~~Nuclear medium effects on~~ Parton distribution functions for spin-1 vector meson

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APCTP Workshop on Nuclear Physics 2022: Physics of Excited Hadrons in the present and future facilities

Outline

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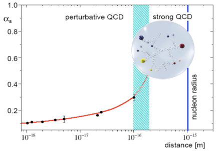
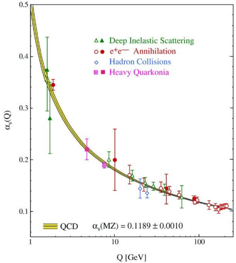
Introduction: Quantum Chromodynamics (QCD)

QCD start out with almost mass-less quarks + mass-less gluons + gauge bosons, their interactions generate the mass of the hadrons more than 95%

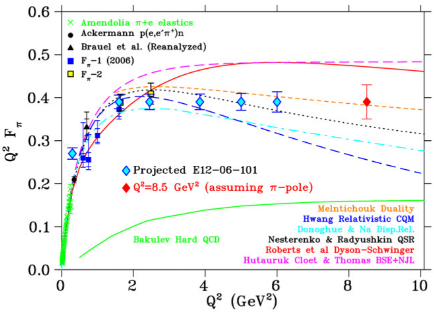


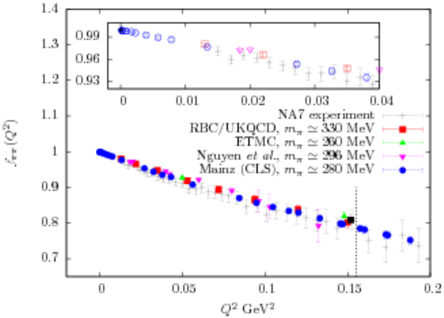


- In short distance scales ($r < 0.1 fm$) $\Rightarrow$ QCD is a theory of weakly coupled quark and gluon $\Rightarrow$ perturbative QCD applicable
- In the low energy limit, at momentum below 1 GeV ($r > 1 fm$) $\Rightarrow$ QCD is governed by quark (color) confinement and dynamical breaking of chiral symmetry $\Rightarrow$ nonperturbative QCD applicable

Motivation: internal structure of hadrons

- QCD, as underlying theory of strong interaction, is unable to *directly* predict structure of hadrons. *The solution*:
  - Lattice QCD: Large momentum effective theory (LAMET) [1]
  - QCD inspired models (mimicking features of QCD) such as NJL model, DSE model, QCD Sum rules, Instanton model, Covariant non-local chiral quark model (NLChQM) and other chiral effective models, Credit: Garth Huber Slide, EIC Meeting 2018 & Ref. [2]





[1] X. Ji, Phys. Rev. Lett. **110**

[2] Bastian. B. Brant, Int. J. Mod. Phys. E**22** (2013)

Motivation: internal structure of hadrons

- To understand the structure of strongly interacting matter, parton distribution function (PDF), elastic form factor (EFF), fragmentation function (FF), generalized parton distribution function and transverse momentum dependent (TMD) are of fundamental importance and provide complementary information
- From experimental side, new experimental data for the hadron will be collected from JLAB (F_π or F_K are expected to be available soon), J-PARC as well as (CERN-SPS) COMPASS, future experiment Electron-Ion Collider (EIC), and EicC (China)

Motivation: Vector meson properties

- The properties of vector mesons from the quark model [3]

Vector mesons

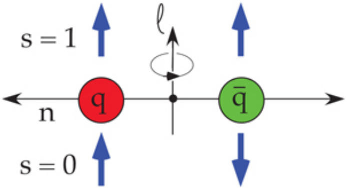
Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c^2)	I^G	J^{PC}	S	C	B'	Mean lifetime (s)	Commonly decays to (>5% of decays)
Charged rho meson[27]	$\rho^+(770)$	$\rho^-(770)$	$u\bar{d}$	775.11 ± 0.34	1^+	1^-	0	Sort ascending	0	$(4.41 \pm 0.02) \times 10^{-24}$ [f][g]	$\pi^\pm + \pi^0$
Neutral rho meson[27]	$\rho^0(770)$	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	775.26 ± 0.25	1^+	1^{--}	0	0	0	$(4.45 \pm 0.03) \times 10^{-24}$ [f][g]	$\pi^+ + \pi^-$
Omega meson[28]	$\omega(782)$	Self	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	782.65 ± 0.12	0^-	1^{--}	0	0	0	$(7.75 \pm 0.07) \times 10^{-23}$ [f]	$\pi^+ + \pi^0 + \pi^-$ or $\pi^0 + \gamma$
Phi meson[29]	$\phi(1020)$	Self	$s\bar{s}$	1 019.461 ± 0.019	0^-	1^{--}	0	0	0	$(1.54 \pm 0.01) \times 10^{-22}$ [f]	$K^+ + K^-$ or $K^0_S + K^0_L$ or $(\rho + \pi)$ / $(\pi^+ + \pi^0 + \pi^-)$
J/Psi[30]	J/ψ	Self	$c\bar{c}$	3 096.916 ± 0.011	0^-	1^{--}	0	0	0	$(7.09 \pm 0.21) \times 10^{-21}$ [f]	See J/ψ(1S) decay modes
Upsilon meson[31]	$\Upsilon(1S)$	Self	$b\bar{b}$	9 460.30 ± 0.26	0^-	1^{--}	0	0	0	$(1.22 \pm 0.03) \times 10^{-20}$ [f]	See Υ(1S) decay modes
Kaon[32]	K^{*+}	K^{*-}	$u\bar{s}$	891.66 ± 0.26	$\frac{1}{2}$	1^-	1	0	0	$(3.26 \pm 0.06) \times 10^{-23}$ [f][g]	See $K^*(892)$ decay modes
Kaon[32]	K^{*0}	$\bar{K}^{*0}$	$d\bar{s}$	895.81 ± 0.19	$\frac{1}{2}$	1^-	1	0	0	$(1.39 \pm 0.02) \times 10^{-23}$ [f]	See $K^*(892)$ decay modes
D meson[33]	$D^{*+}(2010)$	$D^{*-}(2010)$	$c\bar{d}$	2 010.26 ± 0.07	$\frac{1}{2}$	1^-	0	+1	0	$(7.89 \pm 0.17) \times 10^{-21}$ [f]	$D^0 + \pi^+$ or $D^+ + \pi^0$
D meson[34]	$D^{*0}(2007)$	$\bar{D}^{*0}(2007)$	$c\bar{u}$	2 006.96 ± 0.10	$\frac{1}{2}$	1^-	0	+1	0	$>3.1 \times 10^{-22}$ [f]	$D^0 + \pi^0$ or $D^0 + \gamma$
Strange D meson[35]	D^{*+}_s	D^{*-}_s	$c\bar{s}$	2 112.1 ± 0.4	0	1^-	+1	+1	0	$>3.4 \times 10^{-22}$ [f]	$D^{*+} + \gamma$ or $D^{*+} + \pi^0$
B meson[36]	B^{*+}	B^{*-}	$u\bar{b}$	5 325.2 ± 0.4	$\frac{1}{2}$	1^-	0	0	+1	Unknown	$B^+ + \gamma$
B meson[36]	B^{*0}	$\bar{B}^{*0}$	$d\bar{b}$	5 325.2 ± 0.4	$\frac{1}{2}$	1^-	0	0	+1	Unknown	$B^0 + \gamma$
Strange B meson[37]	B^{*0}_s	$\bar{B}^{*0}_s$	$s\bar{b}$	$5\,415.4^{+2.4}_{-2.1}$	0	1^-	−1	0	+1	Unknown	$B^0_s + \gamma$
Charmed B meson†	B^{*+}_c	B^{*-}_c	$c\bar{b}$	Unknown	0	1^-	0	+1	+1	Unknown	Unknown

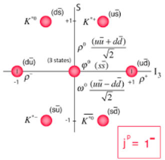
[f] ^ PDG reports the resonance width (Γ). Here the conversion $\tau = \hbar/\Gamma$ is given instead.
[g] ^ The exact value depends on the method used. See the given reference for detail.

[3] Credit: Wikipedia

Motivation: Light meson masses

- We believe that meson mass depends on the constituent quark mass, i.e. $m_{K[u\bar{s}]}(0.495\text{ GeV}) > m_{\pi[u\bar{d}]}(0.140\text{ GeV})$
- However, this does not happen for $m_{\rho(u\bar{d})}(0.776\text{ GeV}) > m_{\pi(u\bar{d})}(0.140\text{ GeV})$
- This is because of the difference quark spin orientation (spin-spin interactions),i.e. $\vec{S} = \vec{S}_1 + \vec{S}_2$ and then $\vec{S}_1.\vec{S}_2 = \frac{1}{2}[\vec{S}^2 - \vec{S}_1^2 - \vec{S}_s^2] = \frac{1}{2}[\vec{S}^2 - \frac{3}{4}]$
- For vector mesons $\vec{S}^2 = S(S+1) = 2 ==> \vec{S}_1.\vec{S}_2 = +1/4$ and for pseudoscalar mesons $\vec{S}^2 = 0 ==> \vec{S}_1.\vec{S}_2 = -3/4$





MESONS PROPERTIES IN THE BSE-NJL MODEL

Mesons in the BSE-NJL Model

The three flavor NJL Lagrangian [4] – containing only four fermion interactions

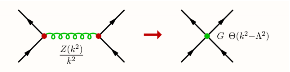
$$\mathscr{L}_{NJL} = \bar{\psi}[i\partial\!\!\!/ - \hat{m}_q]\psi + G_\pi \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2\right] - G_\rho \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2\right] - G_\omega(\bar{\psi}\gamma^\mu\psi)^2 \quad (1)$$

- $\psi = (u, d, s)^T$ denotes the quark field with the flavor components
- G_π and G_ρ and G_ω are four-fermion coupling constants
- $\lambda_1, \cdots, \lambda_8$ are Gell-Mann matrices in flavor space and $\lambda_0 \equiv \sqrt{\frac{2}{3}}1$
- $\hat{m}_q = \mathrm{diag}(m_u, m_d, m_s)$ denotes the current quark matrix

[4] PTPH, Ian Cloet, and A. Thomas, PRC94 (2016), 035201

Mesons in the BSE-NJL Model

- In the NJL model, the gluon exchange is replaced by four-fermion contact interaction by integrating out the gluon field and absorbing into the coupling constant $\Longleftrightarrow$ *quark effective theory*



- NJL model has a lack of confinement (it can be simply seen quark propagator has a pole). Therefore we regularize using the proper−time regularization to simulate confinement [5]

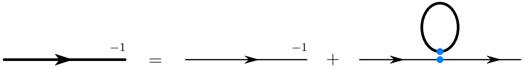
$$\frac{1}{\mathscr{X}^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\tau^{(n-1)}e^{-\tau\mathscr{X}} \to \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau\tau^{(n-1)}e^{-\tau\mathscr{X}} \tag{2}$$

where $\Lambda_{IR} \sim \Lambda_{QCD} \sim 0.24$ GeV and Λ_{UV} is determined.

[5] Ian C. Cloet, PRC**90** (2014), **PTPH**, Ian Cloet, and A. Thomas, PRC**94** (2016)

MESONS IN THE BSE-NJL MODEL

- NJL Gap Equation is determined using quark propagator in momentum space $S_q^{-1}(p) = \not{p} - M_q + i\epsilon$

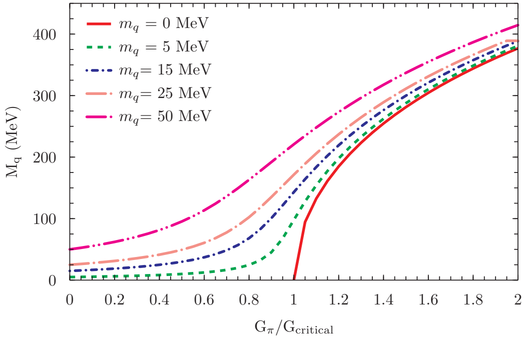


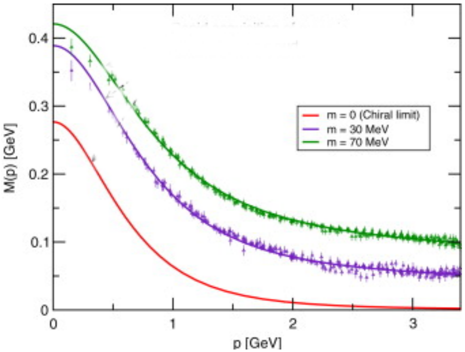
$$M_q = m_q + M_q \frac{3G_\pi}{\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2} = m_q - 2G_\pi \langle \bar{\psi}\psi \rangle \quad (3)$$

- Chiral quark condensates is defined by $\langle \bar{\psi}\psi \rangle = -\frac{3M_q}{2\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2}$
- Mass is generated through interaction vacuum $\rightarrow \langle \bar{\psi}\psi \rangle \neq 0$

NJL Gap Equation

- The NJL constituent [6] quark mass is a constant up to certain $p \sim 0.6$ GeV and it drops in higher p region



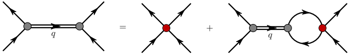


- The NJL model can be used for low momentum p and low energy E

[6] **PTPH**, Ian Cloet, and A. Thomas, PRC**94** (2016)

Bethe Salpeter Equation for the mesons

Mesons in the NJL model are dressed quark-dressed antiquark bound states whose properties are determined by solving the BSE



- In the NJL model, $\mathcal{T}$-matrix is given by

$$\mathcal{T}(q) = \mathcal{K} + \int \frac{d^4k}{(2\pi)^4} \mathcal{K}\, S(q+k)\mathcal{T}(q)S(k)$$

- The solution to the BSE in the mesons

$$\mathcal{T}_\alpha(q)_{ab,cd} = [\Gamma\lambda_\alpha]_{ab}\, t_\alpha(q) \left[\bar{\Gamma}\lambda_\alpha^\dagger\right]_{cd} \cdots \Gamma = \{\gamma_5, \gamma^\mu, \gamma_5\gamma^\mu\} \tag{4}$$

- The reduced t-matrix in this channel take a form

$$t_\alpha(q) = \frac{-2i\,G_\pi}{1+2G_\pi\Pi_\pi(q^2)}, \quad t_\beta^{\mu\nu}(q) = \frac{-2i\,G_\rho}{1+2G_\rho\Pi_\beta(q^2)}\left(g^{\mu\nu} + 2G_\rho\Pi_\beta(q^2)\frac{q^\mu q^\nu}{q^2}\right) \tag{5}$$

Bethe Salpeter Equation of the mesons

- The bubble diagrams appearing read

$$\begin{aligned}\Pi_\pi(q^2) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_l(k+q)\right],\\ \Pi_K(q^2) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_s(k+q)\right],\\ \Pi_\rho(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma^\mu S_a(k)\gamma^\nu S_a(k+q)\right] \end{aligned} \quad (6)$$

- The meson masses is given by the pole of the t-matrix

$$1 + 2G_\pi \Pi_\pi(k^2 = m_\pi^2) = 0, \quad 1 + 2G_\pi \Pi_K(k^2 = m_K^2) = 0, \quad 1 + 2G_\rho \Pi_\rho(k^2 = m_\rho^2) = 0 \quad (7)$$

Meson masses

- The meson masses are defined by the pole in the corresponding t-matrix and therefore the meson masses are given by

$$
\begin{aligned}
m_\pi^2 &= \left[\frac{m}{M_l}\right]\frac{2}{G_\pi \mathscr{I}_{ll}(m_\pi^2)} \\
m_K^2 &= \left[\frac{m_s}{M_s}+\frac{m}{M_l}\right]\frac{1}{G_\pi \mathscr{I}_{ls}(m_K^2)} + (M_s - M_l)^2 , \\
m_\rho^2 &= \left[\frac{m}{M_l}\right]\frac{2}{G_\rho \mathscr{I}_{ll}(m_\rho^2)} ,
\end{aligned} \tag{8}
$$

where $\mathcal{I}_{ll}$ and $\mathcal{I}_{ls}$ in the proper− time regularization scheme are defined by

$$
\mathcal{I}_{ab}(k^2) = \frac{3}{\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(x(x-1)k^2 + xM_b^2 + (1-x)M_a^2)} \tag{9}
$$

The meson-quark-quark coupling constants and meson decay constants

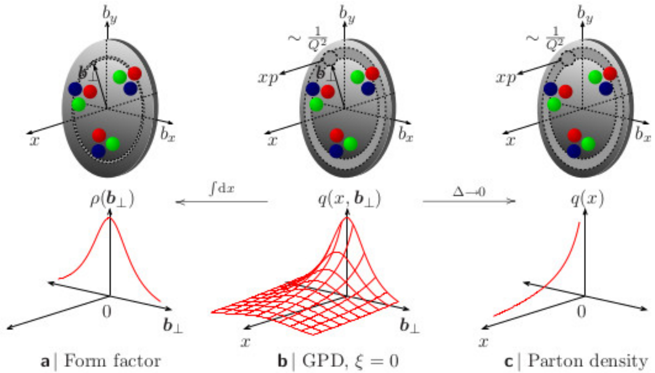
- The residue at a pole in the $\bar{q}q$ $t-$matrix defines the effective meson-quark-antiquark coupling constants:

$$Z_\pi = g^2_{\pi qq} = -\frac{\partial \Pi_\pi(q^2)}{\partial q^2}\Big|_{q^2=m^2_\pi}, \qquad Z_K = g^2_{Kqq} = -\frac{\partial \Pi_K(q^2)}{\partial q^2}\Big|_{q^2=m^2_K}$$
$$Z_\rho = g^2_{\rho qq} = -\frac{\partial \Pi_\rho(q^2)}{\partial q^2}\Big|_{q^2=m^2_\rho} \qquad (10)$$

- Meson decay constant in the proper$-$time regularization is given by

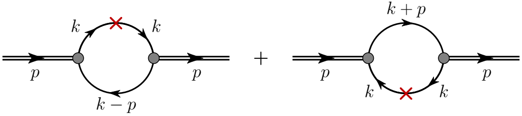
$$f_\pi = \frac{N_C\sqrt{Z_\pi}M}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+M^2)}$$
$$f_K = \frac{N_C\sqrt{Z_K}}{4\pi^2}[(1-x)M_2 + xM_1]\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+xM_2^2-(x-1)M_1^2)} \qquad (11)$$

PARTON DISTRIBUTION OF THE VECTOR MESON IN THE BSE-NJL MODEL



Parton Distribution of the vector meson in the BSE-NJL model

The valence quark distribution functions of the ρ-meson in the NJL model are given by the two Feynman diagrams



- ρ-meson Bethe-Salpeter vertex and conjugate vertex functions are given by

$$\bar{\Gamma}^{(\lambda),i}_{\gamma\delta}\Gamma^{(\lambda),i}_{\alpha\beta} = \left[iZ_\rho \tau_i \gamma^\mu \epsilon^*_{\lambda\mu}\right]_{\gamma\delta}\left[iZ_\rho \tau_i \gamma^\nu \epsilon_{\lambda\nu}\right]_{\alpha\beta} \tag{12}$$

- Operator insertion has a form $\Gamma\delta\left(x - \frac{k^+}{p^+}\right)$, where $\Gamma \equiv \{\gamma^+, \gamma^+\gamma^1\gamma_5, \gamma^+\gamma^2\gamma_5, \gamma^+\gamma_5\}$

$$\langle\Gamma\rangle^{\mu\nu}(x, \mathbf{k}_T) = -\frac{3iZ\rho^2}{p^+}\int\frac{dk^+dk^-}{(2\pi)^4}\delta\left(x - \frac{k^+}{p^+}\right)Tr_D\left[\gamma^\mu S(k)\Gamma\gamma^\nu S(k-p)\right] \tag{13}$$

Parton Distribution of the vector meson in the BSE-NJL model

The methods of covariant integration for expectation values of local operators can be used within the proper-time regularization scheme by first applying a Mellin transformation, which gives the n-th moment of $\langle\Gamma\rangle^{\mu\nu}$ as

$$\langle\Gamma\rangle_n^{\mu\nu}(\mathbf{k}_T) \equiv \int_0^1 dx\, x^{n-1}\, \langle\Gamma\rangle^{\mu\nu}(x,\mathbf{k}_T), \tag{14}$$

- The coefficient functions that contribute at leading-twist are

$$\begin{aligned}
\langle\gamma^+\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \frac{1}{2} Tr_D\left[\gamma^+\Phi^{(\lambda)S}(x,\mathbf{k}_T)\right], \\
\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \frac{1}{2} Tr_D\left[\gamma^+\gamma_5\Phi^{(\lambda)S}(x,\mathbf{k}_T)\right], \\
\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \frac{1}{2} Tr_D\left[-i\sigma^{+i}\Phi^{(\lambda)S}(x,\mathbf{k}_T)\right]
\end{aligned} \tag{15}$$

Parton Distribution of the vector meson in the BSE-NJL model

The coefficient functions that contribute at leading-twist are

$$\begin{aligned}
\langle\gamma^+\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \epsilon^*_{(\lambda)\mu}(p)\,\langle\gamma^+\rangle^{\mu\nu}(x,\mathbf{k}_T)\epsilon_{(\lambda)\nu}(p),\\
\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \epsilon^*_{(\lambda)\mu}(p)\,\langle\gamma^+\gamma_5\rangle^{\mu\nu}(x,\mathbf{k}_T)\epsilon_{(\lambda)\nu}(p),\\
\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \epsilon^*_{(\lambda)\mu}(p)\,\langle\gamma^+\gamma^i\gamma_5\rangle^{\mu\nu}(x,\mathbf{k}_T)\epsilon_{(\lambda)\nu}(p) \qquad (16)
\end{aligned}$$

where,

$$\begin{aligned}
\epsilon^{*\mu}_{(\lambda)}(p)\epsilon^{\nu}_{(\lambda)}(p) &= \frac{1}{3}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{m_h^2}\right)-\frac{i\lambda}{2m_h}\epsilon^{\mu\nu\alpha\beta}p_\alpha S_\beta(p)\\
&- \frac{3\lambda^2-2}{2}\left[S^\mu(p)S^\nu(p)-\frac{1}{3}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{m_h^2}\right)\right] \qquad (17)
\end{aligned}$$

with $S^\mu(p)=\left(\frac{p^3}{m_h}S_L, S_T, \frac{E_p}{m_h}S_L\right)$ and spin projection $\lambda=\pm1,0$ for $S_T=0$ and $|S_L|=1$ as well as for $S_L=0$ and $|S_T|=1$, there are $\lambda=\pm1,0$

Parton Distribution of the vector meson in the BSE-NJL model

The coefficient functions that contribute at leading-twist are

$$\begin{aligned}
\langle\gamma^+\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &\equiv f(x,\mathbf{k}_T^2)+S_{LL}f_{LL}(x,\mathbf{k}_T^2)\\
&+\frac{S_{LT}.\mathbf{k}_T}{m_h}f_{LT}(x,\mathbf{k}_T^2)+\frac{\mathbf{k}_T.S_{TT}.\mathbf{k}_T}{m_h^2}f_{TT}(x,\mathbf{k}_T^2),\\
\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \lambda\left[S_L g_L(x,\mathbf{k}_T^2)+\frac{\mathbf{k}_T.S_T}{m_h}g_T(x,\mathbf{k}_T^2)\right],\\
\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T) &= \lambda\left[S_T^i h(x,\mathbf{k}_T^2)+S_L\frac{\mathbf{k}_T^i}{m_h}h_L^\perp(x,\mathbf{k}_T^2)\right.\\
&\left.+\frac{1}{2m_h^2}\left(2k_T^i\mathbf{k}_T.S_T-S_T^i\mathbf{k}_T^2\right)h_T^\perp(x,\mathbf{k}_T^2)\right] \qquad (18)
\end{aligned}$$

where $S_{LL}=(3\lambda^2-2)(\frac{1}{6}-\frac{1}{2}S_L^2)$, $S_{LT}^i=(3\lambda^2-2)S_LS_T^i$ and $S_{TT}^{ij}=(3\lambda^2-2)(S_T^iS_T^j-\frac{1}{2}S_T^2\delta^{ij})$

Parton Distribution of the vector meson in the BSE-NJL model

By integrating $\langle\gamma^+\rangle_S^{(\lambda)}(x,\mathbf{k}_T)$, $\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T)$, and $\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x,\mathbf{k}_T)$ over $\mathbf{k}_T$. it gives the four PDFs for spin-1 target

$$
\begin{aligned}
\langle\gamma^+\rangle_S^{(\lambda)}(x) &\equiv f(x) + S_{LL} f_{LL}(x), \\
\langle\gamma^+\gamma_5\rangle_S^{(\lambda)}(x) &\equiv \lambda S_L g(x), \\
\langle\gamma^+\gamma^i\gamma_5\rangle_S^{(\lambda)}(x) &\equiv \lambda S_T^i h(x)
\end{aligned} \tag{19}
$$

The relations between the TMDs and PDFs for spin-1 target is given by

$$
\begin{aligned}
f(x) &= \int d^2\mathbf{k}_T f(x,\mathbf{k}_T^2), \\
g(x) &= \int d^2\mathbf{k}_T g(x,\mathbf{k}_T^2), \\
h(x) &= \int d^2\mathbf{k}_T h(x,\mathbf{k}_T^2), \\
f_{LL}(x) &= \int d^2\mathbf{k}_T f_{LL}(x,\mathbf{k}_T^2)
\end{aligned} \tag{20}
$$

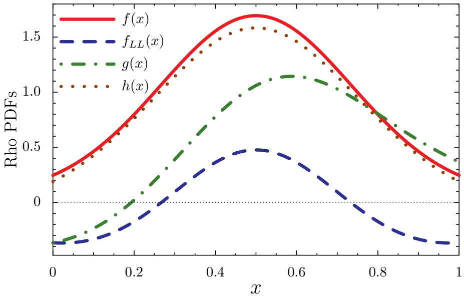
Parton Distribution of the vector meson in the BSE-NJL model

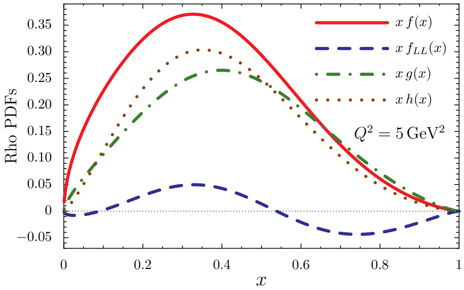
The $f(x)$, $g(x)$, $h(x)$, and f_{LL} in the proper-time regularization scheme are written as

$$
\begin{aligned}
f(x) &= \frac{Z_\rho^2}{2\pi^2}\int \frac{d\tau}{\tau}\left[1 + x(1-x)[1+2x(1-x)]m_\rho^2\tau\right] \\
&\times \; e^{-\tau(M_u^2 - x(1-x)m_\rho^2)}, \\
g(x) &= \frac{3Z_\rho^2}{4\pi^2}\int d\tau\left[\frac{2x-1}{\tau} + x(1-x)m_\rho^2\right] \\
&\times \; e^{-\tau(M_u^2 - x(1-x)m_\rho^2)}, \\
h(x) &= \frac{3Z_\rho^2 M_u}{4\pi^2 m_\rho}\int d\tau\left[\frac{1}{\tau} + 2x(1-x)m_\rho^2\right] \\
&\times \; e^{-\tau(M_u^2 - x(1-x)m_\rho^2)}, \\
f_{LL}(x) &= -\frac{3Z_\rho^2}{4\pi^2}\int \frac{d\tau}{\tau}\left[1 - 6x(1-x) + x(1-x)(1-2x)^2 m_\rho^2\tau\right] \\
&\times \; e^{-\tau(M_u^2 - x(1-x)m_\rho^2)}
\end{aligned}
\tag{21}
$$

Results for the vector meson PDFs in the BSE-NJL model

Results for the unpolarized $f(x)$, helicity $g(x)$, tranversity $h(x)$ and $f_{LL}(x)$ tensor polarized PDFs of the ρ-meson. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs ($xf(x)$, $xh(x)$, $xg(x)$ and $xf_{LL}(x)$) of the ρ-meson after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution [7] and Ref. [8]



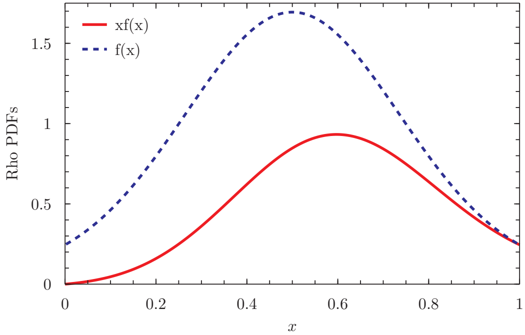


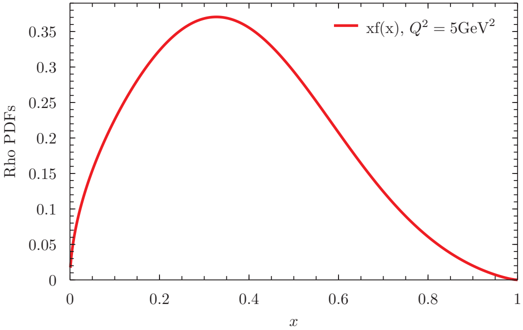
[7] W. Bentz, PRC **96** (2017)

[8] **PTPH**, Nam Seung-il and W. Bentz et. al, in preparation (2022)

Results for the vector meson PDFs in the BSE-NJL model

Results for the unpolarized ($f(x)$) of the vector mesons. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs of the vector mesons after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution [9].

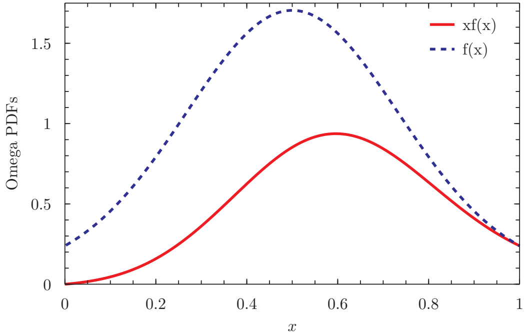


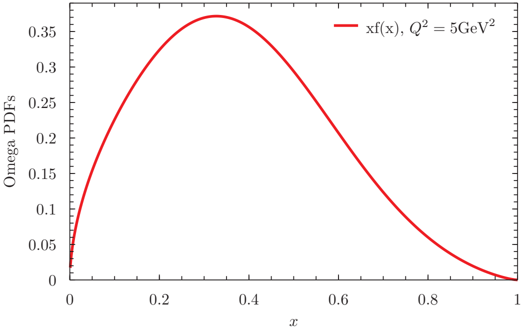


[9] **PTPH**, Nam Seung-il and W. Bentz et. al, in preparation (2022)

Results for the vector meson PDFs in the BSE-NJL model

Results for the unpolarized ($f(x)$) of the vector mesons. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs of the vector mesons after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution [10].

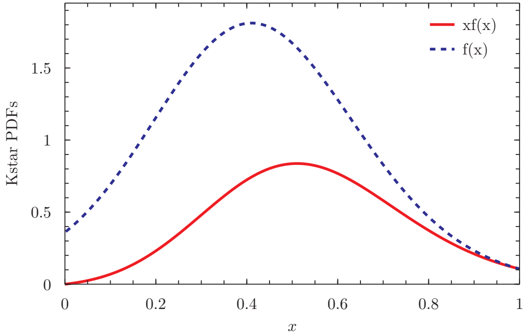


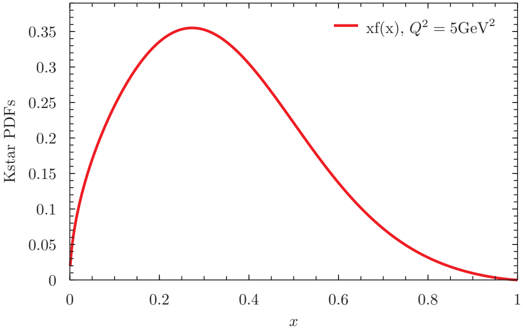


[10] **PTPH**, Nam Seung-il and W. Bentz et. al, in preparation (2022)

Results for the vector meson PDFs in the BSE-NJL model

Results for the unpolarized ($f(x)$) of the vector mesons. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs of the vector mesons after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution [11].

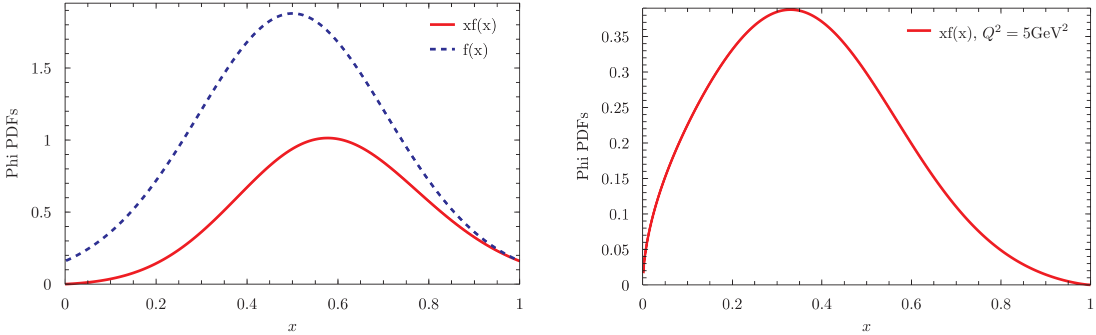




[11] **PTPH**, Nam Seung-il and W. Bentz et. al, in preparation (2022)

Results for the vector meson PDFs in the BSE-NJL model

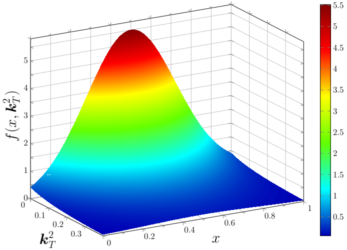
Results for the unpolarized ($f(x)$) of the vector mesons. The left panel is result for the model scale $Q_0^2 = 0.16\ GeV^2$. The right panel is respectively the PDFs of the vector mesons after evolving at the scale $Q^2 = 5\ GeV^2$ using the non-singlet DGLAP evolution [12].

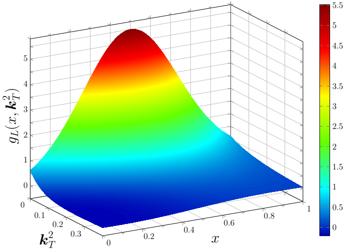


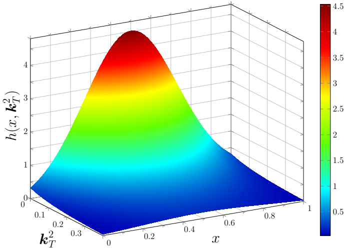
[12] **PTPH**, Nam Seung-il and W. Bentz et. al, in preparation (2022)

Results for TMDs for the ρ-meson in the BSE-NJL model

Results for the TMDs ($f(x, \mathbf{k}_T^2)$), helicity ($h(x, \mathbf{k}_T^2)$), and tranversity ($g(x, \mathbf{k}_T^2)$) of the ρ-meson.

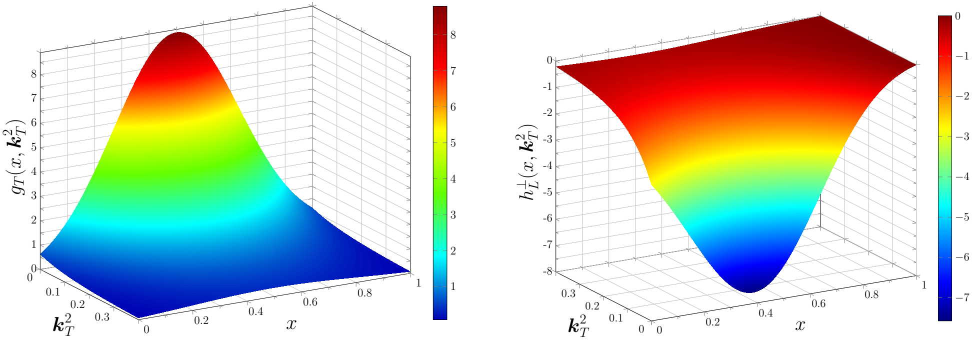






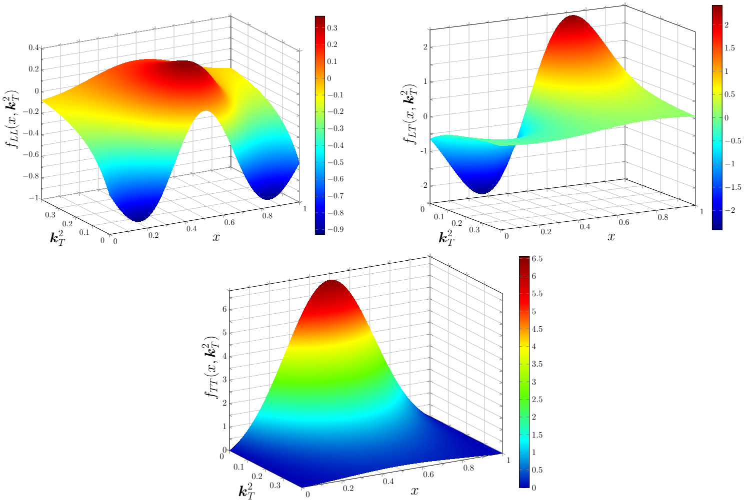
Results for TMDs for the ρ-meson in the BSE-NJL model

Results for the TMDs ($h_L^\perp(x, \mathbf{k}_T^2)$), and ($g_T(x, \mathbf{k}_T^2)$) of the ρ-meson.



Results for TMDs for the ρ-meson in the BSE-NJL model

Results for the TMDs ($f_{LL}(x, \mathbf{k}_T^2)$),($f_{LT}(x, \mathbf{k}_T^2)$), and ($f_{TT}(x, \mathbf{k}_T^2)$) of the ρ-meson.



Summary and Outlook

- We have presented the parton distribution of the vector meson in the confining NJL model.
- We showed our preliminary results on unpolarized valence quark distribution for the light vector mesons. Unfortunately no data to compare at the moment. Hoping that the EIC and EicC mayalso measure the light vector meson unpolarized pdf in the future
- On-going work, we are now calculating the GPD for the pion, kaon and vector-mesons in the NLChQM model (NJL momentum-dependent). The result for the vector meson GPDs in the BSE-NJL model is STILL IN PROGRESS...
- In the future, we are also going to calculate the quasi-PDF for the vector mesons and vacuum and medium fragmentation functions of vector mesons.

THANK YOU VERY MUCH FOR YOUR ATTENTION

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