What can we learn on GPDs from lattice QCD

twist-2 & twist-3

Martha Constantinou

Temple University

APCTP Focus Program in Nuclear Physics 2022: Hadron Physics Opportunities with JLab Energy and Luminosity Upgrade.

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Collaborators

Twist-2 GPDs

- **C. Alexandrou**
  Univ. of Cyprus/Cyprus Institute
- **K. Cichy**
  Adam Mickiewicz University
- **K. Hadjiyiannakou**
  Cyprus Institute
- **K. Jansen**
  DESY, Zeuthen
- **A. Scapellato**
  Temple University
- **F. Steffens**
  University of Bonn

Twist-3 GPDs

- **S. Bhattacharya**
  Brookhaven National Lab
- **K. Cichy**
  Adam Mickiewicz University
- **Jack Dodson**
  Temple University
- **A. Metz**
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Relevant publications

Twist-2 GPDs

- *Unpolarized and helicity generalized parton distributions of the proton within lattice QCD*
  C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens,
  PRL 125 (2020) 26, 262001, [arXiv:2008.10573]

- *Transversity GPDs of the proton from lattice QCD*
  C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens,
  PRD 105 (2022) 3, 034501 [arXiv:2108.10789]

Twist-3 GPDs

- *First Lattice QCD Study of Proton Twist-3 GPDs*
  S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens,
  PoS (LATTICE2021) 054, [arXiv:2112.05538]

Generalized Parton Distributions

★ GPDs provide information on spatial distribution of partons inside the hadron, and its mechanical properties (OAM, pressure, etc.)
[M. Burkardt, PRD62 071503 (2000), hep-ph/0005108] [M. V. Polyakov, PLB555 (2003) 57, hep-ph/0210165]

★ GPDs are not well-constrained experimentally (DVCS, DVMP):

- x-dependence extraction is challenging
- independent measurements to disentangle GPDs
- limited coverage of kinematic region
- data on certain GPDs
- indirectly related to GPDs through the Compton FFs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)

★ twist-3 contributions:

- Related to certain spin-orbit correlations
- $\widetilde{H} + \widetilde{G}_2$ related to tomography of F⊥ acting on the active q in DIS off a transversely polarized N right after the virtual photon absorbing
- $G_2(x, \xi, t)$ related to L_q :

$$L_q = -\int_{-1}^{1} dx\, x\, G_2^q(x, \xi, t = 0)$$

Twist-classification of GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

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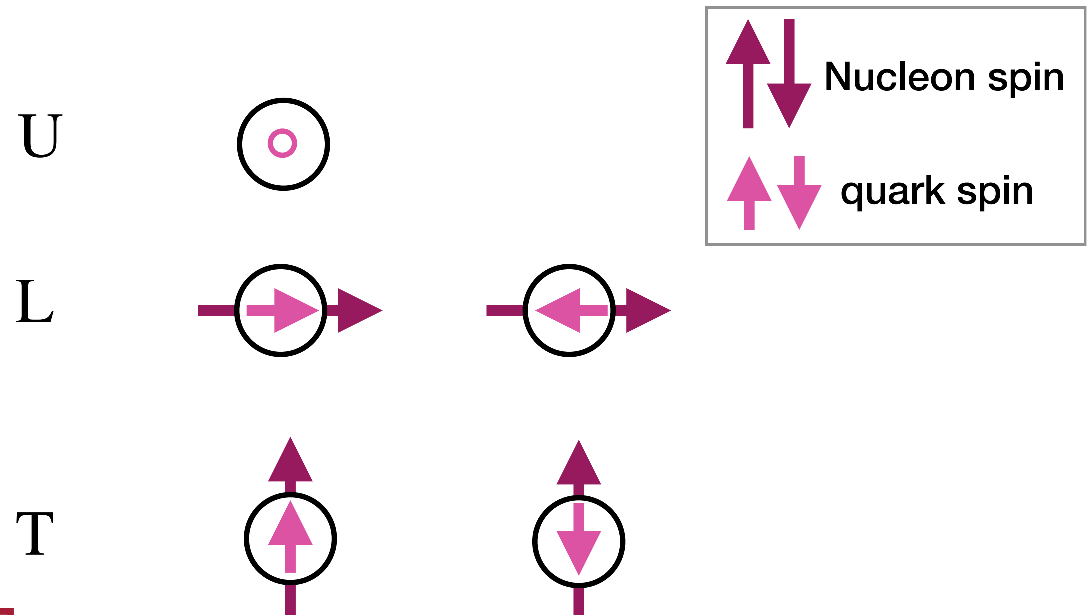
Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

Probabilistic interpretation



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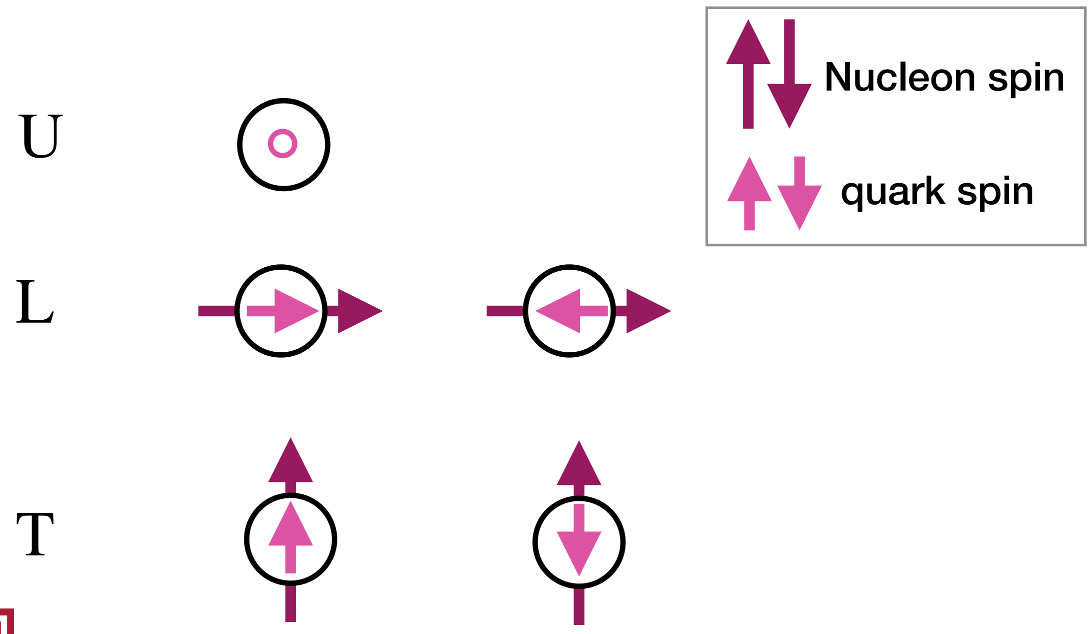
Twist-3 $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

(Selected)

Probabilistic interpretation



- ★ Lack density interpretation, but not-negligible
- ★ Contain info on quark-gluon-quark correlators
- ★ Physical interpretation, e.g., transverse force
- ★ Kinematically suppressed
  Difficult to isolate experimentally
- ★ Theoretically: contain $\delta(x)$ singularities

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma\overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t)\;-i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\}+\frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big)\Bigg\}$$

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local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\\text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big)\Bigg\}$$

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★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\langle N(P')|O_V^{\mu}(x)|N(P)\rangle=\overline{U}(P')\left\{\gamma^\mu H(x,\xi,t)+\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P)+\text{ht}\,,$$

$$\langle N(P')|O_A^{\mu}(x)|N(P)\rangle=\overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t)+\frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P)+\text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle=\overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t)+\frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t)+\frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t)+\frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P)+\text{ht}$$

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Wilson line

$$\langle N(P')|O_V^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5 \widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

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Generalized Form Factors

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★ Advantages

- Frame independence
- Several values of momentum transfer with same computational cost
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- x dependence is integrated out
- GFFs are skewness independence
- Geometrical twist classification (coincides with dynamical twist of scattering processes only at leading order)
- Signal-to-noise ratio decays with the addition of covariant derivatives
- Power-divergent mixing for high Mellin moments (derivatives > 3)
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Form Factors & Generalizations

★ Ultra-local operators (FFS)

$$\langle N(P')|\overline{q}(0)\gamma^\mu q(0)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}F_2(t)\right\}U(P)\,,$$

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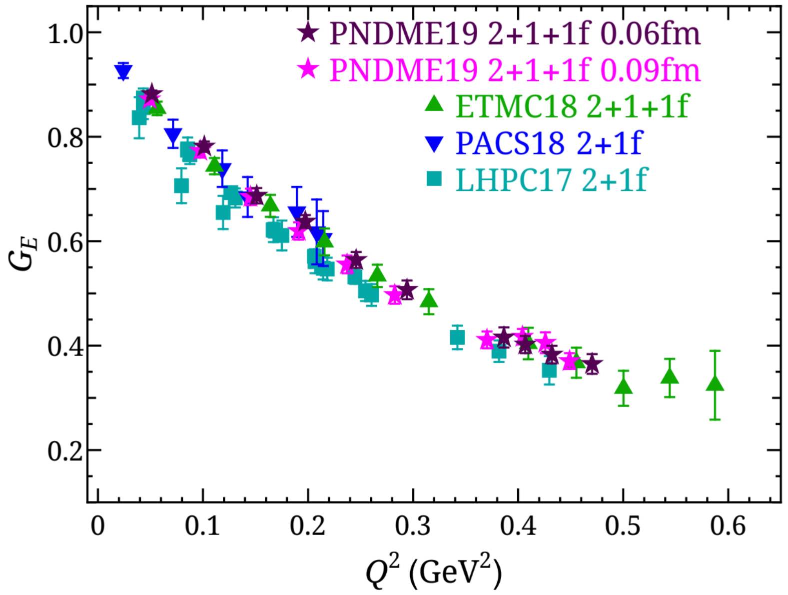

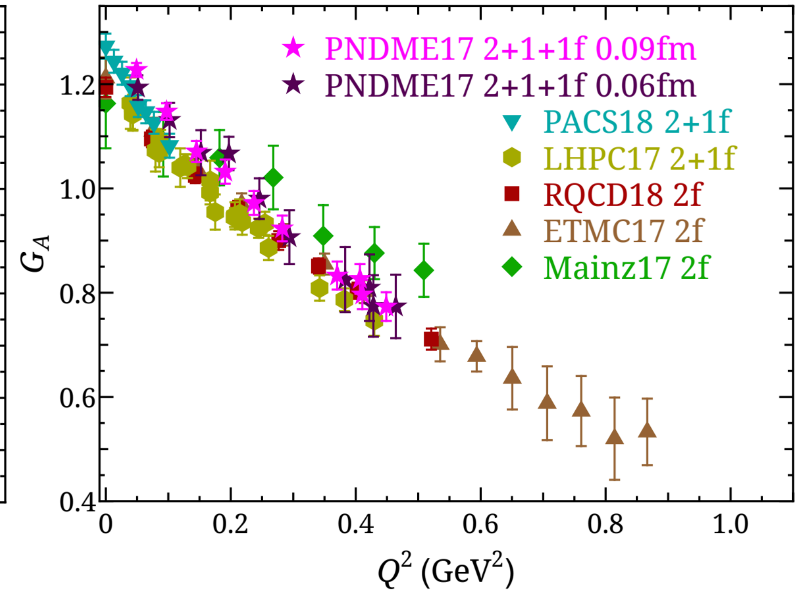

[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908, arXiv:2006.08636]

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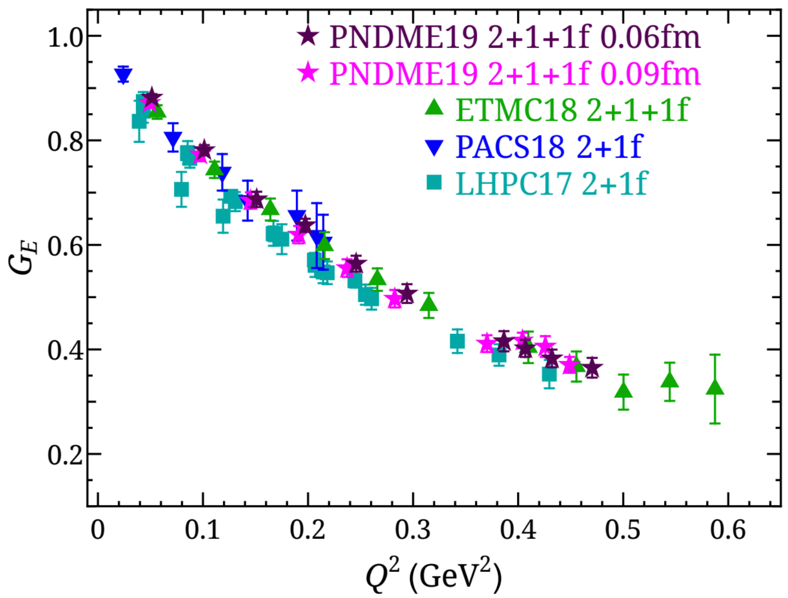

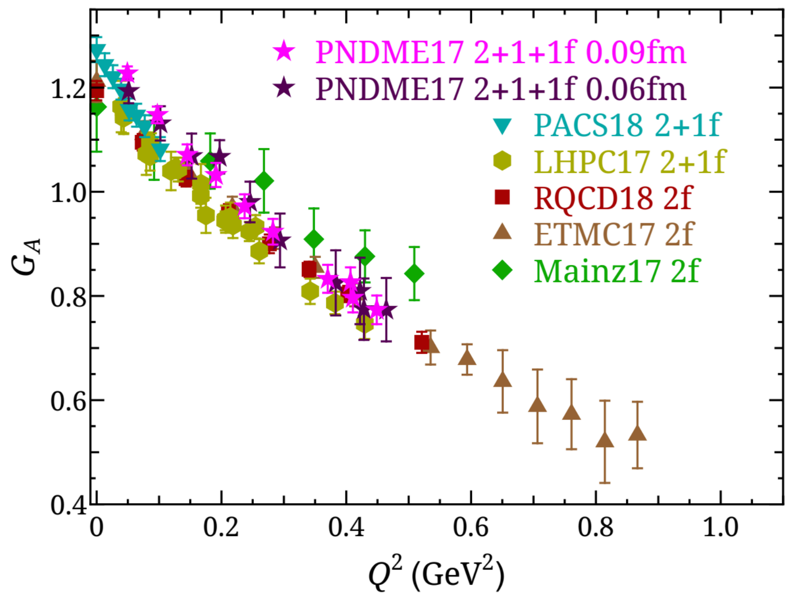

- Simulations at physical point available by multiple groups
- Precision data era
- Towards control of systematic uncertainties

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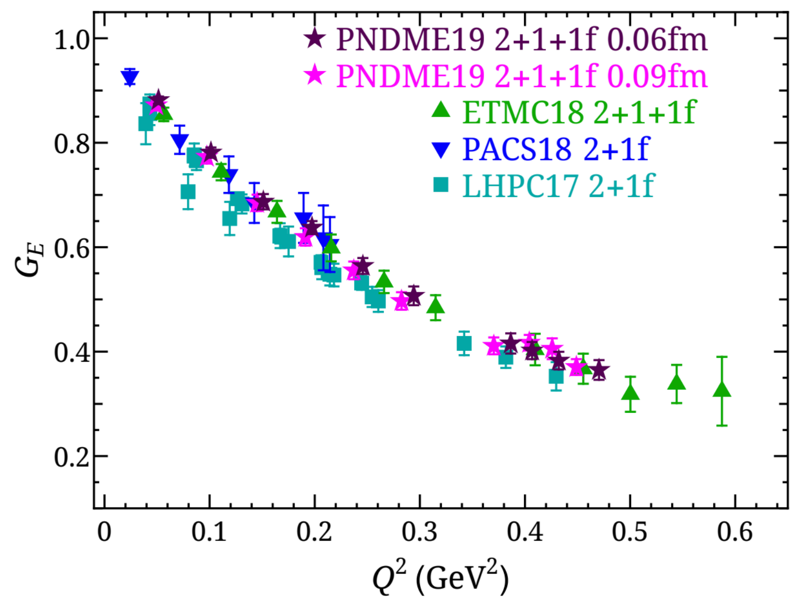

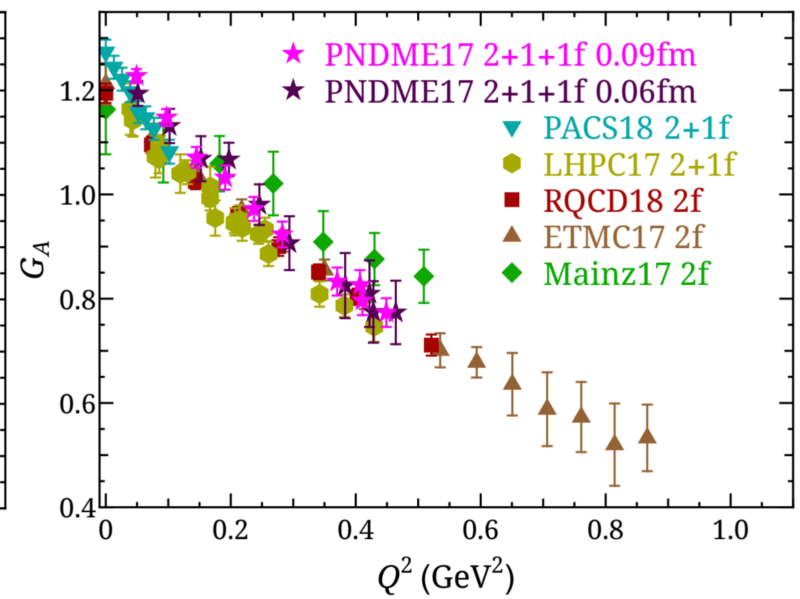

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$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\Big[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}\Big]u_N(p,s)$$

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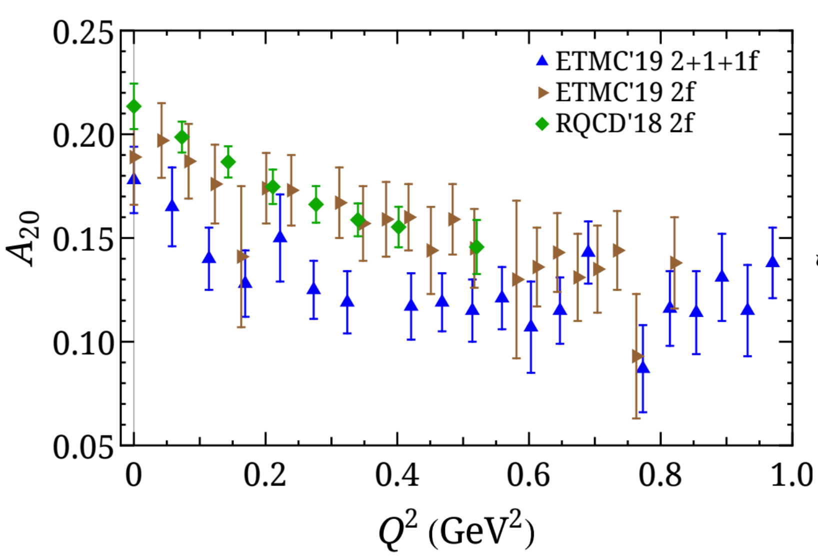

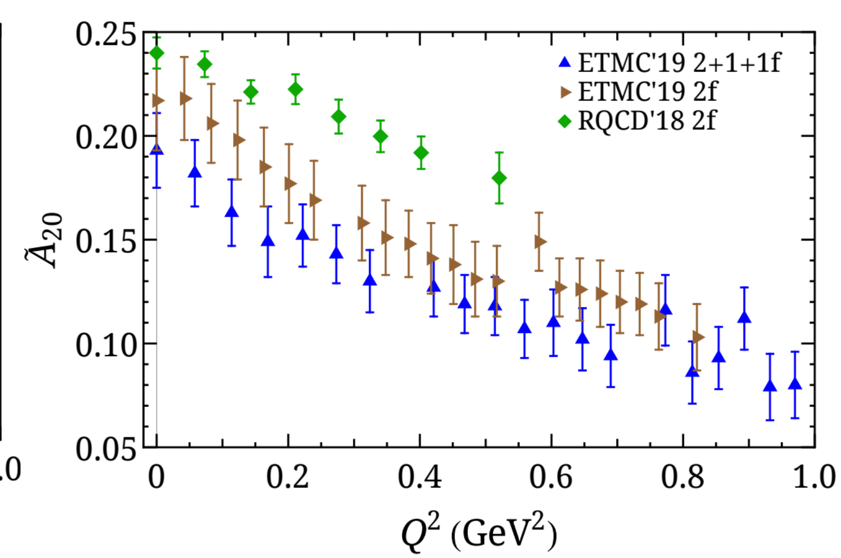

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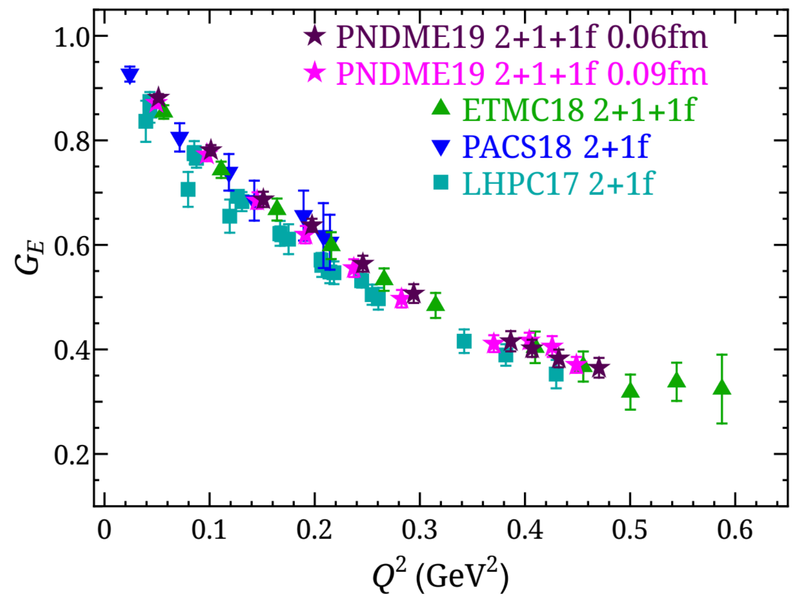

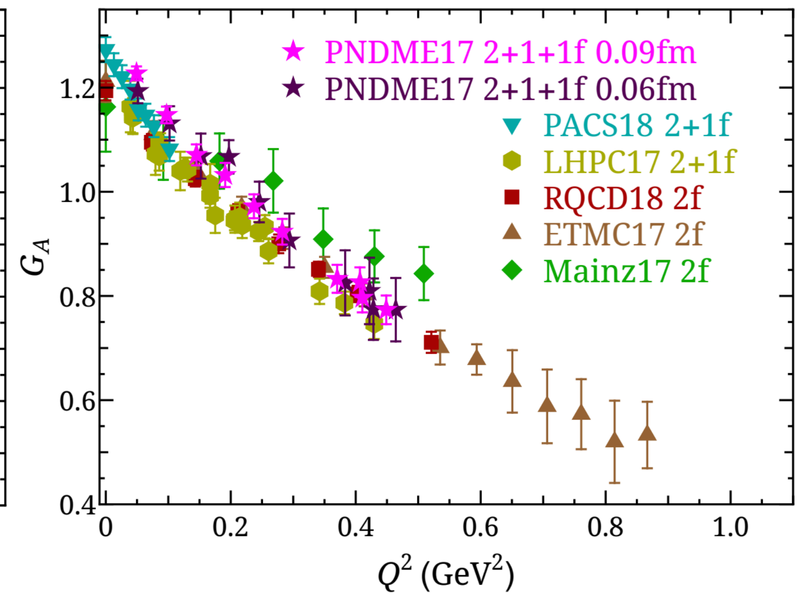

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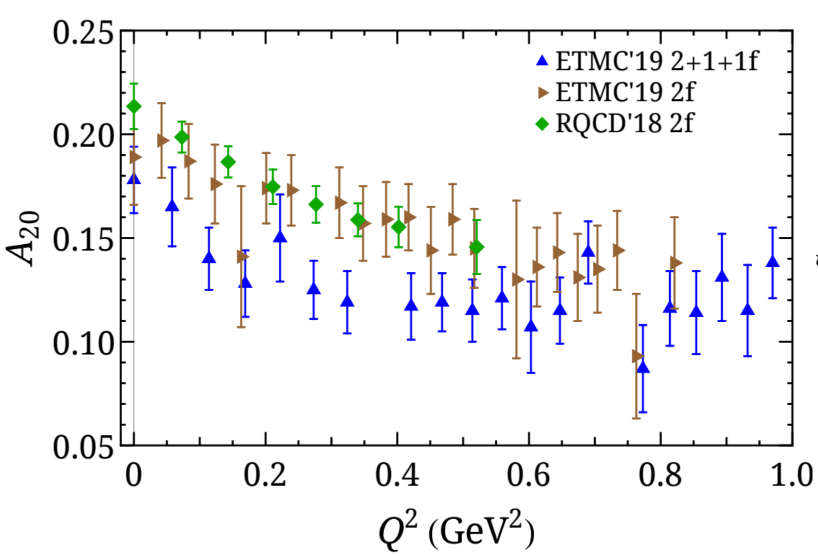

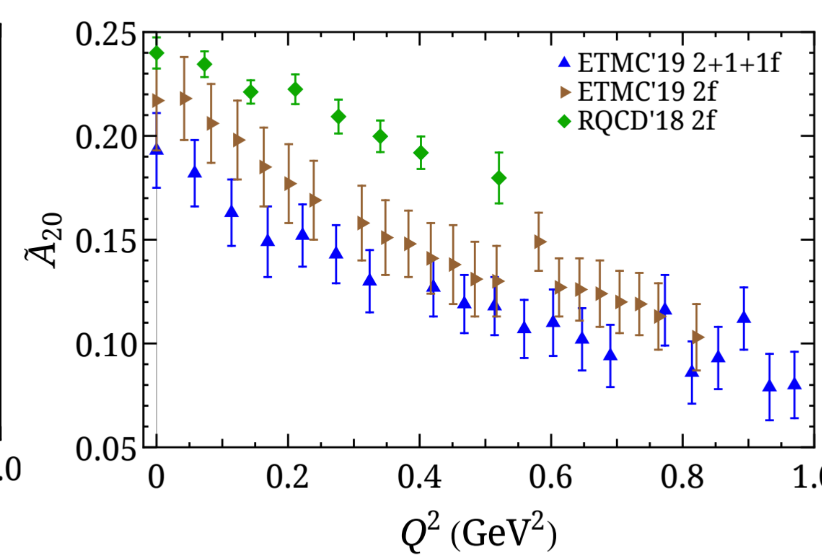

- Lesser studied compared to FFs at physical point
- Decay of signal-to-noise ratio

[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908, arXiv:2006.08636]

GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -\,Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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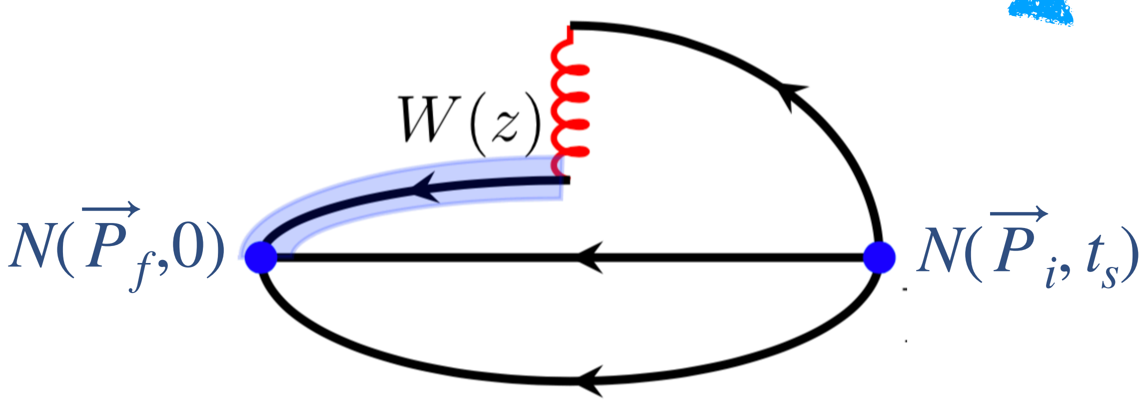

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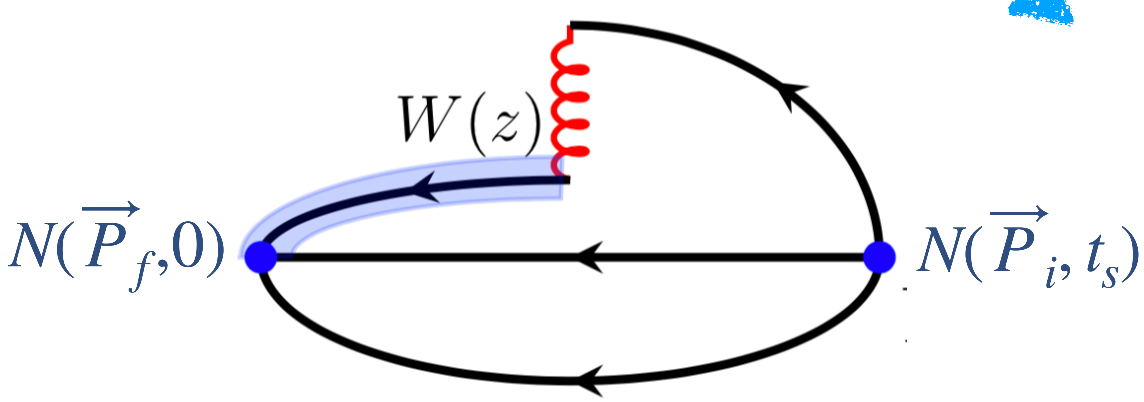
[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$



$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

Variables of the calculation:

- length of the Wilson line (z)
- nucleon momentum boost (P_3)
- momentum transfer (t)
- skewness (ξ)

Access of GPDs on a Euclidean Lattice

Contact with light-cone distributions (PDFs, GPDs, TMDs):

★ Matching procedure in large momentum ET **(LaMET)** X. Ji, Sci. China Phys. Mech. Astron. 57, 1407 (2014), arXiv:1404.6680]

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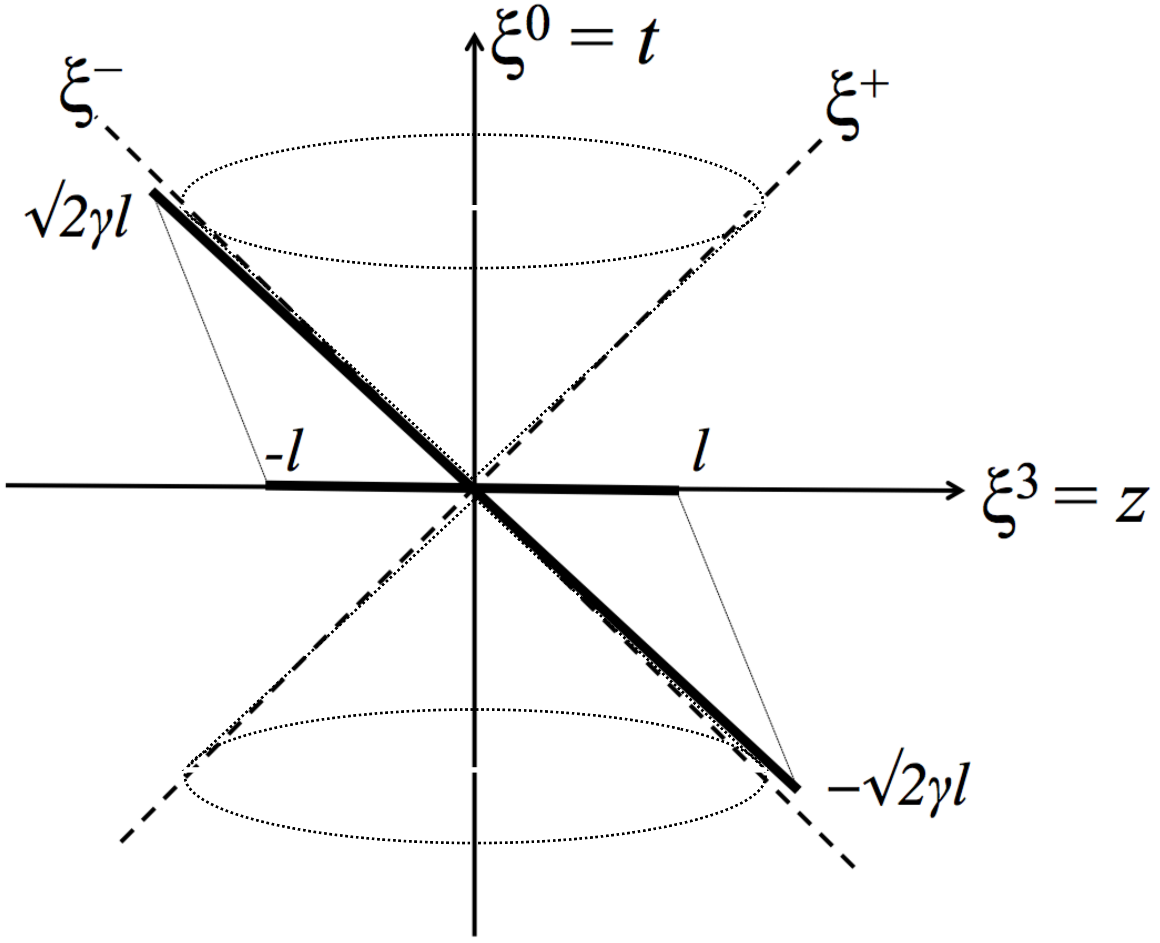

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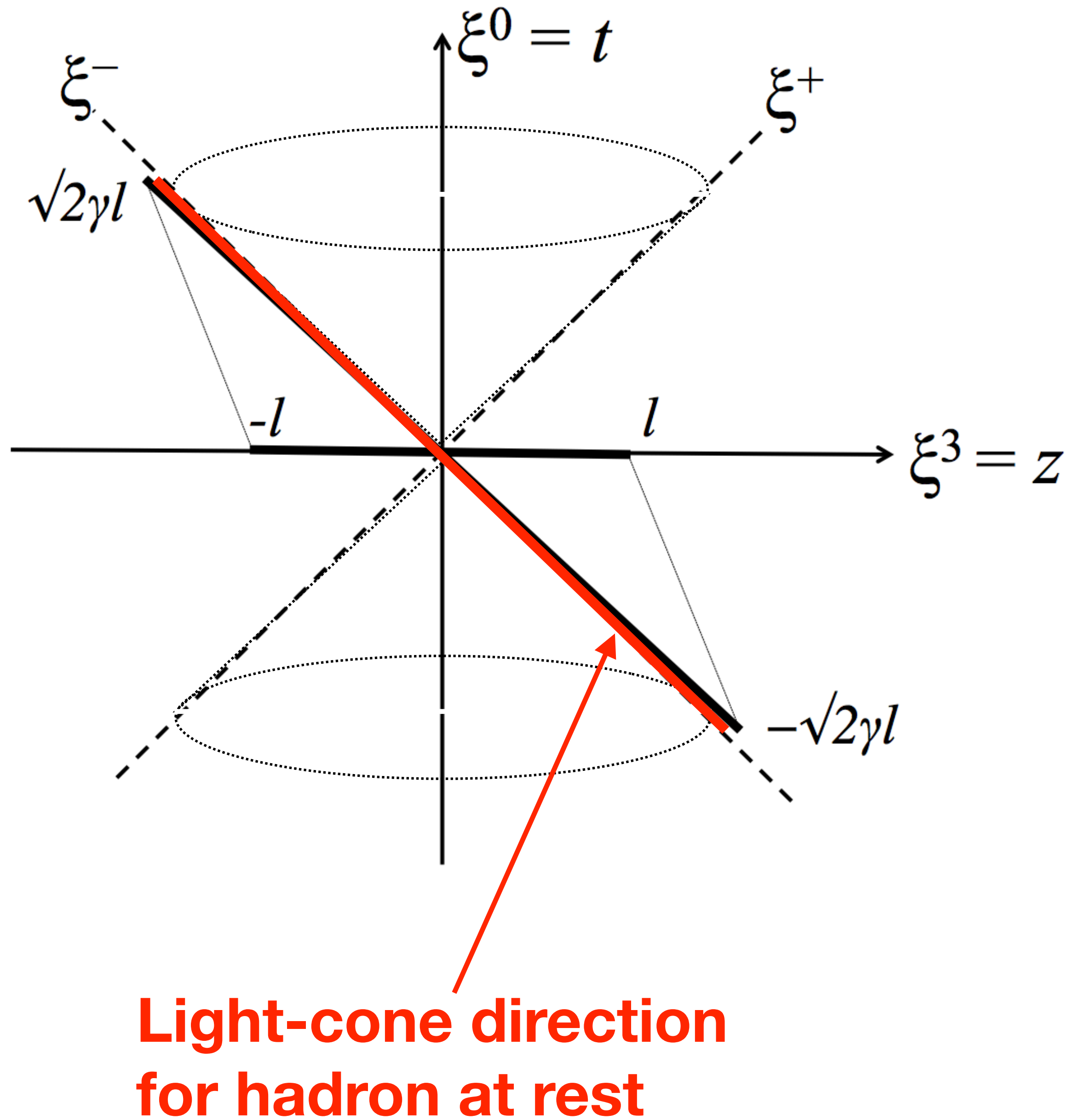

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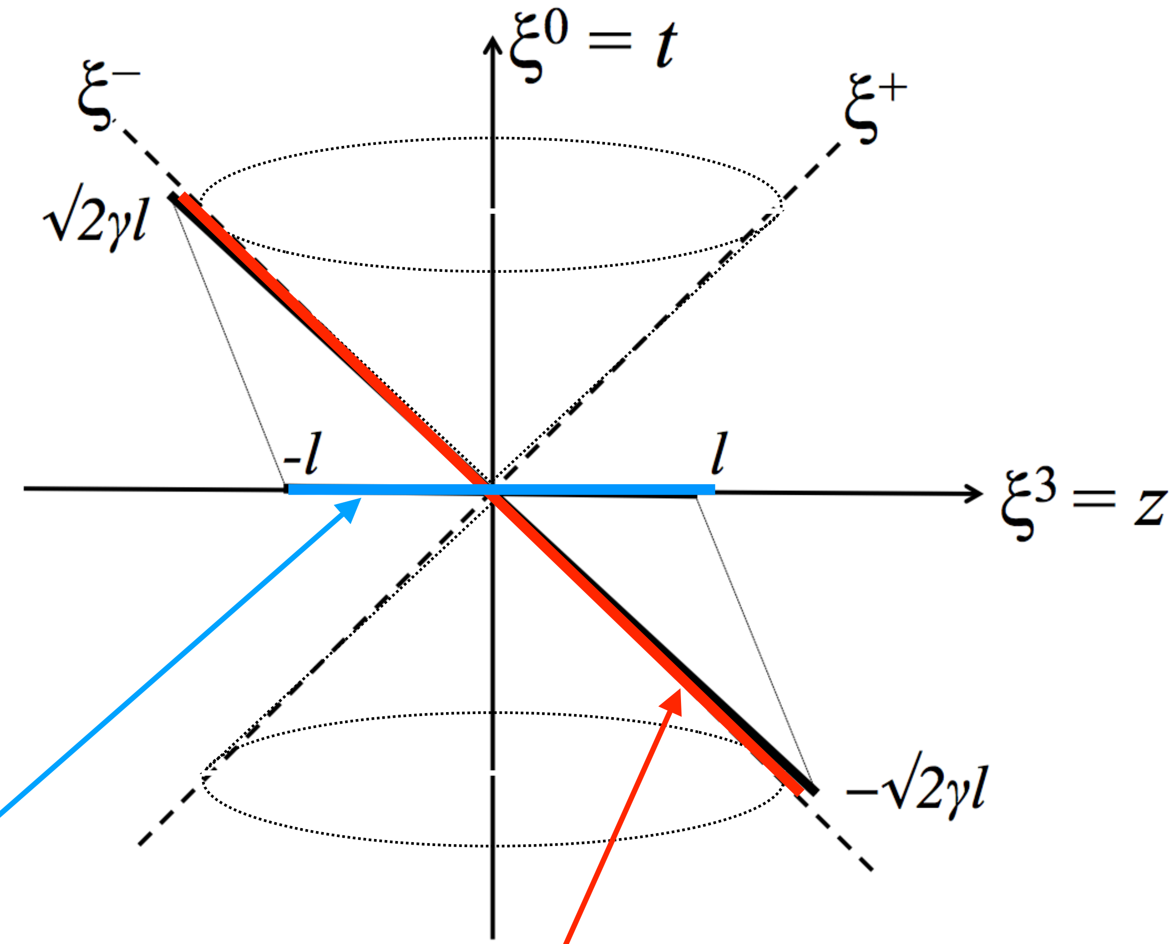

Spatial (z) direction for high-momentum hadron

Light-cone direction for hadron at rest

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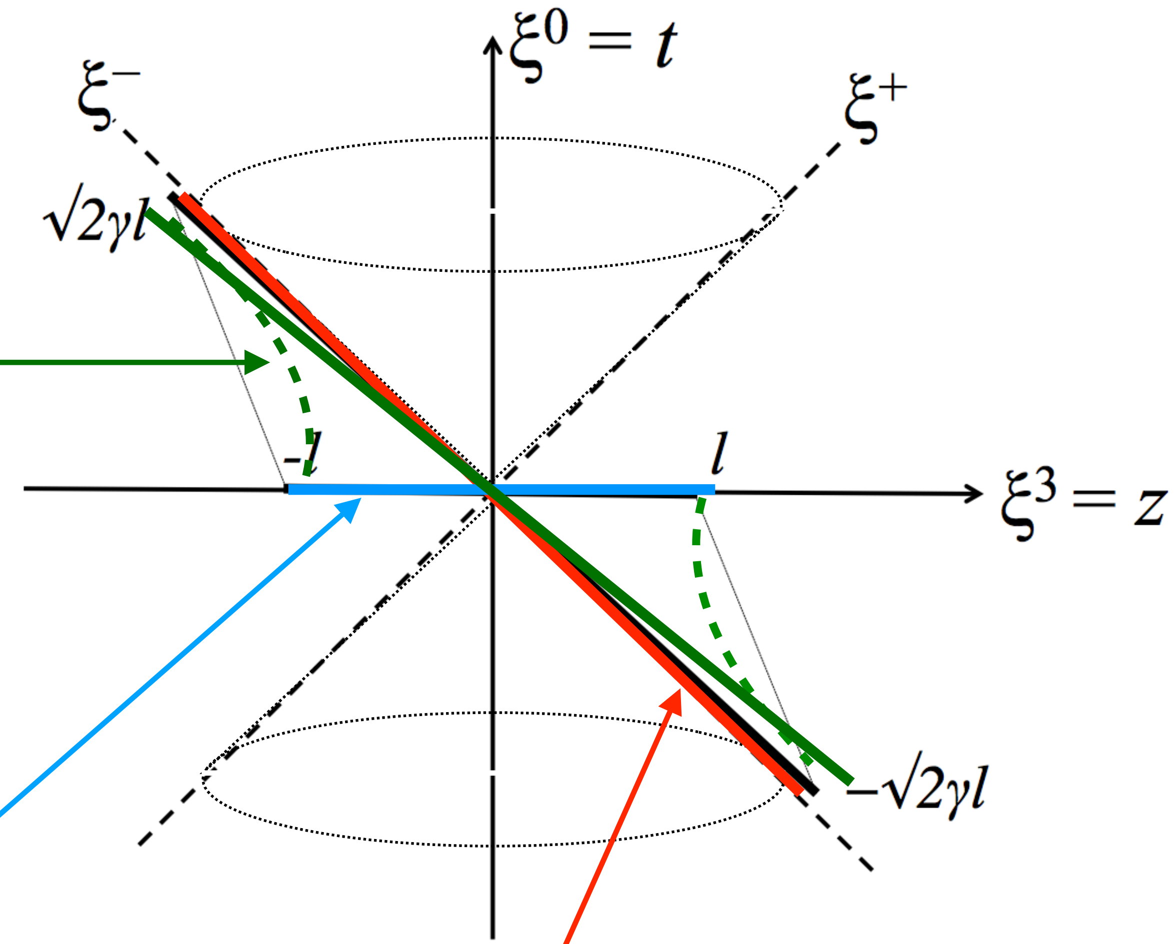
Matching between quasi-GPDs and light-cone GPDs: length increased by the boost factor γ, goes to ∞ in IMF



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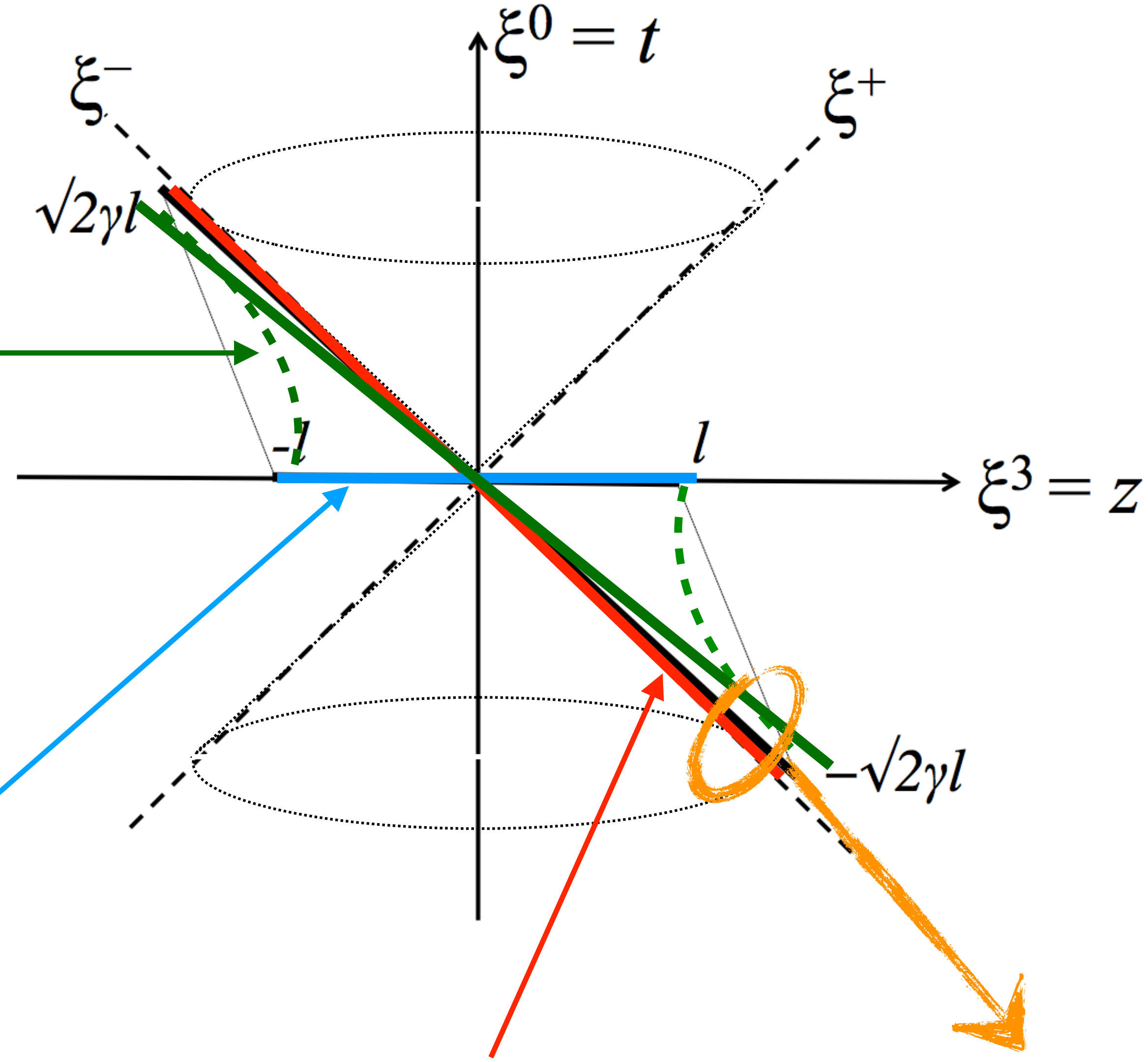


Spatial (z) direction for high-momentum hadron

Light-cone direction for hadron at rest

Difference $\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, m_N^2/P_3^2\right)$ reduced as P_3 increases

quasi distribution approach

hadronic matrix elements

1

quasi distribution approach

$$C^{2pt} = \langle N | N \rangle \qquad C^{3pt} = \langle N | \overline{\psi}(z) \Gamma \mathscr{A}(z,0) \psi(0) | N \rangle$$

hadronic matrix elements — 1

Identification of ground state — 2

quasi distribution approach

$$R_\Gamma(\mathscr{P}_\kappa, \mathbf{P_f}, \mathbf{P_i}; t, \tau) = \frac{C^{3pt}_\Gamma(\mathscr{P}_\kappa, \mathbf{P_f}, \mathbf{P_i}; t, \tau\,)}{C^{2pt}(\mathscr{P}_0, \mathbf{P_f}; t)} \times \sqrt{\frac{C^{2pt}(\mathscr{P}_0, \mathbf{P_i}; t-\tau)C^{2pt}(\mathscr{P}_0, \mathbf{P_f}; \tau)C^{2pt}(\mathscr{P}_0, \mathbf{P_f}; t)}{C^{2pt}(\mathscr{P}_0, \mathbf{P_f}; t-\tau)C^{2pt}(\mathscr{P}_0, \mathbf{P_i}; \tau)C^{2pt}(\mathscr{P}_0, \mathbf{P_i}; t)}} \xrightarrow[\tau \gg a]{t-\tau \gg a} h_{\mathcal{O},\mathscr{P}}(z, t, \xi, P_3)$$

hadronic matrix elements
1

Identification of ground state
2

quasi distribution approach

Renormalization
3

$$h^R_{\mathcal{O},\mathcal{P}}(z,t,\xi,P_3,\mu) = Z_{\mathcal{O}}(z,\mu)\, h_{\mathcal{O},\mathcal{P}}(z,t,\xi,P_3)$$

[M. Constantinou , H. Panagopoulos, Phys. Rev. D96, 054506 (2017), arXiv:1705.11193] hadronic matrix elements

1

Identification of ground state

2

quasi distribution approach

3

Renormalization

4

"form factors" disentanglement

$$h^R_{\gamma^\mu,\mathscr{P}}(z,t,\xi,P_3,\mu) = = \langle\langle\gamma_\mu\rangle\rangle F_H(z,P_3,t,\xi) - i\frac{\langle\langle\sigma_{\rho\mu}\rangle\rangle\, Q_\rho}{2m} F_E(z,P_3,t,\xi)$$

$$h^R_{\gamma^\mu\gamma^5,\mathscr{P}}(z,t,\xi,P_3,\mu) = \langle\langle\gamma_\mu\gamma_5\rangle\rangle F_{\widetilde{H}}(z,P_3,t,\xi) - i\langle\langle\gamma_5\rangle\rangle\frac{Q_\mu}{2m} F_{\widetilde{E}}(z,P_3,t,\xi)$$

quasi distribution approach

1 hadronic matrix elements

2 Identification of ground state

3 Renormalization

4 "form factors" disentanglement

5 x-dependence reconstruction

$$H_q(x, t, \xi, \mu, P_3) = \int \frac{dz}{4\pi} e^{-ixP_3 z} F_H(z, P_3, t, \xi, \mu)$$

In this work: Backus-Gilbert

hadronic matrix elements — 1

Identification of ground state — 2

Renormalization — 3

"form factors" disentanglement — 4

x-dependence reconstruction — 5

Matching to light-cone GPDs — 6

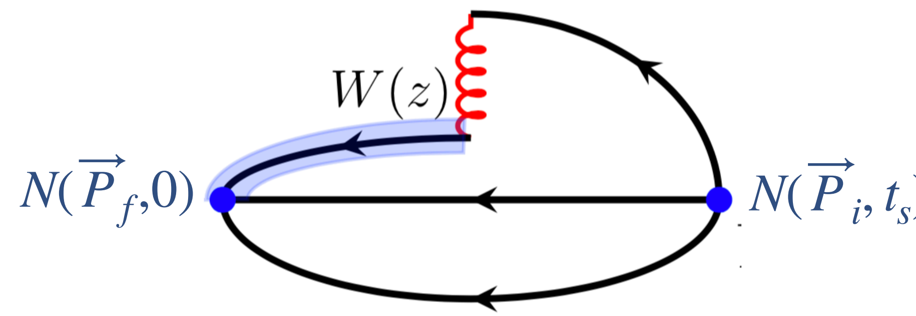
quasi distribution approach

$$H_q(x,t,\xi,\mu_0,(\mu_0)_3,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C_H\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{yP_3},\frac{(\mu_0)_3}{yP_3},r\right) F_H(y,t,\xi,\mu) + \mathcal{O}\left(\frac{m^2}{P_3^2},\frac{t}{P_3^2},\frac{\Lambda^2_{\rm QCD}}{x^2P_3^2}\right)$$

Parameters of calculation

★ Nf=2+1+1 twisted mass (TM) fermions & clover term

Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm



★ Proton Momentum:

P_3 [GeV]	$\vec{Q}\times\frac{L}{2\pi}$	$-t$ [GeV^2]	ξ	$N_{\rm confs}$	$N_{\rm meas}$
0.83	(0,2,0)	0.69	0	519	4152
1.25	(0,2,0)	0.69	0	1315	42080
1.67	(0,2,0)	0.69	0	1753	112192
1.25	(0,2,2)	1.39	1/3	417	40032
1.25	(0,2,-2)	1.39	-1/3	417	40032

★ Excited states: T_{sink}=1.12 fm

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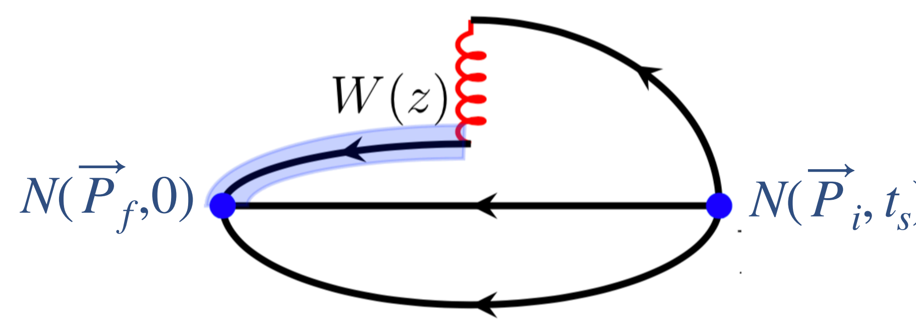

★ Proton Momentum:

zero skewness (first three rows)

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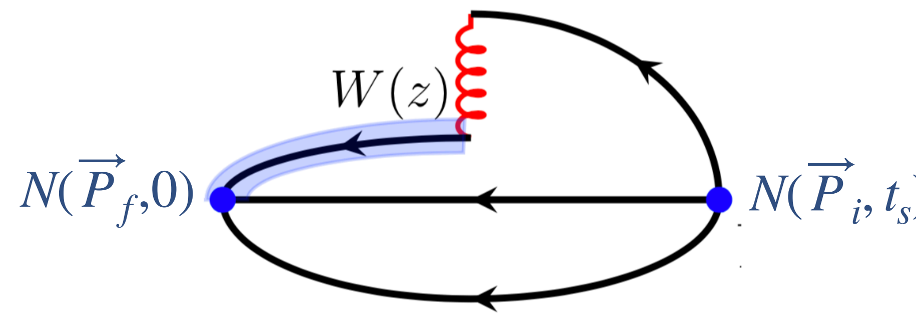

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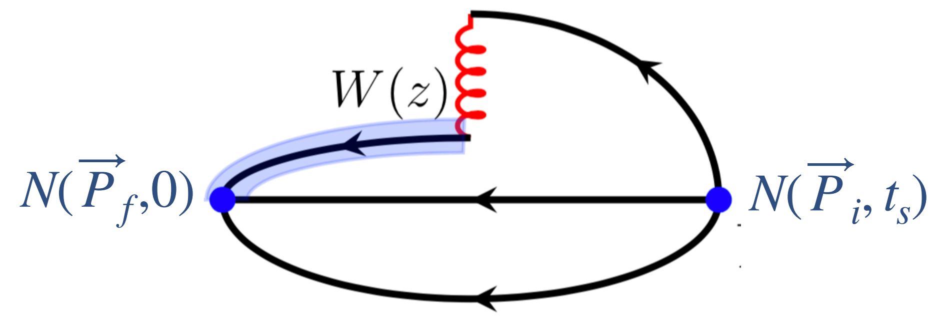

★ Proton Momentum:

zero skewness → rows 1–3; nonzero skewness → rows 4–5

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★ Excited states: $T_{\rm sink}$=1.12 fm

Decomposition of matrix elements (twist-2)

$$\Pi_{\gamma^0}(\mathcal{P}_0; P_f, P_i) = C\Bigg[F_H(Q^2)\left(\frac{E_f E_i}{2m^2} + \frac{E_f+E_i}{4m} + \frac{P_{f\,\rho} P_{i\,\rho}}{4m^2} + \frac{1}{4}\right)$$
$$+F_E(Q^2)\left(P_{f\,\rho} P_{i\,\rho}\left(\frac{E_f+E_i}{8m^3} + \frac{1}{4m^2}\right) - \frac{(E_f-E_i)^2}{8m^2} + \frac{E_f+E_i}{8m} + \frac{1}{4}\right)\Bigg],$$

$$\Pi_{\gamma^0}(\mathcal{P}_j; P_f, P_i) = i\,\epsilon_{j\,0\,\rho\,\tau} C\left[F_H(Q^2)\frac{P_{f\,\rho} P_{i\,\tau}}{4m^2} + F_E(Q^2)\left(\frac{(E_f+E_i)P_{f\,\rho} P_{i\,\tau}}{8m^3} + \frac{P_{f\,\rho} P_{i\,\tau} - P_{f\,\tau} P_{i\,\rho}}{8m^2}\right)\right],$$

$$\Pi_{\gamma^3\gamma^5}(\mathcal{P}_j; P_f, P_i) = i\,C\Bigg[F_{\widetilde{H}}(Q^2)\left(\delta_{j3}\left(\frac{E_f+E_i}{4m} - \frac{P_{f\,\rho} P_{i\,\rho}}{4m^2} + \frac{1}{4}\right) + \frac{P_{i\,3} P_{f\,j} + P_{f\,3} P_{i\,j}}{4m^2}\right)$$
$$-F_{\widetilde{E}}(Q^2)\frac{(P_{f\,3} - P_{i\,3})(-E_f P_{i\,j} + E_i P_{f\,j} + m(P_{f\,j} - P_{i\,j}))}{8m^3}\Bigg],$$

Decomposition of matrix elements (twist-2)

Unpolarized

$$\Pi_{\gamma^0}(\mathcal{P}_0;P_f,P_i) = C\Bigg[F_H(Q^2)\left(\frac{E_f E_i}{2m^2}+\frac{E_f+E_i}{4m}+\frac{P_{f\,\rho}\,P_{i\,\rho}}{4m^2}+\frac{1}{4}\right)$$

$$+F_E(Q^2)\left(P_{f\,\rho}\,P_{i\,\rho}\left(\frac{E_f+E_i}{8m^3}+\frac{1}{4m^2}\right)-\frac{(E_f-E_i)^2}{8m^2}+\frac{E_f+E_i}{8m}+\frac{1}{4}\right)\Bigg],$$

$$\Pi_{\gamma^0}(\mathcal{P}_j;P_f,P_i) = i\,\epsilon_{j\,0\,\rho\,\tau}C\left[F_H(Q^2)\frac{P_{f\,\rho}\,P_{i\,\tau}}{4m^2}+F_E(Q^2)\left(\frac{(E_f+E_i)P_{f\,\rho}\,P_{i\,\tau}}{8m^3}+\frac{P_{f\,\rho}\,P_{i\,\tau}-P_{f\,\tau}\,P_{i\,\rho}}{8m^2}\right)\right],$$

$$\Pi_{\gamma^3\gamma^5}(\mathcal{P}_j;P_f,P_i) = i\,C\Bigg[F_{\widetilde{H}}(Q^2)\left(\delta_{j3}\left(\frac{E_f+E_i}{4m}-\frac{P_{f\,\rho}\,P_{i\,\rho}}{4m^2}+\frac{1}{4}\right)+\frac{P_{i\,3}\,P_{f\,j}+P_{f\,3}\,P_{i\,j}}{4m^2}\right)$$

$$-F_{\widetilde{E}}(Q^2)\frac{(P_{f\,3}-P_{i\,3})(-E_f P_{i\,j}+E_i P_{f\,j}+m(P_{f\,j}-P_{i\,j}))}{8m^3}\Bigg],$$

Decomposition of matrix elements (twist-2)

Unpolarized

$$\Pi_{\gamma^0}(\mathcal{P}_0; P_f, P_i) = C\Bigg[F_H(Q^2)\left(\frac{E_f E_i}{2m^2} + \frac{E_f + E_i}{4m} + \frac{P_{f\,\rho} P_{i\rho}}{4m^2} + \frac{1}{4}\right)$$
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Helicity

$$\Pi_{\gamma^3\gamma^5}(\mathcal{P}_j; P_f, P_i) = i\,C\Bigg[F_{\widetilde{H}}(Q^2)\left(\delta_{j3}\left(\frac{E_f + E_i}{4m} - \frac{P_{f\,\rho} P_{i\rho}}{4m^2} + \frac{1}{4}\right) + \frac{P_{i3} P_{f\,j} + P_{f\,3} P_{ij}}{4m^2}\right)$$
$$-F_{\widetilde{E}}(Q^2)\frac{(P_{f\,3} - P_{i3})(-E_f P_{ij} + E_i P_{f\,j} + m(P_{f\,j} - P_{ij}))}{8m^3}\Bigg],$$

Decomposition of matrix elements (twist-2)

Unpolarized

$$\Pi_{\gamma^0}(\mathcal{P}_0;P_f,P_i) = C\Bigg[F_H(Q^2)\left(\frac{E_f E_i}{2m^2}+\frac{E_f+E_i}{4m}+\frac{P_{f\,\rho}\,P_{i\rho}}{4m^2}+\frac{1}{4}\right)$$
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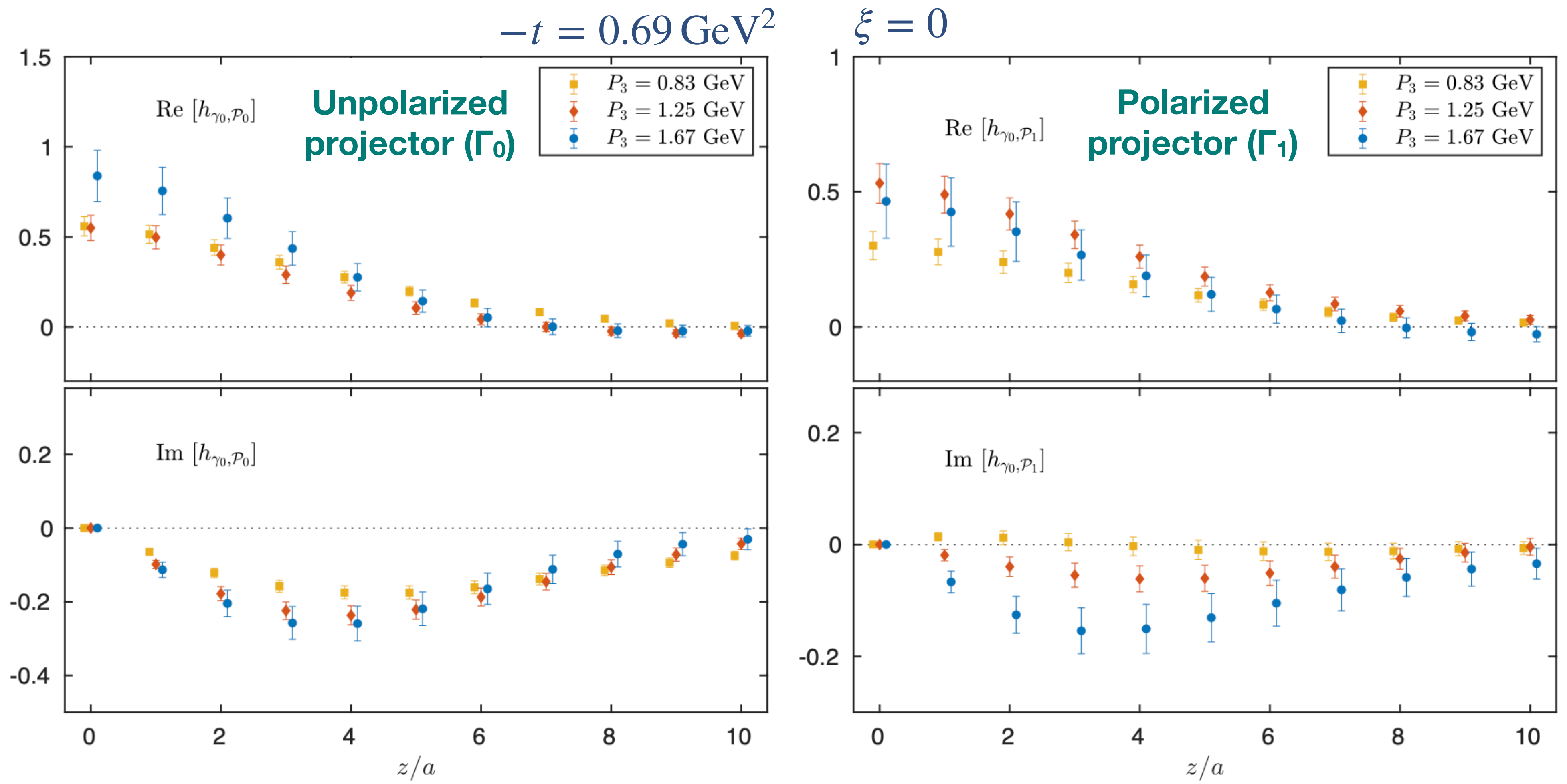
$$-F_{\tilde{E}}(Q^2)\frac{(P_{f\,3}-P_{i3})(-E_f P_{ij}+E_i P_{f\,j}+m(P_{f\,j}-P_{ij}))}{8m^3}\Bigg],$$

- **Kinematic factors defined by calculation setup**
- **Decomposition similar to the decomposition of form factors**
- **Zero skewness: $F_{\tilde{E}}(Q^2)$ inaccessible**

Example: Matrix elements for unpolarized GPDs



Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64

- ★ Statistics compared to 0.83 GeV: x10 (1.25 GeV), x30 (1.78 GeV)
- ★ P_3 = 1.78 GeV : similar contributions from both matrix elements
- ★ P_3 - dependence non-negligible and can propagate to GPDs
- ★ F_E more noisy than F_H

What can we currently do in lattice QCD?

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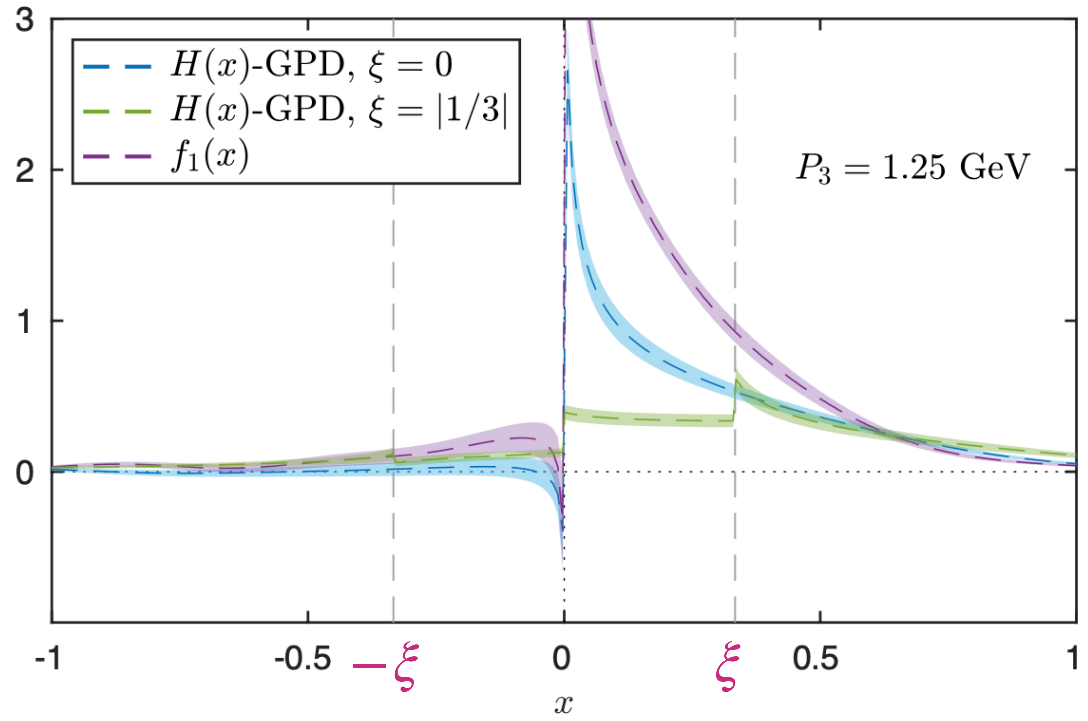

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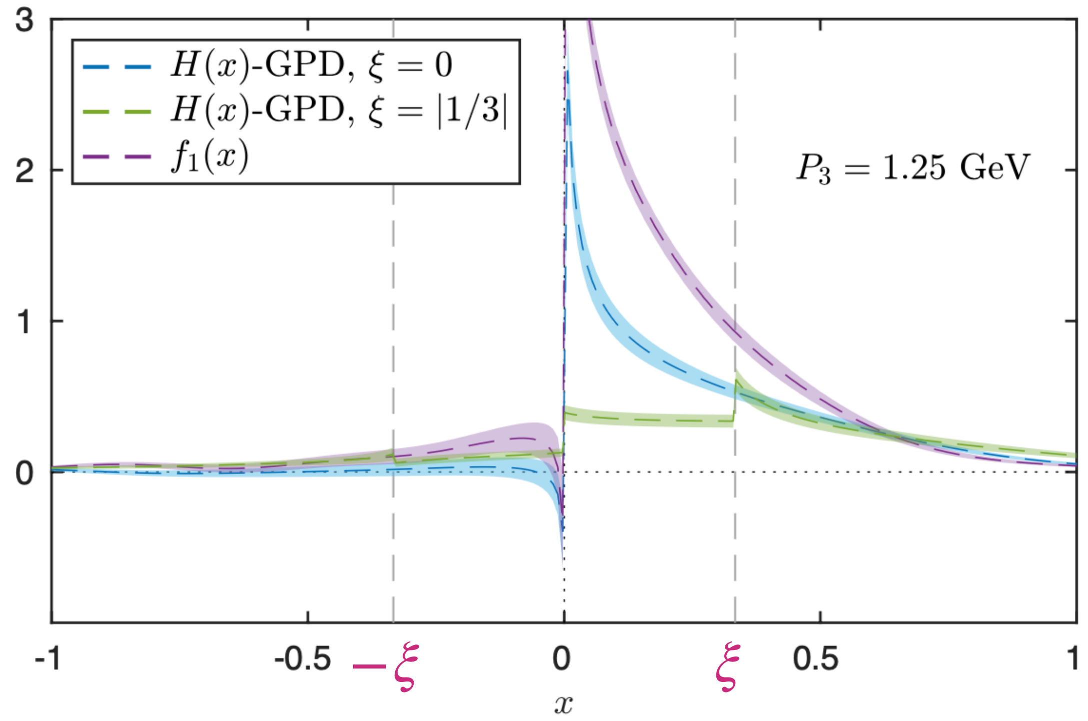

★ ERBL/DGLAP: Qualitative differences

★ $\xi = \pm x$ inaccessible (formalism breaks down)

★ $x \to 1$ region: qualitatively comparison with power counting analysis [F. Yuan, PRD69 (2004) 051501, hep-ph/0311288]

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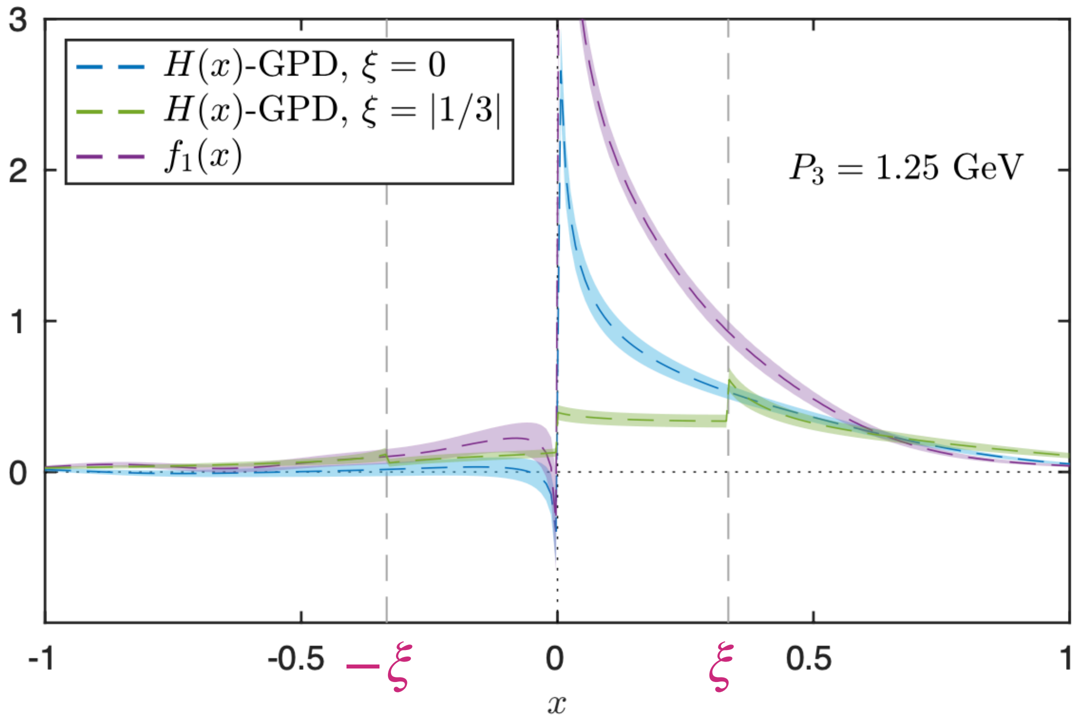

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- ★ $x \to 1$ region: qualitatively comparison with power counting analysis [F. Yuan, PRD69 (2004) 051501, hep-ph/0311288]
  - ✦ t-dependence vanishes at large-x
  - ✦ $H(x,0)$ asymptotically equal to $f_1(x)$

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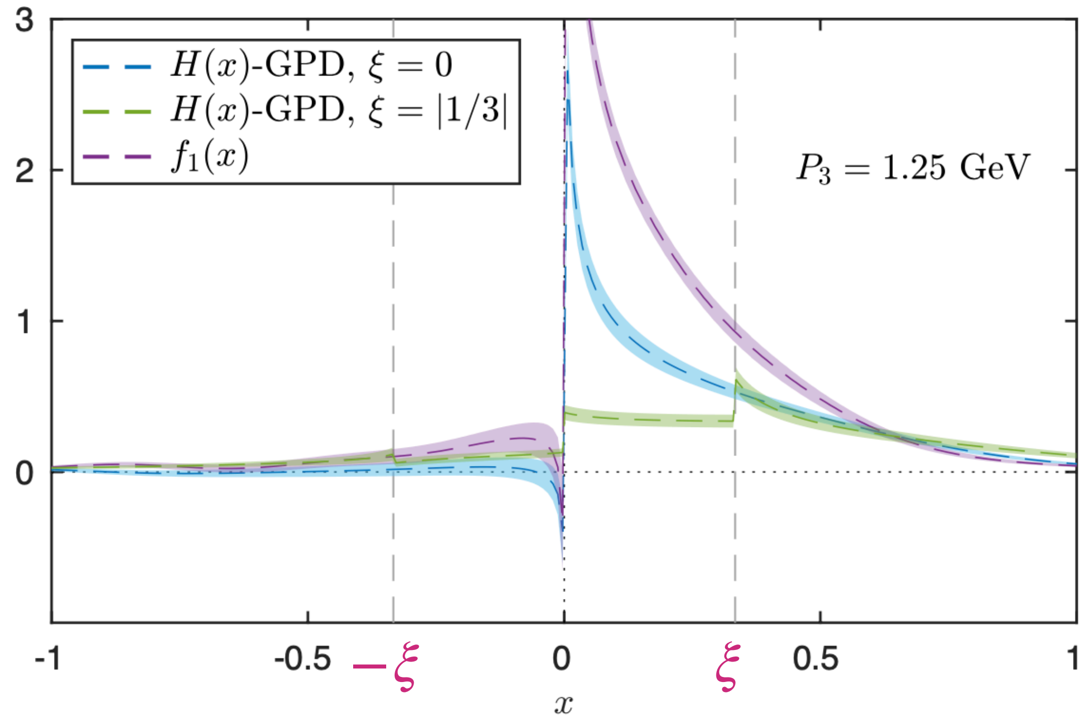

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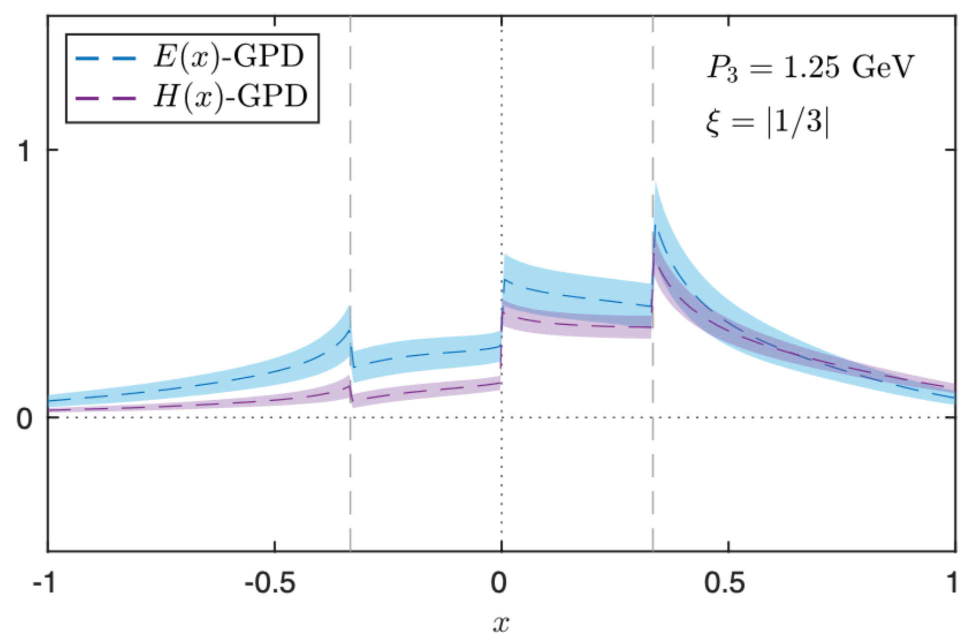

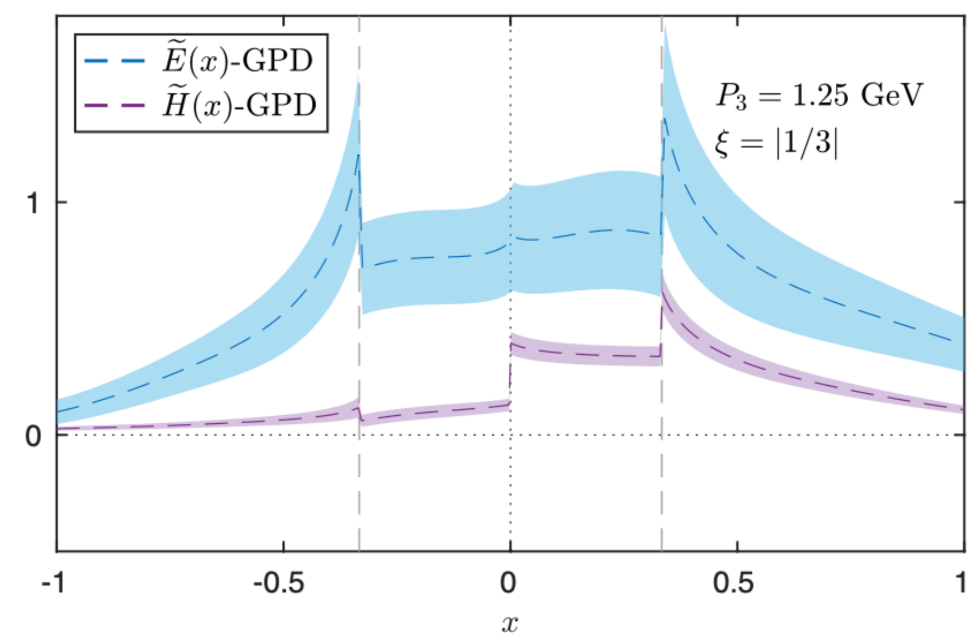

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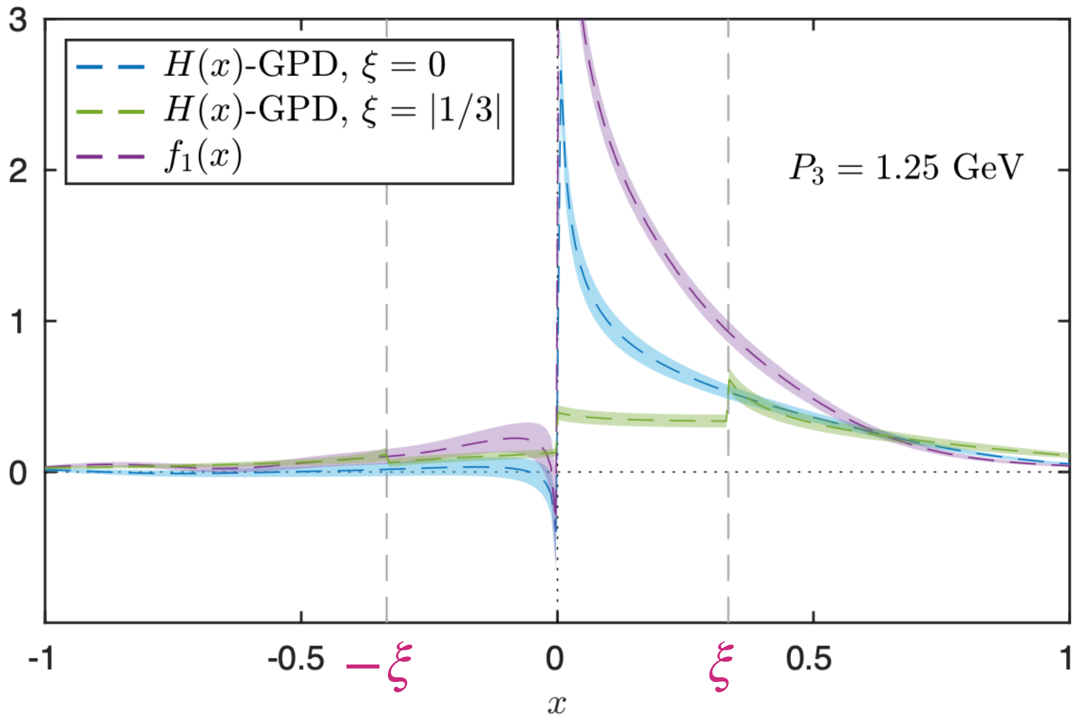

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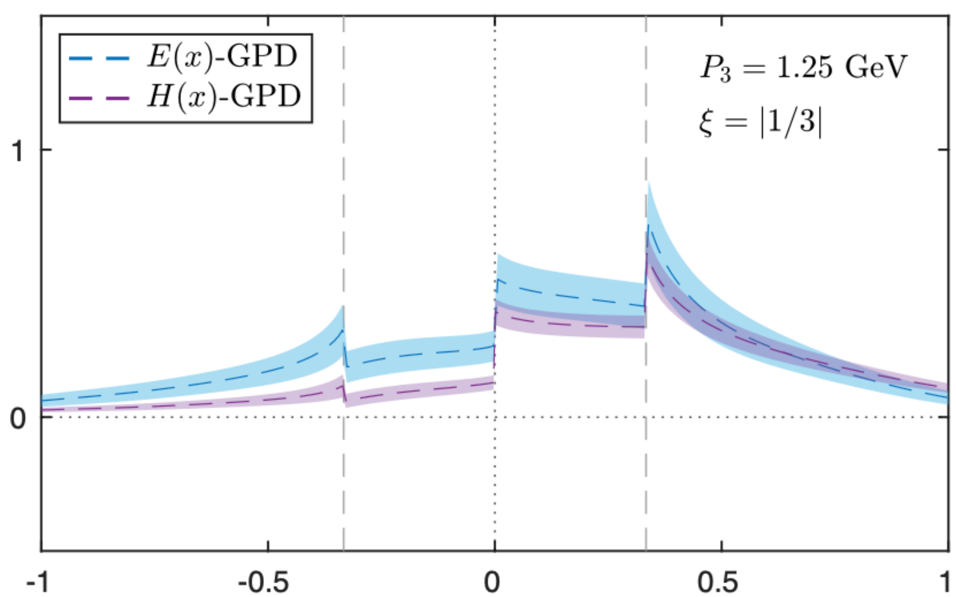
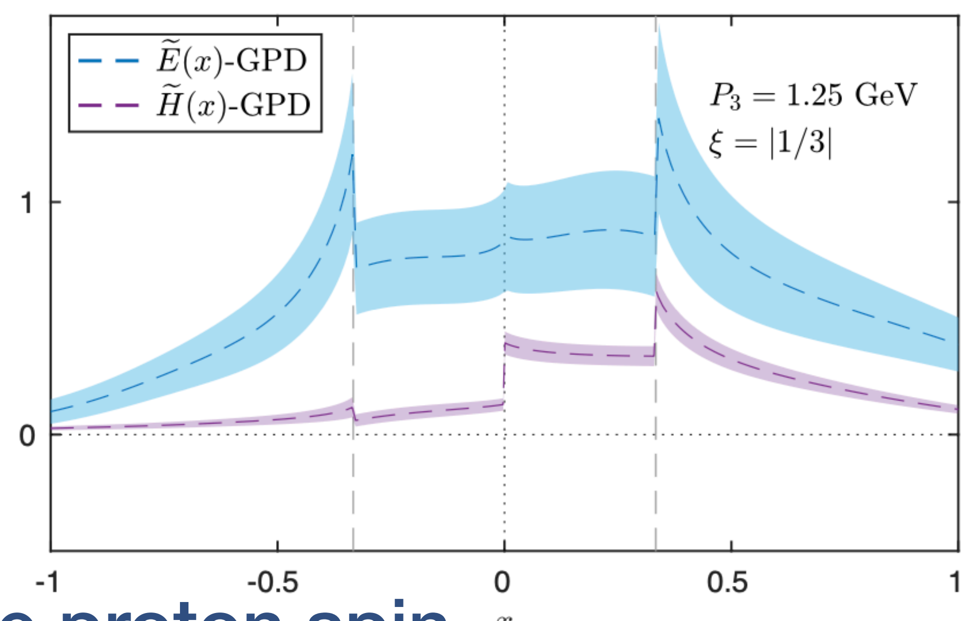
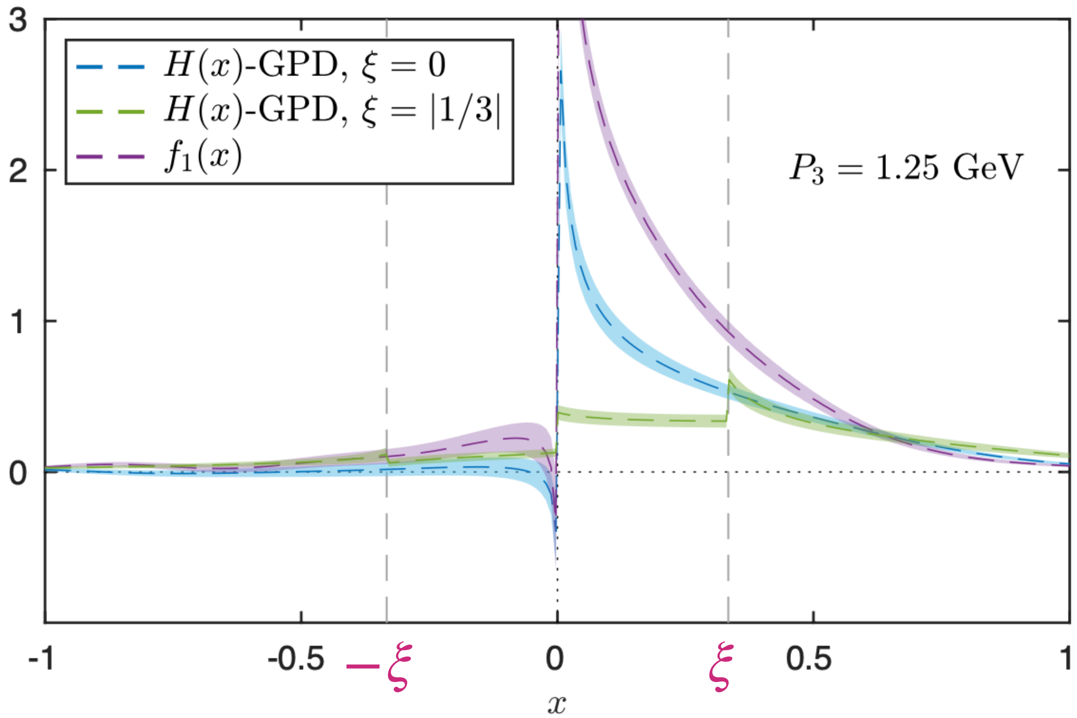




- ★ important contribution in the proton spin

$$\int_{-1}^{+1} dx\, x^2 H^q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_{20}^q(t)\,, \qquad \int_{-1}^{+1} dx\, x^2 E^q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_{20}^q(t)$$

What can we currently do in lattice QCD?

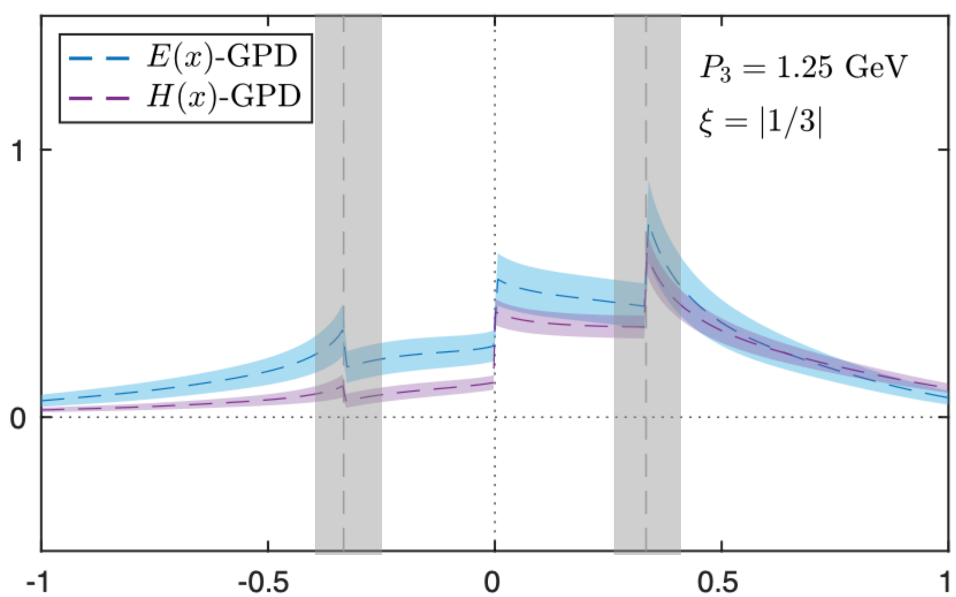


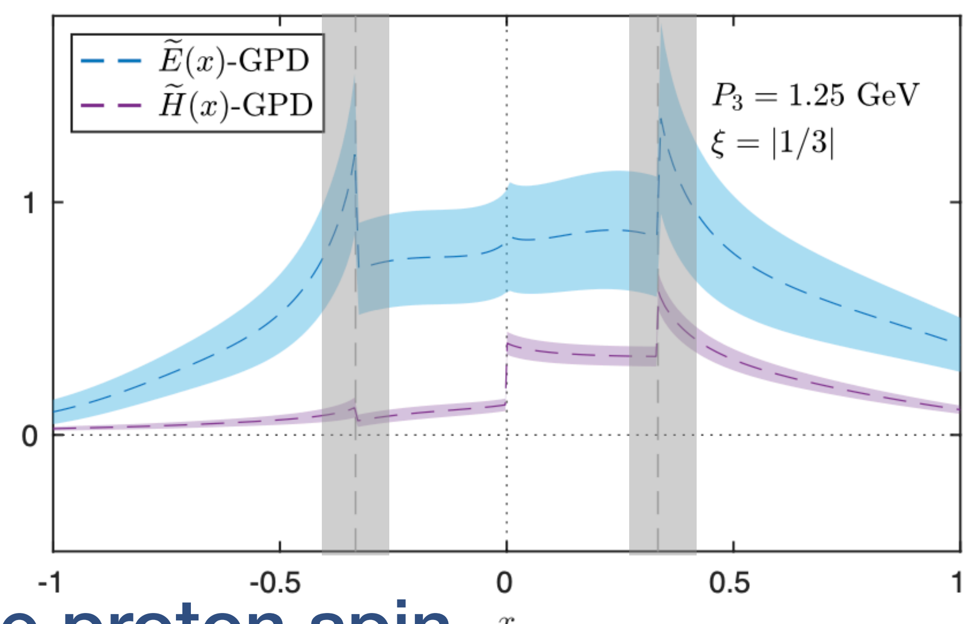
★ ERBL/DGLAP: Qualitative differences

★ $\xi = \pm x$ inaccessible (formalism breaks down)

★ $x \to 1$ region: qualitatively comparison with power counting analysis [F. Yuan, PRD69 (2004) 051501, hep-ph/0311288]

- t-dependence vanishes at large-x
- $H(x,0)$ asymptotically equal to $f_1(x)$





★ important contribution in the proton spin

$$\int_{-1}^{+1} dx\, x^2 H^q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_{20}^q(t)\,, \qquad \int_{-1}^{+1} dx\, x^2 E^q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_{20}^q(t)$$

What can we currently learn from lattice results?

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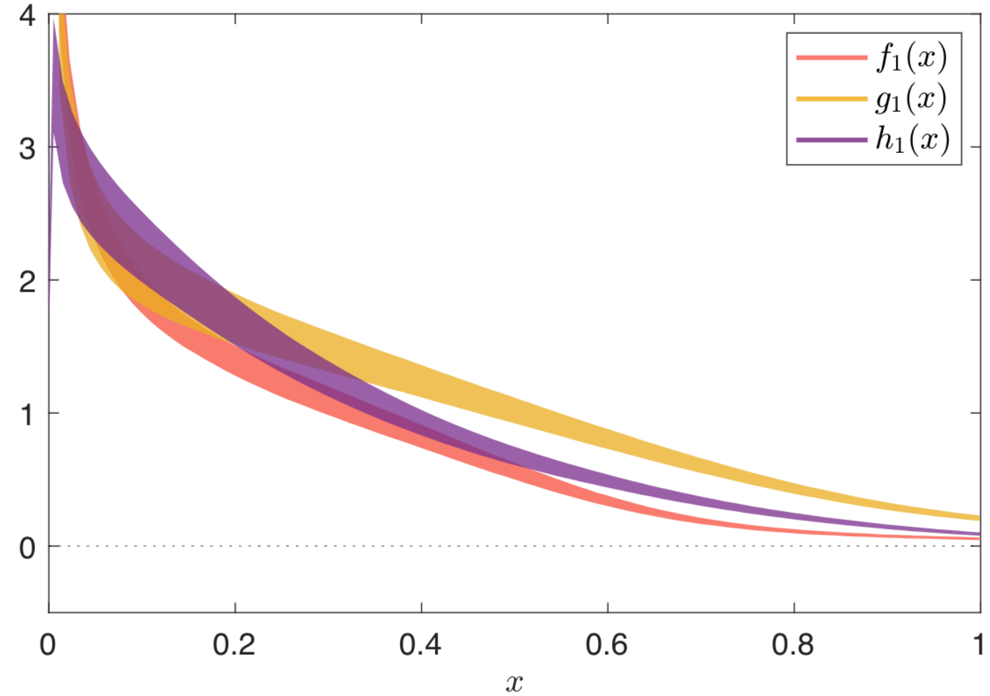
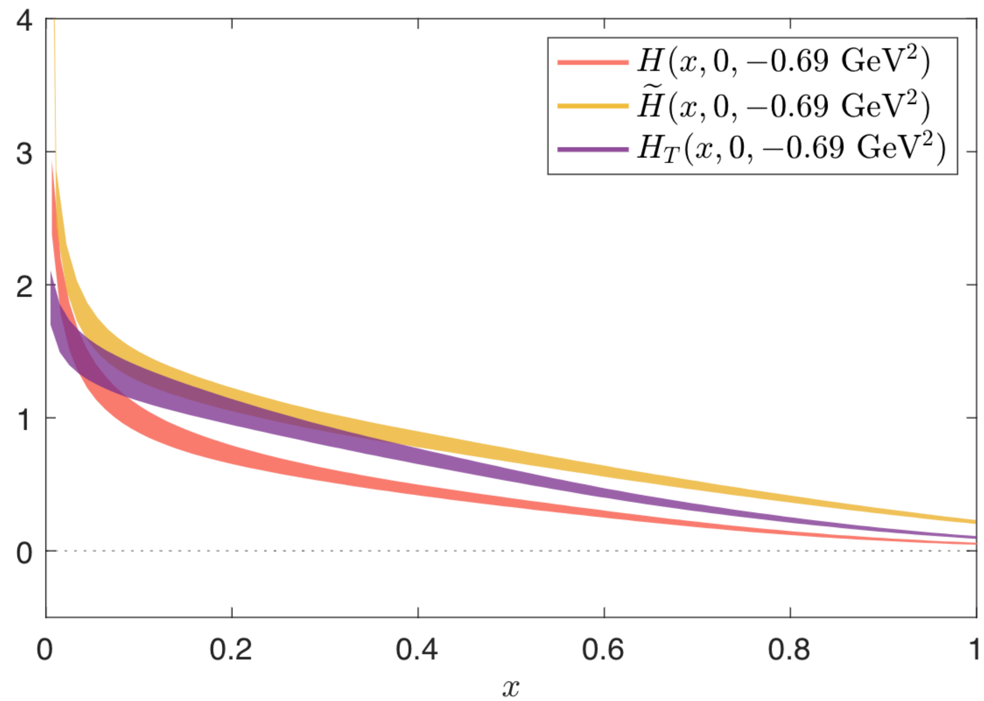
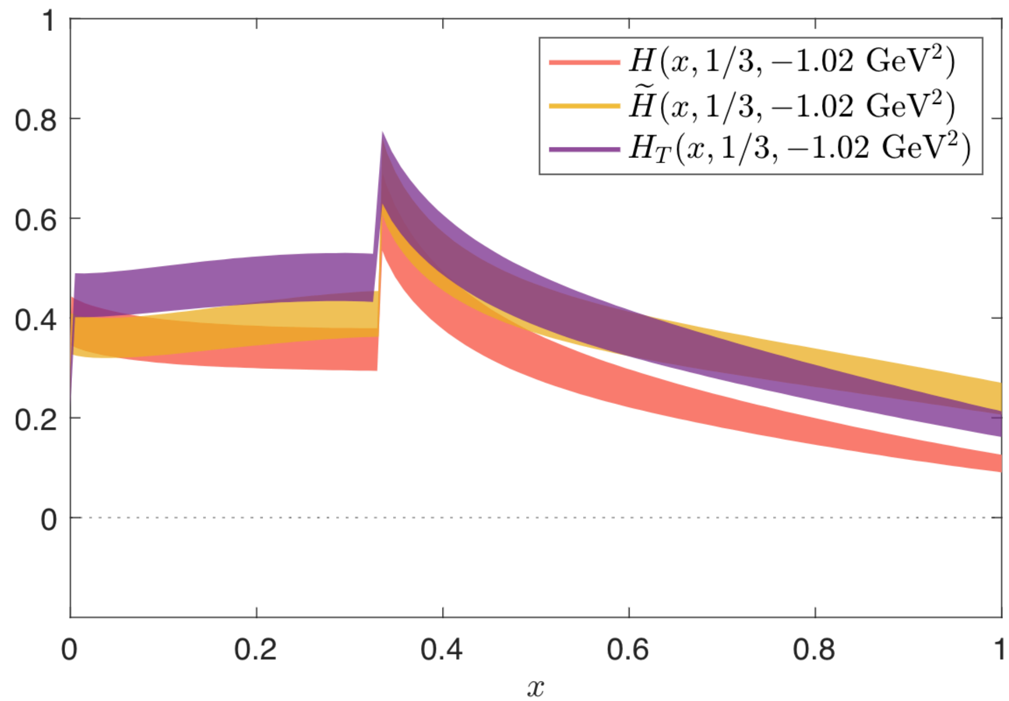
★ Qualitative understanding of GPDs and their relations

★ Qualitative understanding of ERBL and DGLAP regions



★ Relations can be identified for the t-dependence of GPDs





What can we currently check using lattice results?

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★ Understanding of systematic effects through sum rules

$$\int_{-1}^{1} dx\, H_T(x,\xi,t) = \int_{-\infty}^{\infty} dx\, H_{Tq}(x,\xi,t,P_3) = A_{T10}(t)\,,$$

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What can we currently check using lattice results?

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★ Sum rules exist for quasi-GPDs
[S. Bhattacharya et al., PRD 102, 054021 (2020)]

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★ Lattice data on transversity GPDs

$$\int_{-2}^{2} dx H_{Tq}(x,0,-0.69\,\mathrm{GeV}^2,P_3)=\{0.65(4),\,0.64(6),\,0.81(10)\}\,,\quad \int_{-2}^{2} dx H_{Tq}(x,\tfrac{1}{3},-1.02\,\mathrm{GeV}^2,1.25\,\mathrm{GeV})=0.49(5)\,,$$

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- lowest moments the same between quasi-GPDs and GPDs
- Values of moments decrease as t increases
- Higher moments suppressed compared to the lowest

Twist-3 GPDs

Twist-3 GPDs ($\gamma^\mu \gamma^5$)

★ **Transverse matrix element of axial operator**

$$\widetilde{F}^\mu = P^\mu \frac{\tilde{h}^+}{P^+} \widetilde{H} + P^\mu \frac{\tilde{e}^+}{P^+} \widetilde{E}$$

$$h^\mu = \bar{u}(p')\,\gamma^\mu\,u(p)\,, \qquad e^\mu = \bar{u}(p')\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\,u(p)\,, \qquad b = \bar{u}(p')\,u(p)\,,$$

$$\tilde{h}^\mu = \bar{u}(p')\,\gamma^\mu\gamma_5\,u(p)\,, \qquad \tilde{e}^\mu = \frac{\Delta^\mu}{2m}\,\tilde{b}\,, \qquad \tilde{b} = \bar{u}(p')\,\gamma_5\,u(p)$$

$$+ \Delta_\perp^\mu \frac{\tilde{b}}{2m} (\widetilde{E} + \widetilde{G}_1) + \tilde{h}_\perp^\mu (\widetilde{H} + \widetilde{G}_2) + \Delta_\perp^\mu \frac{\tilde{h}^+}{P^+} \widetilde{G}_3 + \tilde{\Delta}_\perp^\mu \frac{h^+}{P^+} \widetilde{G}_4$$

$\mu = 1, 2$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

Twist-3 GPDs ($\gamma^\mu\gamma^5$)

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$$\widetilde{F}^\mu = P^\mu \frac{\tilde{h}^+}{P^+}\widetilde{H} + P^\mu \frac{\tilde{e}^+}{P^+}\widetilde{E} + \Delta^\mu_\perp \frac{\tilde{b}}{2m}(\widetilde{E}+\widetilde{G}_1) + \tilde{h}^\mu_\perp(\widetilde{H}+\widetilde{G}_2) + \Delta^\mu_\perp \frac{\tilde{h}^+}{P^+}\widetilde{G}_3 + \tilde{\Delta}^\mu_\perp \frac{h^+}{P^+}\widetilde{G}_4$$

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★ Sum Rules (generalization of Burkhardt-Cottingham)

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0$$

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

★ Sum Rules (generalization of Efremov-Leader-Teryaev)

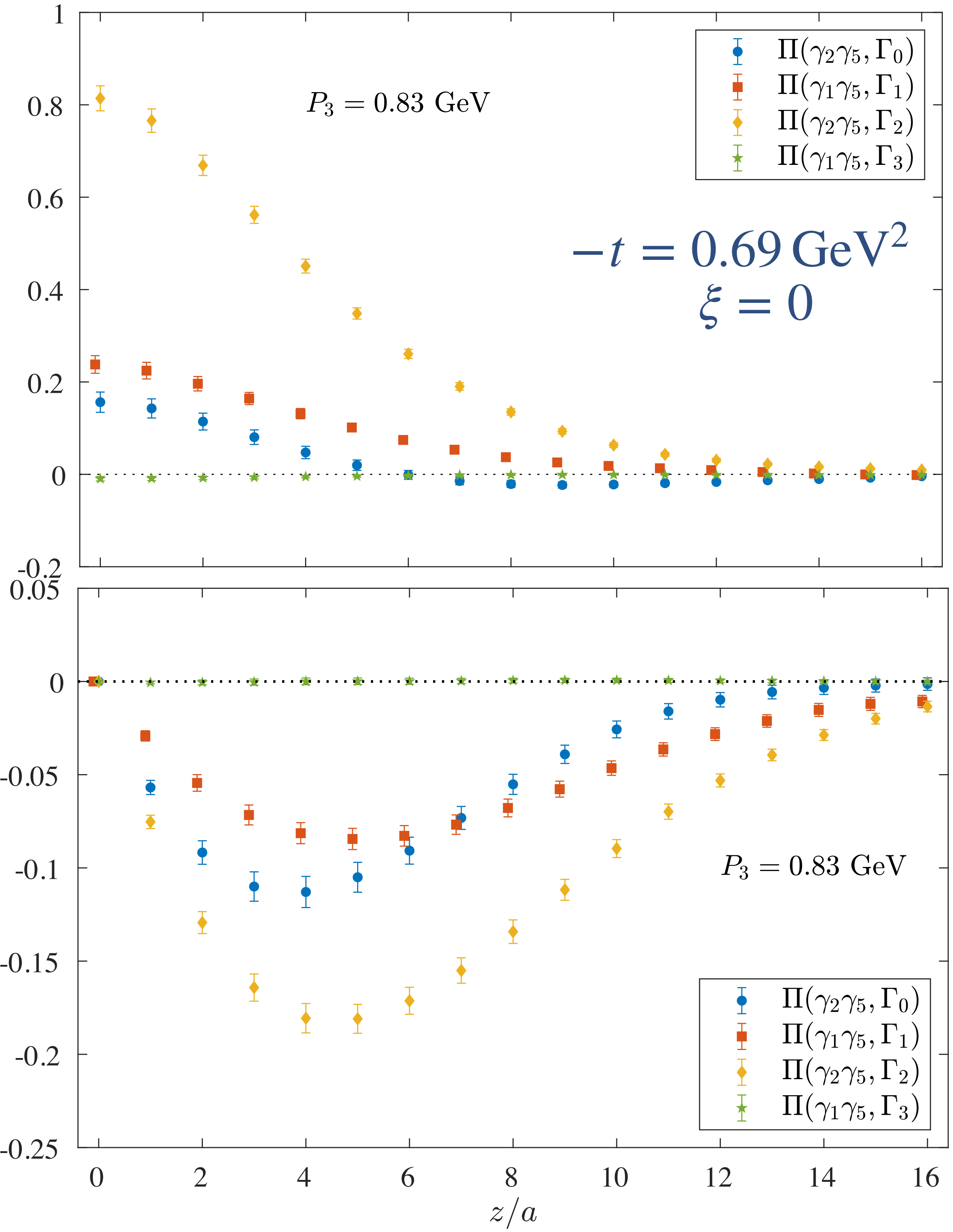
[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_1(x,0,t) = \frac{1}{2}\left[F_2(t) - \int_{-1}^{1} dx\, x\, \widetilde{E}(x,0,t)\right], \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_2(x,0,t) = \frac{1}{2}\left[-\frac{t}{4m^2}F_2(t) - \widetilde{A}_{20}(t)\right],$$

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = 0 \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4}G_E(t)$$

F_2 : Pauli FF
G_E : electric FF
$\widetilde{A}_{20}, \widetilde{B}_{20}$: axial GFFs

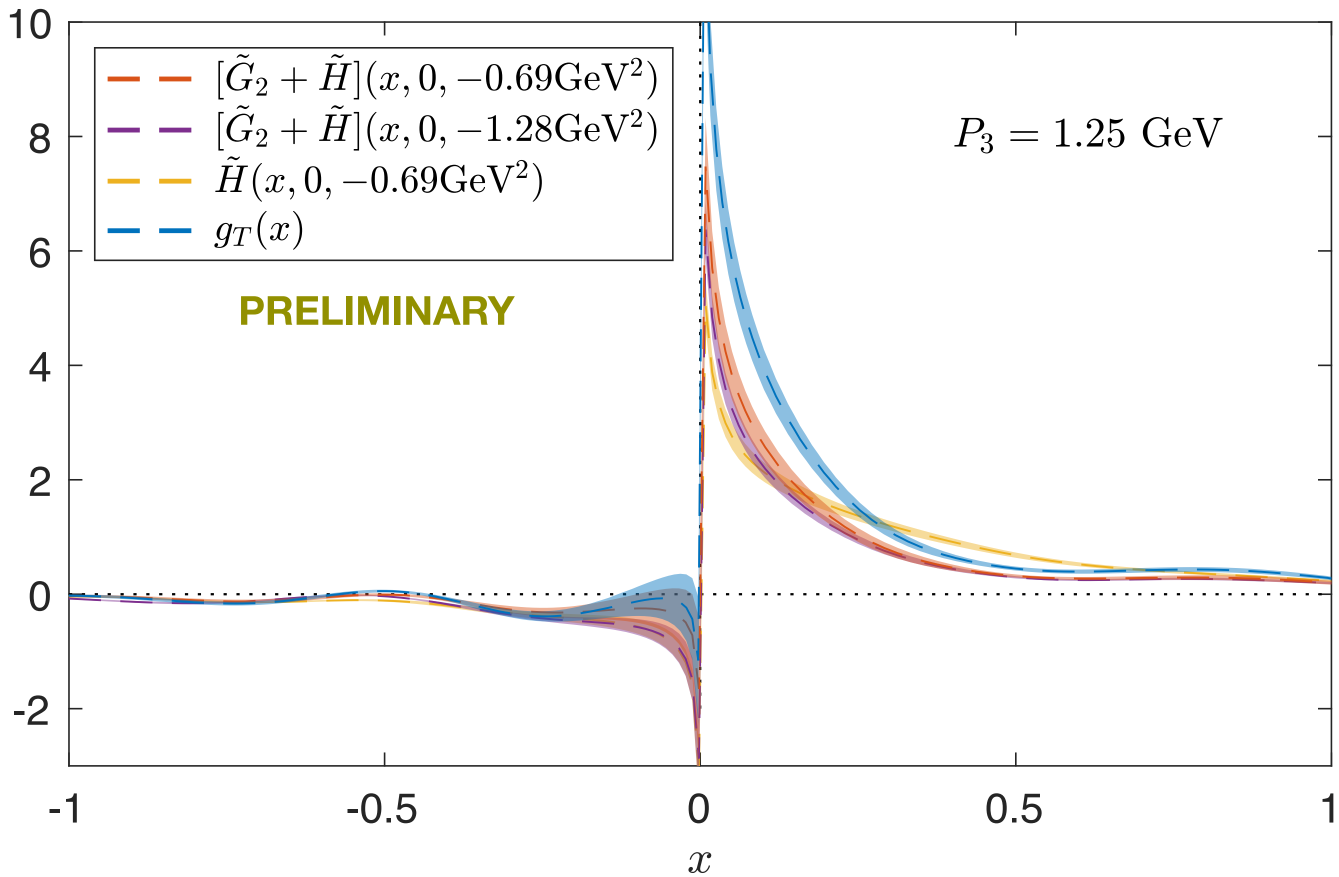
Bare matrix elements (ME)

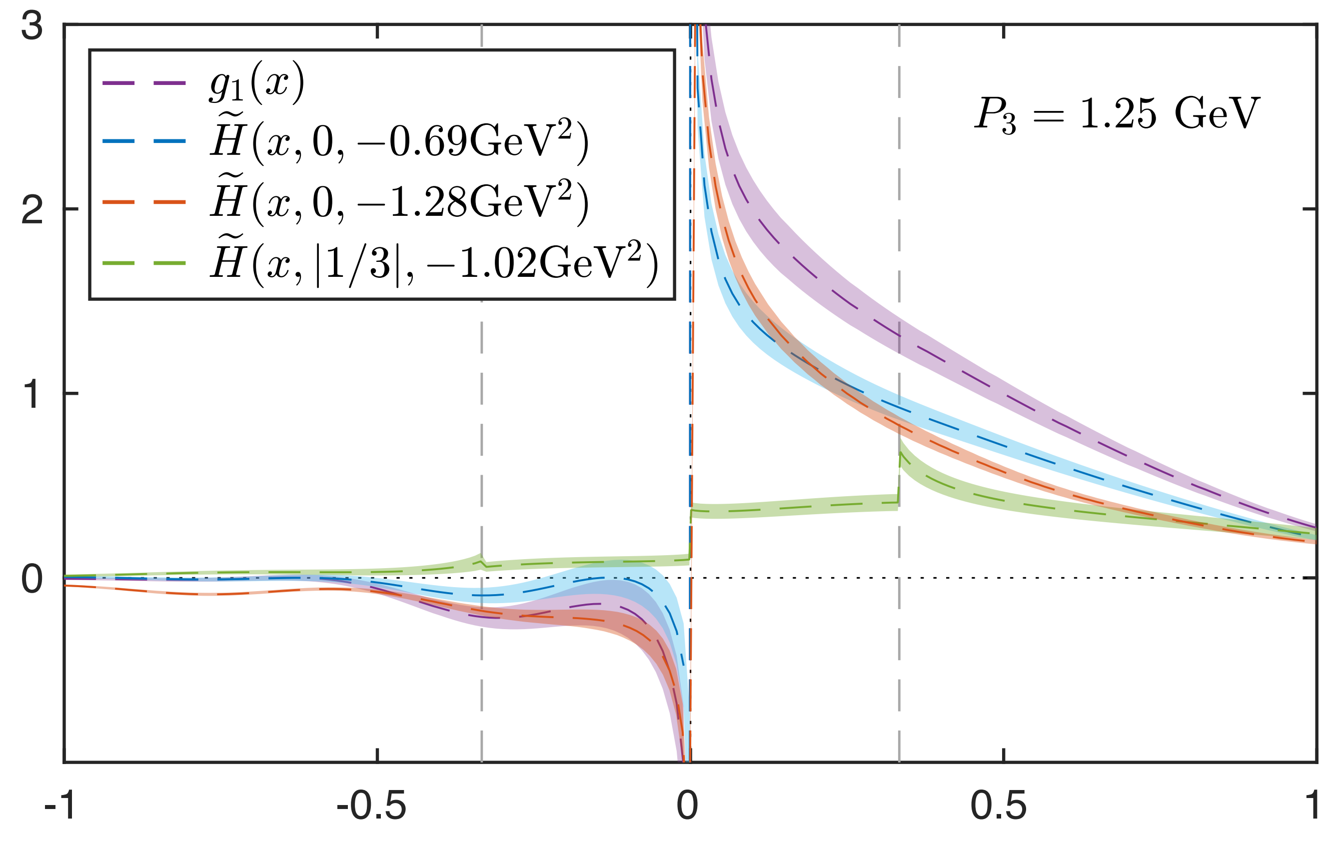


- ★ $\Pi(\gamma^2\gamma^5, \Gamma_0)$ & $\Pi(\gamma^2\gamma^5, \Gamma_2)$: **disentangle** $\widetilde{H} + \widetilde{G}_2$, $\widetilde{G}_4$
- ★ $\Pi(\gamma^1\gamma^5, \Gamma_1)$ **and** $\widetilde{H} + \widetilde{G}_2$: **disentangle** $\widetilde{E} + \widetilde{G}_1$
- ★ $\Pi(\gamma^1\gamma^5, \Gamma_3)$ **gives** $\widetilde{G}_3$

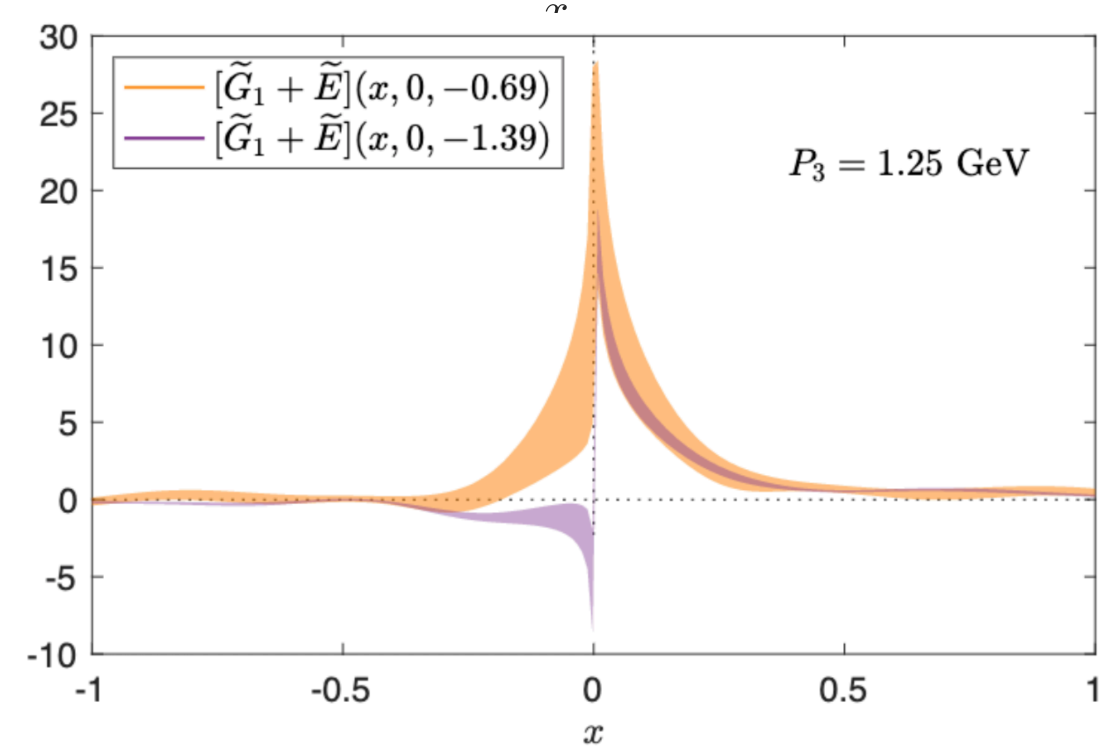
- ★ **Similar picture for $P_3 = 1.25$ GeV**
- ★ **Real part of ME: dominant**
- ★ $\widetilde{G}_3$ **is kinematically suppressed**

x-dependence of twist-3 GPDs





- ★ $g_T(x)$: **dominant distribution**
- ★ $\widetilde{H} + \widetilde{G}_2$ **similar in magnitude to** $\widetilde{H}$
- ★ $\widetilde{G}_2$ **is expected to be small**



Concluding Remarks

★ Computationally expensive calculation
multi-dimensionality

★ Promising results on $x-$dependence of proton GPDs
signal comparable to PDFs

★ Sources of systematic uncertainties need to be addressed
volume effects, continuum limit, pion mass dependence, …

★ Exploration beyond leading twist is feasible

★ Synergy with phenomenology is anticipated

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Thank you

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TMD Collaboration TMD Topical Collab.

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