Light-Front Quark Model Analysis of Radially Excited Pseudo-scalar and Vector Mesons

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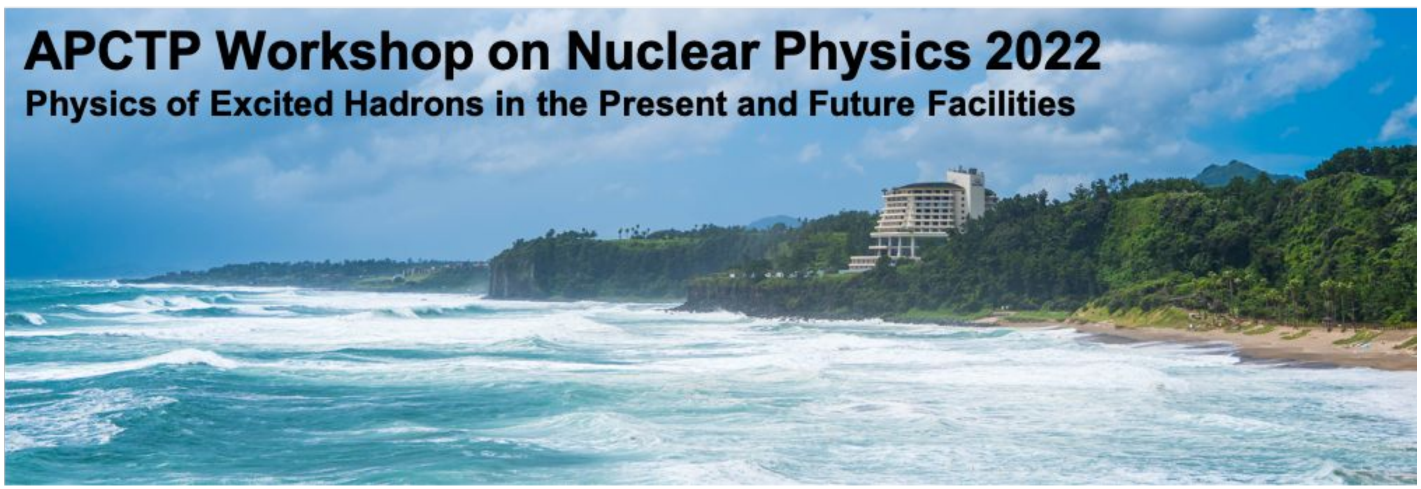


July 15, 2022

Outline

- Link between QCD and LFQM in ‘tHooft Model
- LFQM Framework
  - Bakamjian-Thomas Construction
- LFQM for Spectroscopy and Phenomenology
  - Brief History of Progress
- Universality of Intrinsic Physical Observables
  - Decay Constant vs. Distribution Amplitude
- Application to Radially Excited 0^- and 1^- Mesons
  - Mixing of 1S and 2S Basis States

Large N_C QCD in 1+1 dim. ('tHooft Model)

G,'tHooft, NPB75,461(74)

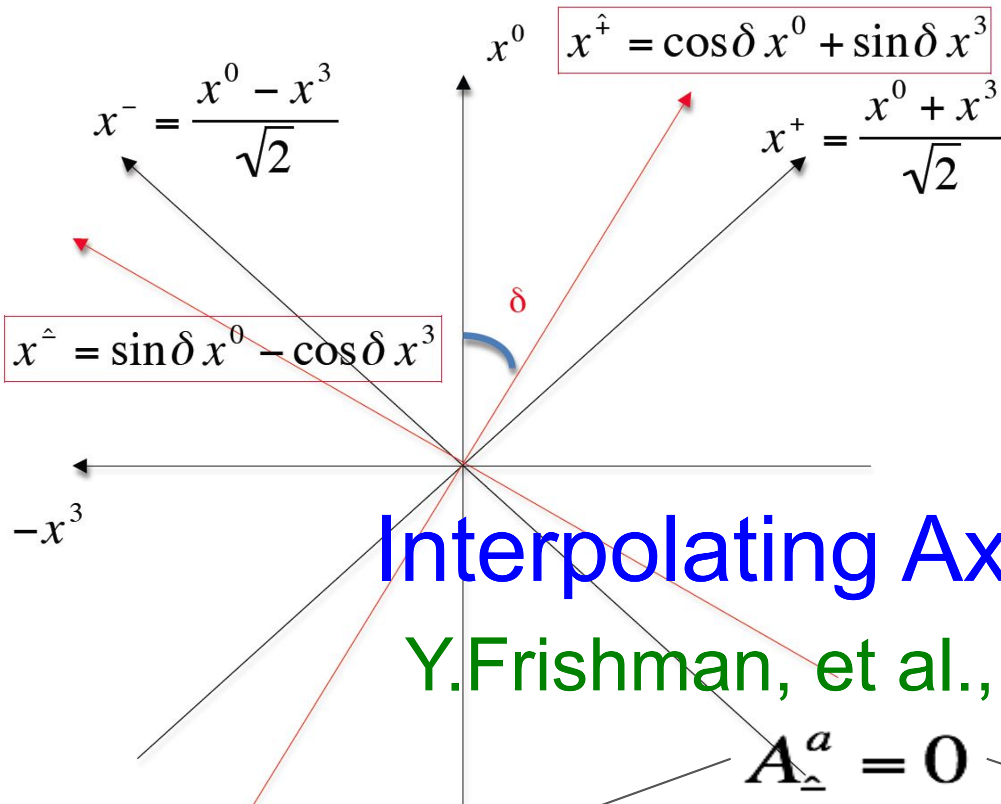
$$\mathcal{L} = -\frac{1}{4} F^a_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + g f^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

'tHooft Coupling $\lambda = \frac{g^2 (N_c - 1/N_c)}{4\pi}$ **and mass** m

$$g \to 0, N_C \to \infty; \lambda \to finite$$



Interpolating Axial Gauge

Y.Frishman, et al., PRD15(75)

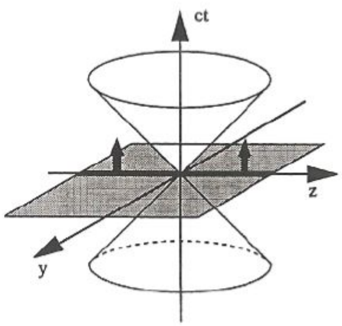
$$\boxed{\begin{array}{c} (IFD) \quad 0 \le \delta \le \frac{\pi}{4} \quad (LFD) \\ 1 \ge \quad C \equiv \cos(2\delta) \quad \ge 0 \end{array}}$$

$$A^a_{\hat{-}} = 0$$

$$A^a_1 = 0 \qquad\qquad A^+_a = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\hat{-}} A^a_{\hat{+}})^2 + \bar{\psi}(i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}}\partial_{\hat{-}} - m)\psi$$

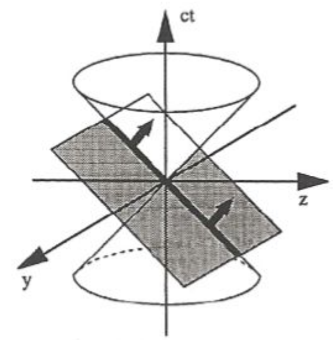
Dirac's Proposition for Relativistic Dynamics



The instant form

1949

$$
\begin{array}{ccc}
Equal\ t & & Equal\ \tau \\
p^0 & \leftrightarrow & p^- = p^0 - p^3 \\
(p^1, p^2) & \leftrightarrow & \vec{p}_\perp \\
p^3 & \leftrightarrow & p^+ = p^0 + p^3
\end{array}
$$

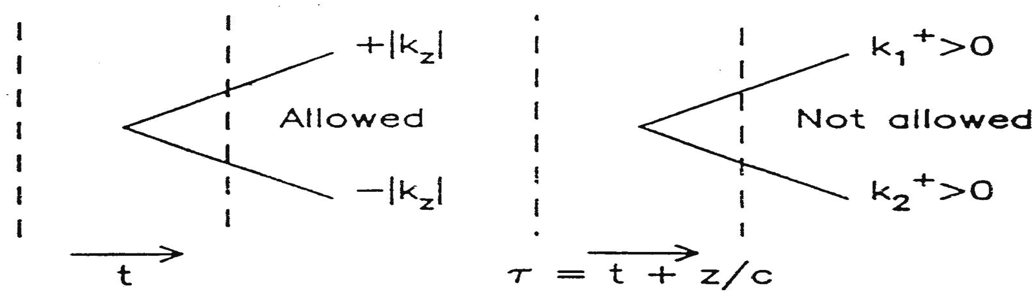


The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



(a) (b)

Except zero-modes

$$k_1^+ = k_2^+ = 0$$

IFD

Instant Form Dynamics

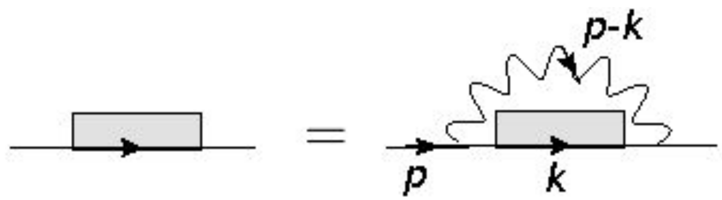
LFD

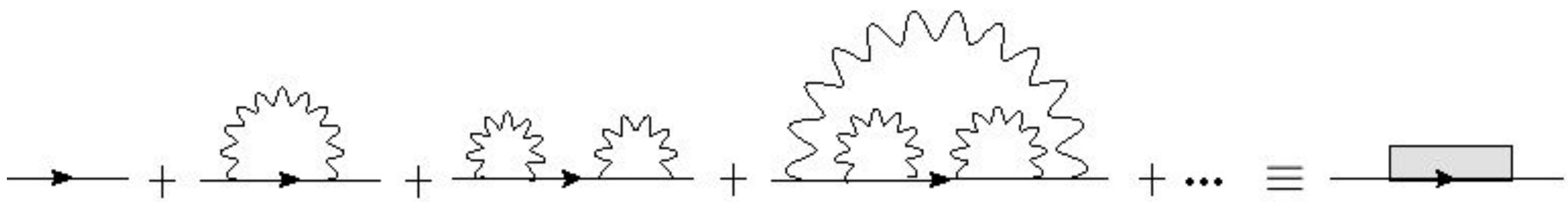
Light-Front Dynamics

Interpolating 'tHooft model between instant and front forms

B.Ma and C.Ji, PRD194,036004(2021)

Mass Gap Equation

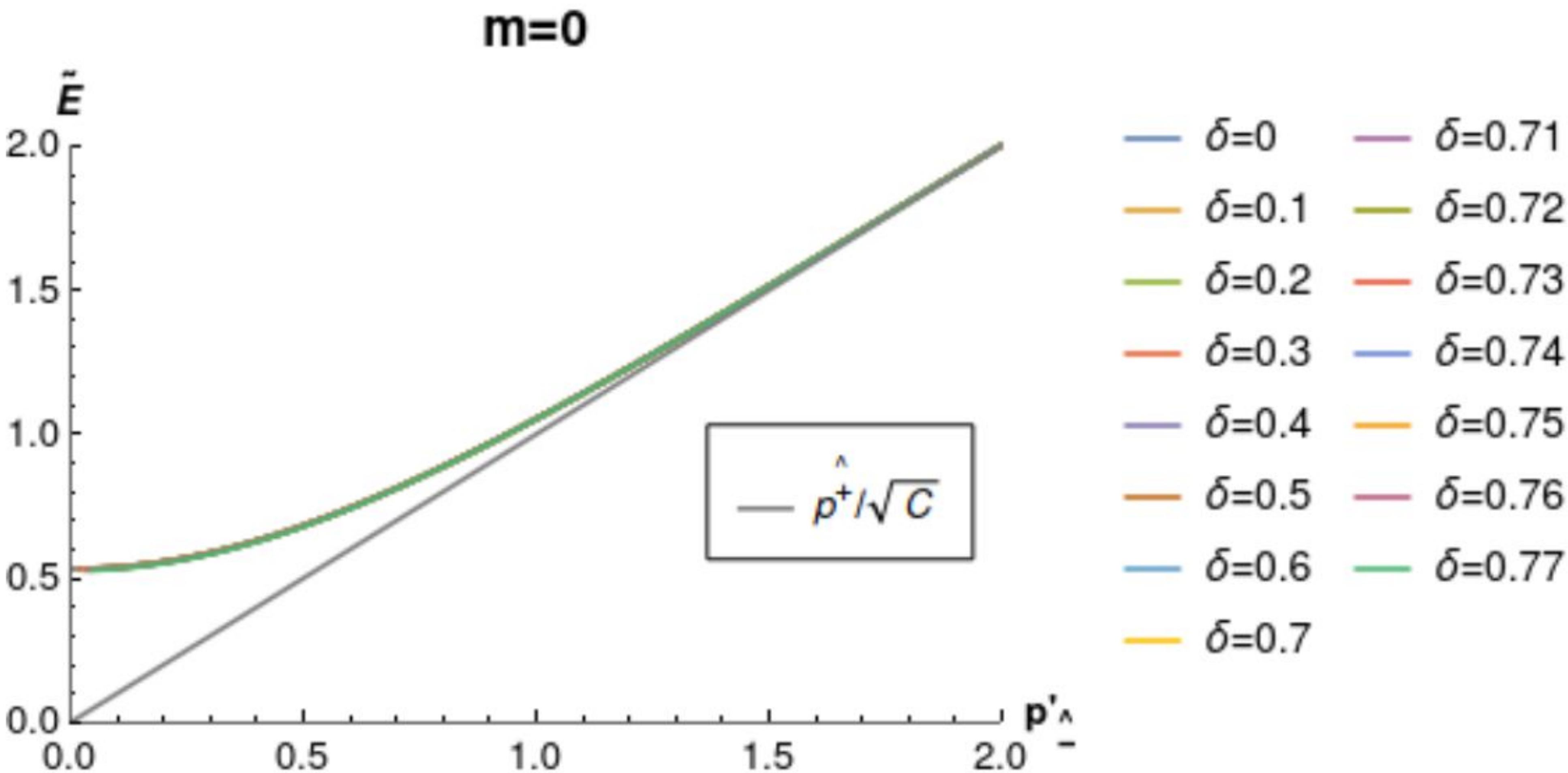




$\equiv i\,\Sigma(p)$

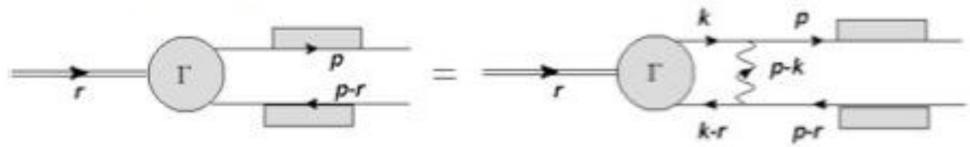
$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}}\frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

Mass Gap Solution

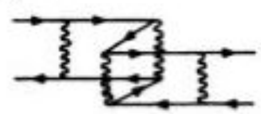


BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\fint \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}}-k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$



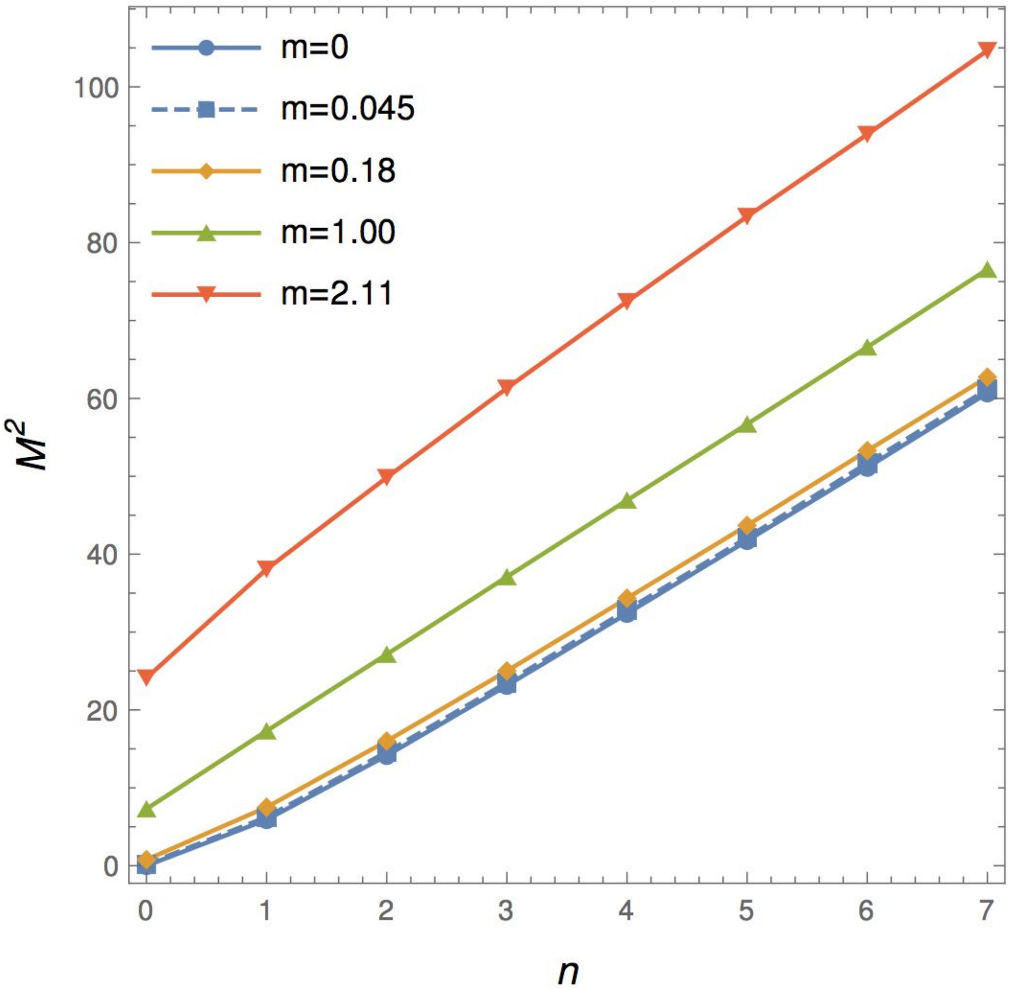
$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}}-r_{\hat{-}}) + E(p_{\hat{-}}-r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}},p_{\hat{-}})$$
$$= \lambda\fint \frac{dk_{\hat{-}}}{(p_{\hat{-}}-k_{\hat{-}})^2}\left[C(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}},k_{\hat{-}}) - S(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}},k_{\hat{-}})\right],$$
$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}}-r_{\hat{-}}) + E(p_{\hat{-}}-r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}},p_{\hat{-}})$$
$$= \lambda\fint \frac{dk_{\hat{-}}}{(p_{\hat{-}}-k_{\hat{-}})^2}\left[C(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}},k_{\hat{-}}) - S(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}},k_{\hat{-}})\right].$$



LFD $$\left[\mathcal{M}^2 - \frac{m^2-2\lambda}{x} - \frac{m^2-2\lambda}{1-x}\right]\phi(x) = -2\lambda\fint_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

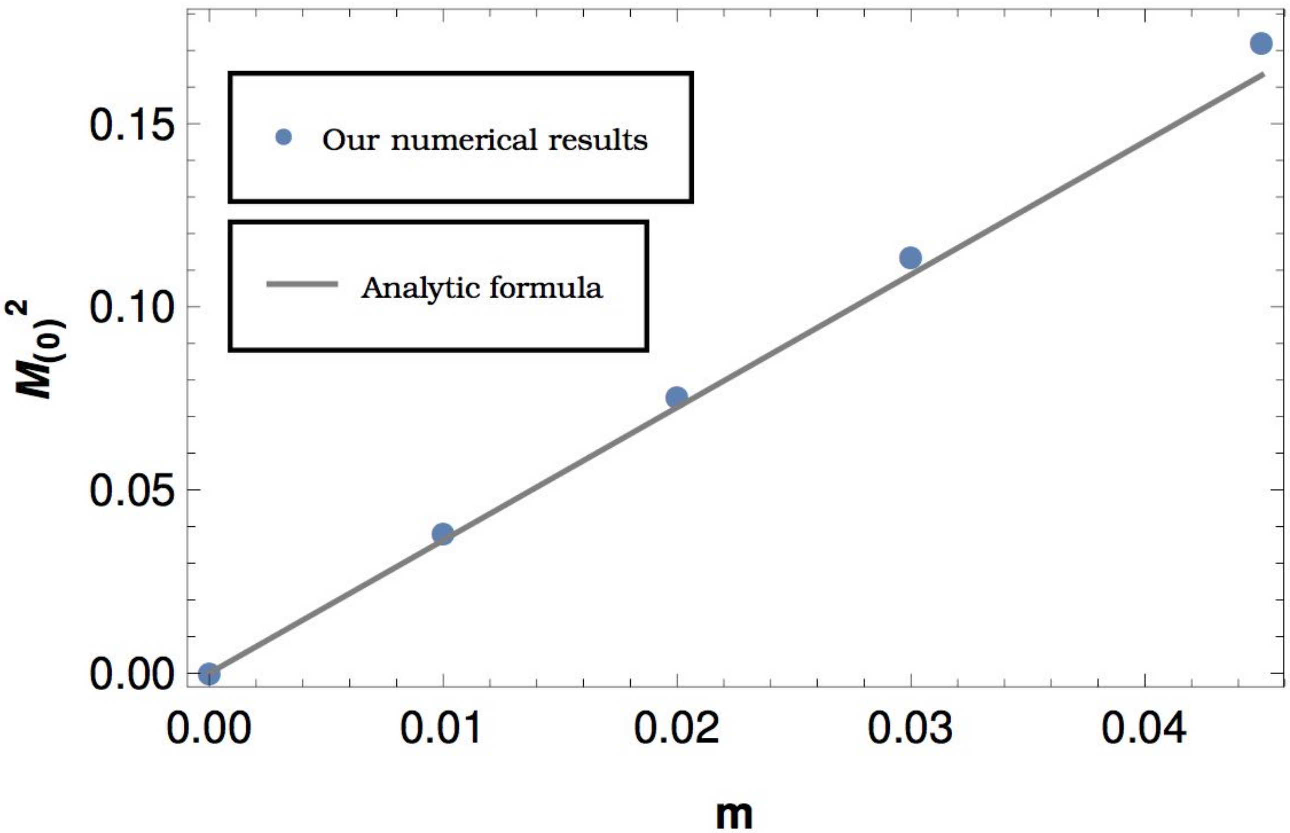
Meson Spectroscopy

- G.'tHooft, NPB75, 461(74) - LFD



- M.Li, et al., JPG13, 915(87) - IFD (rest frame)
- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)

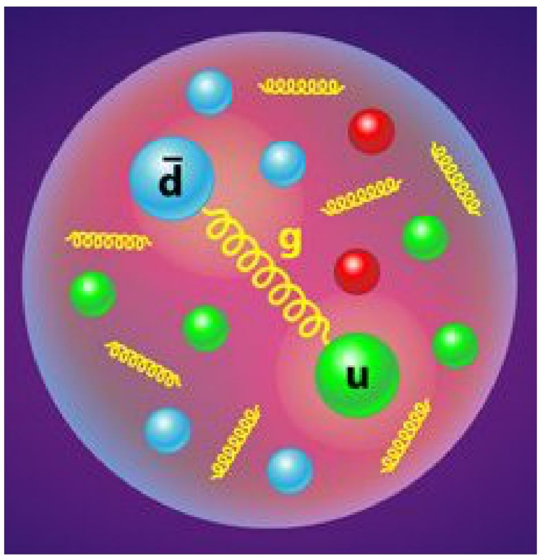
Gell-Mann - Oaks - Renner Relation



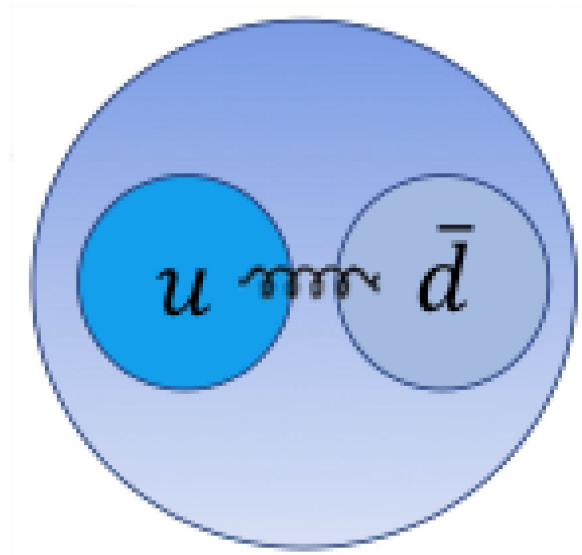
$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

Take advantage of LFD and Construct the Light-Front Quark Model (LFQM)



QCD



LFQM

Bakamjian-Thomas Construction in LFD

B. Bakamjian and L. H. Thomas, Phys. Rev. 92, 1300 (1953).

Add interactions to the non-interacting representations without spoiling the Poincare Algebra satisfied by the interacting physical system.

$$M := M_0 + V$$

with

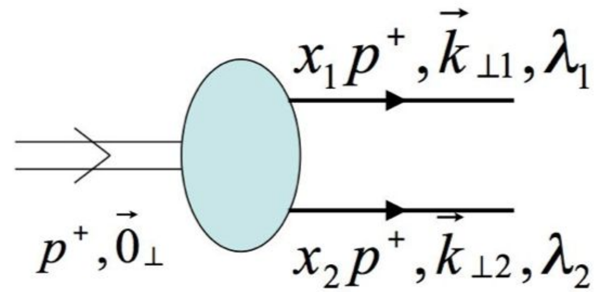
$$[\mathbf{E}_\perp, V]_- = [K^3, V]_- = [\mathbf{j}_{f0}, V]_- = [\mathbf{P}_\perp, V]_- = [P^+, V]_- = 0.$$

Effective Constituent Quark Model for Low Q^2

$$|Meson\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \ldots$$
$$\approx \Psi_{Q\bar{Q}}|Q\bar{Q}\rangle,$$

where

$$|Q\rangle = \psi^{Q}_{q}|q\rangle + \psi^{Q}_{qg}|qg\rangle + \ldots$$
$$|\bar{Q}\rangle = \psi^{\bar{Q}}_{\bar{q}}|\bar{q}\rangle + \psi^{\bar{Q}}_{\bar{q}g}|\bar{q}g\rangle + \ldots$$



$$\Psi_{Q\bar{Q}}(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i})\chi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Radial
(Dependent on the model potential)

H = T + V
V includes Coulomb, Confinement, Spin-Spin,Spin-Orbit interactions.

Spin-Orbit
(Interaction independent Melosh transformation)

$$J^{PC} = 0^{++}(f_0, a_0, \ldots)$$
$$0^{-+}(\pi, K, \eta, \eta', \ldots)$$
$$1^{--}(\rho, K^*, \omega, \phi, \ldots)$$
$$\ldots$$

LFQM for Spectroscopy and Phenomenology

- H.-M.Choi & C.Ji, PRD59,074015('99)
- H.-M.Choi, NCSU PhD Dissertation,hep-ph/991127
- Radiative,Semileptonic,Nonleptonic Decays,etc.
- H.-M.Choi,C.Ji,Z.Li & H.-Y.Ryu,PRC92,055203('15)
- Consistency with Chiral Symmetry
- Twist 2 and 3 DAs of 0^- and 1^- Mesons
- H.-M.Choi,H.-Y.Ryu & C.Ji,PRD96,056008('17)
- Spacelike and Timelike Form Factors
- H.-M.Choi & C.Ji, PRD102,036005('20)
- Chiral Anomaly and Pion Properties (H.-M.Choi's talk S-9)
- A.J.Arifi,H.-M.Choi,C.Ji &Y.Oh,2205.04075[hep-ph],PRD('22)
- Radially Excited 0^- and 1^- Mesons (A.J.Arifi's talk S-12)

PHYSICAL REVIEW C **92**, 055203 (2015)

Variational analysis of mass spectra and decay constants ···

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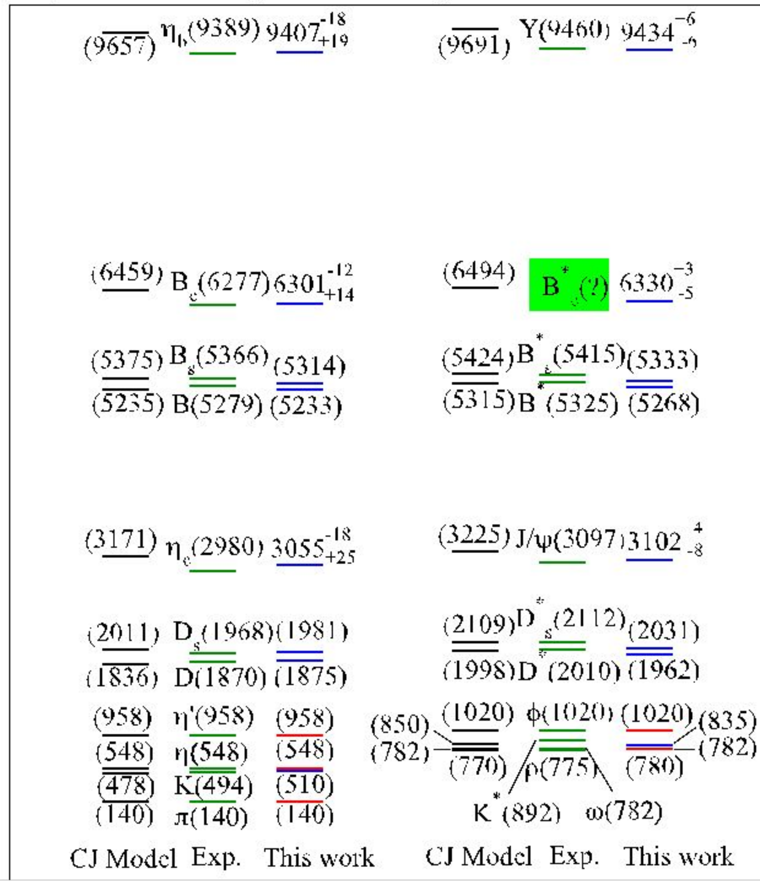

PHYSICAL REVIEW D **102**, 036005 (2020)

Chiral anomaly and the pion properties in the light-front quark model

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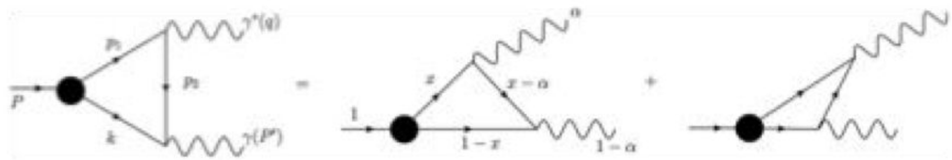


$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{\pi}{4}\alpha_{\rm em}^2 M_\pi^3 |F_{\pi\gamma}(0)|^2$$

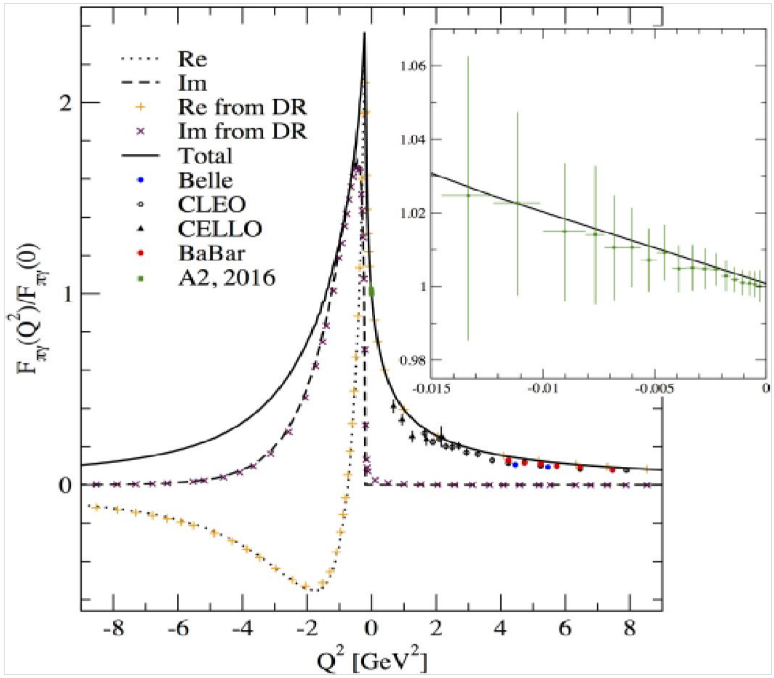
$$\Psi^{\pi}_{Q\bar{Q}} \equiv \Psi_\pi(x_i, \mathbf{k}_{i\perp}, \lambda_i) = \phi_R(x_i, \mathbf{k}_{i\perp})\chi(x_i, \mathbf{k}_{i\perp}, \lambda_i),$$

$$\chi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) = \mathcal{N}\bar{u}_{\lambda_1}(k_1)\Gamma_\pi v_{\lambda_2}(k_2),$$

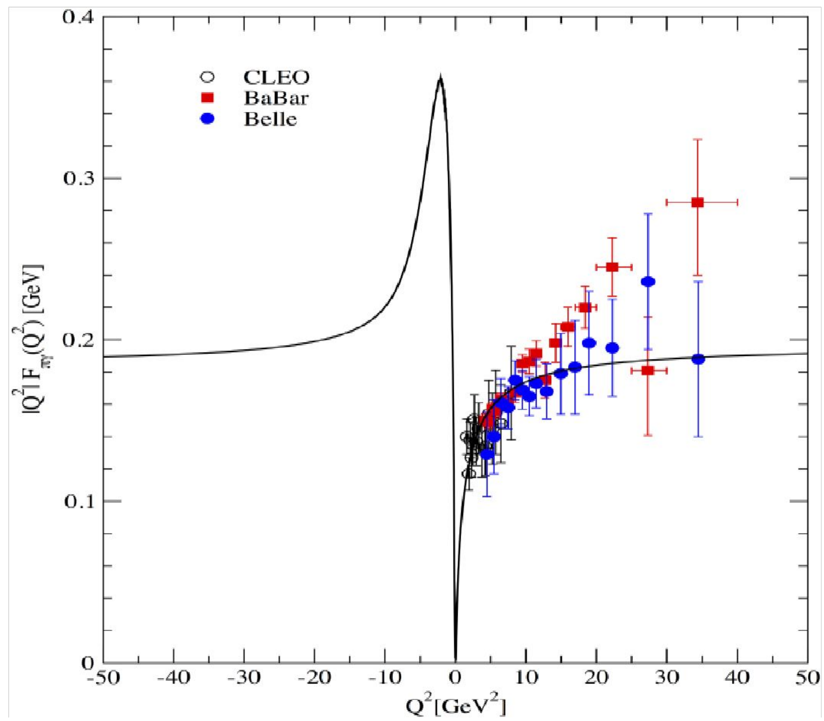
$$\Gamma_\pi = (A_\pi + B_\pi \not{P})\gamma_5$$

$$F_{\pi\gamma}(q^2) = \frac{e_u^2 - e_d^2}{\sqrt{2}}\frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\mathbf{k}_\perp \frac{\psi_\pi(x, \mathbf{k}_\perp)}{M_0^2 - q^2}$$

Both spacelike and timelike form factors can be computed in LFQM.



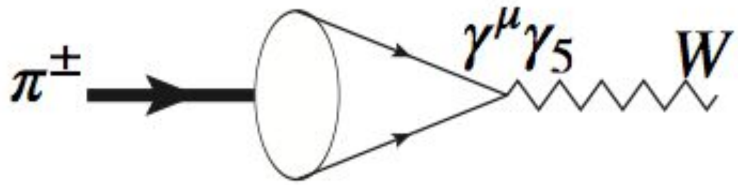
H.-M. Choi, H.-Y. Ryu, C.-R.Ji, PRD96, 056008 (2017)

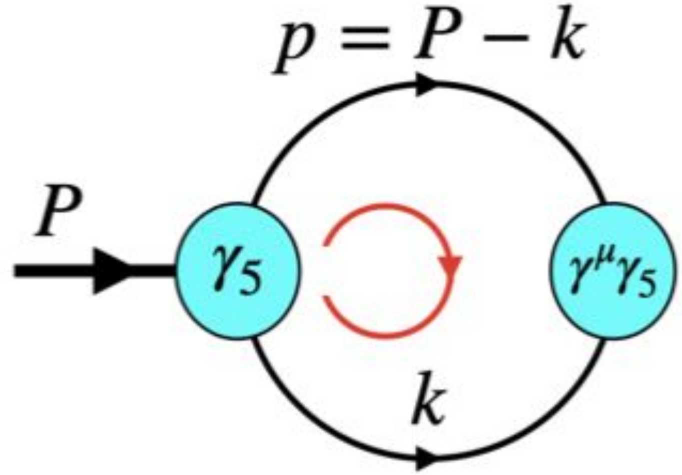


Universality of Decay Constants

- The decay constants must be independent of the reference frames.
- The decay constants must be independent of the component of the current used for the computation.
- DAs are classified with the twist expansion and may depend on the component of current used for the computation.

$$\langle 0 \,|\, \bar{q}\gamma^{\mu}(1-\gamma_5)q \,|\, \pi(P)\rangle = i f_{\pi} P^{\mu}$$





$$\begin{aligned} A_A^{\mu} &= \langle 0|\, \bar{q}(z)\gamma^{\mu}\gamma_5 q(-z)\,|P\rangle \qquad\qquad \zeta = 2x-1 \qquad z^+ = \mathbf{z}_{\perp} = 0 \\ &= i f_P \int_0^1 dx\; \mathrm{e}^{i\zeta P\cdot z}\left[P^{\mu}\phi_{2;P}(x) \;+\frac{M^2 z^{\mu}}{P^+ z^-}\Big(\psi_{4;P}(x) - \phi_{2;P}(x)\Big)\right] \end{aligned}$$

$$A_A^{(+,\perp)} = i f_P P^{(+,\perp)} \int_0^1 dx\ \mathrm{e}^{i\zeta P\cdot z} \phi_{2;P}(x)$$

$$A_A^{-} = i f_P P^{-} \int_0^1 dx\ \mathrm{e}^{i\zeta P\cdot z} \psi_{4;P}(x)$$

$$\boxed{\begin{aligned} \phi_{2;P}(x) &= \frac{2\sqrt{6}}{f_P} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2+\mathbf{k}_\perp^2}} \mathcal{A} \\ \psi_{4;P}(x) &= \frac{2\sqrt{6}}{f_P} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2+\mathbf{k}_\perp^2}} \times \frac{\mathcal{A}}{M_0^2} \left[\frac{\mathbf{k}_\perp^2 \mathcal{A}'/\mathcal{A} + m_1 m_2}{x(1-x)} \right] \end{aligned}}$$

$$\mathcal{A} = (1-x)m_1 + x m_2$$

$$\mathcal{A}' = \mathcal{A}(m_1 \leftrightarrow m_2)$$

$$M_0^2 = \frac{\mathbf{k}_\perp^2 + m_1^2}{x} + \frac{\mathbf{k}_\perp^2 + m_2^2}{1-x}$$

Proof of Component Independence

$$\int_0^1 dx \frac{\phi(x,k_\perp)}{\sqrt{A^2+k_\perp^2}}\left[A-\frac{k_\perp^2 A' + m_1 m_2 A}{x(1-x)M_0^2}\right] = \int_{-\infty}^{+\infty} dk_z k_z f(k_\perp^2, k_z^2, m_1, m_2) = 0$$

Note

$$x = \frac{\sqrt{m_1^2+k_\perp^2+k_z^2}+k_z}{\sqrt{m_1^2+k_\perp^2+k_z^2}+\sqrt{m_2^2+k_\perp^2+k_z^2}}$$

Mixing effects on 1S and 2S state heavy mesons in the light-front quark model

A.J.Arifi,H.-M.Choi,C.Ji & Y.Oh,2205.04075[hep-ph],PRD('22)

$$H_{q\bar{q}}\left|\Psi_{q\bar{q}}\right\rangle = M_{q\bar{q}}\left|\Psi_{q\bar{q}}\right\rangle,$$

$$H_{q\bar{q}} = H_0 + V_{q\bar{q}} = \sqrt{m_q^2+\mathbf{k}^2} + \sqrt{m_{\bar{q}}^2+\mathbf{k}^2} + V_{q\bar{q}},$$

$$\boxed{\begin{gathered} V_{q\bar{q}} = V_{\mathrm{Conf}} + V_{\mathrm{Coul}} + V_{\mathrm{Hyp}} \\ V_{\mathrm{Conf}} = a + br \qquad V_{\mathrm{Coul}} = -\frac{4\alpha_s}{3r} \qquad V_{\mathrm{Hyp}} = \frac{2}{3}\frac{\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}}}{m_q m_{\bar{q}}}\nabla^2 V_{\mathrm{Coul}} \end{gathered}}$$

$$\Psi_{nS}^{JJ_z}(x,\mathbf{k}_\perp,\lambda_i) = \Phi_{nS}(x,\mathbf{k}_\perp)\,\mathcal{R}_{\lambda_q\lambda_{\bar{q}}}^{JJ_z}(x,\mathbf{k}_\perp),$$

$$\begin{pmatrix}\Phi_{1S}\\ \Phi_{2S}\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\phi_{1S}\\ \phi_{2S}\end{pmatrix},$$

$$\phi_{1S}(x,\mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}e^{-\mathbf{k}^2/2\beta^2},$$

$$\phi_{2S}(x,\mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\sqrt{6}\beta^{7/2}}\left(2\mathbf{k}^2-3\beta^2\right)\sqrt{\frac{\partial k_z}{\partial x}}e^{-\mathbf{k}^2/2\beta^2},$$

$$\frac{\partial \left\langle \Psi_{q\bar{q}} \right| (H_0 + V_0) \left| \Psi_{q\bar{q}} \right\rangle}{\partial \beta} = 0$$

$$\begin{aligned}
M_{q\bar{q}}^{1S} &= \frac{\beta}{\sqrt{\pi}} \sum_{i=q,\bar{q}} \Bigg\{ z_i e^{z_i/2} \left[\frac{1}{3} c_2^2 (3 - z_i) z_i \; K_2\left(\frac{z_i}{2}\right) + \frac{1}{6}\left(9 - 3c_1^2 + 2c_2^2 z_i^2 - 6\sqrt{6} c_1 c_2\right) K_1\left(\frac{z_i}{2}\right) \right] \\
&\qquad + \sqrt{\pi}\left(\sqrt{6} c_1 c_2 - 3c_2^2\right) U\left(-1/2, -2, z_i\right) \Bigg\} \\
&\quad + a + \frac{b}{\beta\sqrt{\pi}}\left(3 - c_1^2 - 2\sqrt{\frac{2}{3}} c_1 c_2\right) - \frac{4\alpha_s \beta}{9\sqrt{\pi}}\left(5 + c_1^2 + 6\sqrt{\frac{2}{3}} c_1 c_2\right) + \frac{16\alpha_s \beta^3 \left\langle \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \right\rangle}{9 m_q m_{\bar{q}} \sqrt{\pi}} (3 - c_1^2 + 2\sqrt{6} c_1 c_2), \\
M_{q\bar{q}}^{2S} &= M_{q\bar{q}}^{1S}(c_1 \to -c_2, c_2 \to c_1),
\end{aligned}$$

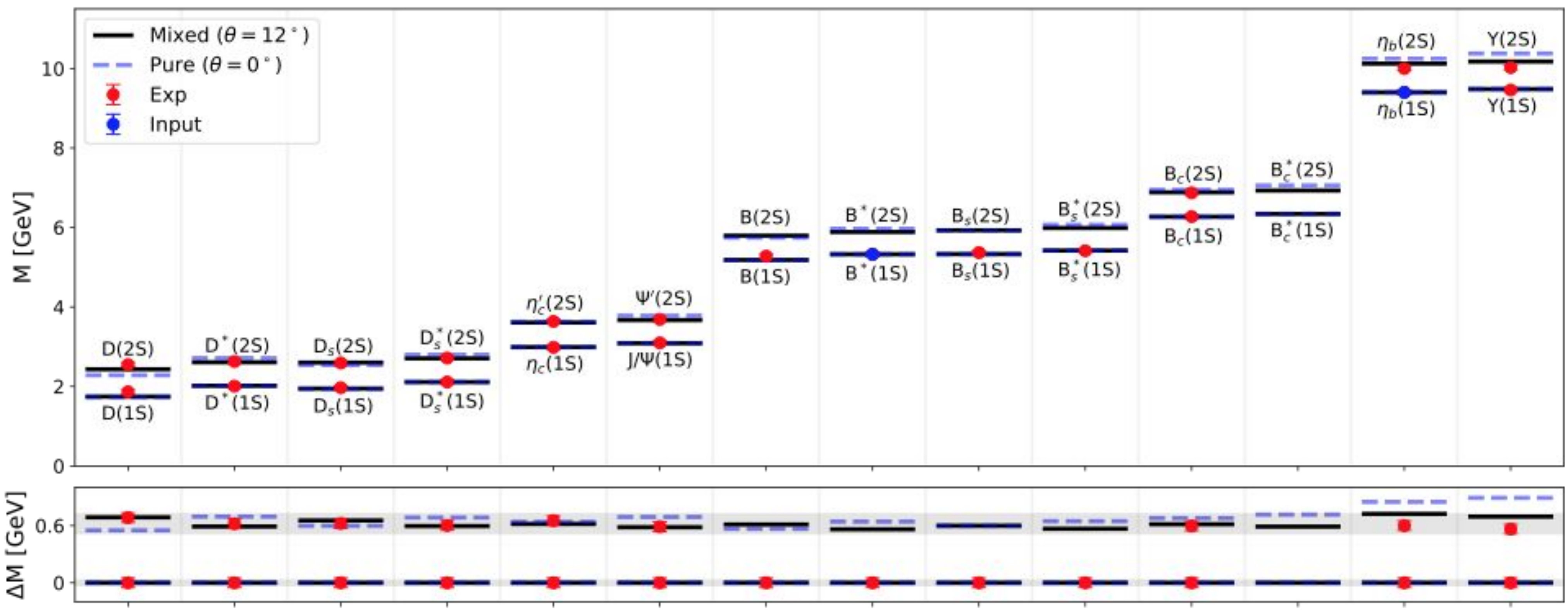
$$\Delta M_{P(V)} = M_{P(V)}^{2S} - M_{P(V)}^{1S}$$

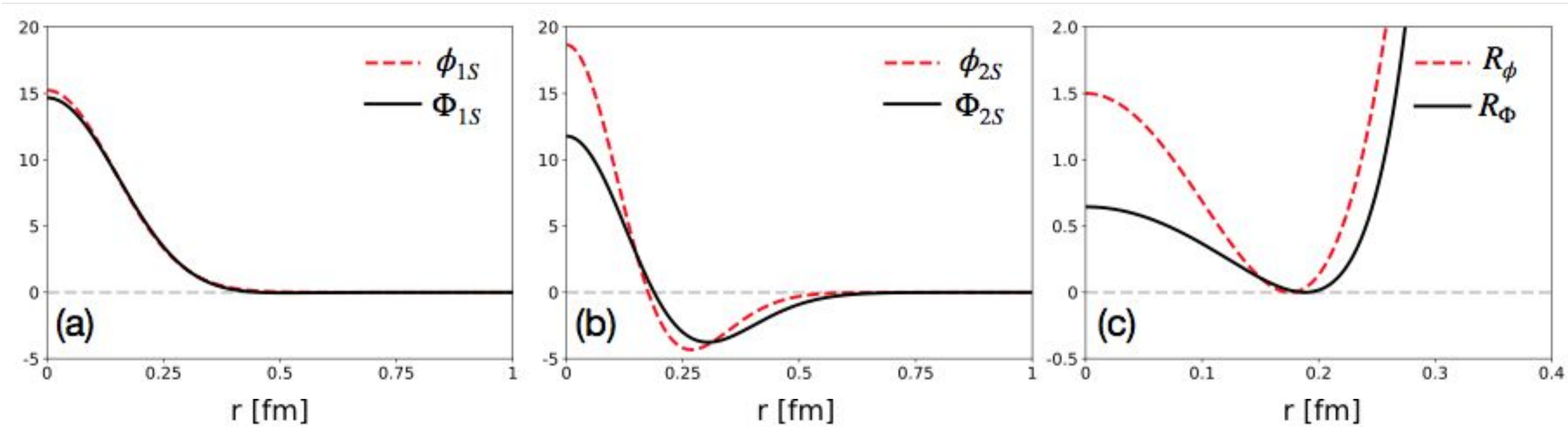
$$\Delta M_P - \Delta M_V = \Delta M_P^{\mathrm{Hyp}} - \Delta M_V^{\mathrm{Hyp}} = C\left(2\sqrt{6}\sin 2\theta - \cos 2\theta\right)$$

$$\Delta M_P - \Delta M_V = \Delta M_P^{\mathrm{Hyp}} - \Delta M_V^{\mathrm{Hyp}}$$

$$= C\left(2\sqrt{6}\sin 2\theta - \cos 2\theta\right)$$

$$\Delta M_P > \Delta M_V \qquad \frac{1}{2}\cot^{-1}(2\sqrt{6}) < \theta < \frac{\pi}{4}$$

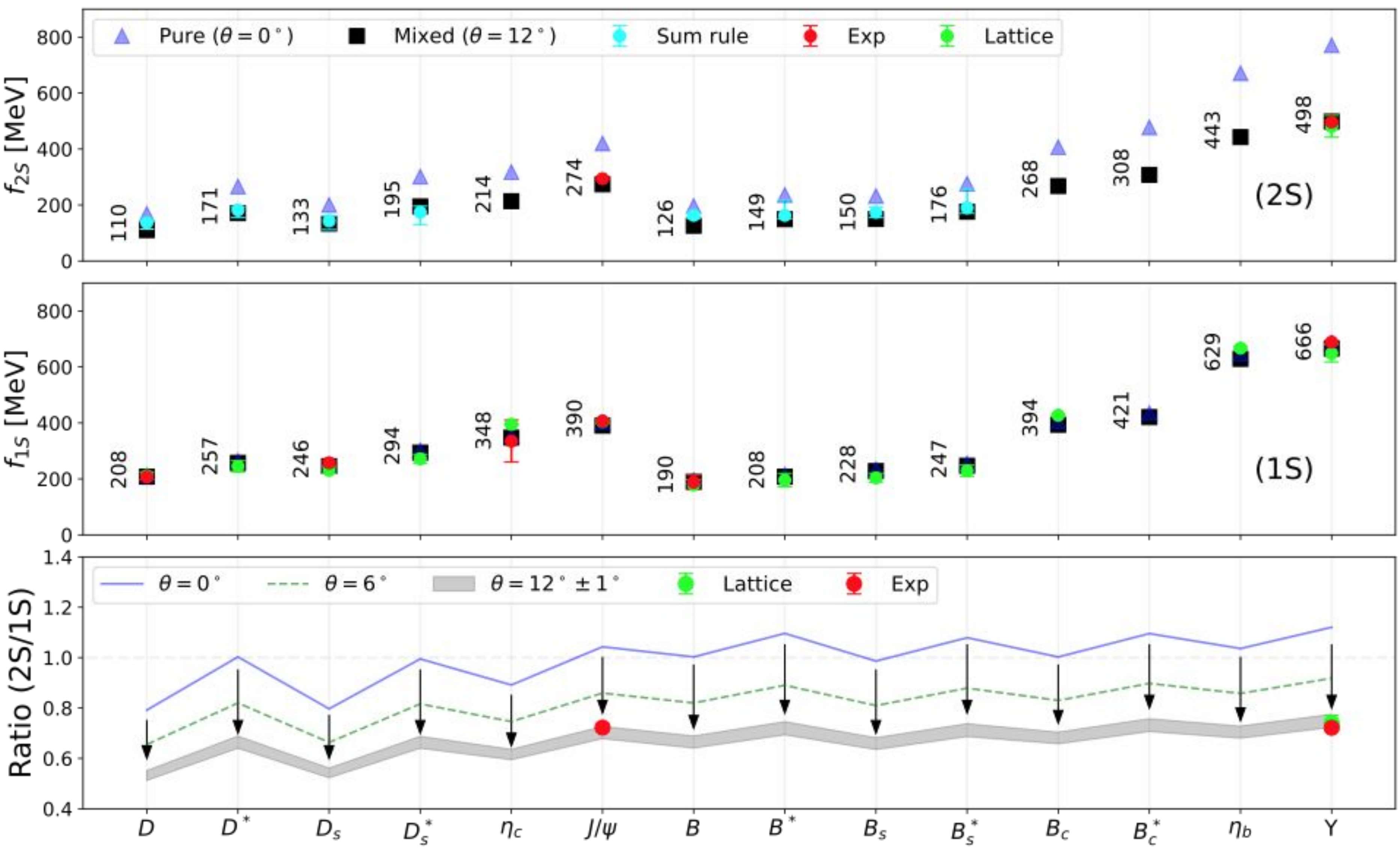




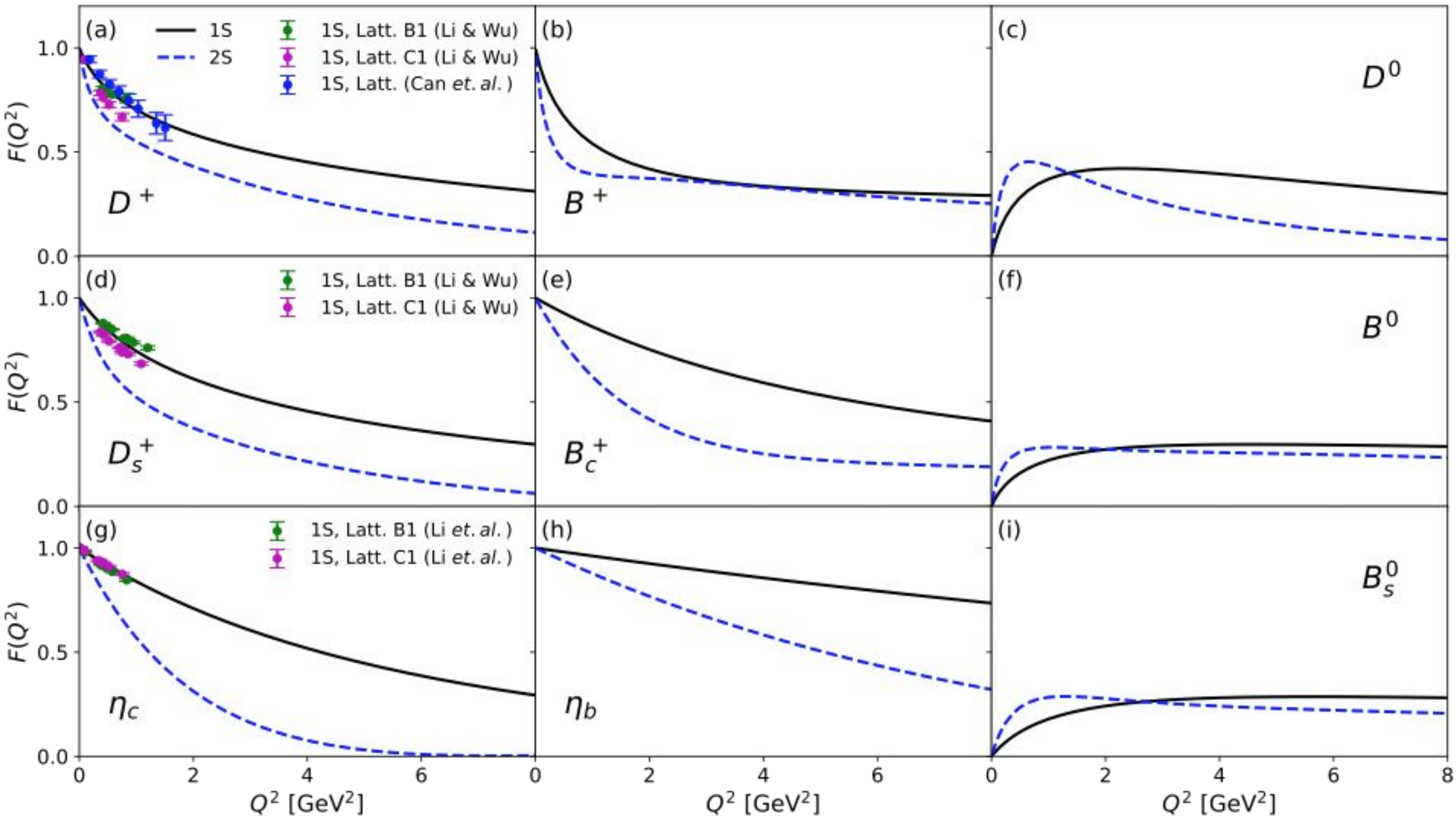
$$f_{1S} > f_{2S}$$

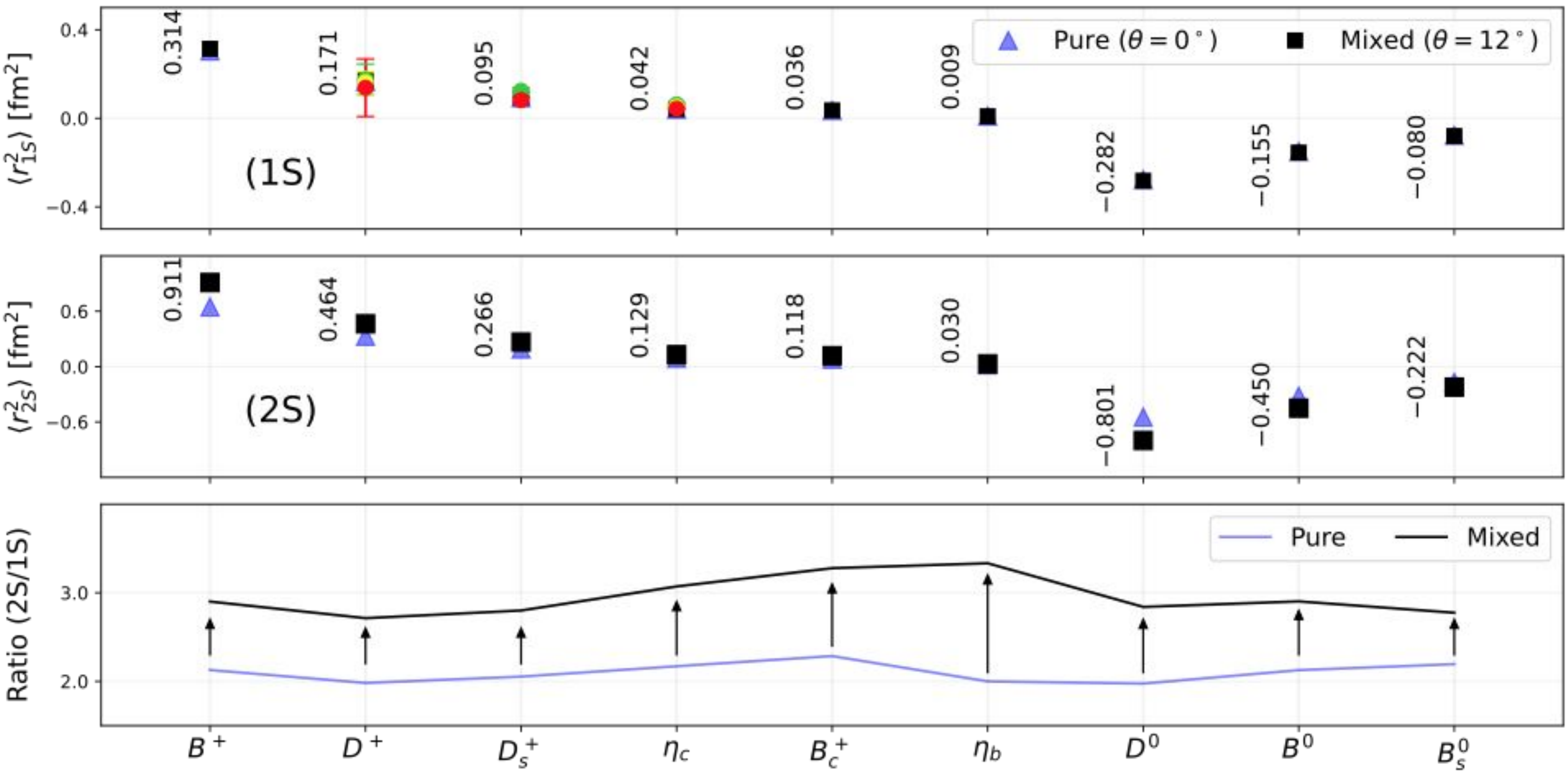
$$R_\phi = \phi_{2S}^2 / \phi_{1S}^2$$

$$R_\Phi = \Phi_{2S}^2 / \Phi_{1S}^2$$



Pure ($\theta = 0^\circ$)
Mixed ($\theta = 12^\circ$)
Sum rule
Exp
Lattice
f_{2S} [MeV]
(2S)
110
171
133
195
214
274
126
149
150
176
268
308
443
498
f_{1S} [MeV]
(1S)
208
257
246
294
348
390
190
208
228
247
394
421
629
666
Ratio (2S/1S)
$\theta = 0^\circ$
$\theta = 6^\circ$
$\theta = 12^\circ \pm 1^\circ$
Lattice
Exp
D
D^*
D_s
D_s^*
η_c
J/ψ
B
B^*
B_s
B_s^*
B_c
B_c^*
η_b
Υ


(a)
1S
2S
1S, Latt. B1 (Li & Wu)
1S, Latt. C1 (Li & Wu)
1S, Latt. (Can et. al.)
D^+
(b)
B^+
(c)
D^0
(d)
1S, Latt. B1 (Li & Wu)
1S, Latt. C1 (Li & Wu)
D_s^+
(e)
B_c^+
(f)
B^0
(g)
1S, Latt. B1 (Li et. al.)
1S, Latt. C1 (Li et. al.)
η_c
(h)
η_b
(i)
B_s^0
$F(Q^2)$
Q^2 [GeV^2]



Pure ($\theta = 0^\circ$)
Mixed ($\theta = 12^\circ$)
$\langle r^2_{1S} \rangle$ [fm^2]
(1S)
0.314
0.171
0.095
0.042
0.036
0.009
−0.282
−0.155
−0.080
$\langle r^2_{2S} \rangle$ [fm^2]
(2S)
0.911
0.464
0.266
0.129
0.118
0.030
−0.801
−0.450
−0.222
Ratio (2S/1S)
Pure
Mixed
B^+
D^+
D_s^+
η_c
B_c^+
η_b
D^0
B^0
B_s^0

Conclusion and Outlook

- ‘tHooft model provides a useful tool to grasp the basic idea of linking between fundamental QCD and phenomenological LFQM.
- LFQM takes advantage of distinguished features of LFD such as the boost invariance and the cleaner vacuum properties.
- Bakamjian-Thomas construction allows to build the LFQM based on the non-interacting representations in modeling the interacting systems without spoiling the Poincare invariance of the intrinsic physical observables of the interacting systems.
- Extension of ground states to the radially excited states requires the mixing between 1S and 2S basis states for the empirical hierarchy of the mass difference between 1S ad 2S states and the decay constants of the heavy pseudoscalar and vector mesons.
- Similar investigations for the light meson sectors are in progress and extension to the baryon systems are under consideration.