Applications of the gauge/gravity duality to hadron physics

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Outline

Introduction to holographic QCD
 Regge theory in holographic QCD
 Holographic Baryons

1. Introduction to holographic QCD

Gauge/gravity duality

- Gauge/gravity duality: at large N_c, strongly coupled field theory ↔ classical higher dimensional gravity
- Well known example: N = 4 Super Yang-Mills
 ↔ type IIB supergravity on AdS₅ × S⁵ ("AdS/CFT")
- ▶ Relatively easy classical analysis of strongly coupled phenomena ⇒ apply to QCD?
- There are possible issues (QCD not conformal, no SUSY, and $N_c = 3$ is not that large ...) but since solving QCD is hard, it's worth trying and it turns out to work quite nicely!

Gauge/gravity duality for QCD

Basic features:

- Geometrization of RG flow: running of the gauge coupling in QCD \leftrightarrow non-AdS geometry
- \blacktriangleright \Rightarrow confinement scale, broken conformal symmetry
- ► Thermodynamics of QCD ↔ thermodynamics of a planar bulk black hole



• Operators $O_i(x^{\mu}) \leftrightarrow$ classical bulk fields $\phi_i(x^{\mu}, r)$

 $Z_{\text{grav}}(\phi_i|_{\text{bdry}} = J_i(x^{\mu})) = \int \mathcal{D}\phi_i e^{iS_{QCD} + i\int d^4 \times J^i(x^{\mu})O_i(x^{\mu})}$ $\models \text{ E.g. } \bar{\psi}^j \psi^i \leftrightarrow \phi^{ij} \qquad T_{\mu\nu} \leftrightarrow g_{\mu\nu} \qquad J_{\mu} \leftrightarrow A_{\mu}$

- Condensates in QCD $\langle O_i \rangle \leftrightarrow$ nontrivial extrema $\phi_i(r)$ in the bulk, e.g. black hole "hair"

Gauge/gravity duality for QCD: approaches

Top-down: models directly based on string theory

- Concrete, fixed string models in 10/11 d with brane configurations
- Control on what dual field theory is (it's not QCD though)
- E.g., D3-D7 models and
 Witten-Sakai-Sugimoto model: D4-D8-D8

Bottom-up: models constructed "by hand"

- Follow generic ideas of holography, inspiration from top-down
- Introduce fields for most important operators (marginal)
- \blacktriangleright Lots of freedom \rightarrow effective 5d description, no link to specific dual theory, comparison with QCD data essential
- Either a fixed geometry (AdS) or dynamical gravity
- Examples: hard/soft wall models, light-front holography

This talk: rich bottom-up models; lots of input from string theory

- Lots of parameters to be fitted from data
- Goal: mimic QCD as closely as possible

Improved Holographic QCD

"Improved holographic QCD" (IHQCD): string-inspired bottom-up model for pure Yang-Mills [Review: Gursoy, Kiritsis, Mazzanti, Michalogiorgakis, Nitti arXiv:1006.5461]

 $S_{\rm IHQCD} = M^3 N_c^2 \int d^5 x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial \lambda)^2}{\lambda^2} + V_g(\lambda) \right]$

• $\lambda = e^{\phi} \leftrightarrow \operatorname{Tr}(F^2)$

• Near UV bdry, $\lambda pprox$ 't Hooft coupling $g^2 N_c$

• $g_{\mu\nu} \leftrightarrow T_{\mu\nu}$

The only non-trivial input V_{g} needs to be determined by comparing to QCD physics

- Strong coupling (IR) physics: qualitative properties of QCD
- Weak coupling (UV) physics: perturbation theory
- Remaining degrees of freedom: lattice data (thermodynamics and glueball spectra)

Adding quarks (V-QCD)

Two bulk scalars: $\lambda \leftrightarrow \text{Tr}(F^2)$, $\tau \leftrightarrow \bar{\psi}\psi$ Gauge field(s) $A_{\mu} \leftrightarrow \bar{\psi}\gamma_{\mu}\psi$

- Motivated by a brane setup
- Duals for all operators with $D \leqslant 4$

$$S_{V-QCD} = N_c^2 M^3 \int d^5 x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial \lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$-N_f N_c M^3 \int d^5 x \ V_{f0}(\lambda) e^{-\tau^2}$$
$$\times \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a \tau \partial_b \tau + w(\lambda) F_{ab})}$$

• Chiral symmetry breaking \leftrightarrow condensation of τ in the bulk Similar approach to pure glue, but more degrees of freedom: additional asymptotics constraints, more data ...



2. Regge theory in holographic QCD

Pomeron and Deep Inelastic Scattering

(Soft) Pomeron: Regge trajectory with vacuum quantum numbers

- Dominates elastic *pp* scattering at high *s* and small angles: $A(s,t) \sim s^{\alpha(t)}$ $\alpha(t) \approx 1.08 + 0.25t$
- Total QCD cross section $\sigma \sim \frac{1}{s} \text{Im} \mathcal{A}(s, 0) \sim s^{\alpha(0)-1}$
- QCD origin: nonperturbative exchange of soft gluons

Pomeron in Deep Inelastic Scattering: take large s and soft photon virtuality $Q^2\sim {\rm GeV}^2$

 \Rightarrow parton momentum fraction $x = \frac{Q^2}{s} \ll 1$

 Regime where gluon parton distributions dominate

Data : $\sigma(x, Q^2) \sim x^{1-\alpha(t=0,Q^2)}$

 Observed Q² dependence due to multiple Regge trajectories?
 Soft+hard Pomeron?

Soft physics, difficult in QCD, open questions – maybe holography can shed light on this?



Pomeron in holographic QCD

Evaluate hadronic tensor $\sim \langle p' | T\{j(x)j(0)\} | p \rangle$ following dictionary

 Witten diagram – dominated by bulk "graviton" trajectory – high spin J cousins of the 5D graviton

Holographic QCD: choose the IHQCD model

- High J fields need to be added Ansatz with 5 parameters to be fitted
- Details of proton unimportant in Regge limit
 - Reduces to proton-graviton couplings
 - Enough to use scalar fields

Holographic Regge physics:

Schrödinger problem in the bulk

 $\left[-\partial_z^2+V_J(z)\right]h_{\mu_1\cdots\mu_J}^{(n)}=0$

Summing over J, Sommerfeld-Watson transformation ⇒

Regge trajectories \leftrightarrow eigenmodes

[Ballon-Bayona, Carcassés Quevedo, Costa, Djurić 1508.00008] [Ballon-Bayona, Carcassés Quevedo, Costa 1704.08280]_{11/21}





Fitting the DIS data



Results, checks and predictions

Regge trajectories from the fit:

- Two leading trajectories: hard and soft Pomeron
- Reproduce the exponent in $\sigma(x, Q^2) \sim x^{-\epsilon(Q^2)}$

Further checks and predictions:

- $\gamma^* \gamma \to X$ can be fitted by the same Pomeron kernel
- Similar approach works (with minor adjustments) for deeply virtual Compton scattering







[Amorim, Carcassés Quevedo, Costa 2106.09439]13/21

Extending to meson trajectories

Meson trajectories also expected to be important \Rightarrow extend the model to include quarks \Rightarrow use V-QCD

- First task: fit the model properly to meson masses
- Regge physics: both gluonic (closed string) and mesonic (open string) trajectories
- Fit: include 3 gluonic and

a mesonic trajectory



Ratio	R _{pred.}	R _{obs.}	$ R_{pred.} - R_{obs.} /R_{obs.} $
$m_{\rho(1450)}/m_{\rho}$	1.662	1.890	0.121
$m_{\rho(1700)}/m_{\rho}$	2.141	2.219	0.035
$m_{\rho(2000)}/m_{\rho}$	2.559	2.580	0.008
$m_{\rho(2270)}/m_{\rho}$	2.940	2.922	0.006
$m_{\omega(782)}/m_{\rho}$	1	1.010	0.010
$m_{\omega(1420)}/m_{\rho}$	1.662	1.832	0.093
$m_{\omega(1650)}/m_{\rho}$	2.141	2.154	0.006
$m_{\omega(1960)}/m_{\rho}$	2.559	2.528	0.012
$m_{\omega(2205)}/m_{\rho}$	2.940	2.844	0.034
$m_{\omega(2290)}/m_{\rho}$	3.289	2.954	0.127
$m_{\omega(2330)}/m_{\rho}$	3.610	3.005	0.201
$m_{a_1(1260)}/m_{\rho}$	1.591	1.587	0.003
$m_{a_1(1930)}/m_{\rho}$	2.095	2.489	0.158
$m_{a_1(2095)}/m_{\rho}$	2.523	2.702	0.066
$m_{a_1(2270)}/m_{\rho}$	2.916	2.928	0.004
$m_{f_1(1285)}/m_{\rho}$	1.653	1.654	0.001
$m_{f_1(1420)}/m_{\rho}$	2.128	1.840	0.157
$m_{f_1(1970)}/m_{\rho}$	2.543	2.542	0.0004
$m_{f_1(2310)}/m_{\rho}$	2.930	2.980	0.017
m_{π}/m_{ρ}	0.1740	0.1741	0.0006
$m_{\pi(1300)}/m_{\rho}$	1.731	1.677	0.032
$m_{\pi(1800)}/m_{\rho}$	2.337	2.337	5×10^{-5}
$m_{\pi(2070)}/m_{\rho}$	2.785	2.670	0.043
$m_{\pi(2360)}/m_{\rho}$	3.173	3.044	0.042
$m_{a_0(1450)}/m_{ ho}$	0.685	1.901	0.640
$m_{a_0(2020)}/m_{\rho}$	1.492	2.612	0.429

[Amorim, Costa, MJ 2102.11296] 14/21

Fit results

- Exchange kernels determined by fixing the Pomeron intercept and high J meson masses
- Couplings of *p* and *γ* to the trajectories fitted to *σ*'s
- ▶ 8 parameter fit to 199 data points with $\chi^2_{d.o.f} \approx 0.74$

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 $\sqrt{s}(GeV)$

300

250

200

100

50

0

(qri) 0 $\sigma(\gamma p \to X)$



Simultaneous fit of all σ 's requires including the meson trajectory! 15/21

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3. Baryons in holographic QCD

Baryons in large N_c QCD

Baryons are color singlet states of N_c quarks

- Massive, $M \propto N_c$ in the large N_c limit [Witten NPB 160 (1979) 57]
- $\blacktriangleright \Rightarrow$ Effective description as solitons of the pion field

Baryon = topological soliton solution of the chiral pion Lagrangian (with nontrivial winding number $\pi_3(SU(N_f))$ of the flavor group)

$$\mathcal{L}_{\pi} = \frac{f_{\pi}^2}{4} \operatorname{Tr} \{ L_{\mu} L^{\mu} \} \quad L_{\mu} = U^{\dagger} \partial_{\mu} U \quad U = \exp(i\pi^a T^a / f_{\pi})$$

carrying a nonzero baryon charge

$$N_B = \frac{1}{24\pi^2} \int d^3x \ \epsilon_{ijk} \mathrm{Tr}\{L^i L^j L^k\}$$

Such solutions exist, but they have zero size at lowest energy...

Fix: add a higher derivative term

$$\mathcal{L}_{\pi} = \frac{f_{\pi}^2}{4} \text{Tr} \{ L_{\mu} L^{\mu} \} + \frac{1}{32e^2} \text{Tr} \left[L_{\mu}, L_{\nu} \right] [L^{\mu}, L^{\nu}]$$

Now solution stabilized: the skyrmion

The holographic baryon

Holographic description of the baryon may be useful

- To understand the properties and structure of baryons
- To model dense nuclear matter (and neutron stars)
- For use in scattering applications

Recall the standard $\mathsf{AdS}/\mathsf{CFT}$ duality:

 $\mathcal{N}=4$ SYM is dual to IIB sugra on $AdS_5{\times}S^5$

- Baryons are objects where N_c fundamental strings (↔ quarks) can end
- Obtained by wrapping a D5 brane over the S⁵ [Witten hep-th/9805112]
- For AdS₅, boils down to solitons of 5D gauge fields in the bulk

$$\mathcal{L}_{\rm B} \sim -\kappa \int d^5 x \, {
m Tr} F_{\mu
u}^2$$

- Similar to the BPST instanton in 4D Yang-Mills
- "Upgrade" of the Skyrmion picture!
- Writing $A_{\mu} \sim i \partial_{\mu} U U^{\dagger}$,
 - Effective Skyrmion Lagrangian reproduced at the bdry
 - The topological instanton number maps to the baryon number $_{18/21}$



Example: solitons in a hard wall model

Hard wall model: AdS₅ with "hard" cutoff at $z = z_{IR}$ with $N_f = 2$

$$S = -M_5^3 \int d^5 x \sqrt{-\det g} \left[\left(F_{\mu\nu}^{(L)} \right)^2 + (L \leftrightarrow R) \right] + \frac{iN_c}{24\pi^2} \int A_L \wedge F^{(L)} \wedge F^{(L)} - (L \leftrightarrow R) + \cdots$$

$$(A_L)_j^a = -(A_R)_j^a = f_1(x)\epsilon_{jak}x_k$$

+ f_2(x)(r^2\delta_{aj} - x_ax_j) + f_3(x)x_jx_a

 $(A_L)_{\mu}|_{z_{\mathrm{IR}}} = (A_R)_{\mu}|_{z_{\mathrm{IR}}}$



[Pomarol, Wulzer 0712.3276, 0807.0316]

	Experiment	AdS_5	Deviation
m_{ρ}	775	850	+10%
m_{a_1}	1230	1390	+13%
m_{ω}	782	850	+9%
F_{ρ}	153	175	+14%
$F_{ ho}/F_{\omega}$	0.88	0.90	+2%
F_{π}	87	91	+5%
$g_{\rho\pi\pi}$	6.0	5.4	-10%
L_9	$6.9 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$	-10%
L_{10}	$-5.5\cdot10^{-3}$	$-6.2\cdot10^{-3}$	+12%
M_N	940	1180	+25%
$\sqrt{\langle r_{E,S}^2 \rangle}$ (fm)	0.79	0.87	+21%
g_A	1.25	0.98	-21%
$\mu_p - \mu_n$	4.7	3.7	-22%
$\Gamma(\omega \rightarrow \pi \gamma)$	0.75	0.82	+10%
$\Gamma(\omega \rightarrow 3\pi)$	7.5	7.1	-6%
$\Gamma(\rho \rightarrow \pi \gamma)$	0.068	0.072	+5%
$\Gamma(\omega \to \pi \mu \mu)$	$8.2 \cdot 10^{-4}$	$7.4\cdot10^{-4}$	-9%
$\Gamma(\omega \rightarrow \pi ee)$	$6.5 \cdot 10^{-3}$	$7.4 \cdot 10^{-3}$	+14%

[See also: baryons in WSS model] [Kim, Sin, Zahed arXiv:0708.1469; Hata, Sakai, Sugimoto, Yamato hep-th/0701280]19/21

Solitons in V-QCD

Towards a realistic holographic description of baryons

- Use V-QCD: a model with gluons and (dynamical) quarks
- The model is able to (simultaneously) describe both QCD thermodynamics and meson mass spectra
- Solve numerically
 - 1. The fully backreacted background quark and gluon sectors simultaneously
 - 2. The soliton in the probe limit
- Highly technical and challenging problem!



Conclusion

Holographic analysis of the Pomeron in DIS and other processes

- Described through higher spin fields in bulk gravity
- "Hard" and "soft" Pomeron arise from the same (graviton, closed string) trajectory in the bulk
- Including a meson trajectory facilitated description of total cross sections at high s

Holographic baryons

- Arise from solitons of gauge fields in the bulk
- Results in simple hard-wall model compare well with experimental data
- Analysis of solitons in a more realistic model (V-QCD) in progress
- Several possible directions to explore in the future!

Backup slides

Constraining the potentials

In the UV ($\lambda \rightarrow 0$):

► UV expansions of potentials matched with perturbative QCD beta functions ⇒ asymptotic freedom and logarithmic flow of the coupling and quark mass, as in QCD [Gürsoy, Kiritsis arXiv:0707.1324; MJ, Kiritsis arXiv:1112.1261]

In the IR $(\lambda \rightarrow \infty)$: various qualitative constraints

- Linear confinement, discrete glueball & meson spectrum, linear radial trajectories
- Existence of a "good" IR singularity
- Correct behavior at large quark masses
- Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (i.e, first guesses usually work!)

[Gürsoy, Kiritsis, Nitti arXiv:0707.1349; MJ, Kiritsis arXiv:1112.1261; Arean, latrakis,

MJ, Kiritsis arXiv:1309.2286, arXiv:1609.08922; MJ arXiv:1501.07272]

Final task: determine the potentials in the middle, $\lambda = \mathcal{O}(1)$

Qualitative comparison to lattice/experimental data

Ansatz for potentials, (x = 1)

$$\begin{split} V_{g}(\lambda) &= 12 \left[1 + V_{1}\lambda + \frac{V_{2}\lambda^{2}}{1 + \lambda/\lambda_{0}} + V_{\mathrm{IR}}e^{-\lambda_{0}/\lambda}(\lambda/\lambda_{0})^{4/3}\sqrt{\log(1 + \lambda/\lambda_{0})} \right] \\ V_{f0} &= W_{0} + W_{1}\lambda + \frac{W_{2}\lambda^{2}}{1 + \lambda/\lambda_{0}} + W_{\mathrm{IR}}e^{-\lambda_{0}/\lambda}(\lambda/\lambda_{0})^{2} \\ &\frac{1}{\kappa(\lambda)} = \kappa_{0} \left[1 + \kappa_{1}\lambda + \bar{\kappa}_{0}\left(1 + \frac{\bar{\kappa}_{1}\lambda_{0}}{\lambda}\right)e^{-\lambda_{0}/\lambda}\frac{(\lambda/\lambda_{0})^{4/3}}{\sqrt{\log(1 + \lambda/\lambda_{0})}} \right] \\ &\frac{1}{w(\lambda)} = w_{0} \left[1 + \frac{w_{1}\lambda/\lambda_{0}}{1 + \lambda/\lambda_{0}} + \bar{w}_{0}e^{-\lambda_{0}/\lambda w_{s}}\frac{(w_{s}\lambda/\lambda_{0})^{4/3}}{\log(1 + w_{s}\lambda/\lambda_{0})} \right] \\ &V_{1} = \frac{11}{27\pi^{2}}, \quad V_{2} = \frac{4619}{46656\pi^{4}}; \quad W_{1} = \frac{8 + 3W_{0}}{9\pi^{2}} \\ &W_{2} = \frac{6488 + 999W_{0}}{15552\pi^{4}}; \quad \kappa_{0} = \frac{3}{2} - \frac{W_{0}}{8}, \qquad \kappa_{1} = \frac{11}{24\pi^{2}} \end{split}$$

Fitting: glue sector



- Determine precise form of $V_g(\lambda)$ with UV and IR asymptotics fixed (at $N_f = 0$)
- Follow roughly the strategy in [Gürsoy, Kiritsis, Mazzanti, Nitti arXiv:0903.2859]
- Stiff fit to large N_c YM lattice data [Panero, arXiv:0907.3719]

Constraining the flavors at $\mu \approx 0$

Stiff fit to lattice data near $\mu = 0$ (many parameters, but results insensitive to them) [MJ, Jokela, Remes, arXiv:1809.07770]

- Many parameters already fixed by requiring qualitative agreement with QCD
- Good description of lattice data nontrivial result!

Interaction measure, 2+1 flavors

Lattice data: Borsanyi et al. arXiv:1309.5258

Baryon number susceptibility

Lattice data: Borsanyi et al. arXiv:1112.4416



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Comparison to lattice QCD

Equation of state from lattice: interaction measure in Yang-Mills $\Delta/T^4 = (\epsilon - 3p)/T^4$

Trace of the energy-momentum tensor



Glueball spectrum

	IHQCD	Lattice $(N_c = 3)$	Lattice $(N_c = \infty)$
$m_{0^{*++}}/m_{0^{++}}$	1.61	1.56(11)	1.90(17)
$m_{2^{++}}/m_{0^{++}}$	1.36	1.40(4)	1.46(11)

And critical temperature

$$\left(\frac{T_c}{m_{0^{++}}}\right)_{\rm IHQCD} = 0.167, \qquad \left(\frac{T_c}{m_{0^{++}}}\right)_{\rm lattice} = 0.177(7)$$

[Gürsoy, Kiritsis, Mazzanti, Nitti arXiv:0903.2859]

Good agreement with both spectrum and finite temperature EoS!