Matching of Fracture Functions for SIDIS in Target Fragmentation Region

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Based on JHEP 11 (2021) 038 with Kai-Bao Chen, Jian-Ping Ma

Matching of Fracture Functions for SIDIS in Target Fragmentation Region

- Introduction
  - SIDIS and Target Fragmentation Region
  - Fracture Functions
- Matching of Fracture Functions
  - Unpolarized/Longitudinal Fracture Functions — Twist-2 level
  - Transversely Polarized Fracture Functions
    - Single transverse-spin asymmetry(T-odd)
    - Double-spin asymmetry(T-even)

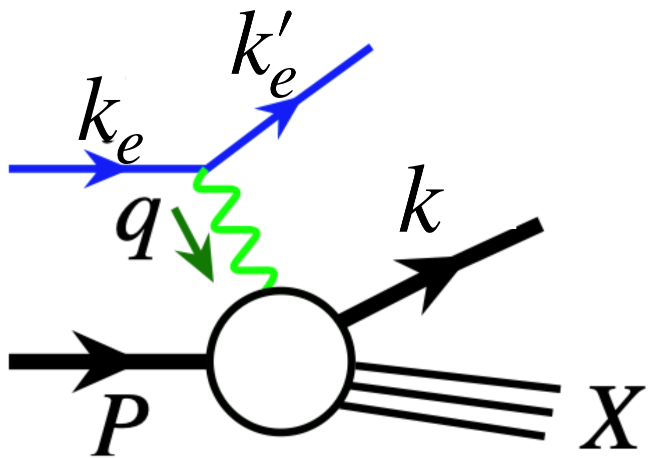
    Twist-3 level

- Summary & Outlook

Introduction

Semi-Inclusive Deep Inelastic Scattering(SIDIS)

$$e(k_e,\lambda_e)+h(P,s)\to e(k_e')+h'(k)+X$$



$$Q^2=-q^2\gg\Lambda_{QCD}^2$$

$$x_B=\frac{Q^2}{2p\cdot q}$$

- Important process to probe the partonic structure of the hadrons:

    3D structure, nucleon spin structure……

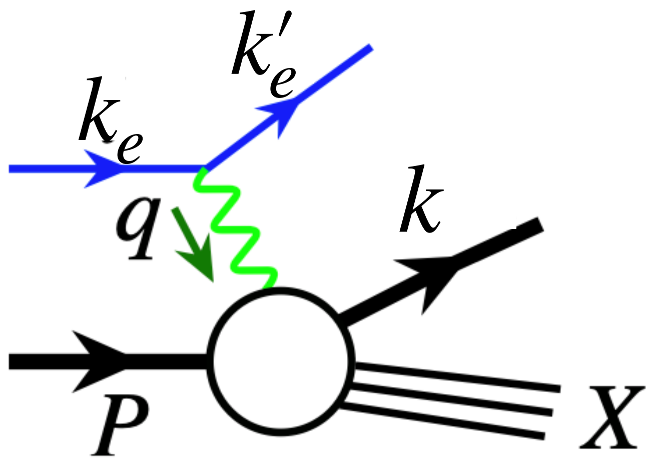
- Experiments: HEMERS, COMPASS, JLab@12GeV, U.S. Eic, EicC…

- QCD Factorizations in specific kinematics.

Introduction

Semi-Inclusive Deep Inelastic Scattering(SIDIS)

$$e(k_e, \lambda_e) + h(P, s) \to e(k_e') + h'(k) + X$$

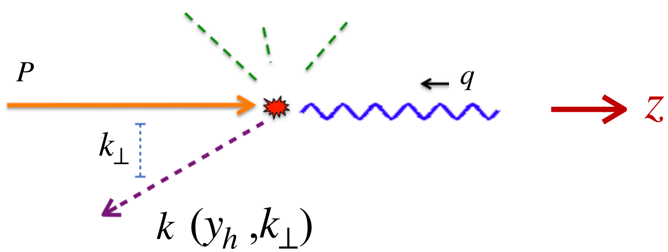


$$Q^2 = -q^2 \gg \Lambda_{QCD}^2$$

$$x_B = \frac{Q^2}{2p \cdot q}$$

- Different kinematical regions lead to different QCD factorizations and different probes.

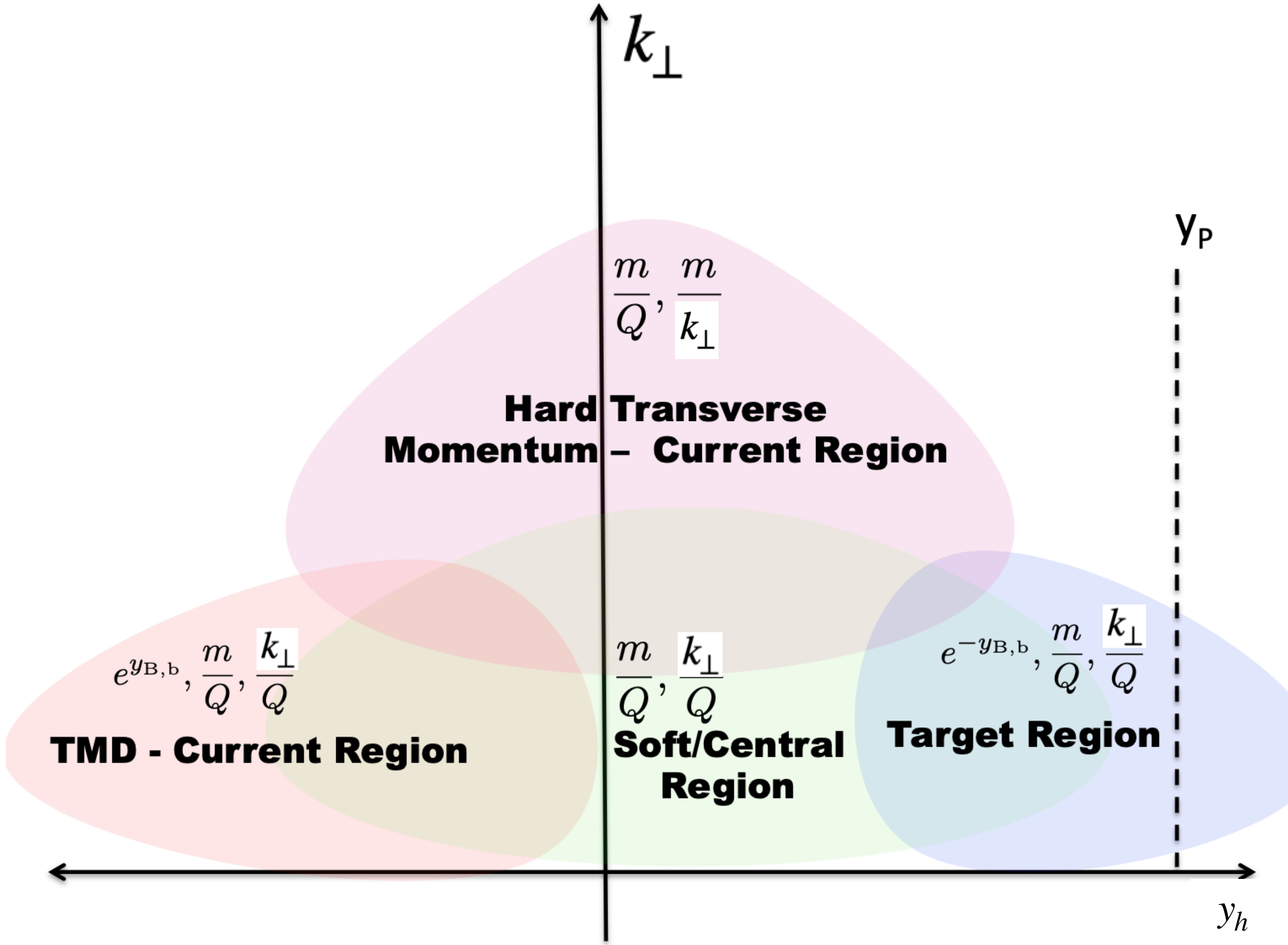
Photon-proton frame:



- Classifications according to the momentum of the detected hadron.

Introduction

Sketch of the kinematic regions of produced hadron



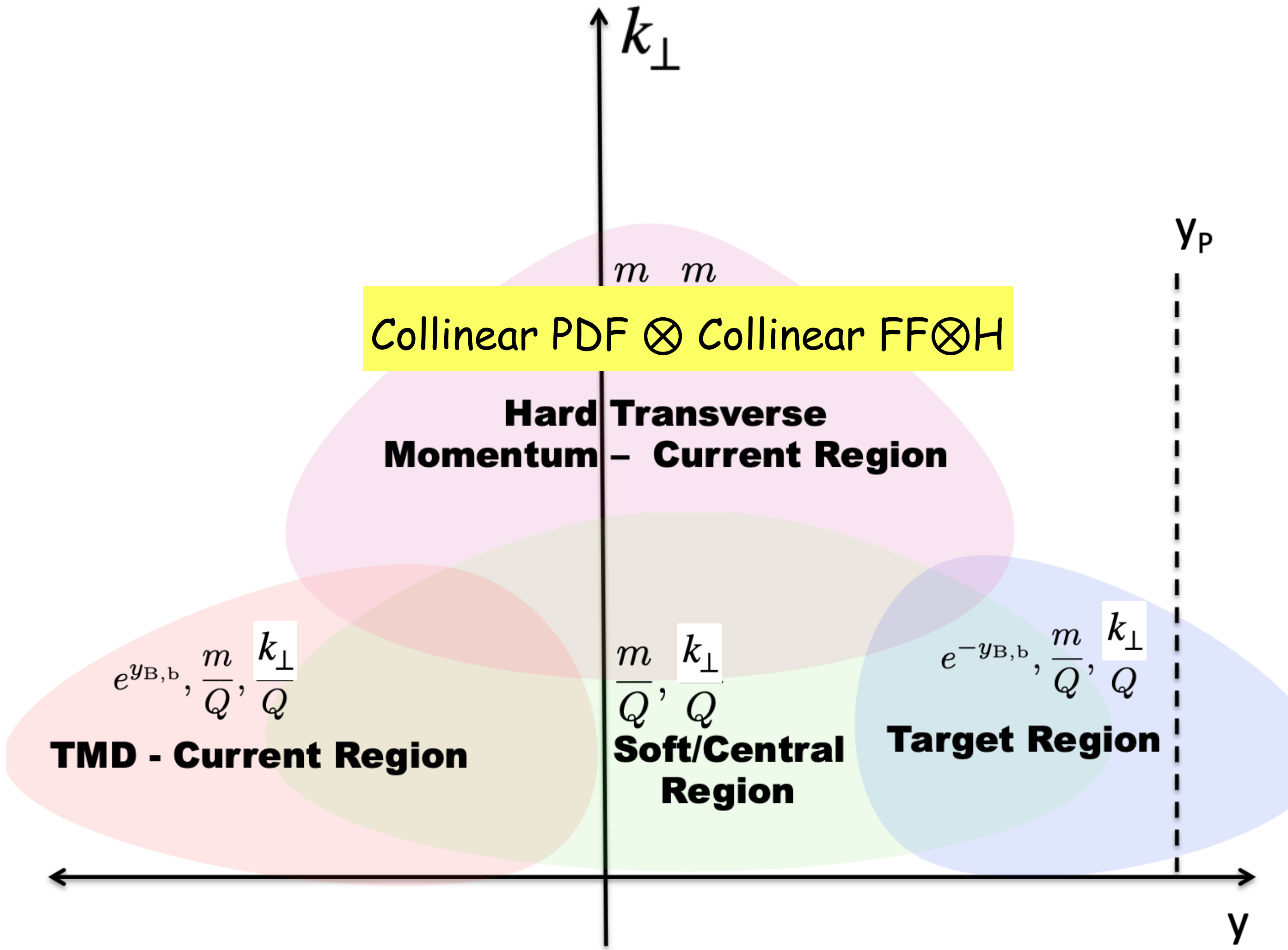
y_h : rapidity

[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

$k_\perp$: transverse momentum

Introduction

Sketch of the kinematic regions of produced hadron



y_h : rapidity

$k_{h\perp}$: transverse momentum

[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

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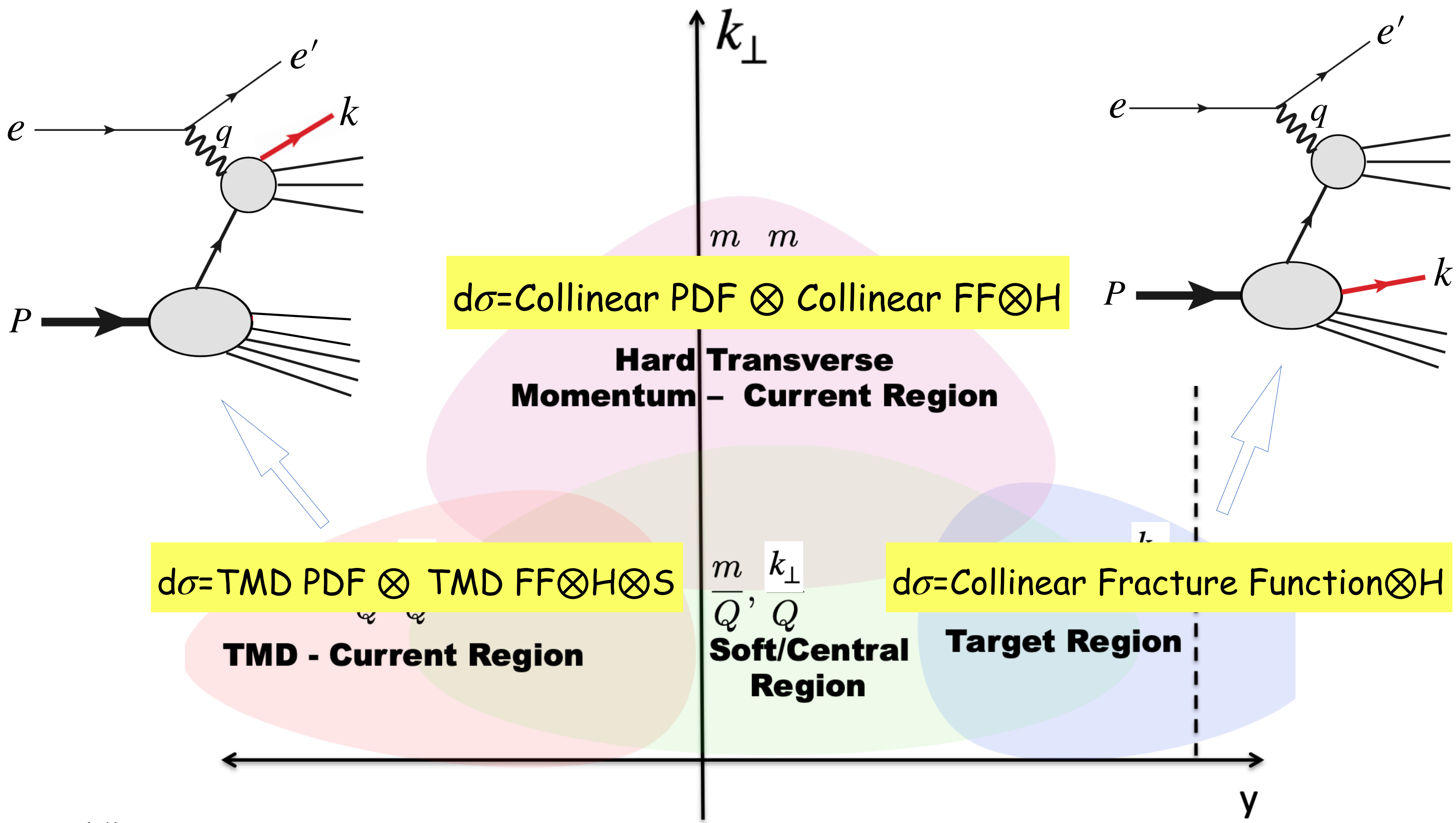

y_h : rapidity

$k_\perp$: transverse momentum

[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

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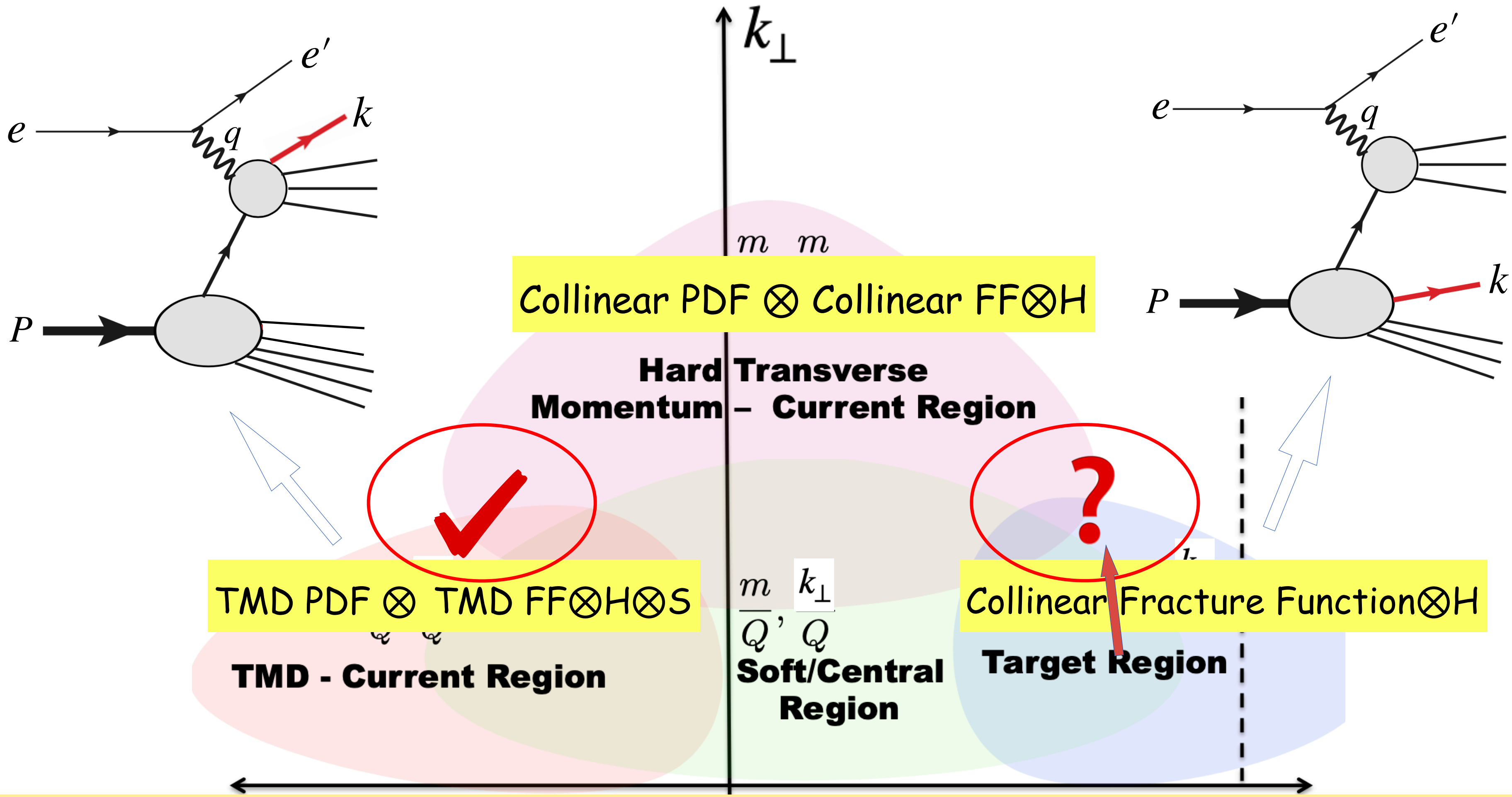
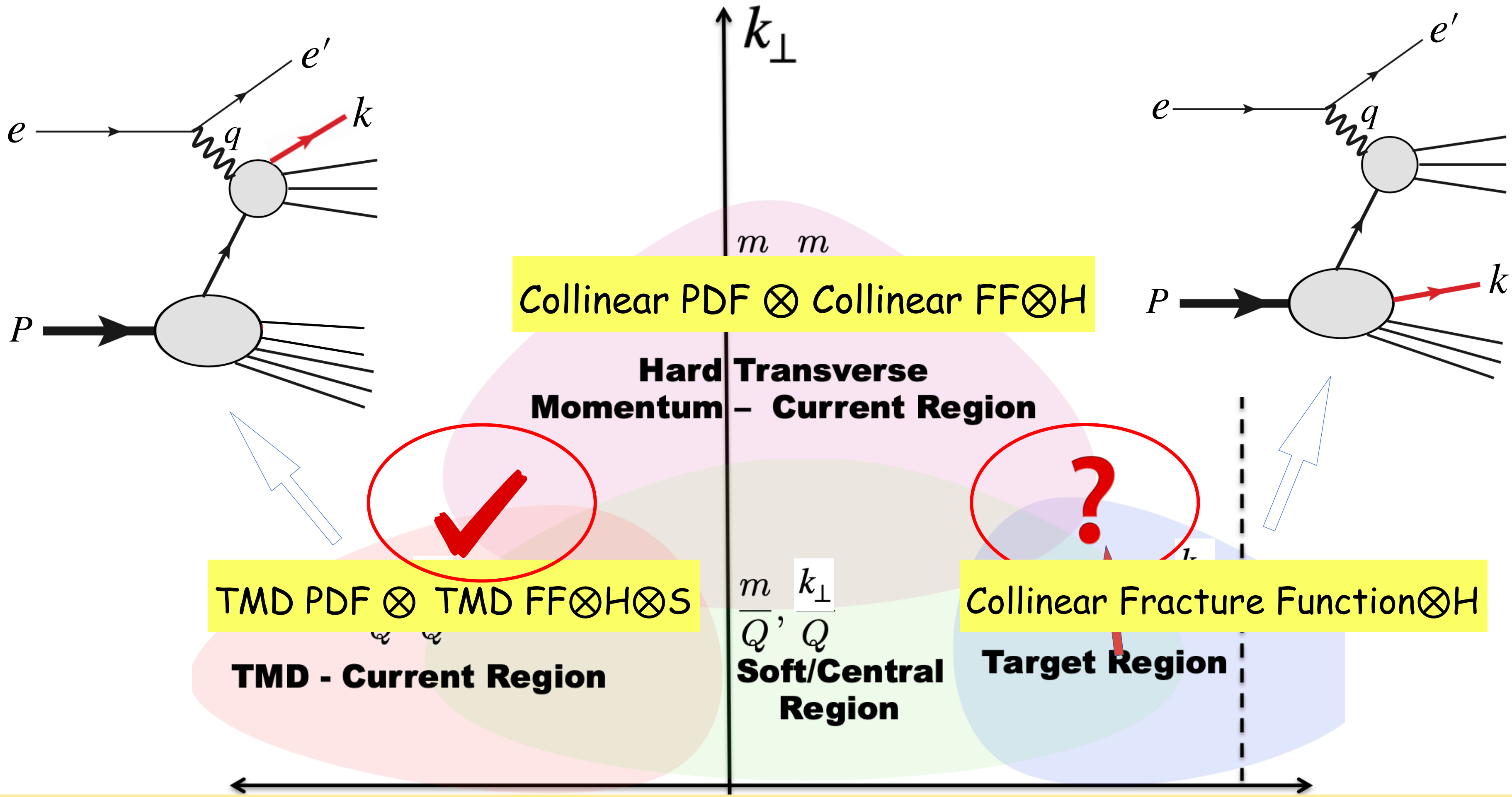


Study the factorization of the quark Collinear Fracture Functions at $\Lambda_{QCD} \ll k_\perp \ll Q$ for SIDIS in the target fragmentation region($y_h \gg \mathcal{O}(\ln k_\perp/Q)$) at tree level .

[K.B Chen, J.P. Ma, X.B. Tong JHEP 11 (2021) 038]

Introduction

Sketch of the kinematic regions of produced hadron



Collinear fracture function=Collinear (multi)parton distribution ⊗ Collinear FF⊗H

$\Lambda_{QCD} \ll k_\perp \ll Q$

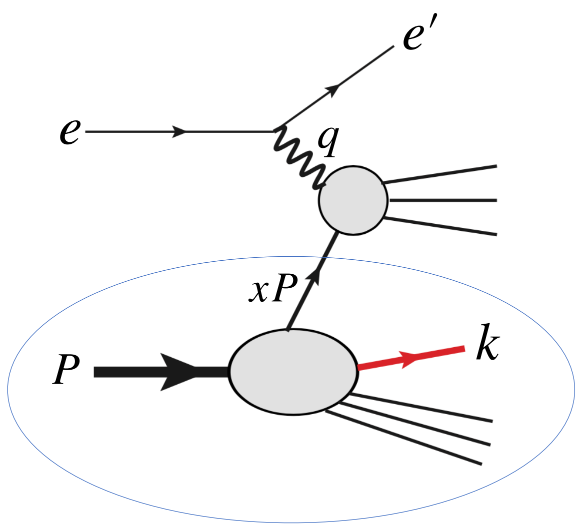
[K.B Chen, J.P. Ma, X.B. Tong JHEP 11 (2021) 038]

Introduction

SIDIS in Target Fragmentation Region(TFR) at $k_\perp \ll Q$

$$d\sigma = \text{Collinear Fracture Function} \otimes H$$

Proof(twist-2) : J.C. Collins Phys. Rev. D 57 (1998) 3051



$F(x, \xi, k_\perp)$

- Fracture function(Diffractive parton distribution):
    - Parton distributions of an initial hadron in the presence of an almost collinear particle observed in the final state.

- Introduced by [Trentadue-Veneziano Phys. Lett. B 323 (1994) 201]
- Operator definition for quark/gluon given in [Berera-Soper Phys. Rev. D 53 (1996) 6162].
- TMD quark fracture function: [Anselmino-Barone-Kotzinian, Phys. Lett. B 699 (2011) 108].
- Other applications: hadron collisions

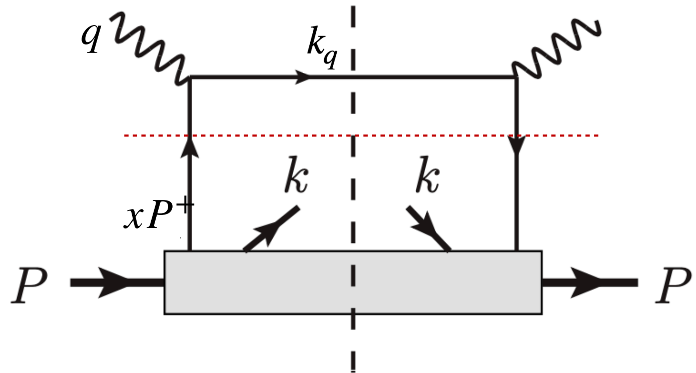
e.g. Chen-Ma-Tong, JHEP 10 (2019) *285*
Ceccopieri -Trentadue, Phys. Lett. B 668 (2008) 319
Ceccopieri, Phys. Lett. B 703 (2011) 491

Introduction

SIDIS in Target Fragmentation Region(TFR) at $k_\perp \ll Q$

- Hadron Tensor

$$W^{\mu\nu} = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{iq\cdot x} \langle P, s | J^\mu(x) | k, X\rangle \langle X, k | J^\nu(0) | P, s\rangle.$$



Leading Order

- NLO calculation:
  D. Graudenz, Nucl. Phys. B 432 (1994) 351
- All-order proof(twist-2):
  J.C. Collins Phys. Rev. D 57 (1998) 3051

Density matrix of the Quark Collinear Fracture Function:

$$P^\mu = (P^+, P^-, 0_\perp)$$
$$k^\mu = (\xi P^+, k^-, k_\perp)$$

$$\mathcal{M}_{Fij}(x, \xi, k_\perp) = \int \frac{d\lambda}{2\pi} e^{-ixP^+\lambda} \sum_X \langle h_A(P,s) | [\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)]_j | X h_B(k)\rangle \langle h_B(k) X \,| [\mathcal{L}_n(0)\psi(0)]_i |\, h_A(P,s)\rangle$$

- "Hybrids" between the PDF and FF $\quad \mathcal{L}_n(x) = \mathrm{P}\exp\left\{ -ig_s \int_0^\infty d\lambda G^+(\lambda n + x)\right\} \quad n^\mu = (0,1,0,0)$

Introduction

SIDIS in Target Fragmentation Region(TFR) at $k_\perp \ll Q$

- Density matrix of the Quark Fracture Function:

$$\mathcal{M}_{Fij}(x,\xi,k_\perp)$$
$$= \int \frac{d\lambda}{2\pi} e^{-ixP^+\lambda} \sum_X \langle h_A(P,s)|[\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)]_j|X h_B(k)\rangle \langle h_B(k)X\,|[\mathcal{L}_n(0)\psi(0)]_i|\,h_A(P,s)\rangle$$

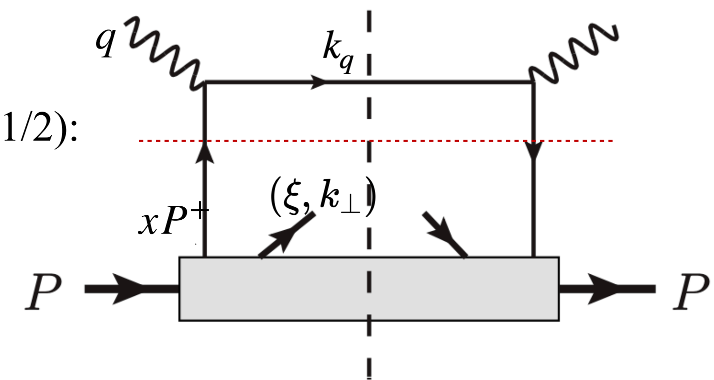
At the leading power,

$$= \frac{1}{2N_c}(\gamma^-)_{ij}\left[F_q(x,\xi,k_\perp) + \epsilon_\perp^{\mu\nu}k_{\perp\mu}s_{\perp\nu}\frac{1}{m_A}F_{qT}(x,\xi,k_\perp)\right]$$
$$+ \frac{1}{2N_c}(\gamma_5\gamma^-)_{ij}\left[s_L\Delta F_q(x,\xi,k_\perp) + k_\perp\cdot s_\perp\frac{1}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\right] + \cdots$$

[M. Anselmino et al Phys. Lett. B 699 (2011) 108]

- Chirality even
- Only consider the polarization of the initial hadron(spin-1/2):

$$s^\mu = s_L\frac{\bar n^\mu P^+ - n^\mu P^-}{m_A} + s_\perp^\mu$$

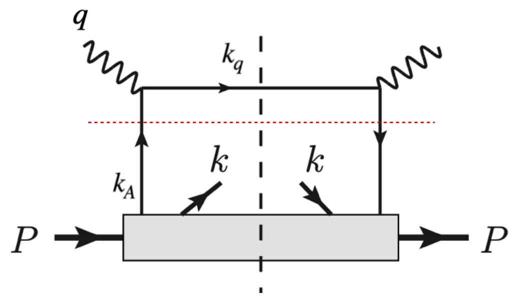


Introduction

SIDIS in Target Fragmentation Region(TFR) at $k_\perp \ll Q$

$$e\,(k_e,\lambda_e) + h_A(P,s) \to e\,(k'_e,\lambda_e) + h_B(k) + X$$

$$Q^2 = -q^2, \quad x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot k_e}$$



Differential Cross-section

$$\frac{d\sigma}{dx_B dy d\xi d\phi_s d^2k_\perp} = \frac{\alpha^2}{2(2\pi)^3\xi y Q^2}\Big[(2-2y+y^2)\Big(F_q(x_B,\xi,k_\perp) + \frac{|k_\perp||s_\perp|}{m_A}\sin(\phi_s-\phi_h)F_{qT}(x_B,\xi,k_\perp)\Big)$$
$$+\,y(2-y)\lambda_e\Big(s_L\Delta F_q(x_B,\xi,k_\perp) - \frac{|k_\perp||s_\perp|}{m_A}\cos(\phi_s-\phi_h)\Delta F_{qT}(x_B,\xi,k_\perp)\Big)\Big]$$

Single transverse spin asymmetry(SSA):

$$A_{UT} = \frac{d\sigma(\vec{s}_\perp) - d\sigma(-\vec{s}_\perp)}{d\sigma(\vec{s}_\perp) + d\sigma(-\vec{s}_\perp)} \propto F_{qT}$$

Double spin asymmetry(DSA):

$$A_{LL} \propto \Delta F_q$$

$$A_{LT} = \frac{d\sigma^+(\vec{s}_\perp) - d\sigma^+(-\vec{s}_\perp) - (d\sigma^-(\vec{s}_\perp) - d\sigma^-(-\vec{s}_\perp))}{d\sigma^+(\vec{s}_\perp) + d\sigma^+(-\vec{s}_\perp) + d\sigma^-(\vec{s}_\perp) + d\sigma^-(-\vec{s}_\perp)} \propto \Delta F_{qT}$$

Matching of Fracture Function for SIDIS in the Target Fragmentation Region

- Introduction
  - SIDIS and the Target Fragmentation region
  - Fracture Functions
- Matching of Fracture Functions
  - Unpolarized/Longitudinal Fracture Function Twist-2 level
  - Transversely Polarized Fracture Function
    - Single transverse-spin asymmetry
    - Double spin asymmetry

    Twist-3 level

- Conclusion&Outlook

Matching of Fracture Function

In the region, $\Lambda_{QCD} \ll k_\perp \ll Q$, the $k_\perp$-dependence is perturbative calculable:

$$\mathcal{M}_{Fij}(x,\xi,k_\perp) = \frac{1}{2N_c}\left(\gamma^-\right)_{ij}\left[F_q(x,\xi,k_\perp) + \epsilon_\perp^{\mu\nu}k_{\perp\mu}s_{\perp\nu}\frac{1}{m_A}F_{qT}(x,\xi,k_\perp)\right]$$
$$+\frac{1}{2N_c}\left(\gamma_5\gamma^-\right)_{ij}\left[s_L\Delta F_q(x,\xi,k_\perp) + k_\perp\cdot s_\perp\frac{1}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\right] + \cdots$$

We study the factorizations by collinear expansions at tree level :

- $F_q, \Delta F_q$: Unpolarized or longitudinal spin

Twist-2 matching: $\frac{1}{k_\perp^2}$ ⊗ twist-2 FF ⊗ twist-2 parton distributions

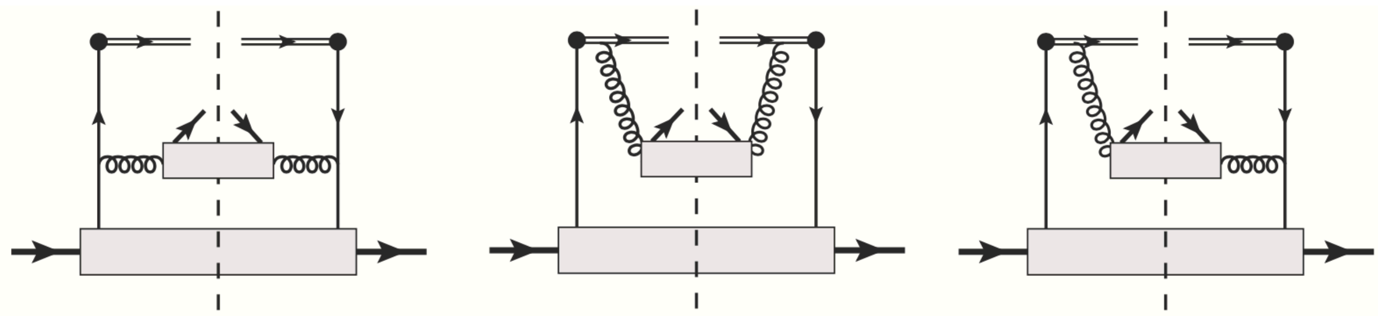
- $F_{qT}, \Delta F_{qT}$: Transverse spin

Twist-3 matching: $\frac{1}{(k_\perp^2)^2}$ ⊗ twist-2 FF ⊗ twist-3 parton distributions

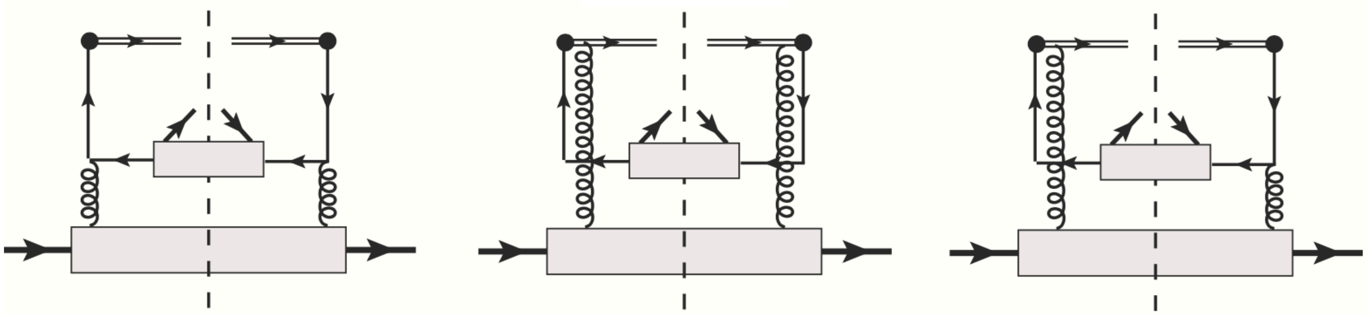
Helicity conservation -> No twist-3 FF ⊗ twist-2 distributions

Matching of Fracture Function

Unpolarized/Longitudinal Fracture Function



$\langle \psi(0)\bar{\psi}(x) \rangle$



$\langle G_a^\mu(x) G_b^\nu(0) \rangle$

At tree level :

$$F_q(x,\xi,k_\perp) = g_s^2 \frac{1}{k_\perp^2} \int \frac{dz}{z^2} \left[2C_F d_g(z) q(y) \frac{z^2}{y^2}(x^2+y^2) + d_{\bar{q}}(z) g(y) \frac{\xi}{zy^3}(z^2x^2+\xi^2) \right]$$

$$\Delta F_q(x,\xi,k_\perp) = g_s^2 \frac{1}{k_\perp^2} \int \frac{dz}{z^2} \left[2C_F d_g(z) \Delta q(y) \frac{z^2}{y^2}(x^2+y^2) + d_{\bar{q}}(z) \Delta g(y) \frac{\xi}{y^2}(xz-\xi) \right] \qquad y = x + \frac{\xi}{z}$$

Provide the access to the unpolarized/longitudinal PDF in TFR

Matching of Fracture Function

In the region, $\Lambda_{QCD} \ll k_\perp \ll Q$, the $k_\perp$-dependence is perturbative calculable:

$$\mathcal{M}_{Fij}(x,\xi,k_\perp) = \frac{1}{2N_c}\left(\gamma^-\right)_{ij}\left[F_q(x,\xi,k_\perp) + \epsilon_\perp^{\mu\nu} k_{\perp\mu} s_{\perp\nu} \frac{1}{m_A} F_{qT}(x,\xi,k_\perp)\right] + \frac{1}{2N_c}\left(\gamma_5\gamma^-\right)_{ij}\left[s_L \Delta F_q(x,\xi,k_\perp) + k_\perp \cdot s_\perp \frac{1}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\right] + \cdots$$

We study the factorizations by collinear expansions at tree level :

- $F_q, \Delta F_q$: Unpolarized or longitudinal spin

Twist-2 matching: $\frac{1}{k_\perp^2}$ $\otimes$ twist-2 FF $\otimes$ twist-2 parton distributions

Twist-3 matching is none-trivial even at tree level !

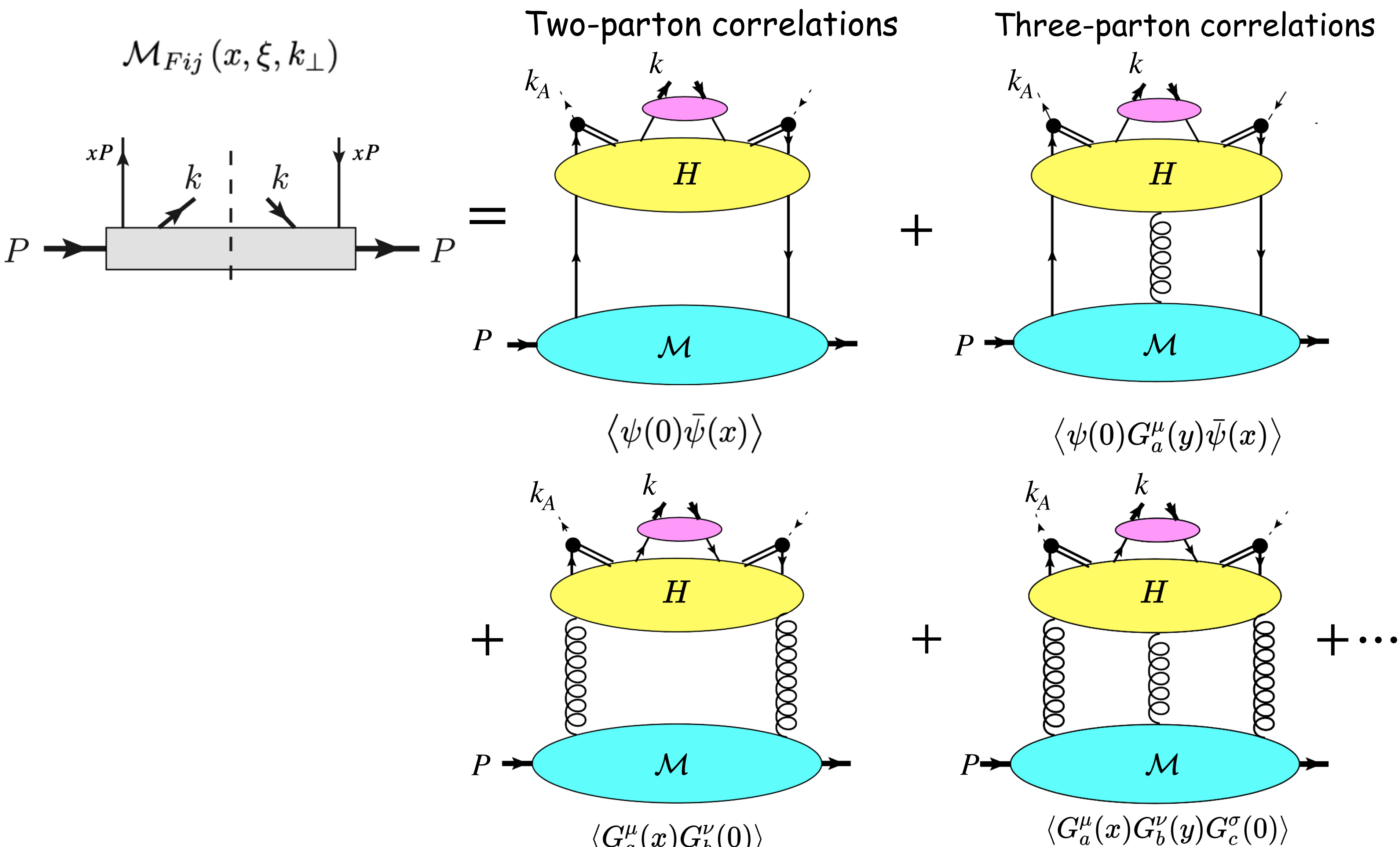
- $F_{qT}, \Delta F_{qT}$: Transverse spin

Twist-3 matching: $\frac{1}{(k_\perp^2)^2}$ $\otimes$ twist-2 FF $\otimes$ twist-3 parton distributions

It is not easy to obtain the gauge invariant results : multi-parton correlations

Matching of Fracture Function

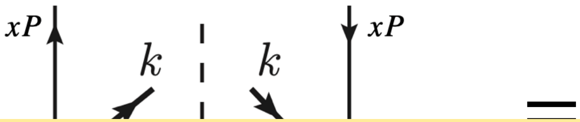
Transversely polarized Fracture Function



Matching of Fracture Function

Transversely polarized Fracture Function

$\mathcal{M}_{Fij}\left(x,\xi,k_\perp\right)$



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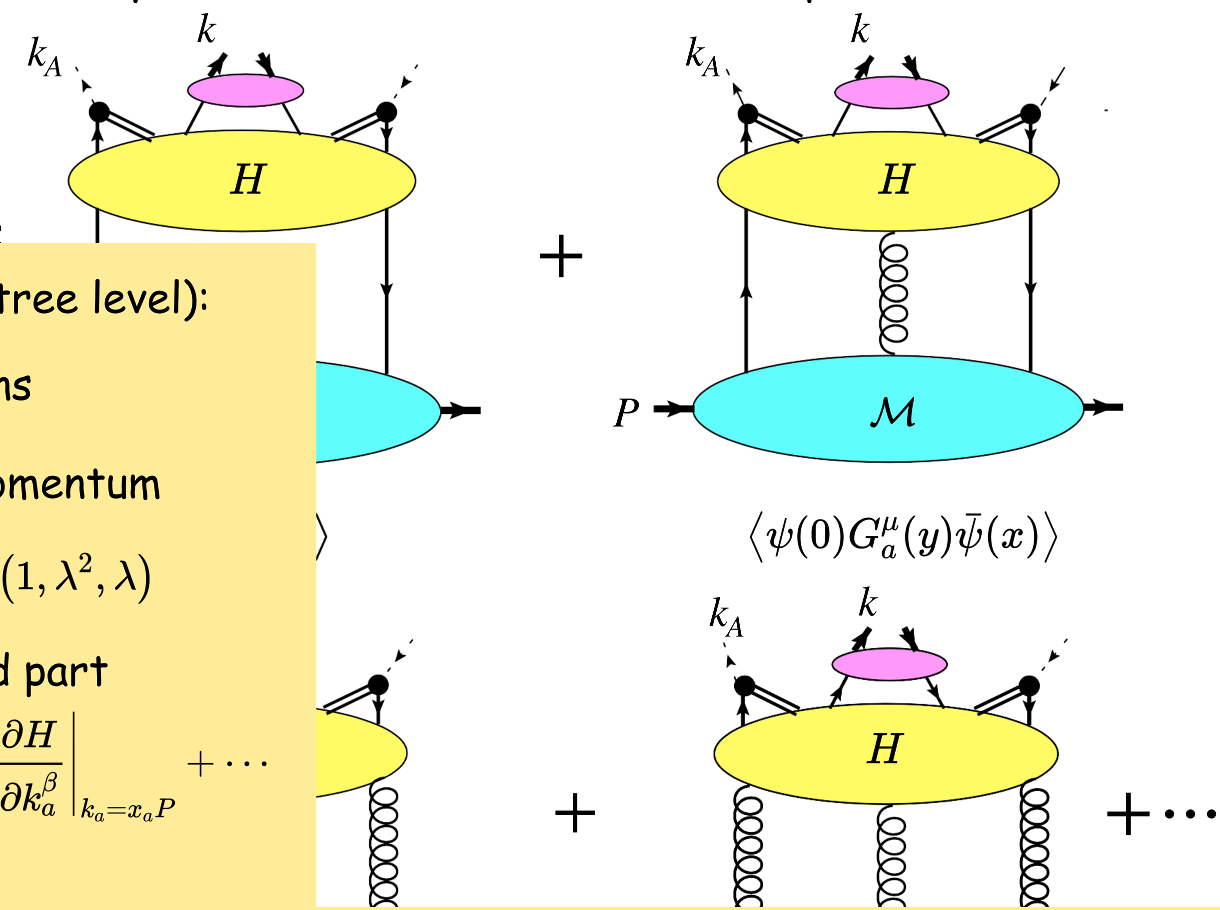
Two-parton correlations

Three-parton correlations



$\langle\psi(0)G^{\mu}_{a}(y)\bar{\psi}(x)\rangle$

Procedures of collinear matching(tree level):

1. Write down the correlations

2. Power counting of field/momentum

$$k^{\mu}_{q}\sim k^{\mu}_{g}\sim\left(1,\lambda^2,\lambda\right)\quad G^{\rho}\sim\left(1,\lambda^2,\lambda\right)$$

3. Collinear expansion of hard part

$$H\left(k_a\right)=H\left(x_aP\right)+\left(k_a-x_aP\right)^{\beta}\left.\frac{\partial H}{\partial k_a^{\beta}}\right|_{k_a=x_aP}+\cdots$$

4. Keep the terms at twist-3

$$\langle\bar{\psi}\Gamma\partial_\perp\psi\rangle,\langle\bar{\psi}\Gamma\partial_\perp G^{+}\bar{\psi}\rangle,\langle\bar{\psi}\Gamma G^{\perp}\psi\rangle,\langle\bar{\psi}\Gamma G^{+}\partial_\perp\psi\rangle,\langle G^{+\perp}G^{\perp}\partial_\perp G^{+}\rangle,\cdots$$

5. Rearrange to twist-3 distributions:

Gauge invariance

What twist-3 distributions do we expect?

Matching of Fracture Function

Set of Twist-3 distributions

A pair of quark fields: e.g., Chen-Ma- Zhang, Phys. Lett. B 754 (2016)

$$q_T(x)s_\perp^\mu = P^+ \int \frac{d\lambda}{4\pi} e^{-ix\lambda P^+} \langle h_A|\bar{\psi}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma_\perp^\mu\gamma_5\mathcal{L}_n(0)\psi(0)|h_A\rangle,$$

$$-iq_\partial(x)s_\perp^\mu = \int \frac{d\lambda}{4\pi} e^{-ix\lambda P^+} \langle h_A|\bar{\psi}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma^+\gamma_5\partial_\perp^\mu\left(\mathcal{L}_n\psi\right)(0)|h_A\rangle,$$

$$-iq'_\partial(x)\epsilon_\perp^{\mu\nu}s_{\perp\nu} = \int \frac{d\lambda}{4\pi} e^{-ix\lambda P^+} \left\langle h_A\left|\bar{\psi}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma^+\partial_\perp^\mu\left(\mathcal{L}_n\psi\right)(0)\right|h_A\right\rangle$$

A pair of quark fields with one gluon field strength tenor(Chirality even):

Qiu-Sterman, Phys. Rev. D 59 (1999) 014004

$$T_F(x_1,x_2)\,\epsilon_\perp^{\mu\nu}s_{\perp\nu} = g_s\int \frac{d\lambda_1 d\lambda_2}{4\pi} e^{-i\lambda_2(x_2-x_1)p^+ - i\lambda_1 x_1 p^+}\left\langle h_A\left|\bar{\psi}\left(\lambda_1 n\right)\gamma^+ G^{+\mu}\left(\lambda_2 n\right)\psi(0)\right|h_A\right\rangle$$

$$T_\Delta(x_1,x_2)\,s_\perp^\mu = -ig_s\int \frac{d\lambda_1 d\lambda_2}{4\pi} e^{-i\lambda_2(x_2-x_1)p^+ - i\lambda_1 x_1 p^+}\left\langle h_A\left|\bar{\psi}\left(\lambda_1 n\right)\gamma_5\gamma^+ G^{+\mu}\left(\lambda_2 n\right)\psi(0)\right|h_A\right\rangle$$

$$T_F(x_1,x_2) = T_F(x_2,x_1), \quad T_\Delta(x_1,x_2) = -T_\Delta(x_2,x_1)$$

Matching of Fracture Function

Set of Twist-3 distributions

Set of twist-3 gluon distributions: Ji, Phys. Lett. B 289, 137 (1992).

$$\frac{i^3 g_s}{P^+}\int\frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi}e^{i\lambda_1 x_1 P^+ + i\lambda_2(x_2-x_1)P^+}\langle h_A|G^{a,+\alpha}(\lambda_1 n)G^{c,+\gamma}(\lambda_2 n)G^{b,+\beta}(0)|h_A\rangle \qquad \text{F-type}$$

$$= \frac{N_c}{(N_c^2-1)(N_c^2-4)}d^{abc}O^{\alpha\beta\gamma}(x_1,x_2) - \frac{i}{N_c(N_c^2-1)}f^{abc}N^{\alpha\beta\gamma}(x_1,x_2),$$

From the Bose-symmetry and covariance Beppu-Koike-Tanaka-Yoshida, Phys. Rev. D 82 (2010) 054005

$$O^{\alpha\beta\gamma}(x_1,x_2) = -2i\left[O(x_1,x_2)g_\perp^{\alpha\beta}\tilde{s}_\perp^\gamma + O(x_2,x_2-x_1)g_\perp^{\beta\gamma}\tilde{s}_\perp^\alpha + O(x_1,x_1-x_2)g_\perp^{\gamma\alpha}\tilde{s}_\perp^\beta\right],$$

$$N^{\alpha\beta\gamma}(x_1,x_2) = -2i\left[N(x_1,x_2)g_\perp^{\alpha\beta}\tilde{s}_\perp^\gamma - N(x_2,x_2-x_1)g_\perp^{\beta\gamma}\tilde{s}_\perp^\alpha - N(x_1,x_1-x_2)g_\perp^{\gamma\alpha}\tilde{s}_\perp^\beta\right],$$

with the properties:

$$O(x_1,x_2) = O(x_2,x_1), \quad O(x_1,x_2) = O(-x_1,-x_2),$$

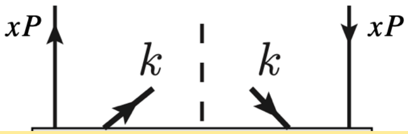
$$N(x_1,x_2) = N(x_2,x_1), \quad N(x_1,x_2) = -N(-x_1,-x_2).$$

All the other twist-3 gluon distributions can be determined by $N(x_1,x_2)$ $O(x_1,x_2)$

Matching of Fracture Function

Transversely polarized Fracture Function

$\mathcal{M}_{Fij}(x,\xi,k_\perp)$



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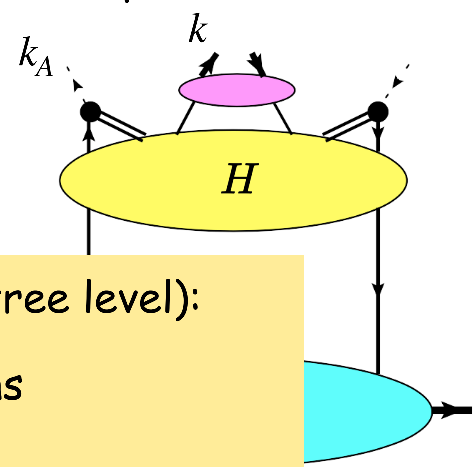
Two-parton correlations



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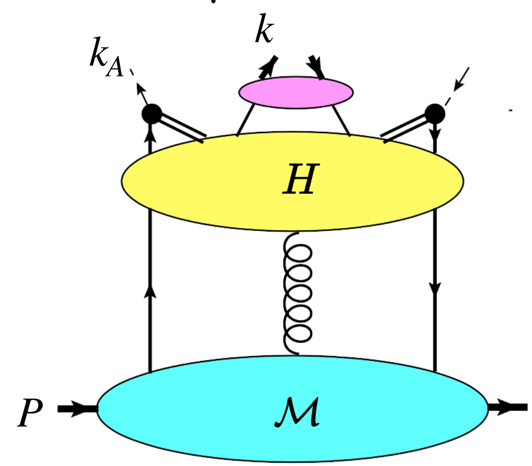

$\langle\psi(0)G_a^\mu(y)\bar{\psi}(x)\rangle$

k_A k

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5. Rearrange to twist-3 distributions:

Gauge invariance

Gauge invariance- difficult and non-trivial:

(a) Ward identities?

(b)QCD equation of motion

To what extent, can (a) help us?

Matching of Fracture Function

Transversely polarized Fracture Function

$$\mathcal{M}_{Fij}(x,k) = \frac{1}{2N_c}\left(\gamma^-\right)_{ij}\left[F_q(x,\xi,k_\perp) + \epsilon_\perp^{\mu\nu} k_{\perp\mu} s_{\perp\nu} \frac{1}{m_A} F_{qT}(x,\xi,k_\perp)\right]$$
$$+ \frac{1}{2N_c}\left(\gamma_5\gamma^-\right)_{ij}\left[s_L \Delta F_q(x,\xi,k_\perp) + k_\perp \cdot s_\perp \frac{1}{m_A} \Delta F_{qT}(x,\xi,k_\perp)\right] + \cdots$$

Another key point in this twist-3 matching:

- F_{qT} : Single transverse-spin asymmetry(T-odd) → Absorptive parts are required

e.g., Qiu-Sterman, Phys. Rev. D 59 (1999) 014004
Phys. Rev. Lett. 67 (1991) 2264
Efremov-Teryaev, Phys. Lett. B 150 (1985)
Hagiwara-Hikasa-Kai, Phys.Rev.D 27 (1983)

- ΔF_{qT} : Double spin asymmetry(T-even) → No absorptive part is required

$$\mathcal{M}_{Fij}\left(x,\xi,k_\perp\right)$$
$$= \int \frac{d\lambda}{2\pi} e^{-ixP^+\lambda} \sum_X \langle h_A(P,s)|[\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)]_j|X h_B(k)\rangle \langle h_B(k) X\,|[\mathcal{L}_n(0)\psi(0)]_i|\, h_A(P,s)\rangle$$

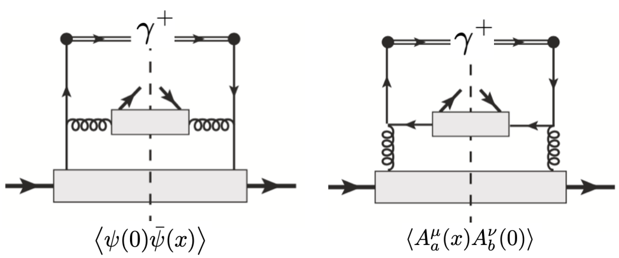
Matching of Fracture Function

- F_{qT} : Single transverse-spin asymmetry(T-odd)

  - Propagator: $$\frac{1}{x \pm i\epsilon} = P\frac{1}{x} \mp i\pi\delta(x)$$
    - Make the internal particle on shell->Ward identity!
    - Constrain the partonic momenta

    absorptive part , "pole"

  - Two-parton correlations do not contribute to SSA(or from PT symmetry)



  - Three-parton correlations

    Quark-gluon correlations:
    - hard pole
    - soft fermion pole
    - soft gluon pole

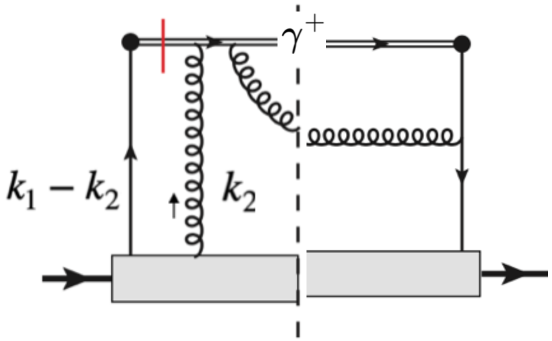
    Three-gluon correlations:
    - soft gluon pole

Matching of Fracture Function

Example:

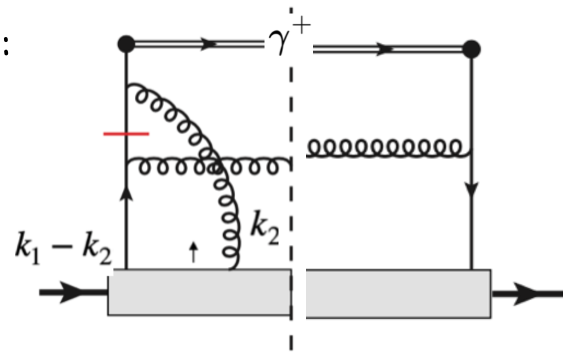
SSA: Quark-gluon correlations

Hard pole:



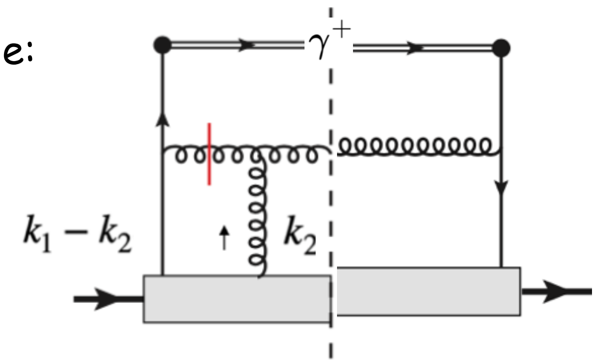
$$\frac{1}{k_g^+ - k_2^+ + i\epsilon} \to -i\pi\delta\left(k_g^+ - k_2^+\right)$$

Soft-fermion pole:



$$\frac{1}{(k_1 - k_2 - k_g)^2 + i\epsilon} \to -i\pi\frac{1}{2k_g^-}\delta\left(k_1^+ - k_2^+\right)$$

Soft gluon pole:



$$\frac{1}{(k_2 - k_g)^2 + i\epsilon} \to -i\pi\frac{1}{2k_g^-}\delta\left(k_2^+\right)$$

Matching of Fracture Function

SSA: Quark-gluon correlations

$$\frac{1}{m_A} F_{qT}\Big|_{qG\bar{q}} = \int \frac{dz}{z^2} d_g(z) d^4k_A d^4k_1 \mathcal{H}^{a,\rho}_{L,kl}(k_A, k_1) \mathcal{M}^a_{\rho,lk}(k_A, k_1)$$

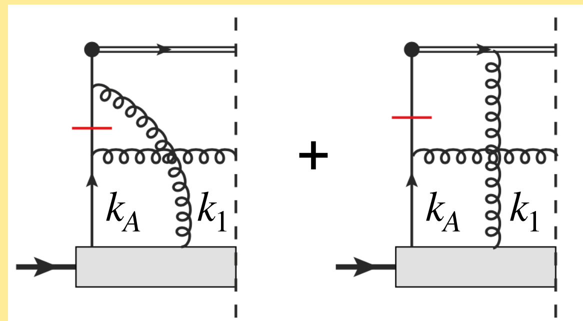
$$\langle \bar{\psi}_k G^{a,\rho} \psi_l \rangle$$

- Four Dirac structures up to twist-3:

$$\langle \bar{\psi}\gamma^+ G^\rho \psi \rangle \quad \langle \bar{\psi}\gamma^+\gamma_5 G^\rho \psi \rangle \quad \langle \bar{\psi}\gamma^\mu_\perp G^+ \psi \rangle \quad \langle \bar{\psi}\gamma^\mu_\perp \gamma_5 G^+ \psi \rangle$$

- Hard pole&Soft fermion pole ($k_1^+ \neq 0$)
  - The poles put the internal momenta on shell,

e.g.,



$$k_1^\mu H_\mu(k_A, k_1) = 0$$

  - Promise the absence of the contributions from $\gamma_\perp, \gamma_\perp\gamma_5$: $k_1^+ H^-(\hat{k}_A, \hat{k}_1) = 0$
  - Promise the cancellation of gauge-variant terms from $\gamma^+, \gamma^+\gamma_5$ after the expansion -> T_F, T_Δ

e.g.

$$\frac{1}{m_A} F_{qT}\Big|_L = g_s \int dk_A^+ dk_1^+ \frac{dz}{z^2} d_g(z) \int \frac{d\lambda_1 d\lambda_2}{2(2\pi)^2} e^{-i\lambda_1 k_A^+ + i\lambda_2 k_1^+} \Big\{ i\frac{\partial H_L^-}{\partial k_{1\perp}^\alpha}(\hat{k}_A, \hat{k}_1) \langle h_A | \bar{q}(\lambda_1)\gamma^+ \Big[\partial_\perp^\alpha G^+ - \partial^+ G_\perp^\alpha\Big](\lambda_2 n) q(0) | h_A \rangle - i\frac{\partial H_L^-}{\partial k_{A\perp}^\alpha}(\hat{k}_A, \hat{k}_1) \langle h_A | \left(\partial_\perp^\alpha \bar{q}\right)(\lambda_1)\gamma^+ G^+(\lambda_2 n) q(0) | h_A \rangle + \Big[k_1^+ \frac{\partial H_L^-}{\partial k_{1\perp}^\alpha}(\hat{k}_A, \hat{k}_1) + H_{L\perp\alpha}(\hat{k}_A, \hat{k}_1)\Big] \langle h_A | \bar{q}(\lambda_1)\gamma^+ G_\perp^\alpha(\lambda_2 n) q(0) | h_A \rangle \Big\},$$

  - HP,SFP contributions come from $\gamma^+, \gamma^+\gamma_5$.

Matching of Fracture Function

SSA: Quark-gluon correlations

$$\frac{1}{m_A} F_{qT}\Big|_{qG\bar{q}} = \int \frac{dz}{z^2} d_g(z) d^4k_A d^4k_1 \mathcal{H}^{a,\rho}_{L,kl}(k_A, k_1) \mathcal{M}^a_{\rho,lk}(k_A, k_1)$$

$$\langle \bar{\psi}_k G^{a,\rho} \psi_l \rangle$$

- Four Dirac structures up to twist-3:

$$\langle \bar{\psi}\gamma^+ G^\rho \psi \rangle \quad \langle \bar{\psi}\gamma^+ \gamma_5 G^\rho \psi \rangle \quad \langle \bar{\psi}\gamma^\mu_\perp G^+ \psi \rangle \quad \langle \bar{\psi}\gamma^\mu_\perp \gamma_5 G^+ \psi \rangle$$

- Soft gluon pole ($k_1^+ = 0$)
  - The identity still holds but is not useful.

$$k_1^\mu H_\mu(k_A, k_1) = 0$$

e.g. $k_1^+ H^-(\hat{k}_A, \hat{k}_1) = 0$

  - The terms from $\gamma_\perp$ cancel between the diagrams and their mirrors.
  - Similarly, the gauge-variant terms from γ^+ vanish -> $T_F(y,y)$ and $dT_F(y,y)/dy$:

$$\begin{aligned}\frac{1}{m_A} F_{qT}\Big|_L = g_s \int dk_A^+ dk_1^+ \frac{dz}{z^2} d_g(z) \int \frac{d\lambda_1 d\lambda_2}{2(2\pi)^2} e^{-i\lambda_1 k_A^+ + i\lambda_2 k_1^+} \Big\{ \\ i\frac{\partial H_L^-}{\partial k^\alpha_{1\perp}}(\hat{k}_A, \hat{k}_1) \langle h_A | \bar{q}(\lambda_1)\gamma^+ \Big[\partial^\alpha_\perp G^+ - \partial^+ G^\alpha_\perp\Big](\lambda_2 n) q(0) | h_A \rangle \\ - i\frac{\partial H_L^-}{\partial k^\alpha_{A\perp}}(\hat{k}_A, \hat{k}_1) \langle h_A | \left(\partial^\alpha_\perp \bar{q}\right)(\lambda_1)\gamma^+ G^+(\lambda_2 n) q(0) | h_A \rangle \\ + \Big[k_1^+ \frac{\partial H_L^-}{\partial k^\alpha_{1\perp}}(\hat{k}_A, \hat{k}_1) + H_{L\perp\alpha}(\hat{k}_A, \hat{k}_1)\Big] \langle h_A | \bar{q}(\lambda_1)\gamma^+ G^\alpha_\perp(\lambda_2 n) q(0) | h_A \rangle \Big\},\end{aligned}$$

Matching of Fracture Function

SSA: Quark-gluon correlations

$$\frac{1}{m_A} F_{qT}\Big|_{qG\bar{q}} = \int \frac{dz}{z^2} d_g(z) d^4k_A d^4k_1 \mathcal{H}^{a,\rho}_{L,kl}(k_A, k_1) \mathcal{M}^a_{\rho,lk}(k_A, k_1)$$

$$\langle \bar{\psi}_k G^{a,\rho} \psi_l \rangle$$

- Four Dirac structures up to twist-3:

$$\langle \bar{\psi}\gamma^+ G^\rho \psi \rangle \quad \langle \bar{\psi}\gamma^+\gamma_5 G^\rho \psi \rangle \quad \langle \bar{\psi}\gamma_\perp^\mu G^+ \psi \rangle \quad \langle \bar{\psi}\gamma_\perp^\mu \gamma_5 G^+ \psi \rangle$$

- Soft gluon pole ($k_1^+ = 0$)
  - However, $\gamma^+\gamma_5$ and $\gamma_\perp\gamma_5$ terms are not gauge invariant and not zero, respectively.

$$F_{qT}\Big|_{\gamma^+\gamma_5} \sim \langle (\partial_\perp^\alpha \bar{q})(\xi_1^-)\gamma^+\gamma_5 G^+(\xi_2^-) q(0) \rangle$$

$$F_{qT}\Big|_{\gamma_\perp\gamma_5} \sim \langle \bar{q}(\xi_1^-)\gamma_\perp^\mu \gamma_5 G^+(\xi_2^-) q(0) \rangle$$

$$\to T_F(y,y) \sim \langle \bar{\psi}\gamma^+ G^{+\perp}\psi \rangle$$

  - Solution of QCD-EOM:

$$\psi_-(x) = \frac{1}{2}\mathcal{L}_n^\dagger(x)\int_0^\infty d\lambda \big(\mathcal{L}_n \gamma^+ \gamma_\perp^\mu D_\mu \psi_+\big)(\lambda n + x)$$

  - SGP contributions come from γ^+, $\gamma^+\gamma_5$, $\gamma_\perp\gamma_5$.
    - Different from the cases in current region: e.g.

Eguchi-Koike-Tanaka, Nucl. Phys. B 763 (2007) 198

Matching of Fracture Function

Factorization:

SSA: Quark-gluon correlations

Hard pole:
$$\frac{1}{m_A} F_{qT}\left(x,\xi,k_\perp\right)\Big|_{HP} = g_s^2 \frac{N_c}{\left(k_\perp^2\right)^2} \int \frac{dz}{z^2} d_g(z) \frac{z^2}{y} \left(\xi T_\Delta(y,x) - (\xi + 2xz) T_F(y,x)\right)$$

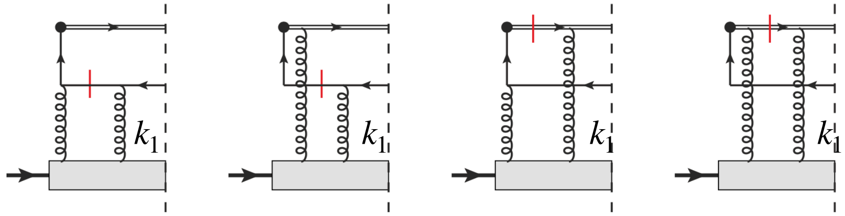
Soft-fermion pole:
$$\frac{1}{m_A} F_{qT}\left(x,\xi,k_\perp\right)\Big|_{SFP} = g_s^2 \frac{1}{N_c} \frac{1}{\left(k_\perp^2\right)^2} \int \frac{dz}{z^2} d_g(z) \frac{x\xi z}{y^4} \Big[(xz - \xi) T_F(y,0) - (\xi + xz) T_\Delta(y,0)\Big]$$

Soft gluon pole:
$$\frac{1}{m_A} F_{qT}(x,\xi,k_\perp)\Big|_{SGP} = \frac{g_s^2 N_c}{(k_\perp^2)^2} \int \frac{dz}{z^2} d_g(z) \frac{1}{y^3} \Big[z^3(y^3 + 3x^2 y - 2x^3) T_F(y,y) - y\xi z^2 (y^2 + x^2) \frac{\partial T_F(y,y)}{\partial y}\Big].$$

$$y = x + \xi/z.$$

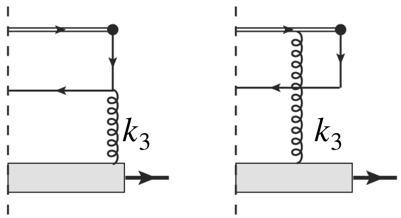
Matching of Fracture Function

SSA: Three-gluon correlations



(a)

+ Mirrors



(b)

- Soft-gluon poles($k_1^+ = 0$):
  - Ward identities from (b) are useful; Those from (a) are not:

$$\langle G^{a,\mu} G^{b,\nu} G^{c,\sigma} \rangle$$

$$k_3^{\mu_3} H^{abc}_{\mu_1\mu_2\mu_3}(k_1, k_2, k_3) = 0.$$

$$\langle G^{a,\mu} G^{b,\nu} G^{c,+\sigma} \rangle$$

  - Gauge-invariant results are obtained by sum of the diagrams and their mirror diagrams.
  - Factorization:

Methodology : e.g. Beppu-Koike-Tanaka-Yoshida, Phys. Rev. D 82 (2010) 054005

$$\frac{1}{m_A} F_{qT}(x, \xi, k_\perp)|_{3G}$$
$$= \frac{4\pi g_s^2 \xi^2}{(k_\perp^2)^2} \int \frac{dz}{z^2} d_{\bar{q}}(z) \frac{1}{zy^5} \Big[2(\xi^2 + 2x^2z^2 - \xi xz)(N(y,y) - O(y,y))$$
$$- 2(\xi^2 + 4x^2z^2 - 3\xi xz)(N(y,0) + O(y,0)) + y(\xi - xz)^2 \frac{d}{dy}(N(y,0) + O(y,0))$$
$$-y(\xi^2 + x^2z^2) \frac{d}{dy}(N(y,y) - O(y,y)) \Big] \quad y = x + \xi/z.$$

Matching of Fracture Function

Transversely polarized Fracture Function

$$\mathcal{M}_{Fij}(x,k) = \frac{1}{2N_c}\left(\gamma^-\right)_{ij}\left[F_q(x,\xi,k_\perp) + \epsilon_\perp^{\mu\nu}k_{\perp\mu}s_{\perp\nu}\frac{1}{m_A}F_{qT}(x,\xi,k_\perp)\right]$$
$$+\frac{1}{2N_c}\left(\gamma_5\gamma^-\right)_{ij}\left[s_L\Delta F_q(x,\xi,k_\perp) + k_\perp\cdot s_\perp\frac{1}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\right] + \cdots$$

Another key point in this twist-3 matching:

- F_{qT} : Single transverse-spin asymmetry(T-odd) ⟶ Absorptive parts are required

e.g., Qiu-Sterman, Phys. Rev. D 59 (1999) 014004
Phys. Rev. Lett. 67 (1991) 2264
Efremov-Teryaev, Phys. Lett. B 150 (1985)
Hagiwara-Hikasa-Kai, Phys.Rev.D 27 (1983)

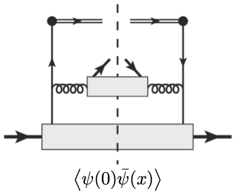
- ΔF_{qT} : Double spin asymmetry(T-even) ⟶ No absorptive part is required

$$\Delta F_{qT} = \int \frac{d\lambda}{4\pi}e^{-ixP^+\lambda}\sum_X \langle h_A(P,s)|[\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma^+\gamma_5]_i|Xh_B(k)\rangle\langle h_B(k)X|[\mathcal{L}_n(0)\psi(0)]_i|h_A(P,s)\rangle$$

Matching of Fracture Function

DSA: Quark-gluon correlations

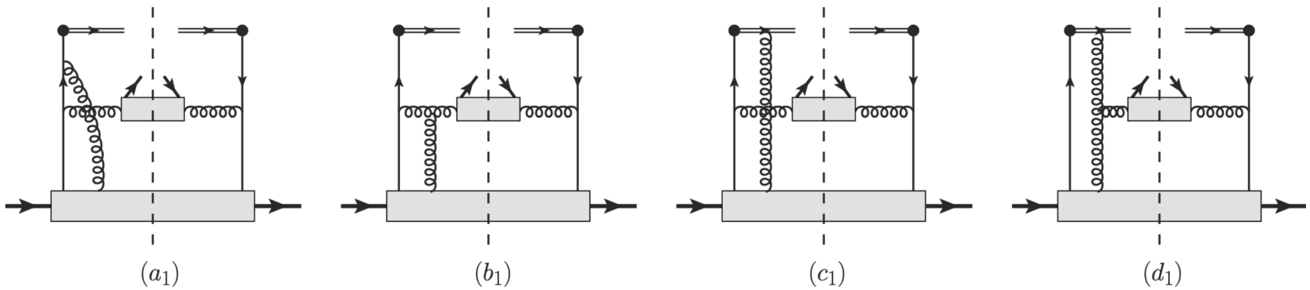
- Two-parton correlations:



$$\frac{2k_\perp \cdot s_\perp}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\Big|_{2a}$$
$$= \frac{k_{\perp\mu}}{(k_\perp^2)^2}\int \frac{dz}{z^2} d_g(z)\Big[H_{2p,T}(x,\xi)\underbrace{\int\frac{d\lambda}{4\pi}P^+ e^{-iyP^+}\langle h_A|\bar{q}(\lambda n)\gamma_\perp^\mu\gamma_5 q(0)|h_A\rangle}_{q_T(x)}$$
$$+ H_{2p,\partial}(x,\xi)\underbrace{\int\frac{d\lambda}{4\pi} ie^{-iyP^+}\langle h_A|\bar{q}(\lambda n)\gamma^+\gamma_5\partial_\perp^\mu q(0)|h_A\rangle}_{q_\partial(x)}\Big],$$

- g_s order of the gauge link comes from three-parton correlation

- Three-parton correlations:



+ Mirrors

- Four Dirac structures up to twist-3(Feynman gauge):

$$\left\langle \bar{\psi}\gamma^+ G^\rho \psi\right\rangle \quad \left\langle \bar{\psi}\gamma^+\gamma_5 G^\rho \psi\right\rangle \quad \left\langle \bar{\psi}\gamma_\perp^\mu G^+ \psi\right\rangle \quad \left\langle \bar{\psi}\gamma_\perp^\mu\gamma_5 G^+ \psi\right\rangle$$

- No Ward identities can be applied here.

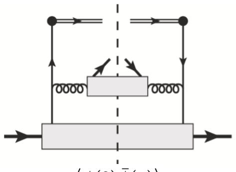
- In LC gauge $G^+ = 0$, the twist-3 distributions are trivially obtained:

$$\left\langle \bar{\psi}\gamma^+ G^\perp \psi\right\rangle \to T_F \quad \left\langle \bar{\psi}\gamma^+\gamma_5 G^\perp \psi\right\rangle \to T_\Delta \quad \text{since} \quad G^{+\mu} = \partial^+ G^\mu$$

Matching of Fracture Function

DSA: Quark-gluon correlations

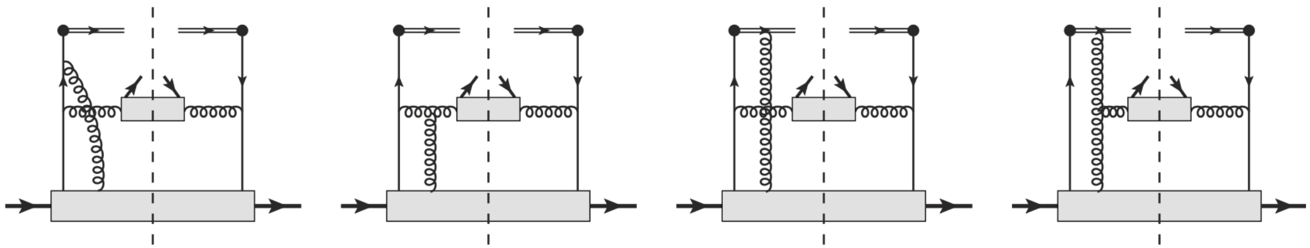
- Two-parton correlations:



$$\frac{2k_\perp\cdot s_\perp}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\Big|_{2a}$$
$$=\frac{k_{\perp\mu}}{(k_\perp^2)^2}\int\frac{dz}{z^2}d_g(z)\Big[H_{2p,T}(x,\xi)\underbrace{\int\frac{d\lambda}{4\pi}P^+e^{-iyP^+}\langle h_A|\bar{q}(\lambda n)\gamma_\perp^\mu\gamma_5 q(0)|h_A\rangle}_{q_T(x)}$$
$$+H_{2p,\partial}(x,\xi)\underbrace{\int\frac{d\lambda}{4\pi}ie^{-iyP^+}\langle h_A|\bar{q}(\lambda n)\gamma^+\gamma_5\partial_\perp^\mu q(0)|h_A\rangle}_{q_\partial(x)}\Big],$$

- g_s order of the gauge link comes from three-parton correlation

- Three-parton correlations:



+ Mirrors

- Four Dirac structures up to twist-3(Feynman gauge):

$$\langle\bar{\psi}\gamma^+G^\rho\psi\rangle\quad\langle\bar{\psi}\gamma^+\gamma_5G^\rho\psi\rangle\quad\langle\bar{\psi}\gamma_\perp^\mu G^+\psi\rangle\quad\langle\bar{\psi}\gamma_\perp^\mu\gamma_5G^+\psi\rangle$$

- In Feynman gauge, need to show the cancellation of much gauge-variant terms with different Dirac structure.

$$G^{+\perp}=\partial^+G^\perp-\partial^\perp G^++\mathcal{O}(g_s)$$

  - Solution of QCD-EOM help!

$$\psi_-(x)=\frac{1}{2}\mathcal{L}_n^\dagger(x)\int_0^\infty d\lambda\big(\mathcal{L}_n\gamma^+\gamma_\perp^\mu D_\mu\psi_+\big)(\lambda n+x)$$

See the details in Chen-Ma-Tong, JHEP 11 (2021) 038

Matching of Fracture Function

DSA: Quark-gluon correlations

- Factorization:

$$\frac{1}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\Big|_{q\bar{q}+qG\bar{q}} = \frac{1}{2(k_\perp^2)^2}\int\frac{dz}{z^2}d_g(z)\Big[\Big(H_{2p,T}(x,\xi)q_T(y)+H_{2p,\partial}(x,\xi)q_\partial(y)\Big) + \frac{2}{\pi}\int dx_2\Big(T_F(y,x_2)H(x,\xi,x_2)+T_\Delta(y,x_2)H_A(x,\xi,x_2)\Big)\Big],$$

- Hard part:

$$H_{2p,\partial}(x,\xi) = -8g_s^2C_F\frac{\xi z^2(y^2+2x^2)}{y^3}, \quad H_{2p,T}(x,\xi) = 8g_s^2C_F\frac{\xi x^2z^2}{y^2},$$

$$H(x,\xi,x_2) = \frac{2g_s^2\xi z^2}{yx_2(y-x_2)}\left[C_A\frac{x^2-x_2y}{x_2-x}+2C_F\frac{1}{y}(xy-x_2(x+y))\right], \qquad y=x+\xi/z.$$

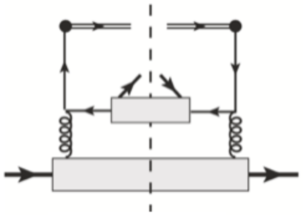
$$H_A(x,\xi,x_2) = \frac{2g_s^2\xi z^2}{yx_2(x_2-y)}\left[C_A\frac{(x^2+x_2y)(x_2+y-2x)}{(x_2-x)(x_2-y)}+2C_F\frac{1}{y}(x(2x-y)+x_2(x+y))\right]$$

- We check the results in both LC gauge and Feynman gauge.

Matching of Fracture Function

DSA: Pure Gluonic correlations (Feynman Gauge)

- Two-gluon correlations:



$\langle G^{a,\mu} G^{b,\nu} \rangle$

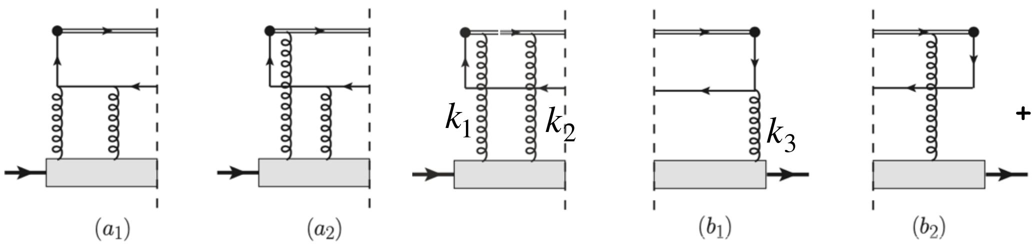
$$\frac{1}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\Big|_{2G} = \frac{16\pi\alpha_s}{(k_\perp^2)^2}\int \frac{dz}{z^2} d_{\bar q}(z)\frac{\xi^2 xz}{y^2} g_{3T}(y),$$

$$g_{3T}(x)\epsilon_\perp^{\mu\nu} s_{\perp\nu} = \frac{i}{x}\int\frac{d\lambda}{2\pi} e^{-ix\lambda P^+}\langle h_A\left|G^{+-}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\mathcal{L}_n(0)G^{+\mu}(0)\right|h_A\rangle$$

- The QCD EOM yields

$$g_{3T}(x) = -\frac{1}{x^2}\int dx_2\left\{\frac{1}{\pi}T_F(x_2+x,x)+P\left(\frac{1}{x-x_2}\right)\Big[N(x,x-x_2)-N(x,x_2)+2N(x_2,x_2-y)\Big]\right\}$$

- Three-gluon correlations:



+ Mirrors

$\langle G^{a,\mu} G^{b,\nu} G^{c,\sigma} \rangle$

$\Downarrow$ $k_3^{\mu_3} H^{abc}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3) = 0.$

$\langle G^{a,\mu} G^{b,\nu} G^{c,+\sigma} \rangle$

- d^{abc}: QED Ward identities

$$k_1^{\mu_1} H^{abc}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)\Big|_d = 0,$$

$$k_2^{\mu_2} H^{abc}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)\Big|_d = 0$$

- f^{abc}: No other Ward identities; Complicated but straightforward

Need to do the collinear expansion to $\mathcal{O}(\lambda^2)$

More than 15 terms appear, e.g.

$$\frac{\partial^2 H^{--\sigma}}{\partial k_{1\perp}^\alpha \partial k_{2\perp}^\beta}\langle G^{c,+\sigma}(0)\partial^\beta A^{b,+}(\lambda_2)\partial^\alpha A^{a,+}(\lambda_1)\rangle, \quad \frac{\partial H^{-\nu\mu}}{\partial k_{2\perp}^\rho}\langle G^{c,+\mu}(0)\partial^\rho(\partial^+ A^{b,\nu})(\lambda_2)A^{a,+}(\lambda_1)\rangle \quad \ldots\ldots$$

Matching of Fracture Function

DSA: Pure Gluonic correlations

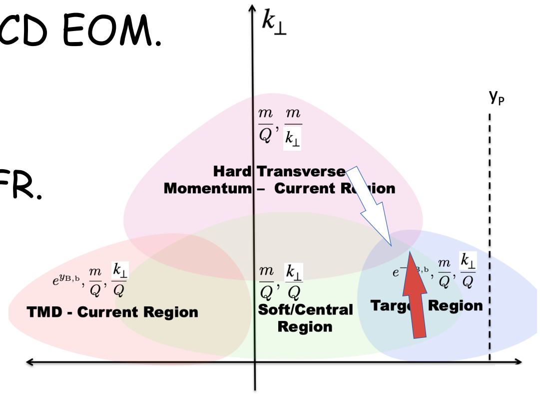
Factorization:

$$
\begin{aligned}
&\frac{1}{m_A}\Delta F_{qT}(x,\xi,k_\perp)\Big|_{2G+3G} \\
&= \frac{-16\pi\alpha_s\xi^2}{(k_\perp^2)^2}\int\frac{dzdx_2}{z^2y^3}d_{\bar{q}}(z)\Bigg\{\frac{xz}{\pi y}T_F(x_2,x_2+y)-\frac{2xz}{y(y-x_2)}\Big[N(y,x_2) \\
&\quad -N(y-x_2,y)+2N(y-x_2,-x_2)\Big]+\frac{1}{x_2^2(y-x_2)}\Big(x_2\xi\Big[O(y-x_2,y) \\
&\quad -N(y-x_2,y)\Big]+(2\xi y+yx_2z-y^2z-\xi x_2)\Big[N(y,x_2)+O(y,x_2)\Big] \\
&\quad +y(z(y+x_2)-2\xi)\Big[N(y-x_2,-x_2)+O(y-x_2,-x_2)\Big]\Big)\Bigg\}.
\end{aligned}
$$

- We check the results in both LC gauge and Feynman gauge.

Summary&Outlook

- We studied the factorization of the quark collinear fracture functions at $\Lambda_{QCD} \ll k_\perp \ll Q$ for SIDIS in the TFR at tree level:
  - Twist-2 matching for the unpolarized/longitudinal part.
  - Twist-3 matching for the transversely polarized parts(SSA,DSA).
    - Gauge invariance in Feynman gauge by Ward identities&QCD EOM.
    - Help the modeling of fracture function.
    - Provide the access to the twist-2/3 parton distributions in TFR.



[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

- Outlook:
  - One-loop extension. twist-2: Chen-Ma-Tong, arXiv: 220x:xxxx
  - To full understand the interplay between the target and current regions, further study on the current region at large forward rapidity is needed.
  - Numerical/Phenomenological studies;
  - May help the kinematic regime estimation in SIDIS.

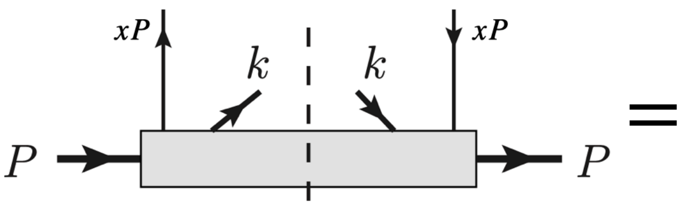
e.g., M. Boglione et al, arXiv: 2201.12197
M. Boglione et al, JHEP 10 (2019) 122
M. Boglione et al, Phys.Lett.B 766 (2017) 245-253

Thanks for your attention!

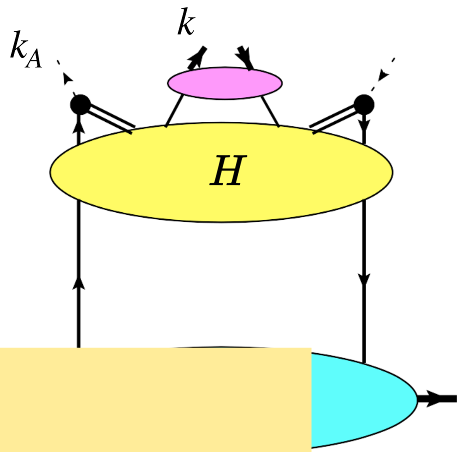
Backup

Transversely polarized Fracture Function

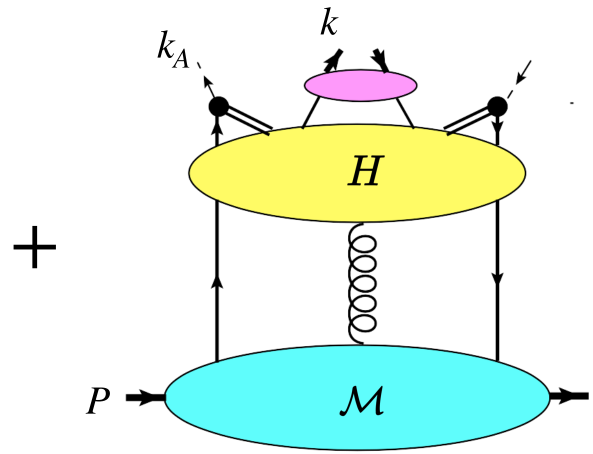
$\mathcal{M}_{Fij}(x, \xi, k_\perp)$



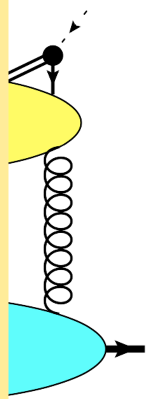
Two-parton correlations

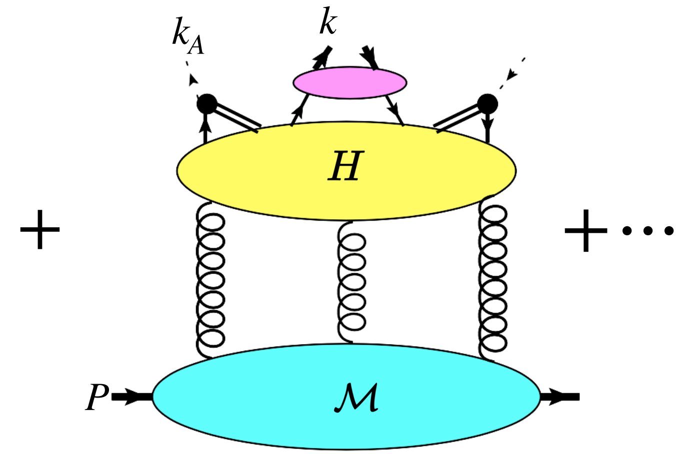


Three-parton correlations



$\langle \psi(0) A_a^\mu(y) \bar{\psi}(x) \rangle$





$\langle A_a^\mu(x) A_b^\nu(y) A_c^\sigma(0) \rangle$

No twist-3 FF ⊗ twist-2 distributions

Chirality even

twist-2 quark/gluons distributions:

$$\int \frac{dy}{2\pi} e^{-iyxP^+} \langle h_A | \left(\bar{\psi}(yn) \mathcal{L}_n^\dagger(yn) \right)_\beta \left(\mathcal{L}_n(0) \psi(0) \right)_\alpha | h_A \rangle = \frac{1}{2N_c} \left[q(x)\gamma^- + s_L \Delta q(x) \gamma_5 \gamma^- + h_1(x) \gamma_5 \gamma \cdot s_\perp \gamma^- \right]_{\alpha\beta} + \cdots$$

$$\frac{1}{xP^+} \int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle h_A | \left(G^{+\mu}(\lambda n) \mathcal{L}_n^\dagger(\lambda n) \right)^a \left(\mathcal{L}_n(0) G^{+\nu}(0) \right)^a | h_A \rangle = -\frac{1}{2} g_\perp^{\mu\nu} g(x) - \frac{i}{2} \epsilon_\perp^{\mu\nu} s_L \Delta g(x) + \cdots .$$

- Twist-2 Factorization at $\Lambda_{QCD} \ll k_\perp \ll Q$

$$F_{q,h_B/h_A}(x,\xi,k_\perp) = \sum_{a,b}\int \frac{dydz}{yz^2} d_{h_B/b}(z) H_{ba}(x/y,\xi/(yz),k_\perp/z) f_{a/h_A}(y)$$

$$\Delta F_{q,h_B/h_A}(x,\xi,k_\perp) = \sum_{a,b}\int \frac{dydz}{yz^2} d_{h_B/b}(z) \Delta H_{ba}(x/y,\xi/(yz),k_\perp/z) \Delta f_{a/h_A}(y)$$

- 6 hard coefficients contribute at one loop level

$$g/q, \quad q/q, \quad \bar{q}/q, \quad \bar{q}/g\,. \quad g/g, \quad \bar{q}/\bar{q}$$

- Over 70 independent perturbative diagrams contribute
- Evolution equation: Same as DGLAP Eq.