Transverse Momentum Dependent Parton Distributions: an Overview

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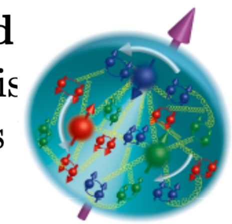
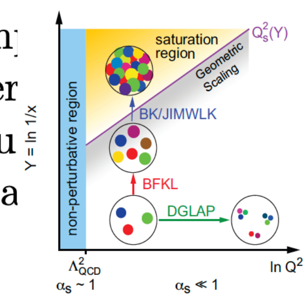
Big questions for hadron physics in the following decades

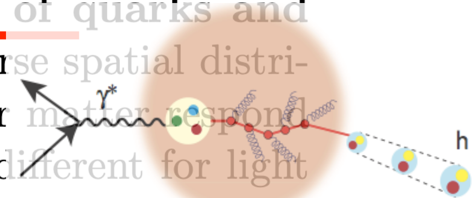
- **How are the sea quarks and gluons, and their spins, distributed and momentum inside the nucleon?** How are these quark and gluon dis correlated with overall nucleon properties, such as spin direction? What is the orbital motion of sea quarks and gluons in building the nucleon spin?



- **Where does the saturation of gluon densities set in?** Is there a sim that separates this region from that of more dilute quark-gluon matter do the distributions of quarks and gluons change as one crosses the bou this saturation produce matter of universal properties in the nucleon a viewed at nearly the speed of light?



- **How does the nuclear environment affect the distribution of quarks and gluons and their interactions in nuclei?** How does the transverse spatial distribution of gluons compare to that in the nucleon? How does nuclear matter respond to a fast moving color charge passing through it? Is this response different for light and heavy quarks?

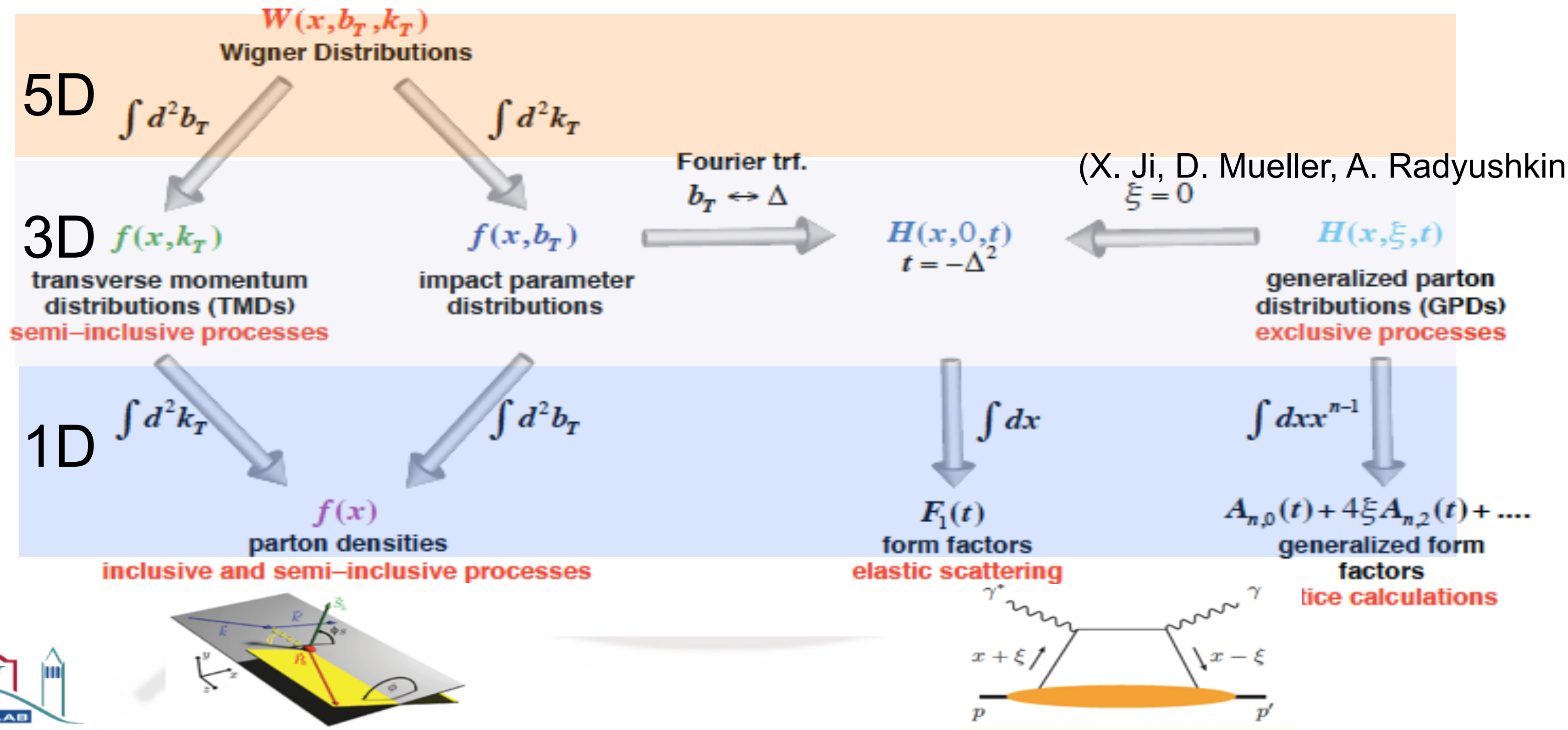


New ways to look at partons

- We not only need to know that partons have long. momentum, but must have transverse degrees of freedom as well
- Partons in transverse coordinate space
  - Generalized parton distributions (GPDs)
- Partons in transverse momentum space
  - Transverse-momentum distributions (TMDs)
- Both? Wigner distributions!

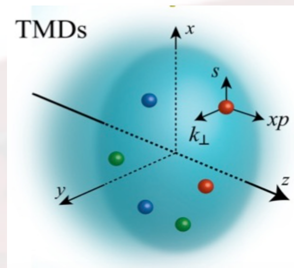
Unified view of the Nucleon

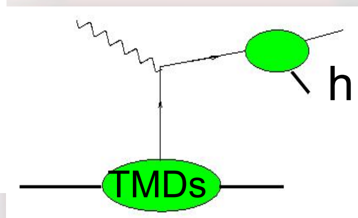
❑ Wigner distributions (Belitsky, Ji, Yuan)



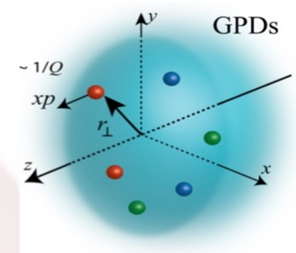
Zoo of TMDs & GPDs

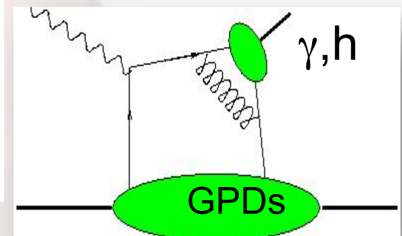
	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1, h_{1T}^{\perp}$





	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$





- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in **a large kinematic domain of** x_B**, t,** Q^2 (GPD) and x_B, P_T, Q^2, **z** (TMD)

What can we learn

- 3D Imaging of partons inside the nucleon (non-trivial correlations)
  - Try to answer more detailed questions as Rutherford was doing for atomic matter more than 100 years ago
- QCD dynamics involved in these processes
  - Transverse momentum distributions: universality, factorization, evolutions,…
  - Small-x: BFKL vs Sudakov?

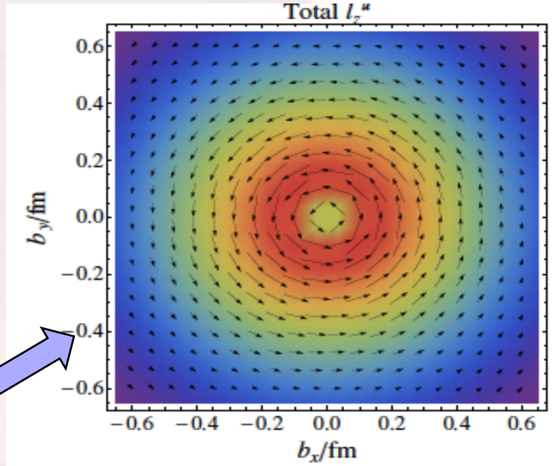
Parton's orbital motion through the Wigner Distributions

Phase space distribution:

Projection onto p (x) to get the momentum (probability) density

Quark orbital angular momentum

$$L(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$



Well defined in QCD:
Ji, Xiong, Yuan, PRL, 2012; PRD, 2013
Lorce, Pasquini, Xiong, Yuan, PRD, 2012
Lorce-Pasquini 2011
Hatta 2011

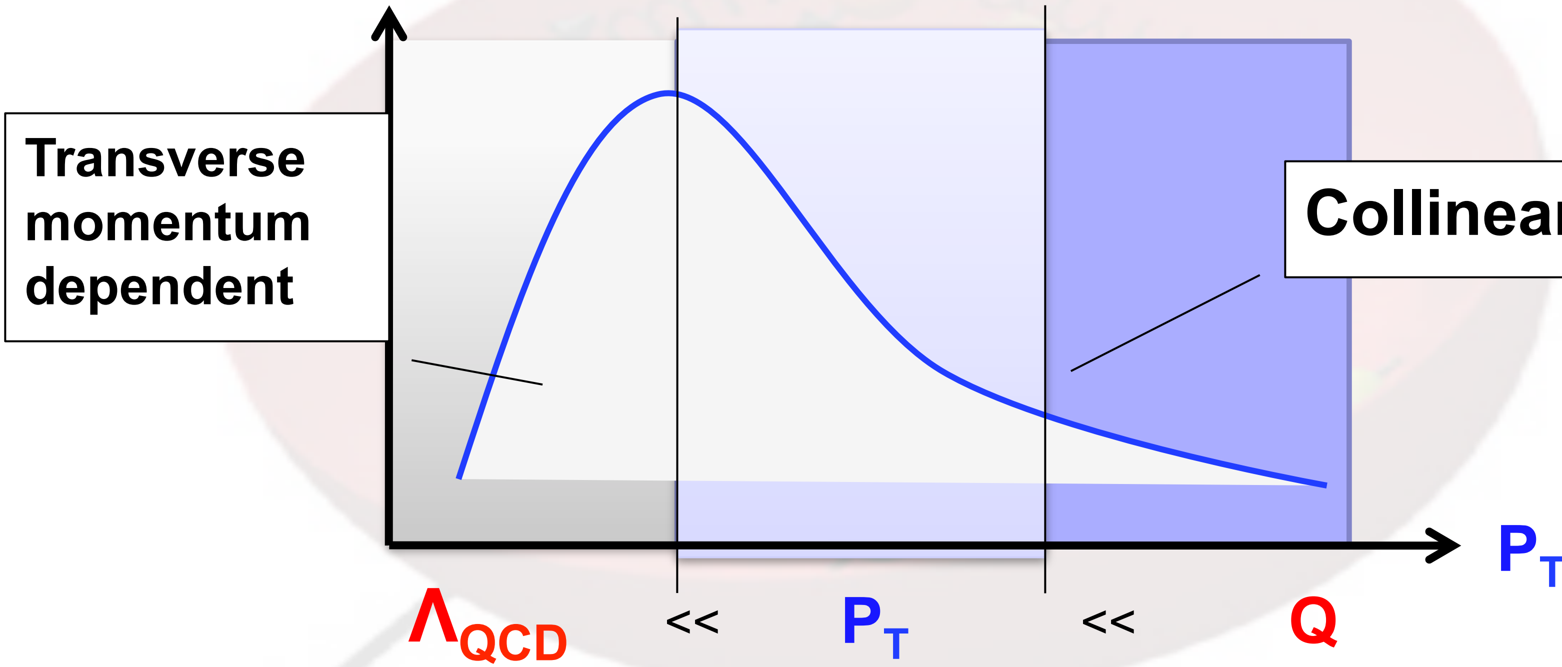
Transverse-momentum-dependent (TMD) Parton distributions

- Generalize Feynman parton distribution q(x) by including the transverse momentum dependence

  $q(x,k_T)$

- At small k_T, the transverse-momentum dependence is generated by soft non-perturbative physics.
- At large k_T, the k-dependence can be calculated in perturbative QCD and falls like powers of $1/k_T^2$

A unified picture (leading pt/Q)



Collins-Soper-Sterman 1985;
Ji-Qiu-Vogelsang-Yuan,2006

TMD Parton Distributions: definition

- The definition contains explicitly the gauge links

$$f(x,k_\perp) = \frac{1}{2}\int\frac{d\xi^- d^2\xi_\perp}{(2\pi)^3}e^{-i(\xi^- k^+ - \vec{\xi}_\perp\cdot\vec{k}_\perp)} \times\langle PS|\overline{\psi}(\xi^-,\xi_\perp)L^\dagger_{\xi_\perp}(\xi^-)\gamma^+ L_0(0)\psi(0)|PS\rangle$$

Collins-Soper 1981,
Collins 2002,
Belitsky-Ji-Yuan 2002

- The polarization and kt dependence provide rich structure in the quark and gluon distributions

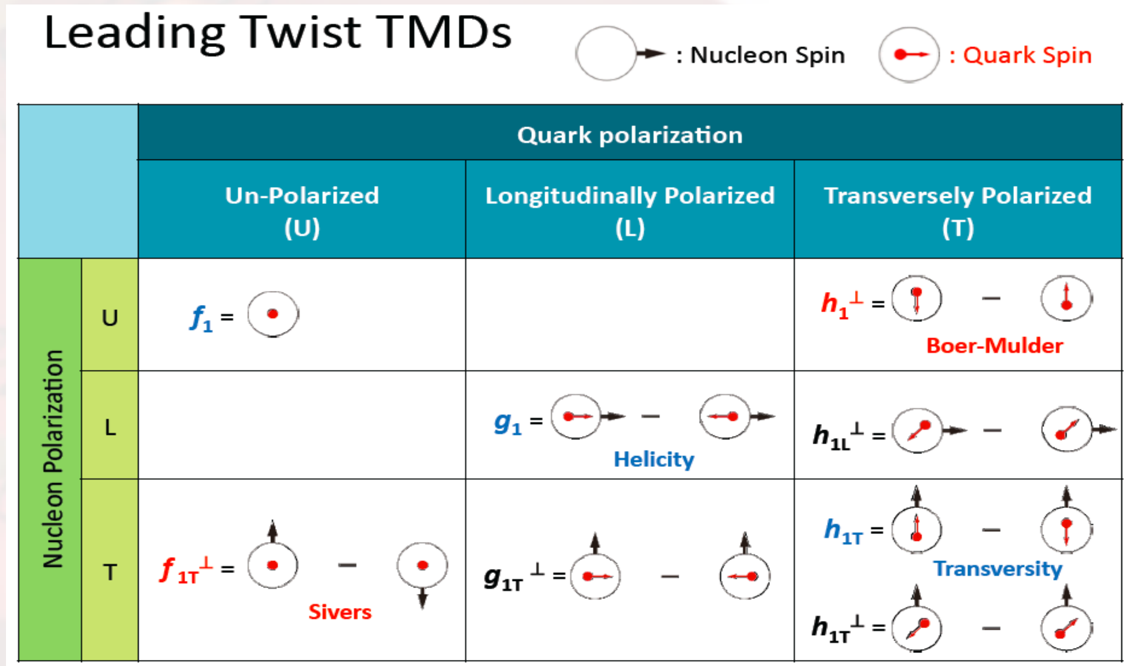
Transverse momentum dependent parton distribution

Straightforward extension

- Spin average, helicity, and transversity distributions

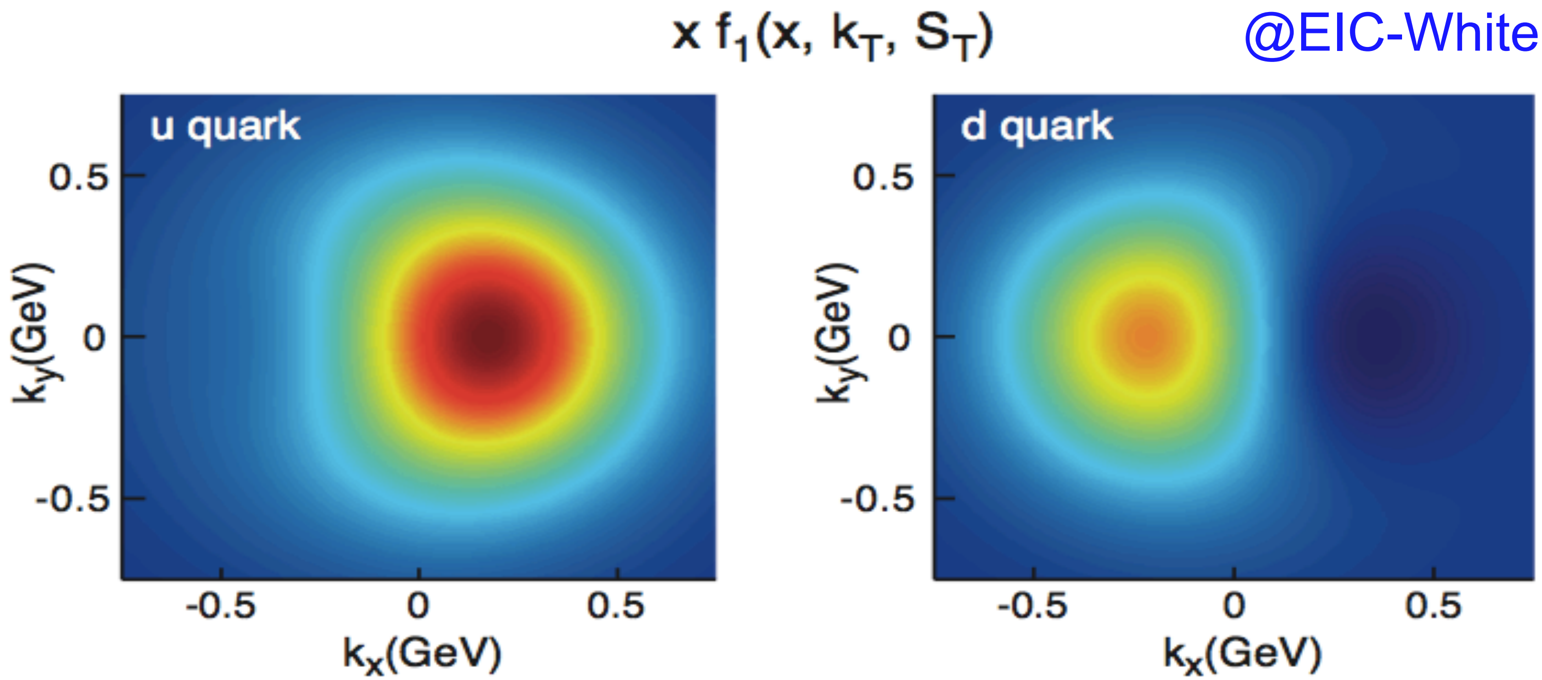
P_T-spin correlations

- Nontrivial distributions, $S_T X P_T$
- In quark model, depends on S- and P-wave interference



Mulders-Tangerman 95, Boer-Mulders 98;
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, 2007

Alex Prokudin
@EIC-Whitepaper

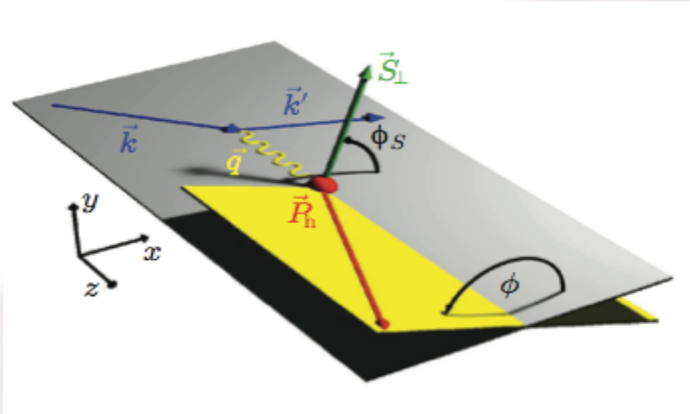


Quark Sivers function leads to an azimuthal asymmetric distribution of quark in the transverse plane

Where can we learn TMDs

- Semi-inclusive hadron production in deep inelastic scattering (SIDIS)
- Drell-Yan lepton pair, photon pair productions in pp scattering
- Dijet correlation in DIS
- Relevant e+e- annihilation processes
- …

TMDs at JLab and EIC: Semi-inclusive DIS



quark distribution
$\otimes$
fragmentation

- Novel Single Spin Asymmetries

Collins: $A_{UT}^{\sin(\phi+\phi_S)} \propto S_\perp \frac{\sum_{q,\bar{q}} e_q^2 \delta q(x)\, H_1^\perp(z)}{\sum_{q,\bar{q}} e_q^2 q(x)\, D_1(z)}$ $\quad z \overset{lab}{=} \frac{E_h}{\nu}$

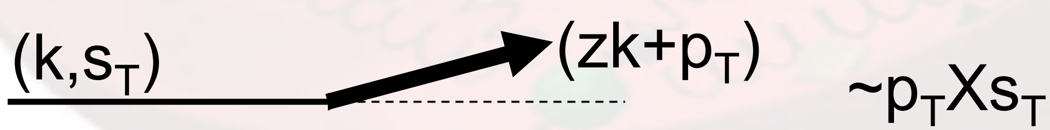
Sivers: $A_{UT}^{\sin(\phi-\phi_S)} \propto S_\perp \frac{\sum_{q,\bar{q}} e_q^2 f_{1T}^{\perp,q}(x) \cdot D_1(z)}{\sum_{q,\bar{q}} e_q^2 q(x)\, D_1(z)}$

U: unpolarized beam
T: transversely polarized target

Two major contributions

- Sivers effect in the distribution

- Collins effect in the fragmentation

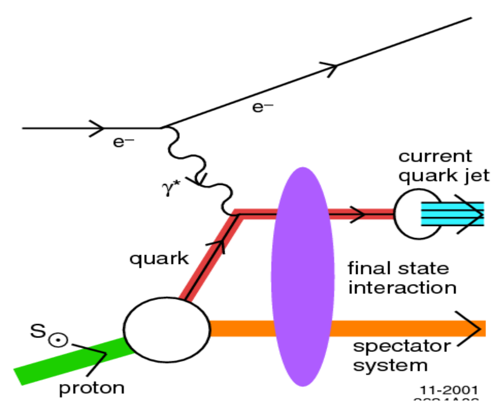


- Other contributions…

Sivers effect is different

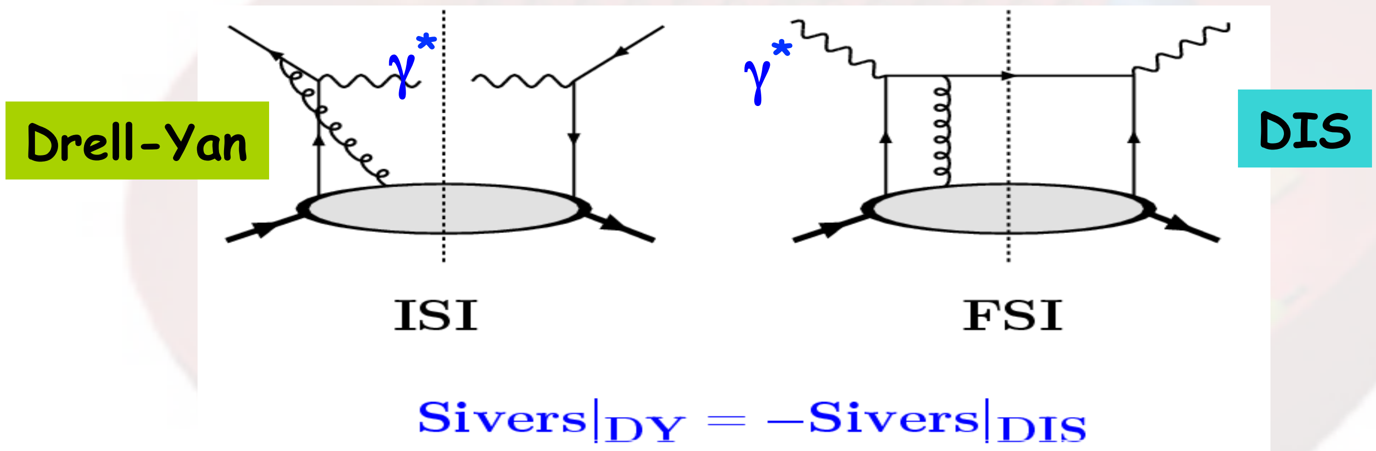
- It is the final state interaction providing the phase to a nonzero SSA
- Non-universality in general
- Only in special case, we have "Special Universality"

Brodsky,Hwang,Schmidt 02
Collins, 02;
Ji,Yuan,02;
Belitsky,Ji,Yuan,02



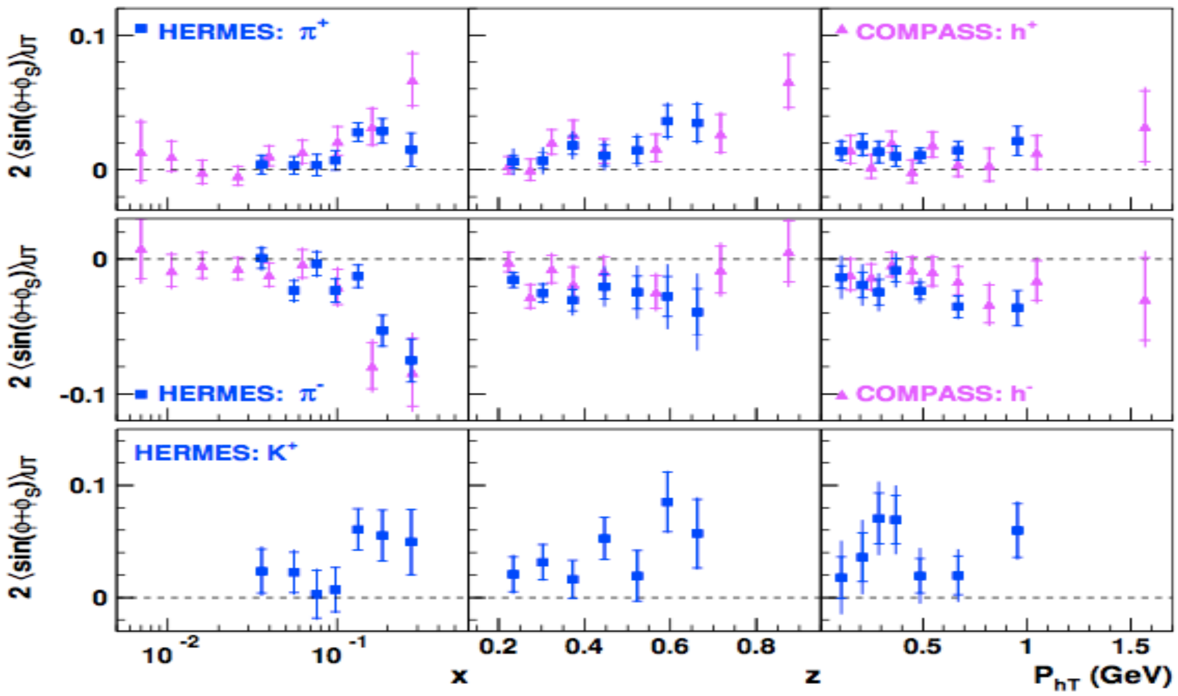
DIS and Drell-Yan

- Initial state vs. final state interactions



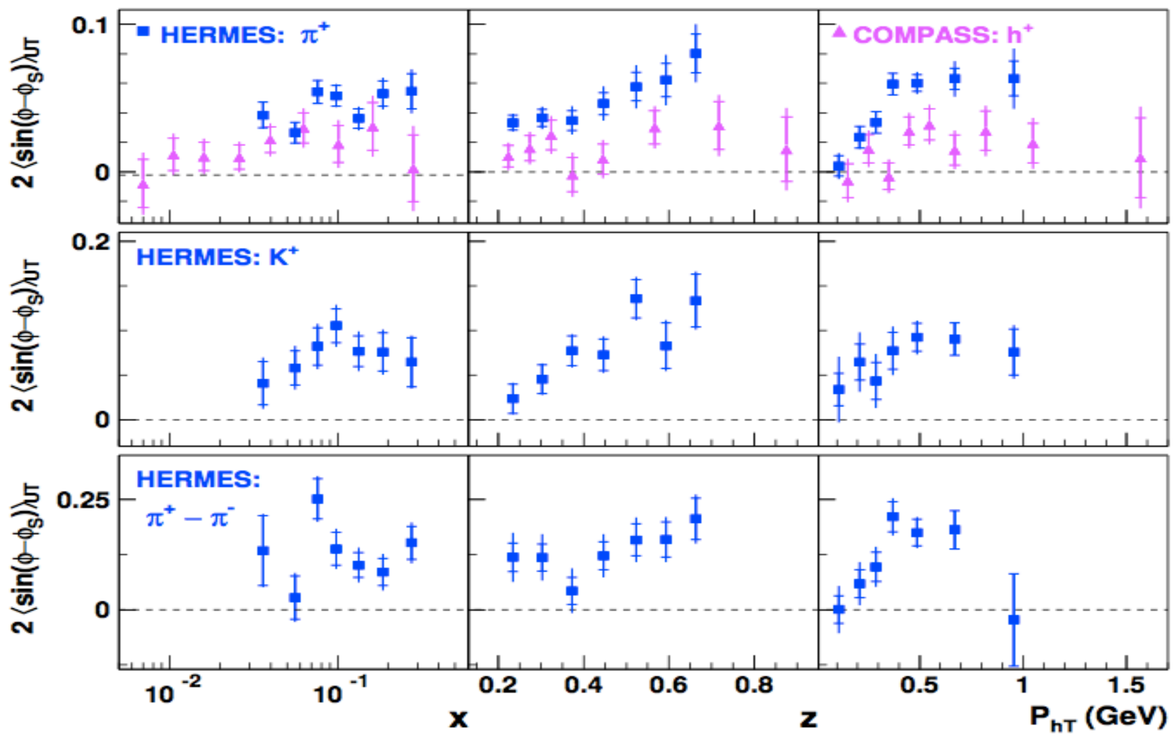
- "Universality": QCD prediction

Collins asymmetries in SIDIS



Summarized in the EIC Write-up

Sivers asymmetries in SIDIS



TMD predictions rely on

- Non-perturbative TMDs constrained from experiments
- QCD evolutions, in particular, respect to the hard momentum scale Q
  - Strong theory/phenomenological efforts in the last few years
  - Need more exp. data/lattice calculations

Tremendous progress has been made in last few years!

Soft gluon radiation leads to Sudakov Logarithms

Sudakov, 1956; Collins-Soper-Sterman 1985

- Differential cross section depends on $Q_1 = q_T$, where $Q^2 >> Q_1^2 >> \Lambda^2_{QCD}$

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i-1} \frac{Q^2}{Q_1^2} + \cdots$$

- Resummation of these large logs
  - In terms of transverse momentum dependent parton distributions and fragmentation functions and apply to
  - Semi-inclusive hadron production in DIS, Drell-Yan type of hard processes in pp collisions, e.g., Higgs, Z/W boson, …

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Collins-Soper-Sterman Resummation

- Large Logs are resummed by solving the energy evolution equation of the TMDs

$$\frac{\partial}{\partial \ln Q} f(k_\perp, Q) = (K(q_\perp, \mu) + G(Q, \mu)) \otimes f(k_\perp, Q)$$

- K and G obey the renormalization group eq.

$$\frac{\partial}{\partial \ln \mu} K = -\gamma_K = \frac{\partial}{\partial \ln \mu} G$$

(Collins-Soper 81, Collins-Soper-Sterman 85)

Solving the evolution equations

$$\widetilde{f}_q^{(sub.)}(x,b,\zeta^2=\rho Q^2;\mu_F=Q) = e^{-S^q_{pert}(Q,b_*)-S^q_{\mathrm{NP}}(Q,b)}\widetilde{\mathcal{F}}_q(\alpha_s(Q);\rho)$$
$$\times\sum_i C_{q/i}(\mu_b/\mu)\otimes f_i(x,\mu)\ ,$$

Sudakov form factor (perturbative)

Non-perturbative input

- Universal C-function

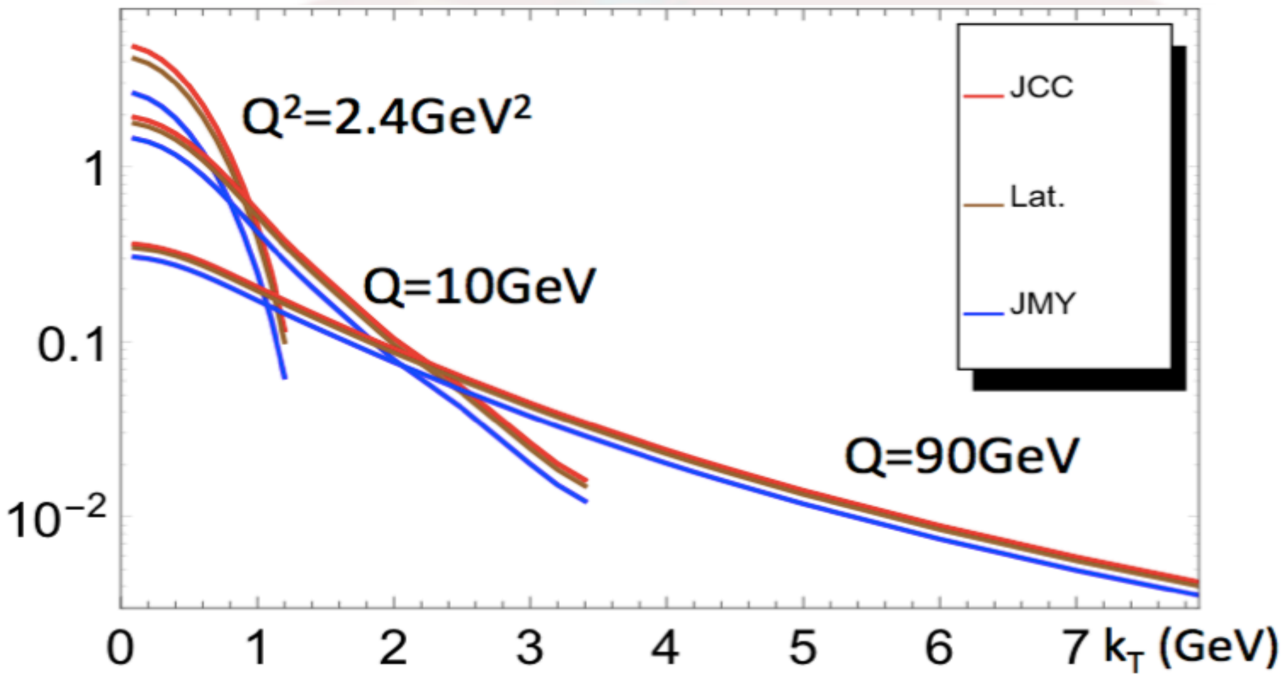
$$C_{q/q'}(x) = \delta_{qq'}\left[\delta(1-x)+\frac{\alpha_s}{2\pi}C_F(1-x)\right]$$

- Scheme-dept.

$$\widetilde{\mathcal{F}}_q^{\mathrm{JCC}}(\alpha_s(Q)) = 1+\mathcal{O}(\alpha_s^2)$$

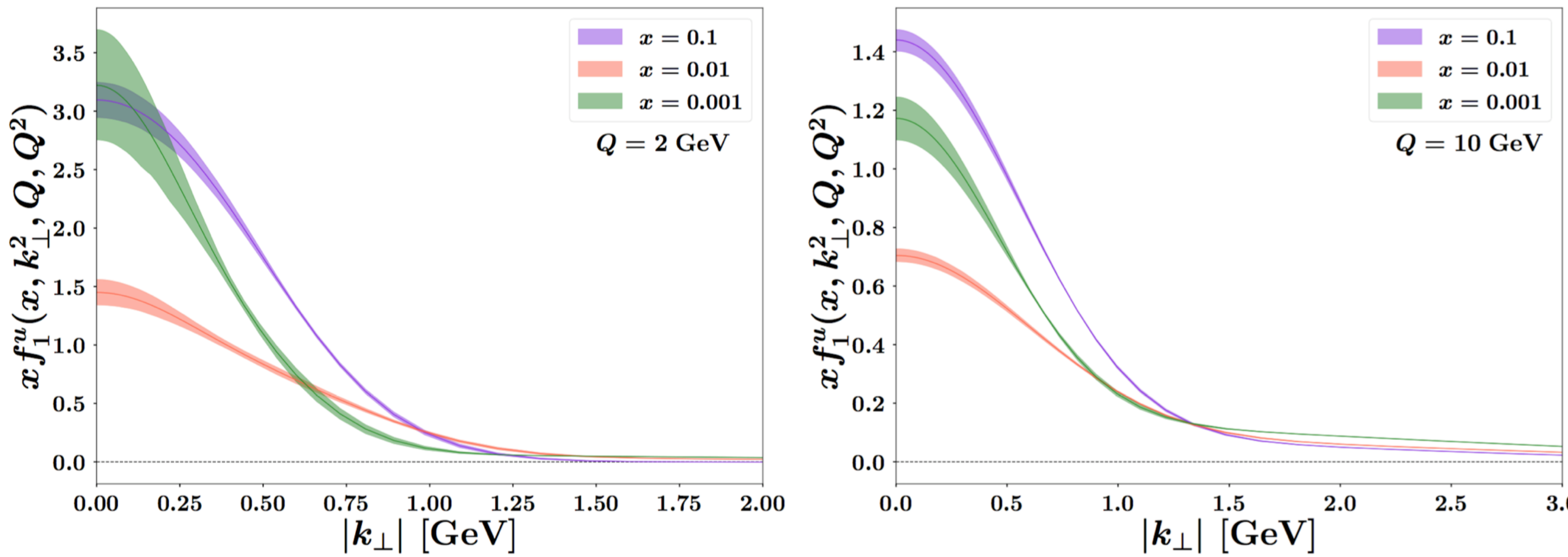
$$\widetilde{\mathcal{F}}_q^{\mathrm{JMY}}(\alpha_s(Q);\rho) = 1+\frac{\alpha_s}{2\pi}C_F\left(\ln\rho-\frac{\ln^2\rho}{2}-\frac{\pi^2}{2}-2\right)$$

Unpolarized quark distribution



Prokudin-Sun-Yuan, 2014; Sun-Issac-Yuan-Yuan, 2014

Precision theory advances are available: Bacchetta et al., MAP Coll. , 2206.07598

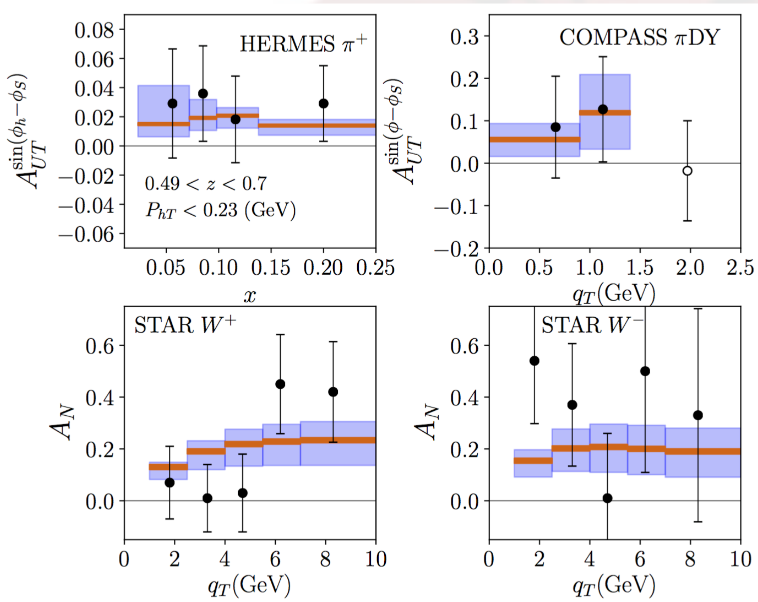


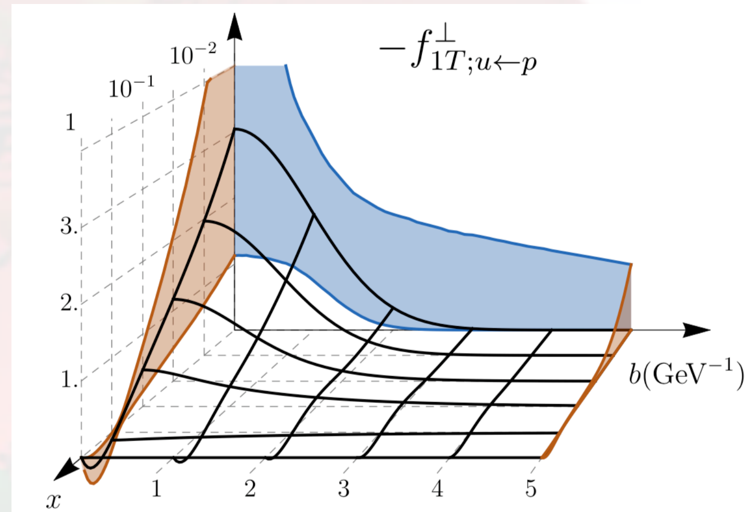
- Including DY and SIDIS data

See also: SV 17,19, PV 17,19, …

Precision theory advances are available

- N^{3}LO analysis of Sivers asymmetries

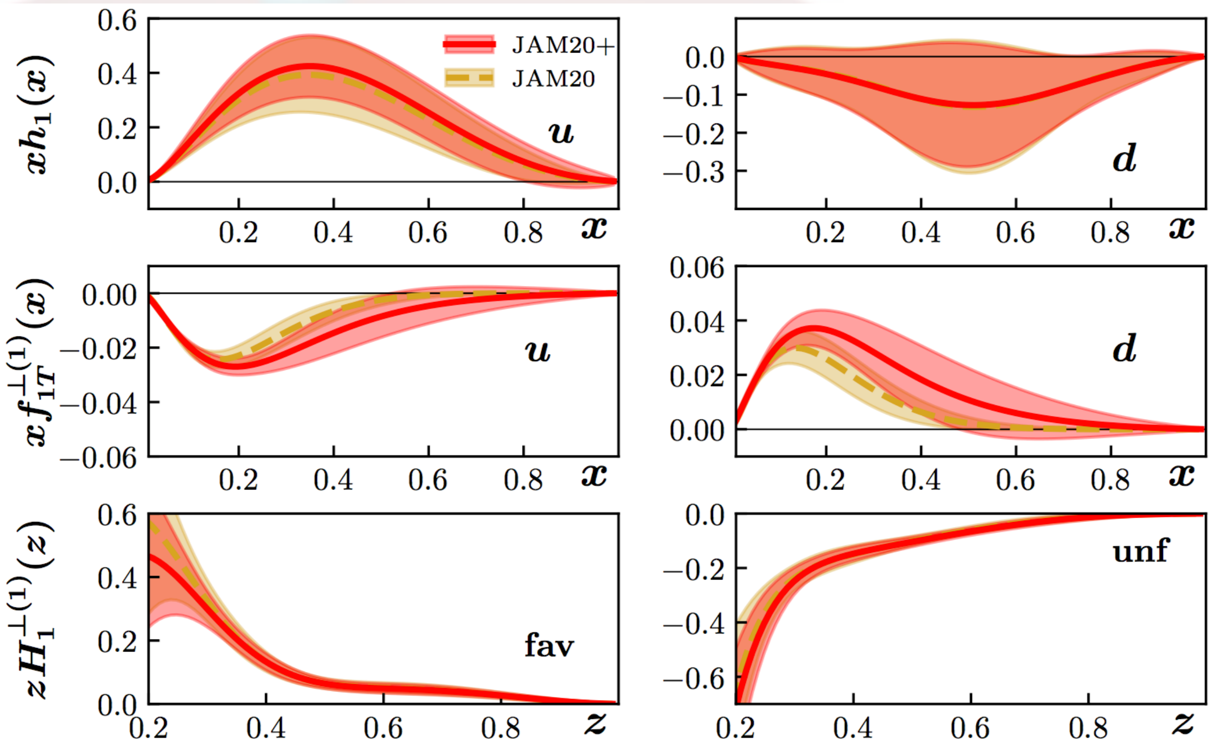




Bury, Prokudin, Vladimirov, 2012.05135
See also, JAM coll., 2002.08384; Bacchetta, Delcarro, Pisano, Radici, 2004.14278; Echevarria, Kang, Terry, 2009.10710

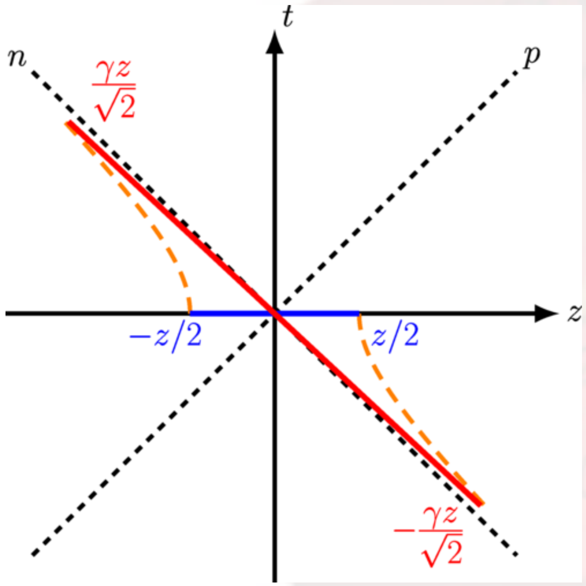
Global analysis of SSAs

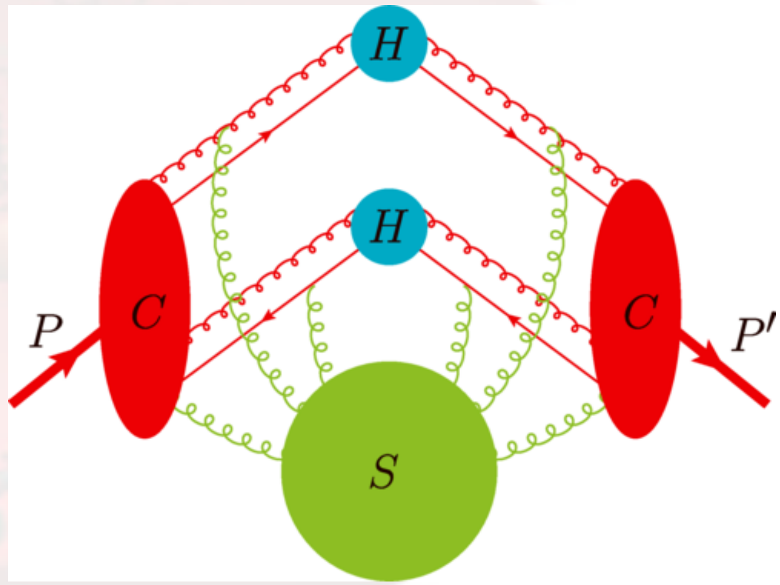
Gamberg et al., JAM Coll., 2205.00999



Computational tool: large momentum effective theory (LaMET)

Ji 2013





Ji, Liu, Liu, Zhang, Zhao, Rev. Mod. Phys. 2021

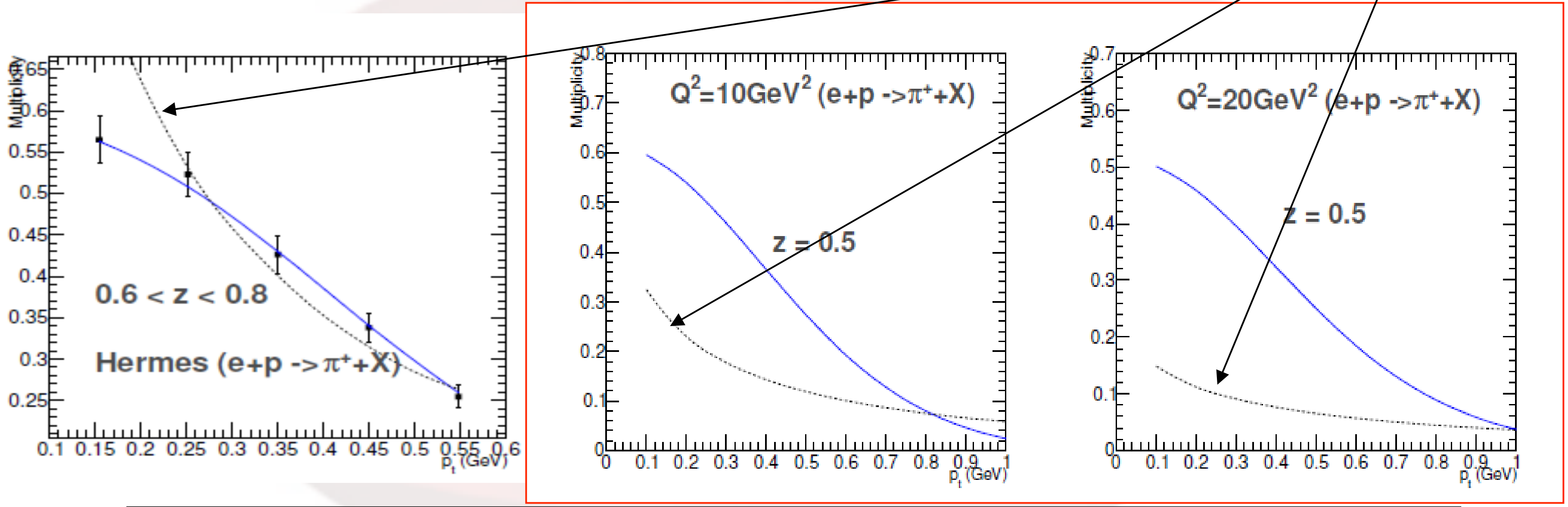
See also, Yong Zhao's following talk

Comments on the JLab-Upgrade

- Energy range covers the gap between Jlab-12 GeV and EIC
- Precision data will help to identify the physics behind the TMD physics, matching to collinear computations

Remaining issues in applying QCD factorizations (TMD, collinear) in SIDIS

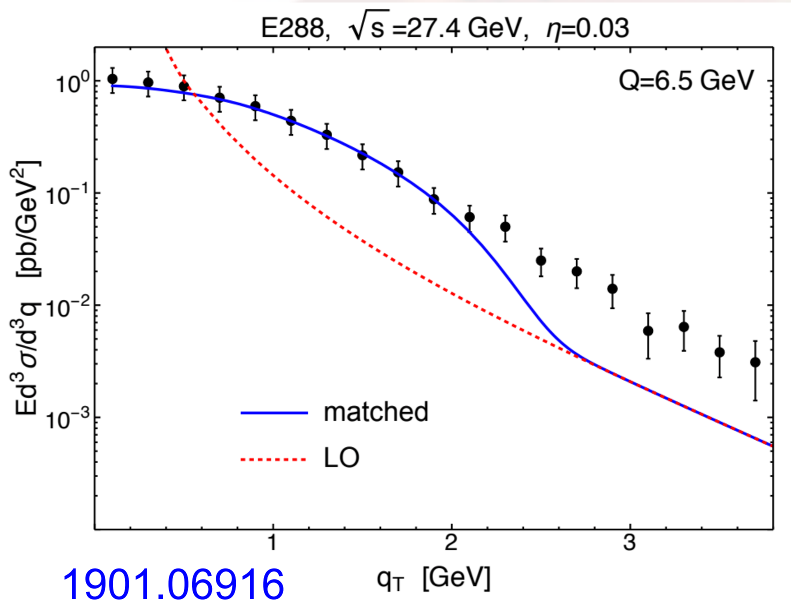
$$\frac{d\sigma}{dx_B dy dz_h d^2\vec{P}_{h\perp}} = \sigma_0^{(\mathrm{DIS})}\left[\frac{1}{z_h^2}\int\frac{d^2b}{(2\pi)^2}e^{i\vec{P}_{h\perp}\cdot\vec{b}/z_h}\widetilde{F}_{UU}(Q;b) + Y_{UU}(Q;P_{h\perp})\right]$$



The Q^2 is smaller, and the Y piece is more important.

P.Sun's talk at the INT workshop, Feb. 2014; Sun-Issac-Yuan-Yuan, 1406.3073

Matching to collinear calculations provides opportunity to study QCD dynamics



1901.06916

- Bacchetta, et al., 1901.06916 for Drell-Yan processes
- SIDIS, similar studies
  - Sun, Isaacson, C.P. Yuan, FY, 1406.3073
  - Boglione, Hernandez, Melis, Prokudin, 1412.1383
  - Wang, Gonzalez-Hernandez, Rogers, Sato, 1903.01529
  - Liu, Qiu, 1907.06136
- Wide kinematics of EIC/JLab is an ideal place to systematically study this!