

# Excited hadrons at Belle (II) experiment



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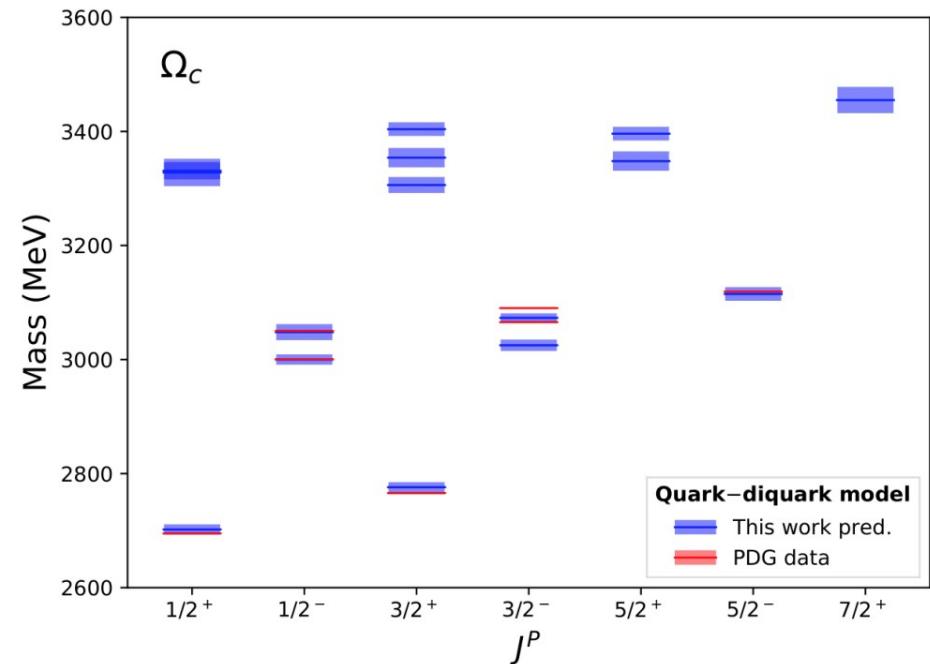
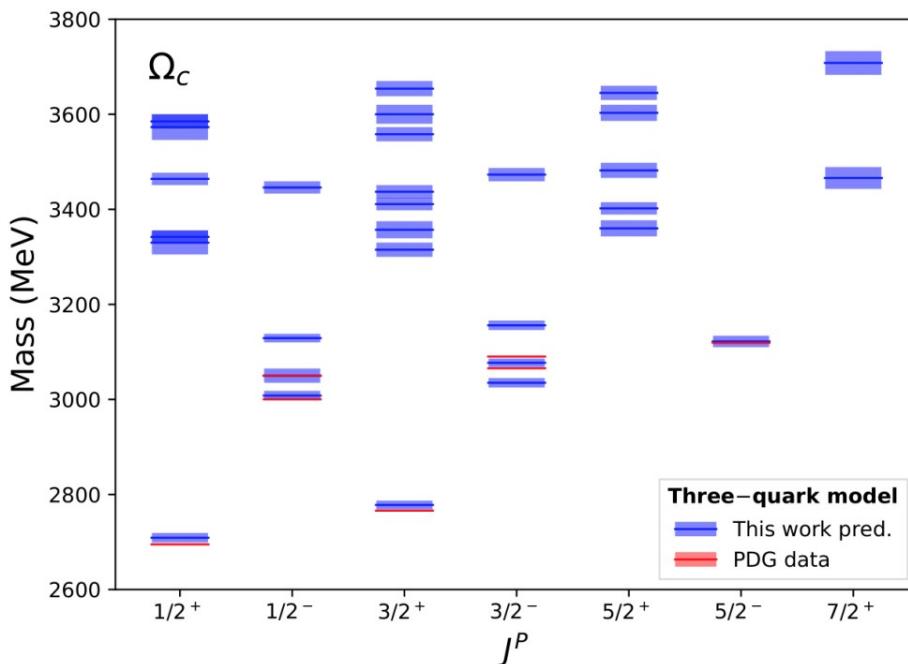
KNU member

# Advertisement:

## Theoretical work at Kyungpook National University

H. Garcia-Tecocoatzi, A. Giachino, J. Li, A. Ramirez-Morales, E. Santopinto,  
"Strong decay widths and mass spectra of charmed baryons", arXiv:2205.07049 (2022)

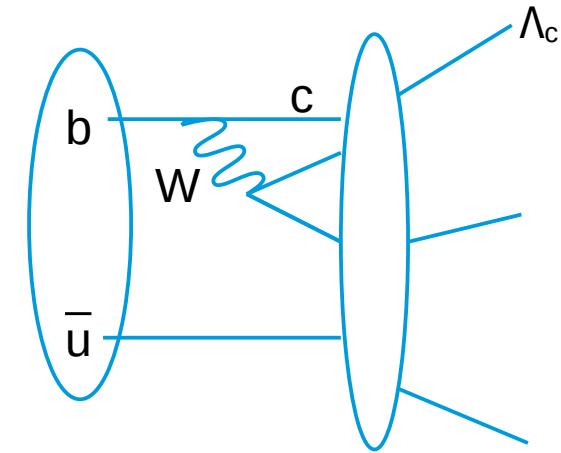
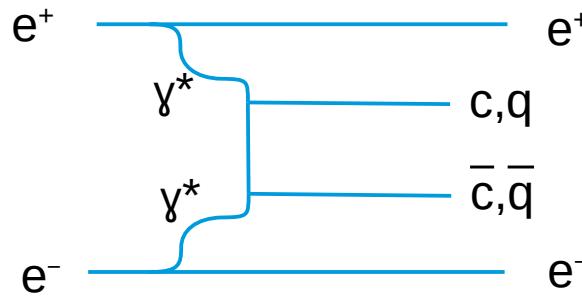
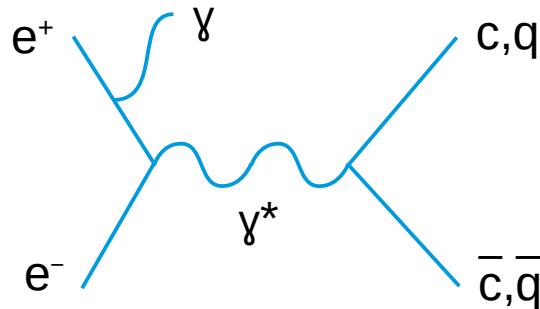
Calculate masses and widths of charmed baryons up to the D-wave states in a constituent quark model.



# Outline

- Hadron production at Belle.
- Observation and subsequent theoretical and experimental development for the new multiple strange  $\Omega(2012)^-$ .
- Various theoretical inputs for experimental charmed baryon study.
- Future studies.

# Production of baryons at lepton collider



Direct production

Resonance or fragmentation.  
Possible initial state radiation.  
Hyprons can produced.

2-phonon process for  
resonance production

Indirect production from  
cascade decays

# Some of recent (< 1.3 y) hadron results from Belle (total 26)

$e^+e^- \rightarrow \eta\phi$  via ISR (to be submitted to PRD)

$\Lambda_c \rightarrow \Sigma^+\gamma, \Xi_c^0 \rightarrow \Xi^0\gamma$  (to be submitted to PRD)

**Peak structure in  $\Lambda_c \rightarrow pK^-\pi^+$**  (to be submitted to PRL)

$\gamma\gamma \rightarrow \chi_{c2}(1P) \rightarrow J/\psi\gamma$  (to be submitted to JHEP)

**$\Omega(2012) \rightarrow \Xi(1530)\bar{K}$**  (arXiv:2207.03090)

2-hadron correlation in  $e^+e^-$  collisions (arXiv:2206.09440)

New baryon  $\Lambda_c(2910)$  in B decay (arXiv:2206.08822)

$\Xi_c^0 \rightarrow \Lambda_c\pi^-$  (arXiv:2206.08527)

Search for  $X(3872) \rightarrow \pi^+\pi^-\pi^0$  (arXiv:2206.08592)

Exotics in  $\gamma\gamma \rightarrow \gamma\psi(2S)$  (PRD 105 (2022) 112011)

$\Lambda_c \rightarrow p\eta'$  (JHEP 03 (2022) 090)

Search for  $X_{cc\bar{s}\bar{s}}$  in  $Ds^{(*)+}D\bar{s}^{(*)+}$  (PRD105 (2022) 032002)

$\Xi_c^0 \rightarrow \Lambda K_S^0, \Sigma^0 K_S^0, \Sigma^+ K^-$  (PRD105 (2022) L011102)

$e^+e^- \rightarrow Y(1,2S)\eta, Y(1S)\eta'$  (PRD104 (2021) 112006)

$\Lambda_c \rightarrow p\omega$  (PRD104 (2021) 072008)

**$\Omega(2012)$  in  $\Omega_c$  decay** (PRD104 (2021) 052005)

Mass and width of  $\Sigma_c^{(*)+}$  (PRD104 (2021) 052003)

$\Xi_c^0 \rightarrow \Xi^- l^+ \nu_l, \Xi^- \pi^+$  (PRL127 (2021) 121803)

Search for  $\eta_{c2}(1D)$  in  $e^+e^- \rightarrow \gamma\eta_{c2}(1D)$

(PRD104 (2021) 012012)

$\Xi_c^0 \rightarrow \Lambda\bar{K}^{*0}, \Sigma^0\bar{K}^{*0}, \Sigma^+\bar{K}^{*-}$  (JHEP 06 (2021) 160)

Energy dependence of  $e^+e^- \rightarrow B^{(*)}\bar{B}^{(*)}$   
(JHEP 06 (2021) 137)

**Spin-parity measurement of  $\Xi_c(2970)$**

(PRD103 (2021) L111101)

$\Xi_c^0 \rightarrow \Xi^0 K^+ K^-$  (PRD103 (2021) 112002)

$\Lambda_c \rightarrow p\eta$  and  $p\pi^0$  (PRD103 (2021) 072004)

**$\Lambda_c \rightarrow \Lambda\eta\pi^+$  decay and  $\Lambda(1670)$**

(PRD103 (2021) 052005)

Evidence of  $\gamma\gamma^* \rightarrow X(3872)$

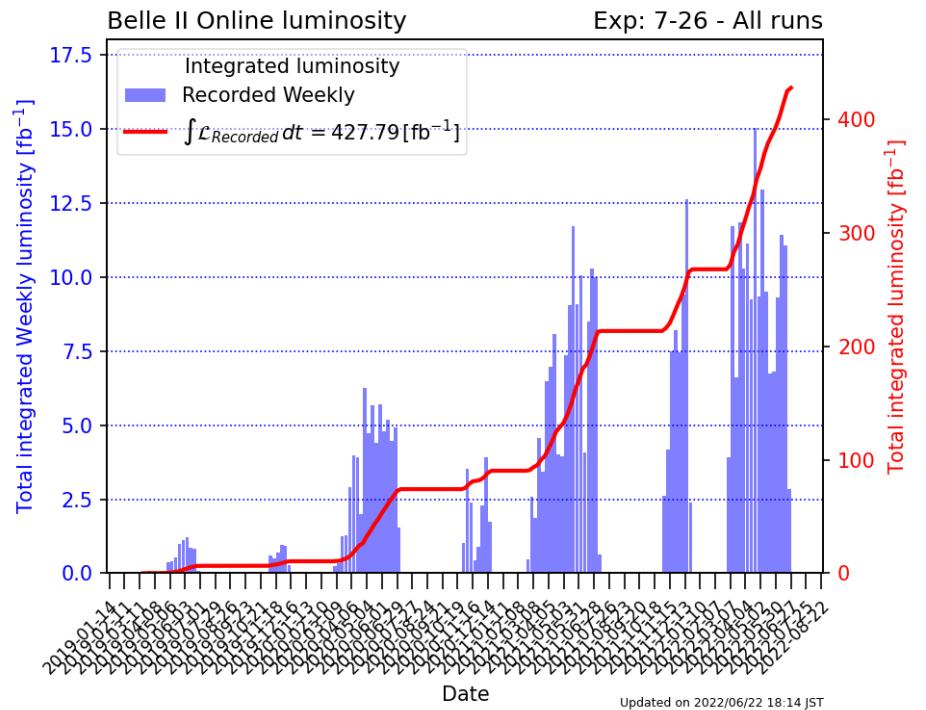
(PRL126 (2021) 122001)

**... more past and coming ...**

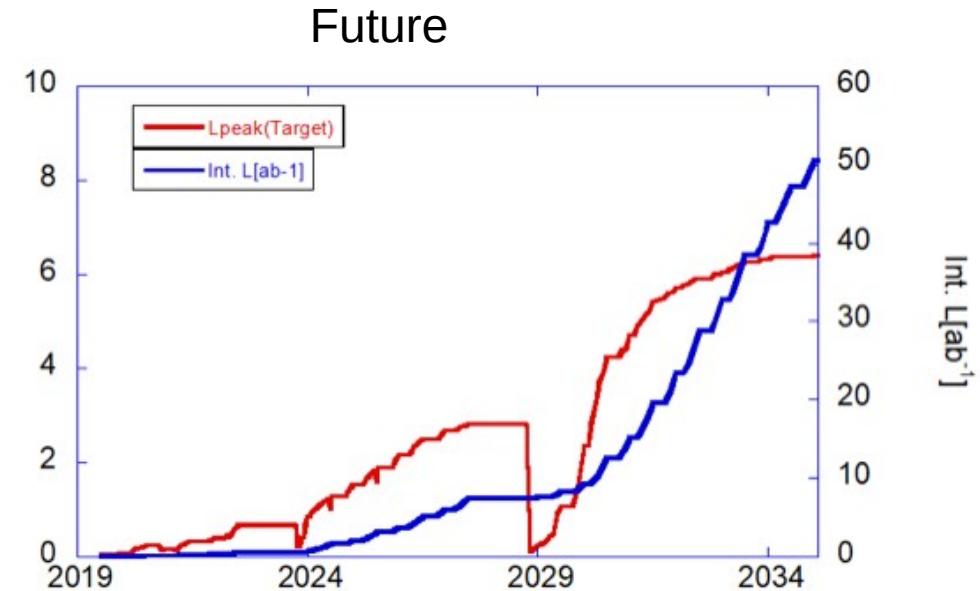
I am covering a tiny fraction of them  
(>20 hadron results per year!) ,  
colored in blue, mainly on strange  
baryon  $\Omega(2012)$ .

# Future Belle II ( $50 \times$ Belle Lum, better recon.)

Past



Future



Latest peak record:  $4.65 \times 10^{34} \text{ cm}^{-2} \text{s}^{-1}$  on June 22, 2022

$\Omega(2012)$  : example to show the importance  
of direct interplay between theorist and  
experimentalist.

# Predictions of excited $\Omega$ before observation (Theory 1)

Quark Model:

K.-T. Chao, N. Isgur, and G. Karl, Phys. Rev. D 23, 155 (1981).- Non-relativistic, SU(6), predicts  $M(3/2^-)=2020$  MeV, Gamma=3.9 MeV, decay is  $\Xi K$  only.

R. N. Faustov and V. O. Galkin, Strange baryon spectroscopy in the relativistic quark model, Phys. Rev. D 92, 054005 (2015). - Relativistic (2038 MeV)

Skyrme model:

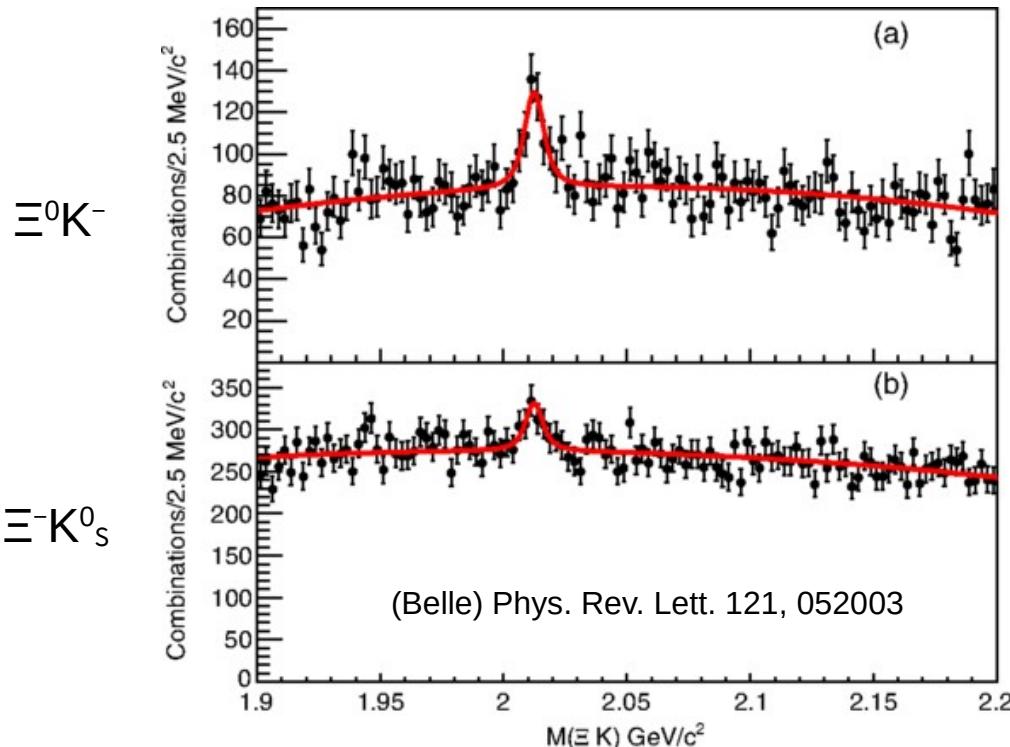
Yongseok Oh,  $\Xi$  and  $\Omega$  baryons in the Skyrme model, Phys. Rev. D 75, 074002 (2007).  
 $M(\Omega(J^P:3/2^-))=1978$  MeV

Chiral Unitarity Approach:

Sourav Sarkar et al., Baryonic resonances from baryon decuplet-meson octet interaction, Nuclear Physics A 750 (2005), 294-323 – mass and width ( 2141 – i38) MeV

Xu Si-Qi, et al., The  $\Xi^* \bar{K}$  and  $\Omega \eta$  Interaction Within a Chiral Unitary Approach, Commun. Theor. Phys. 65, (2016) 53 – different subtraction  $a(\mu)$  in renormalization can affect mass

# Observation of an excited $\Omega^-$ baryon (Theory 1 $\Rightarrow$ Exp 1)



Mass:

$$2012.4 \pm 0.7(\text{stat}) \pm 0.6(\text{syst}) \text{ MeV}/c^2$$

$$\Gamma = 6.4^{+2.5}_{-2.0}(\text{stat}) \pm 1.6(\text{syst}) \text{ MeV.}$$

$$\mathcal{R} = \frac{\mathcal{B}(\Omega^{*-} \rightarrow \Xi^0 K^-)}{\mathcal{B}(\Omega^{*-} \rightarrow \Xi^- \bar{K}^0)} = 1.2 \pm 0.3$$

Narrow width can be explained with d-wave only decay  $\Rightarrow J^P = 3/2^-$  preferred.

## After the observation (Exp 1 $\Rightarrow$ Theory 2(explanation) )

Quark model:

Ming-Sheng Liu, Kai-Lei Wang, Qi-Fang Lü, and Xian-Hui Zhong,  $\Omega$  baryon spectrum and their decays in a constituent quark model, Phys. Rev. D 101, 016002 (2020). - no 3 body width,  $\frac{1}{2}-$  not completely ruled out.

Hadronic molecule:

R.Pavao and E.Oset, Coupled channels dynamics in the generation of the  $\Omega(2012)$  resonance, Eur. Phys. J. C78, 857 (2018);arXiv:1808.01950. -  $\Gamma(\bar{K}\pi\Xi) \sim 3\text{MeV}$ , similar to  $\Gamma(\bar{K}\Xi)$ .

Y.H.Lin and B.S.Zou, Hadronic molecular assignment for the newly observed  $\Omega^*$  state, Phys. Rev. D98, 056013 (2018);arXiv:1807.00997. -  $\Gamma(\bar{K}\pi\Xi) = 6 \times \Gamma(\bar{K}\Xi)$ . (2.4 and 0.4 MeV)

## Response of Theory 2. $\Rightarrow$ Exp 2

S. Jia et al. (Belle Collaboration), Search for  $\Omega(2012) \rightarrow K\Xi(1530) \rightarrow K\pi\Xi$  at Belle,  
Phys. Rev. D 100, 032006 (2019)

$$\mathcal{R}_{\Xi K}^{\Xi\pi K} = \frac{\mathcal{B}(\Omega(2012) \rightarrow \Xi(1530)(\rightarrow \Xi\pi)K)}{\mathcal{B}(\Omega(2012) \rightarrow \Xi K)} < 11.9\%$$

at 90% C.L.

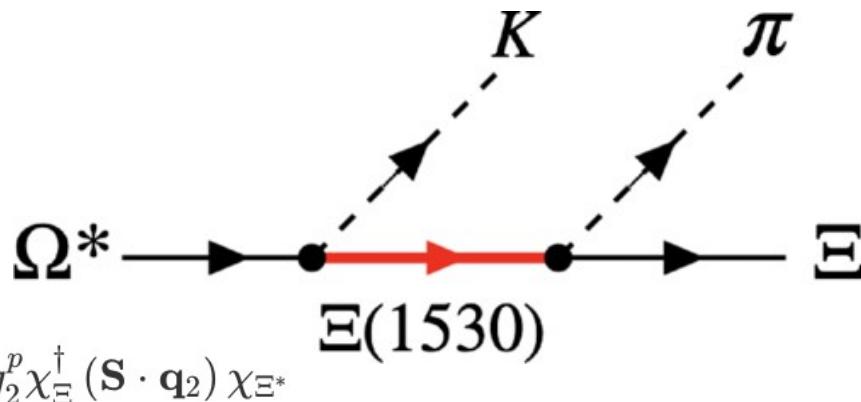
Using theoretical relations between BF ratios:

$$\mathcal{R}_{\Xi^-\bar{K}^0}^{\Xi^-\pi^+K^-} : \mathcal{R}_{\Xi^-\bar{K}^0}^{\Xi^-\pi^0\bar{K}^0} : \mathcal{R}_{\Xi^0K^-}^{\Xi^0\pi^-\bar{K}^0} : \mathcal{R}_{\Xi^0K^-}^{\Xi^0\pi^0K^-} = 1 : \frac{1}{2} : \frac{1}{1.2} : \frac{1}{2.4}.$$

It seems that  $\Omega^*(2012)$  Is not a molecular ?

## Experiment 2 $\Rightarrow$ Theory 3.1 (quark model)

A. J. Arifi, D. Suenaga, A. Hosaka, and Y. Oh, Strong decays of multi-strangeness baryon resonances in the quark model, Phys. Rev. D 105, 094006 (2022). - Relativistic correction, 3-body decay computed,  $\Gamma(\Omega \rightarrow \Xi^-\bar{K}\pi)/\Gamma(\Omega \rightarrow \Xi^-\bar{K})=4.5\%$ .



$$-i\mathcal{T}_{\Xi^*}\mathcal{T}_{\Xi\pi} = g_2^p \chi_{\Xi}^\dagger (\mathbf{S} \cdot \mathbf{q}_2) \chi_{\Xi^*}$$

$$-i\mathcal{T} = -i \frac{\mathcal{T}_{\Xi(1530)\rightarrow\Xi\pi} \mathcal{T}_{\Omega\rightarrow\Xi(1530)\bar{K}}}{m_{\Xi\pi} - M_{\Xi(1530)} + \frac{i}{2}\Gamma_{\Xi(1530)}}$$

$$\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M_i^3} \int |\overline{\mathcal{T}}|^2 dm_{\Xi\pi}^2 dm_{\pi\bar{K}}^2$$

See also:

Xuejie Liu, Hongxia Huang, Jialun Ping, and Dianyong Chen, Investigating  $\Omega(2012)$  as a molecular state., Phys. Rev. C 103 025202 (2021).

- Not suitable for a molecule state,  $\Xi^*$  and  $\bar{K}$  are repulsive.

$$-iT_{\Omega^*\rightarrow\Xi^*\bar{K}}^{(s)} = g_1^s \chi_{\Xi^*}^\dagger \chi_\Omega,$$

$$-iT_{\Omega^*\rightarrow\Xi^*\bar{K}}^{(d)} = g_1^d \chi_{\Xi^*}^\dagger (V_{ij} q_{1i} q_{1j}) \chi_\Omega,$$

## Experiment 2 $\Rightarrow$ Theory 3.2 (Hadronic molecule)

Yong-Hui Lin, Fei Wang, and Bing-Song Zou, Reanalysis of the newly observed  $\Omega^*$  state in a hadronic molecule model, Phys. Rev. D 102, 074025 ( 2020). -  $1/2^+$  or  $3/2^+$  or  $\bar{K}\Xi(1530)$  molecule.

Thomas Gutsche and Valery E Lyubovitskij, Strong decays of the hadronic molecule  $\Omega^*(2012)$  , J. Phys. G: Nucl. Part. Phys. 48 025001 (2021).- explain the BF ratio if big mixing between  $\Xi K$  and  $\Omega\eta$  component.

Natsumi Ikeno, Genaro Toledo, and Eulogio Oset, Molecular picture for the  $\Omega(2012)$  revisited, Phys. Rev. D 101, 094016 (2020). - molecular picture of  $\bar{K}\Xi^*$ ,  $\eta\Omega$ ,  $\bar{K}\Xi$  interactions.

Jun-Xu Lu, Chun-Hua Zeng, En Wang, Ju-Jun Xie & Li-Sheng Geng, Revisiting the  $\Omega(2012)$  as a hadronic molecule and its strong decays, Eur. Phys. J. C 80, 361 (2020). - from the coupled channels interactions of the  $\bar{K}\Xi^*(1530)$  and  $\eta\Omega$  in s-wave and  $\bar{K}\Xi$  in d-wave.

## Three couplings

Unitarized scattering amplitude

$$T = V + VGT = [1 - VG]^{-1}V$$

Lippmann-Schwinger

Tree level transition  
from chiral  
Lagrangians

$$V = \begin{pmatrix} 0 & 3F & \alpha q_{\text{on}}^2 \\ 3F & 0 & \beta q_{\text{on}}^2 \\ \alpha q_{\text{on}}^2 & \beta q_{\text{on}}^2 & 0 \end{pmatrix} \quad \begin{matrix} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{matrix}$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0);$$

$$q_{\text{on}} = \frac{\lambda^{1/2}(s, m_{\bar{K}}^2, m_{\Xi}^2)}{2\sqrt{s}},$$

Energies of  $\bar{K}$  and  $\eta$

Loop function, diagonal

Close to the pole:

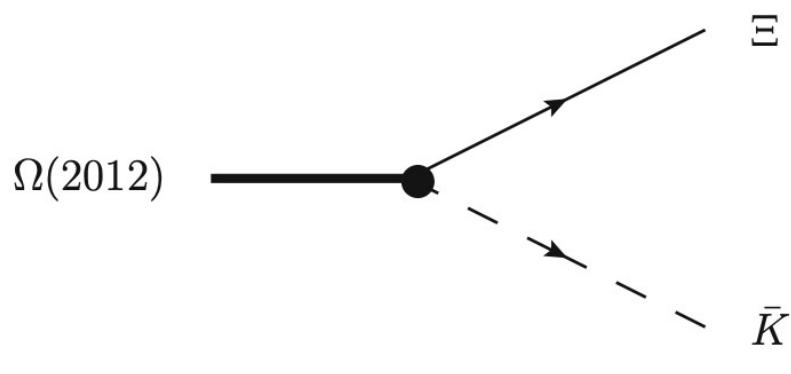
$$z_R = M_R - i\Gamma_R/2,$$

$$T_{ij} = \frac{g_i g_j}{z - z_R}$$

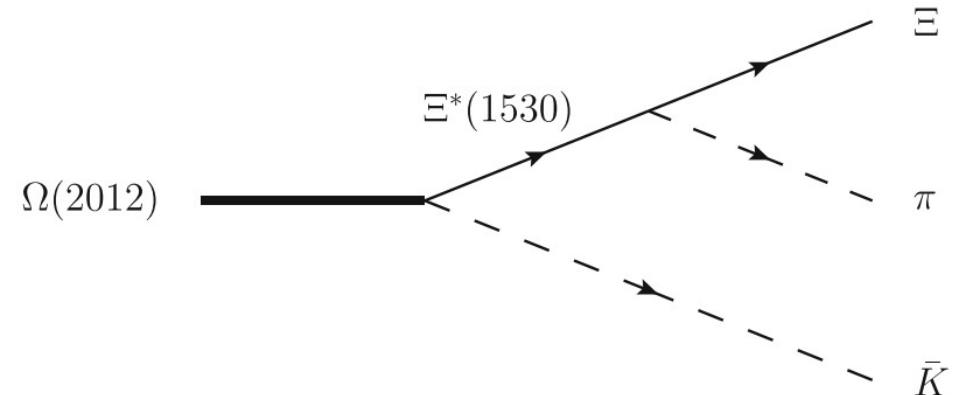
$$g_i^2 = \lim_{z \rightarrow z_R} (z - z_R) T_{ii}$$

$$g_j = g_i \left. \frac{T_{ij}}{T_{ii}} \right|_{z=z_R}$$

## Decay calculation



$$\Gamma_{\Omega(2012) \rightarrow \bar{K}\Xi} = \frac{|g_{\Omega^*\bar{K}\Xi}|^2}{2\pi} \frac{M_\Xi}{M} q_{\bar{K}}$$



$$\frac{d\Gamma_{\Omega(2012) \rightarrow \bar{K}\pi\Xi}}{dM_{\pi\Xi}} = \frac{M_{\pi\Xi}}{\pi^2 M} \frac{|g_{\Omega^*\bar{K}\Xi^*}|^2 p_{\bar{K}} \tilde{\Gamma}_{\Xi^*}}{4(M_{\pi\Xi} - M_{\Xi^*})^2 + \tilde{\Gamma}_{\Xi^*}^2}$$

Experimental mass and width  $\Rightarrow$  determination of  $g_{ij}, \alpha, \beta$

## Fitted parameters from Exp 1

$$R = R_{\bar{K}\Xi}^{\bar{K}\Xi\pi}$$

$q_{\max}$ (MeV)	$\alpha$ ( $10^{-8}$ MeV $^{-3}$ )	$\beta$ ( $10^{-8}$ MeV $^{-3}$ )	$(M_R, \Gamma_R)$ (MeV)	$R$ (%)
735	$-6.6 \pm 0.8$	$16.5 \pm 0.8$	$(2012.3 \pm 0.4, 8.3 \pm 0.6)$	11.88
750	$-9.9 \pm 0.5$	$18.5 \pm 0.5$	$(2012.2 \pm 0.4, 7.8 \pm 0.8)$	10.50
800	$-17.5 \pm 0.6$	$20.6 \pm 0.5$	$(2012.4 \pm 0.5, 6.4 \pm 1.3)$	11.90
850	$-20.2 \pm 1.0$	$19.6 \pm 0.8$	$(2012.4 \pm 0.5, 6.4 \pm 1.1)$	9.00
900	$-20.8 \pm 1.7$	$17.5 \pm 1.1$	$(2012.4 \pm 0.5, 6.4 \pm 1.3)$	7.22

Useful for further study of production and decays.



	$g_{\Omega^* \bar{K}\Xi^*}$	$g_{\Omega^* \eta\Omega}$	$g_{\Omega^* \bar{K}\Xi}$
	(1.826, -0.064)	(3.350, 0.159)	(-0.419, -0.040)
	(1.796, -0.128)	(3.448, 0.298)	(-0.399, -0.109)
	(1.574, 0.188)	(3.590, -0.313)	(-0.307, 0.201)
	(1.386, 0.090)	(3.777, -0.151)	(-0.353, 0.109)
	(1.251, 0.063)	(3.853, -0.111)	(-0.363, 0.082)

## Theory 3.2 $\Rightarrow$ Theory 3.3 ( $\Omega(2012)$ from $\Omega_c$ )

Chun-Hua Zeng, Jun-Xu Lu, En Wang, Ju-Jun Xie, and Li-Sheng Geng, Theoretical study of the  $\Omega(2012)$  state in the  $\Omega_c^0 \rightarrow \pi^+ \Omega(2012)^- \rightarrow \pi^+ (\bar{K}\Xi)^-$  and  $\pi^+ (\bar{K}\Xi\pi)^-$  decays, Phys. Rev. D 102, 076009 (2020). arXiv:2006.15547 - Suggest an experimental study of  $\Omega(2012)$  in charmed baryon decays.

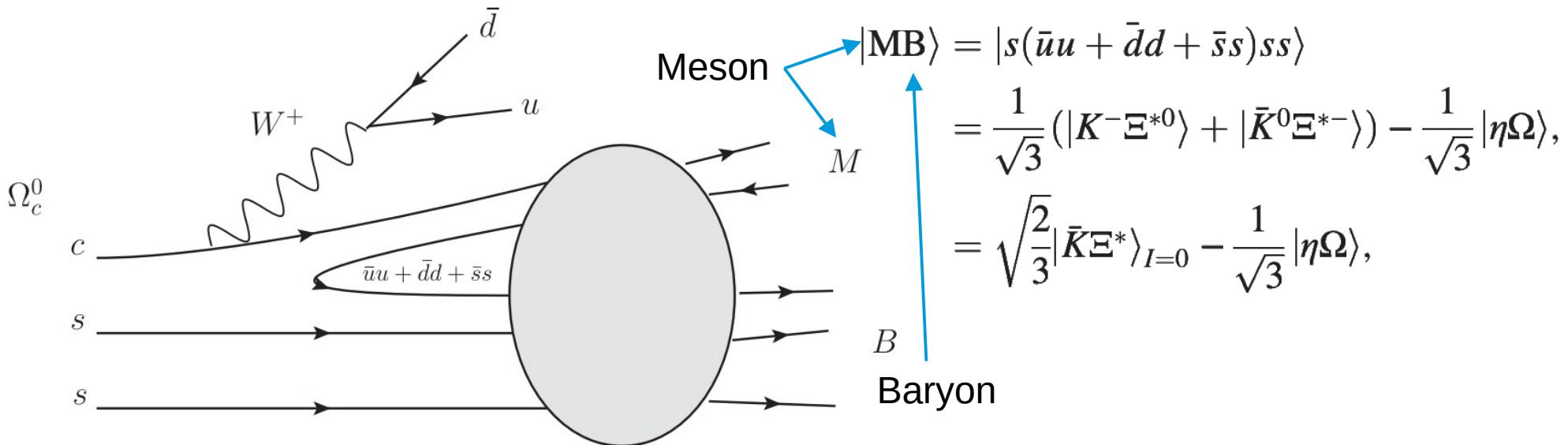
Decay of  $\Omega_c^0 \rightarrow \pi^+ (\bar{K}\Xi\pi)^-$  is from  $\Omega_c^0 \rightarrow \pi^+ (\bar{K}\Xi(1530)^*)^-$  at tree level from molecular model, which will not contribute to the production of the  $\Omega(2012)^-$  from its three body  $(\bar{K}\Xi\pi)^-$  channel.

$$R_{\bar{K}\Xi}^{\bar{K}\Xi\pi} = \frac{\Gamma[\Omega_c^0 \rightarrow \pi^+ \Omega(2012)^- \rightarrow \pi^+ \bar{K}\Xi\pi]}{\Gamma[\Omega_c^0 \rightarrow \pi^+ \Omega(2012)^- \rightarrow \pi^+ \bar{K}\Xi]}$$

TABLE II. Predicted ratio  $R_{\bar{K}\Xi}^{\bar{K}\Xi\pi}$  for different cutoffs.

$\Lambda = q_{\max}(\text{MeV})$	735	750	800	850	900
$R_{\bar{K}\Xi}^{\bar{K}\Xi\pi}(\%)$	13.9	13.8	13.5	10.0	7.3

# Charmed baryon decay to $\Omega(2012)$



$$|\Xi^{*0}\rangle = \frac{1}{\sqrt{3}}|uss + sus + ssu\rangle,$$

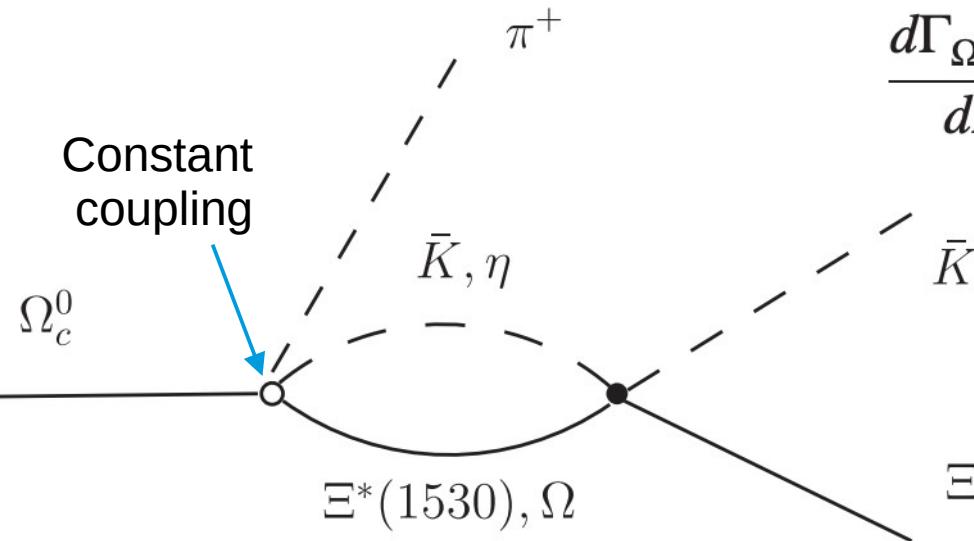
$$|\Xi^{*-}\rangle = \frac{1}{\sqrt{3}}|dss + sds + ssd\rangle,$$

$$|\Omega\rangle = |sss\rangle,$$

$$|\eta\rangle = \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d - \bar{s}s\rangle.$$

# KΞ production

L=1



$$\frac{d\Gamma_{\Omega_c^0 \rightarrow \pi^+ \bar{K}\Xi}}{dM_{\bar{K}\Xi}} = \frac{1}{16\pi^3} \frac{M_\Xi}{M_{\Omega_c^0}} p_\pi^3 p_{\bar{K}} \sum |\mathcal{M}_{\Omega_c^0 \rightarrow \pi^+ \bar{K}\Xi}|^2$$

Coupling constants  $g$ 's from coupled channel study Eur. Phys. J. C 80, 361 (2020).

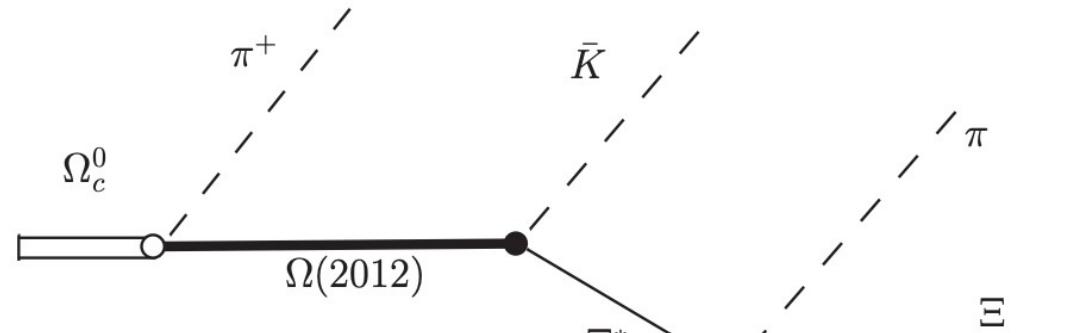
$$\begin{aligned} \mathcal{M}_{\Omega_c^0 \rightarrow \pi \bar{K}\Xi} = V_p & \left( \sqrt{\frac{2}{3}} G_{\bar{K}\Xi^*}(M_{\text{inv}}) \underbrace{t_{\bar{K}\Xi^* \rightarrow \bar{K}\Xi}(M_{\text{inv}})}_{\text{blue bracket}} \right. \\ & - \sqrt{\frac{1}{3}} G_{\eta\Omega}(M_{\text{inv}}) \underbrace{t_{\eta\Omega \rightarrow \bar{K}\Xi}(M_{\text{inv}})}_{\text{blue bracket}} \Big), \end{aligned}$$

$$\frac{g_{\Omega^*\bar{K}\Xi^*} g_{\Omega^*\bar{K}\Xi}}{M_{\text{inv}} - M_{\Omega^*} + i\Gamma_{\Omega^*}/2}$$

$$\frac{g_{\Omega^*\eta\Omega} g_{\Omega^*\bar{K}\Xi}}{M_{\text{inv}} - M_{\Omega^*} + i\Gamma_{\Omega^*}/2}$$

# Three body production of ( $K\Xi\pi$ )<sup>-</sup>

Two consecutive propagators.



$$\mathcal{M}_{\Omega_c^0 \rightarrow \pi^+ \bar{K} \Xi^*} = \frac{g_{\Xi^* \Xi \pi} \bar{p}_\pi \mathcal{M}_{\Omega_c^0 \rightarrow \pi^+ \bar{K} \Xi^*}}{M_{\Xi \pi} - M_{\Xi^*} + i\Gamma_{\Xi^*}/2}$$

$$\begin{aligned} \mathcal{M}_{\Omega_c^0 \rightarrow \pi \bar{K} \Xi^*} &= V_p \left( \sqrt{\frac{2}{3}} [1 + G_{\bar{K} \Xi^*}(M_{\text{inv}}) t_{\bar{K} \Xi^* \rightarrow \bar{K} \Xi^*}(M_{\text{inv}})] \right. \\ &\quad \left. - \sqrt{\frac{1}{3}} G_{\eta \Omega}(M_{\text{inv}}) t_{\eta \Omega \rightarrow \bar{K} \Xi^*}(M_{\text{inv}}) \right). \end{aligned}$$

$\frac{g_{\Omega^* \bar{K} \Xi^*} g_{\Omega^* \bar{K} \Xi^*}}{M_{\text{inv}} - M_{\Omega^*} + i\Gamma_{\Omega^*}/2}$   
 $\frac{g_{\Omega^* \eta \Omega} g_{\Omega^* \bar{K} \Xi^*}}{M_{\text{inv}} - M_{\Omega^*} + i\Gamma_{\Omega^*}/2}$

# Invariant mass of 2 and 3 body $\Omega(2012)$ decay in $\Omega_c$ production

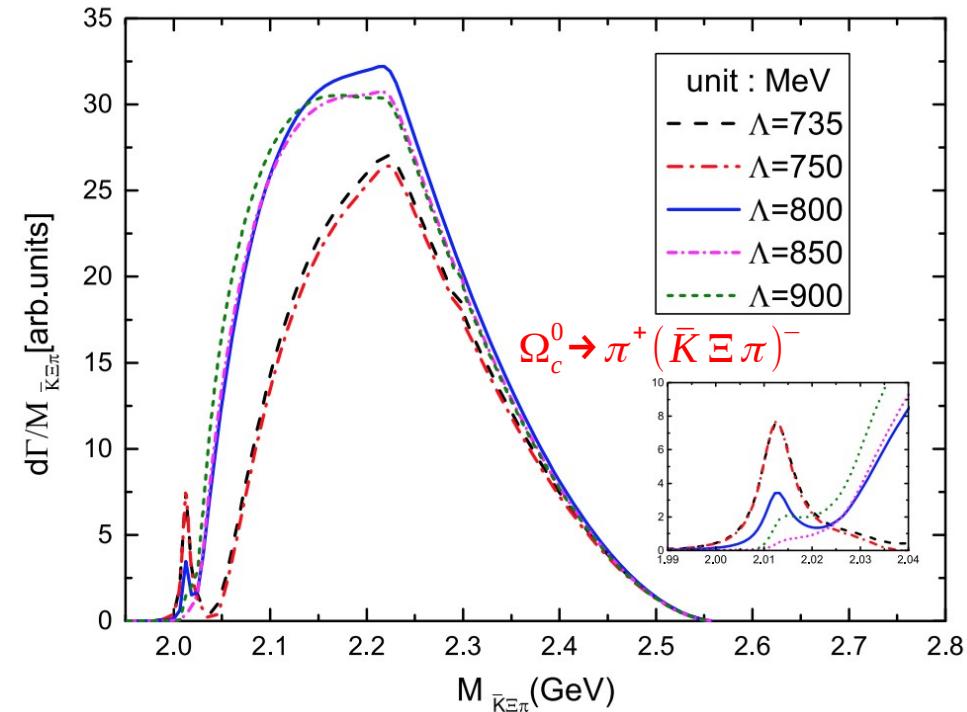
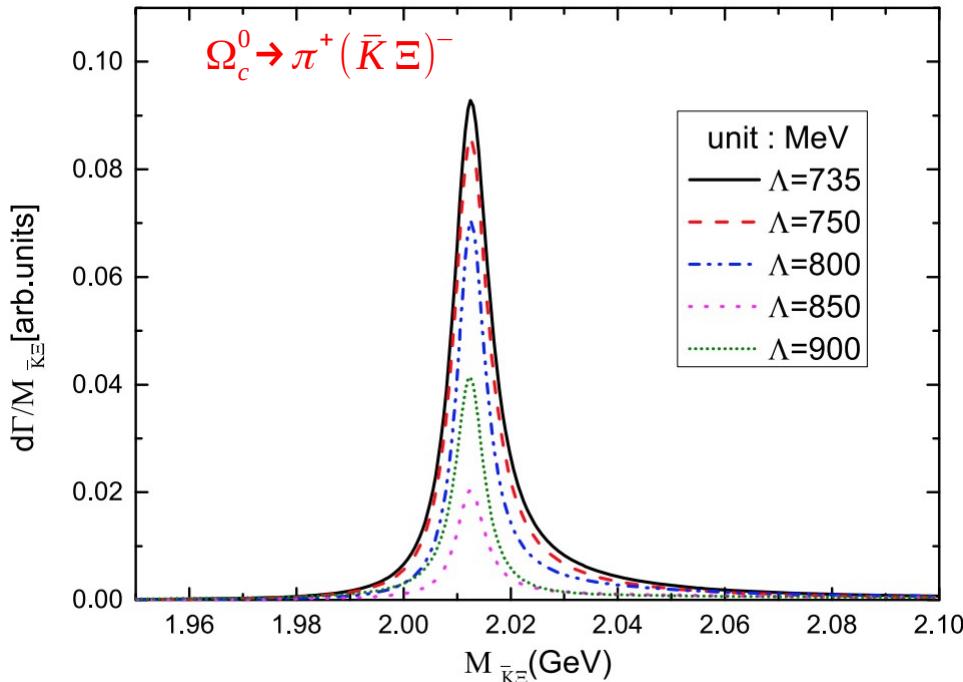
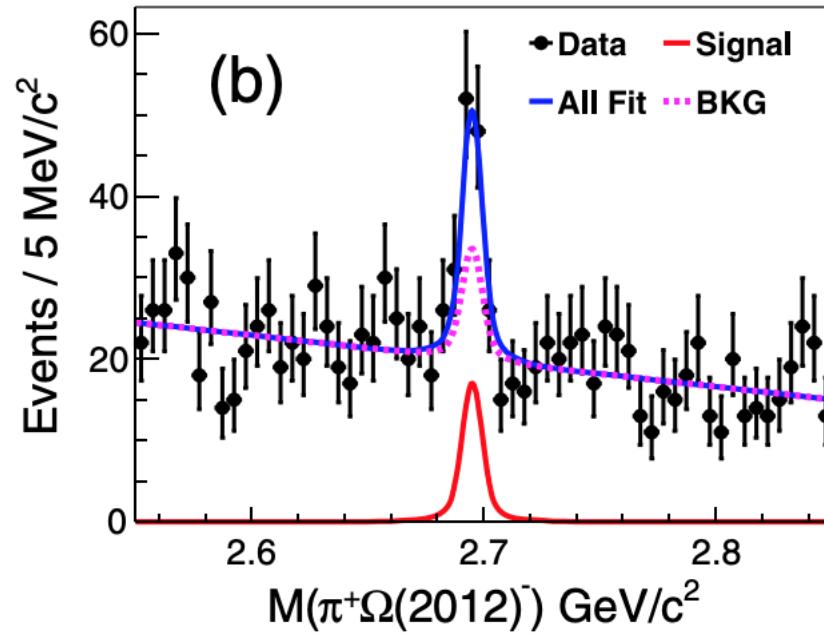
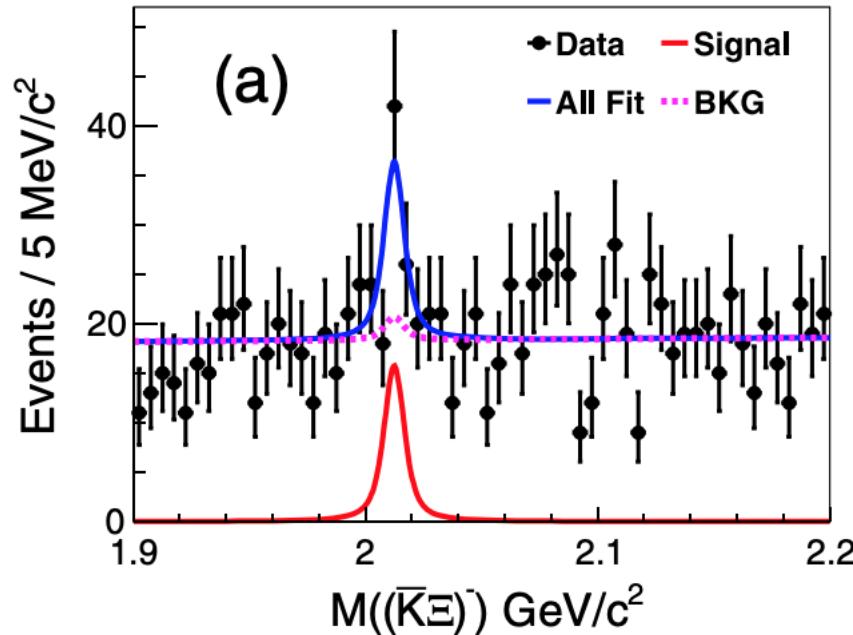


TABLE II: Predicted ratio  $R_{\bar{K}\Xi}^{\bar{K}\Xi\pi}$  for different cutoffs.

$\Lambda = q_{\max} (\text{MeV})$	735	750	800	850	900
$R_{\bar{K}\Xi}^{\bar{K}\Xi\pi} (\%)$	13.9	13.8	13.5	10.0	7.3

Small peak over dominant tree level BG

# Theory 3.3 $\Rightarrow$ Exp 3: Evidence for $\Omega_c^0 \rightarrow \pi^+ [\Omega(2012)^- \rightarrow \pi^+(\bar{K}\Xi)^-]$ from Belle



$$N_{\text{sig}} = 46.6 \pm 12.3$$

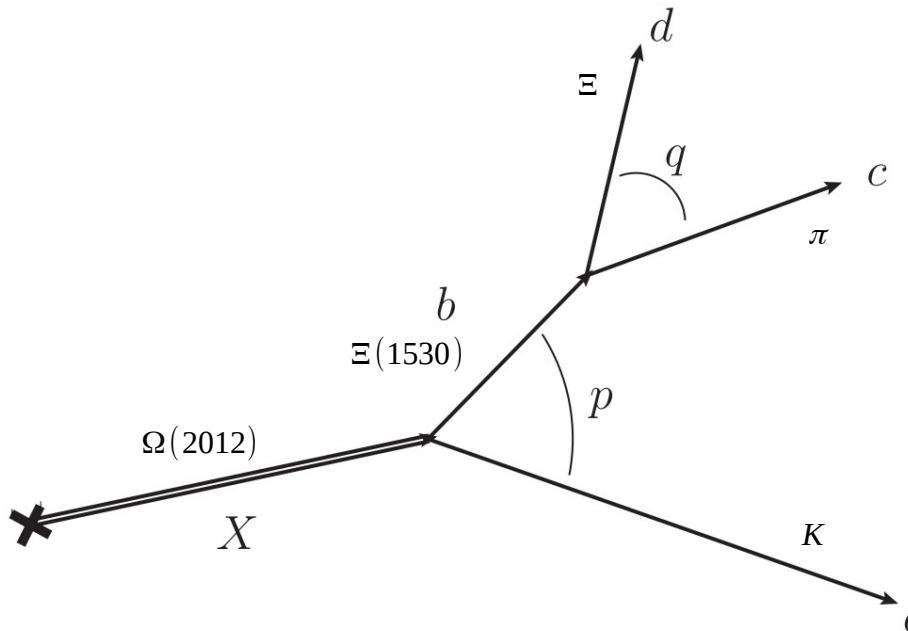
Significance:  $4.2\sigma$

$$\frac{\mathcal{B}(\Omega_c^0 \rightarrow \pi^+ [\Omega(2012)^- \rightarrow (\bar{K}\Xi)^-])}{\mathcal{B}(\Omega_c^0 \rightarrow \pi^+ (\bar{K}\Xi)^-)} = (6.50 \pm 1.22 \text{ (stat.)} \pm 0.94 \text{ (syst.)})\%$$

Phys. Rev. D 104, 052005 (2021)

## Theory 4 (lineshape) $\Rightarrow$ Experiment 4 at Belle for $\Omega(2012)$

Experiment should use the accurate lineshape instead of Breit-Wigner like:



- All decay is near-threshold.
- Intermediate  $b$  and  $X$  has finite width.

The lineshapes of  $X[\Omega(2012)]$  and  $b[\Xi(1530)]$  are completely distorted from the naive BW !

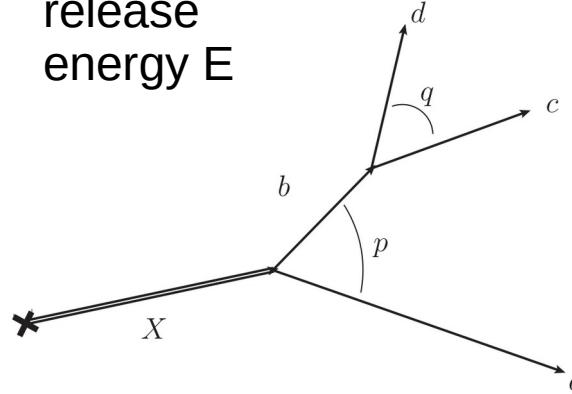
How to calculate them?

Fortunately, **theory 4** came to rescue us.

Much more advanced  
Flatte distribution.

C. HANHART, YU. S. KALASHNIKOVA, AND A. V. NEFEDIEV,  
Lineshapes for composite particles with unstable constituents,  
PHYSICAL REVIEW D 81, 094028 (2010)

For small release energy E



$$g_l = g_{cd}(2\mu_q)^{l+1/2} \text{ in cd system}$$

$\mu$ : reduced mass

$$\kappa_{\text{eff}}(E) = \kappa_1(E) + \kappa_2(E) - \kappa_1(E_X) - \kappa_2(E_X)$$

$$\kappa_1(E) = \frac{1}{\pi\mu_p} \int_0^\infty p^2 dp \times \frac{E_R - E + \frac{p^2}{2\mu_p}}{(E_R - E + \frac{p^2}{2\mu_p})^2 + \frac{g_l^2}{4}(E - \frac{p^2}{2\mu})^{2l+1}}$$

Theory 4 (lineshape)

$$k_{\text{eff}}(E) = \frac{g_l}{2\pi\mu_p} \int_0^{\sqrt{2\mu_p E}} p^2 dp \frac{(E_R - E + \frac{p^2}{2\mu_p})^{(2l+1)/2}}{(E_R - E + \frac{p^2}{2\mu_p})^2 + \frac{g_l^2}{4}(E - \frac{p^2}{2\mu_p})^{2l+1}}$$

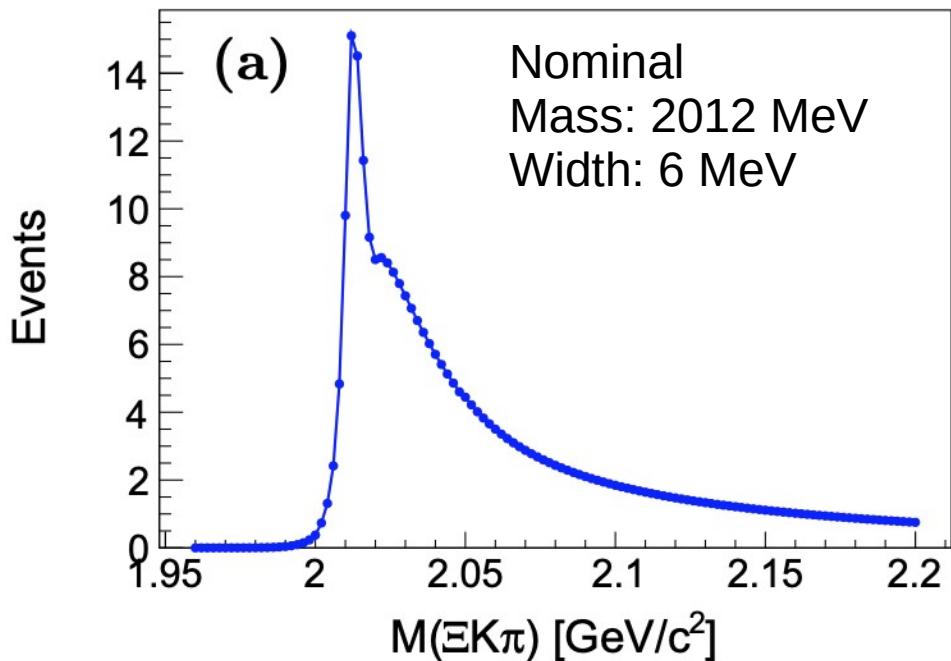
$$= \mathcal{B} \frac{1}{2\pi} \frac{g_{ab} k_{\text{eff}}(E)}{|E - E_X + \frac{1}{2}g_{ab}[\kappa_{\text{eff}}(E) + ik_{\text{eff}}(E)] + \frac{i}{2}\Gamma_0|^2}$$

Self-energy term

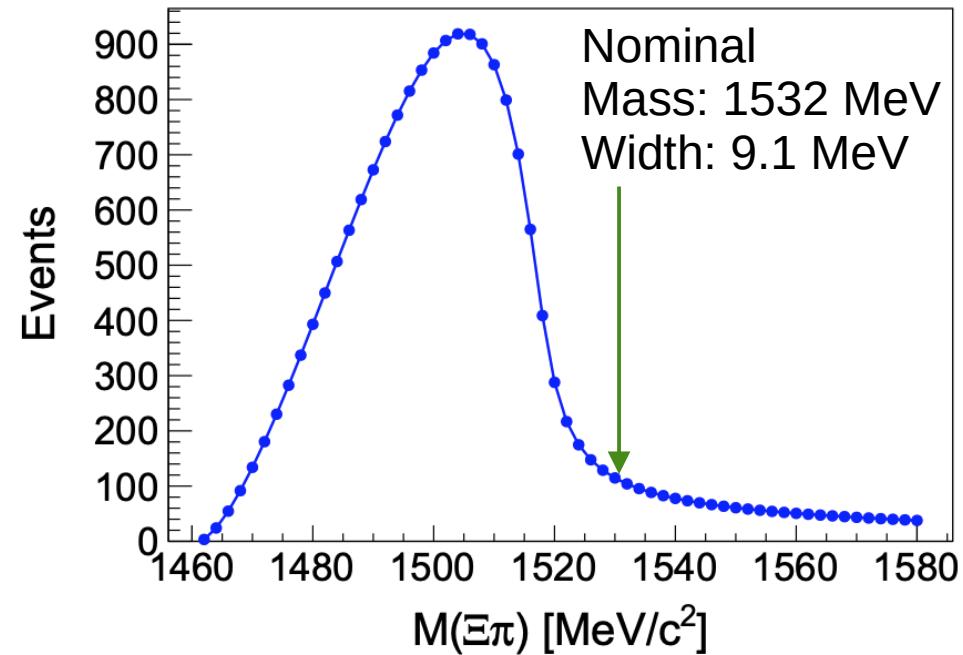
Can add energy term from other channel

$$\kappa_2(E) = -\frac{g_l}{2\pi\mu_p} \int_{\sqrt{2\mu_p E}}^\infty p^2 dp \times \frac{(\frac{p^2}{2\mu_p} - E)^{(2l+1)/2}}{(E_R - E + \frac{p^2}{2\mu_p})^2 + \frac{g_l^2}{4}(E - \frac{p^2}{2\mu})^{2l+1}}$$

# “Correct” lineshapes for $\Xi\pi K$ and $\Xi\pi$ in $\Omega(2012) \rightarrow [\Xi(1530) \rightarrow \Xi\pi]K$

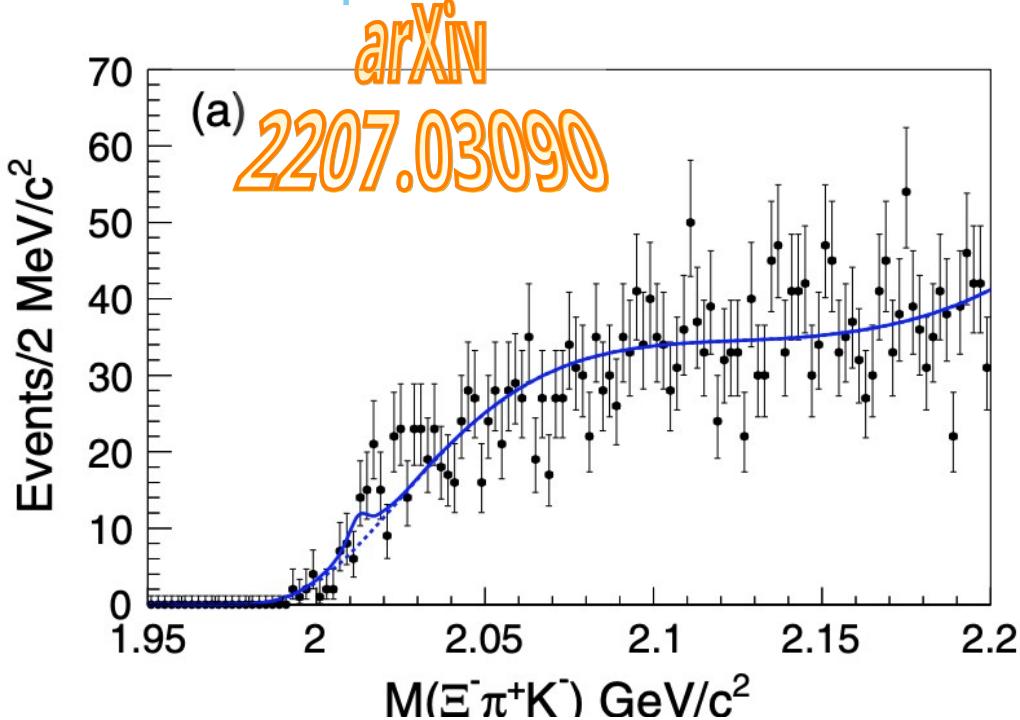


Weird shape with extremely long tail



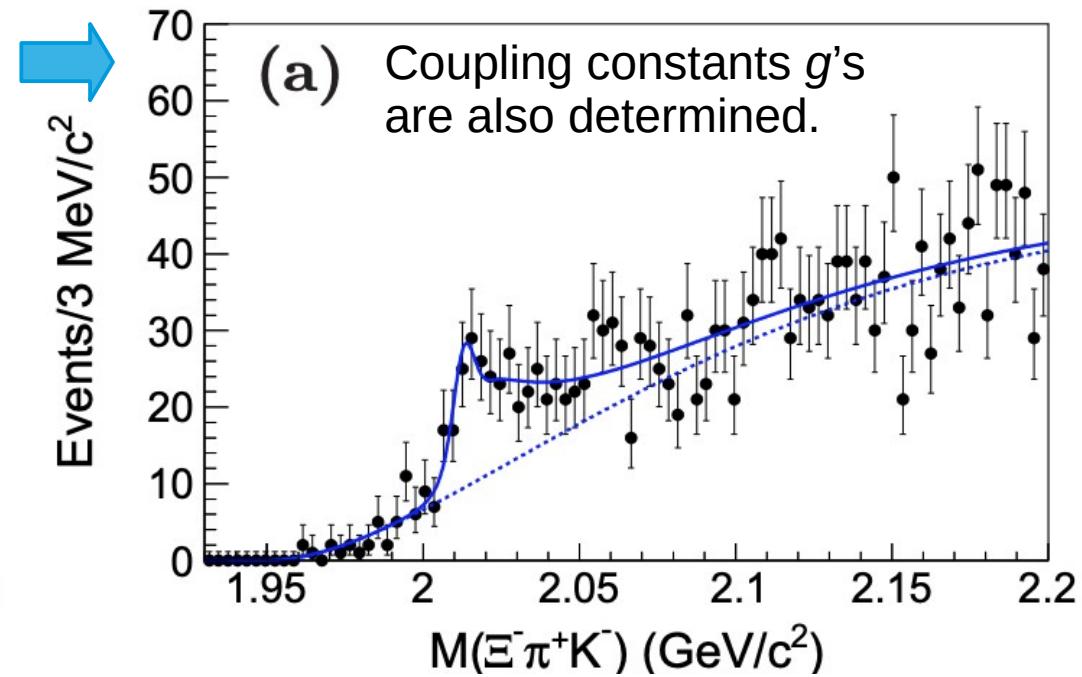
Peak completely off position.

## Experiment 4: Need to Revise selection and fit in Exp. 2 !



$$\mathcal{R}_{\Xi K}^{\Xi \pi K} = \frac{\mathcal{B}(\Omega(2012) \rightarrow \Xi(1530)(\rightarrow \Xi \pi) K)}{\mathcal{B}(\Omega(2012) \rightarrow \Xi K)} < 11.9\%$$

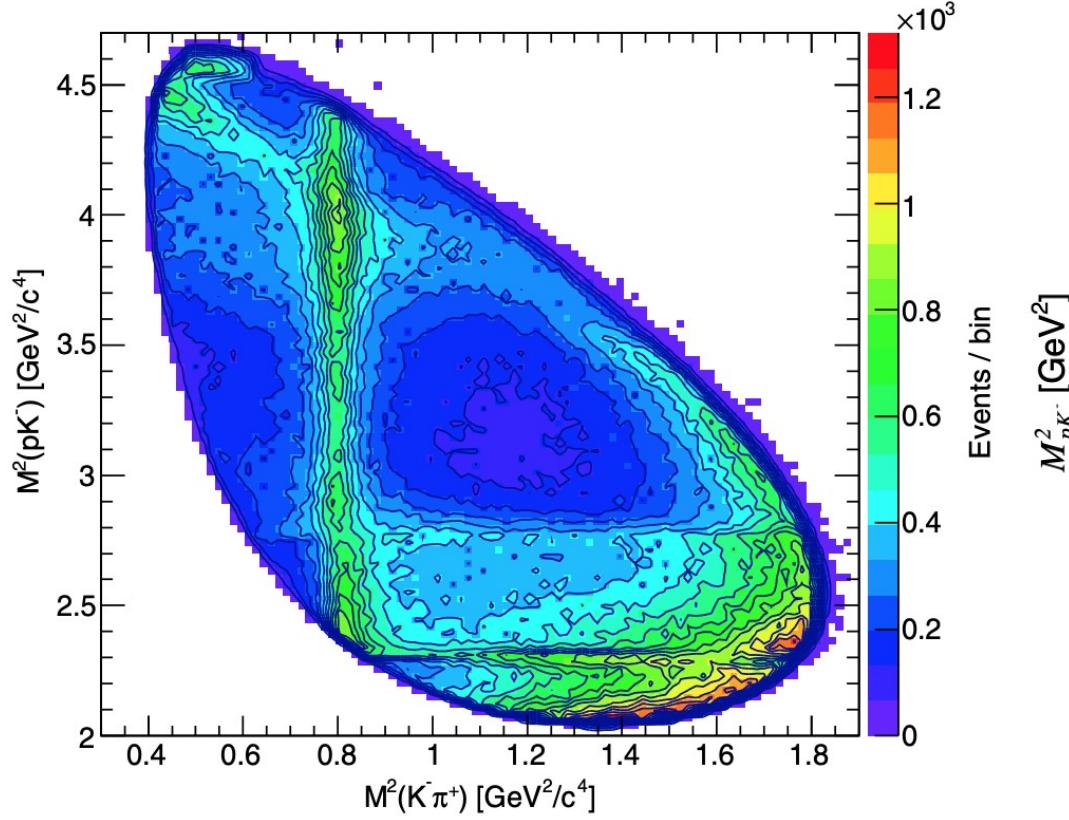
at 90% C.L.



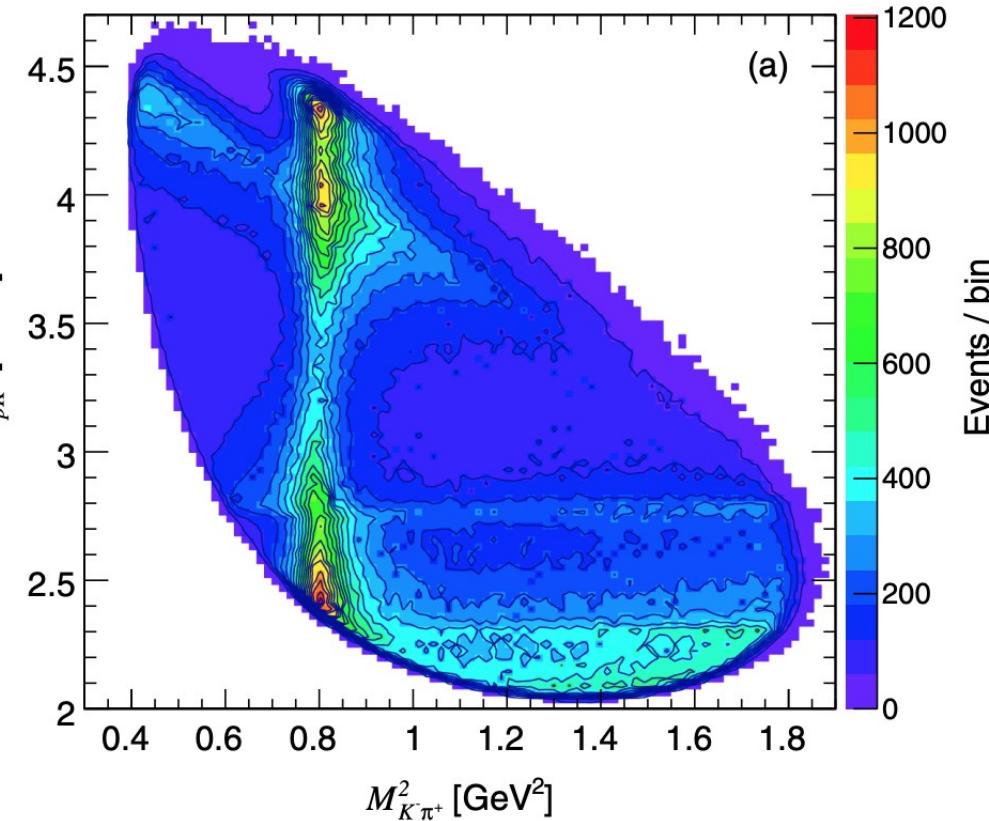
$$\mathcal{R}_{\Xi \bar{K}}^{\Xi \pi \bar{K}} = 0.97 \pm 0.24 \pm 0.07$$

I expect a new round of theory explanation.

# $\Lambda(1670)$ in $\Lambda_c$ decay

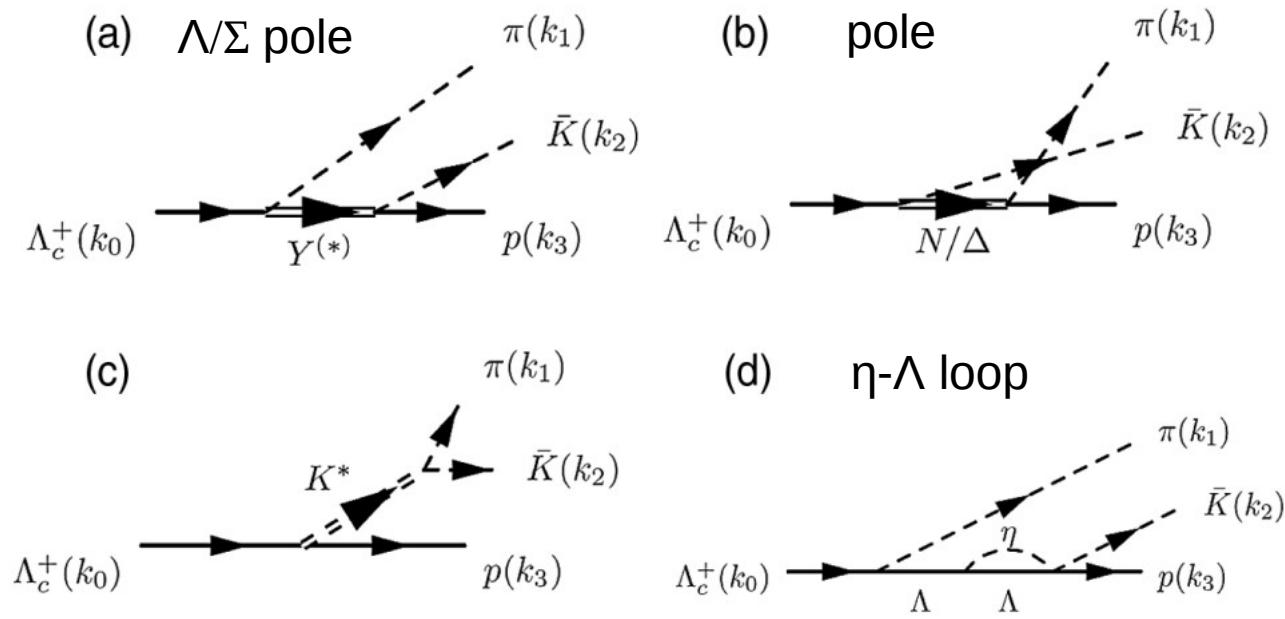


Exp: (Belle) Phys. Rev. Lett. 117, 011801



Theory: Jung Keun Ahn, et al., PHYS.  
REV. D 100, 034027 (2019)

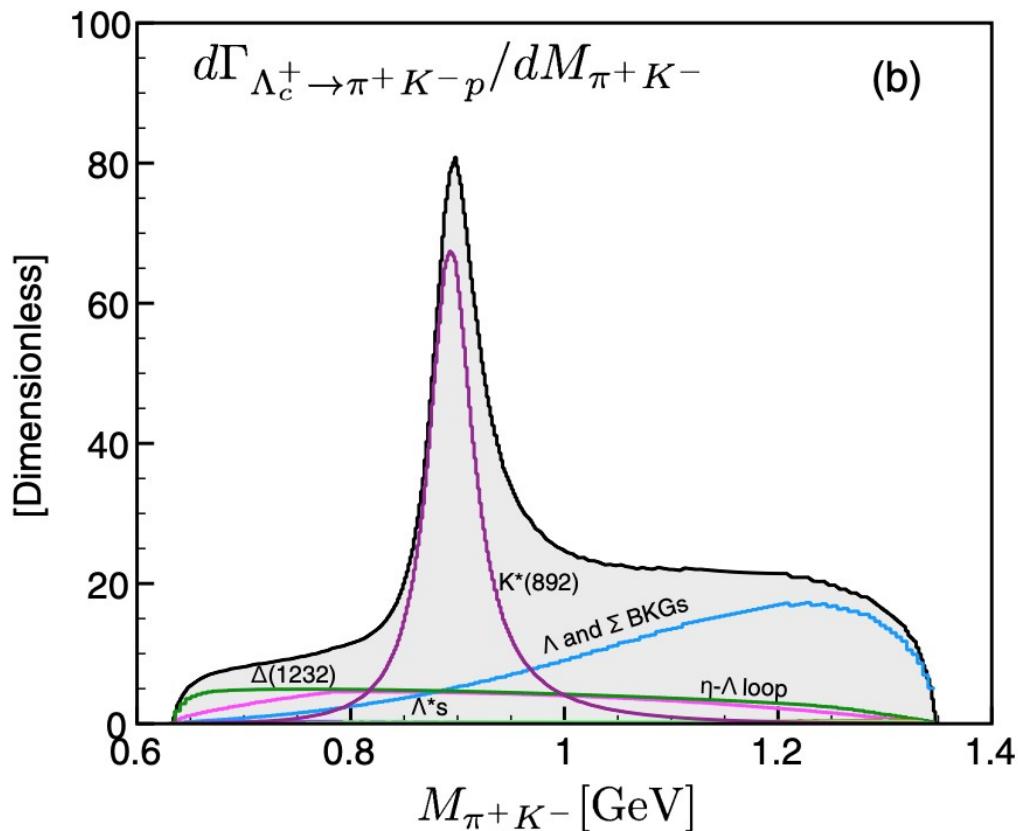
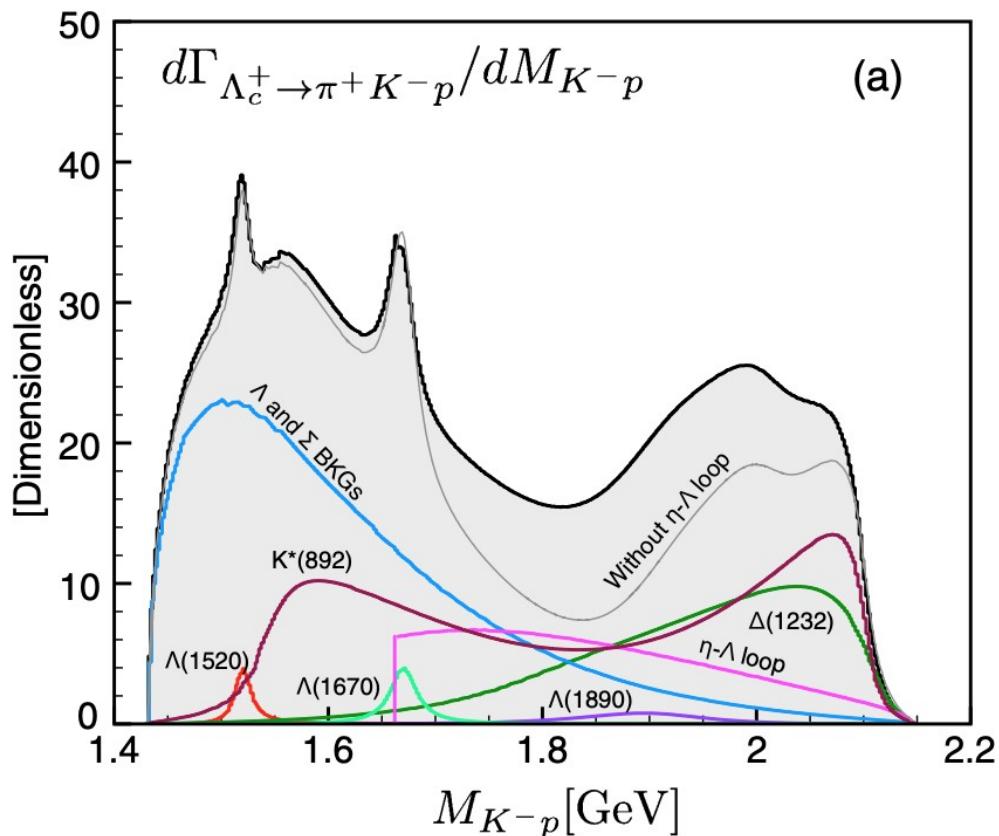
# Theoretical interpretation of $\Lambda_c \rightarrow p K^- \pi^+$



Effective Lagrangians  
and corresponding  
propagators in  
amplitudes.

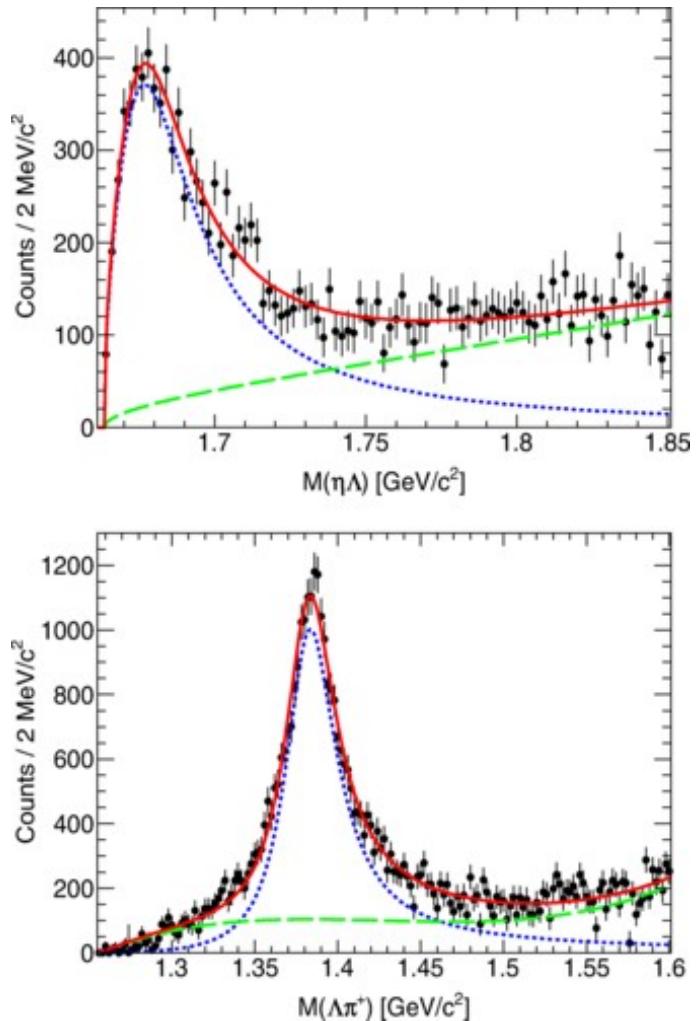
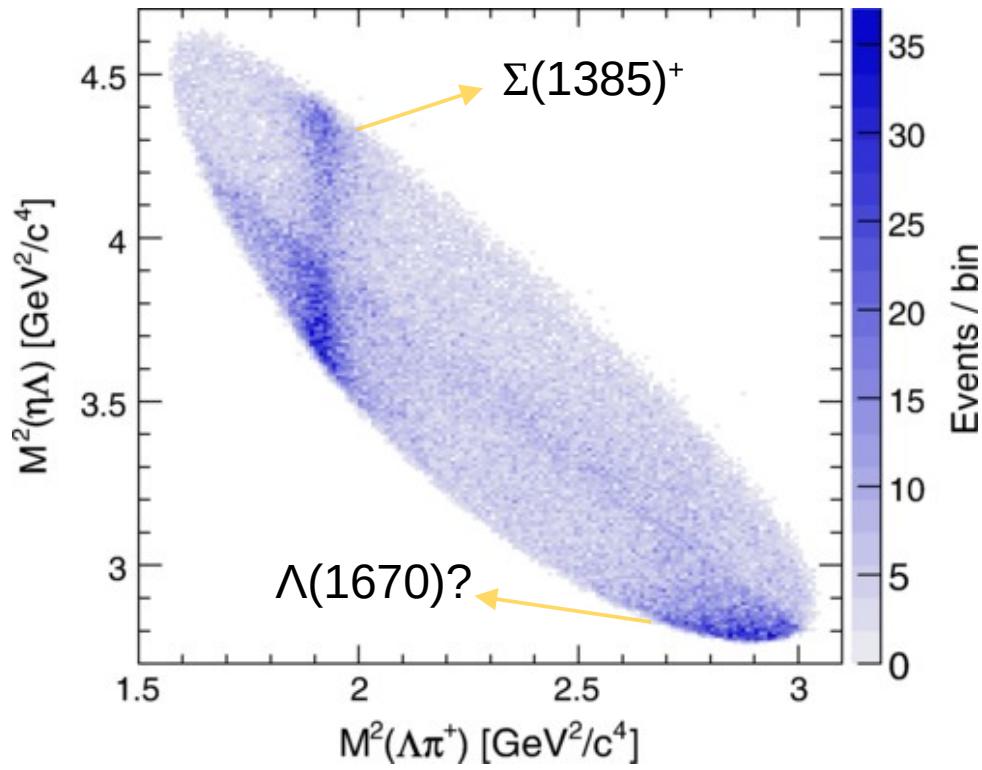
Assume same parity  
conserving and parity  
violating couplings for  
the hyperon  
resonances.

# Theory projections

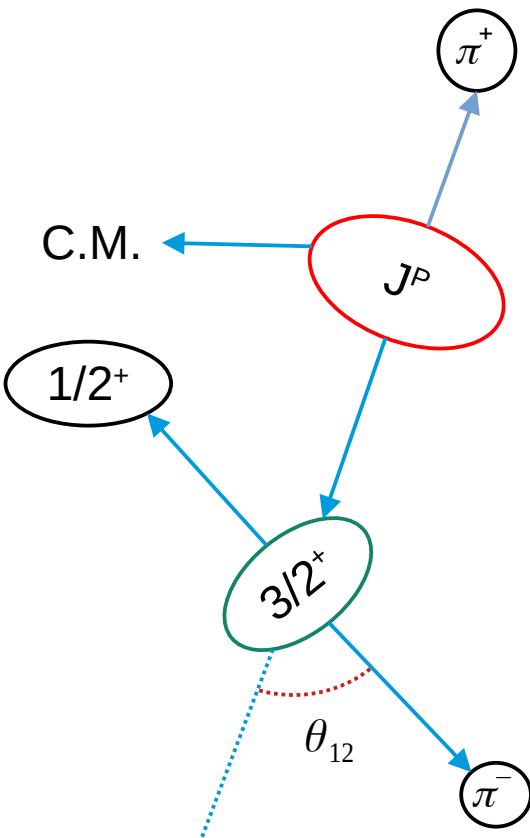


Solid black curve includes interference terms.

# Experiment to theory for $\Lambda(1670)$ and $\Sigma(1385)^+$ in $\Lambda c \rightarrow \eta \Lambda \pi^+$ ?



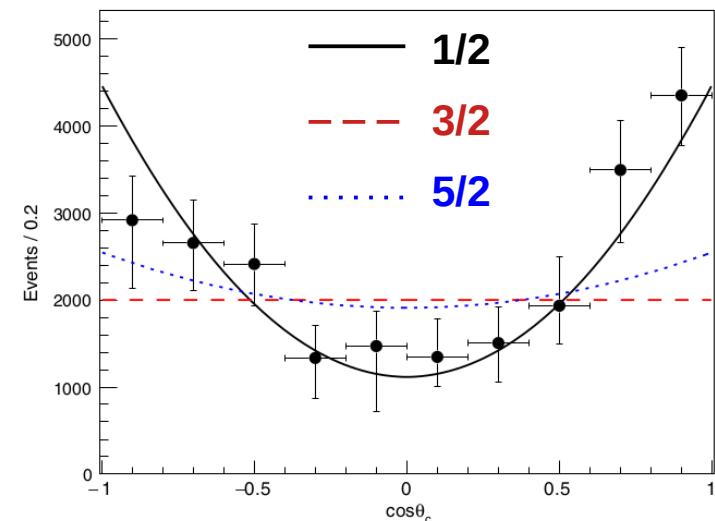
# Angular Distribution of $J^P \rightarrow [3/2^+ \rightarrow 1/2^+ \pi] \pi$



$J(j)^P$	$L$	$W(\theta_{12})$
$1/2(0)^-$	$d$	...
$1/2(1)^-$	$d$	$1 + 3 \cos^2 \theta_{12}$
$1/2(0)^+$	$p$	$1 + 3 \cos^2 \theta_{12}$
$1/2(1)^+$	$p$	$1 + 3 \cos^2 \theta_{12}$
$3/2(1)^-$	$s, d$	1
$3/2(2)^-$	$s, d$	1
$3/2(1)^+$	$p, f$	$1 + 6 \sin^2 \theta_{12}$
$3/2(2)^+$	$p, f$	$1 + 6 \sin^2 \theta_{12}$
$5/2(2)^-$	$d, g$	$1 + (15/4) \sin^2 \theta_{12}$
$5/2(2)^+$	$p, f$	$1 + (1/3) \cos^2 \theta_{12}$
$5/2(3)^+$	$p, f$	$1 + (3/8) \sin^2 \theta_{12}$
$7/2(3)^+$	$f, h$	$1 + 3 \sin^2 \theta_{12}$

Theory: PHYS. REV. D 101, 094023  
(2020)

$J$  determination of  
 $\Xi_c(2970)^+$  in its  
 $\Xi_c(2645)^0[\Xi_c^+\pi^-]\pi^+$  channel

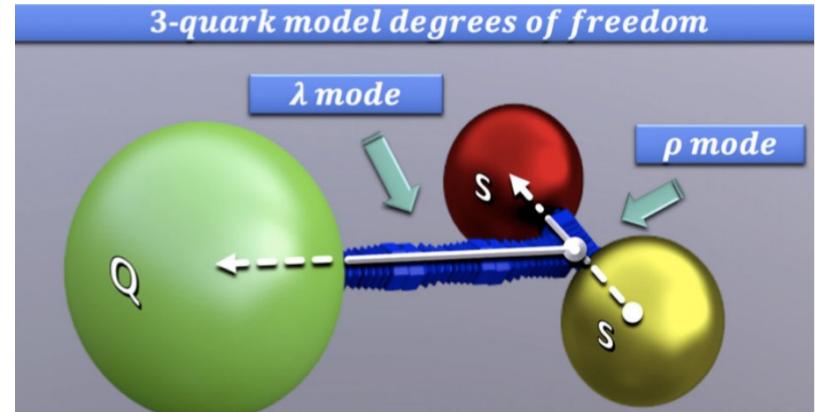


Favors  $J=1/2$   
- consistent with  $1+3\cos^2\theta_c$

Exp: (Belle) Phys. Rev. D 103, 111101

# Radiative transition of charmed baryon

- EM transitions of charmed baryons are observed only for strong decay forbidden states:  $\Xi'_c \rightarrow \Xi_c \gamma$  and  $\Omega_c(2770) \rightarrow \Omega_c \gamma$ .
- Theoretical predictions of observable partial width ( $\sim 300$  keV) for decays from  $\Xi_c(2790)$  and  $\Xi_c(2815)$  to  $\Xi_c \gamma$  (3-10 % level of BR).
- Input of EM decay measurements is crucial for interpretation of  $\lambda$ - $\rho$  modes and theoretical modeling.




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WYZZ [14] K-L. Wang, Y-X. Yao, X-H. Zhong, and Q. Zhao, Phys. Rev. D 96, 116016 (2017)

Mode	$\lambda$ excitation	$\rho$ excitation	$\rho$ excitation	Actual total width [3]	Phys. Rev. D 94, 052011 (2016)
$\Xi_c(2790)^+ \rightarrow \Xi_c^+ \gamma$	4.65	1.39	0.79	$8900 \pm 600 \pm 800$	
$\Xi_c(2790)^0 \rightarrow \Xi_c^0 \gamma$	263	5.57	3.00	$10000 \pm 700 \pm 800$	
$\Xi_c(2815)^+ \rightarrow \Xi_c^+ \gamma$	2.8	1.88	2.81	$2430 \pm 200 \pm 170$	
$\Xi_c(2815)^0 \rightarrow \Xi_c^0 \gamma$	292	7.50	11.2	$2540 \pm 180 \pm 170$	

---

# $\Xi_c(2790)$ & $\Xi_c(2815)$ radiative decay results

Typically interpreted as an HQS doublet with orbital L=1 ( $\lambda$ -mode), with expected  $J^P = 1/2^-$  and  $3/2^-$ .

Clear signal for neutral channel, but not charged.

$$\frac{\mathcal{B}(\Xi_c(2815)^0 \rightarrow \Xi_c^0 \gamma)}{\mathcal{B}(\Xi_c(2815)^0 \rightarrow \Xi_c(2645)^+ \pi^- \rightarrow \Xi_c^0 \pi^+ \pi^-)} = 0.45 \pm 0.05 \pm 0.03$$

$$\frac{\mathcal{B}(\Xi_c(2790)^0 \rightarrow \Xi_c^0 \gamma)}{\mathcal{B}(\Xi_c(2790)^0 \rightarrow \Xi_c^+ \pi^- \rightarrow \Xi_c^+ \gamma \pi^-)} = 0.13 \pm 0.03 \pm 0.02$$

$$\Gamma(\Xi_c(2815)^0 \rightarrow \Xi_c^0 \gamma) = 320 \pm 45^{+45}_{-80} \text{ keV}$$

$$\Gamma(\Xi_c(2790)^0 \rightarrow \Xi_c^0 \gamma) \sim 800 \text{ keV ( uncertainty } \sim 40\%)$$

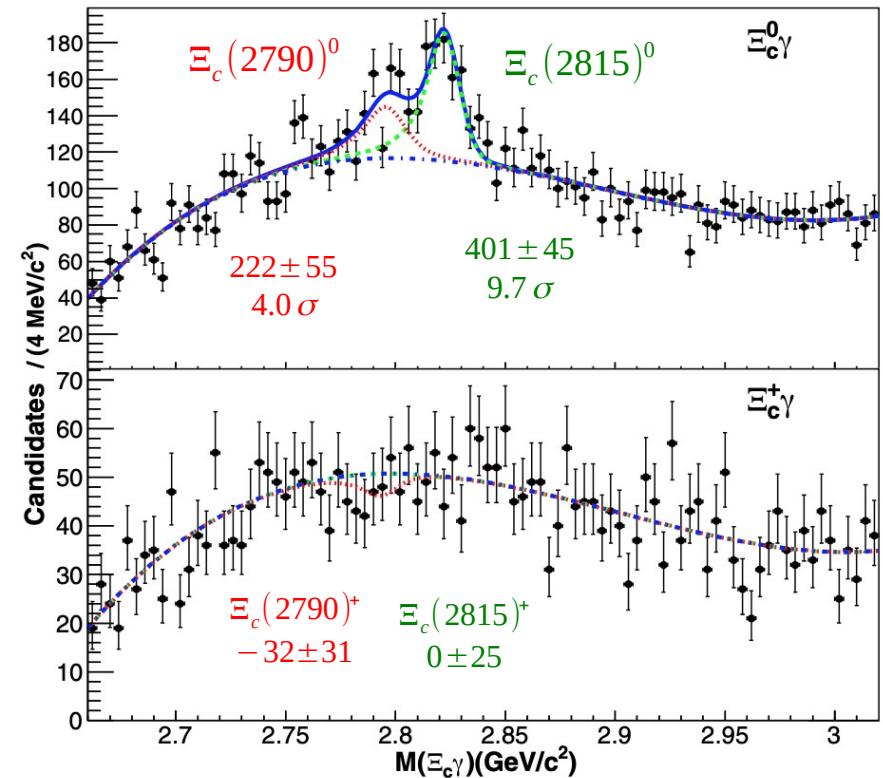
$$\Gamma(\Xi_c(2815)^+ \rightarrow \Xi_c^+ \gamma) < 80 \text{ keV}$$

$$\Gamma(\Xi_c(2790)^+ \rightarrow \Xi_c^+ \gamma) < 350 \text{ keV}$$

Consistent with orbital excitation interpretation.

Phys. Rev. D 96, 116016 (2017)

PRD 102, 071103(R) (2020)



## Summary and Upcoming interplay

We have shown the importance of joint experimental and theoretical effort for studies on  $\Omega(2012)^-$ , excited hyperons in three body  $\Lambda c$  decay,

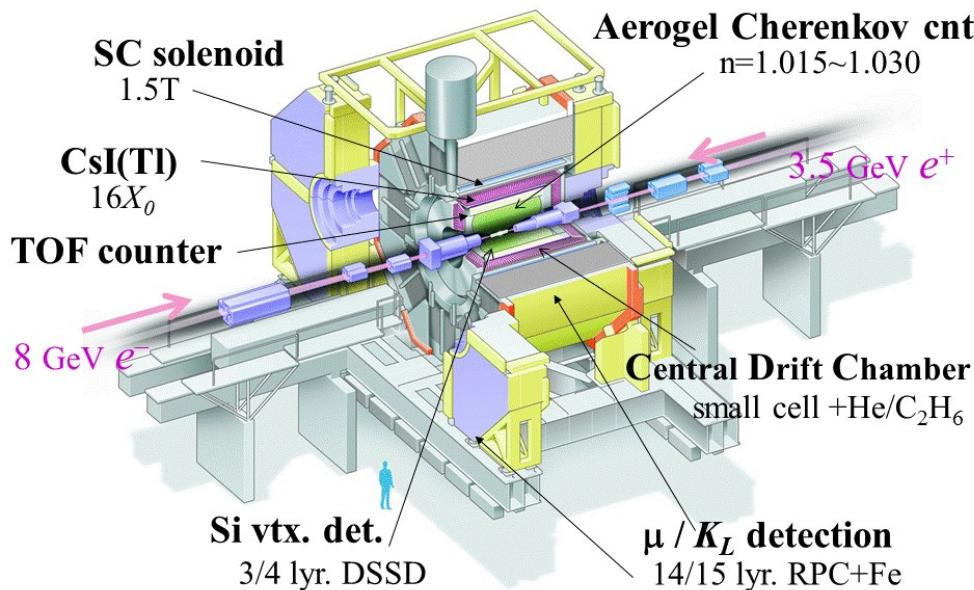
# Need interplay between theorists and experimentalists.

- ◆ Global fits to baryon and meson spectrum. [ ${}^3P_0$  model for charmed baryon; light cone model for mesons]. The common bootstrap and fitting method in experiment is used. Output can guide future experimental search. [ [arXiv:2205.07049](#) ]
- ◆ Double charmed baryon production (ex.  $\bar{\Lambda}_c \Lambda_c^*$ ) from  $e^+e^-$ ; spin-parity study of excited baryon– the initial helicity of excited baryon is known, precise and powerful angular distributions.
- ◆ Theoretical predictions of masses and decay channels of exotic hadrons – ex:  $\Lambda_c \bar{D}^0$  and  $\Sigma_c D^-$  (hidden charm pentaquark); tetraquark and pentaquark with strangeness; collaboration with Italian group at INFN.

# BACKUP

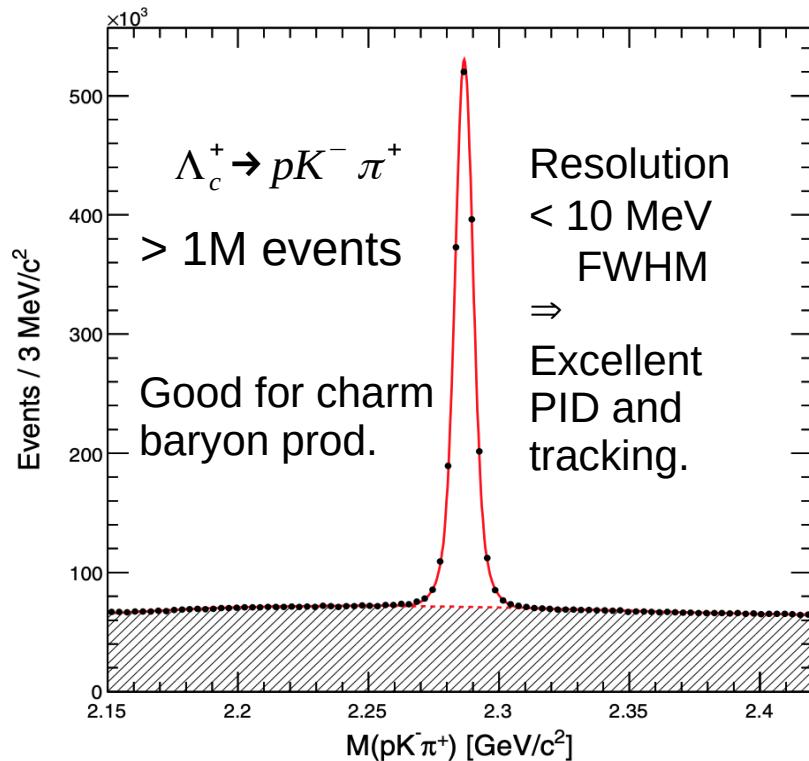
# Belle Detector and charmed baryon production.

## Belle Detector @ 10.58 GeV



Total int. luminosity  $\sim 1 \text{ ab}^{-1}$   
⇒ used in all three studies

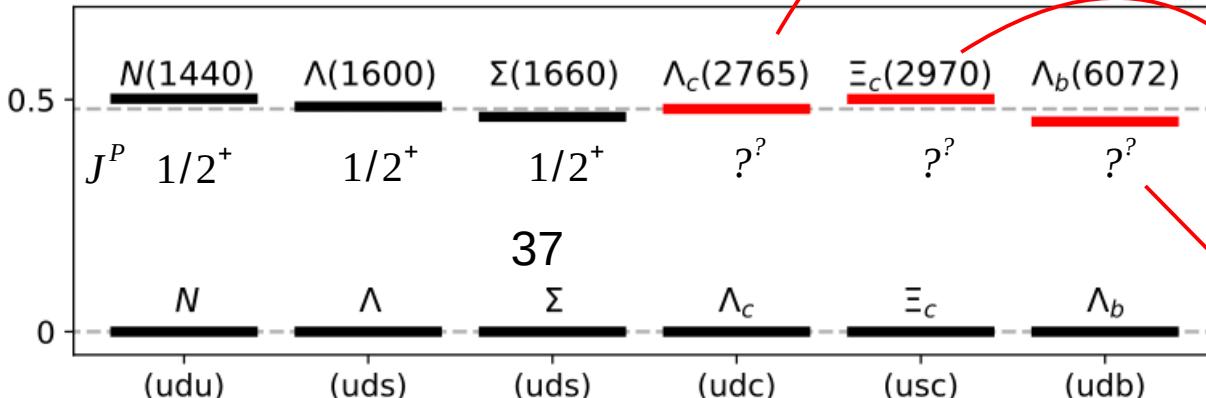
$$e^+ e^- \rightarrow Y(4S): 1.1 \text{ nb}^{-1} \quad e^+ e^- \rightarrow c\bar{c}: 1.3 \text{ nb}^{-1}$$



# $\Xi_c(2970)^+$ in baryon family

First excited  $J^P = \frac{1}{2}^+$  baryons

$\Delta M$  [GeV]

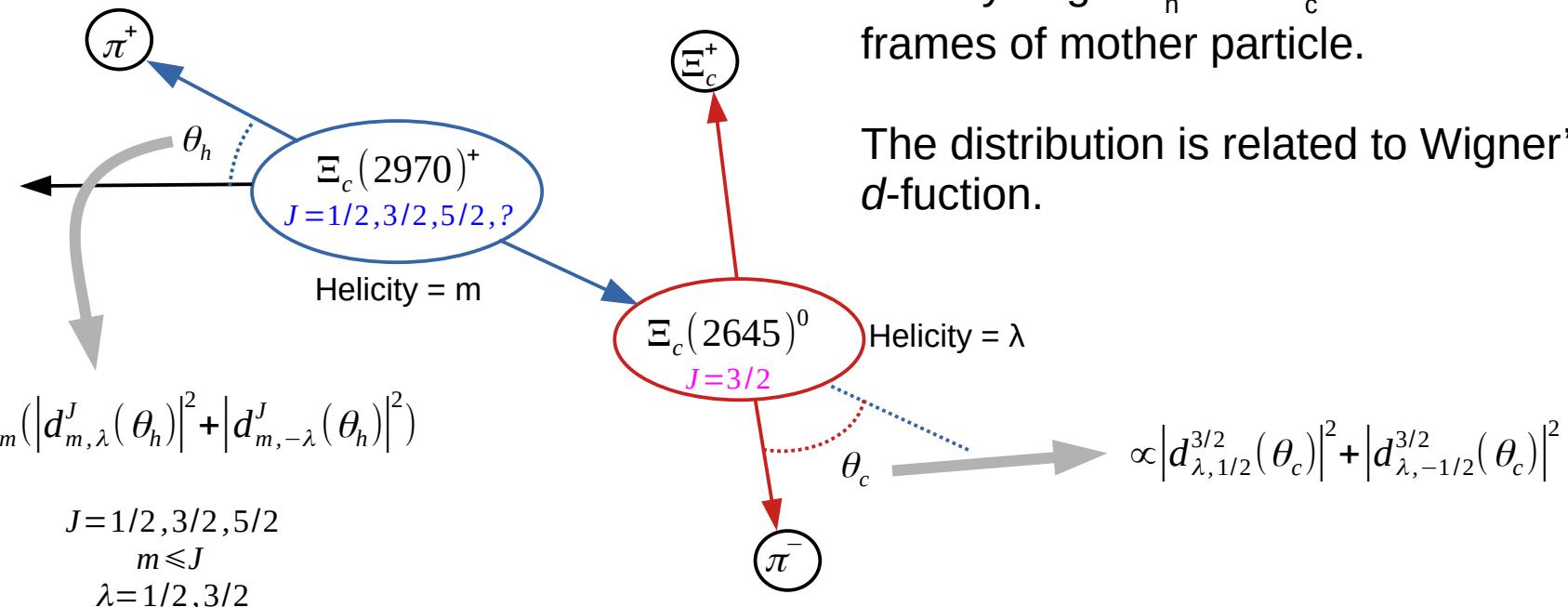


Not been pinned down.  
Theory prediction only.

Nucl. Phys. Rev. 35, 1  
Int J Theor Phys 59, 1129

Belle's work (this talk)  
Phys. Rev. D 103, 111101

LHCb data suggest  $\frac{1}{2}^+$  assignment.  
- Arifi, Hosaka, Nagahiro,  
and Tanida  
Phys. Rev. D 101, 111502



$\rho_{mm}$ : Initial spin density of  $\Xi_c(2970)^+$

Helicity angle  $\theta_h$  and  $\theta_c$  evaluated in rest frames of mother particle.

The distribution is related to Wigner's  $d$ -function.

$$\propto |d_{\lambda,1/2}^{3/2}(\theta_c)|^2 + |d_{\lambda,-1/2}^{3/2}(\theta_c)|^2$$

# Formula of angular distribution ( $\theta_h$ and $\theta_c$ )

$\theta_h$  for  $J \rightarrow 3/2(\Xi_c(2645)) + 0$

Relative fraction of 3/2 polarization in  $\Xi_c(2645)$

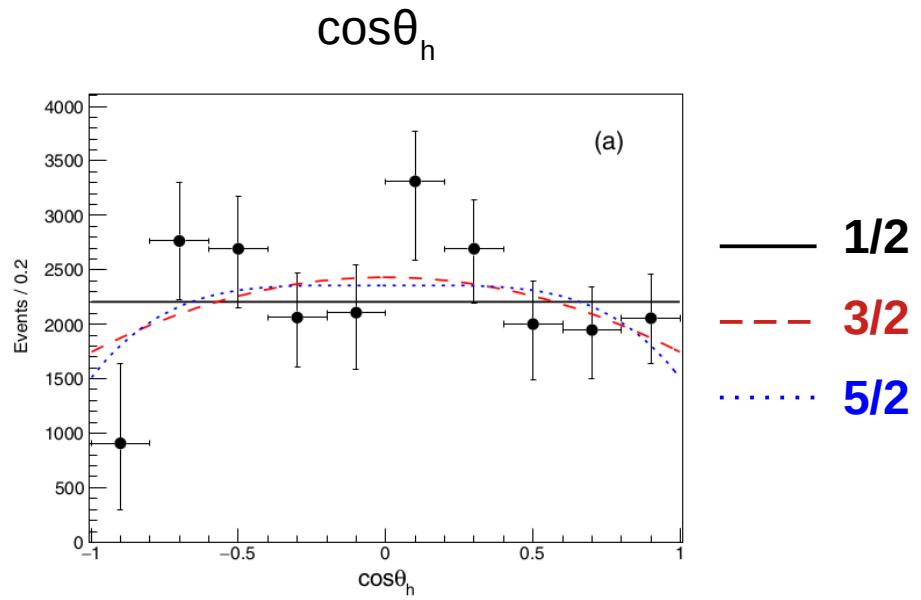
$$W_{\frac{1}{2}} = \text{constant}$$

$$W_{\frac{3}{2}} = \rho_{33} \left\{ 1 + T \left( \frac{3}{2} \cos^2 \theta_h - \frac{1}{2} \right) \right\} + \rho_{11} \left\{ 1 + T \left( -\frac{3}{2} \cos^2 \theta_h + \frac{1}{2} \right) \right\}$$

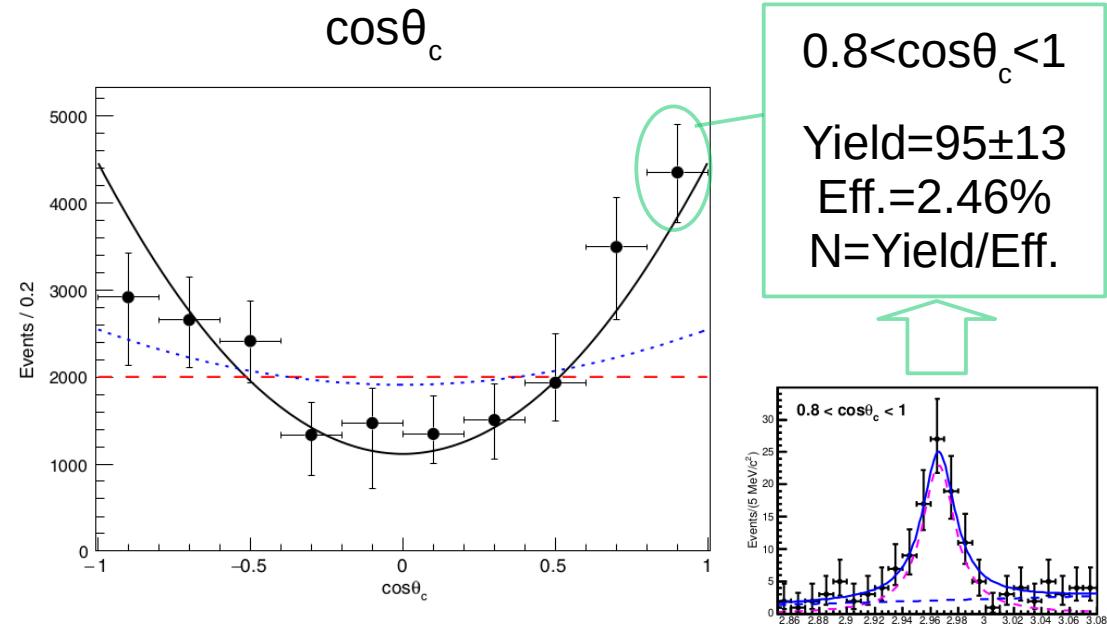
$$\begin{aligned} W_{\frac{5}{2}} = & \frac{3}{32} [\rho_{55} 5 \{(-\cos^4 \theta_h - 2 \cos^2 \theta_h + 3) + T(-5 \cos^4 \theta_h + 6 \cos^2 \theta_h - 1)\} \\ & + \rho_{33} \{(15 \cos^4 \theta_h - 10 \cos^2 \theta_h + 11) + T(75 \cos^4 \theta_h - 66 \cos^2 \theta_h + 7)\}] \\ & + \rho_{11} 2 \{(-5 \cos^4 \theta_h + 10 \cos^2 \theta_h + 3) + T(-25 \cos^4 \theta_h + 18 \cos^2 \theta_h - 1)\}] \end{aligned}$$

$\theta_c$  in  $\Xi_c(2645)(3/2^-) \rightarrow \Xi_c(1/2^+) + \pi(0^-)$

$J^P$ of $\Xi_c(2970)^+$	Partial Wave	Expected Angular Distribution
1/2 <sup>+</sup>	P	$1 + 3 \cos \theta_c^2$
1/2 <sup>-</sup>	D	$1 + 3 \cos \theta_c^2$
3/2 <sup>+</sup>	P	$1 + 6 \sin \theta_c^2$
3/2 <sup>-</sup>	S	1
5/2 <sup>+</sup>	P	$1 + (1/3) \cos \theta_c^2$
5/2 <sup>-</sup>	D	$1 + (15/4) \sin \theta_c^2$



Inconclusive in  $\cos\theta_h$   
Exclusion Level 0.8 (0.5) $\sigma$  for 3/2 (5/2) hypothesis



Favors  $J=1/2$   
over 3/2 (5/2) at 5.1 (4.0) $\sigma$   
- consistent with  $1+3\cos^2\theta_c$

- HQS doublet with brown-muck (light component) spin  $j=1$ :  $J=3/2$  ( $\Xi_c(2645)^0$ ) and  $1/2$  ( $\Xi_c^{'+0}$ )
- The decay rate ratio  $R = \frac{\Gamma(\Xi_c(2970)^+ \rightarrow \Xi_c(2645)^0 \pi^+)}{\Gamma(\Xi_c(2970)^+ \rightarrow \Xi_c^{'+0} \pi^+)}$  is calculable:

PRD 75 (2007) 014006

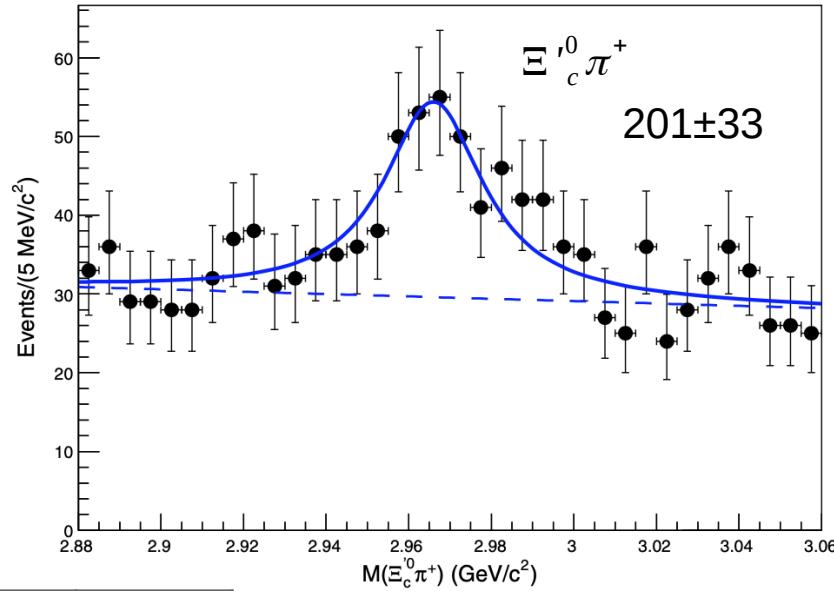
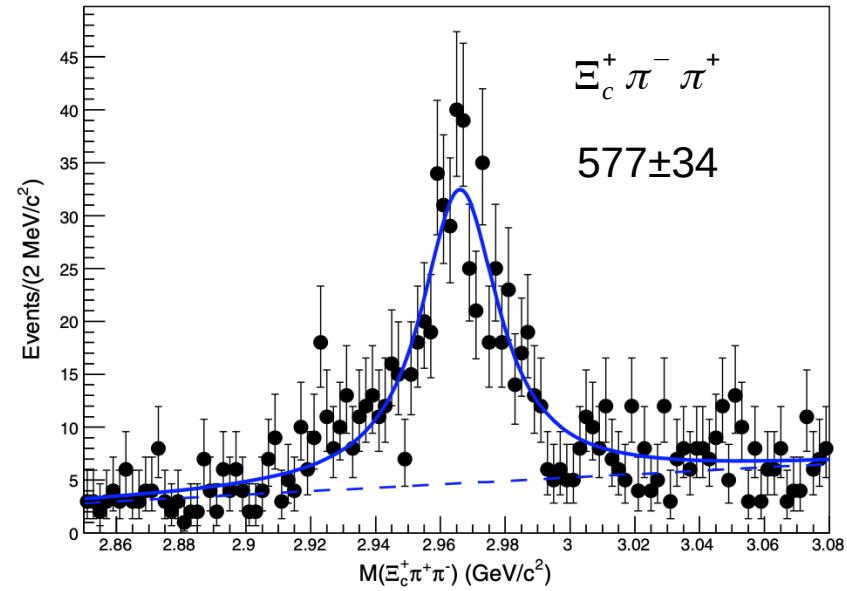
Parity	+	+	-	-
Brown-muck spin $s_\ell$	0	1	0	1
R	1.06	0.26	0	$\ll 1$

Suppressed due to  
D-wave of  
 $\Xi_c(2645)^0 \pi^+$



Determination of Parity and  $s_\ell$

# Results of BR



$s_l$ ( $P=+$ )	0	1
$R$	1.06	0.26

$$R = \frac{\Gamma(\Xi_c(2970)^+ \rightarrow \Xi_c(2645)^0 \pi^+)}{\Gamma(\Xi_c(2970)^+ \rightarrow \Xi_c'^0 \pi^+)} = 1.67 \pm 0.29^{+0.18}_{-0.09} \pm 0.25$$

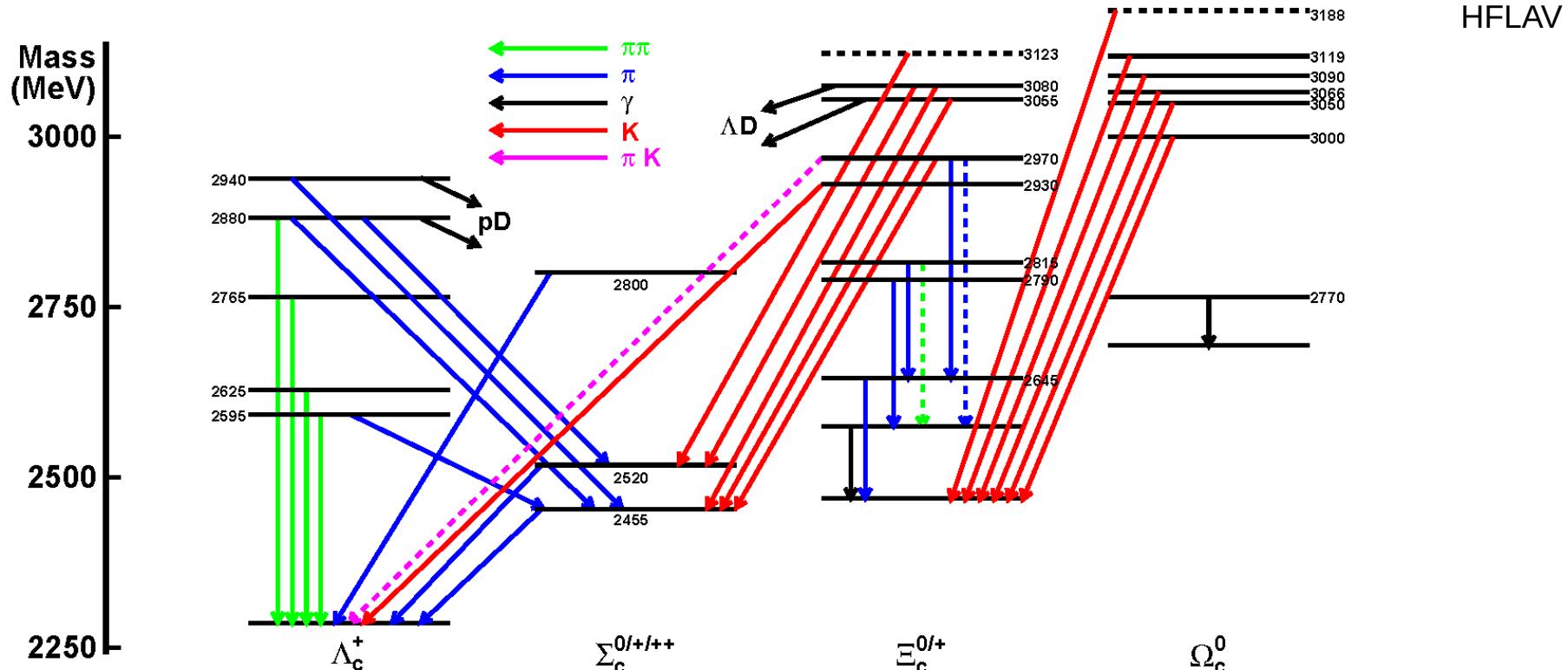
Favors

Due to isospin assumption in sub-BF calculation for  $\Xi_c^{+0}$

Final conclusion:  $J^P(s_l) = 1/2^+(0)$

PRD 103, L111101 (2021)

# Transitions of charmed baryons



Only  $\Xi_c' \rightarrow \Xi_c \gamma$  and  $\Omega_c(2770) \rightarrow \Omega_c \gamma$  for electromagnetic decays.