Recent GPD developments obtained with PARTONS framework

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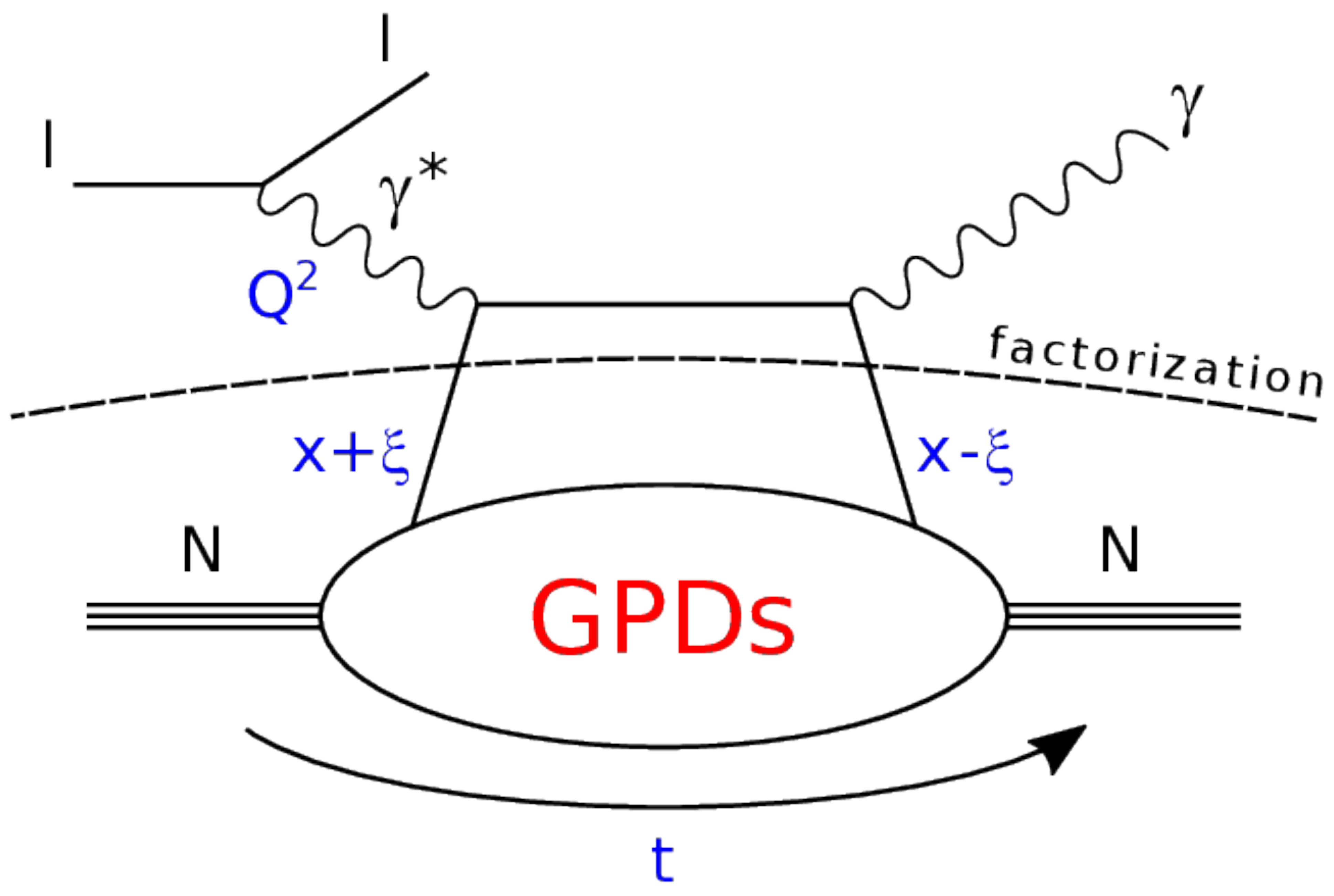
Outline

- Introduction
- Phenomenology at level of DVCS amplitudes (Compton form factors)
- Phenomenology at level of GPDs
- New sources of GPD information
- Tools

1. Introduction

Deeply Virtual Compton Scattering (DVCS)



factorisation for $|t|/Q^2 \ll 1$

Chiral-even GPDs:
(helicity of parton conserved)

$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	*for sum over parton helicities*
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	*for difference over parton helicities*
nucleon helicity conserved	*nucleon helicity changed*	

Reduction to PDF:

$$H(x, \xi = 0, t = 0) \equiv q(x)$$

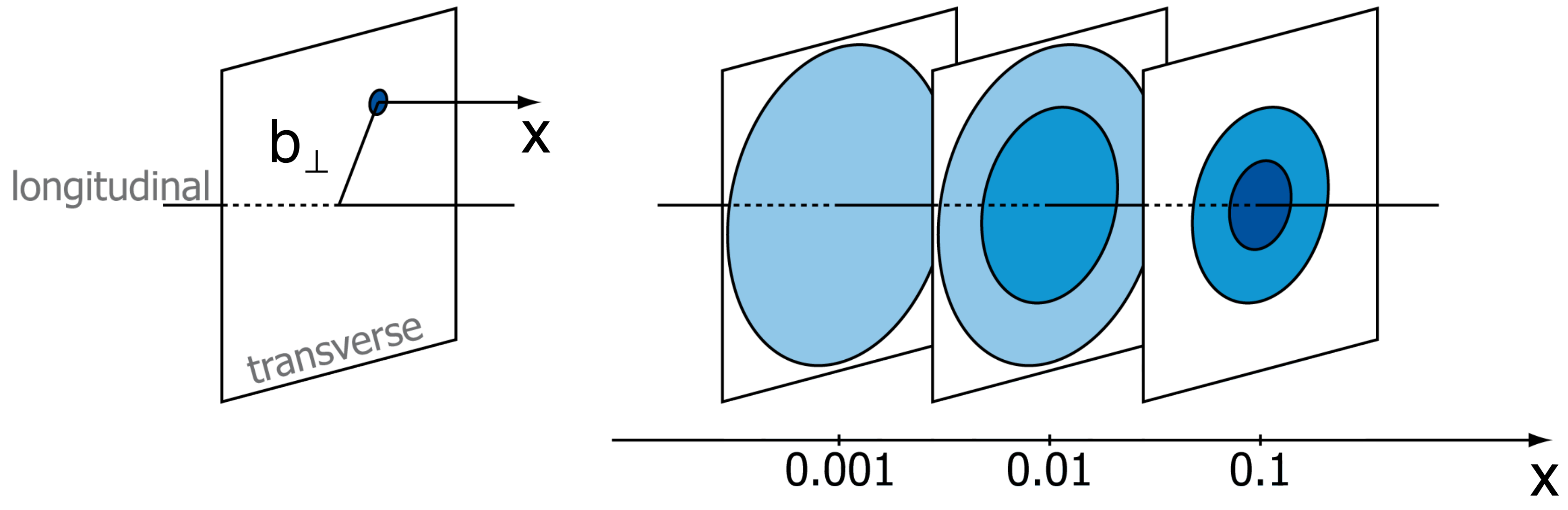
Polynomiality - non-trivial consequence of Lorentz invariance:

$$\mathcal{A}_n(\xi, t) = \int_{-1}^{1} \mathrm{d}x x^n H(x, \xi, t) = \sum_{\substack{j=0 \\ \text{even}}}^{n} \xi^j A_{n,j}(t) + \mathrm{mod}(n, 2)\xi^{n+1} A_{n,n+1}(t)$$

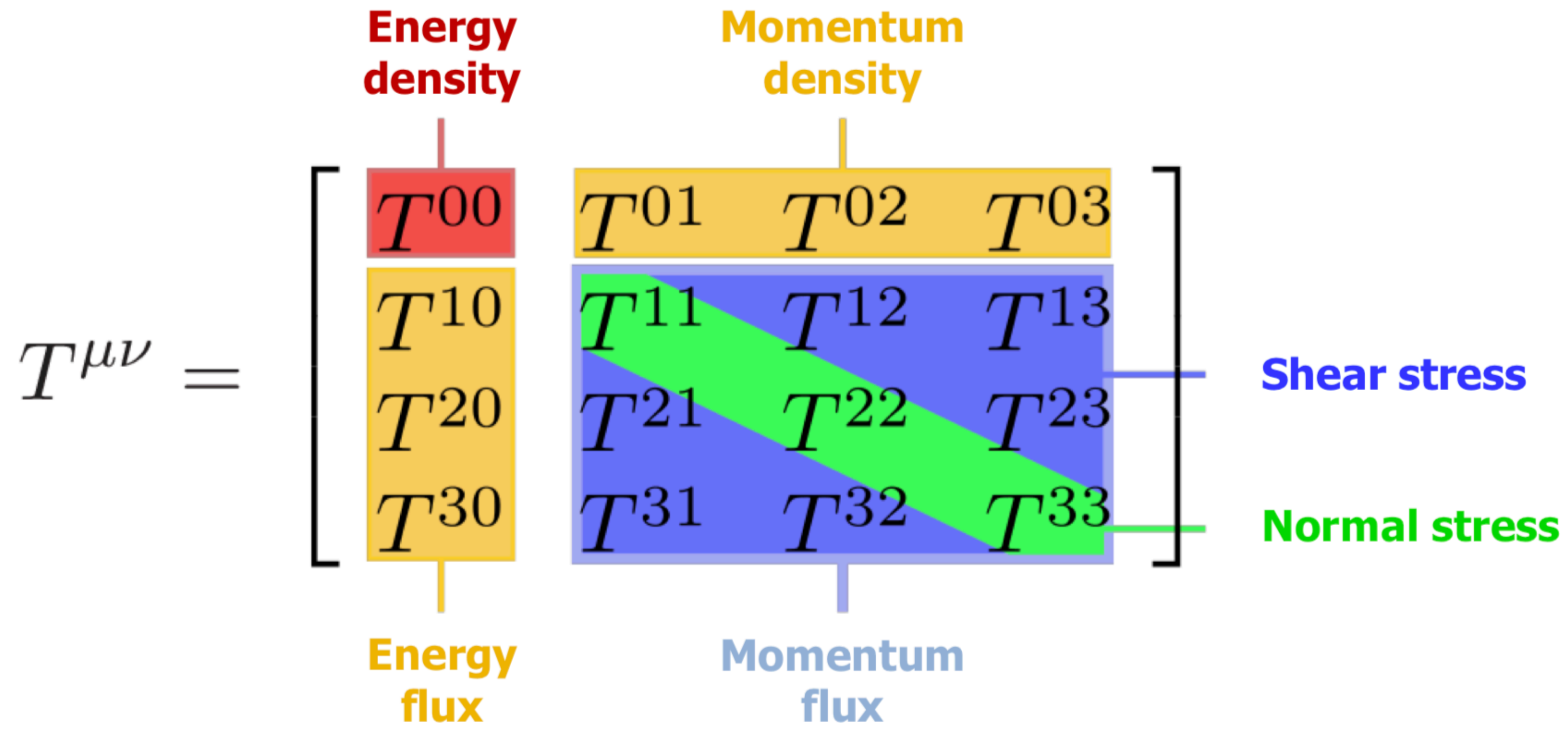
Positivity bounds - positivity of norm in Hilbert space, e.g.:

$$|H(x, \xi, t)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right) \frac{1}{1-\xi^2}}$$

Nucleon tomography:



$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\mathbf{\Delta}}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}} H^q(x, 0, t = -\mathbf{\Delta}^2)$$

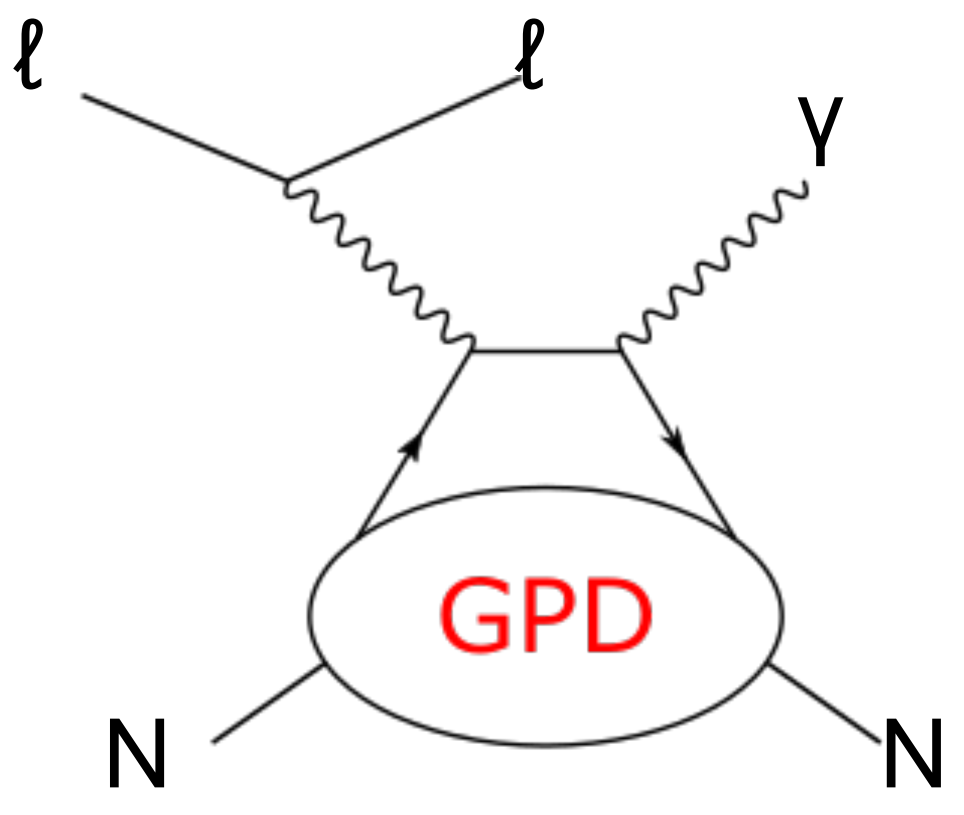


Energy momentum tensor in terms of form factors (OAM and mechanical forces):

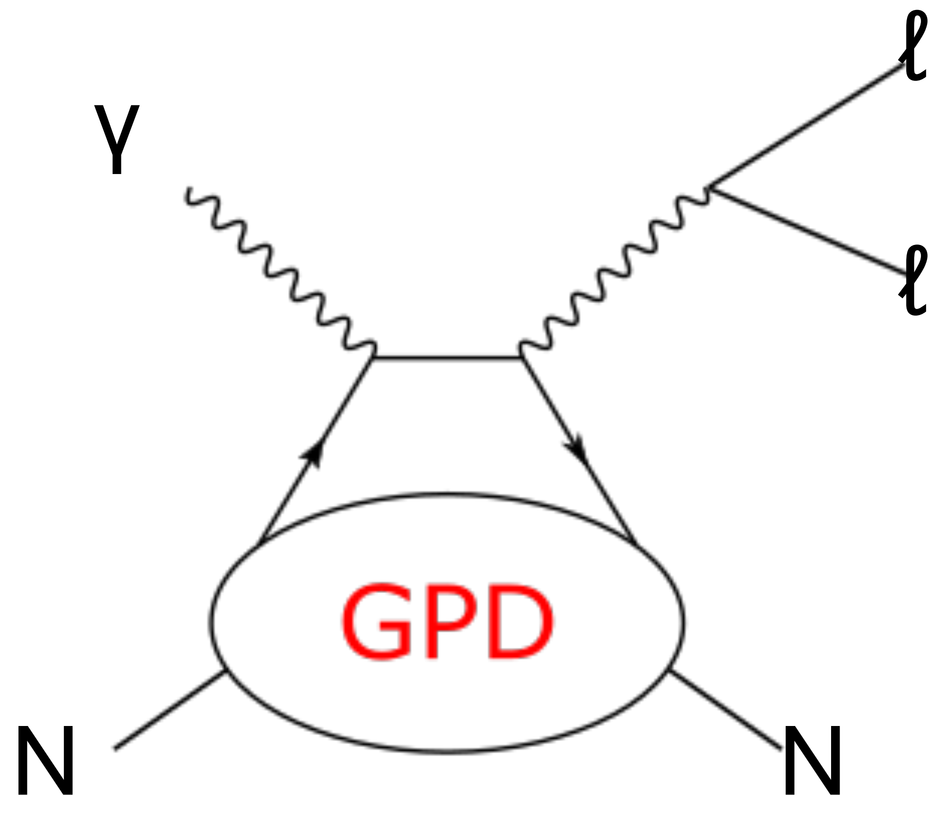
$$\langle p', s'|\widehat{T}^{\mu\nu}|p, s\rangle = \bar{u}(p', s')\left[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu}\bar{C}(t) + \right.$$

$$\left.\frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M}\left[A(t) + B(t) + D(t)\right] + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M}\left[A(t) + B(t) - D(t)\right]\right] u(p, s)$$

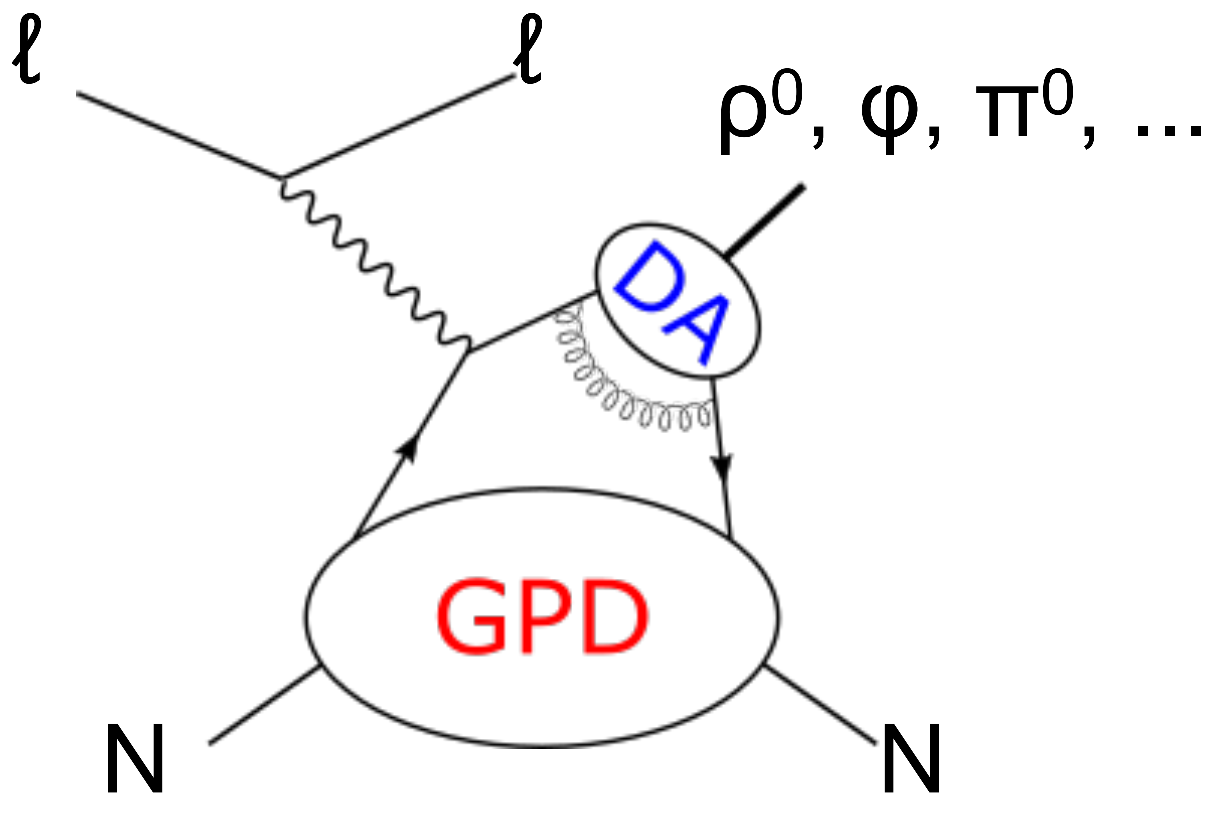
GPDs accessible in various production channels and observables
→ experimental filters



DVCS
Deeply Virtual Compton Scattering



TCS
Timelike Compton Scattering



HEMP
Hard Exclusive Meson Production

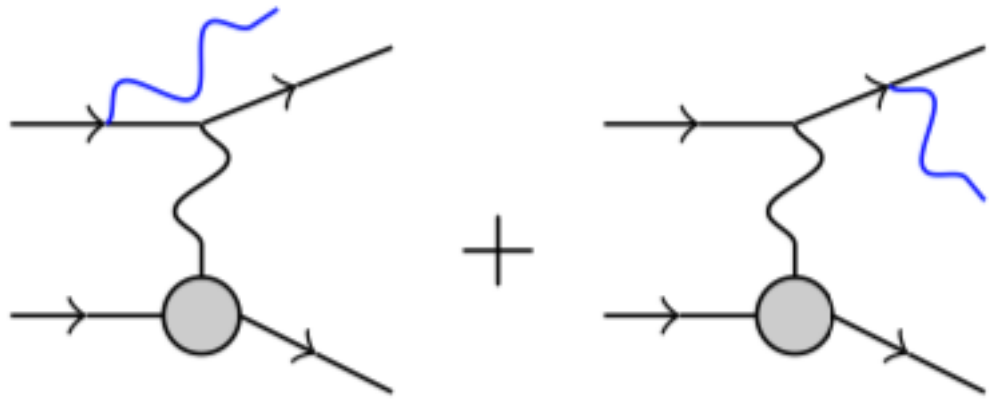
more production channels sensitive to GPDs exist!

DVCS Compton Form Factors

Cross-section for single photon production ($l + N \rightarrow l + N + \gamma$):

$$\sigma \propto |\mathscr{A}|^2 = |\mathscr{A}_{BH} + \mathscr{A}_{DVCS}|^2 = |\mathscr{A}_{BH}|^2 + |\mathscr{A}_{DVCS}|^2 + \mathscr{I}$$

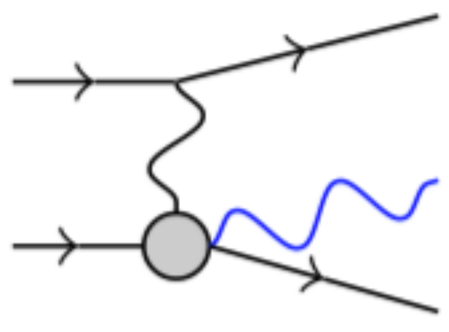
Bethe-Heitler process



calculable within QED

parametrised by elastic FFs

DVCS



calculable within QCD

parametrised by CFFs

For more details and formulae see e.g.:
A. V. Belitsky et al.
NPB 878 (2014) 214

$$\mathrm{Im}\mathscr{H}(\xi, t) \overset{\text{LO}}{=} \pi \sum_q e_q^2 H^{q(+)}(\xi, \xi, t)$$

$$\mathrm{Re}\mathscr{H}(\xi, t) = \mathrm{PV}\int_0^1 \frac{d\xi'}{\pi} \mathrm{Im}\mathscr{H}(\xi', t)\left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) + C_H(t)$$

2. Phenomenology at level of DVCS amplitudes (Compton form factors)

Parametric Ansatz of CFFs

$$G = \{H, E, \widetilde{H}, \widetilde{E}\}$$

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 78 (2018) 11, 890

$$G^q(x,0,t) = \mathrm{pdf}^q_G(x)\;\exp(f^q_G(x)t)$$

$$f^q_G(x) = A^q_G\log(1/x) + B^q_G(1-x)^2 + C^q_G(1-x)x$$

- reduction to PDFs and correspondence to EFFs
- modify "classical" log(1/x) term by $B_G^q(1\text{-}x)^2$ in low-x and by $C_G^q(1\text{-}x)x$ in high-x regions
- polynomials found in analysis of EFF data → good description of data
- allow to use the analytic regularisation prescription
- finite proton size at x → 1

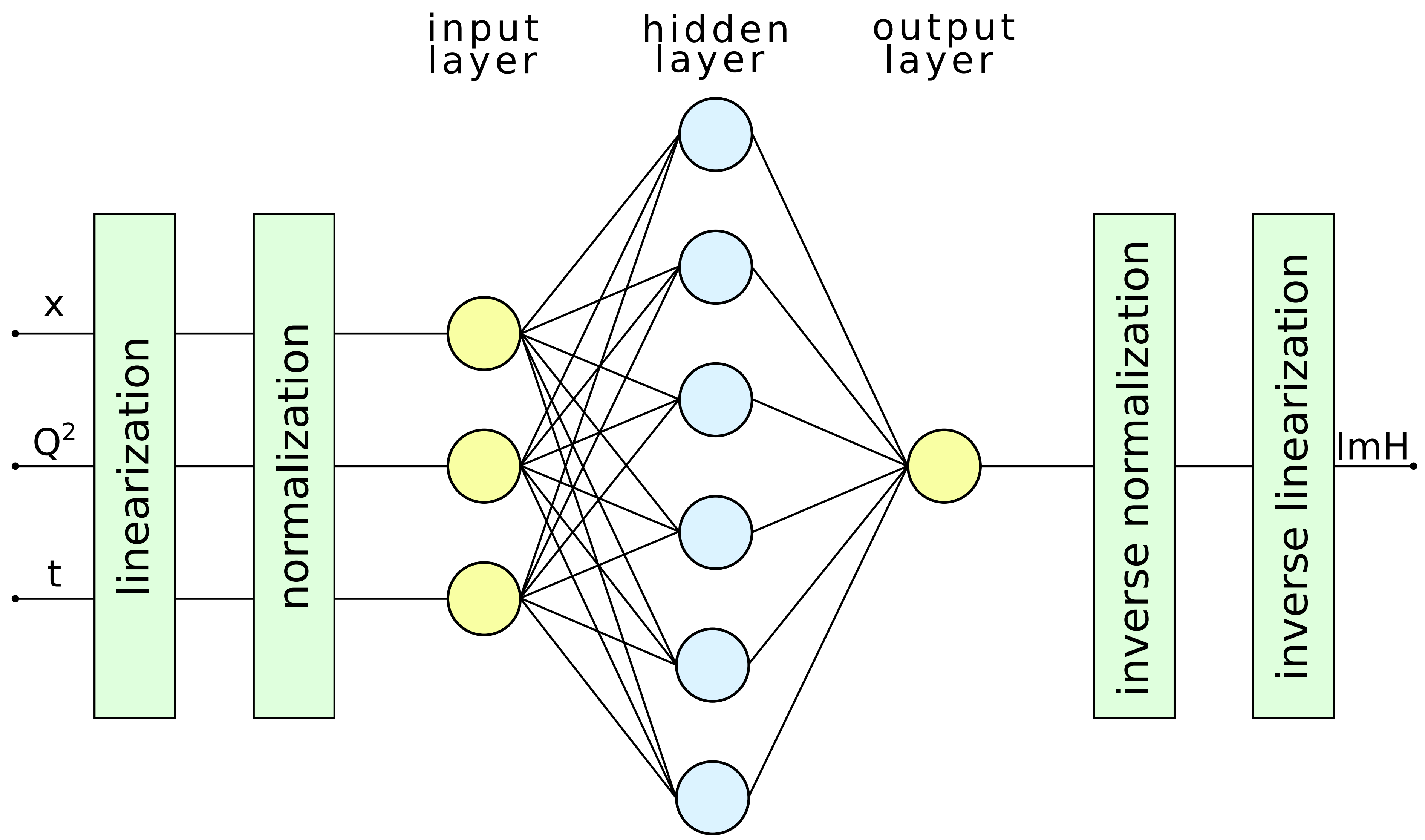
$$G^q(x,x,t) = G^q(x,0,t)\;g^q_G(x,x,t) \qquad g^q_G(x,x,t) = \frac{a^q_G}{(1-x^2)^2}\left(1+t(1-x)(b^q_G + c^q_G\log(1+x))\right)$$

- at x → 0 constant skewness effect
- at x → 1 reproduce power behaviour predicted for GPDs in Phys. Rev. D69, 051501 (2004)
- t-dependence similar to DD-models with (1-x) to avoid any t-dep. at x = 1

$$C^q_G(t) = 2\int_{(0)}^{1}\left(G^{q(+)}(x,x,t) - G^{q(+)}(x,0,t)\right)\frac{1}{x}dx$$

- subtraction constant as analytic continuation of Mellin moments to *j* = -1

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 79 (2019) 7, 614



Features of analysis:

- Independent artificial neural network for each CFF and Re/Im parts
- Functions of x_B, Q^2 and t
- Network size determined using benchmark sample
- No power-behaviour pre-factors
- Trained with genetic algorithm
- Regularisation method based on early stopping criterion
- Replica method for propagation of experimental uncertainties

DVCS data

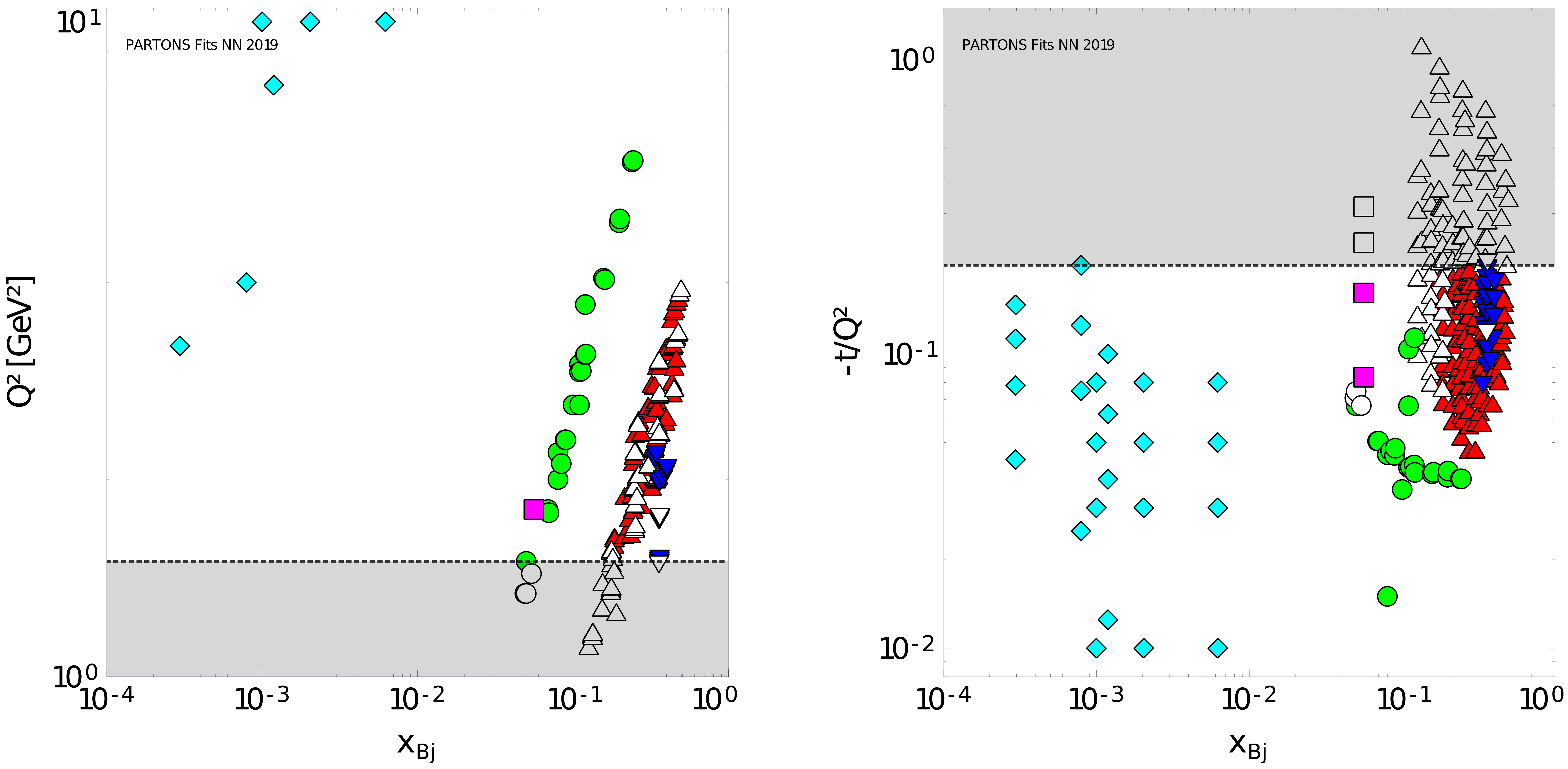
H. Moutarde, PS, J. Wagner, Eur. Phys. J. C 79 (2019) 7, 614

Kinematic cuts
used in our recent analyses:

$$Q^2 > 1.5\ \mathrm{GeV}^2$$

$$-t/Q^2 < 0.2$$

▼ HALL A
▲ CLAS
● HERMES
■ COMPASS
◆ H1 and ZEUS



Note: both analytic and non-analytic Ansätze use specific PDF parametrisations
analytic Ansatz is also fitted to elastic FF data

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 78 (2018) 11, 890

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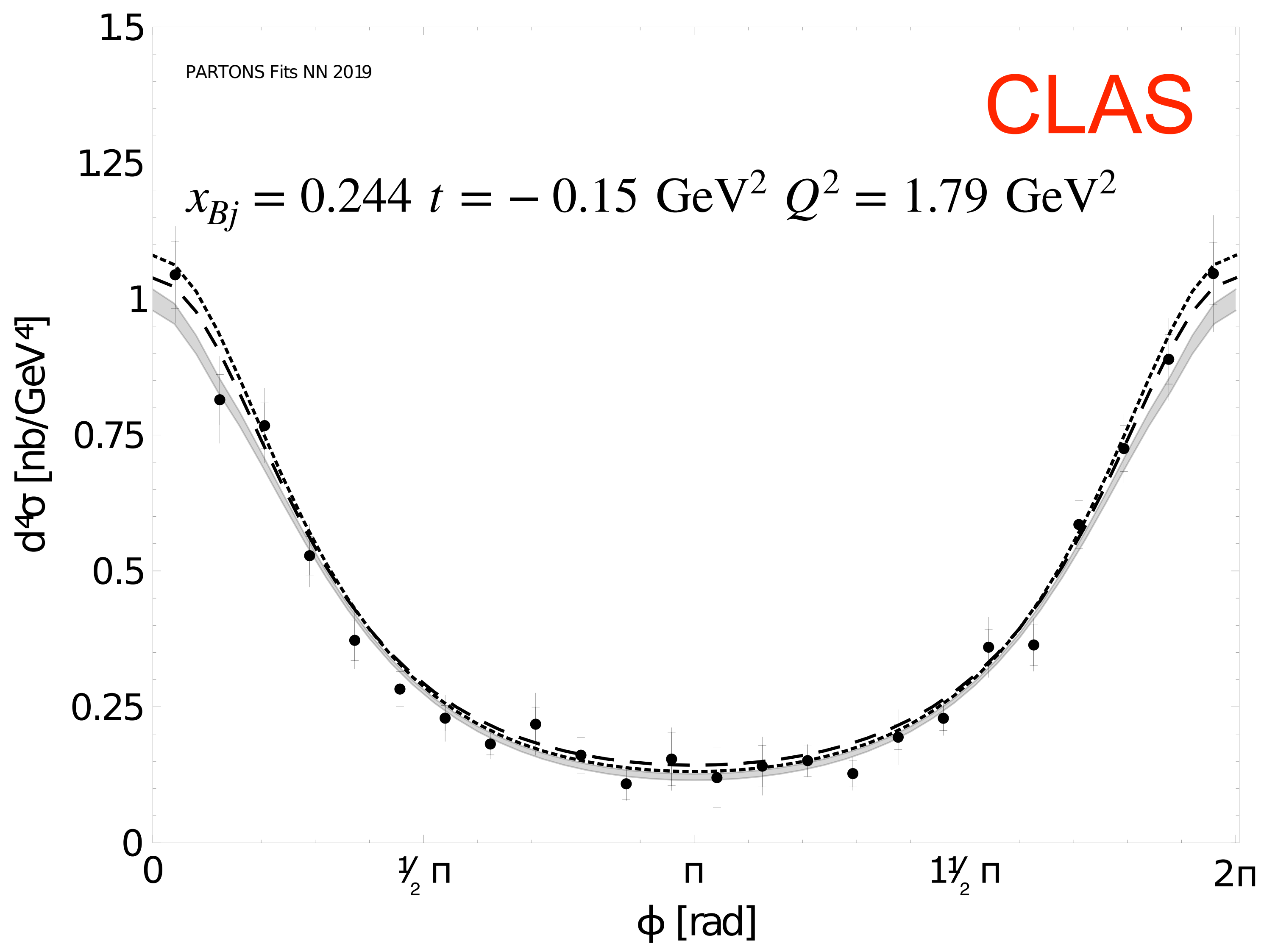
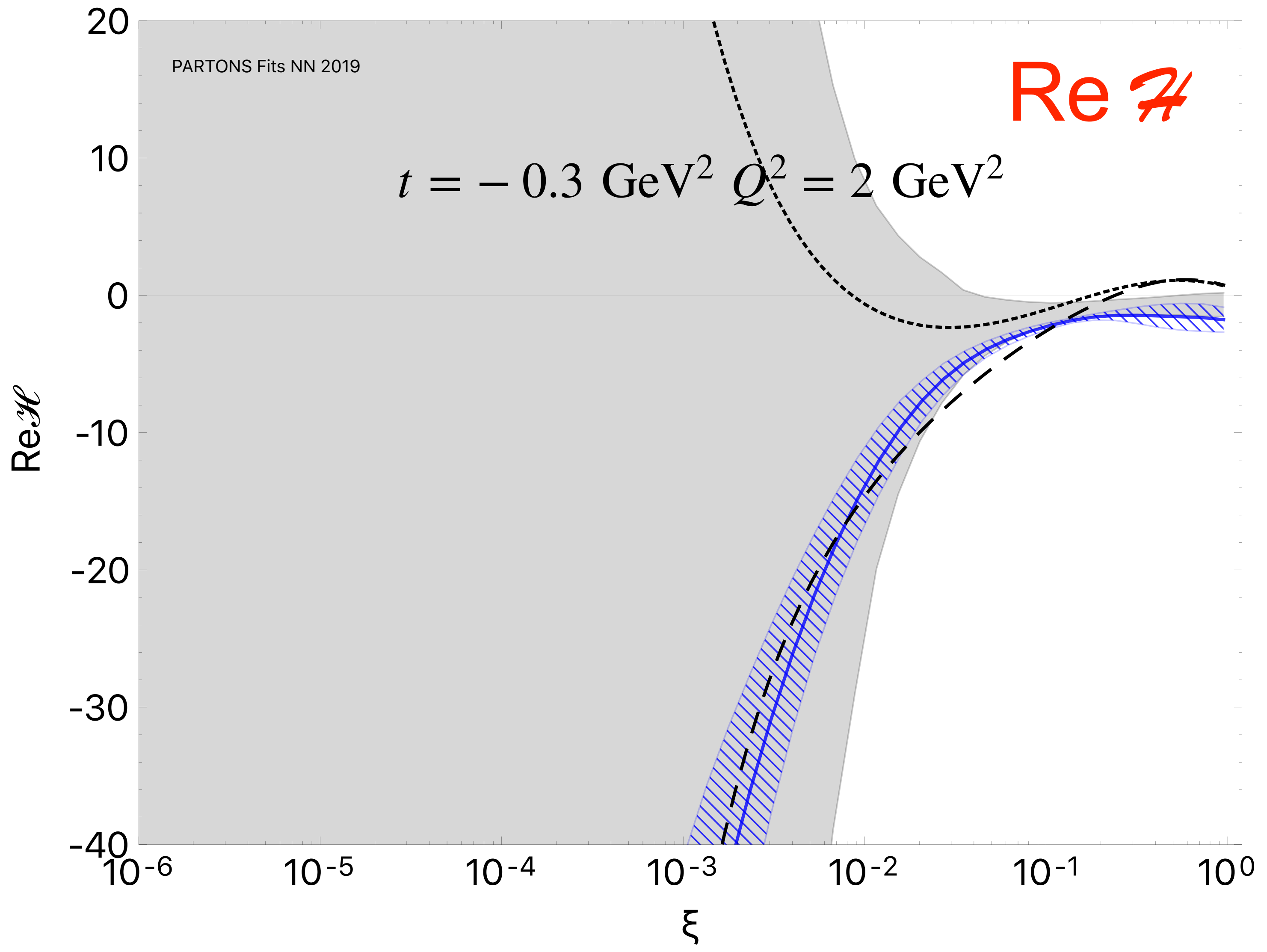
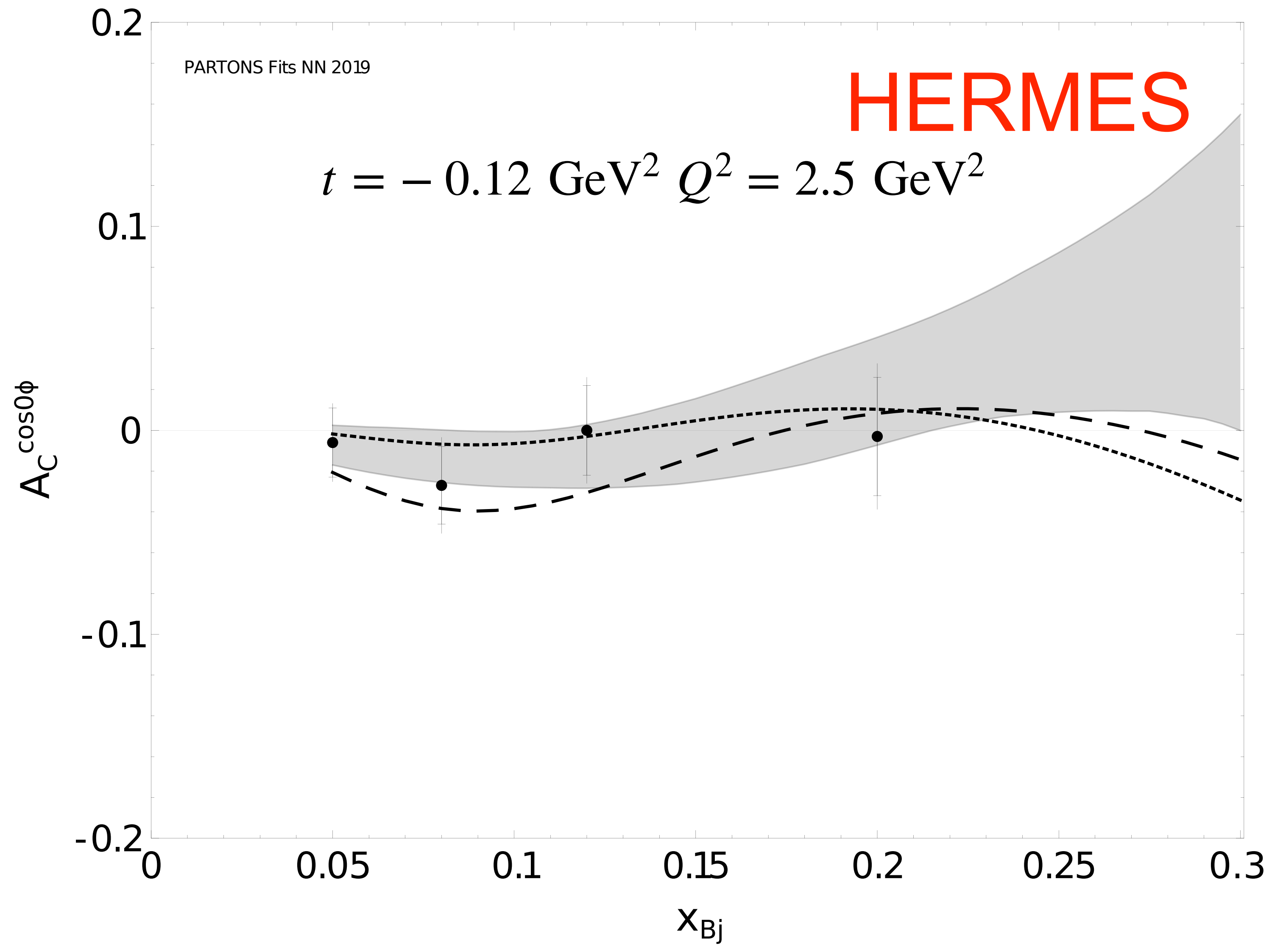
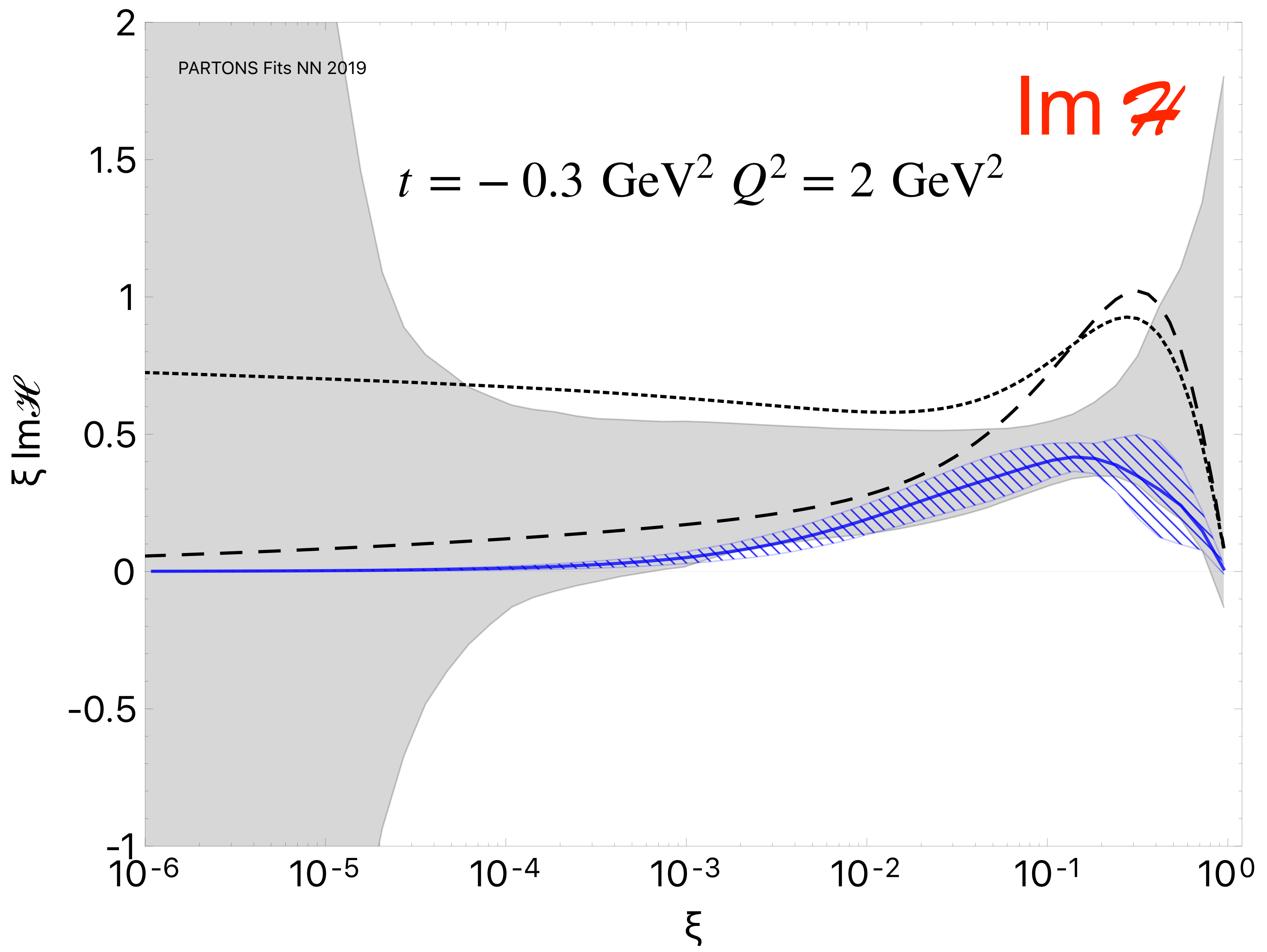
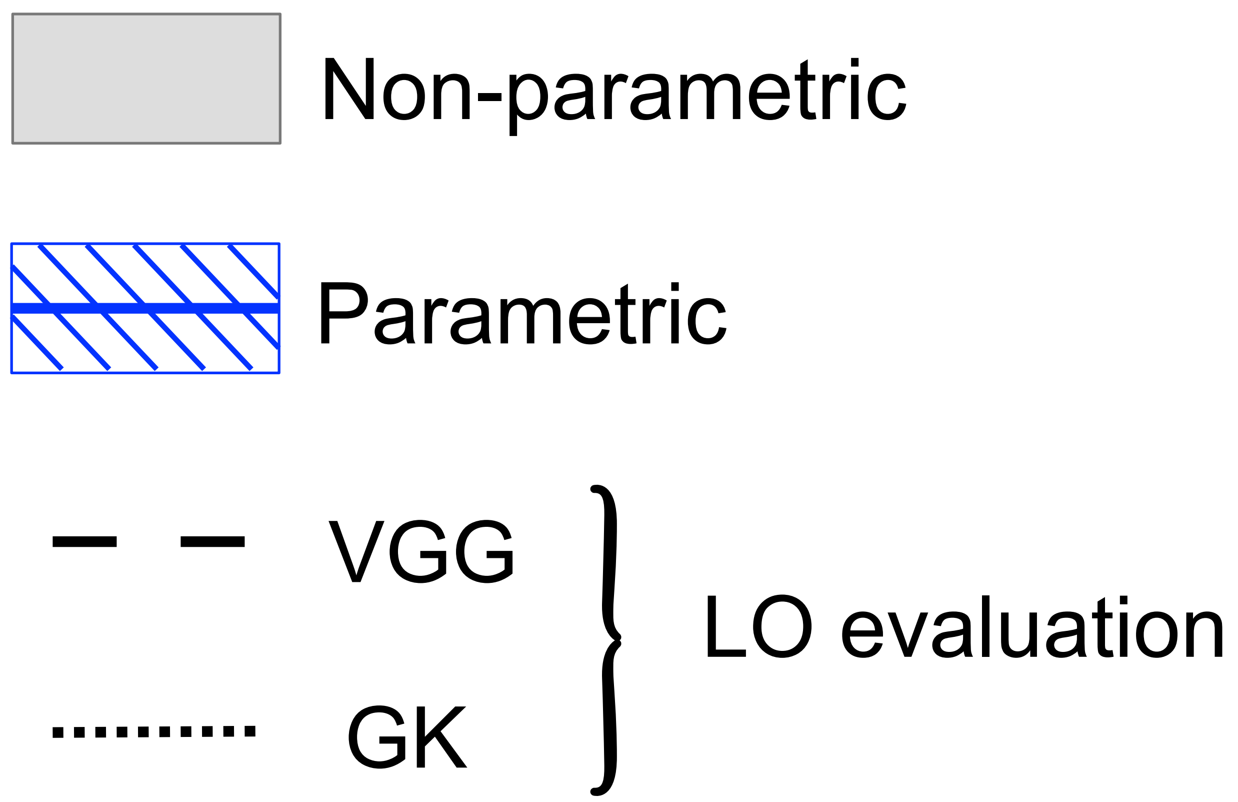
Eur. Phys. J. C 79 (2019) 7, 614







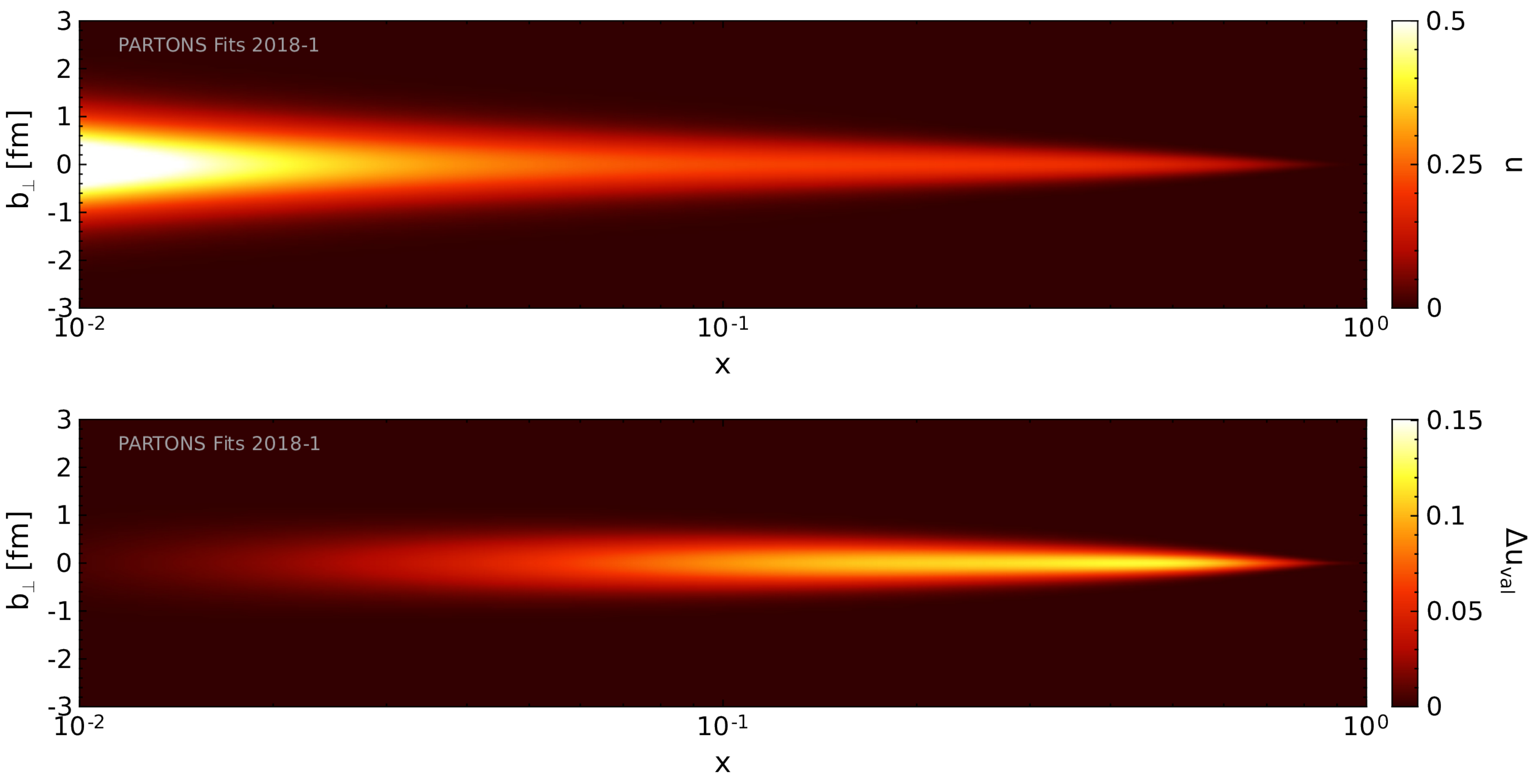


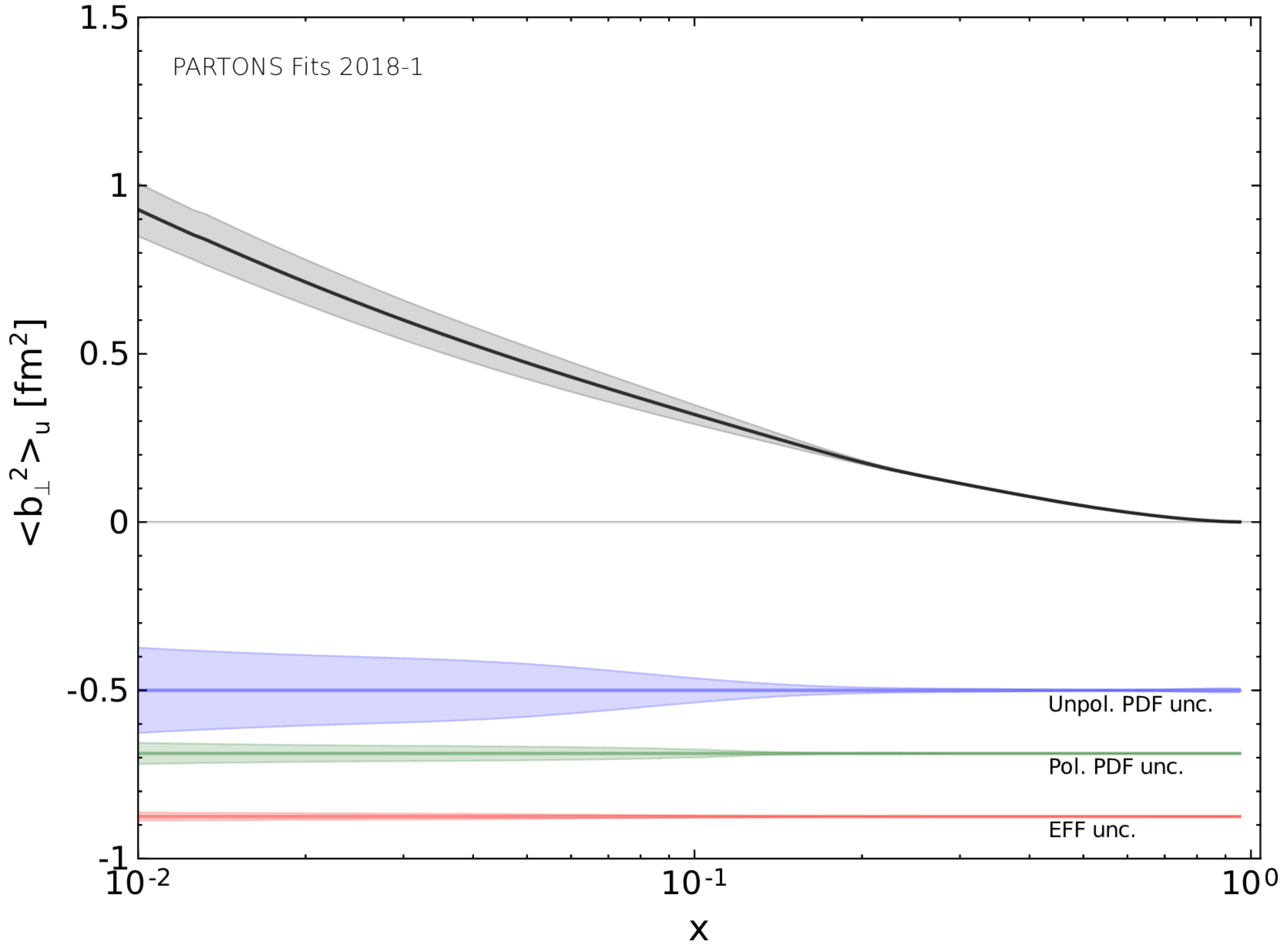


$$\xi \approx x_{Bj}/(2 - x_{Bj})$$

H. Moutarde, PS, J. Wagner, Eur. Phys. J. C 78 (2018) 11, 890

Parametric Ansatz allows us to access nucleon tomography





$$Q^2 = 2\ \mathrm{GeV}^2$$

Subtraction constant

H. Dutrieux et al.,
Eur. Phys. J. C 81 (2021), 300

Non-parametric Ansatz allows us to access EMT FF C

Dispersion relation:

$$\mathcal{C}_H(t,Q^2) = \mathrm{Re}\,\mathcal{H}(\xi,t,Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi'\,\mathrm{Im}\,\mathcal{H}(\xi',t,Q^2)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)$$

Relation between subtraction constant and D-term (z=x/ξ):

$$\mathcal{C}_H(t,Q^2) \overset{LO}{=} 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z\, \frac{D^q_{\mathrm{term}}(z,t,\mu_F^2 \equiv Q^2)}{1-z}$$

Decomposition into Gegenbauer polynomials:

$$D^q_{\mathrm{term}}(z,t,\mu_F^2) = (1-z^2)\sum_{\mathrm{odd}\ n} d_n^q(t,\mu_F^2)\, C_n^{3/2}(z)$$

Finally:

$$\mathcal{C}_H(t,Q^2) \overset{LO}{=} 4\sum_q e_q^2 \sum_{\mathrm{odd}\ n} d_n^q(t,\mu_F^2 \equiv Q^2)$$

Connection to EMT FF:

$$d_1^q(t,\mu_F^2) = 5C_q(t,\mu_F^2)$$

H. Dutrieux et al., Eur. Phys. J. C 81 (2021), 300

Subtraction constant:

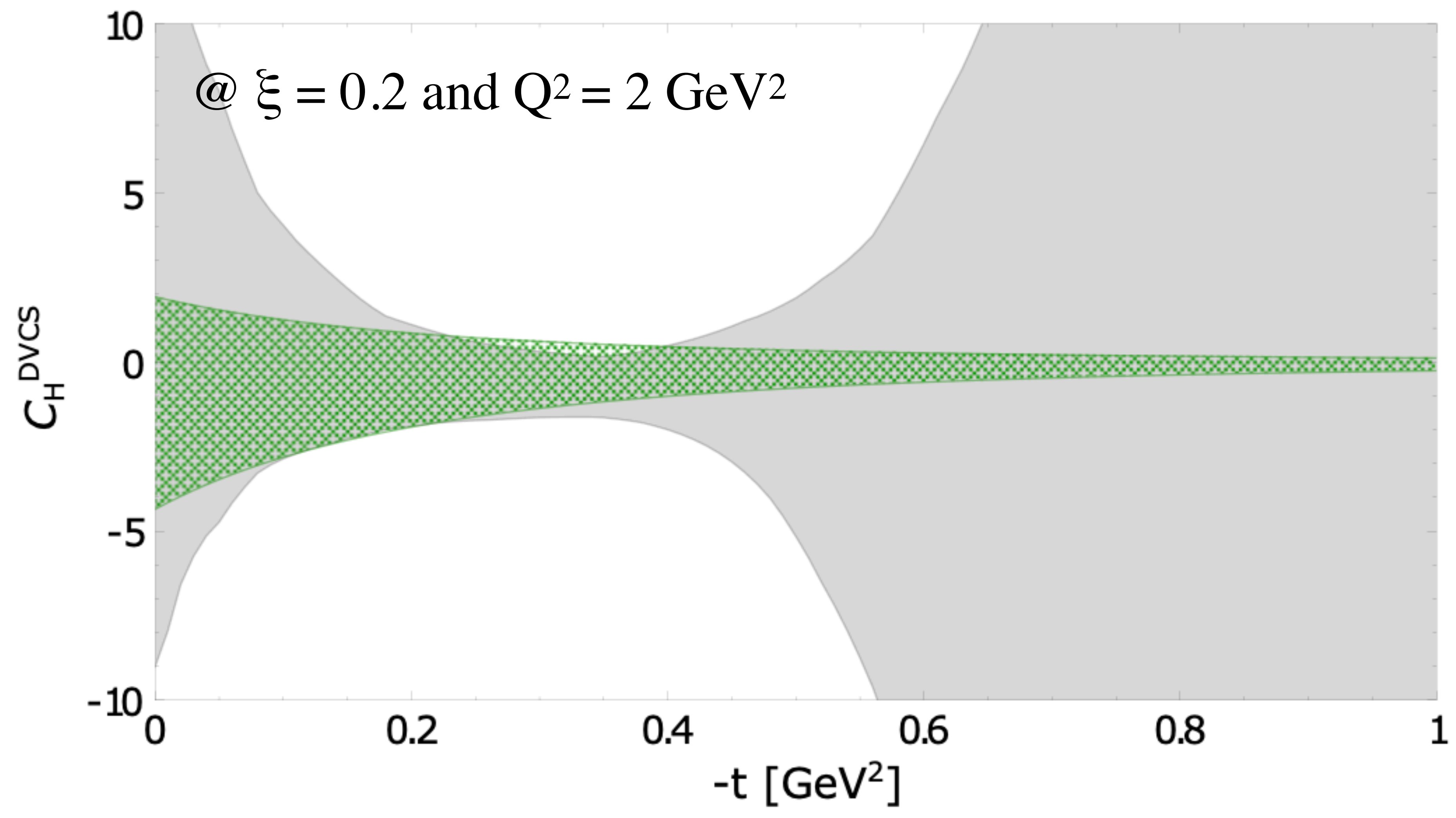
Non-parametric

Model dependent extraction

$$d_1^{uds}(t,\mu_F^2) = d_1^{uds}(\mu_F^2)\left(1-\frac{t}{\Lambda^2}\right)^{-\alpha}$$

$\alpha = 3$

$\Lambda = 0.8$ GeV



Results:

Parameter	Value
$d_1^{uds}(\mu_F^2 = 2$ GeV$^2)$	-0.5 ± 1.2
$d_1^{c}(\mu_F^2 = 2$ GeV$^2)$	-0.0020 ± 0.0053
$d_1^{g}(\mu_F^2 = 2$ GeV$^2)$	-0.6 ± 1.6

Conclussions:

- d_1^{uds} compatible with zero within large statistical uncertainty
- no way to simultaneously fit d_1^{uds} and d_1^{g} or d_3^{uds}
- large model uncertainty of used Ansatz

3. Phenomenology at level of GPDs

Motivation

- Despite a substantial progress in both measurement and description of exclusive processes, and in lattice-QCD the problem of the model dependency of GPDs is still poorly addressed.

- Exceptions:
  - probing nucleon tomography at low-xB
  - extraction of D-term

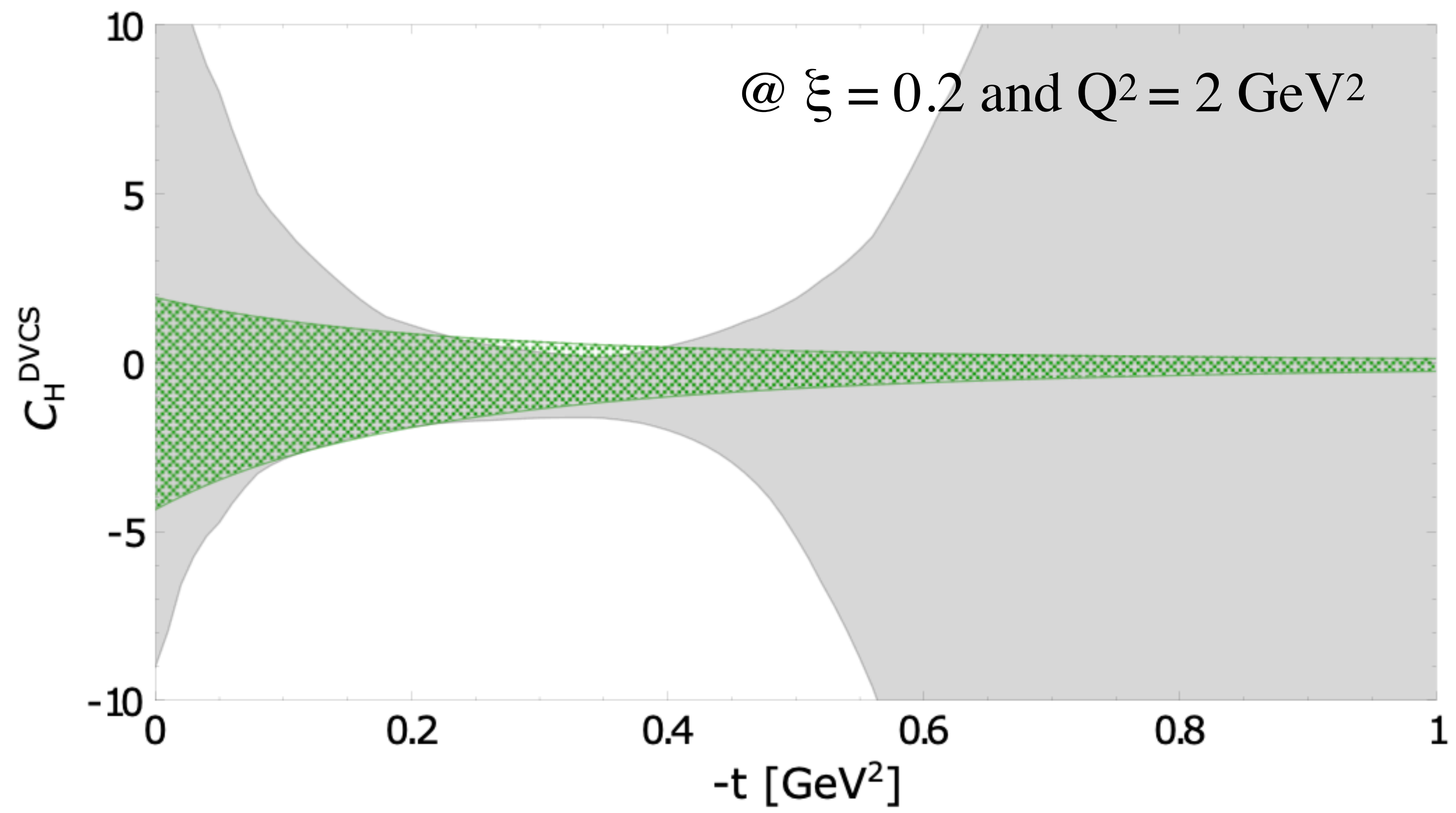
ANN analysis

Model dependent extraction

$$d_1^{uds}(t, \mu_F^2) = d_1^{uds}(\mu_F^2)\left(1 - \frac{t}{\Lambda^2}\right)^{-\alpha}$$

$\alpha = 3$

$\Lambda = 0.8\ \mathrm{GeV}$



- No GPD models that could be considered non-parametric → no tools to study model dependency of the extraction of GPDs, nucleon tomography and orbital angular momentum

Modelling in (β, α)-space

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

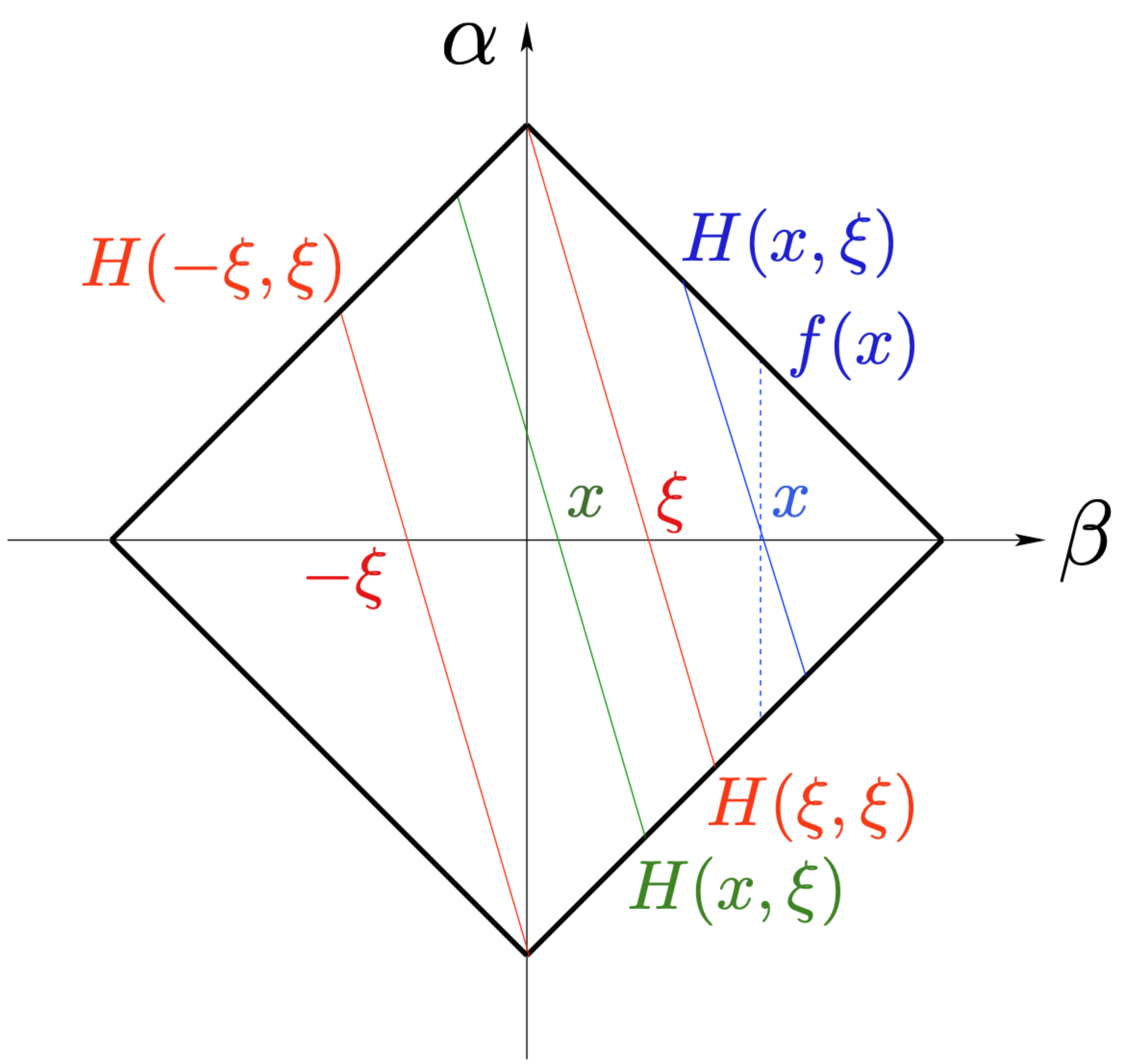
Double distribution:

$$H(x, \xi, t) = \int d\Omega \, F(\beta, \alpha, t)$$

where:

$$d\Omega = d\beta \, d\alpha \, \delta(x - \beta - \alpha\xi)$$

$$|\alpha| + |\beta| \leq 1$$



from PRD83, 076006, 2011

We also consider non-parametric GPD modelling in (x, ξ)-space, see our paper
The drawback of this modelling is that one can not keep PDF singularity for only x=0 and ξ=0

Modelling in (β, α)-space

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

Double distribution: $$(1-x^2)F_C(\beta,\alpha)+(x^2-\xi^2)F_S(\beta,\alpha)+\xi F_D(\beta,\alpha)$$

Classical term:

$$F_C(\beta,\alpha)=f(\beta)h_C(\beta,\alpha)\frac{1}{1-\beta^2}$$

$$f(\beta)=\mathrm{sgn}(\beta)q(|\beta|)$$

$$h_C(\beta,\alpha)=\frac{\mathrm{ANN}_C(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|}\mathrm{d}\alpha\mathrm{ANN}_C(|\beta|,\alpha)}$$

Shadow term:

$$F_S(\beta,\alpha)=f(\beta)h_S(\beta,\alpha)$$

$$f(\beta)=\mathrm{sgn}(\beta)q(|\beta|)$$

$$h_S(\beta,\alpha)/N_S=\frac{\mathrm{ANN}_S(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|}\mathrm{d}\alpha\mathrm{ANN}_S(|\beta|,\alpha)}-\frac{\mathrm{ANN}_{S'}(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|}\mathrm{d}\alpha\mathrm{ANN}_{S'}(|\beta|,\alpha)}.$$

$$\mathrm{ANN}_{S'}(|\beta|,\alpha)\equiv\mathrm{ANN}_C(|\beta|,\alpha)$$

D-term:

$$F_D(\beta,\alpha)=\delta(\beta)D(\alpha)$$

$$D(\alpha)=(1-\alpha^2)\sum_{\substack{i=1\\ \mathrm{odd}}}d_iC_i^{3/2}(\alpha)$$

Modelling in (β, α)-space

V. Bertone et al.,
Phys. Rev. D 103 (2021) 11, 114019

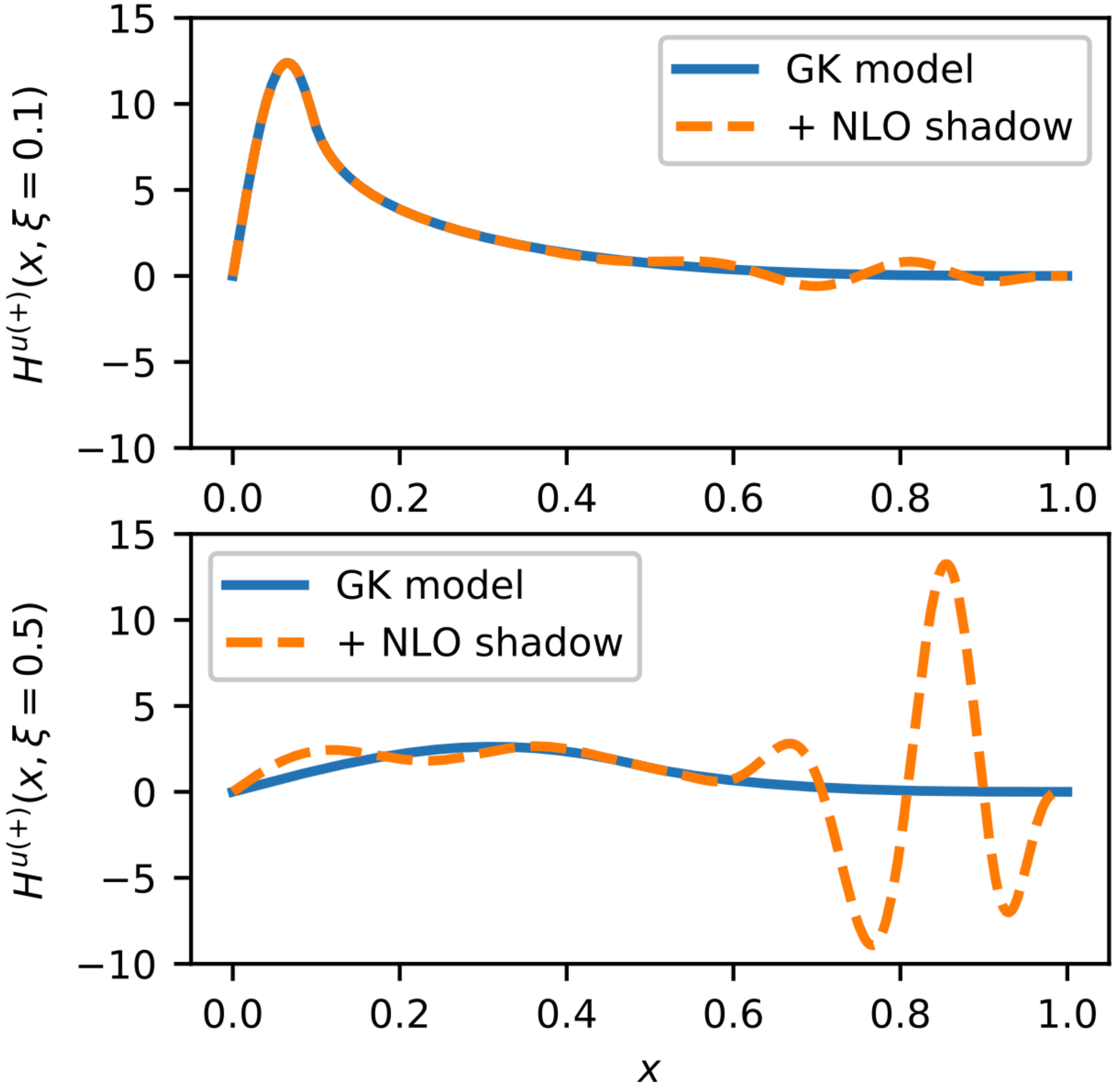
Shadow term is closely related to the so-called shadow GPDs

Shadow GPDs have considerable size and:

- at the initial scale do not contribute to both PDFs and CFFs
- at some other scale they contribute negligibly

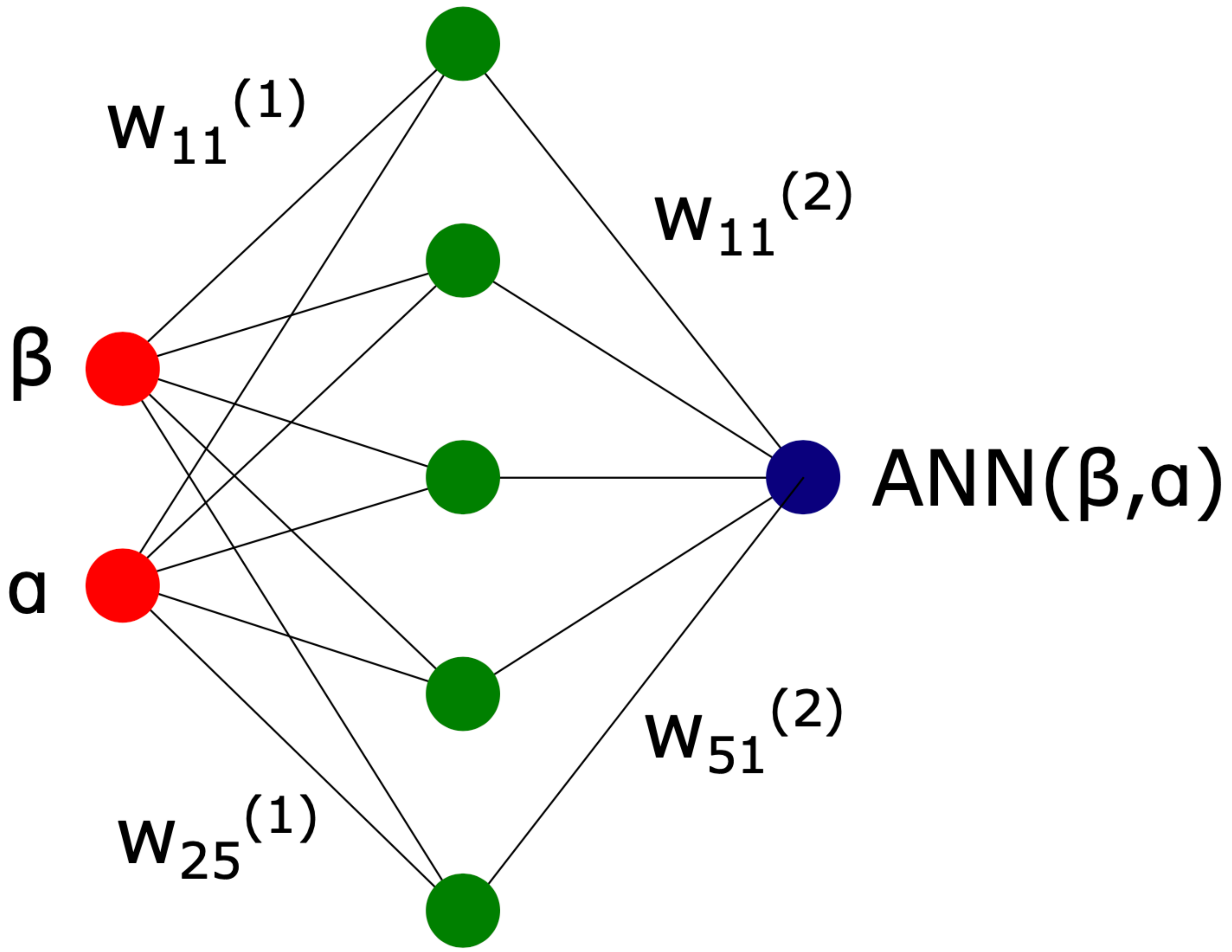
making the deconvolution of CFFs ill-posed

We found such GPDs for both LO and NLO



H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

Our ANNs:



Requirements:

symmetric w.r.t. α

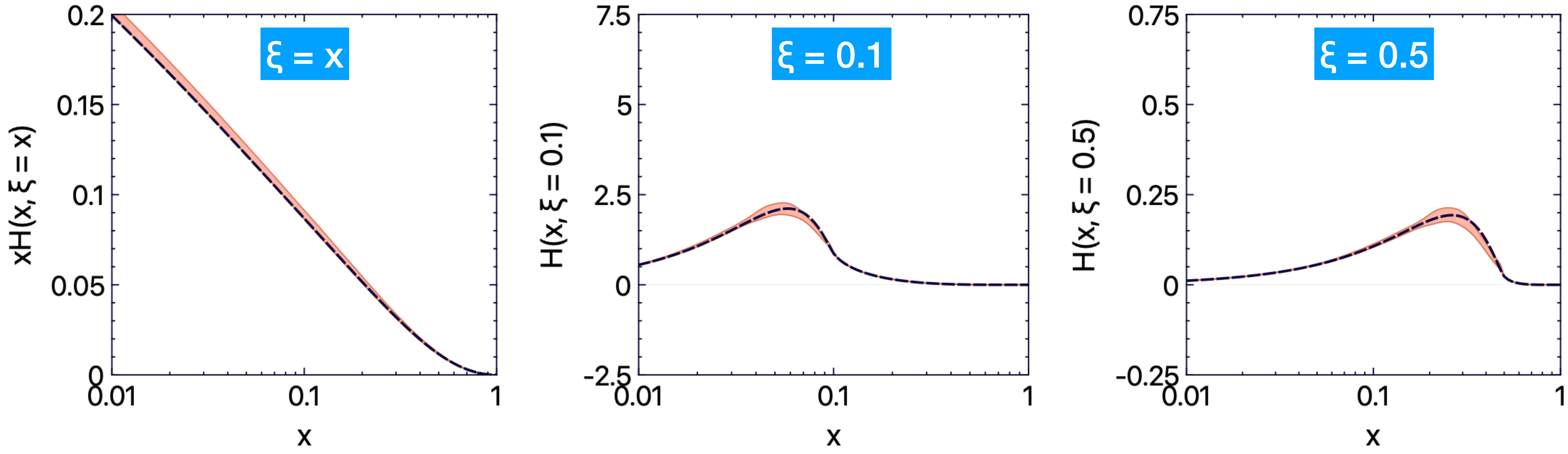
symmetric w.r.t. β

vanishes at $|\alpha| + |\beta| = 1$

Activation function:

$$\left(\varphi_i\left(w_i^{\beta}|\beta| + w_i^{\alpha}\alpha/(1-|\beta|) + b_i\right) - \varphi_i\left(w_i^{\beta}|\beta| + w_i^{\alpha} + b_i\right)\right) + (w^{\alpha} \to -w^{\alpha})$$

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252



Conditions:

- Input: 400 x ≠ ξ points generated with GK model
- Positivity not forced

Technical detail of the analysis:

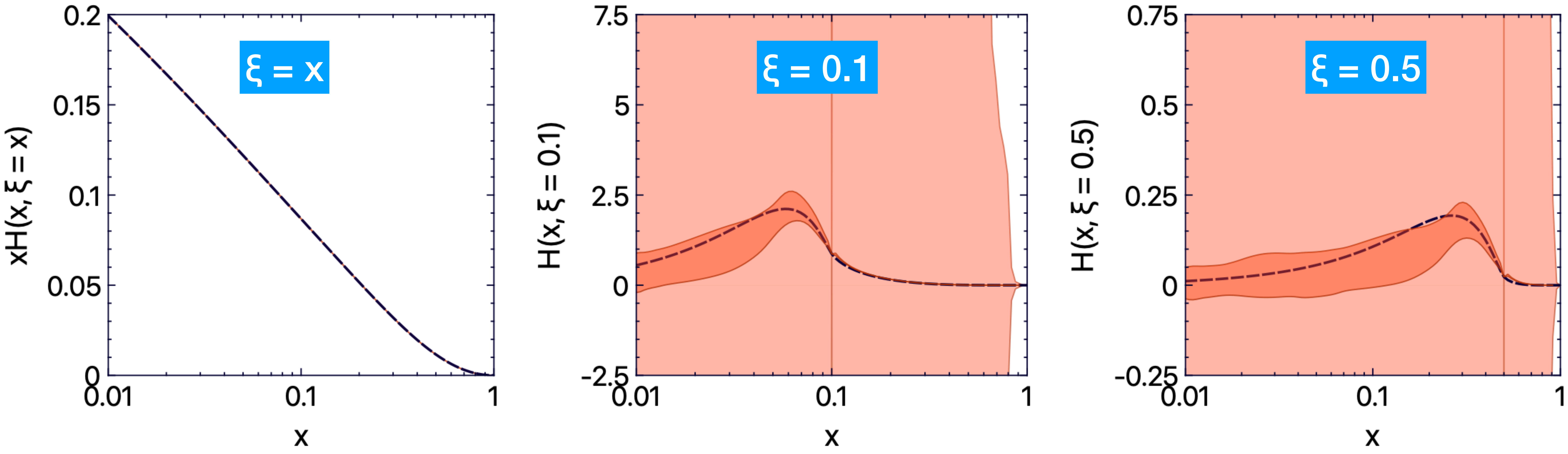
- Minimisation with genetic algorithm
- Replication for estimation of model uncertainties
- “Local” detection of outliers
- Dropout algorithm for regularisation

GK

ANN model 68% CL $F_C + F_S + F_D$

Demonstration of results

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252



GK

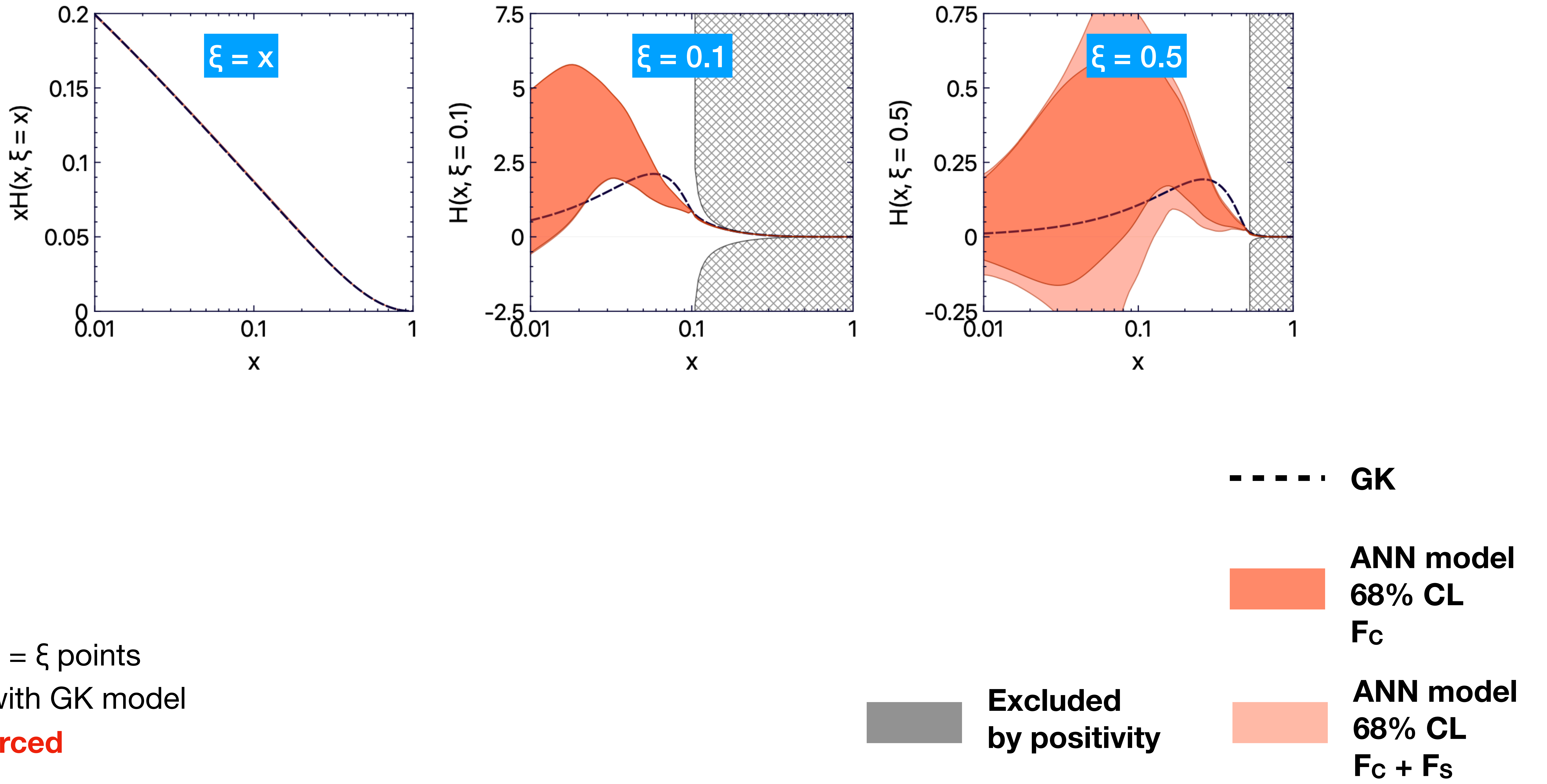
ANN model 68% CL F_C

ANN model 68% CL $F_C + F_S$

Conditions:

- Input: 200 x = ξ points generated with GK model
- Positivity not forced

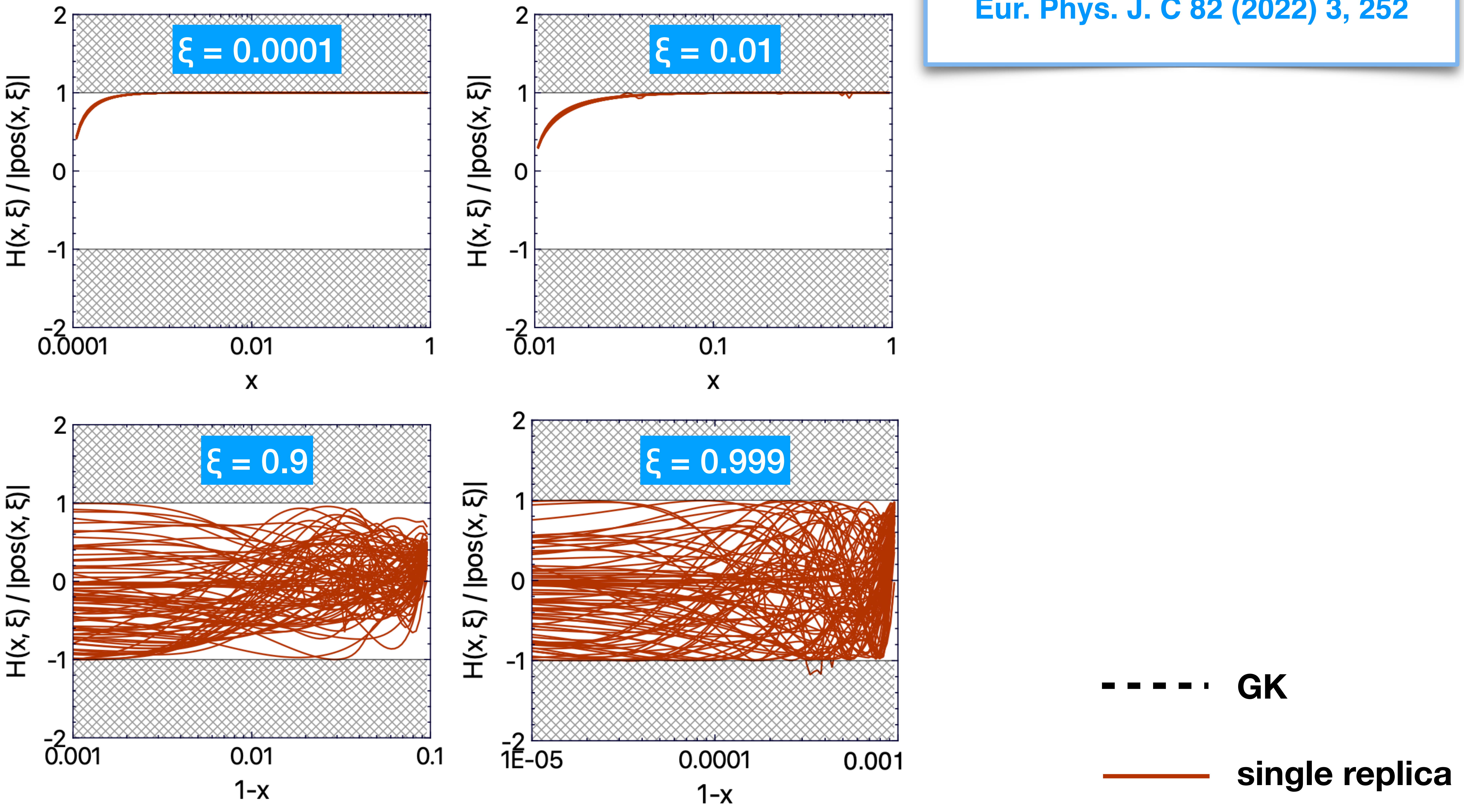
H. Dutrieux et al., Eur. Phys. J. C 82 (2022) 3, 252



Conditions:

- Input: 200 x = ξ points generated with GK model
- Positivity **forced**

Demonstration of results



H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

Conditions:

- Input: 200 x = ξ points generated with GK model
- Positivity **forced**

4. New sources of GPD information

O. Grocholski et al.,
Eur. Phys. J. C 80 (2020) 2, 171

Relation between
DVCS and TCS CFFs:

for more details see:
Mueller, Pire, Szymanowski, Wagner
Phys. Rev. D86, 031502 (2012)

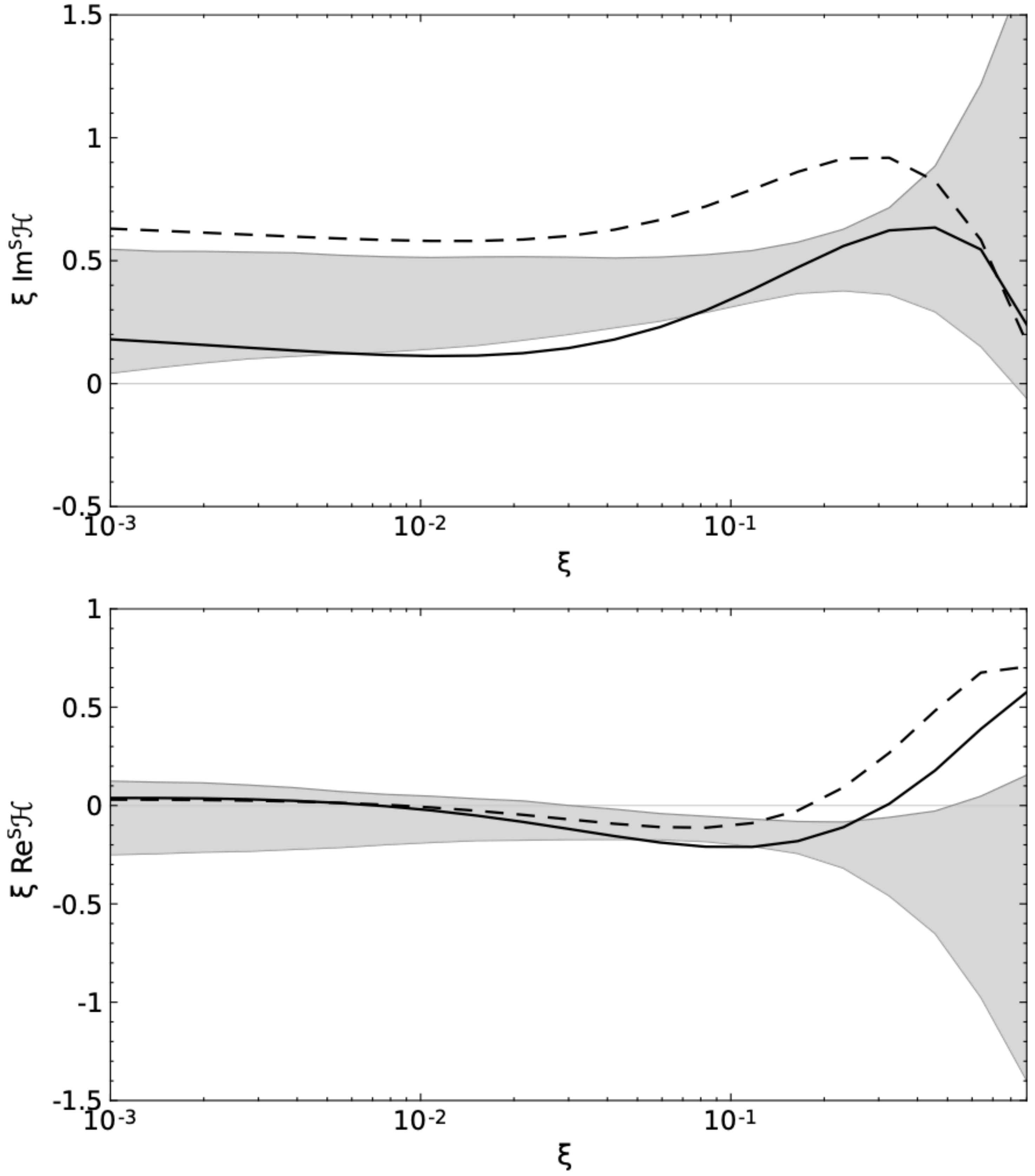
$$
\begin{aligned}
{}^{T}\mathscr{H} &\stackrel{\text{LO}}{=} {}^{S}\mathscr{H}^{*} \\
{}^{T}\widetilde{\mathscr{H}} &\stackrel{\text{LO}}{=} -{}^{S}\widetilde{\mathscr{H}}^{*} \\
{}^{T}\mathscr{H} &\stackrel{\text{NLO}}{=} {}^{S}\mathscr{H}^{*} - i\pi\,\mathscr{Q}^{2}\frac{\partial}{\partial\mathscr{Q}^{2}}{}^{S}\mathscr{H}^{*} \\
{}^{T}\widetilde{\mathscr{H}} &\stackrel{\text{NLO}}{=} -{}^{S}\widetilde{\mathscr{H}}^{*} + i\pi\,\mathscr{Q}^{2}\frac{\partial}{\partial\mathscr{Q}^{2}}{}^{S}\widetilde{\mathscr{H}}^{*}\,.
\end{aligned}
$$

Combined study of DVCS and TCS:

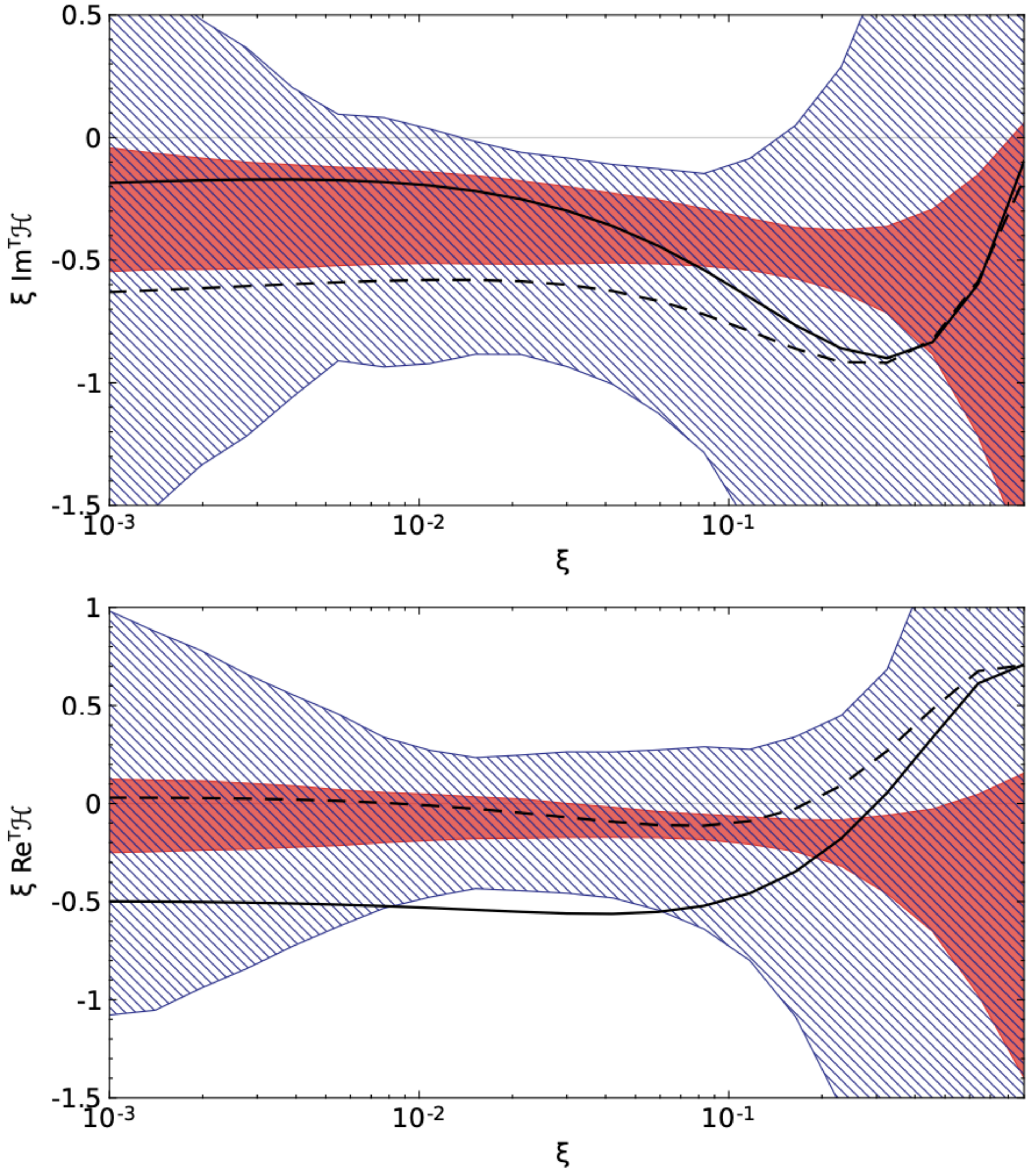
- source of GPD information
- useful to prove universality of GPDs
- impact of NLO corrections
- constrain Q2-dep. of CFFs

O. Grocholski et al.,
Eur. Phys. J. C 80 (2020) 2, 171

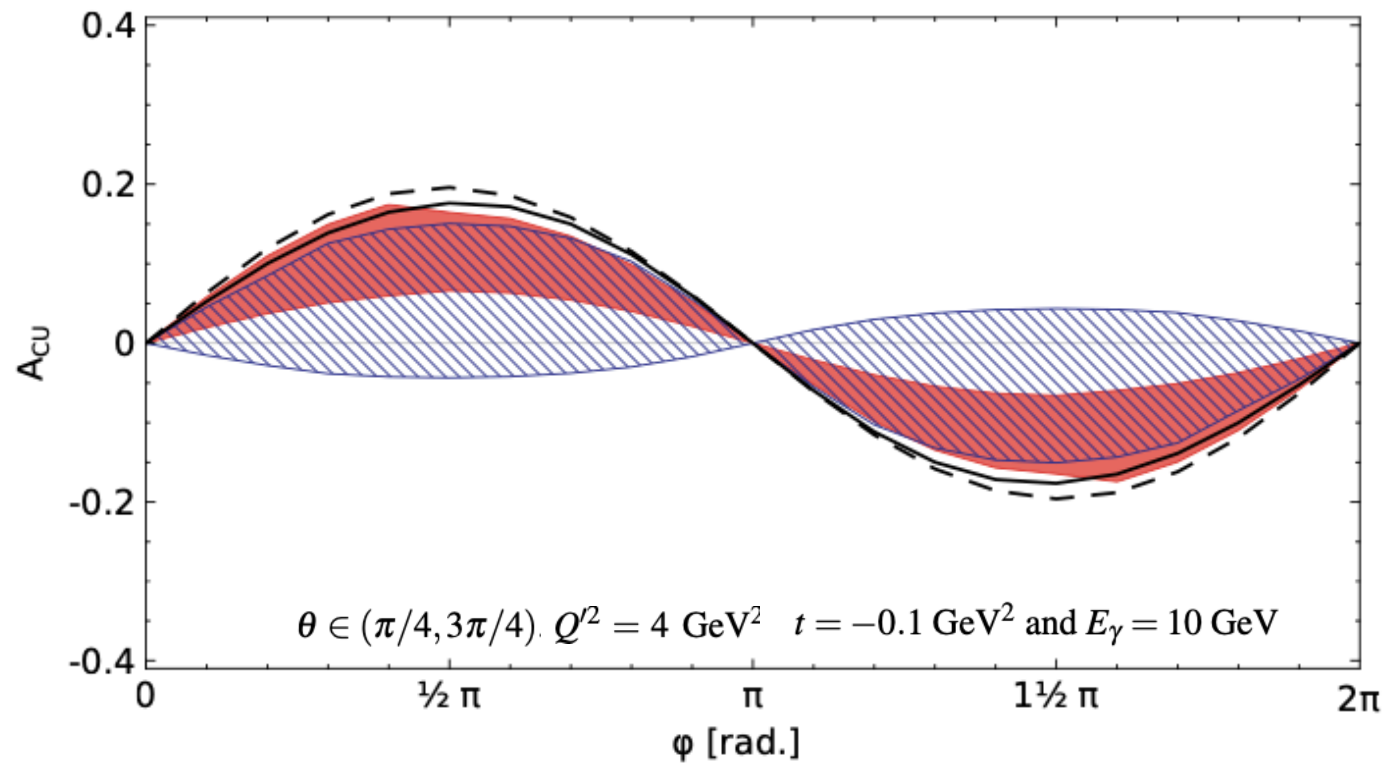
DVCS CFF (Non-parametric):



TCS CFF:



TCS circular beam asymmetry:



DVCS

TCS from DVCS (LO)

TCS from DVCS (NLO)

GK model (LO)

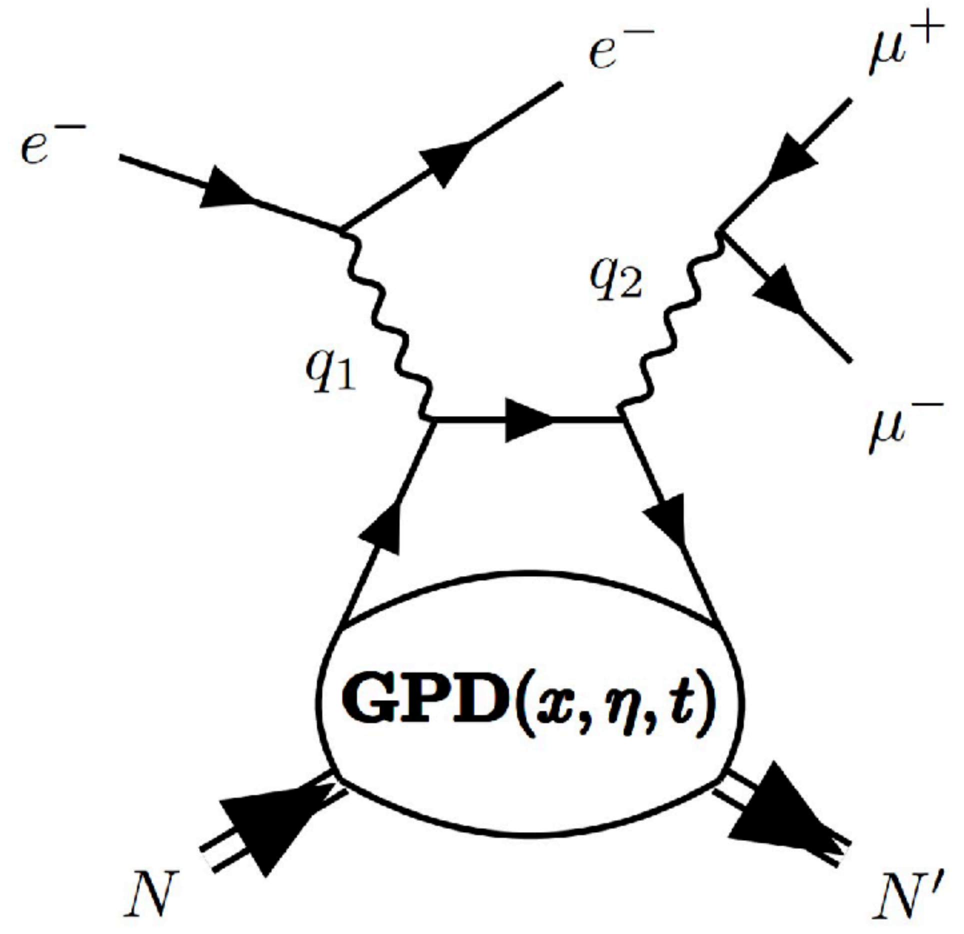
GK model (NLO)

Double DVCS

PRELIMINARY !!!

- The process allows to probe GPDs outside x=ξ line, but is much more challenging experimentally

$$\mathcal{A}_{\mathrm{DDVCS}} \overset{LO}{\sim} \int_{-1}^{1} dx \, \frac{1}{x-\xi+i0} \mathrm{GPD}(x,\eta,t)$$

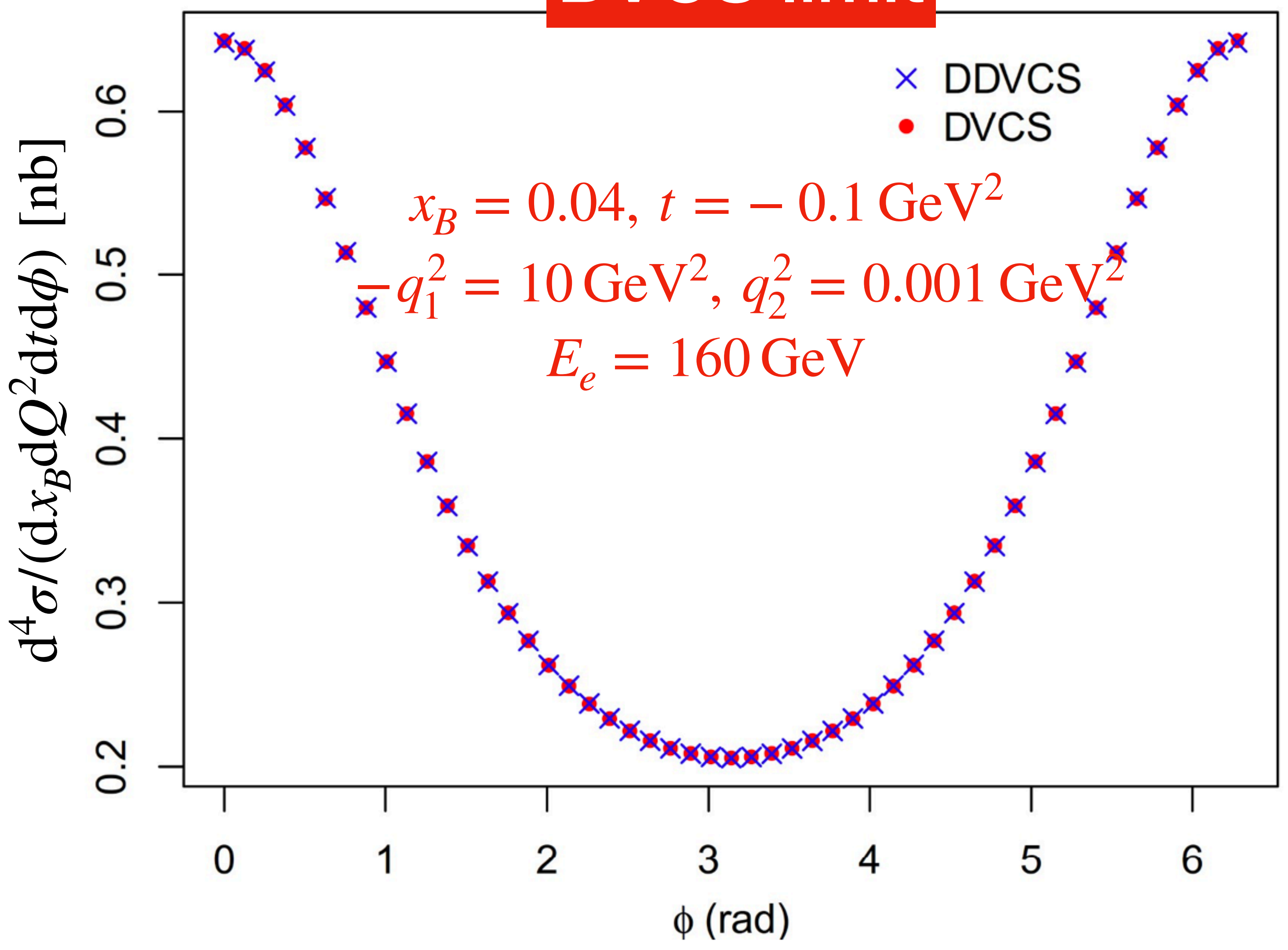


- We are revisiting DDVCS for phenomenological studies, i.e. we reevaluate DDVCS and related BH amplitudes using Kleiss-Stirling technique

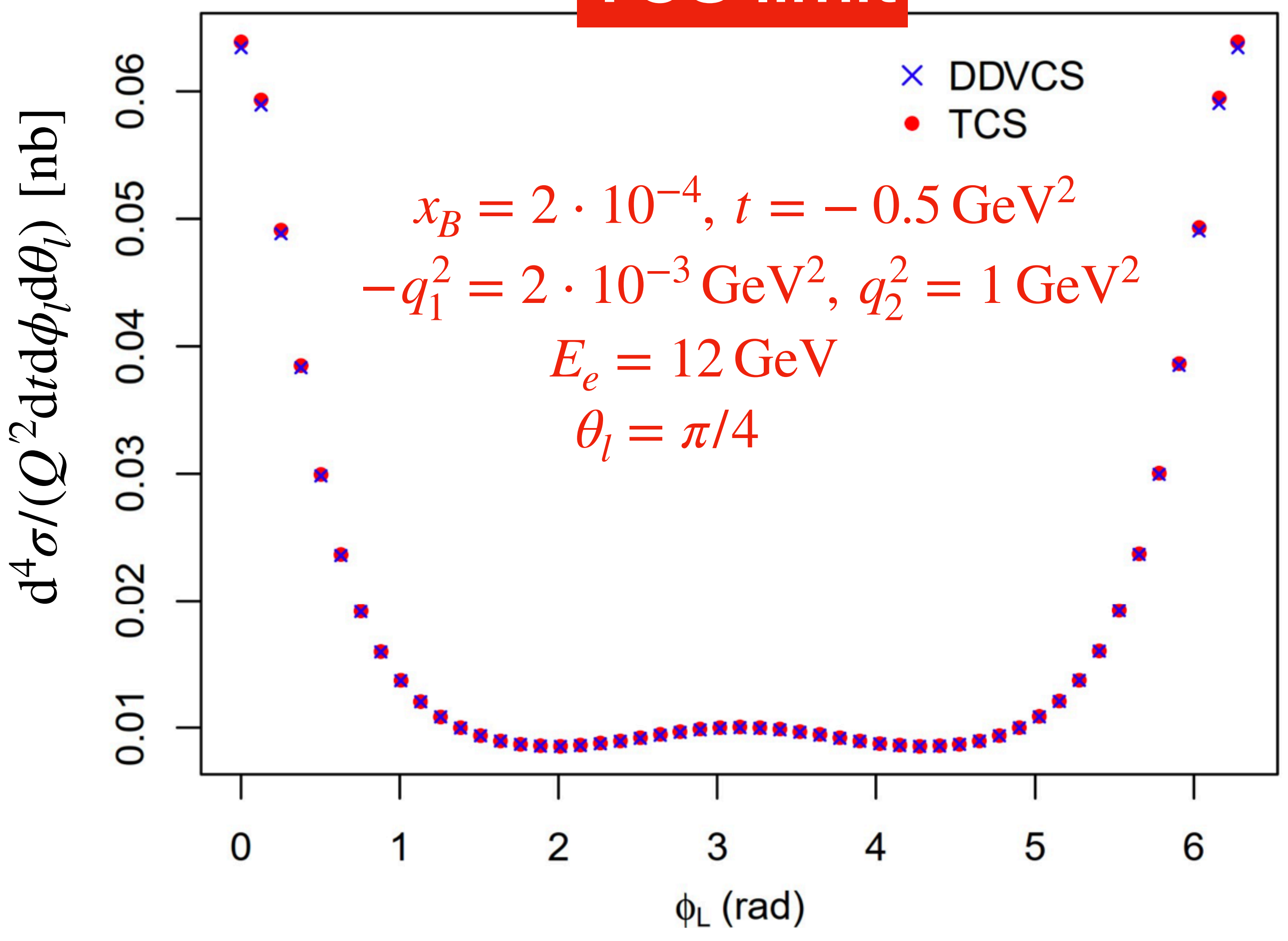
- We plane to release obtained formulae in PARTONS and EpIC MC generator

Preliminary results:
BH cross-section in DVCS and TCS limits

DVCS limit



TCS limit

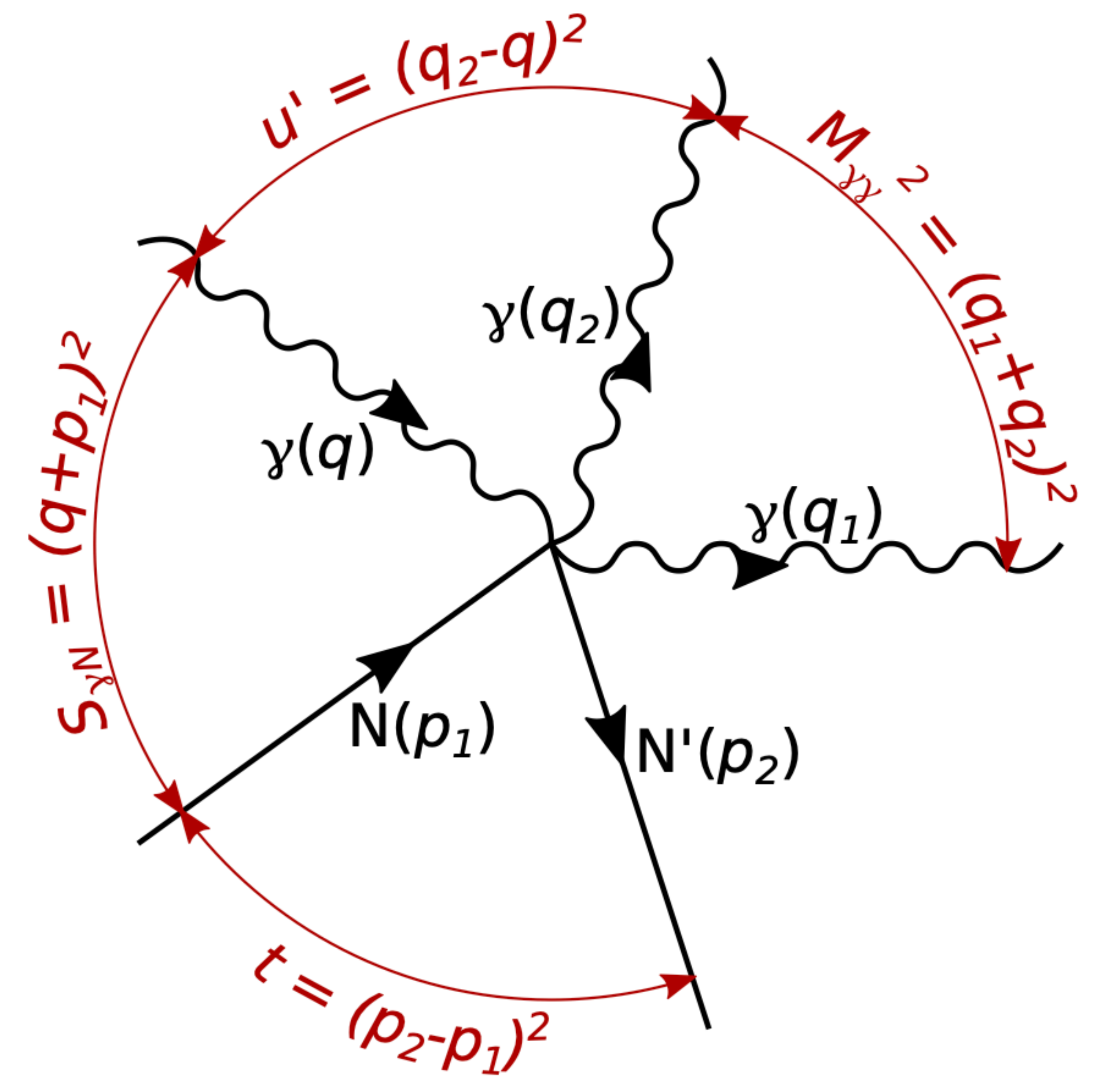


Exclusive diphoton photoproduction

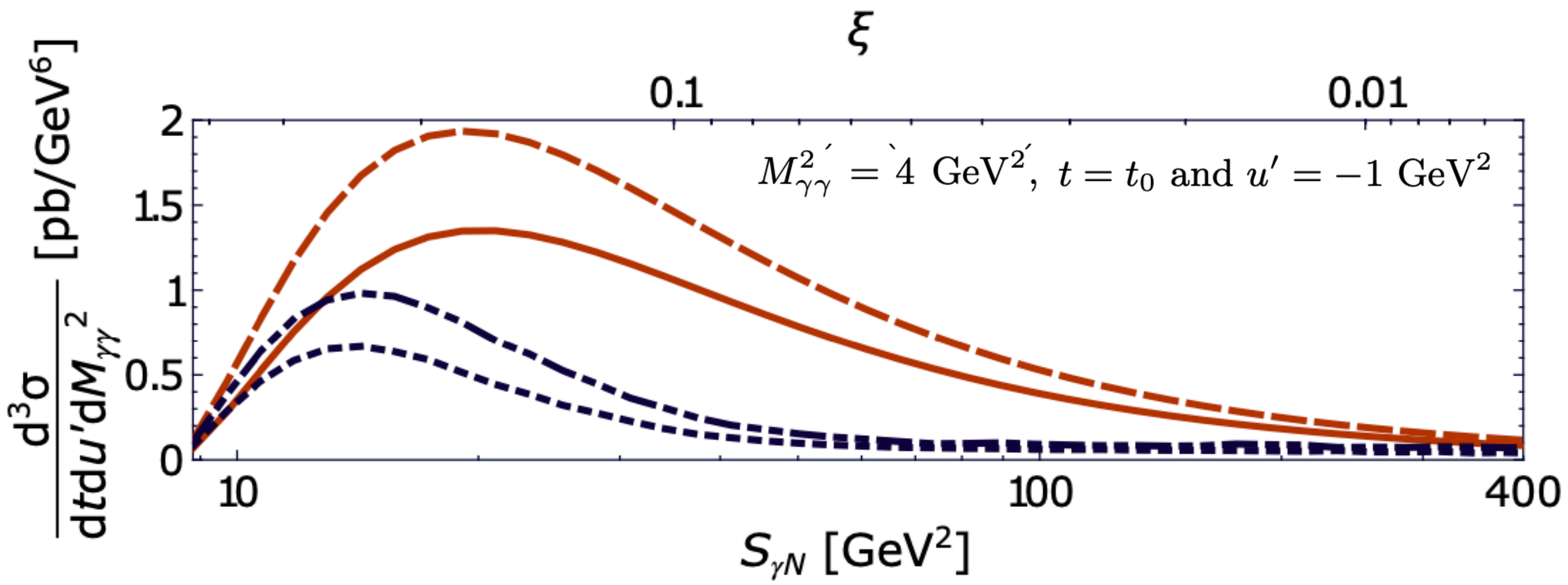
O. Grocholski et al.,
Phys. Rev. D 105 (2022) 9, 094025
Phys. Rev. D 104 (2021) 11, 114006

- Process probes C-odd GPDs
- No contribution of D-term
- No non-perturbative ingredients other than GPDs
- Both LO and NLO description available
- Gluons do not contribute also at NLO

- Description already available in PARTONS (not released yet), soon will be available in EpIC



Cross-section



4. Tools

PARTONS project

B. Berthou et al.,
Eur. Phys. J. C 78 (2018) 6, 478

- PARTONS - open-source framework to study GPDs
  → http://partons.cea.fr
- Come with number of available physics developments implemented
- Written in C++, also available via virtual machines (VirtualBox) and containers (Docker)
- Addition of new developments as easy as possible
- Developed to support effort of GPD community,
  can be used by both theorists and experimentalists
- v3 version of PARTONS is now available!

EpIC MC generator

E. C. Aschenauer et al.,
2205.01762 [hep-ph]

- Novel MC generator called EpIC released
  → *https://pawelsznajder.github.io/epic*

- EpIC is based on PARTONS

- EpIC is characterised by:
  - flexible architecture that utilises a modular programming paradigm
  - a variety of modelling options, including radiative corrections
  - multichannel capability (initial version includes DVCS, TCS and DVMP)

- This is the new tool to be use in the precision era commenced by the new generation of experiments

Summary

- Review of recent results given

- Substantial progress in:

  - understanding fundamental problems, like deconvolution of CFFs, and analysis methods
    → important for extraction of GPDs

  - description of exclusive processes
    → new sources of GPD information

  - modelling of GPD, fulfilling all theory-driven constraints (including positivity)
    → subject not touched enough in the current literature
    → developed in mind for easy inclusion of latticeQCD data

  - addressing the long-standing problem of model dependency of GPDs
    → nontrivial and timely analysis

  - delivering open-source tools for the community
    → to suport both experimentalists and theoreticians

This progress is important for the precision era of GPD extraction allowed by the new generation of experiments