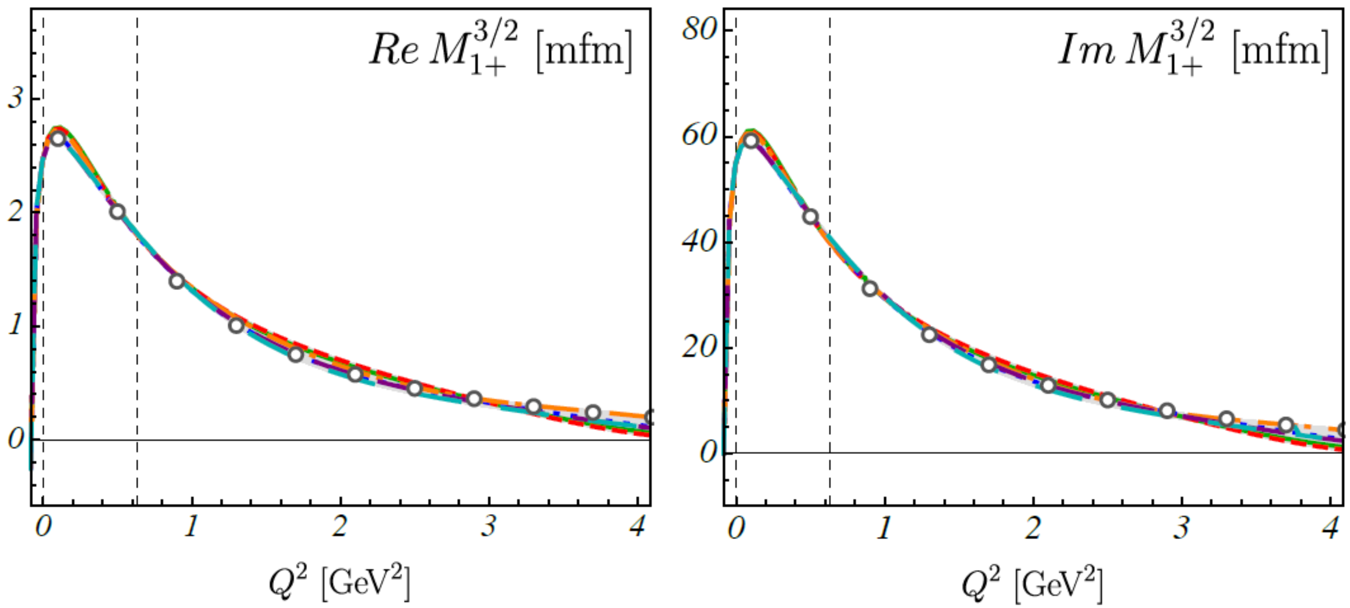


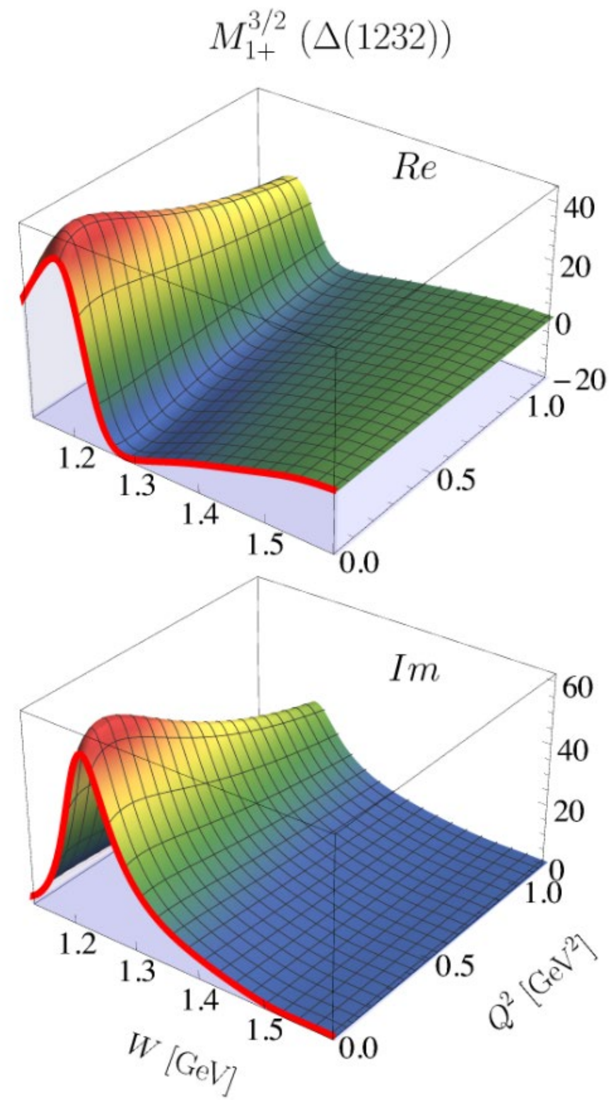
$Q^2 = 0.9\ \mathrm{Gev}^2$, $\pi^0 p$

data: CLAS, Phys. Rev. C (2003) 0301012 [nucl-ex], Phys. Rev. Lett. (2002) 0110007 [hep-ex]

Results (4): Large Multipoles

Prominent multipoles are well determined, even with significantly different fit strategies (e.g., all parameters initially set to zero, no guidance for fit!)



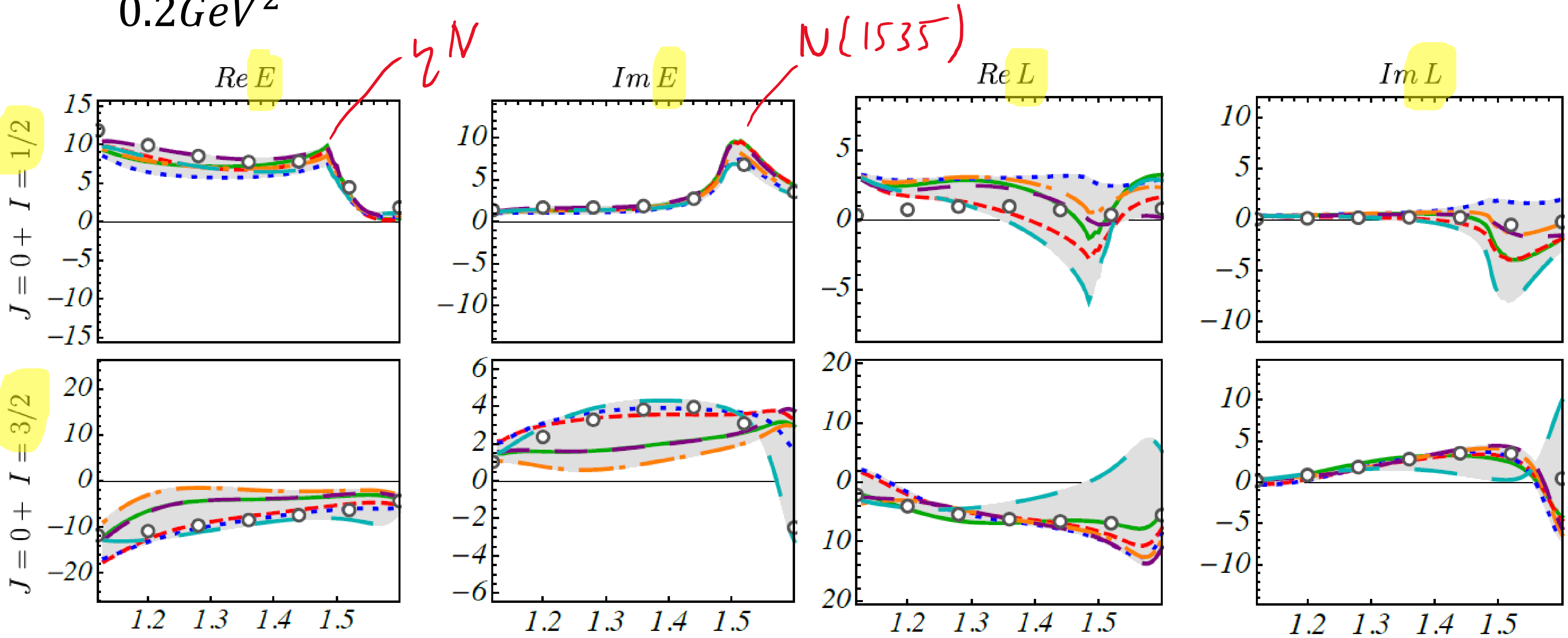


Fit strategies 1-6 together with MAID (open dots) for the magnetic multipole of the $\Delta(1232)$ Drechsel et al., EPJA (2007) [0710.0306 [nucl-th]]

(Strategy 1 only)

Results (5): Other multipoles

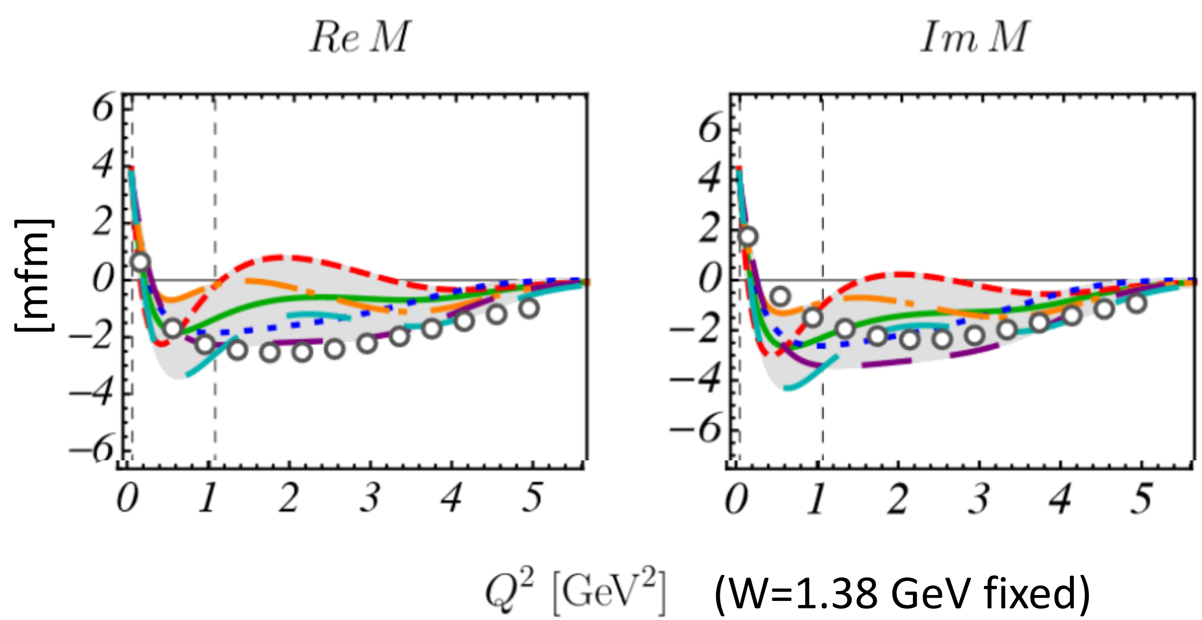
- Less prominent multipoles are sometimes less well determined
- Overall: solutions are still surprisingly close together given vastly different strategies
- Differences from various strategies (different local χ^2minima) much larger than statistical uncertainties; larger than typical MAID uncertainties.
- **Example**: S-wave multipoles [mfm] as function of energy W at fixed $Q^2 = 0.2GeV^2$



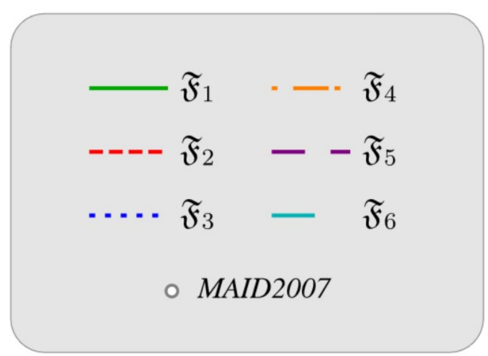
W [GeV]

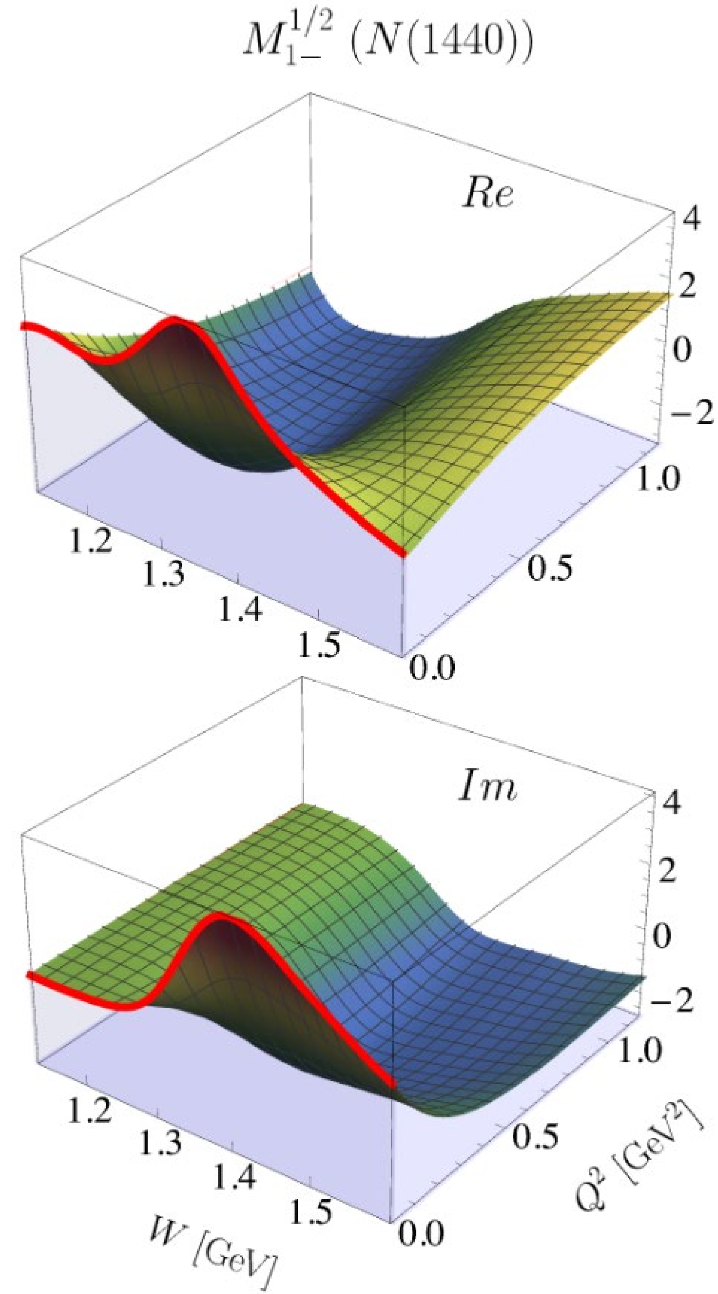
Results (6): Roper Multipole

- Non-trivial structure
- Zero transition
- Helicity coupling still to be extracted



(W=1.38 GeV fixed)





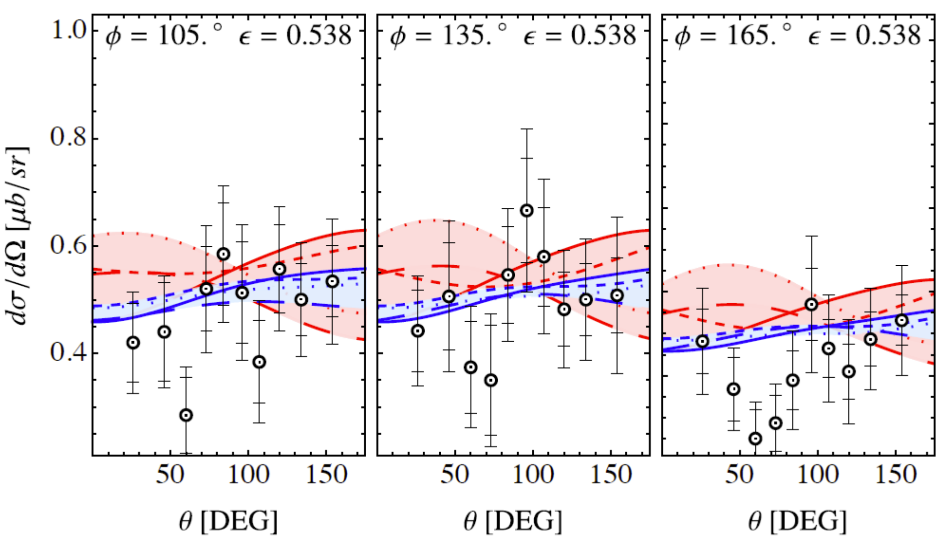
(Strategy 1 only)

η Electroproduction

[M. Mai et al., arXiv: 2111.04774]

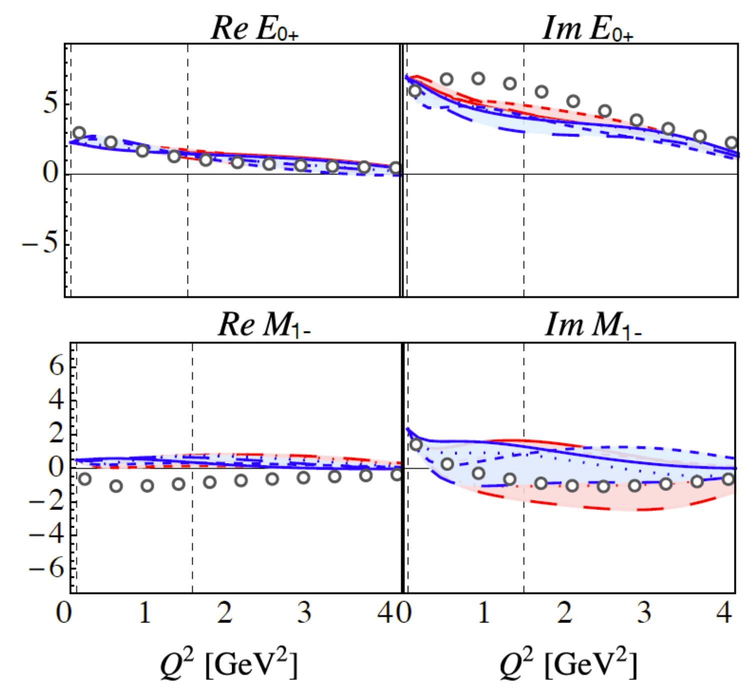
- $N_{data}^{\eta p} = 1,874$ (only $d\sigma/d\Omega$) (84,842 in total)
- kinematic range: $0 < Q^2 < 4$ GeV^2, $1.13 < W < 1.6$ GeV
- 8 different fit strategies: 4 with standard χ^2, 4 with weighted χ^2 to account for the smaller $N_{data}^{\eta p}$ $\rightarrow$ better data description with weighted fit strategies:

Selected fit results: $\gamma^* p \rightarrow \eta p$ at $W = 1.5$ GeV, $Q^2 = 1.2$ GeV^2. Data: Denizli et al. (CLAS) PRC 76 (2007)



Extraction of resonance helicity couplings at the pole is under way

Selected multipoles at $W = 1535$ MeV



Dots: eta-MAID, arXiv:0110034

Outlook

- Statistical challenges
- DCC approaches for three-body physics

Parameterization Dependence

- Can parametrization dependence be avoided? Not if the data is far from being complete enough to represent even a truncated complete electroproduction experiment

L. Tiator et al. Phys. Rev. C (2017), arXiv: 1702.08375
Wunderlich, Svarc et al., arXiv:1708.06840

- Single-Q^2 analysis can decrease parametrization-independence but not remove it (discrete & continuous ambiguities).

Y. Wunderlich, arXiv:2111.09587

- Future: Bias-variance tradeoff: Different statistical criteria (Akaike, Bayesian) to find sweet spot between no. of parameters (flexibility of parametrization) and data accuracy

J. Landay et al., Phys.Rev.C (2017), arXiv: 1610.07547

- Challenges: PWA solution uncertainties dominated by
  - Ambiguities
  - Empty kinematic regions (in W and Q^2)
  - Systematic data uncertainty/consistency

DCC approaches for three-body physics

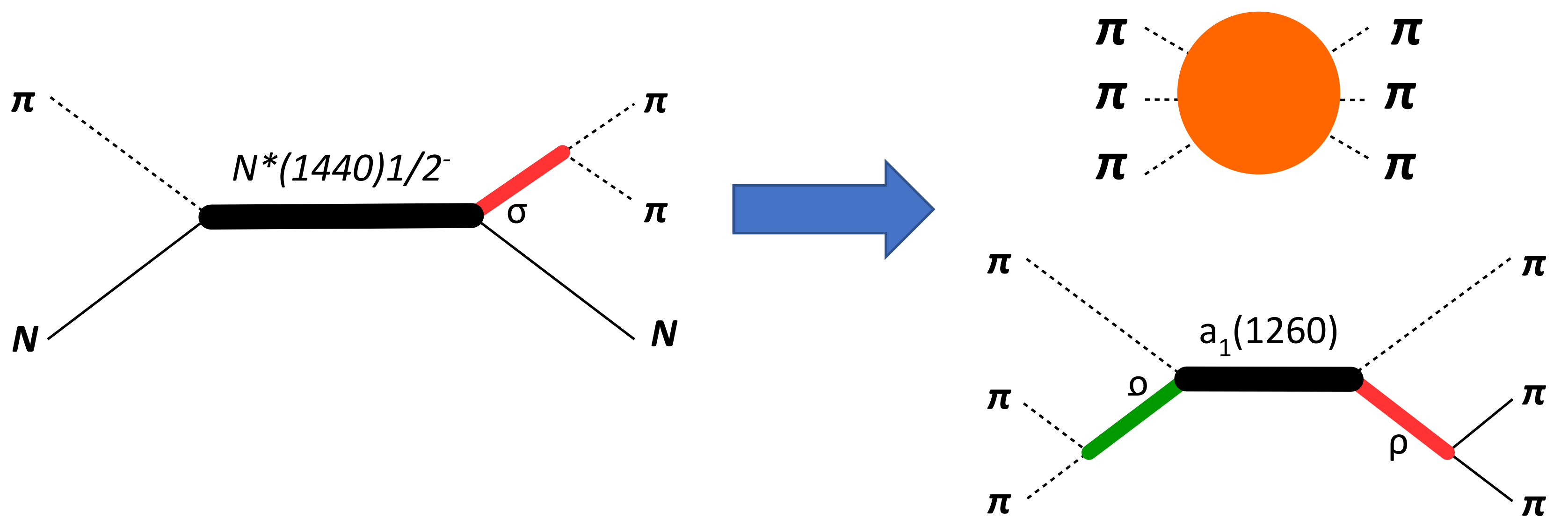
- Strength of DCC approaches: Allows for general three-body amplitudes, ordered and truncated by “isobars”
- DCC approaches well suited for three-body meson physics

ANL/Osaka (Kamano et al.) arXiv:1106.4523

- Three-body unitarity + Dispersive representations to construct a general 3-body amplitude

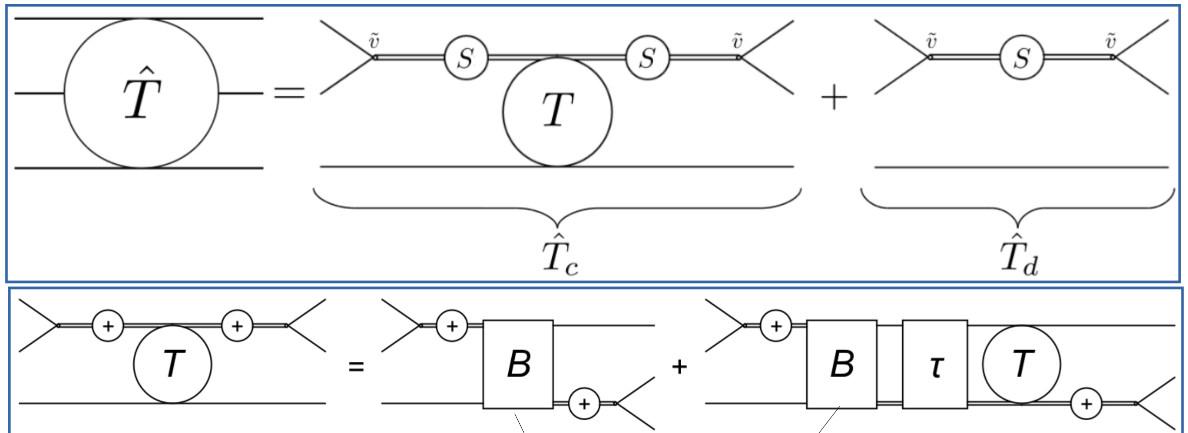
Aaron et al., Phys. Rev. 174 (1968)

M. Mai et al., arXiv:1706.06118



Unitary three-body scattering equation

Mai et al., arXiv:1706.06118

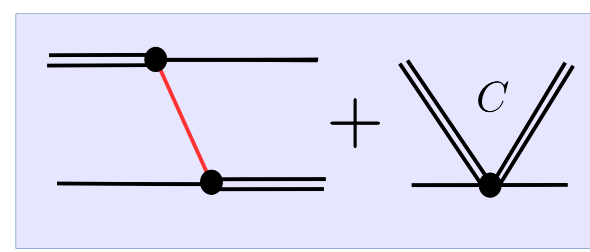


Imaginary parts of ***B, S*** are fixed by **unitarity/matching**

$$\mathrm{Disc}\,B(u) = 2\pi i\lambda^2 \frac{\delta\left(E_Q - \sqrt{m^2+\mathbf{Q}^2}\right)}{2\sqrt{m^2+\mathbf{Q}^2}}$$

- un-subtracted dispersion relation

$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2+\mathbf{Q}^2}\left(E_Q-\sqrt{m^2+\mathbf{Q}^2}+i\epsilon\right)} + C$$



- one-π exchange in TOPT → *RESULT, NOT INPUT !*

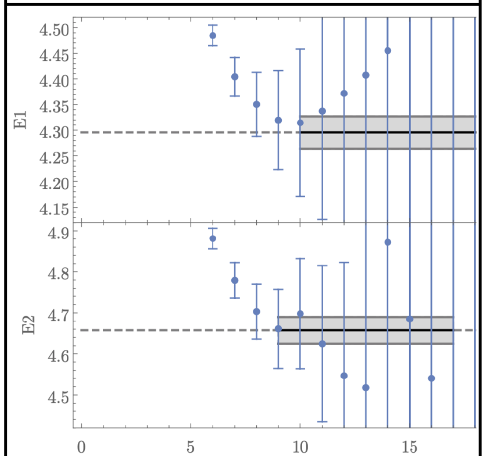
- One can map to field theory but does not have to. Result is a-priori dispersive.

Extraction of $a_1(1260)$ from lQCD

[Mai/GWQCD]

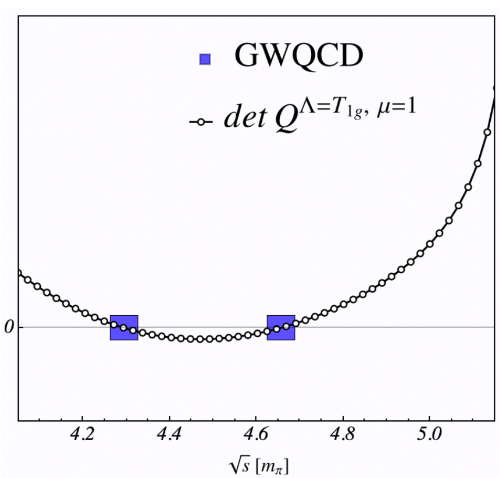
- First-ever three-body resonance from 1st principles
- A step towards understanding axials, Roper, exotics,…]

Finite-volume lattice QCD spectrum



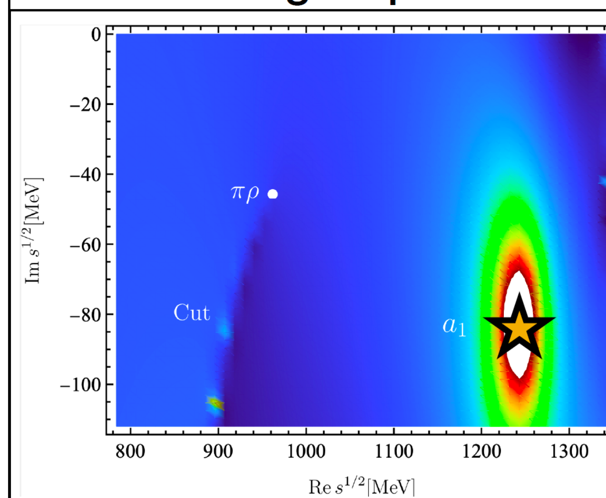
energy eigenvalues

3-body quantization conditions



volume-independent quantities

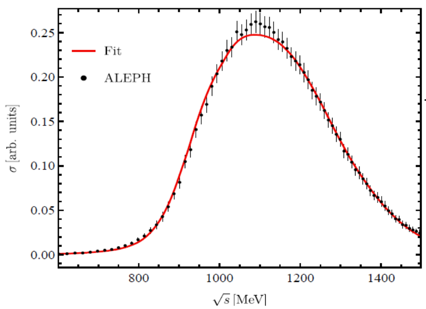
Riemann Surface: scattering amplitude

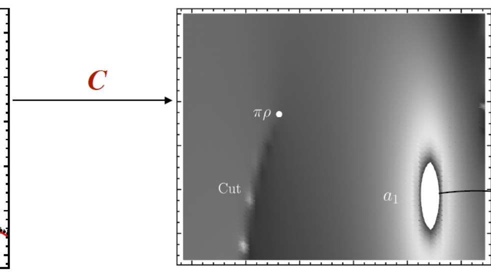


Another talk on axial mesons by S. Clymton on Thursday afternoon, 16:05!

Extraction of $a_1(1260)$ from lQCD

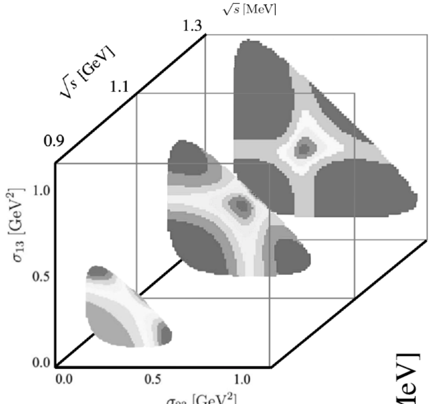
[Mai/GWQCD]



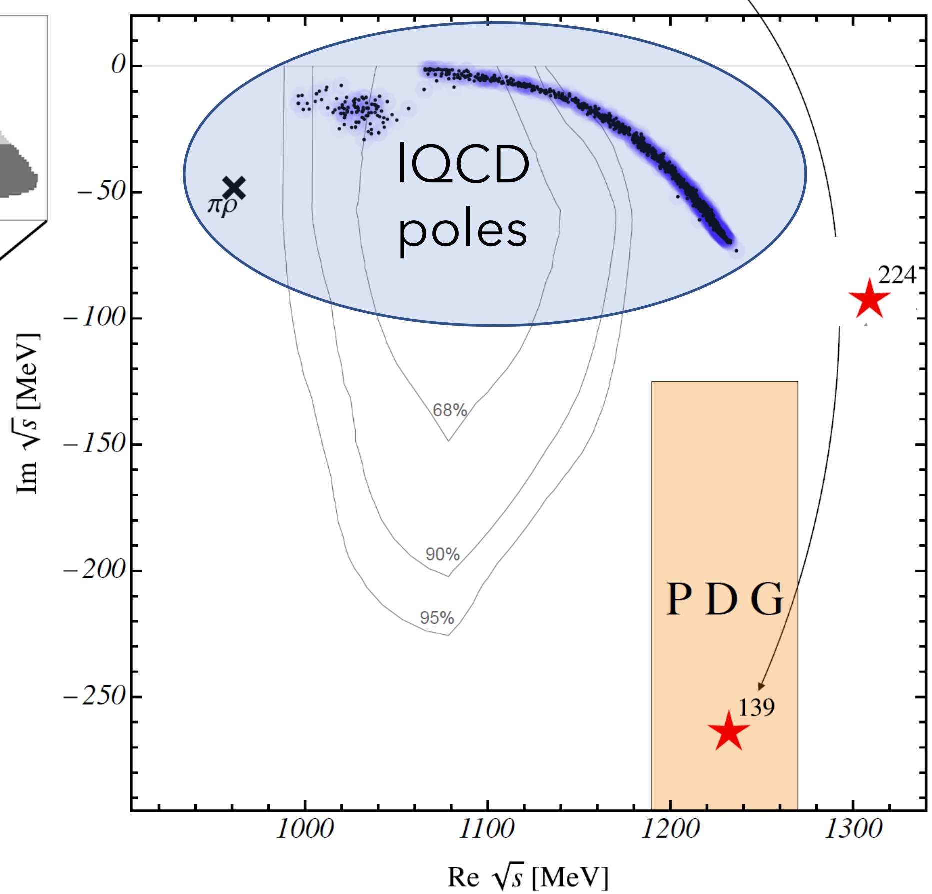


What does phenomenology says?

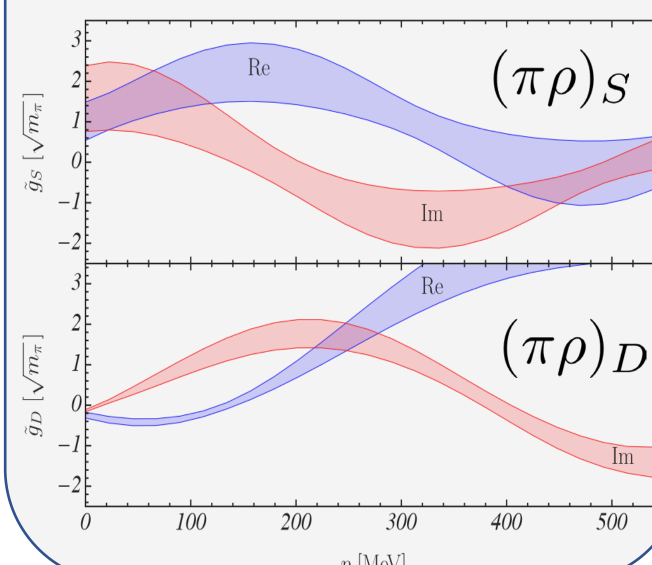
- $\tau\to(\pi\pi\pi)\nu_\tau$ from ALEPH@CERN
- fit to line shape to fix C



[Sadasivan at al., arXiv: 2107.03973, arXiV:2112.03355]



"**Branching ratios**" in 3B decays are momentum - dependent, complex pole residues



Summary

- JBW model: Phenomenology of excited baryons through coupled-channels, two- and three-body dynamics
- Analysis finds/confirms new states in analysis of photo-production data, renewed effort to explore additional reaction channels
- Pion and eta electroproduction analysis
  - Exploration of parameter space through different fit strategies reveals different local minima leading to significantly different multipole content.
  - Yet, prominent multipole well determined, albeit with uncertainties larger than in other analyses.

- Extraction of helicity couplings and single-bin analyses planned
- Upgrade KY electroproduction under way
- Machine Learning: How to find a minimal resonance spectrum through model selection J. Landay et al., Phys.Rev.D (2019), 1810.00075 [nucl-th]

- Unitary dispersive amplitudes for analysis of lQCD data allow access to dynamics of three-body systems (resonances)

(spare slides)

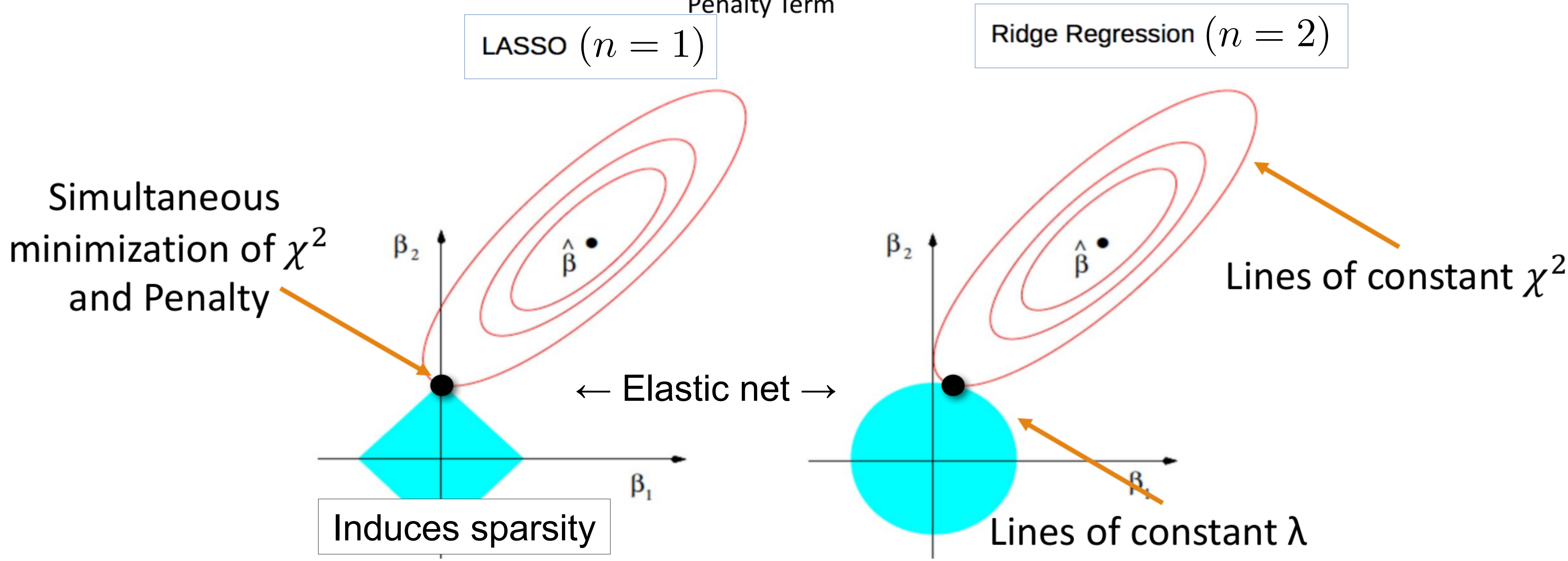
Machine learning and Model selection for baryon spectrum

[J. Landay et al., arXiv:1810.00075, arXiv: 1610.07547]

Least Absolute Shrinkage and Selection Operator (LASSO)

$$\sum_{i=1}^{n}\underbrace{\frac{(y_i - f(x_i,\beta_j))^2}{\sigma_i^2}}_{\text{Normal } X^2} + \lambda \underbrace{\sum_{j=1}^{m}|\beta_j|^{n}}_{\text{Penalty Term}}$$

$\hat{\beta}_i$: Best parameters without penalty
$\beta_i = 0$: Best parameters only penalty



See, e.g.: *The Elements of Statistical Learning*: Data Mining, Inference, and Prediction, T. Hastie, R. Tibshirani, J. Friedman, Springer 2009 second ed.

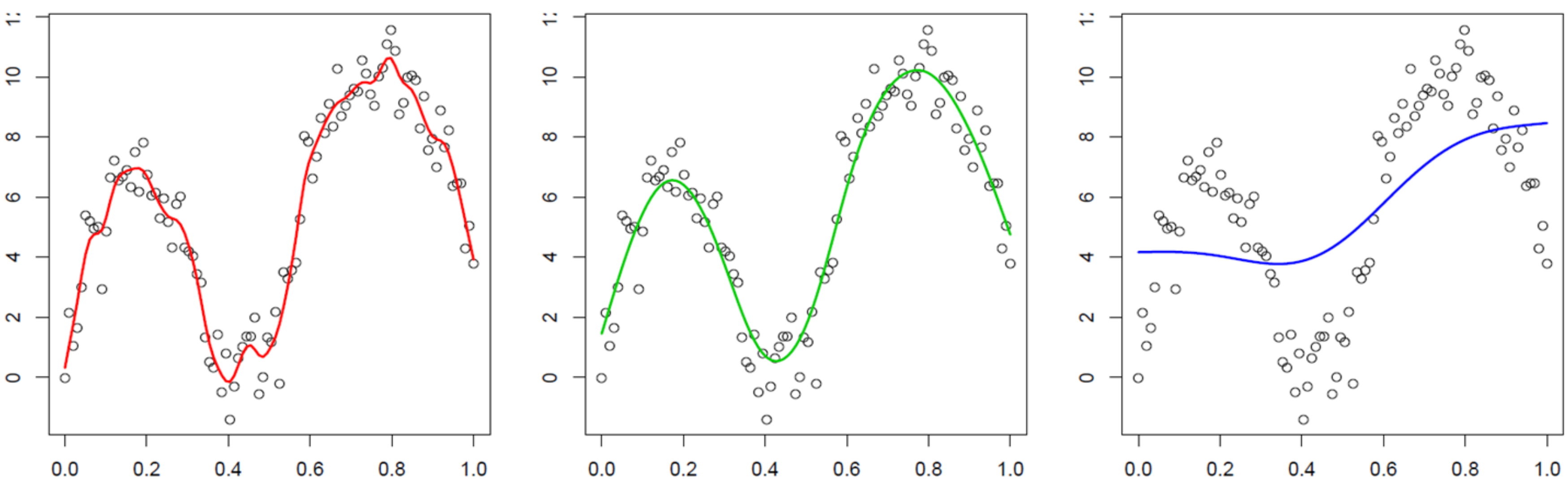
The Sweet Spot for λ

- Example: Smoothing a curve

Smoothning splines use a form of regularization:

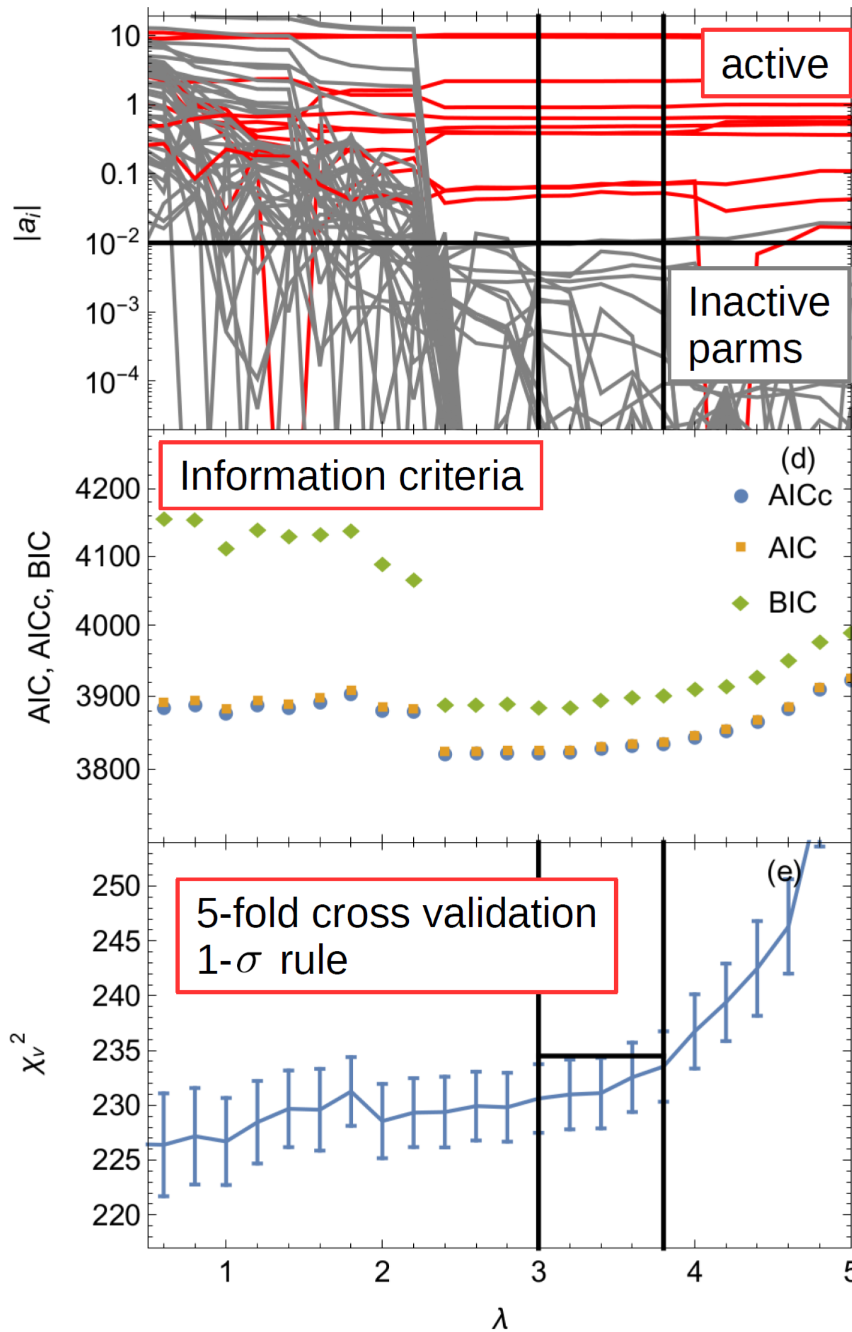
$$\hat{f} = \underset{f}{\operatorname{argmin}} \sum_{i=1}^{n} \big(y_i - f(x_i)\big)^2 + \lambda \cdot \underbrace{\int \big(f''(x)\big)^2 dx}_{R(f)}$$

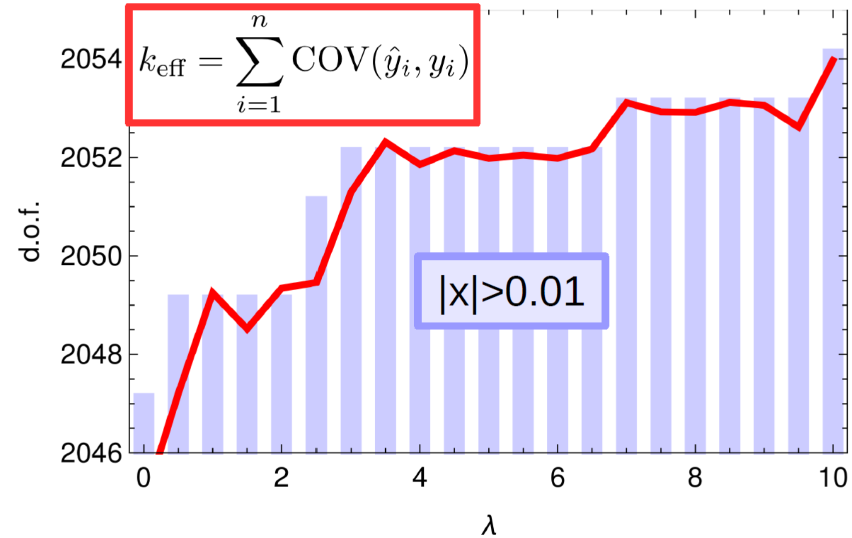
Example with $n = 100$ points:



λ too small　　λ just right　　λ too big

Information theory criteria





$k \to k_{\mathrm{eff}}$ subsequently

$AIC = -2 \max \log(L(\hat{\theta}|data)) + 2k = \chi^2 + 2k$

$AIC_c = AIC + \frac{2k(k+1)}{n-k-1}$

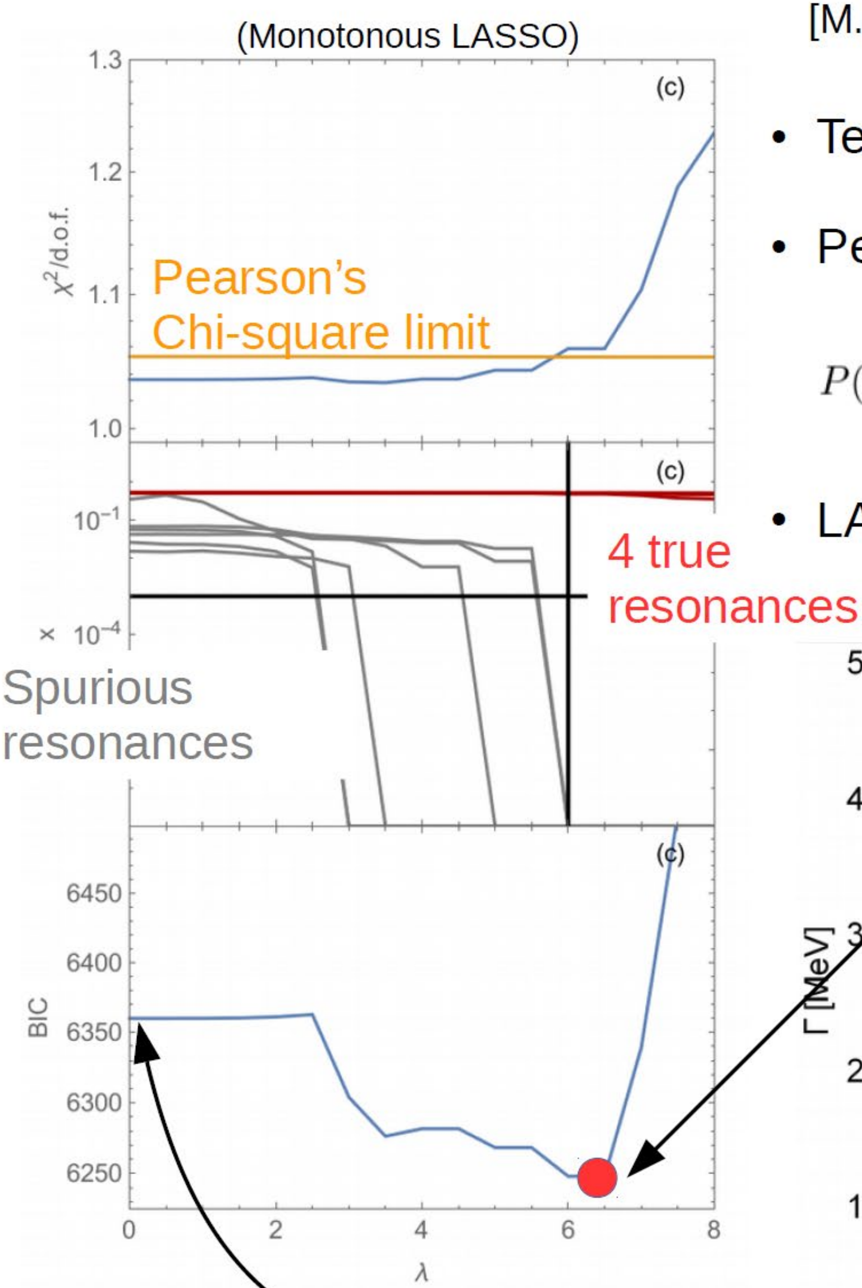
$BIC = -2 \max \log(L(\hat{\theta}|data)) + 2log(n) = \chi^2 + k\, log(n)$

Close relation to Bayesian model comparison (here: $n \gg k$)

See, e.g.: Andrew A. Neath, Joseph E. Cavanaugh, DOI: 10.1002/wics.199

Resonance selection $(K^- p \to K\Xi)$

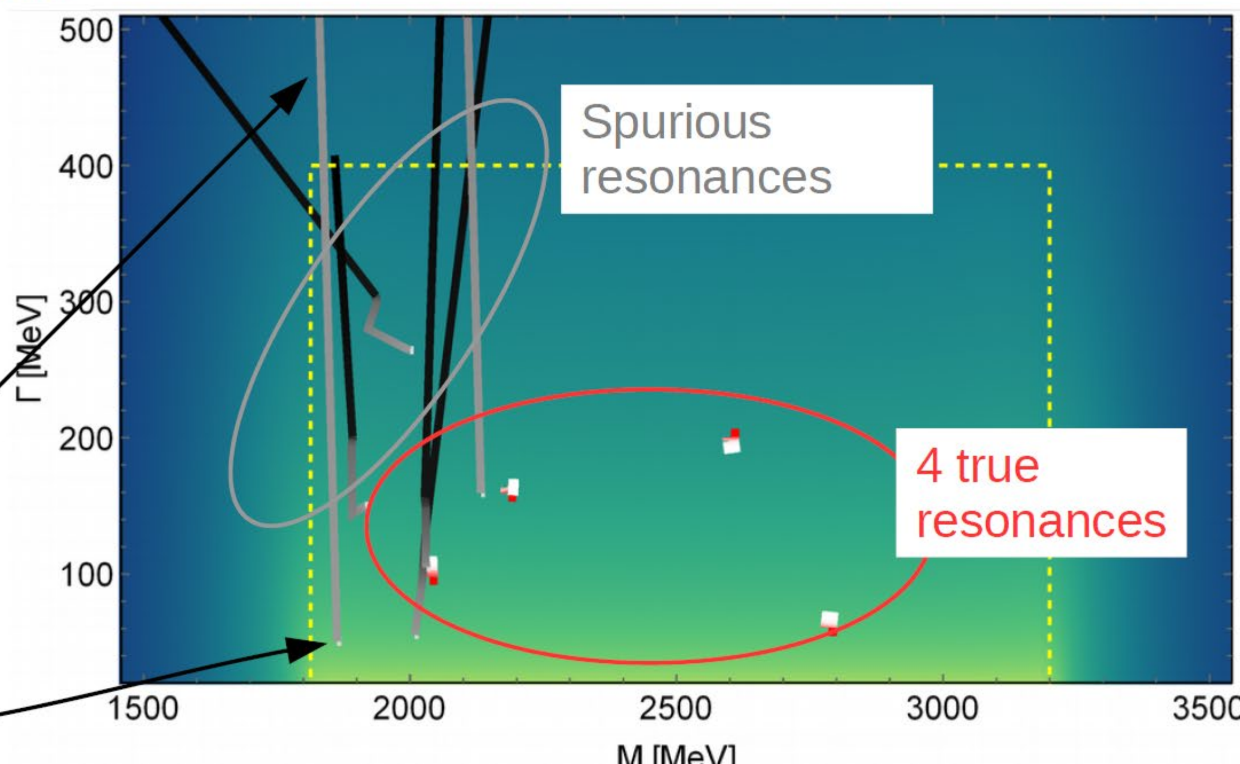
[M.D., J. Landay, H. Haberzettl, M. Mai, K. Nakayama, PRC 2019]



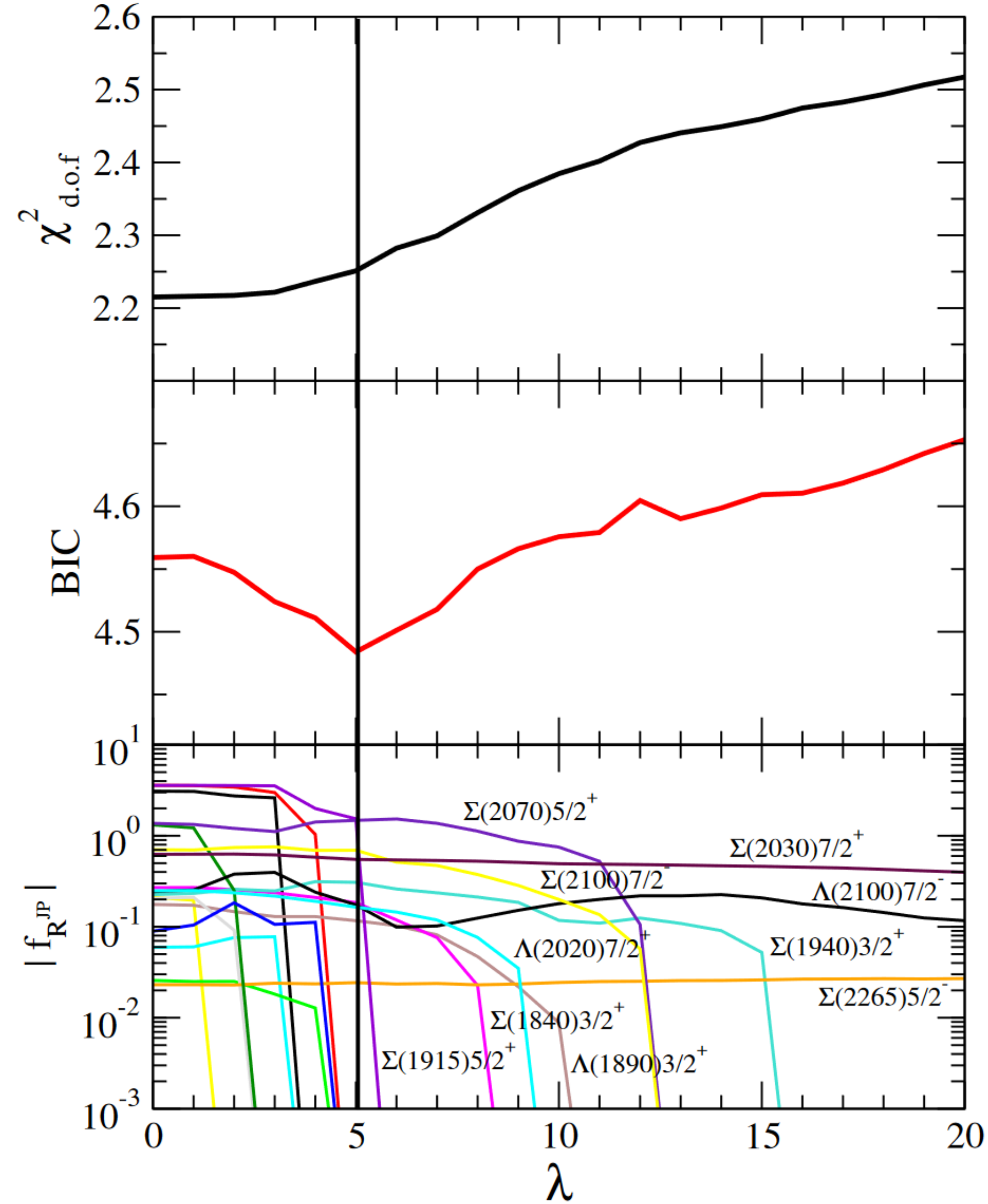
- Ten partial waves; 10 resonances in Ansatz
- Penalty: $\chi^2 = \chi^2_{\text{stat.}} + P$

$$P(\lambda) = \lambda^5 \sum_{i=0}^{9} \int_{m_K+m_\Xi}^{3200} \frac{\partial^2}{\partial W^2}\left(\left| -x_i\, e^{i\Phi_i} \frac{\Gamma_i}{2(W - M_i + i\frac{\Gamma_i}{2})}\right|^2\right) dW$$

- LASSO picks the 4 correct ones



Bayesian Information Criterion for model selection



Data description of selected model