# Radiative Corrections in polarized SiDIS

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# RC without exclusive radiative tail

- RC to unpolarized three-fold cross section $\frac{d\sigma}{dxdydz}$

  A.V. Soroko, N.M. Shumeiko. Sov.J.Nucl.Phys. 49 (1989) 838-844,
  Yad.Fiz. 49 (1989) 1348-1358

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  A.V. Soroko, N.M. Shumeiko. Sov.J.Nucl.Phys. 53 (1991) 628-631
  Yad.Fiz. 53 (1991) 1015-1020

  as an option SIRAD of FORTRAN code POLRAD 2.0

  I.Akushevich, et al. Comp.Phys.Comm. 104 (1997) 201-244

- RC to unpolarized five-fold cross section $\frac{d\sigma}{dxdydzd\phi_h dp_T}$

  I.Akushevich, N.Shumeiko, A.Soroko. Eur.Phys.J. C10 (1999) 681-687

  Basing on this calculation FORTRAN code HAPRAD has been developed.

# RC with exclusive radiative tail

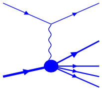
- Exclusive radiative tail contribution to unpolarized five-fold cross section $\frac{d\sigma}{dxdydzd\phi_h dp_T}$

  I.Akushevich, A.Ilyichev, M.Osipenko. Eur.Phys.J. C10 (1999) 681-687

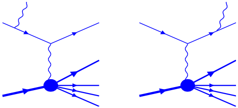
  has been included in FORTRAN code HAPRAD. As it will be shown later, in some kinematical region this contribution is rather important.

# Contribution to the lowest-order RC in SiDIS $ep \to ehX$

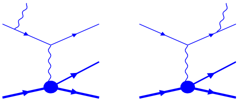
- The lowest order contribution.



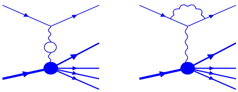
- Real photon emission from lepton line with the inelastic final hadronic state. Contains the infrared divergence.



- Real photon emission from lepton line with the exclusive final hadronic state. Infrared free.



- Additional virtual particle contribution. Last graph contains the infrared divergence.



# Advantages of Model-Independent RC

- The task can be solved exactly.
- Model-Independent RC is rather large because of including so-called leading-order term $\log(Q^2/m^2)$.
- Uncertainties of the model-independent RC come only from fits and models used for structure functions.
- The calculation of model-dependent correction (box-type graphs, real photon emission from hadronic line) requires additional assumptions about hadron interaction, so it has additional pure theoretical uncertainties, which are hard to control.

# Hadronic tensor for SIDIS

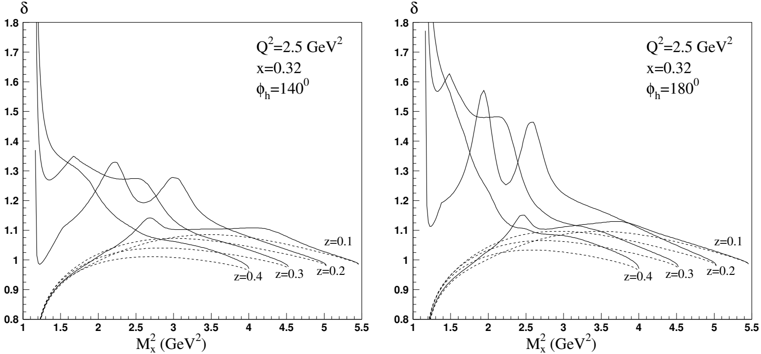
One of the important thing for the numerical estimation of model-independent RC consists in the knowledge of the hadronic tensor structure as well as the structure function parameterization in the rather wide kinematical region both for SiDIS and exclusive final hadronic states.

According to Aram Kotzinian. Nucl.Phys. B441 (1995) 234-248 the hadronic tensor for SiDIS with the initial polarized nucleon reads

$$W_{\mu\nu} = \sum_{a,b=0}^{3} e_\mu^{\gamma(a)} e_\nu^{\gamma(b)} (H_{ab}^{(0)} + \sum_{\rho,i=0}^{3} s^\rho e_\rho^{h(i)} H_{abi}^{(S)}).$$

$e^{\gamma(a,b)}$ ($e^{h(i)}$) are the complete set of the basis vectors for the polarization 4-vectors of the virtual photon (nucleon) in the target rest frame.
Due to the parity and current conservation, hermiticity as well as $ps \equiv 0$, only 5 spin-independent $H_{ab}^{(0)}$ and 13 spin-dependent $H_{abi}^{(S)}$ SF are survived. All the rest of the SF have to be set to zero.
Another set of SF can be found in A. Bacchetta et al. JHEP 0702 (2007) 093

# Importance of exclusive radiative tail




$M_X^2 = (p + q - p_h)^2 = M^2 + (q - p_h)^2 + 2(1 - z)pq$

$M_X^2$-dependence of the RC factor for the semi-inclusive $\pi^+$ electroproduction at fixed proton for lepton beam energy 6 GeV: solid lines show the total correction, dashed lines represent the correction excluding the exclusive radiative tail (I.Akushevich, A.Ilyichev, M.Osipenko, Phys.Lett. B672(2009)35)

# Exclusive radiative tail

Exclusive radiative tail contribution $d\sigma_R^{ex} \sim W_{ex}^{\mu\nu} L_{\mu\nu}^R d\Gamma_R^{ex}$,

Hadronic tensor: $W_{ex}^{\mu\nu} \sim \mathcal{M}^\mu \mathcal{M}^{\nu\dagger}$, $\mathcal{M}^\mu = \bar{u}(p_f)\left(\sum_{i=1}^6 A_i M_i^\mu\right) u(p_i)$

$$M_1^\mu = -\frac{i}{2}\gamma_5\left(\gamma^\mu \not{k} - \not{k}\gamma^\mu\right),$$

$$M_2^\mu = 2i\gamma_5\left[P^\mu k\cdot\left(q-\frac{1}{2}k\right) - \left(q^\mu - \frac{1}{2}k^\mu\right)k\cdot P\right],$$

$$M_3^\mu = -i\gamma_5\left(\gamma^\mu k\cdot q - \not{k}q^\mu\right),$$

$$M_4^\mu = -2i\gamma_5\left(\gamma^\mu k\cdot P - \not{k}P^\mu\right) - 2m_N M_1^\mu,$$

$$M_5^\mu = i\gamma_5\left(k^\mu k\cdot q - q^\mu k^2\right),$$

$$M_6^\mu = -i\gamma_5\left(\not{k}k^\mu - \gamma^\mu k^2\right).$$

Six invariant amplitudes $A_i$ can be extract from MAID 2007.

# Leading, next-to-leading, and exact contributions to RC

By "exactly" calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy. The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where $A$ and $B$ do not depend on the electron mass $m$.

- If only $A$ is kept, this is the leading log approximation.
- If both contributions are kept (i.e., contained $A$ and $B$), this is the calculation with the next-to-leading accuracy, practically equivalent to exact calculation.

Three approaches to extract the leading Log contribution for the SIDIS cross section (i.e., to calculate $A$)

- use our exact formulae, collect all terms that result in leading log after integration over photon angles, combine them into the final expression
- extract the poles that correspond to radiation collinear to initial and final electron, integrate over angles, and find the factozing form traditional for leading log calculations.
- use the method of the electron structure functions.

# Leading log: exatraction from exact formulae

The exact expression for $\sigma_R^F$ is:

$$\sigma_R^F = -\frac{\alpha^3 S S_x^2}{32\pi^2 \lambda_S \lambda_Y} \int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \int_0^{R_{\max}} dR \sum_{i=1}^{9} \left[ \sum_{j=1}^{k_i} \tilde{\mathcal{H}}_i \theta_{ij} \frac{R^{j-2}}{\tilde{Q}^4} - \frac{\theta_{i1} \mathcal{H}_i}{R Q^4} \right],$$

Analysis of the integrand and tracing the origin of the leading log allows us to extract the leading log terms from the exact formulae:

$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{j=1}^{k_i} \theta_{ij}^s R^{j-2} = -4\pi l_m \frac{\sqrt{\lambda_Y}}{S} \frac{1+z_1^2}{z_1(1-z_1)} \theta_i^{Bs}$$

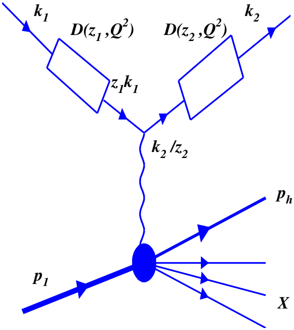
$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{j=1}^{k_i} \theta_{ij}^p R^{j-2} = -4\pi l_m \frac{\sqrt{\lambda_Y}}{X} \frac{1+z_2^2}{1-z_2} \theta_i^{Bp}$$

# Leading log: exatraction the collinear poles

- Born cross section: $\sigma_B(x, Q^2, z, t, \phi_h) = CL_{\mu\nu}(k_1, k_2)W_{\mu\nu}(p, q, p_h)$.
- RC cross section
  $\sigma_{RC}(x, Q^2, z, t, \phi_h) = \int C_p L^{rad}_{\mu\nu}(k_1, k_2, k)W_{\mu\nu}(p, q-k, p_h)d^3k/2k_0$.
- the pole for radiation collinear to the initial electron, $1/k.k_1$:
  $L^{rad}_{\mu\nu}(k_1, k_2, k) = A/k.k_1 + Bm^2/k.k_1^2 + \dots$ and keep only the term
  $A/k.k_1$.
- Substitute $k = (1-z_1)k_1$ in $A$ everywhere including $W_{\mu\nu}(p, q-k, p_h)$,
  neglect terms with $m^2$ in the numerator.
- Integrate $d^3k/k^0/k.k_1$ over angles (resulting in the leading log) and
  remaining integral over photon energy rewrite as the integral over $z_1$.
- Express the RC cross section as
  $\int_{z^1_{min}}^1 C_{pp}dz_1(1+z_1^2)/(1-z_1)f(z_1)L_{\mu\nu}((1-z_1)k_1, k_2)W_{\mu\nu}(p, q-(1-z_1)k_1, p_h)$
- Write the final initial coliiner radiation as:
  $\sigma_{RC}(x, Q^2, z, t, \phi_h) = \int_{z^1_{min}}^1 C_{pp}dz_1(1+z_1^2)/(1-z_1)f(z_1)\sigma_B(\tilde{x}, \tilde{Q}^2, \tilde{z}, \tilde{t}, \tilde{\phi}_h)$
- Obtain formulae for shifted variables and analyze shifted kinematics.
- Make similar calculation for the pole for radiation collinear to the final electron.

# Leading log: electron structure functions

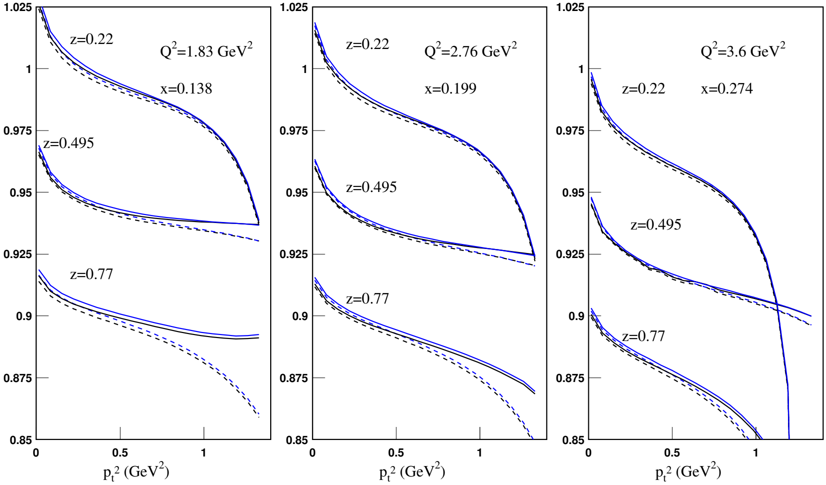
- *The QED radiative corrections to the corresponding cross sections can be written as a contraction of two electron structure functions and the hard part of the cross section.*
- *Traditionally, these radiative corrections include effects caused by loop corrections and soft and hard collinear radiation of photons and e+e- pairs.*
- *This method can be improved by also including effects due to radiation of one noncollinear photon. The corresponding procedure results in a modification of the hard part of the cross section, which takes the lowest-order correction into account exactly and allows going beyond the leading approximation.*



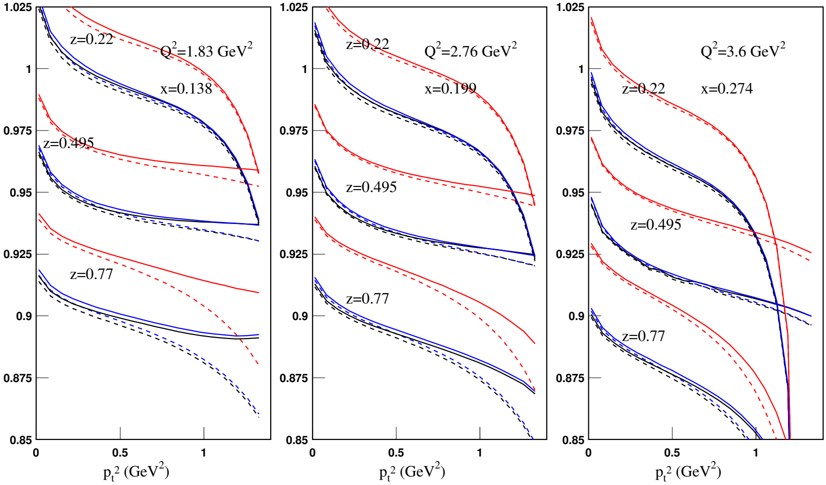

$$\sigma^{in} = \int_{z_1^m}^{1} dz_1 D(z_1, Q^2) \int_{\hat{z}_2^m}^{1} \frac{dz_2}{z_2^2} D(z_2, Q^2) \sqrt{\frac{\hat{\lambda}_Y}{\lambda_Y}} \frac{S_x^2}{\hat{S}_x^2} \hat{\sigma}_t^B$$

with ESF $D(z_{1,2}, Q^2) = D^{\gamma}(z_{1,2}, Q^2) + D_N^{e+e-}(z_{1,2}, Q^2) + D_S^{e+e-}(z_{1,2}, Q^2)$ (see details in Afanasev, Akushevich, Merenkov (2004) Journal of Experimental and Theoretical Physics, 98(3) 403-416, and references therein).

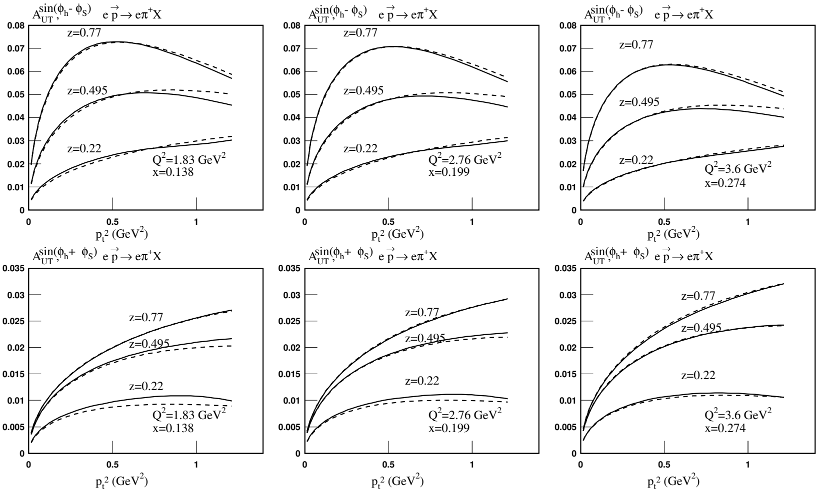
# RC to unpolarized cross section



# RC to unpolarized cross section



# RC to Collins and Sivers asymmetries



# Conclusions

- Basing on Aram's expression we construct the hadronic tensor in the covariant form

$$W_{\mu\nu} = -g^{\perp}_{\mu\nu}\mathcal{H}_1 + p^{\perp}_{\mu}p^{\perp}_{\nu}\mathcal{H}_2 + p^{\perp}_{h\mu}p^{\perp}_{h\nu}\mathcal{H}_3 + (p^{\perp}_{\mu}p^{\perp}_{h\nu} + p^{\perp}_{h\mu}p^{\perp}_{\nu})\mathcal{H}_4$$

$$+i(p^{\perp}_{\mu}p^{\perp}_{h\nu} - p^{\perp}_{h\mu}p^{\perp}_{\nu})\mathcal{H}_5 + (p^{\perp}_{\mu}n_{\nu} + n_{\mu}p^{\perp}_{\nu})\mathcal{H}_6 + i(p^{\perp}_{\mu}n_{\nu} - n_{\mu}p^{\perp}_{\nu})\mathcal{H}_7$$

$$+(p^{\perp}_{h\mu}n_{\nu} + n_{\mu}p^{\perp}_{h\nu})\mathcal{H}_8 + i(p^{\perp}_{h\mu}n_{\nu} - n_{\mu}p^{\perp}_{h\nu})\mathcal{H}_9.$$

  $g^{\perp}_{\mu\nu} = g_{\mu\nu} - q_{\mu}q_{\nu}/q^2$ $p^{\perp}_{\mu} = p_{\mu} - q_{\mu}pq/q^2$ and
  $n^{\mu} = \varepsilon^{\mu\nu\rho\sigma}q_{\nu}p_{\rho}p_{h\sigma}$
  with 9 generalized SF $\mathcal{H}_{1-9}$ and found that the born cross section exactly reproduced the cross section obtained by Alessandro Bacchetta.
- Using Bardin-Shumeiko approach we obtained the analytical expressions for the lowest order model-independent RC to polarized SiDIS. These expressions are written in the most compact, covariant form convenient for the numerical analysis.

# Conclusions

- The obtained above results have been published in I.Akushevich, A.Ilyichev. Phys.Rev. D100 (2019) no.3, 033005
- More than two year ago using WW-SIDIS model for the structure functions
  S. Bastami *et al.*, JHEP **1906**, 007 (2019)
  we start to develop the new version of FORTRAN code HAPRAD that allows to calculate RC to polarized SiDIS coming from the inelastic final hadronic state.
- We completed the theoretical calculation of exclusive radiative tail for polarized particles using the six invariant amplitudes $A_{1-6}$. Fortan code is upgrated and is under final testing. Further dicussion with theoreticians and experimentalsts are necessary to provide cross check and sensitivitity studies. We plan to publish the respective (and highly demanded) paper as soon as possible.