Transverse-Momentum-Dependent Proton Structures from Lattice QCD

APCTP Focus Program in Nuclear Physics 2022: Hadron Physics Opportunities with JLab Energy and Luminosity Upgrade

APCTP, Korea, Jul. 18-23, 2022

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JUL 19, 2022

Argonne NATIONAL LABORATORY

Outline

- TMDs in the non-perturbative region
- Quasi-TMDs in Large-Momentum Effective Theory
- Lattice calculations
- Outlook

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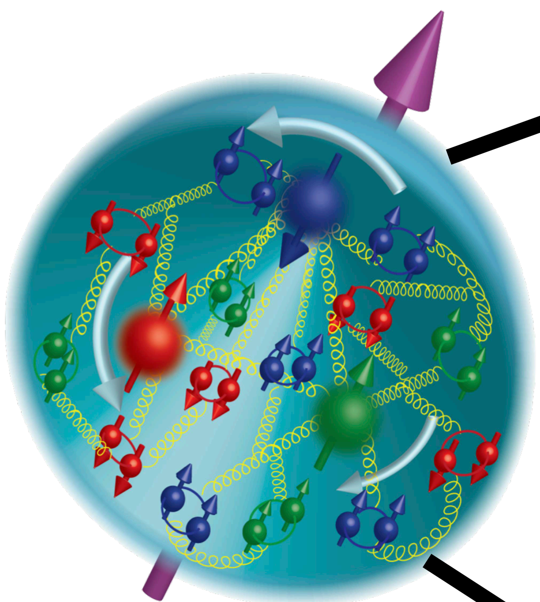
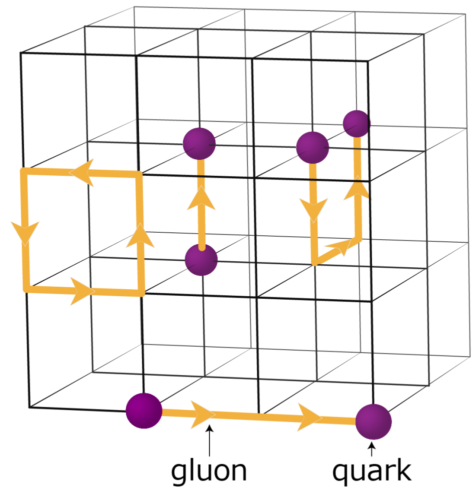
3D Tomography of the Proton

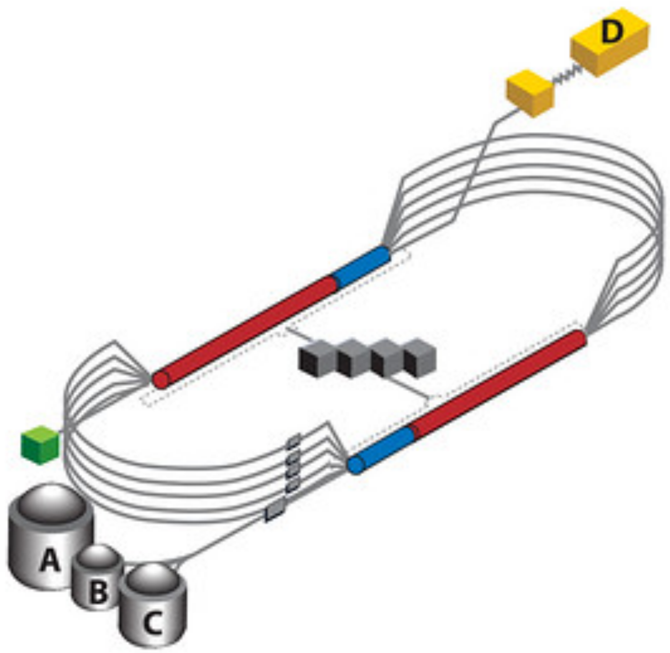


Lattice QCD

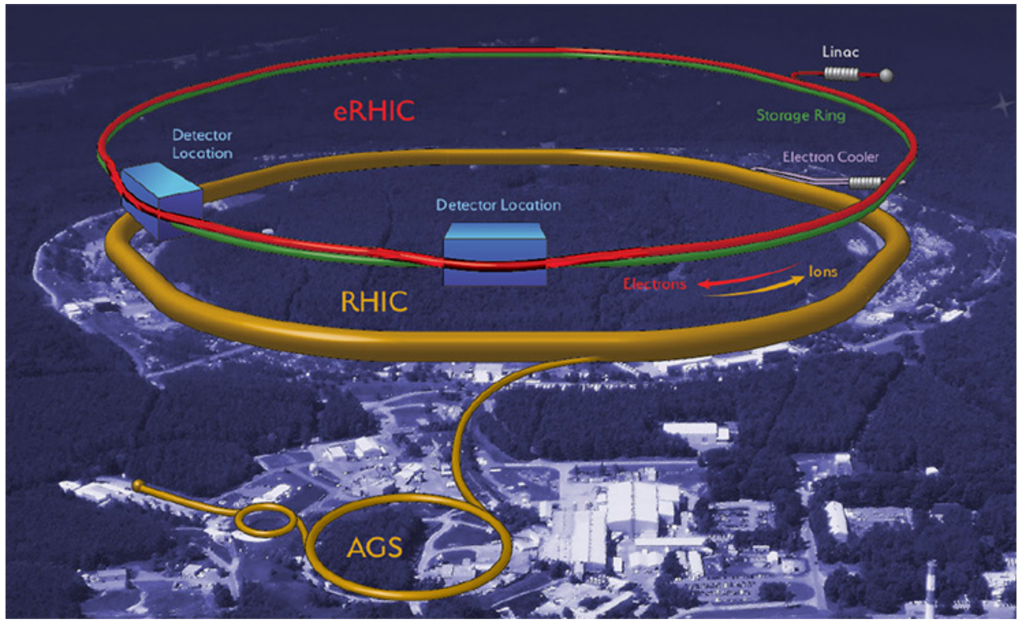


+

Hard Scattering

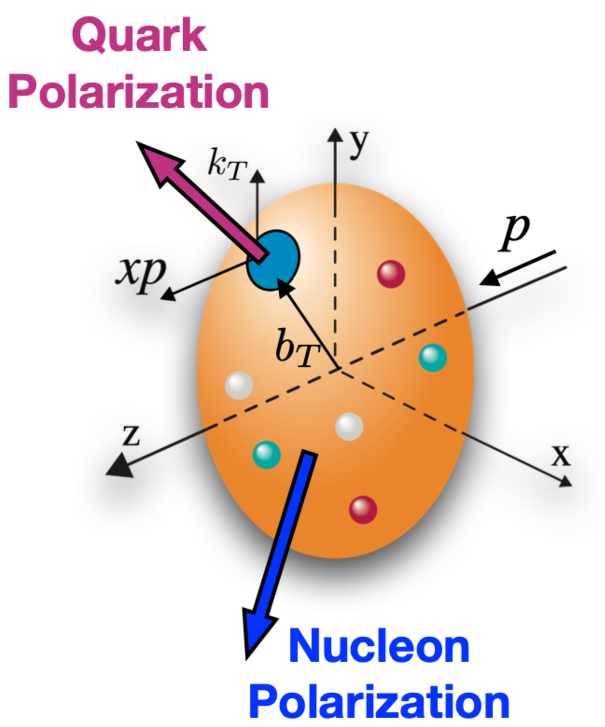


Jefferson Lab 12 GeV



The Electron-Ion Collider

3D Tomography of the Proton



Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

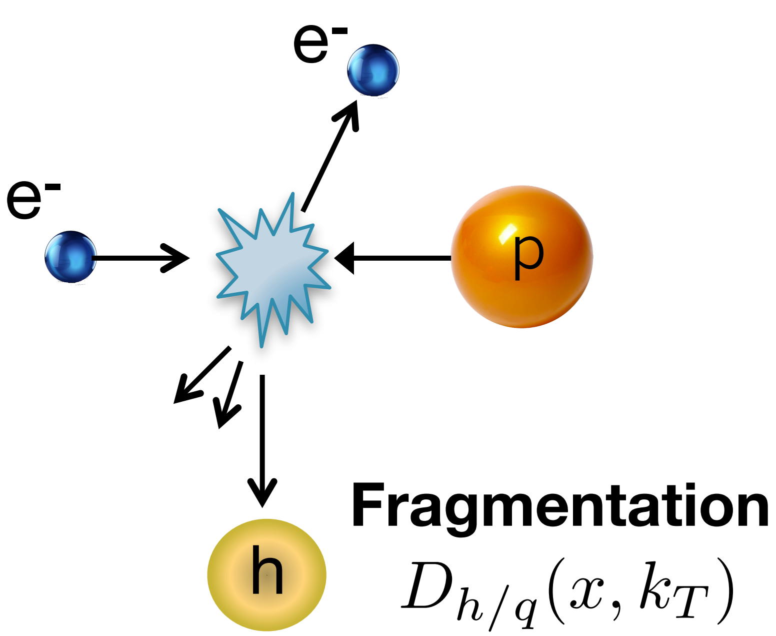
From TMD Handbook, TMD Topical Collaboration, to appear soon.

TMDs from experiments

TMD processes:

Semi-Inclusive DIS

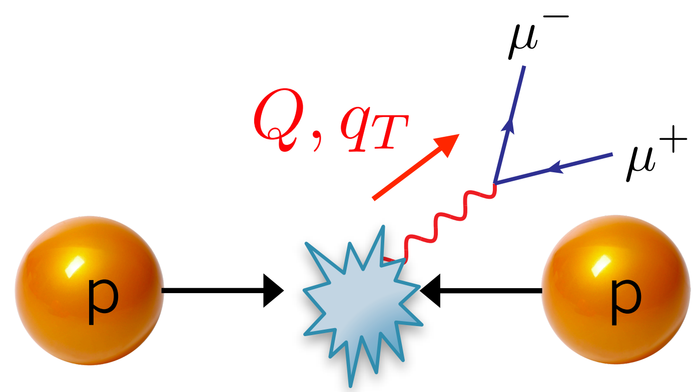
$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



HERMES, COMPASS, JLab, EIC, ...

Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$

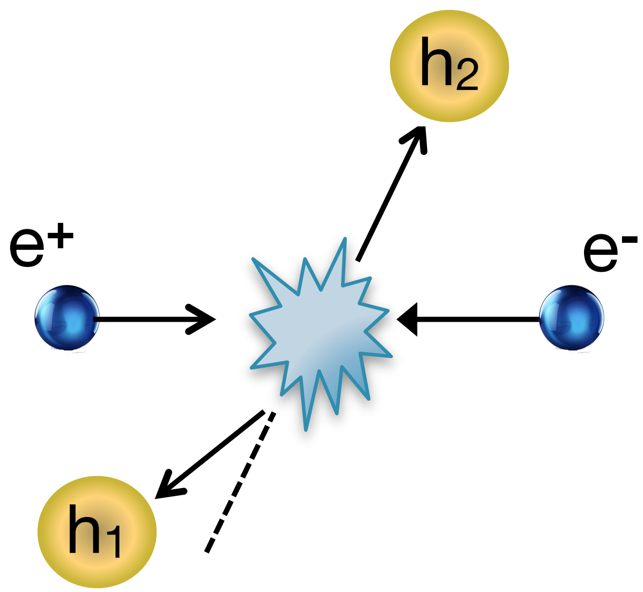


$$q_T \ll Q$$

Fermilab, RHIC, LHC, ...

Dihadron in e^+e^-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$



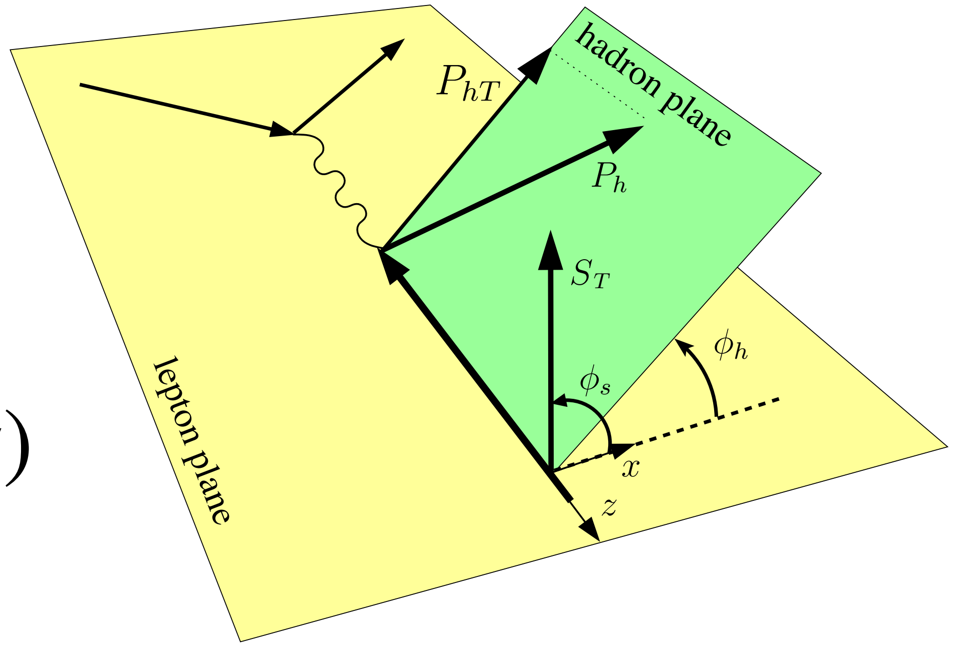
Babar, Belle, BESIII, ...

TMDs from global analyses

Semi-inclusive deep inelastic scattering: $l + p \longrightarrow l + h(P_h) + X$

$$\frac{d\sigma^W}{dxdydz_h d^2\mathbf{P}_{hT}} \sim \int d^2\mathbf{b}_T\ e^{i\mathbf{b}_T\cdot\mathbf{P}_{hT}/z}$$

$$\times f_{i/p}(x, \mathbf{b}_T, Q, Q^2)\ D_{h/i}(z_h, \mathbf{b}_T, Q, Q^2)$$



Kang, Prokudin, Sun and Yuan, PRD **93**, 014009 (2016)

$$f_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = f^{\text{pert}}_{i/p}(x, b^*(b_T), \mu, \zeta)$$

$$\times \left(\frac{\zeta}{Q_0^2}\right)^{g_K(b_T)/2}\quad f^{\text{NP}}_{i/p}(x, b_T)$$

$g_K(b_T)/2$ ⟶ **Collins-Soper kernel (NP part)**

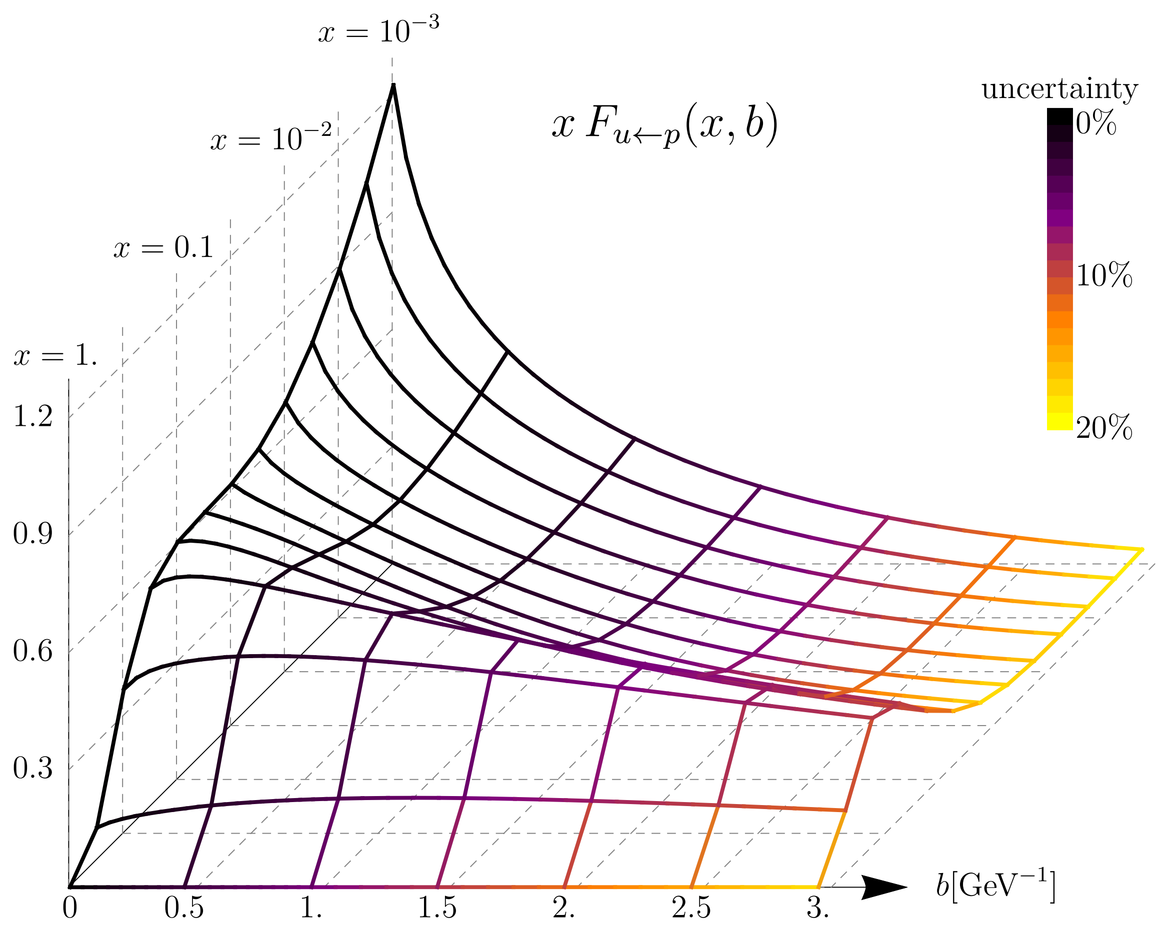
$f^{\text{NP}}_{i/p}(x, b_T)$ ⟶ **Intrinsic TMD**

$Q_0 \sim 1$ GeV

Non-perturbative when $b_T \sim 1/\Lambda_{\text{QCD}}$!

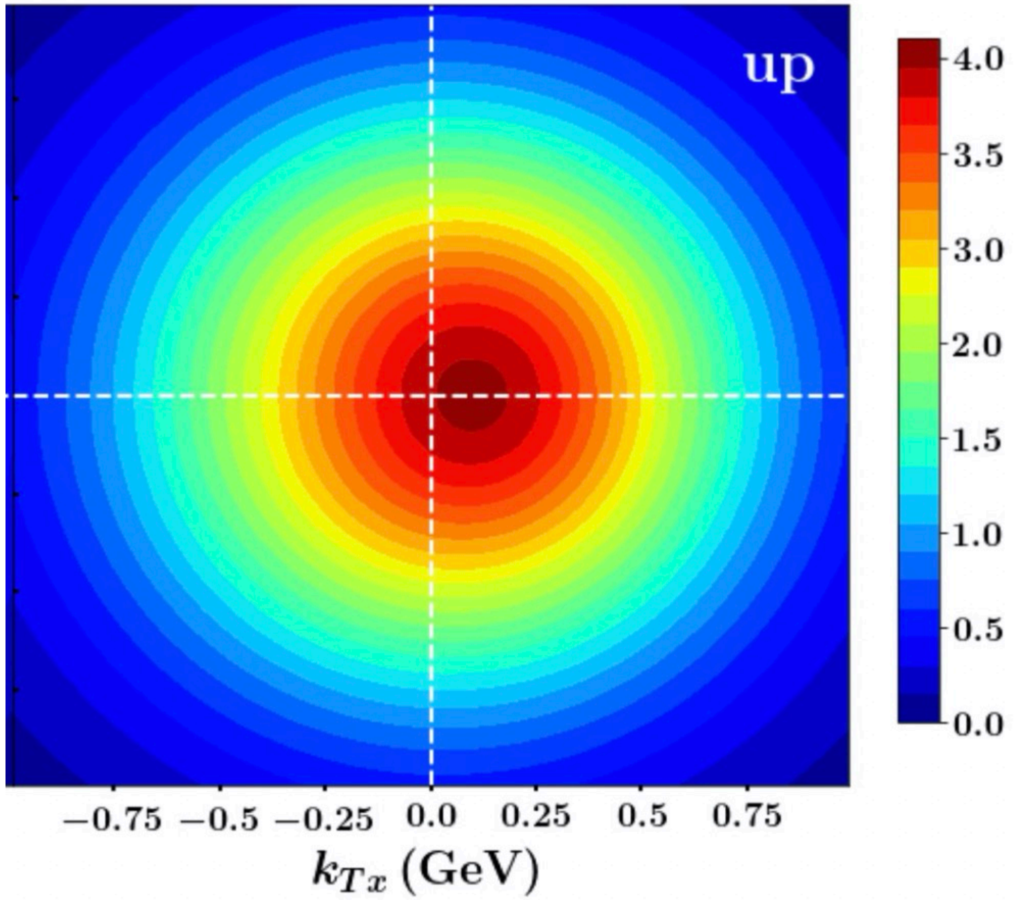
TMDs from global analyses

Unpolarized quark TMD



Scimemi and Vladimirov, JHEP **06** (2020).

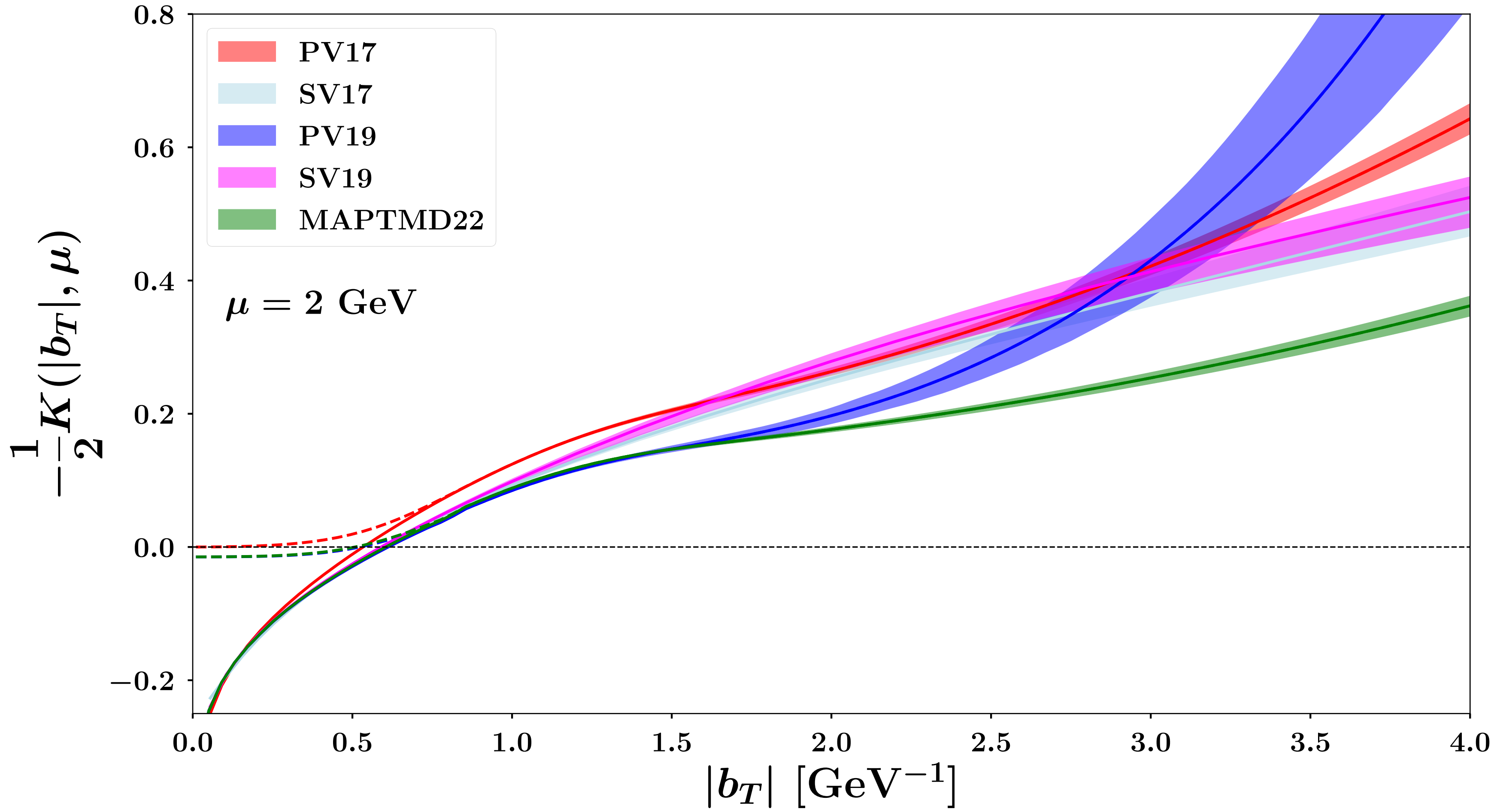
Quark Sivers function



Cammarota, Gamberg, Kang et al. (JAM Collaboration), PRD **102** (2020).

TMDs from global analyses

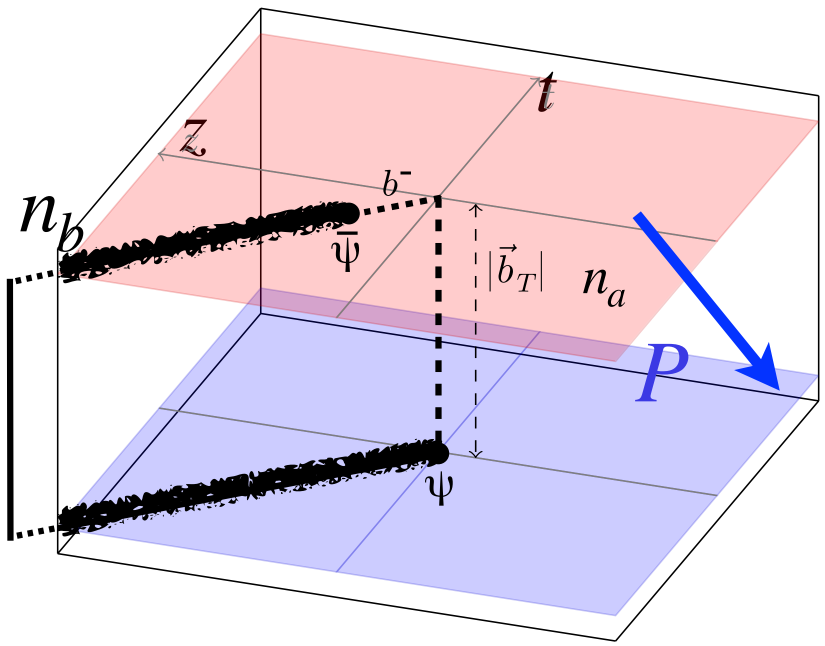
Collins-Soper Kernel $K(b_T, \mu) = K^{\text{pert}}(b_T, \mu) + g_K(b_T)$



Bacchetta, Bertone, Bissolotti, et al., MAP Collaboration, 2206.07598

TMD definition

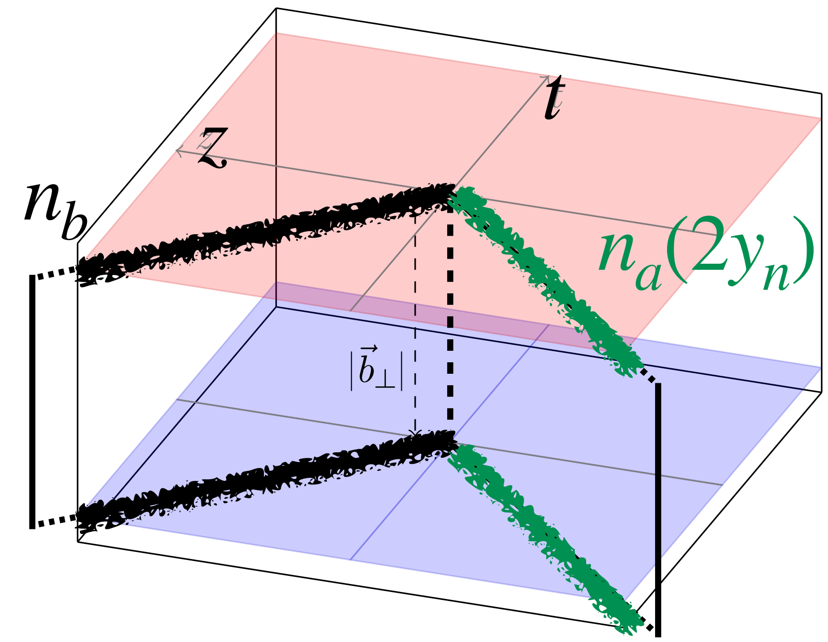
- Beam function:



Hadronic matrix element

$n_b^2 = 0$

- Soft function :



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\rm UV} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

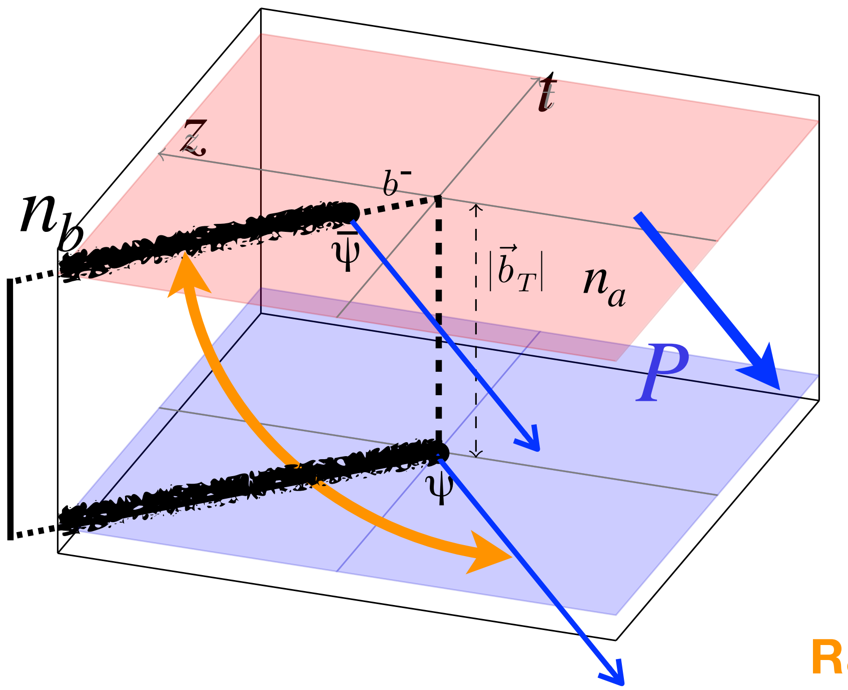
Collins-Soper scale: $\zeta = 2(xP^+e^{-y_n})^2$

Rapidity divergence regulator

First principles calculation of TMDs from the above matrix elements would greatly complement global analyses!

TMD definition

- Beam function:



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Rapidity divergences

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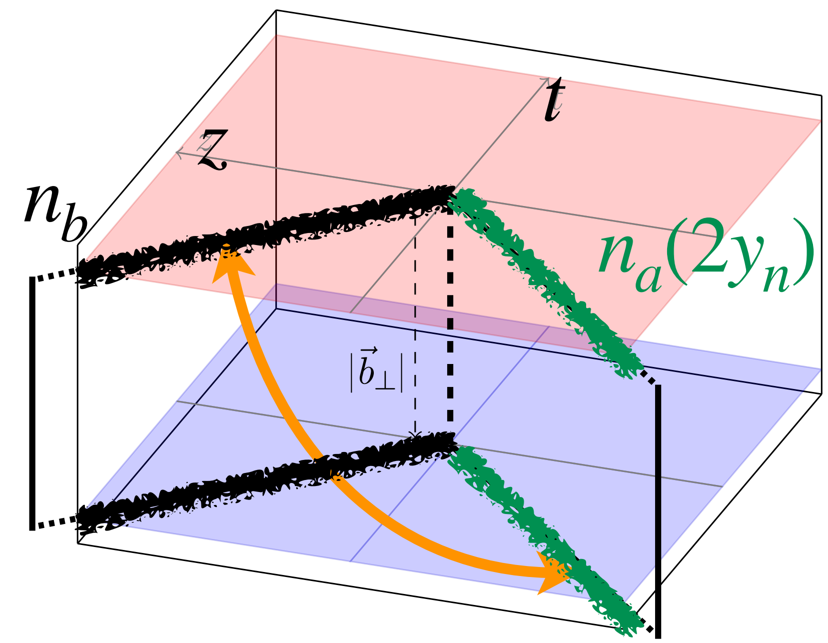

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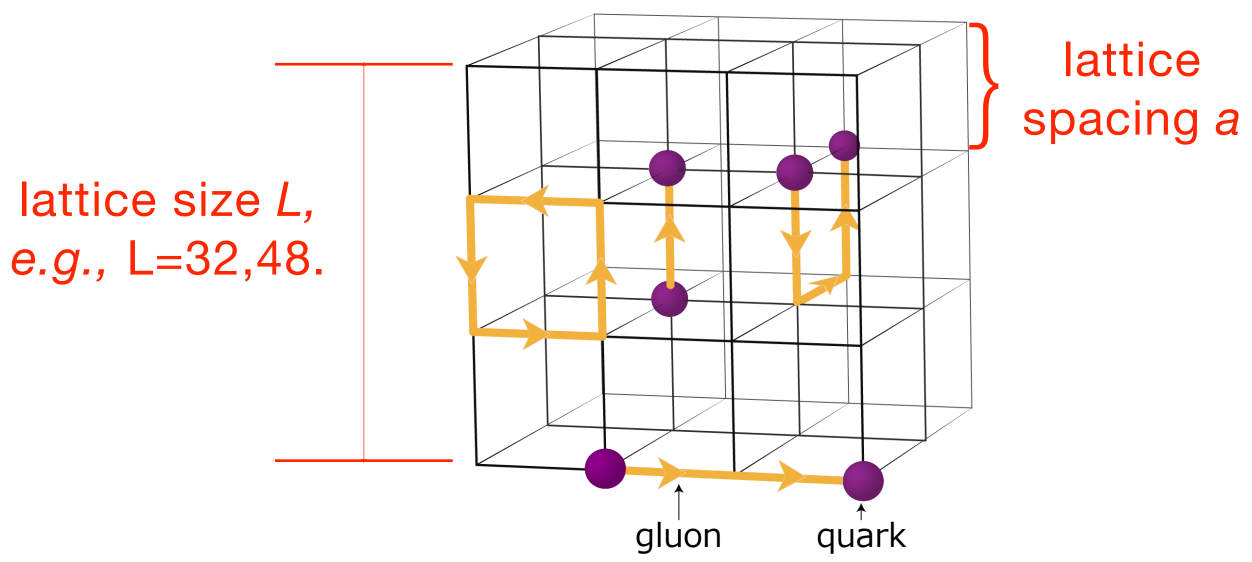
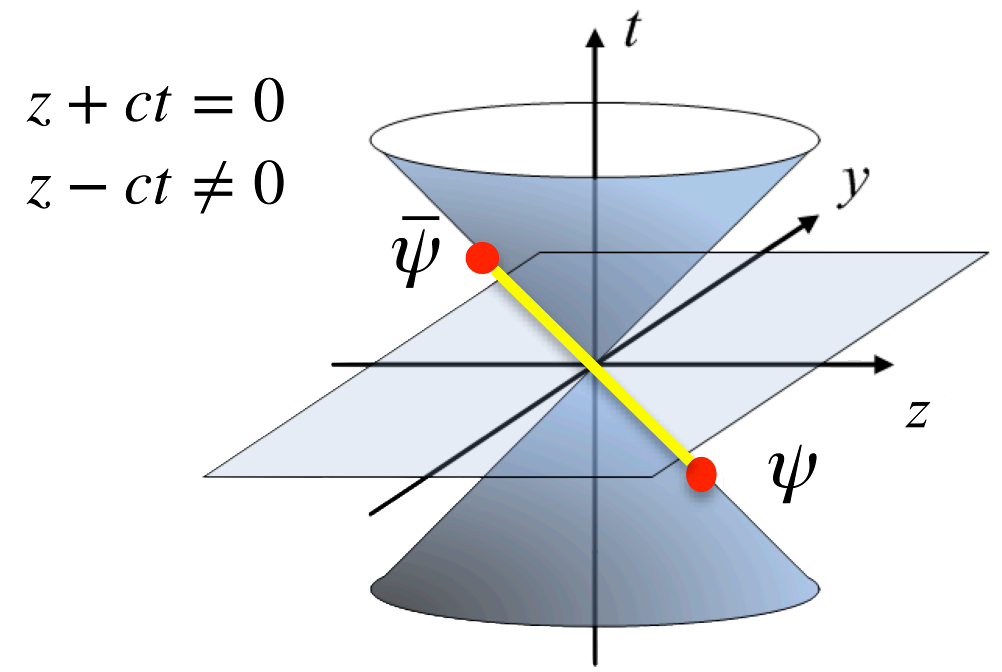
Lattice QCD

Lattice gauge theory: a systematically improvable approach to solve non-perturbative QCD.





Imaginary time: $t \to i\tau$ $\quad O(i\tau) \xrightarrow{?} O(t)$

Simulating real-time dynamics has been extremely difficult due to the issue of analytical continuation. 🙁

Progress in the lattice study of TMDs

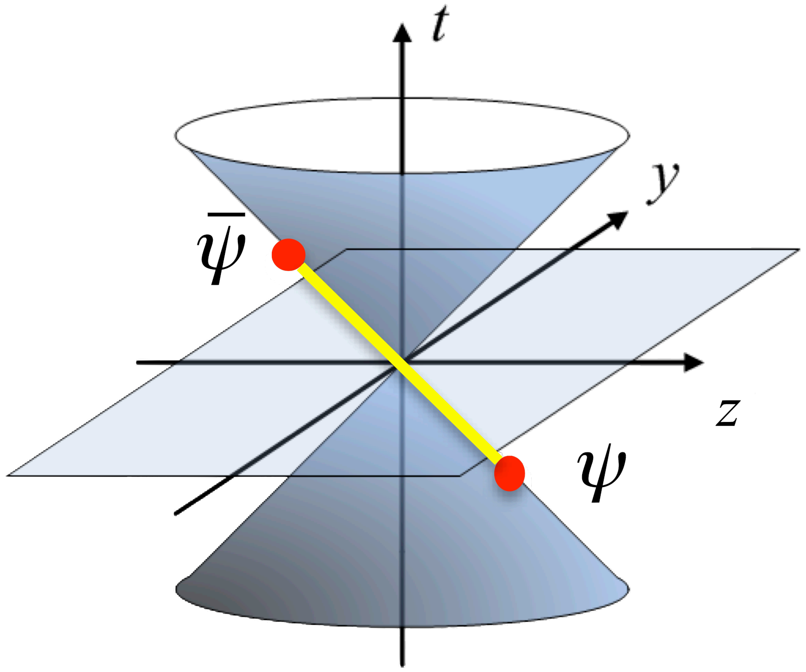
- **Lorentz invariant method**
  - Musch, Hägler, Engelhardt, Negele and Schäfer et al.
  - Primary efforts focused on ratios of TMD x-moments (2009—)

- **Quasi-TMDs**
  - Large-momentum effective theory (Ji, 2013)
  - One-loop studies of quasi beam and soft functions (Ji, Yuan, Scäfer, Liu, Liu, Ebert, Stewart, YZ, Vladimirov, Wang, …, 2015-2022)
  - Method to calculate the Collins-Soper kernel (Ji, Yuan et al., 2015; Ebert, Stewart and YZ, 2018)
  - Method to calculate the soft function, and thus the x and b_T dependence of TMDs (Ji, Liu and Liu, 2019)
  - Derivation of factorization formula (Ebert, Schindler, Stewart and YZ, 2022)
  - First lattice results (SWZ, LPC, ETMC/PKU, SVZES, 2020—)

Large-Momentum Effective Theory (LaMET)

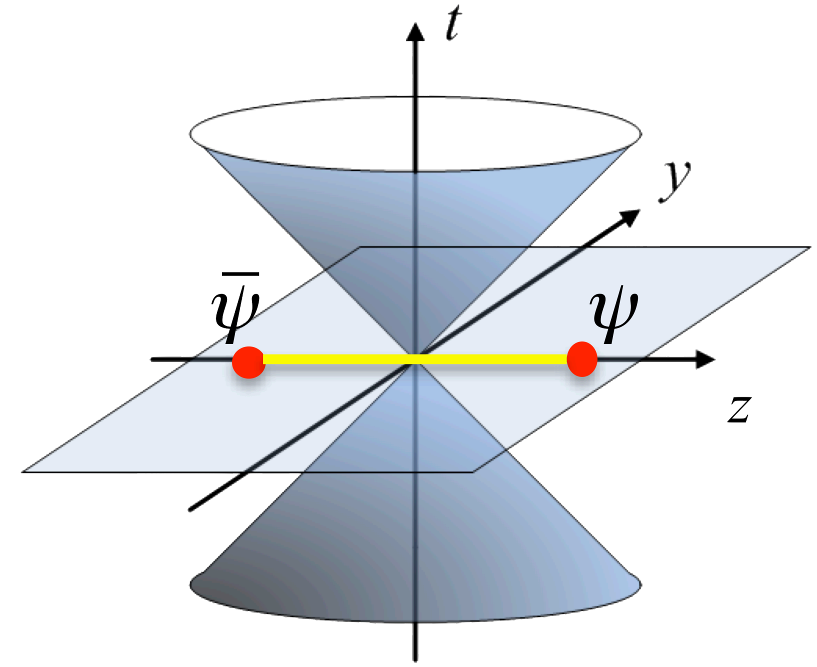
X. Ji, PRL 110 (2013)

$z + ct = 0, \quad z - ct \neq 0$



PDF $f(x)$:
Cannot be calculated on the lattice

$t = 0, \quad z \neq 0$



Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

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Related by Lorentz boost

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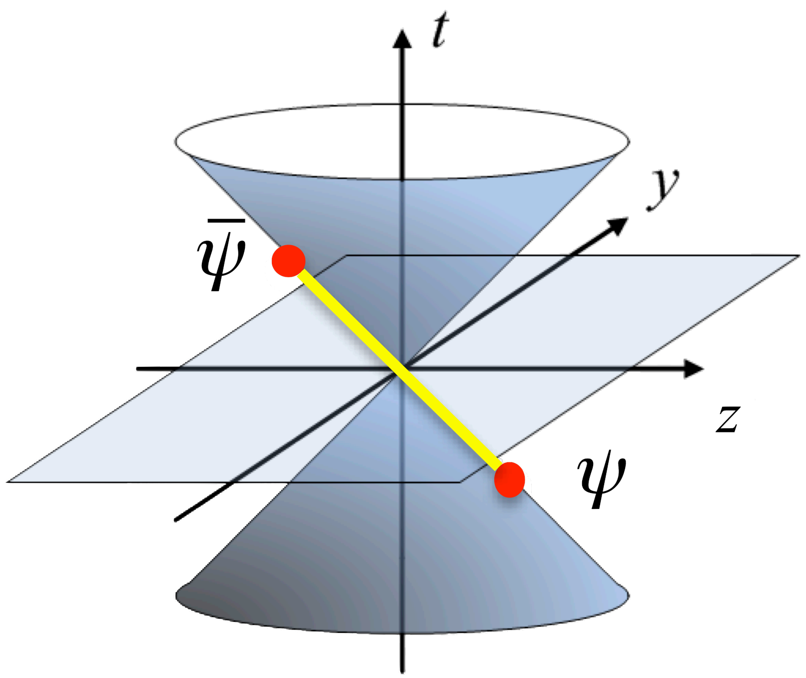

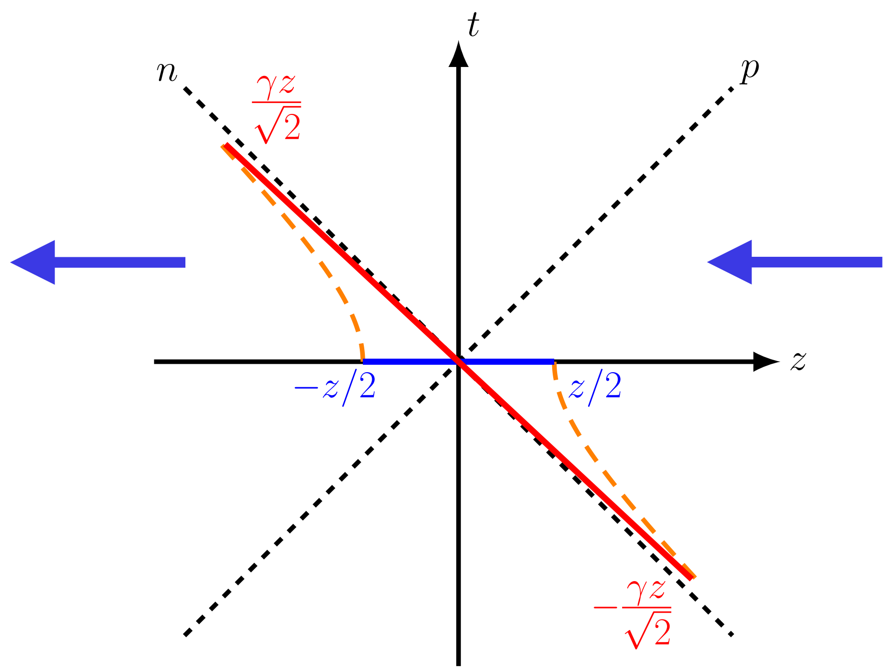

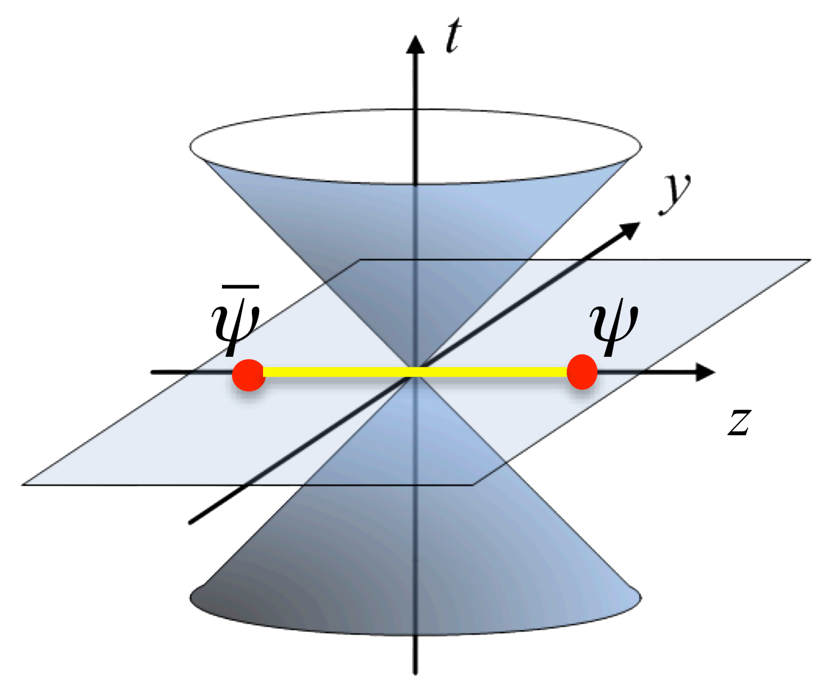

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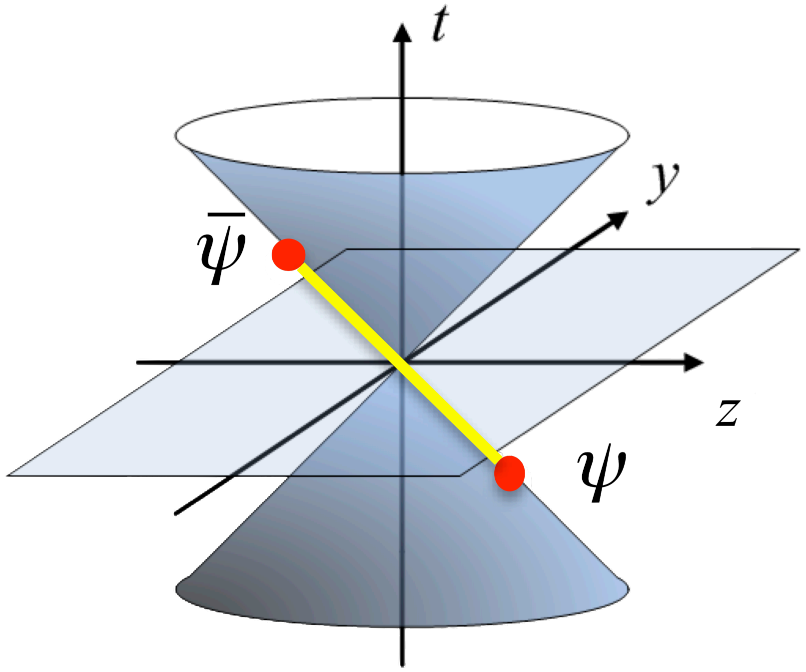

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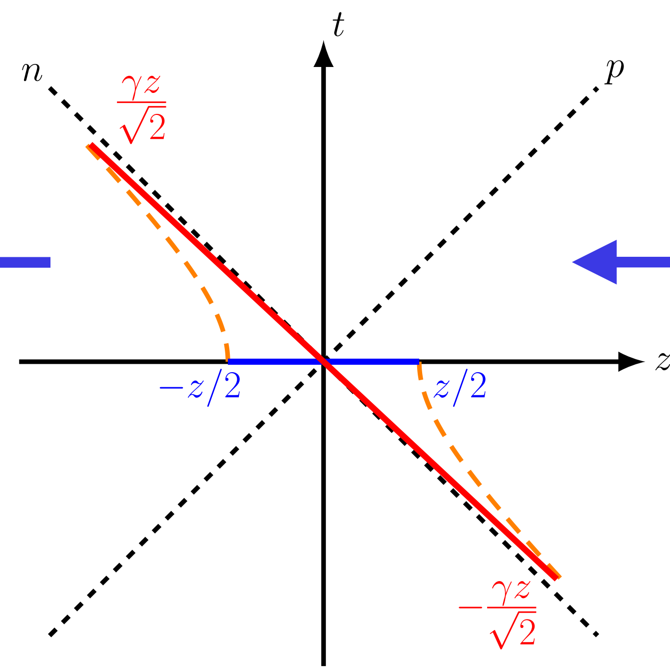
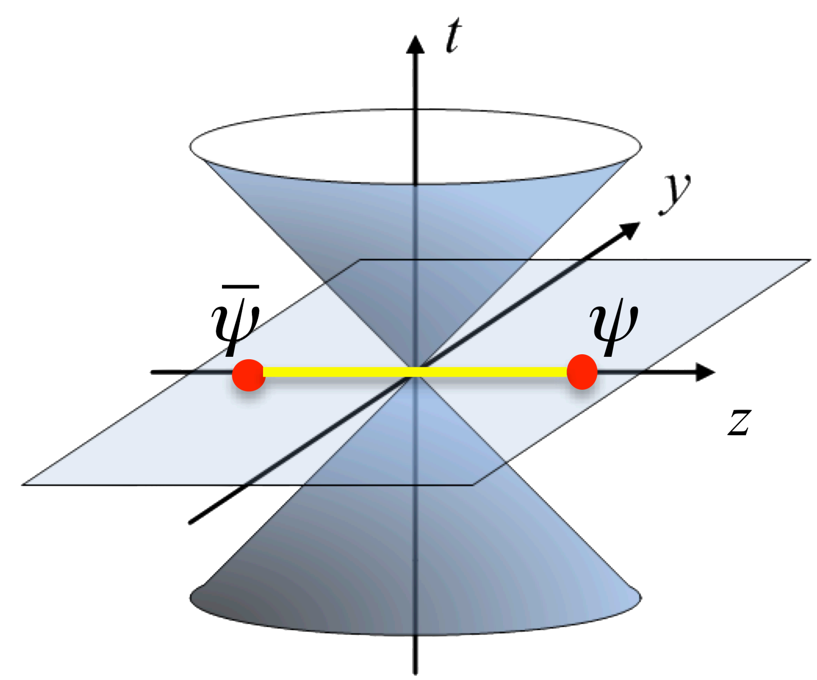


$$\lim_{P^z \to \infty} \tilde{f}(x, P^z) \stackrel{?}{=} f(x)$$

$t = 0, \quad z \neq 0$



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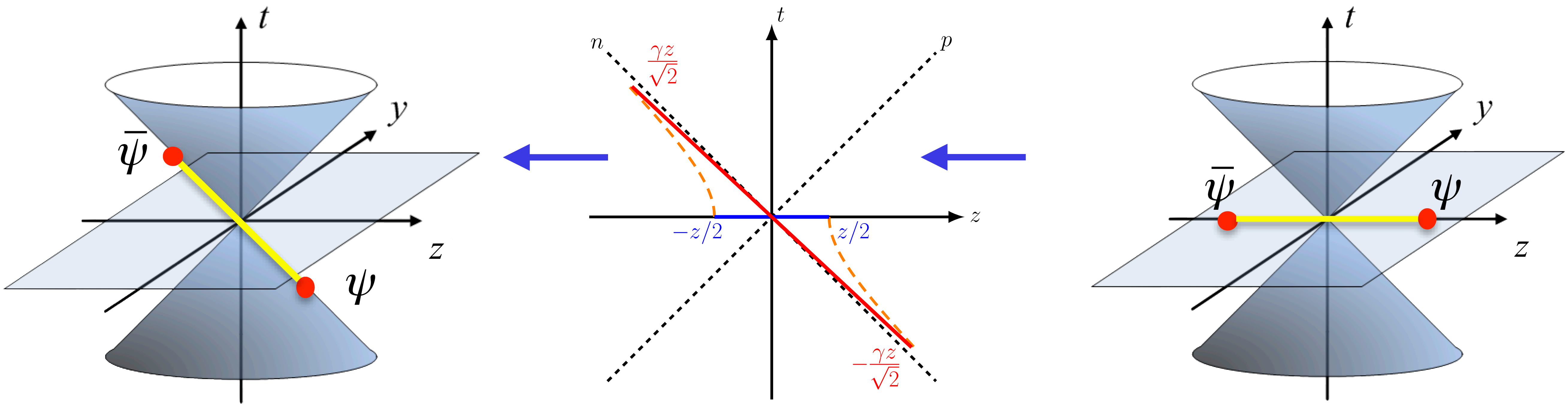

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$$\lim_{P^z\to\infty} \tilde{f}(x, P^z) \stackrel{?}{=} f(x)$$

✘

Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

Large-Momentum Effective Theory (LaMET)

- Quasi-PDF: $P^z \ll \Lambda$;

Λ: the ultraviolet lattice cutoff, $\sim 1/a$

- PDF: $P^z = \infty$, implying $P^z \gg \Lambda$.
  - The limits $P^z \ll \Lambda$ and $P^z \gg \Lambda$ are not usually exchangeable;
  - For $P^z \gg \Lambda_{\rm QCD}$, the infrared (nonperturbative) physics is not affected, which allows for an effective field theory matching.

$$\tilde{f}(x, P^z, \Lambda) = C\left(x, P^z/\mu, \Lambda/P^z\right) \otimes f(x, \mu) + O\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

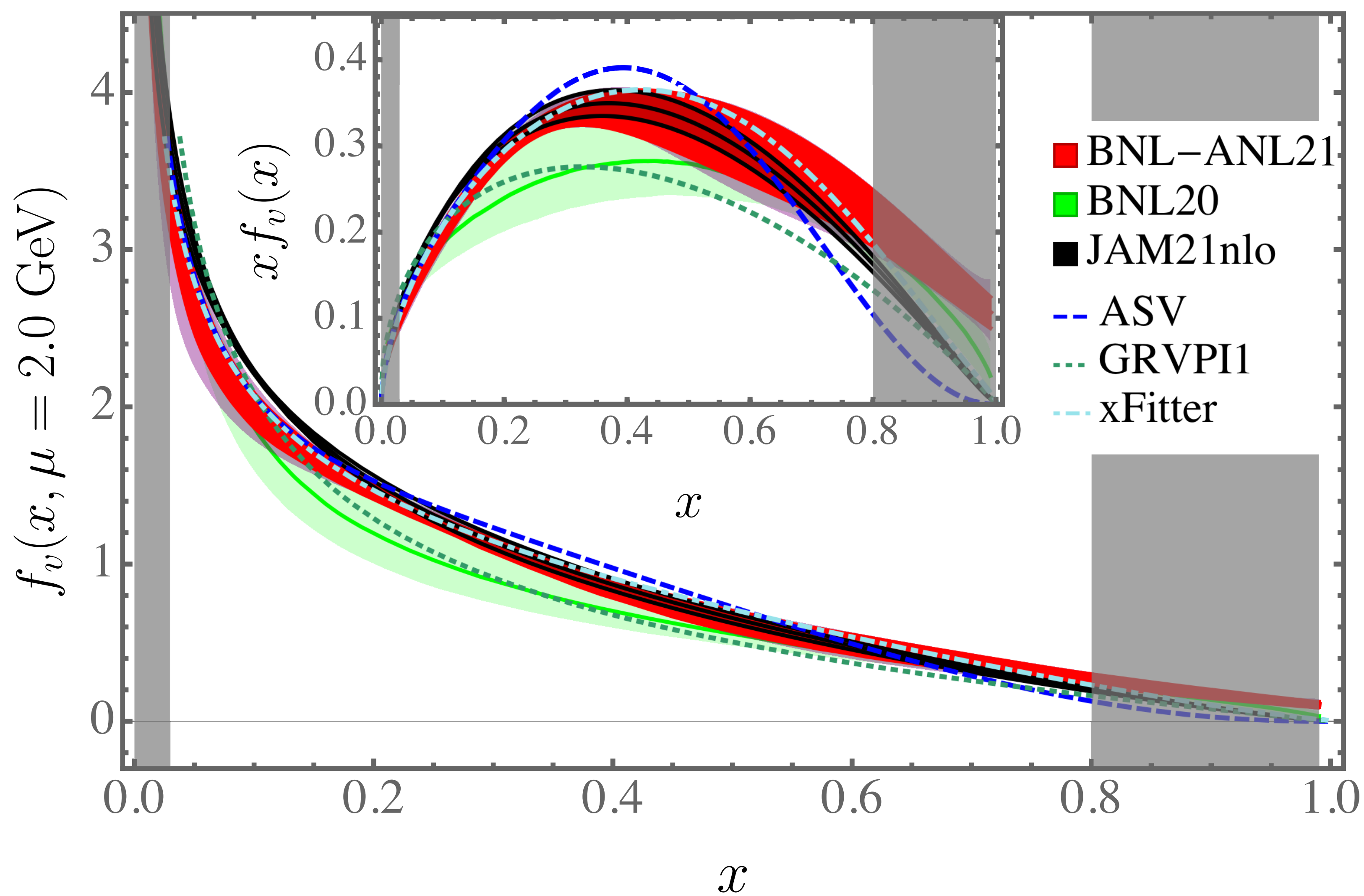
Perturbative matching

Power corrections

- X. Ji, PRL 110 (2013); SCPMA57 (2014).
- X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD 90 (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP 93 (2021).

LaMET calculation of the collinear PDFs

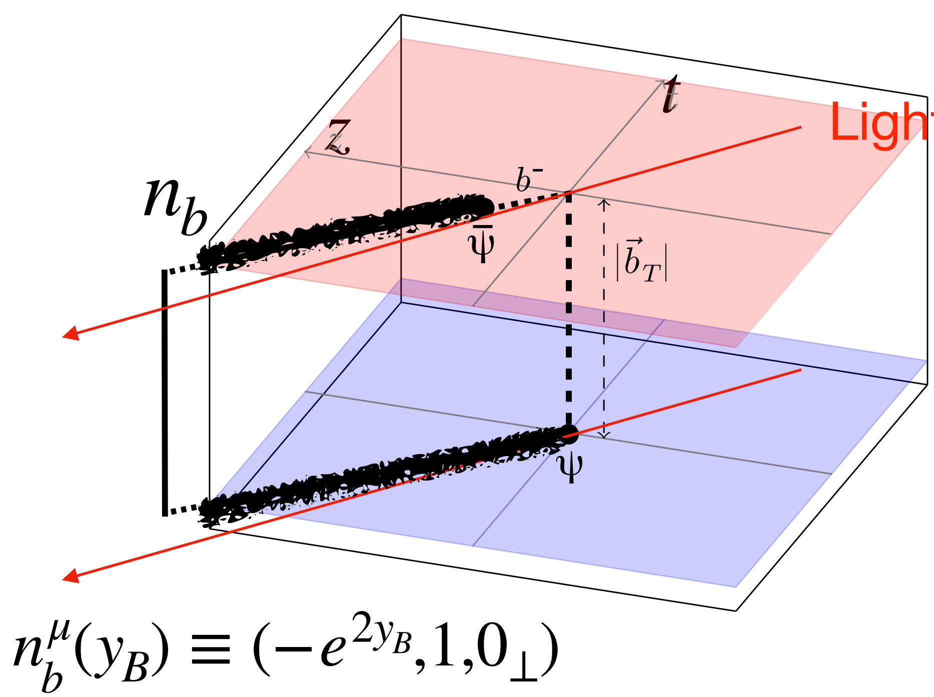
A state-of-the-art calculation of the pion valence quark PDF with fine lattices, large momentum and NNLO matching:



Gao, Hanlon, Mukherjee, Petreczky, Scior, Syritsyn and YZ, PRL **128**, 142003 (2022).

Quasi TMD in the LaMET formalism

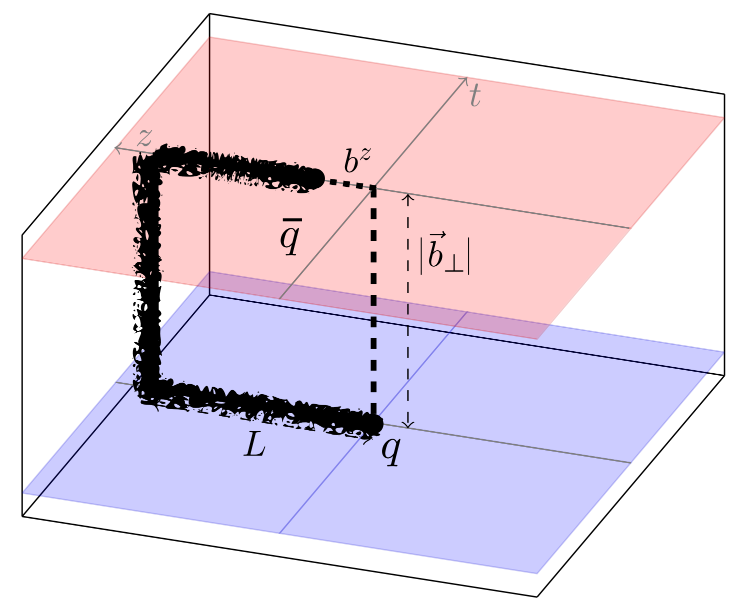
- **Beam function in Collins scheme:**



Spacelike but close-to-lightcone ($y_B \to -\infty$) Wilson lines, not calculable on the lattice ☹️

- **Quasi beam function :**

Lorentz boost and $L \to \infty$

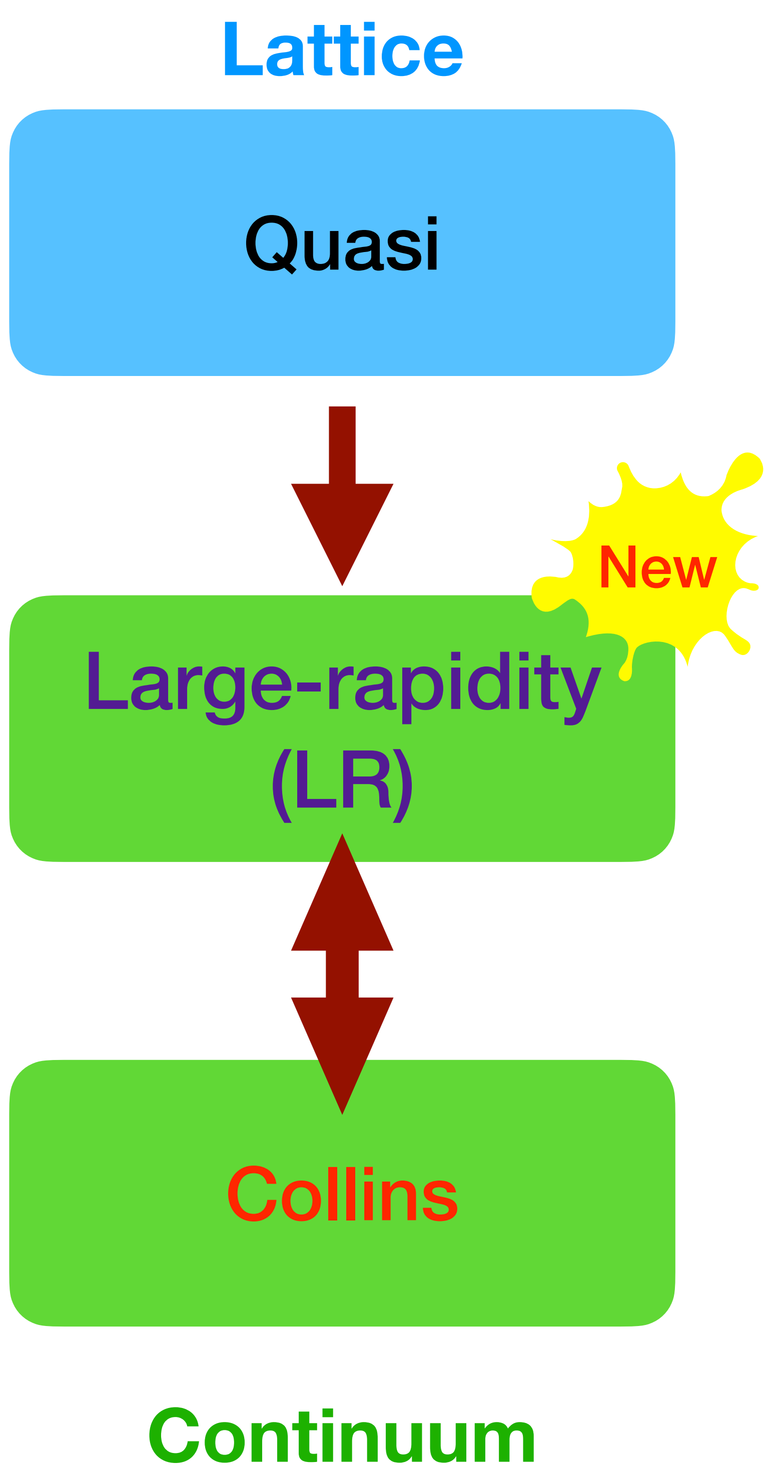


Equal-time Wilson lines, directly calculable on the lattice🙂

Related by Lorentz invariance ($y_{\tilde{P}} = y_P - y_B$), equivalent in the large $\tilde{P}^z$ or $(-y_B)$ expansion.

Ebert, Schindler, Stewart and YZ, JHEP **04**, 178 (2022).

Factorization relation with the TMDs



$$\tilde{f}_i(x, \mathbf{b}_T, \mu, \tilde{\zeta}, \tilde{P}^z) = \lim_{\tilde{P}^z \gg m_N} \lim_{a \to 0} \tilde{Z}_{\rm UV} \frac{\tilde{B}_i}{\sqrt{S^q}}$$

Lorentz invariance $\quad y_{\tilde{P}} = y_P - y_B$

$$f_i^{\rm LR}(x, \mathbf{b}_T, \mu, \zeta, y_P - y_B) = \lim_{-y_B \gg 1} \lim_{\epsilon \to 0} Z_{\rm UV}^{\rm LR} \frac{B_i}{\sqrt{S^q}}$$

Same matrix elements, but different orders of UV limits

Perturbative matching in LaMET!

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\rm UV} \lim_{y_B \to -\infty} \frac{B_i}{\sqrt{S^q}}$$

Ebert, Schindler, Stewart and YZ, JHEP **04**, 178 (2022).

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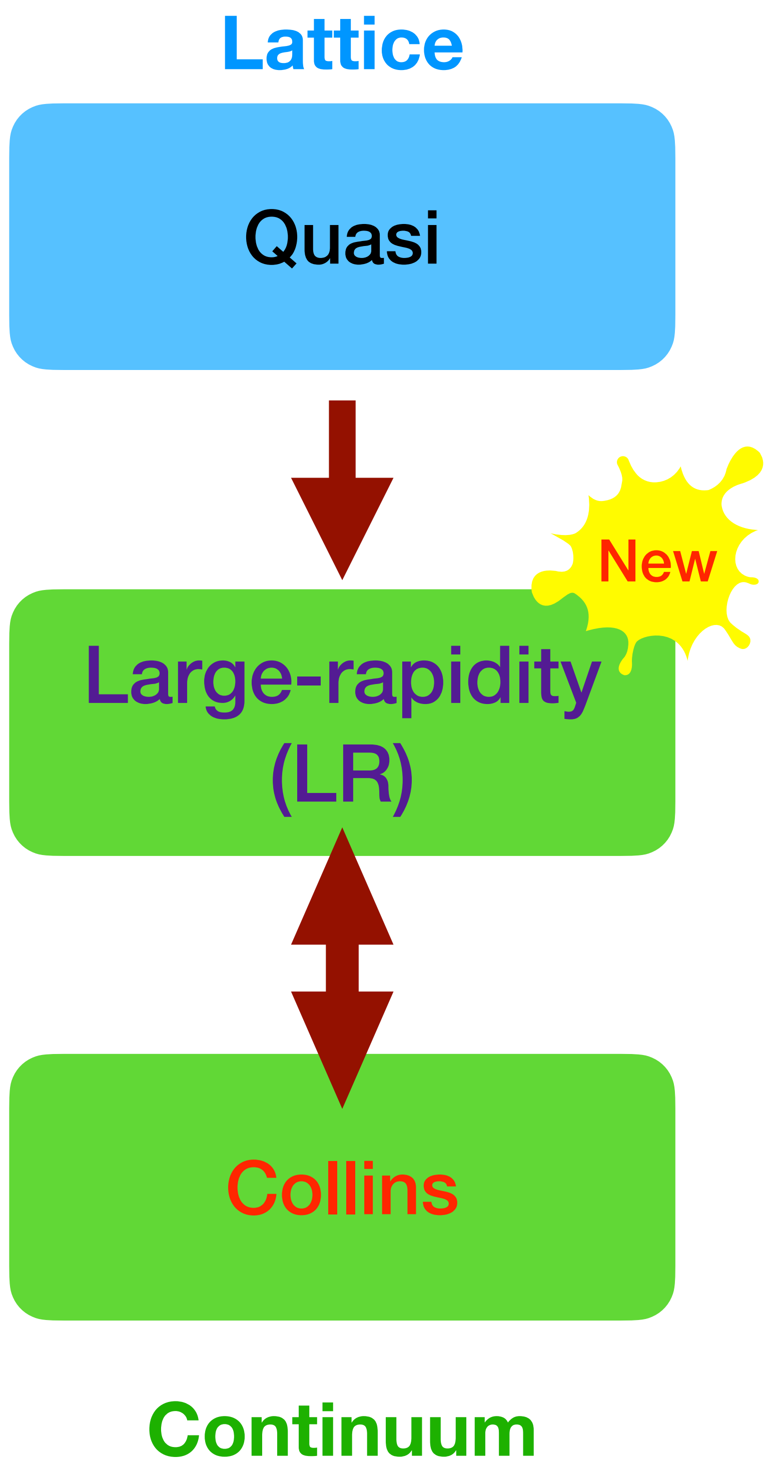


$$\tilde{f}_i(x, \mathbf{b}_T, \mu, \tilde{\zeta}, \tilde{P}^z) = \lim_{\tilde{P}^z \gg m_N} \lim_{a \to 0} \tilde{Z}_{\mathrm{UV}} \frac{\tilde{B}_i}{\sqrt{S^q}}$$

Lorentz invariance

$y_{\tilde{P}} = y_P - y_B$

$$f_i^{\mathrm{LR}}(x, \mathbf{b}_T, \mu, \zeta, y_P - y_B) = \lim_{-y_B \gg 1} \lim_{\epsilon \to 0} Z_{\mathrm{UV}}^{\mathrm{LR}} \frac{B_i}{\sqrt{S^q}}$$

Same matrix elements, but different orders of UV limits

Perturbative matching in LaMET!

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{UV}} \lim_{y_B \to -\infty} \frac{B_i}{\sqrt{S^q}}$$

Ebert, Schindler, Stewart and YZ, JHEP **04,** 178 (2022).

Factorization relation with the TMDs

- **Factorization of quasi-TMD:**

$$\tilde{f}_{q/h}(x, \mathbf{b}_T, \mu, \tilde{\zeta}, x\tilde{P}^z) = C(x\tilde{P}^z, \mu)\exp\left[\frac{1}{2}K^q(\mu, b_T)\ln\frac{\tilde{\zeta}}{\zeta}\right] f_{i/h}(x, \mathbf{b}_T, \mu, \zeta) + \mathcal{O}(y_{\tilde{P}}^{-k} e^{-y_{\tilde{P}}})$$

$\tilde{\zeta} = x^2 m_N^2 e^{2\tilde{y}_P + 2y_B - 2y_n}$

Matching coefficient

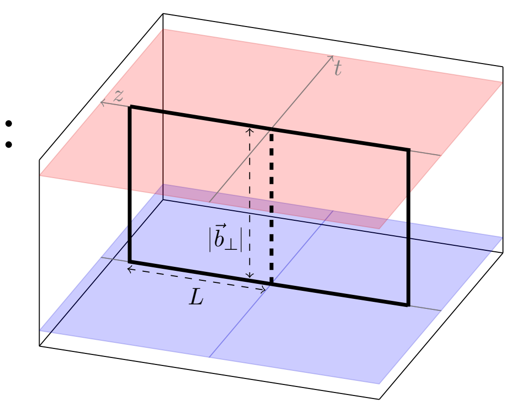
Warning:
soft function still not calculable on the lattice

- **Factorization of naive quasi-TMD:**

$$\tilde{f}_{i/h}^{\text{naive}} = \lim_{a\to 0} \tilde{Z}_{\text{uv}} \tilde{B}_{i/h} \Big/ \sqrt{\tilde{S}^q_{\text{naive}}}$$

$$\frac{\tilde{f}_{i/h}^{\text{naive}}}{\sqrt{S_r^q(b_T, \mu)}} = C(x\tilde{P}^z, \mu)\exp\left[\frac{1}{2}K^q(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/h}\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

$\tilde{S}^q_{\text{naive}}$:



Directly calculable on the lattice!

Reduced soft function ✓

Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020).

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- **Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).**

TMDs from lattice QCD

$$\frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S^q_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f^{[s]}_{i/p}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda^2_{\text{QCD}}}{(x\tilde{P}^z)^2}\right]\right\}$$

Matching coefficient:

- Independent of spin;
- No quark-gluon or flavor mixing, which makes gluon calculation much easier.

- Vladimirov and Schäfer, PRD 101 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
- Ji, Liu, Schäfer and Yuan, PRD 103 (2021).

One-loop matching for gluon TMDs:

Ebert, Schindler, Stewart and YZ, 2205.12369.

TMDs from lattice QCD

$$\frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S^q_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \ \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f^{[s]}_{i/p}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda^2_{\text{QCD}}}{(x\tilde{P}^z)^2}\right]\right\}$$

- ✳ **Collins-Soper kernel;**

$$K(\mu, b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

- ✳ **Flavor separation;**

$$\frac{f^{[s]}_{i/p}(x, \mathbf{b}_T)}{f^{[s']}_{j/p}(x, \mathbf{b}_T)} = \frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T)}{\tilde{f}^{\text{naive}[s']}_{j/p}(x, \mathbf{b}_T)}$$

- ✳ **Spin-dependence, e.g., Sivers function;**
- ✳ **Full TMD kinematic dependence.**

Ji, Liu, Schäfer and Yuan, PRD 103 (2021).

- ✳ **Twist-3 PDFs from small b_T expansion of TMDs.**

Outline

- TMDs in the non-perturbative region
- Quasi-TMDs in Large-Momentum Effective Theory
- **Lattice calculations**
- Outlook

Collins-Soper kernel from lattice QCD

$$K^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}(\mu, xP_2^z)\, \tilde{B}_{\rm ns}(x, \mathbf{b}_T, \mu, P_1^z)}{C_{\rm ns}(\mu, xP_1^z)\, \tilde{B}_{\rm ns}(x, \mathbf{b}_T, \mu, P_2^z)} + \text{power corrections}$$

Studying CS kernel through quasi-TMDs suggested in

- Ji, Sun, Xiong and Yuan, PRD91 (2015);

The concrete formalism first derived in

- Ebert, Stewart and YZ, PRD 99 (2019).

Does not depend on the external hadron state, could be calculated with pion TMD or wave function (vacuum to pion amplitude) for simplicity;

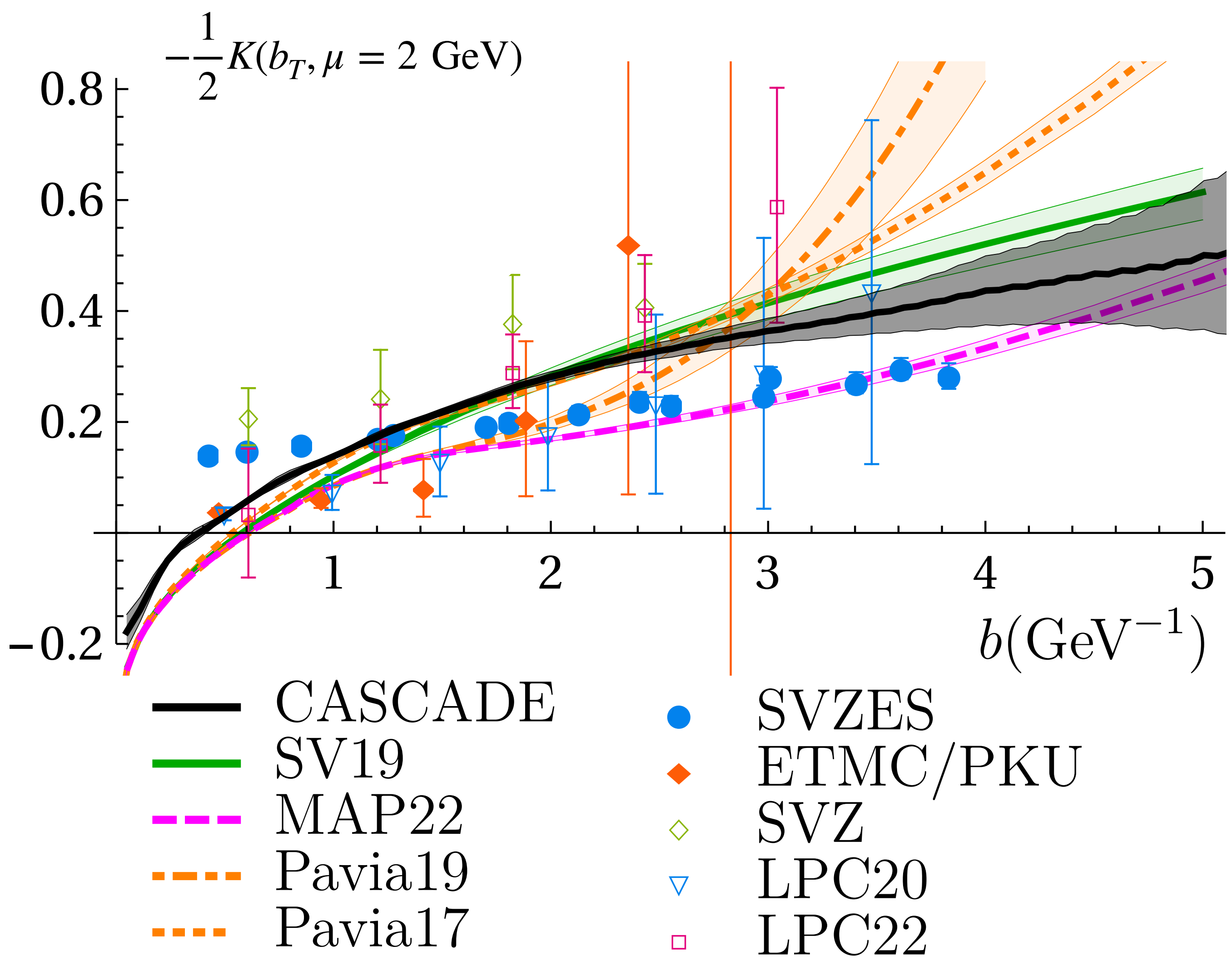
- Shanahan, Wagman and YZ, PRD 102 (2020);
- Ebert, Stewart and YZ, PRD 99 (2019);
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020).

Current status for the Collins-Soper kernel

	Lattice setup	Renormalization	Operator mixing	Fourier transform	Matching	x-plateau search
SWZ20 PRD 102 (2020) Quenched	$a = 0.06$ fm, $m_\pi = 1.2$ GeV, $P^z_{\max} = 2.6$ GeV	Yes	Yes	Yes	LO	Yes
LPC20 PRL 125 (2020)	$a = 0.10$ fm, $m_\pi = 547$ MeV, $P^z_{\max} = 2.11$ GeV	N/A	No (small)	N/A	LO	N/A
SVZES 21 JHEP 08 (2021)	$a = 0.09$ fm, $m_\pi = 422$ MeV, $P^+_{\max} = 2.27$ GeV	N/A	No	N/A	NLO	N/A
PKU/ETMC 21 PRL 128 (2022)	$a = 0.09$ fm, $m_\pi = 827$ MeV, $P^z_{\max} = 3.3$ GeV	N/A	No	N/A	LO	N/A
SWZ21 PRD 104 (2021)	$a = 0.12$ fm, $m_\pi = 580$ MeV, $P^z_{\max} = 1.5$ GeV	Yes	Yes	Yes	NLO	Yes
LPC22 2204.00200	$a = 0.12$ fm, $m_\pi = 670$ MeV, $P^z_{\max} = 2.58$ GeV	Yes	No (small)	Yes	NLO	Yes

Collins Soper kernel

Comparison between lattice results and global fits

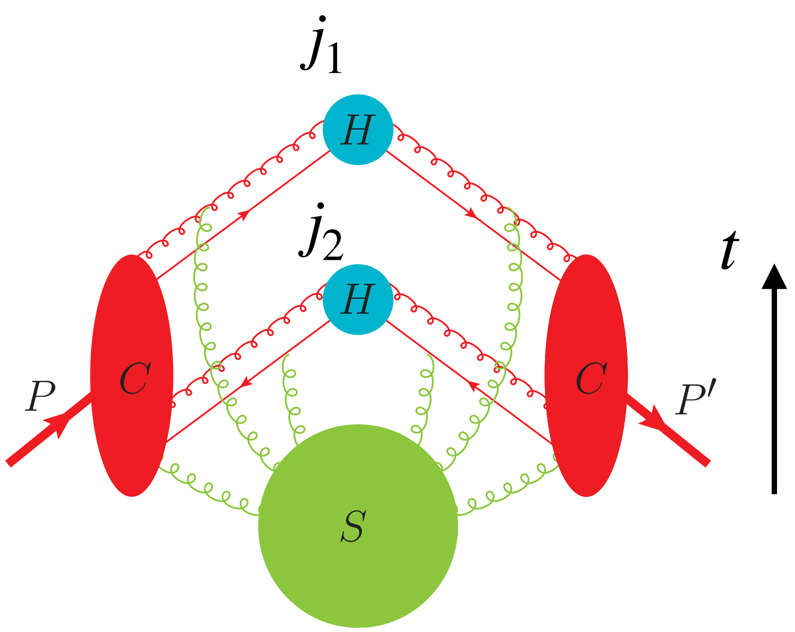


Approach	Collaboration
Quasi-Beam Functions	**SWZ 21** PRD 104 (2021)
Quasi-TMD Wavefunctions	**LPC 20** PRL 125 (2020)
	ETMC/PKU 21 PRL 128 (2022)
	LPC 22 2204.00200
Mellin Moments of Quasi-TMDs	**SVZES 21** JHEP 08 (2021)

MAP22: Bacchetta, Bertone, Bissolotti, et al., 2206.07598
SV19: I. Scimemi and A. Vladimirov, JHEP 06 (2020) 137
Pavia19: A. Bacchetta et al., JHEP 07 (2020) 117
Pavia 17: A. Bacchetta et al., JHEP 06 (2017) 081
CASCADE: Martinez and Vladimirov, 2206.01105

Reduced soft function from LaMET

Light-meson form factor: $F(b_T, P^z) = \langle \pi(-P) | j_1(b_T) j_2(0) | \pi(P) \rangle$



$$\stackrel{P^z \gg m_N}{=} S_q^r(b_T, \mu) \int dx dx' \; H(x, x', \mu)$$

$$\times \Phi^\dagger(x, b_T, P^z) \Phi(x', b_T, P^z)$$

Φ: Quasi-TMD wave function

$$\tilde{\Phi} = \frac{\langle 0 | \mathcal{O}(b^\mu) | \pi(P) \rangle}{\sqrt{\tilde{S}^q_{\text{naive}}}}$$

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD 105, 076014 (2022);
- Deng, Wang and Zeng, 2207.07280.

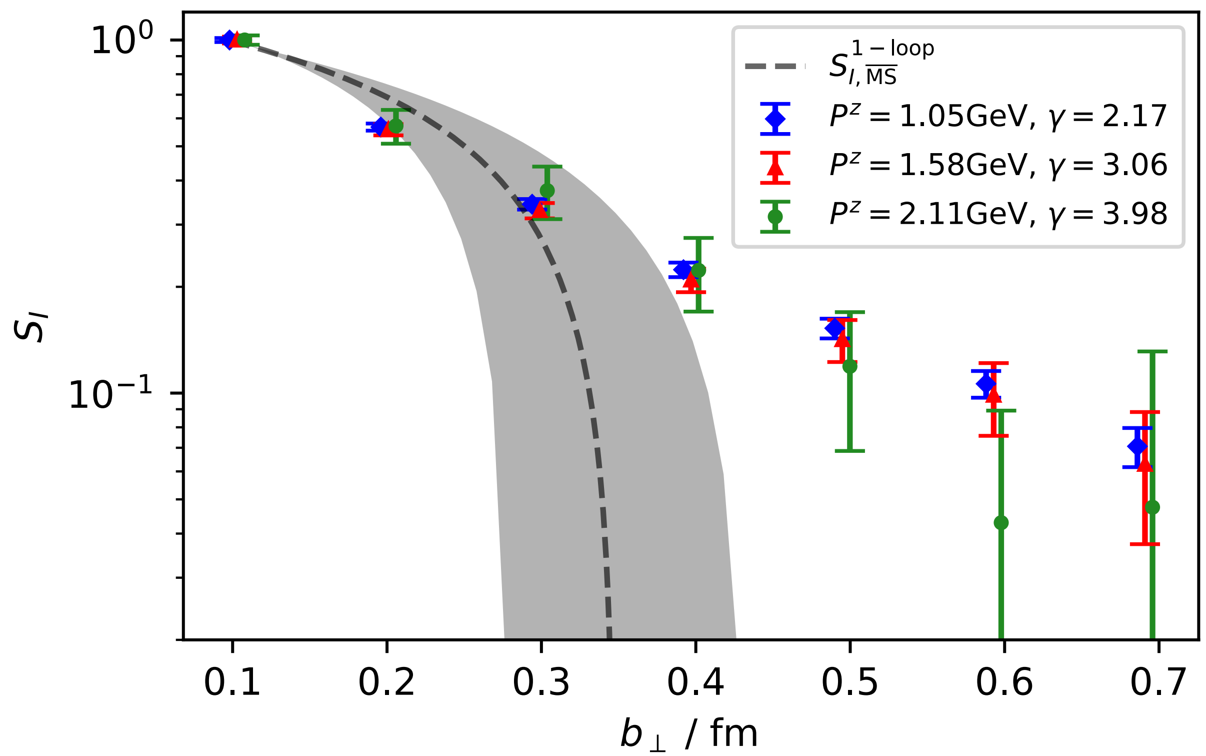
Tree-level approximation:

$$H(x, x', \mu) = 1 + \mathcal{O}(\alpha_s)$$

$$\Rightarrow S_q^r(b_T) = \frac{F(b_T, P^z)}{[\tilde{\Phi}(b^z = 0, b_T, P^z)]^2}$$

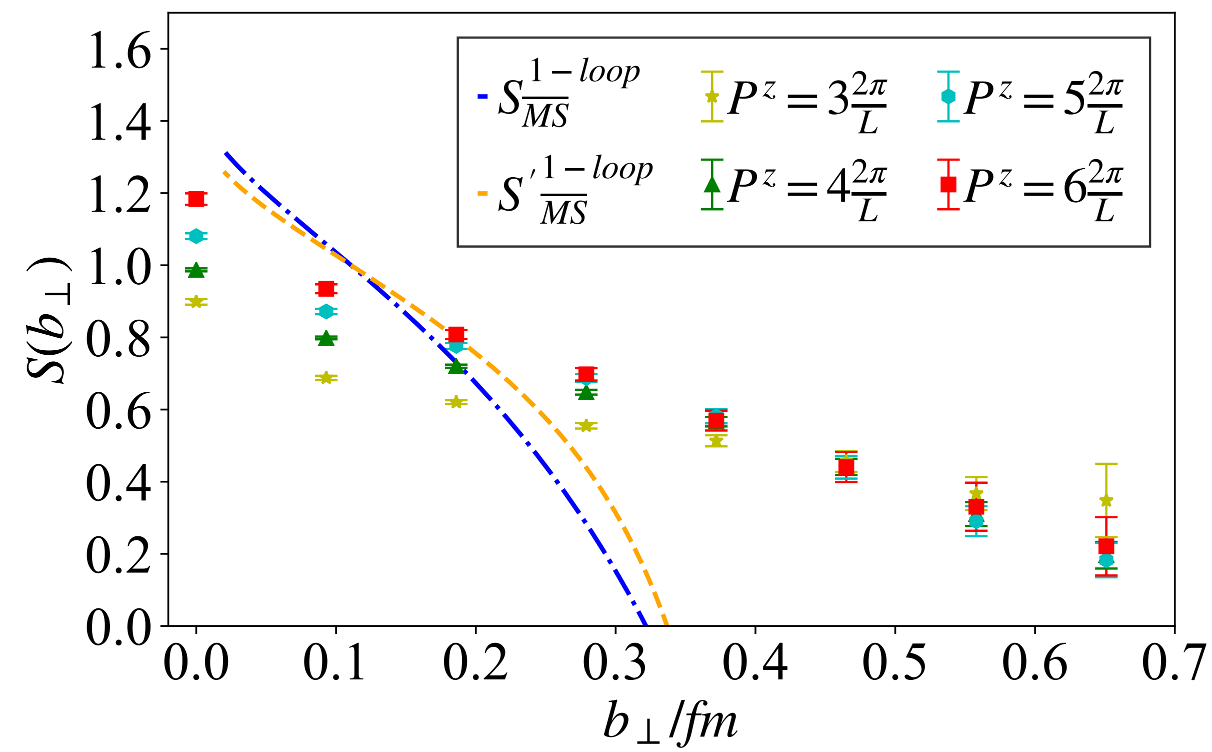
First lattice results with tree-level matching

$a = 0.10$ fm,
$m_\pi = 547$ MeV,
$P^z_{\max} = 2.11$ GeV



Q.-A. Zhang, et al. (LPC), PRL 125 (2020).

$a = 0.09$ fm,
$m_\pi = 827$ MeV,
$P^z_{\max} = 3.3$ GeV



Y. Li et al., PRL 128 (2022).

Beyond tree-level, it is necessary to obtain the *x*-dependence to carry out the convolution.

Conclusion

- The quark and gluon quasi TMDs can be related to the new LR scheme, which can be factorized into the physical TMDs;
- There is no mixing between quarks of different flavors, quark and gluon channels, or different spin structures.
- The method for calculating all the leading-power TMDs is complete;
- Lattice results for the Collins-Soper kernel and soft function are promising, but systematics need to be under control.

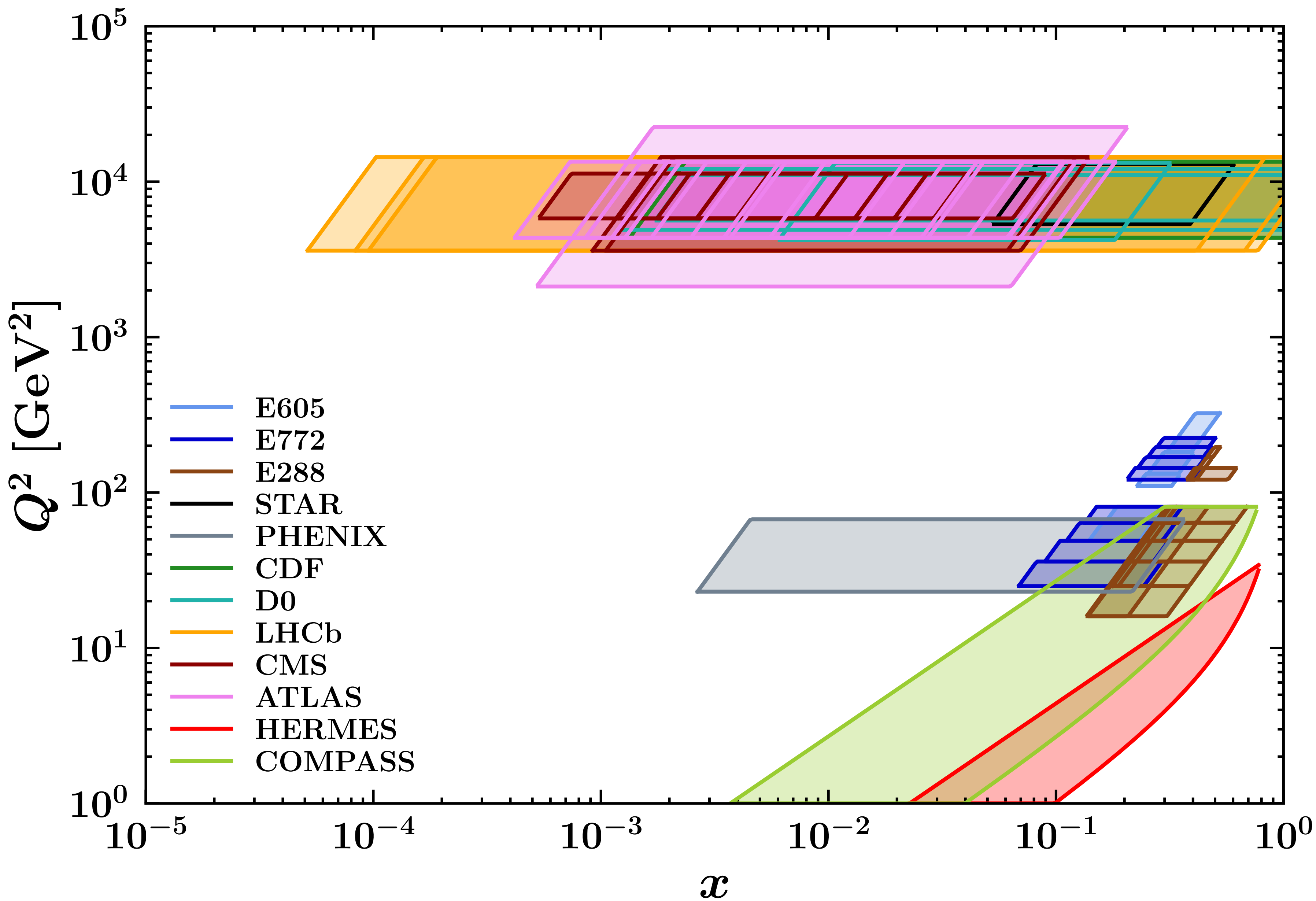
Outlook

Targets for lattice QCD studies:

Observables	Status
Non-perturbative Collins-Soper kernel	✔, improving the systematics
Soft factor	✔, to be under systematic control
Info on spin-dependent TMDs (in ratios)	In progress
Proton v.s. pion TMDs, (x, b_T) (in ratios)	In progress
Flavor dependence of TMDs, (x, b_T) (in ratios)	to be studied
TMDs and TMD wave functions, (x, b_T)	In progress
Gluon TMDs (x, b_T)	to be studied
Wigner distributions/GTMDs (x, b_T)	to be studied

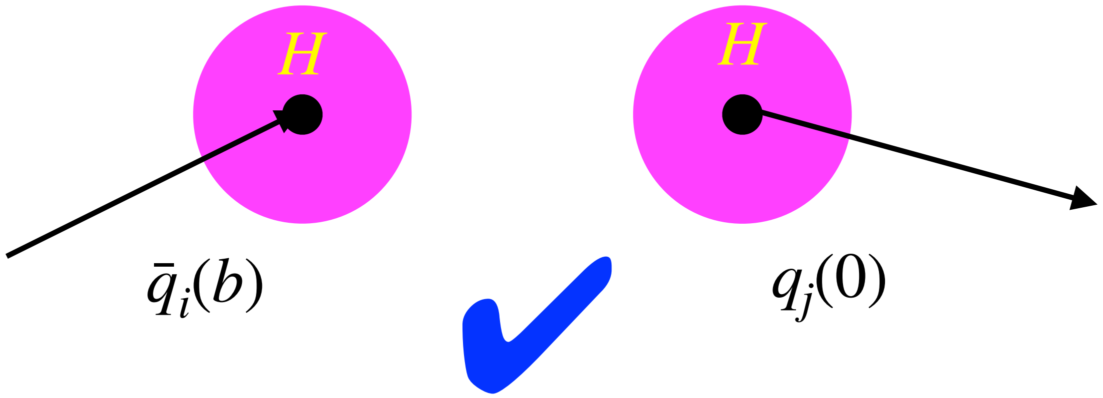
Backup slides

Data used by the MAP collaboration in 2206.07598

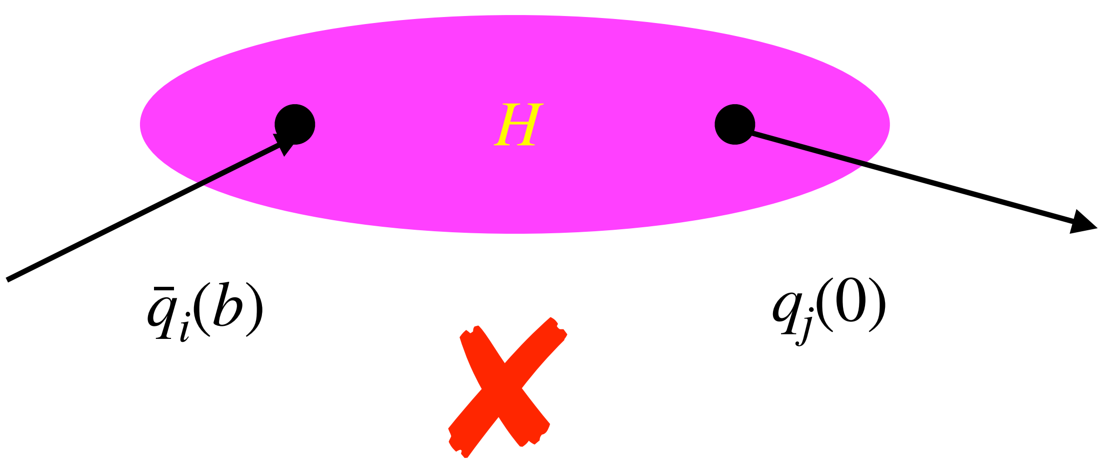


Bacchetta, Bertone, Bissolotti, et al., MAP Collaboration, 2206.07598

Backup slides



i, j **(including spinor indices) remain intact**



$\propto \delta_{ij}$ **Can mix with singlet channel and with gluons**

$$b^2 = -b_z^2 - b_T^2 < b_T^2 \sim 1/\Lambda_{\rm QCD}^2$$

Hard particles cannot propagate that far!