Chiral anomaly and the pion properties in the light-front quark model

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in collaboration with C.-R. Ji

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Outline

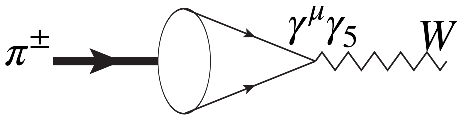
1. Motivation

2. Model Description: Light-Front Quark Model(LFQM)

   - Role of axial-vector coupling in the chiral limit $(M_\pi, m_Q \to 0)$

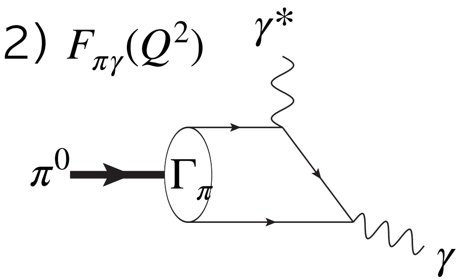
3. Application

1) f_π



$$\Gamma_\pi = (A_\pi + B_\pi \not{P})\gamma_5$$

2) $F_{\pi\gamma}(Q^2)$



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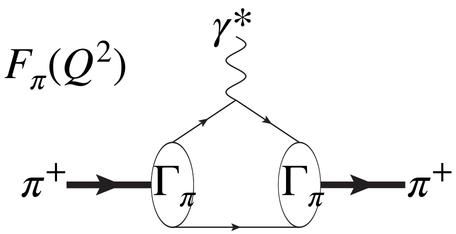

4. Numerical Results

5. Conclusion

1. Motivation

- Pion is the lightest pseudo-Goldstone boson arising from the SSB of the chiral symmetry in QCD.
- $\pi^0 \to \gamma\gamma^*$ is the simplest exclusive process in testing QCD and understanding the structure of the pion.

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- $\pi^0 \to \gamma\gamma^*$ is the simplest exclusive process in testing QCD and understanding the structure of the pion.

Its complete understanding requires

a formulation capable of explaining

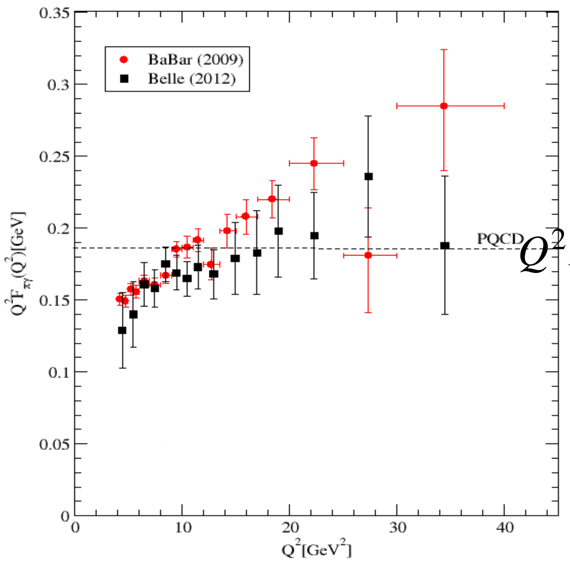
$Q^2 \to 0$ limit and $Q^2 \to \infty$ limit

: Adler-Bell-Jackiw (ABJ) anomaly

(or chiral anomaly), which determines

$$\Gamma_{\pi^0\to\gamma\gamma} \propto |F_{\pi\gamma}(0)|^2$$

$$\boxed{F^{\rm ABJ}_{\pi\gamma}(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}}$$



$$Q^2 F_{\pi\gamma} = f_\pi\sqrt{2}$$

- The purpose of this work is to explore

"the correlation between the nontrivial QCD vacuum effect and the constituent quark mass", through the analysis of

(1) the nonzero axial vector coupling for the consistency with the chiral anomaly

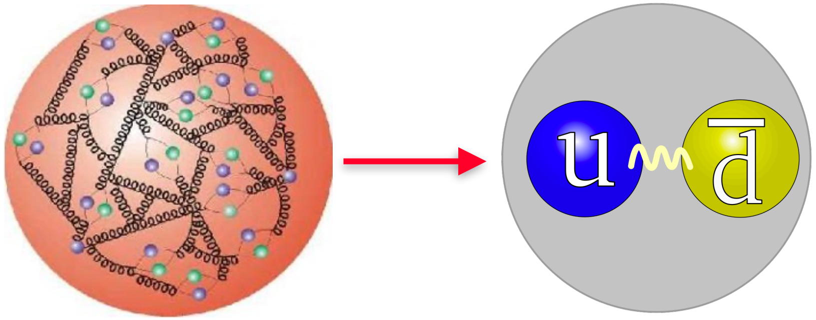
(2) the difference between the constituent quark picture ($M_\pi < 2m$) and the current quark picture ($M_\pi > 2m$)

(3) the quark mass variation effects on $F_{\pi\gamma}(Q^2), F_\pi(Q^2)$

in the LFQM using $\Gamma_\pi = (A_\pi + B_\pi \not{P})\gamma_5$ for the pion spin-orbit structure.

2. Model Description: Light-Front Quark Model (LFQM)

$|\pi\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \cdots \equiv \Psi^{\pi}_{Q\bar{Q}}|Q\bar{Q}\rangle$: mock-hadron approx.



QCD | Quark Model

Higher Fock states
(Non-trivial QCD vacuum) ⟷ Constituent Q and $\bar{Q}$

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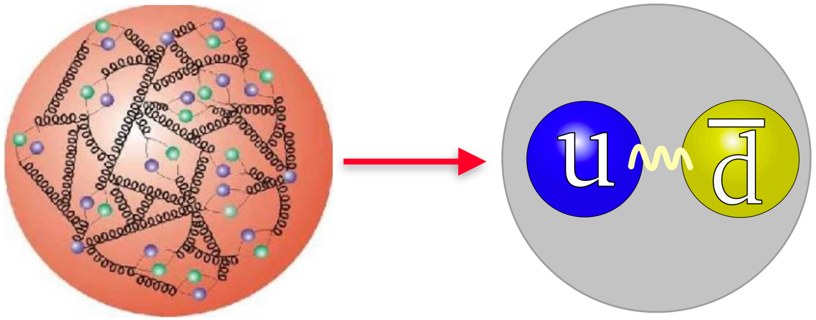


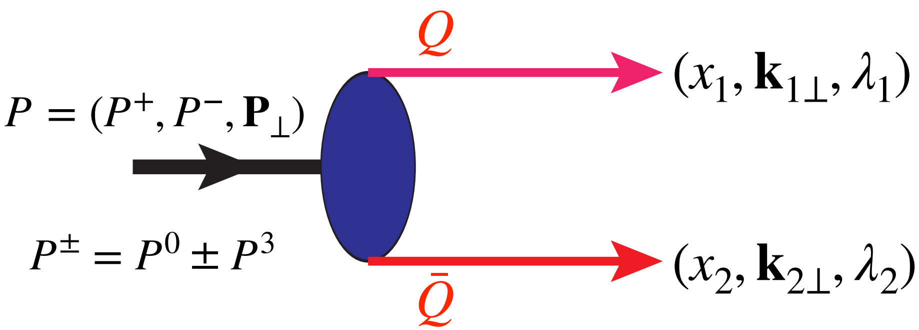
QCD

Quark Model

Higher Fock states

(Non-trivial QCD vacuum) ⟷ Constituent Q and $\bar{Q}$

Pion in LFQM:



$$x_i = \frac{k_i^+}{P^+} \qquad \sum_{i=1}^{2} x_i = 1, \sum_{i=1}^{2} \mathbf{k}_{i\perp} = 0$$

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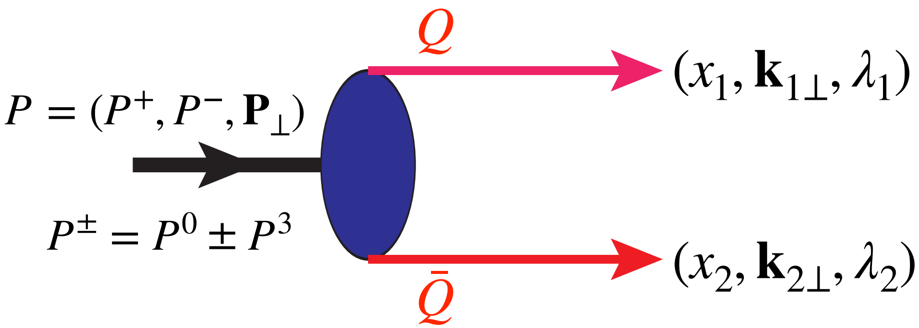

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$$\Psi^{\pi}_{Q\bar{Q}} \equiv \Psi_\pi(x_i, \mathbf{k}_{i\perp}, \lambda_i) = \phi_R(x_i, \mathbf{k}_{i\perp})\chi(x_i, \mathbf{k}_{i\perp}, \lambda_i)$$

Normalization: $\langle \Psi^{\pi}_{Q\bar{Q}} | \Psi^{\pi}_{Q\bar{Q}} \rangle = P_{Q\bar{Q}}$

$$\phi_R(x, \mathbf{k}_\perp) = \sqrt{P_{Q\bar{Q}}}\frac{4\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}e^{-\frac{\vec{k}^2}{2\beta^2}}, \qquad \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}|\phi_R(x, \mathbf{k}_\perp)|^2 = P_{Q\bar{Q}}.$$

$$\{\mathbf{k}_\perp, k_z\} \to \{\mathbf{k}_\perp, x\}$$

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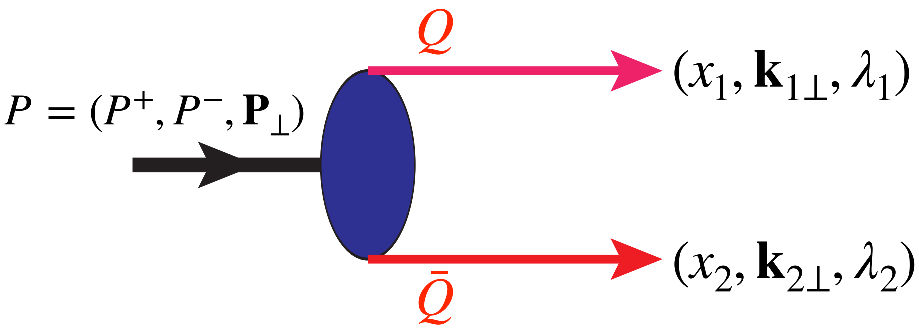

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$\chi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) = \mathcal{N}\bar{u}_{\lambda_1}(k_1)\Gamma_\pi v_{\lambda_2}(k_2)$ satisfying $\langle \chi_{\lambda_1\lambda_2}|\chi_{\lambda_1\lambda_2}\rangle = 1$

$$\boxed{\Gamma_\pi = (A_\pi + B_\pi \not{P})\gamma_5}$$

where we set $A_\pi = M_\pi$, B_π being a free parameter.

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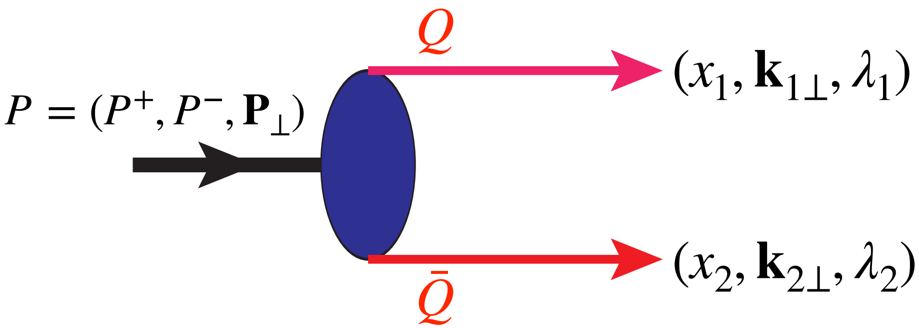

$$M_0^2 = \frac{m_Q^2 + \mathbf{k}_{1\perp}^2}{x_1} + \frac{m_{\bar{Q}}^2 + \mathbf{k}_{2\perp}^2}{x_2}$$

$$\Psi^{\pi}_{Q\bar{Q}} \equiv \Psi_\pi(x_i, \mathbf{k}_{i\perp}, \lambda_i) = \phi_R(x_i, \mathbf{k}_{i\perp})\chi(x_i, \mathbf{k}_{i\perp}, \lambda_i)$$

$$\chi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) = \begin{pmatrix} \chi_{\uparrow\uparrow} & \chi_{\uparrow\downarrow} \\ \chi_{\downarrow\uparrow} & \chi_{\downarrow\downarrow} \end{pmatrix} \propto \begin{pmatrix} -k^L\mathcal{M} & m\mathcal{M} + x(1-x)B_\pi\epsilon_B \\ -m\mathcal{M} - x(1-x)B_\pi\epsilon_B & -k^R\mathcal{M} \end{pmatrix}$$

$$\mathcal{M} = M_\pi + 2B_\pi m,\ \epsilon_B = M_\pi^2 - M_0^2, \quad k^{R(L)} = k_x \pm i k_y$$

$$\chi_{\lambda_1\lambda_2}(x,\mathbf{k}_\perp) = \mathcal{N}\bar{u}_{\lambda_1}(k_1)(M_\pi + B_\pi \not{P})\gamma_5 v_{\lambda_2}(k_2)$$

Chiral limit

$(M_\pi, m \to 0)$

$$\boxed{\chi^{\text{chiral}}_{\lambda_1\lambda_2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \operatorname{sgn}(-\mathrm{B}_\pi)}$$

$$\operatorname{sgn}(-\mathrm{B}_\pi) = -\operatorname{sgn}(\mathrm{B}_\pi)$$

$$\operatorname{sgn}(B_\pi) = \begin{cases} 1 & \text{for } B_\pi > 0 \\ -1 & \text{for } B_\pi < 0 \\ 0 & \text{for } B_\pi = 0 \end{cases}$$

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Zero-binding limit

$(M_\pi \to M_0 \text{ or } \epsilon_B = 0)$ $\Rightarrow$ $\Gamma_\pi \propto \gamma_5$

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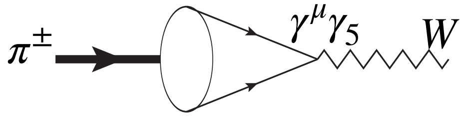
Axial vector coupling (i.e. $B_\pi < 0$) is absolutely required to have non-zero chiral limit!

3. Application

(1) Pion decay constant



$$\Gamma_\pi = (A_\pi + B_\pi \not{P})\gamma_5$$

$$\langle 0 | \bar{q}\gamma^{\mu}(1-\gamma_5)q | \pi(P)\rangle = i f_\pi P^{\mu}$$

Using $\mu = +$

$$f_\pi = 2\sqrt{2N_c}\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\psi_\pi(x,\mathbf{k}_\perp), \qquad \psi_\pi(x,\mathbf{k}_\perp) = \frac{1}{\sqrt{2}}(\chi_{\uparrow\downarrow} - \chi_{\downarrow\uparrow})\phi_{\mathrm{R}}(x,\mathbf{k}_\perp)$$

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Twist-2 pion Distribution amplitude (DA) :

$$\phi_{\pi}(x) = \int^{Q^2}\frac{d^2\mathbf{k}_{\perp}}{16\pi^3}\psi_{\pi}(x,\mathbf{k}_{\perp}), \qquad \int_0^1 dx\, \phi_{\pi}(x) = \frac{f_{\pi}}{2\sqrt{2N_c}}\,.$$

In the chiral limit, we obtain

$$f_{\pi}^{\mathrm{chiral}} = \sqrt{P_{Q\bar{Q}}}\frac{\sqrt{3}\beta}{2^{3/4}\pi^{1/4}}\Gamma(\frac{5}{4}), \qquad \phi_{\pi}^{\mathrm{chiral}}(x) = \frac{2\sqrt{2}f_{\pi}^{\mathrm{chiral}}}{\sqrt{3}\pi}\sqrt{x(1-x)}$$

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Brodsky, de Teramond

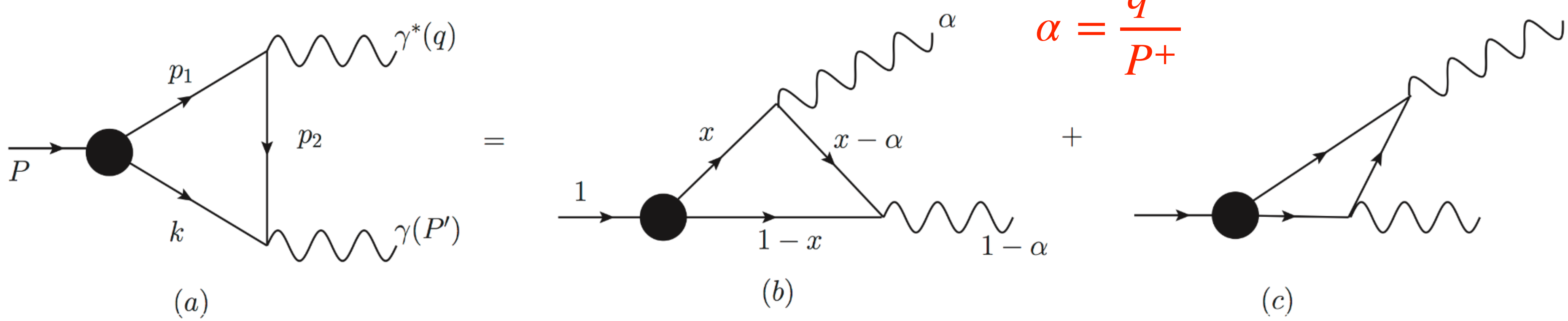
3. Application

(2) $\pi^0 \to \gamma\gamma^*$ transition form factor (TFF)

$$\langle \gamma(P-q) \,|\, J^{\mu}_{\text{em}} \,|\, \pi^0(P) \rangle = ie^2 F_{\pi\gamma}(Q^2)\epsilon^{\mu\nu\rho\sigma} P_\nu \epsilon_\rho q_\sigma,$$

$$q^2 = q^+ q^- - \mathbf{q}_\perp^2$$

$$\alpha = \frac{q^+}{P^+}$$



$(\alpha < x < 1)$ $\quad$ $(0 < x < \alpha)$

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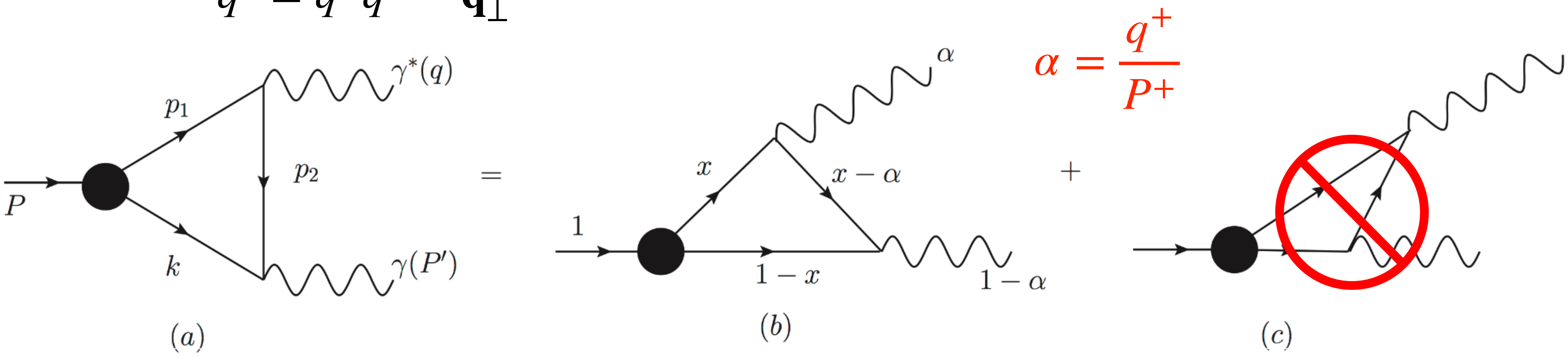

$(\alpha < x < 1)$ $(0 < x < \alpha)$

(1) For $q^+ = 0$ frame $(\alpha = 0)$: spacelike region, i.e. $q^2 = -\mathbf{q}_\perp^2 = -Q^2 < 0$

$$F_{\pi\gamma}(q^2) = \frac{e_u^2 - e_d^2}{\sqrt{2}}\frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 (1-x)dx\int d^2\mathbf{k}_\perp \frac{\psi_\pi(x,\mathbf{k}_\perp)}{\mathbf{k}'^2_\perp + m^2}, \quad \mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$$

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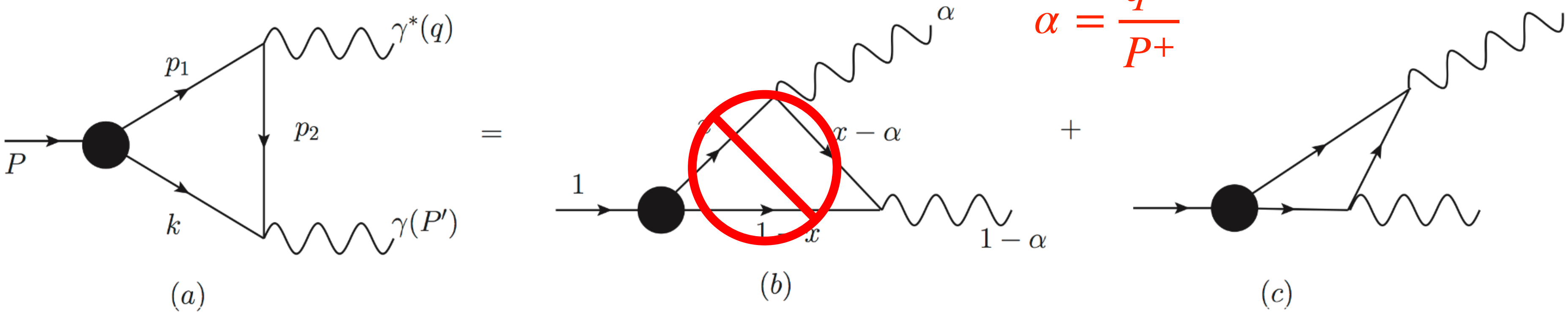

$(\alpha < x < 1)$ $(0 < x < \alpha)$

(2) For $q^+ = P^+$ with $\mathbf{q}_\perp = 0$ frame $(\alpha = 1)$: timelike region, i.e. $q^2 = q^+ q^- > 0$

Choi,Ryu,Ji, PRD 96, 056008(17)

$$F_{\pi\gamma}(q^2) = \frac{e_u^2 - e_d^2}{\sqrt{2}} \frac{\sqrt{2N_c}}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \frac{\psi_\pi(x, \mathbf{k}_\perp)}{M_0^2 - q^2}$$

Simple pole

3. Application

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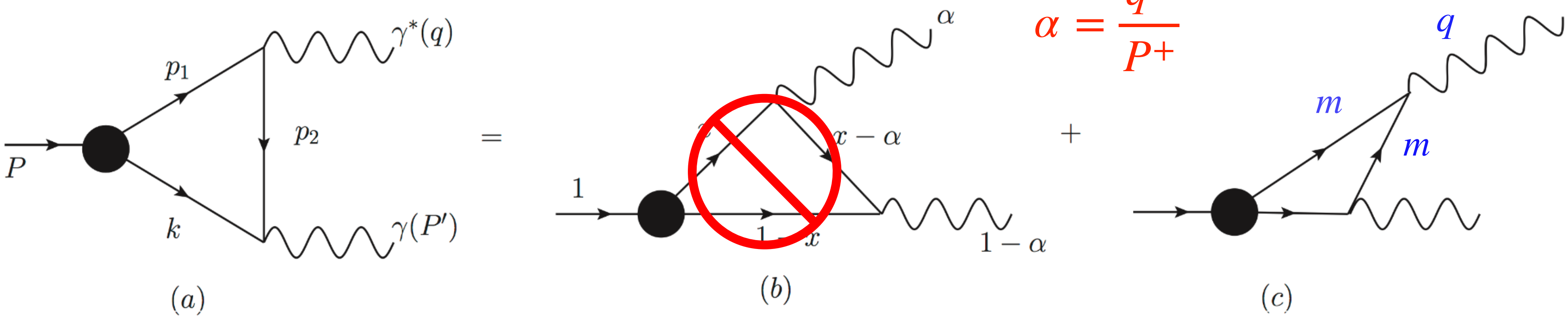

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Choi,Ryu,Ji, PRD 96, 056008(17)

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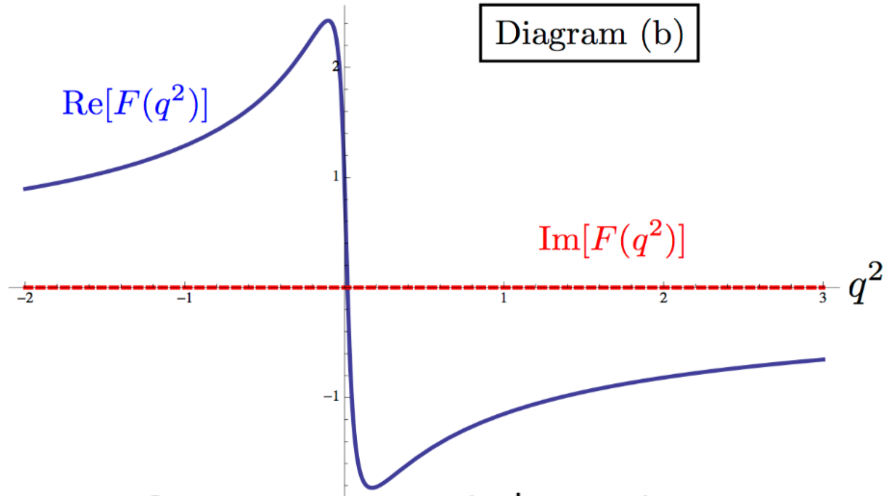
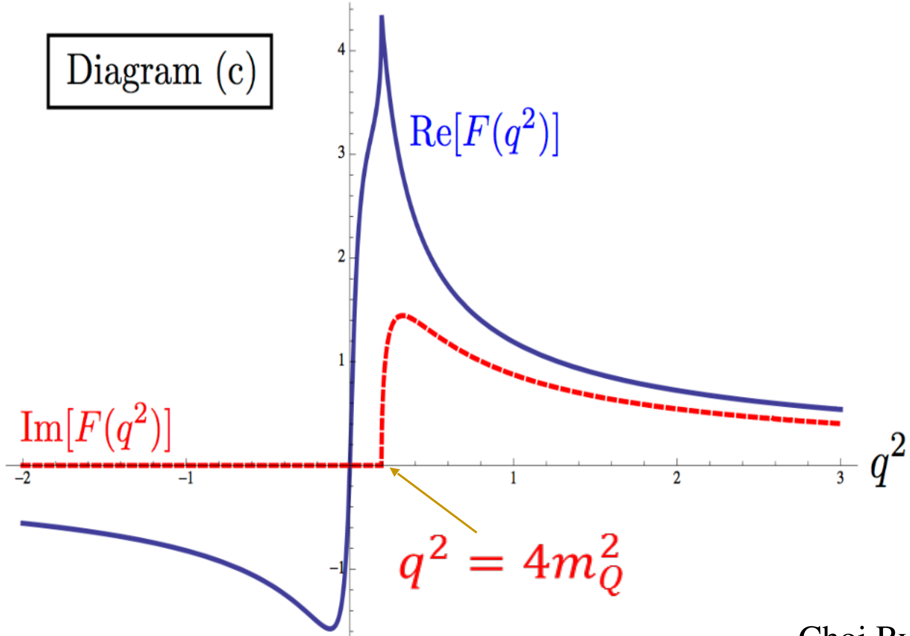
→ Imaginary part starts at $q^2_{\rm th} = 4m^2$

e.g.) Direct vs. DR Calculations



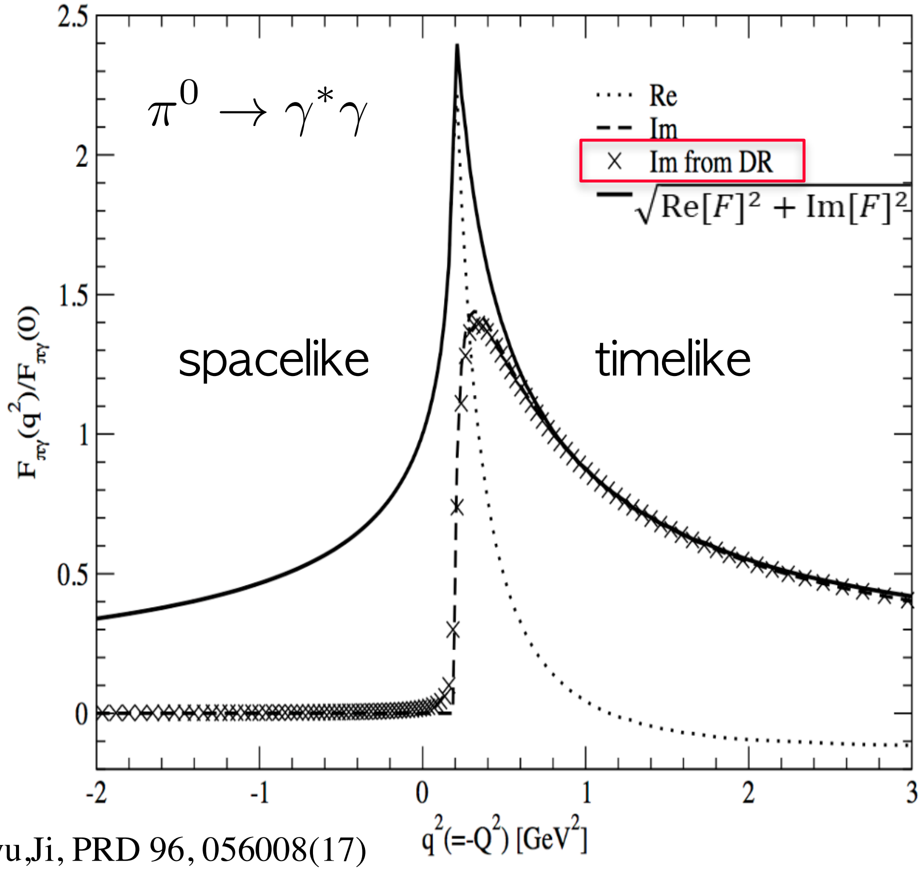
for $0 < \alpha < 1$ ($q^+ \neq 0$)



Dispersion Relation(DR) for
$F(q^2) = \mathrm{Re}\, F(q^2) + i\mathrm{Im}\, F(q^2)$:

$$\mathrm{Re}\, F(q^2) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Im}\, F(q'^2)}{q'^2 - q^2} dq'^2$$

$$\mathrm{Im}\, F(q^2) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Re}\, F(q'^2)}{q'^2 - q^2} dq'^2$$



Choi,Ryu,Ji, PRD 96, 056008(17)

$$F^{Cov}_{(a)}(q^2) = [F^{LF}_{(b)} + F^{LF}_{(c)}]_{0<\alpha<1} = [F^{LF}_{(b)}]_{\alpha=0} = [F^{LF}_{(c)}]_{\alpha=1}$$

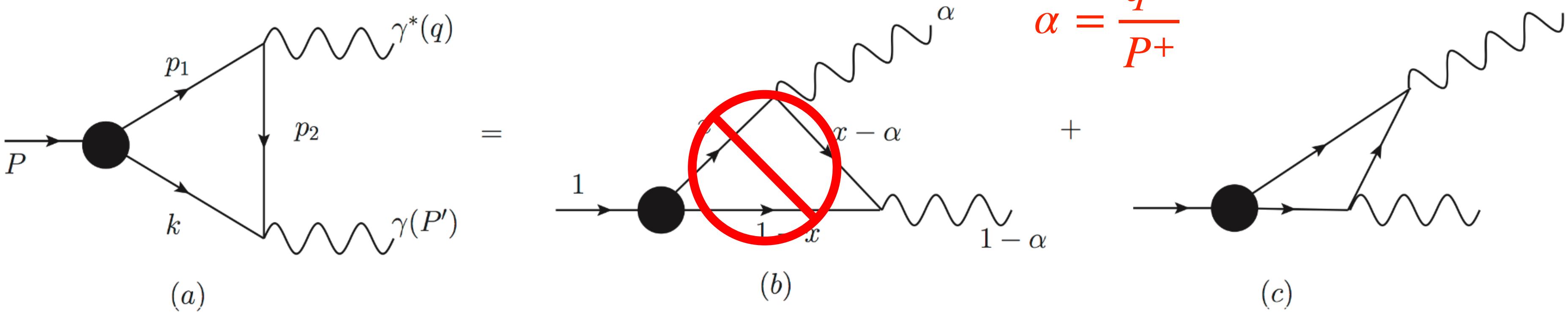
3. Application

(2) $\pi^0 \to \gamma\gamma^*$ transition form factor (TFF)

$$\langle \gamma(P-q)\,|\,J^{\mu}_{\rm em}\,|\,\pi^0(P)\rangle = ie^2 F_{\pi\gamma}(Q^2)\epsilon^{\mu\nu\rho\sigma}P_\nu \epsilon_\rho q_\sigma,$$

$$q^2 = q^+q^- - \mathbf{q}_\perp^2$$

$$\alpha = \frac{q^+}{P^+}$$



$(\alpha < x < 1)$ $(0 < x < \alpha)$

(2) For $q^+ = P^+$ with $\mathbf{q}_\perp = 0$ frame ($\alpha = 1$): timelike region, i.e. $q^2 = q^+q^- > 0$

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$$F_{\pi\gamma}(q^2) = \frac{e_u^2 - e_d^2}{\sqrt{2}} \frac{\sqrt{2N_c}}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \frac{\psi_\pi(x, \mathbf{k}_\perp)}{M_0^2 - q^2}$$

$Q^2 F_{\pi\gamma}(Q^2) \to$ constant as $Q^2 \to \infty$

3. Application

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- TFF at $Q^2 = 0$

$$\Gamma_{\pi^0 \to \gamma\gamma} = \frac{\pi}{4} \alpha_{\rm em}^2 M_\pi^3 |F_{\pi\gamma}(0)|^2$$

$$F_{\pi\gamma}^{\rm ABJ}(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi^{\rm Exp}} \simeq 0.276\ \mathrm{GeV}^{-1}$$

$$F_{\pi\gamma}^{\rm exp}(0) = 0.272(3)\ \mathrm{GeV}^{-1}$$

3. Application

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$$\boxed{F_{\pi\gamma}^{\rm chiral}(0) = \frac{\sqrt{\frac{\pi^3}{32}[\Gamma(\frac{1}{4})]^2 P_{Q\bar{Q}}}}{2\sqrt{2}\pi^2 f_\pi^{\rm chiral}}}$$

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To fit both $(F_{\pi\gamma}^{\mathrm{Exp}}(0), f_\pi^{\mathrm{Exp}})$ $\updownarrow$ $P_{Q\bar{Q}} < 0.1$ is required!

$$\boxed{F_{\pi\gamma}^{\mathrm{chiral}}(0) = \frac{\sqrt{\frac{\pi^3}{32}[\Gamma(\frac{1}{4})]^2 P_{Q\bar{Q}}}}{2\sqrt{2}\pi^2 f_\pi^{\mathrm{chiral}}}}$$

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To fit both

$(F_{\pi\gamma}^{\rm Exp}(0), f_\pi^{\rm Exp})$

$P_{Q\bar{Q}} < 0.1$ is required!

Significant higher Fock-states

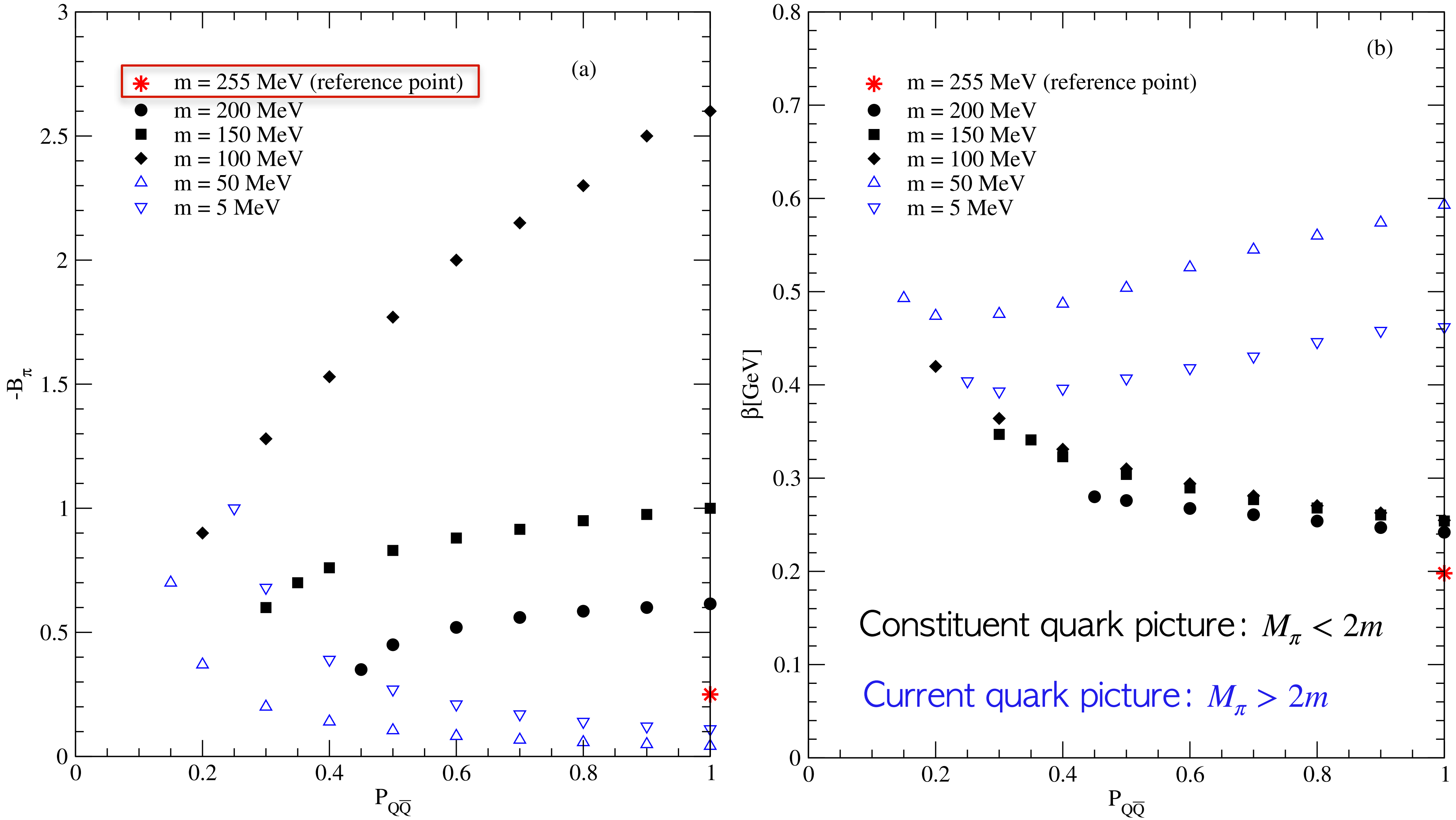
contribute in the chiral limit !

$P_{Q\bar{Q}}$ increases as m increases!

$$\boxed{F_{\pi\gamma}^{\rm chiral}(0) = \frac{\sqrt{\frac{\pi^3}{32}[\Gamma(\frac{1}{4})]^2 P_{Q\bar{Q}}}}{2\sqrt{2}\pi^2 f_\pi^{\rm chiral}}}$$

4. Numerical Results

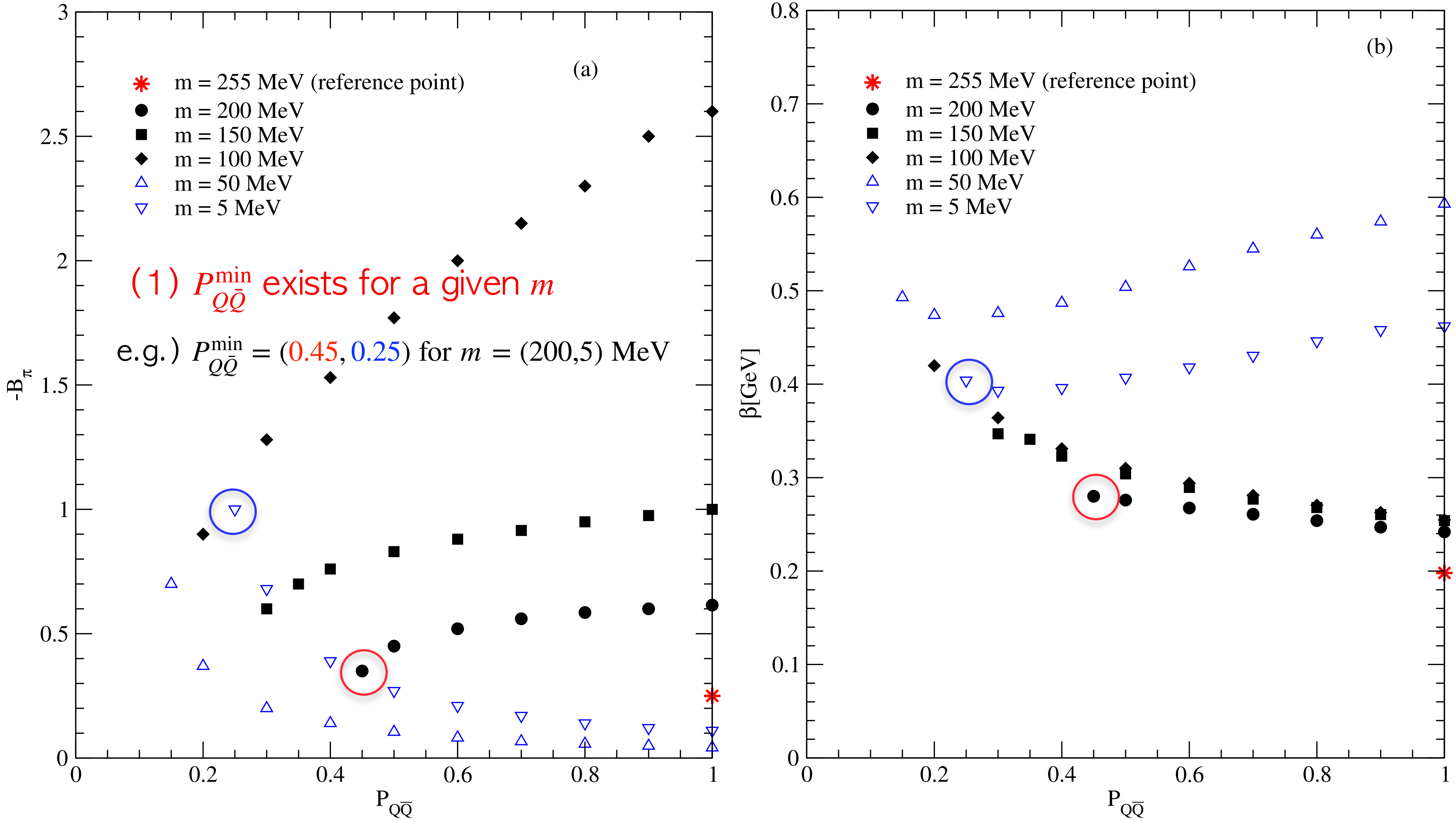
- Possible solution sets for $(-B_\pi \; vs \; P_{Q\bar{Q}})$ and $(\beta \; vs \; P_{Q\bar{Q}})$ satisfying both f_π^{Exp} and $F_{\pi\gamma}^{\mathrm{Exp}}(0)$.



c.f.) $(M_\pi, m) = (135,5)$ MeV satisfies the GMOR relation $M_\pi^2 f_\pi^2 = -2(m_q + m_{\bar{q}})\langle q\bar{q}\rangle$

4. Numerical Results

- Our main findings for the model parameters:



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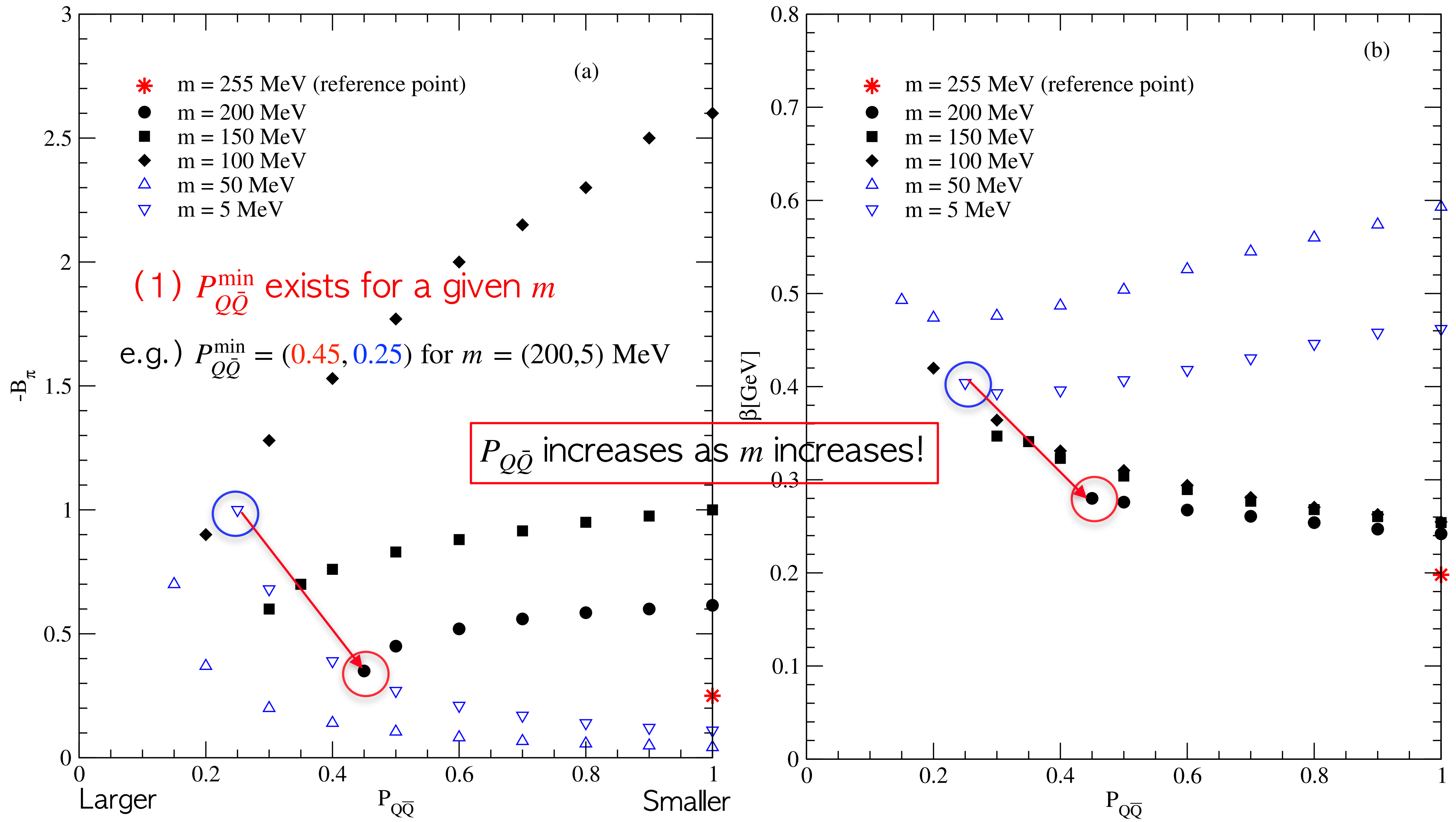

Larger higher Fock contribution

Smaller higher Fock contribution

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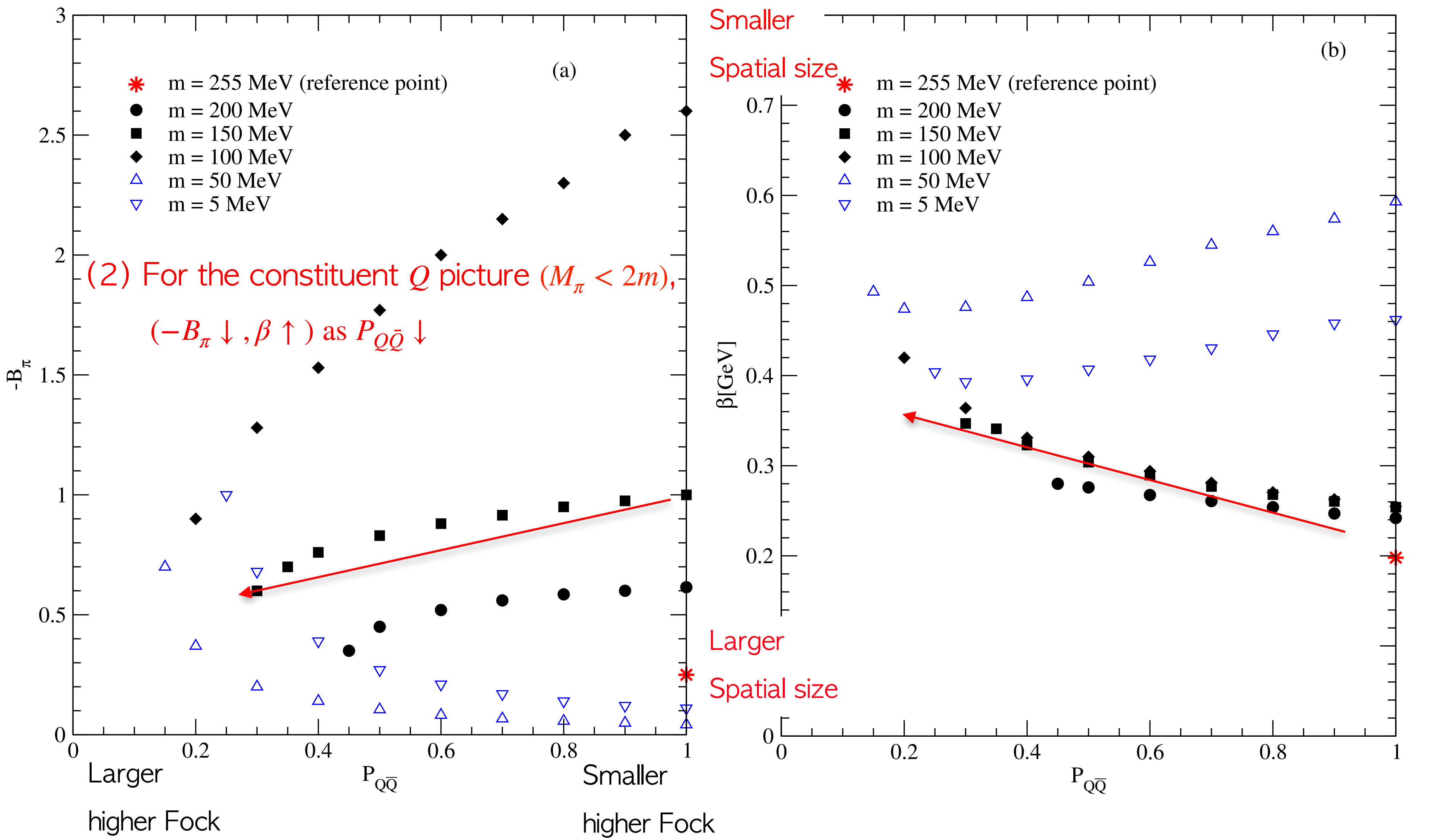

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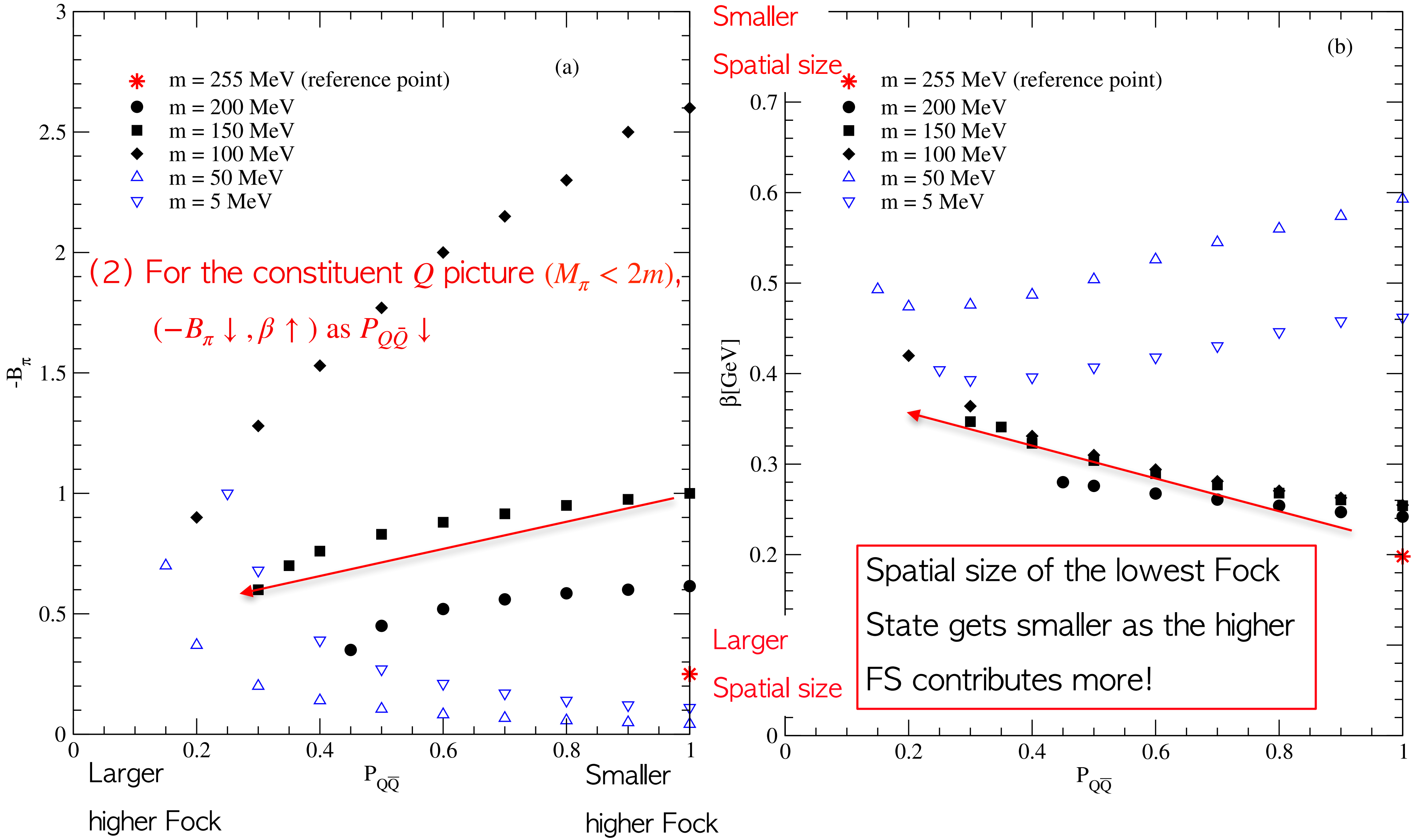

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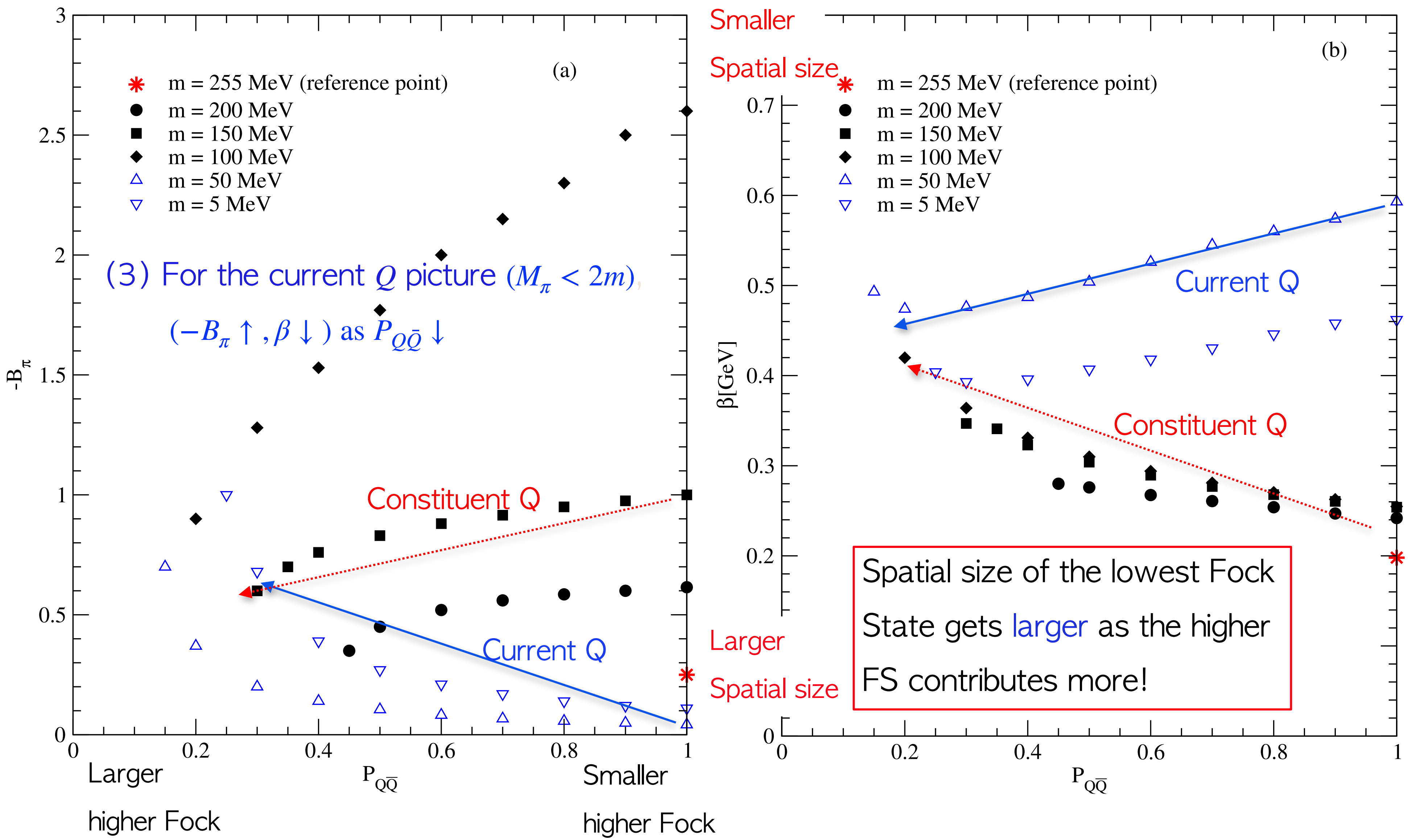
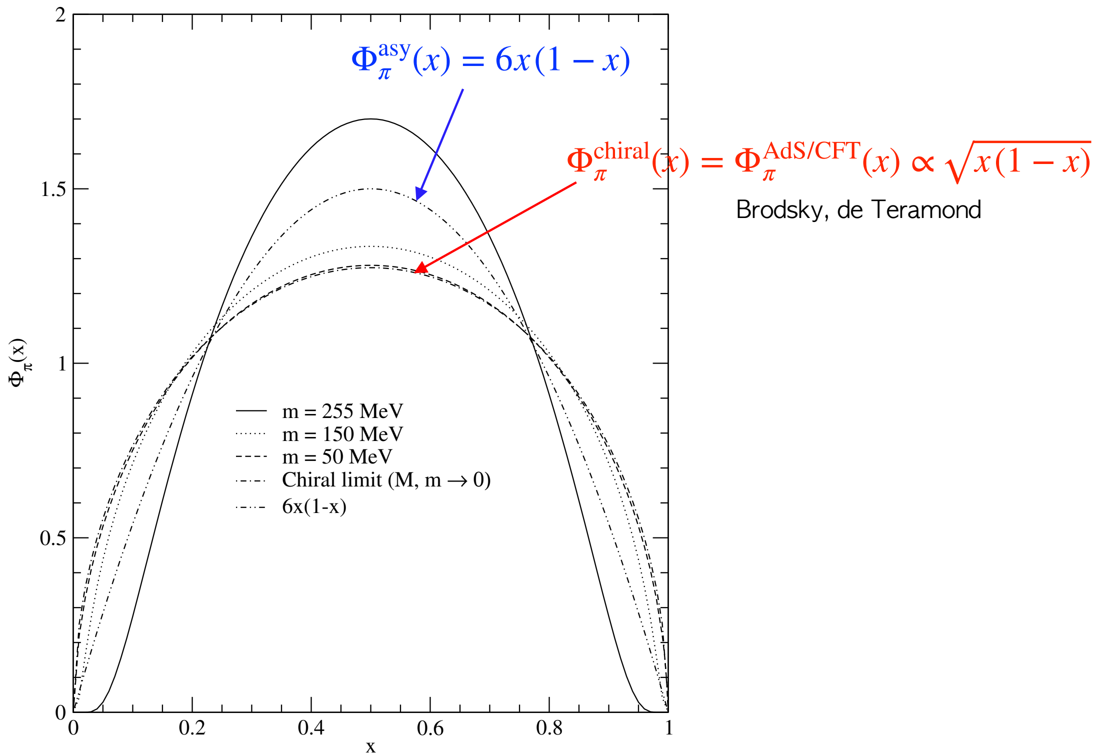
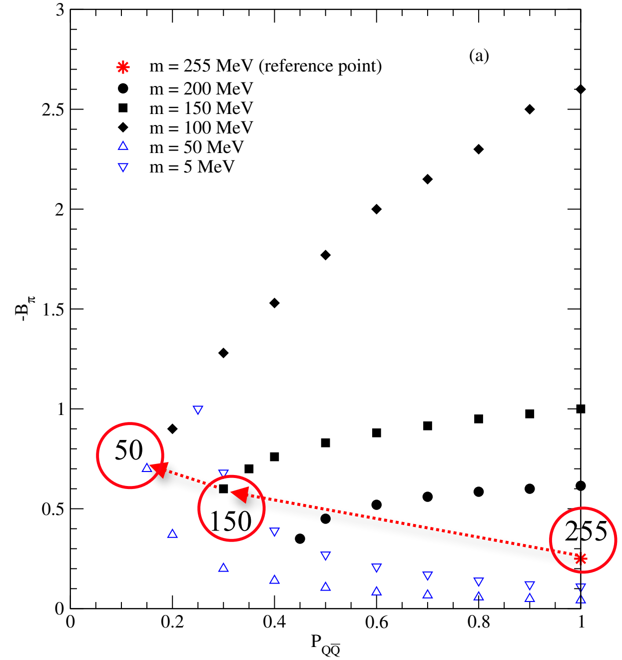


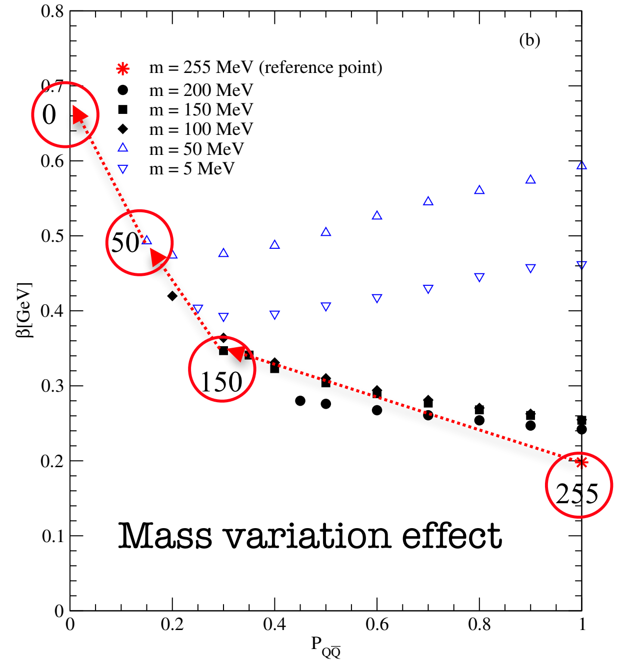
4. Numerical Results

TABLE I: Model parameters (B_π, β) depending on the variation of (M_π, m) and $P_{Q\bar{Q}}$. We denote (M_π, m, β, f_π) in unit of MeV.

(M_π, m)	$P_{Q\bar{Q}}$	B_π	β	$f_\pi^{\rm Th}$	$F_{\pi\gamma}^{\rm Th}(0)$ [GeV^{-1}]
(135,255)	1	-0.25	198.0	130.4	0.271
(135,150)	0.3	-0.60	346.9	130.6	0.272
(135,50)	0.15	-0.7	493.0	130.7	0.271
(0,0)	0.078	< 0	668.5	130.9	0.276
Exp. [44]	–	–	–	130.2(1.7)	0.272(3)







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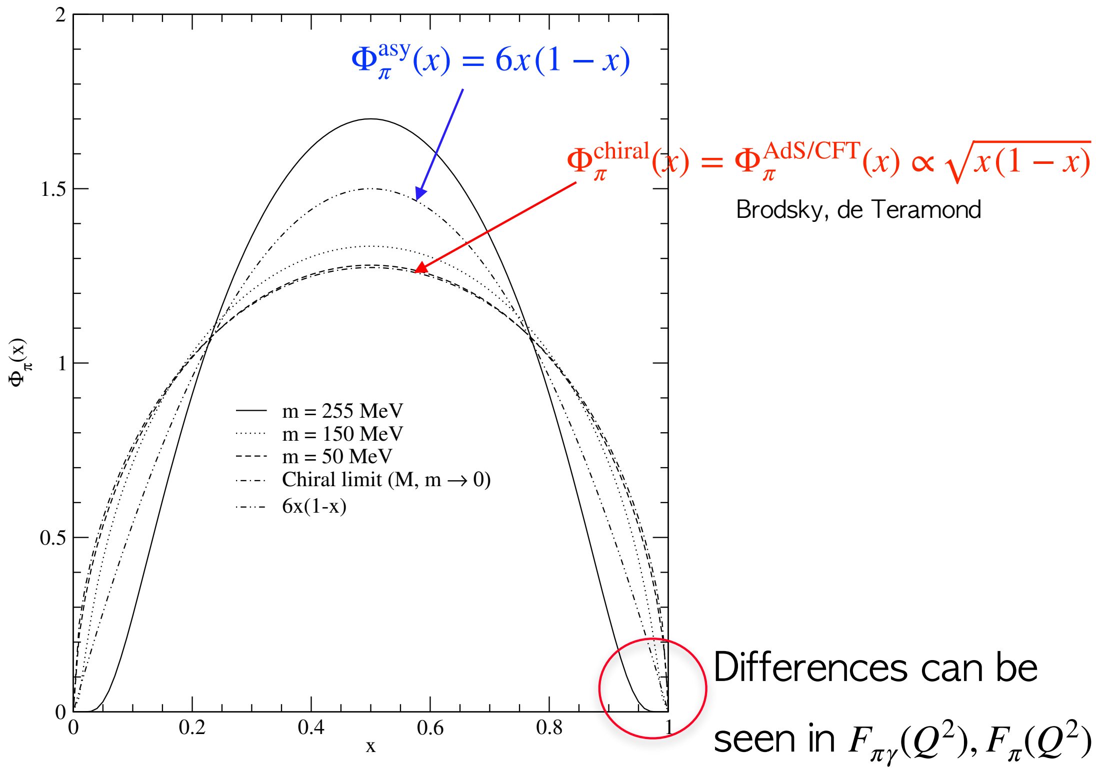
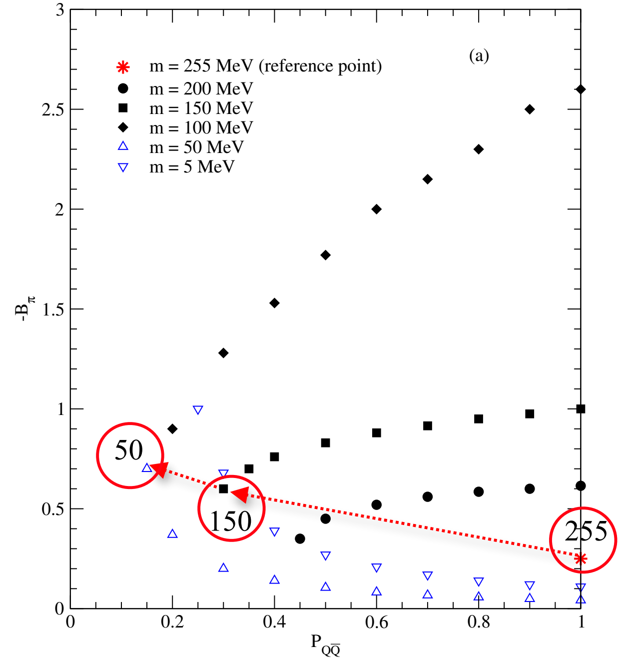
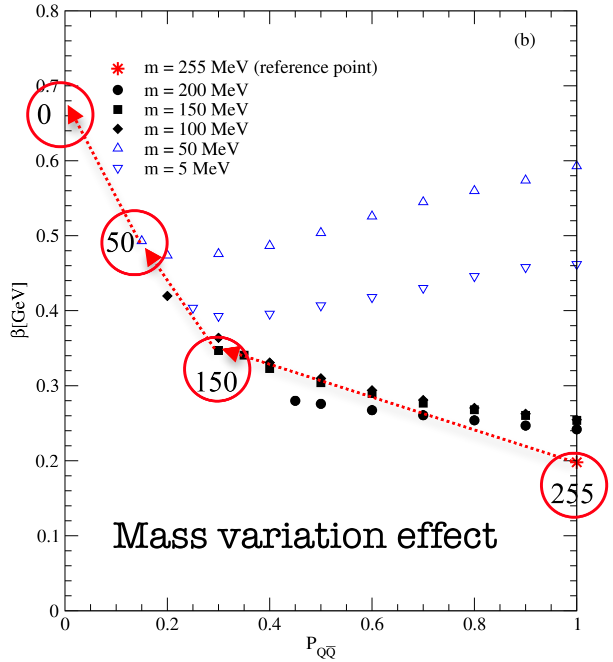


Brodsky, de Teramond

Differences can be seen in $F_{\pi\gamma}(Q^2), F_\pi(Q^2)$





- Estimation of the quark mass variation effect on Q^2 evolution of $F_{\pi\gamma}(Q^2)$ and $F_\pi(Q^2)$

As a first attempt to estimate the quark mass variation effect, we use the mixing between $m_{\rm ref}$ and $m(<m_{\rm ref})$ via

$$\langle \Psi^\pi_{m'} | \Psi^\pi_m \rangle = \delta_{m'm}\sqrt{P_{m'}P_m} = \delta_{m'm}P_m \qquad m_{\rm ref} = m = 255 \text{ MeV and } P_{Q\bar{Q}} = P_{m_{\rm ref}} = 1$$

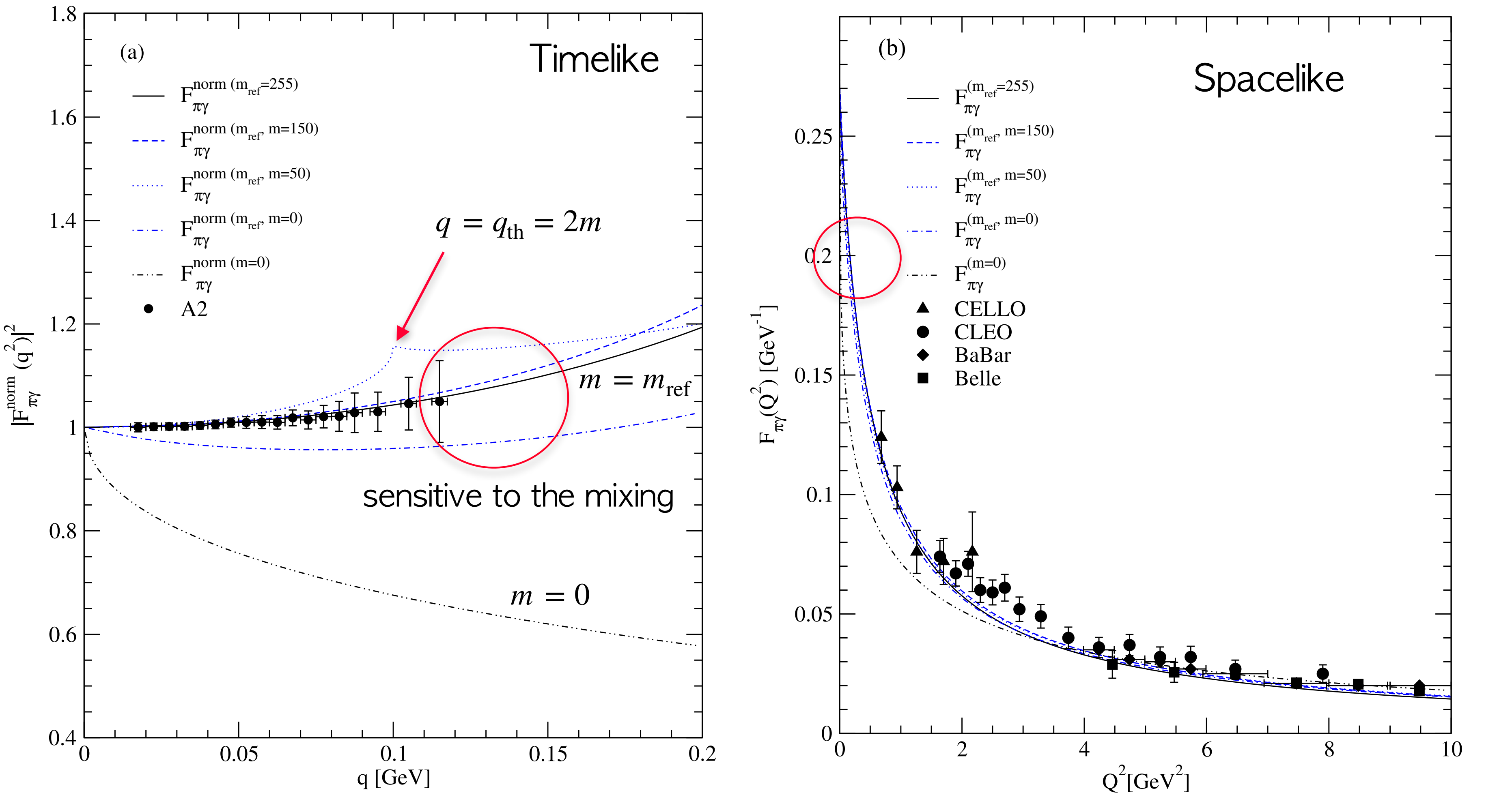
e.g.) Prescription of the mixing between $m_{\rm ref}$ and $m = 150$ MeV

$$F^{(m_{\rm ref}, m=150)}_{\pi\gamma}(Q^2) = \frac{\sqrt{1-\tilde{P}_m}F^{(m_{\rm ref})}_{\pi\gamma}(Q^2) + \sqrt{\tilde{P}_m}F^{(m=150)}_{\pi\gamma}(Q^2)}{\sqrt{1-\tilde{P}_m} + \sqrt{\tilde{P}_m}}, \text{ with } F^{(m_{\rm ref}, m)}_{\pi\gamma}(0) = F^{\rm Exp}_{\pi\gamma}(0)\,.$$

$$F^{(m_{\rm ref}, m=150)}_{\pi}(Q^2) = (1-\tilde{P}_m)F^{(m_{\rm ref})}_{\pi}(Q^2) + \tilde{P}_m F^{(m=150)}_{\pi}(Q^2), \text{ with } F^{(m_{\rm ref}, m)}_{\pi}(0) = 1.$$

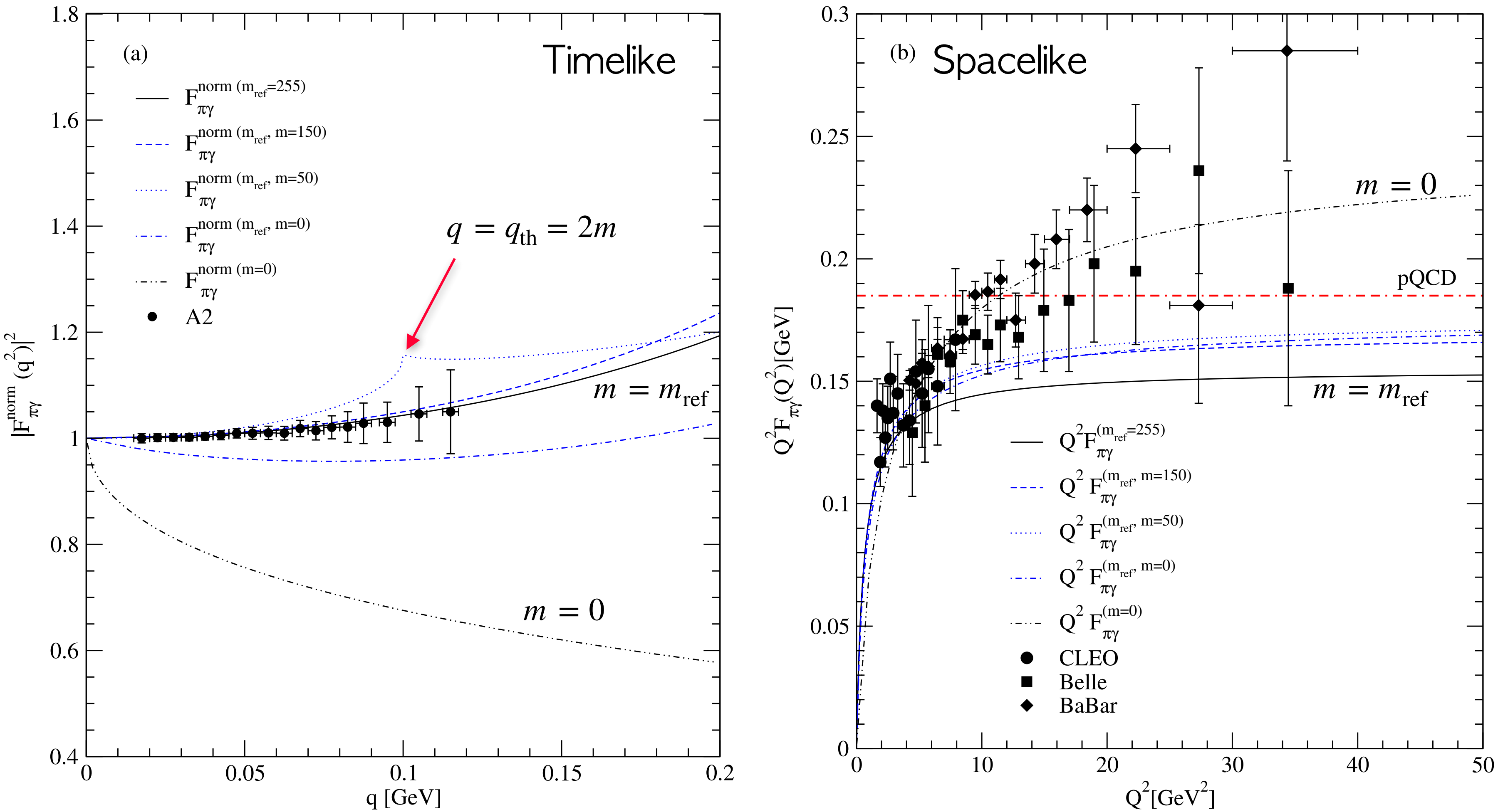
$$\tilde{P}_m = \frac{P_m}{(P_{m_{\rm ref}} + P_m)} = \frac{0.3}{1.3} \approx 0.23 \ \ : \text{renormalized probability}$$

• Quark mass variation effect on $F_{\pi\gamma}(Q^2)$



• In constraining the model parameters, the analysis in timelike region is important.

- Quark mass variation effect on $F_{\pi\gamma}(Q^2)$



The standard LFQM prediction with the invariant mass scheme.

$$\chi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) \propto \bar{u}_{\lambda_1}(k_1)\gamma_5 v_{\lambda_2}(k_2)$$

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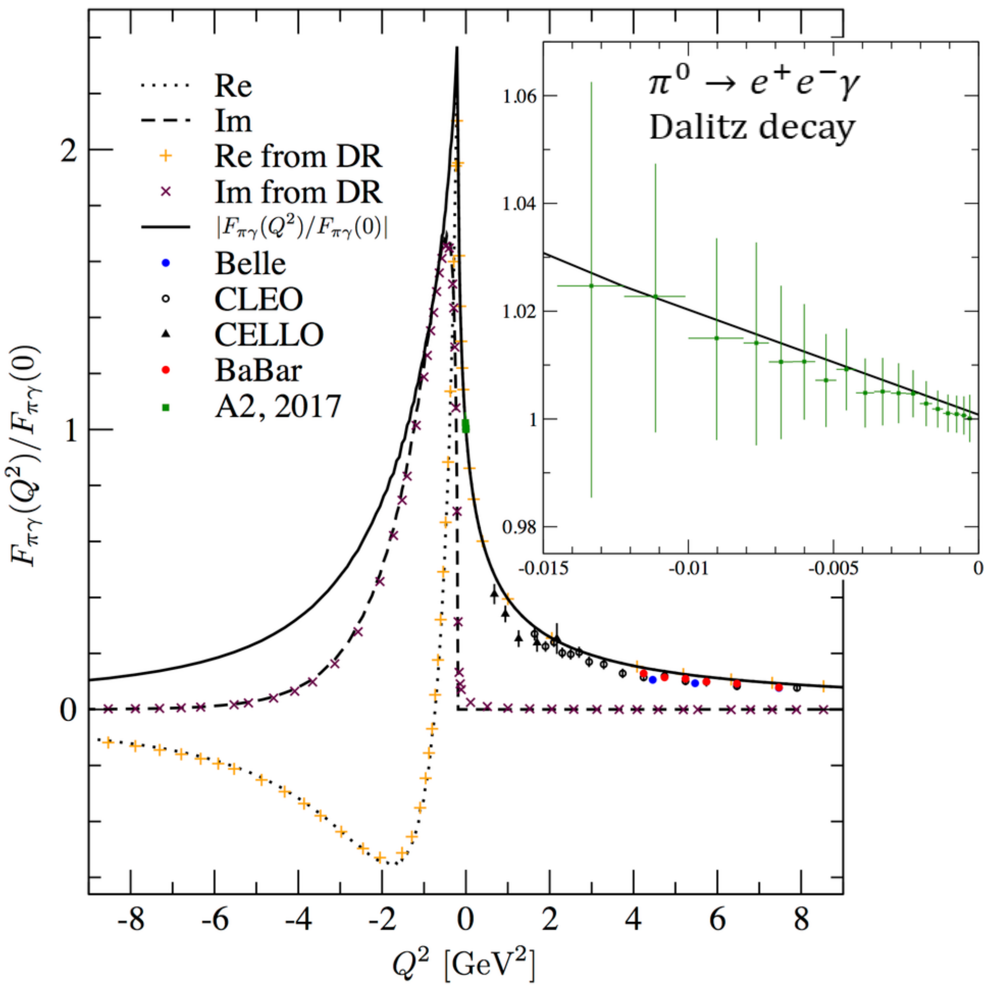
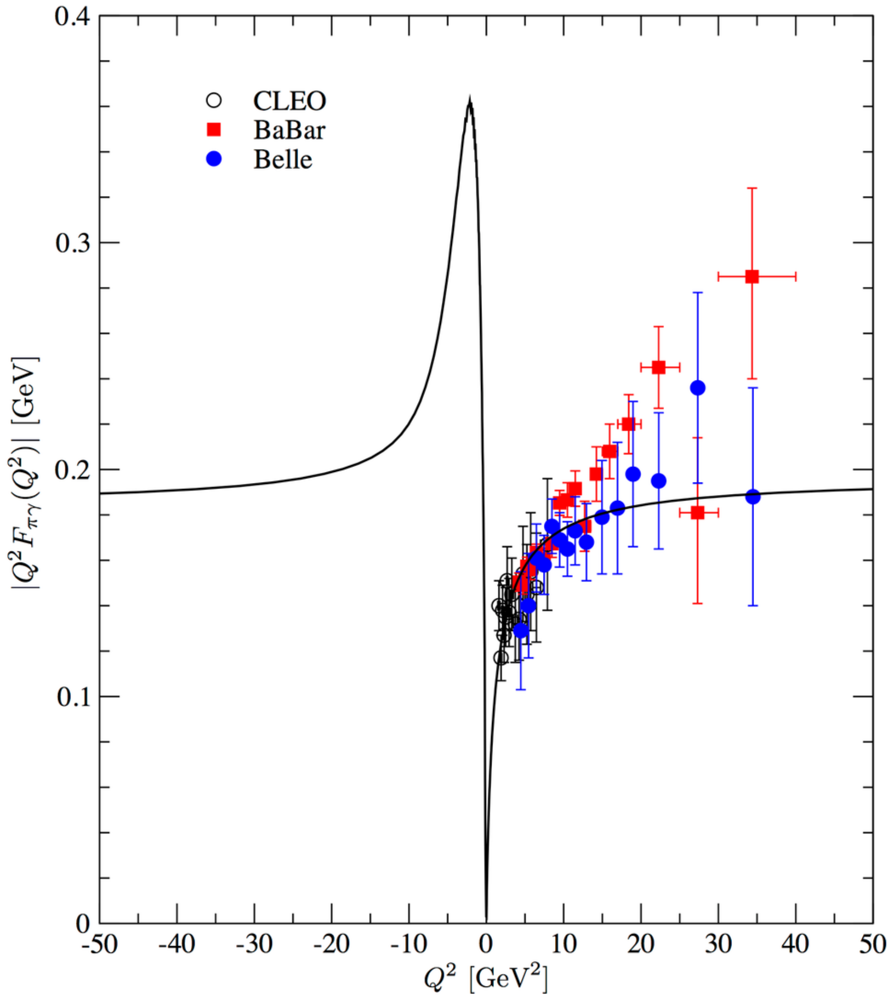
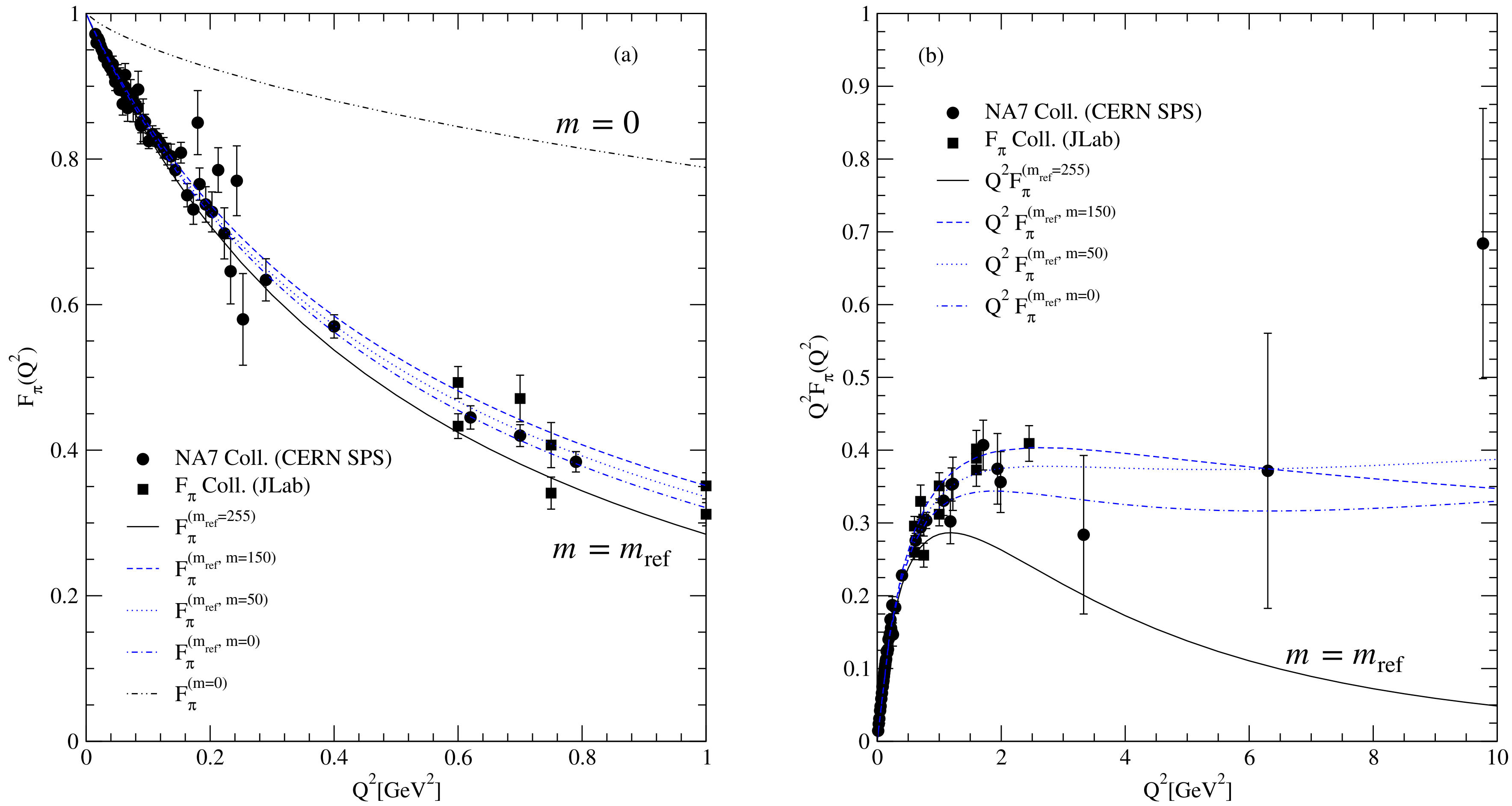




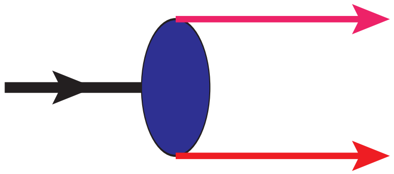
• Quark mass variation effect on $F_\pi(Q^2)$



Shows the necessity of the quark mass evolution effect

5. Conclusion

- We explored the link between the chiral sym. of QCD and the numerical results of the LFQM analyzing $f_\pi, F_{\pi\gamma}(Q^2), F_\pi(Q^2)$.



$$\Gamma_\pi = (M_\pi + B_\pi \not{P})\gamma_5$$

- Axial-vector coupling with $B_\pi < 0$ is essential to describe the correct chiral limit expression in the LFQM.

- Our chiral limit results for f_π and $\phi_\pi(x)$ are exactly the same as AdS/CFT predictions.

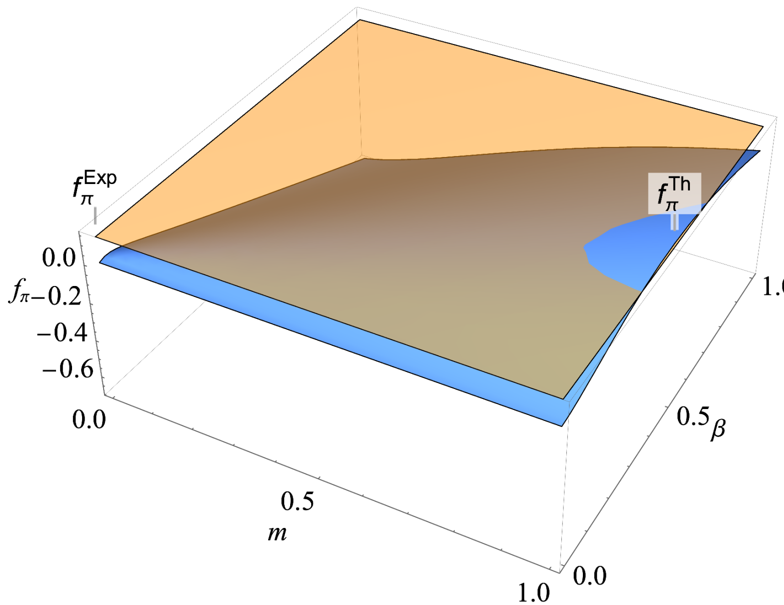
- In constraining the model parameters, we found that the chiral anomaly plays a critical role and the analysis of $F_{\pi\gamma}(q^2)$ in timelike region is important.

- Our results indicate that the constituent quark picture is very effective in describing both $F_{\pi\gamma}(Q^2), F_\pi(Q^2)$ in the low energy regime, but the quark mass evolution seems inevitable as Q^2 grows.

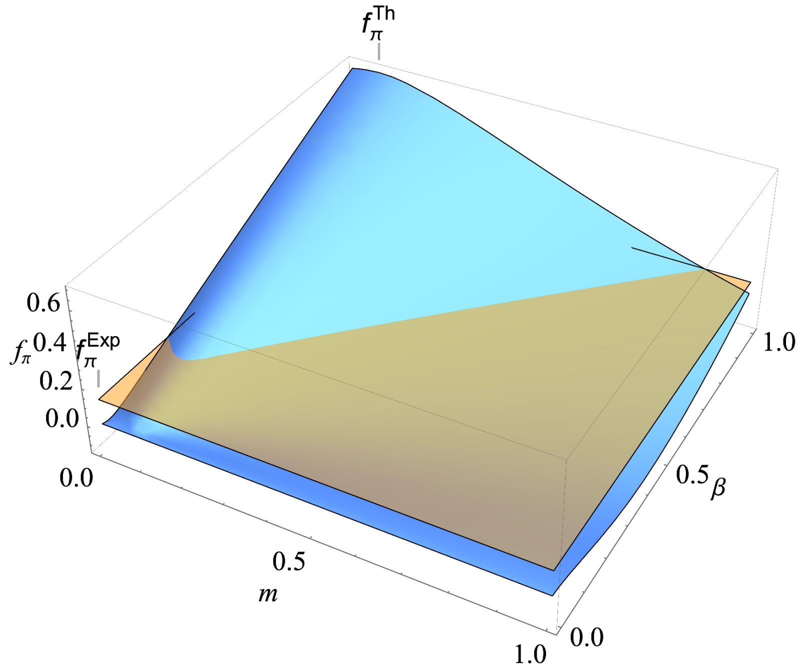
4. Numerical Results

- Check sign problem of B_π in $\Gamma_\pi = (M_\pi + B_\pi \not{P})\gamma_5$

$B_\pi = 1$



$\boxed{B_\pi = -1}$



- Possible solution sets for (m, β) satisfying $f_\pi^{\mathrm{Th}} = f_\pi^{\mathrm{Exp}} = 130.2(2)$ MeV

4. Numerical Results

- Check sign problem of B_π in $\Gamma_\pi = (M_\pi + B_\pi \not{P})\gamma_5$

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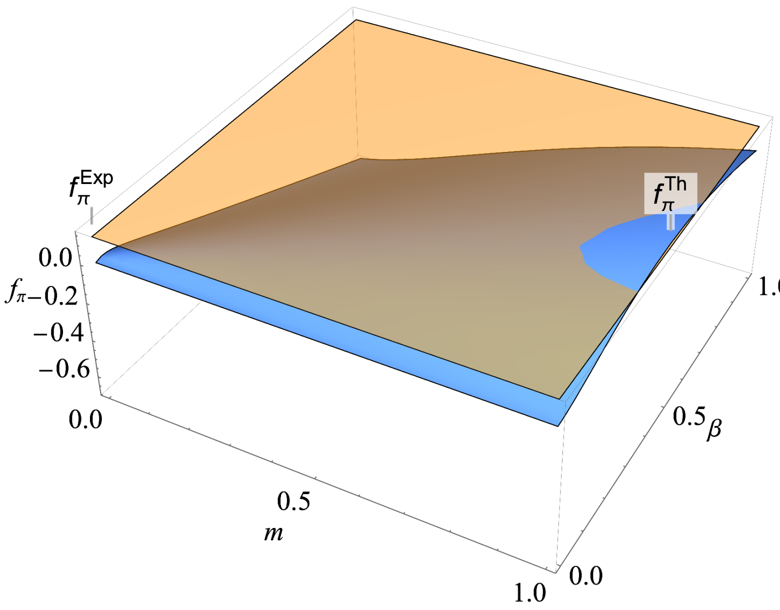

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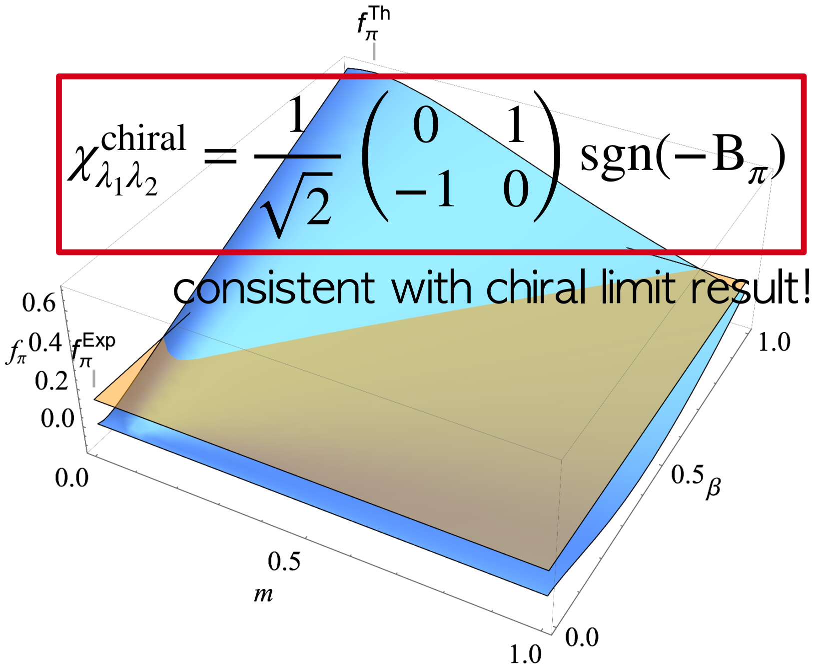

$$\boxed{\chi^{\text{chiral}}_{\lambda_1\lambda_2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathrm{sgn}(-\mathrm{B}_\pi)}$$

consistent with chiral limit result!

- Possible solution sets for (m, β) satisfying $f_\pi^{\mathrm{Th}} = f_\pi^{\mathrm{Exp}} = 130.2(2)\ \mathrm{MeV}$