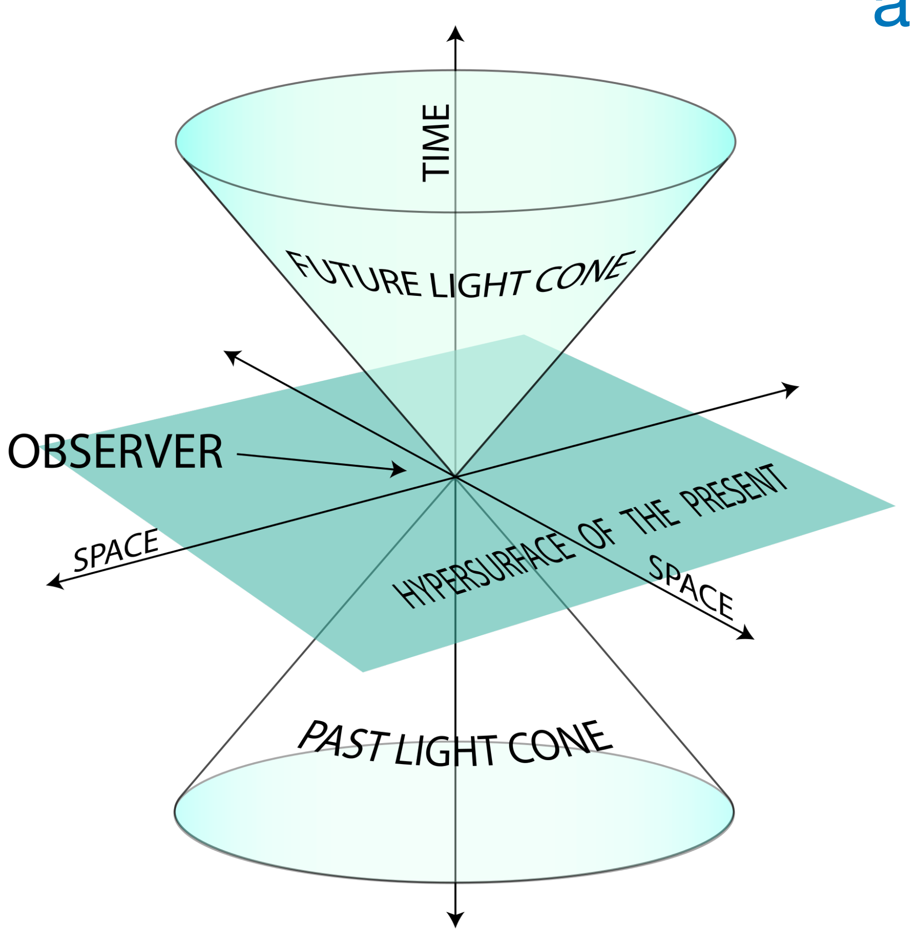
APCTP Focus Program in Nuclear Physics 2022 :
Hadron Physics Opportunities with JLab Energy and Luminosity Upgrade

APCTP
21 July 2022

Analysis of Virtual Meson Production in a (1+1)-dimensional Scalar Field Model

Yongwoo Choi
Korea University and Inha University

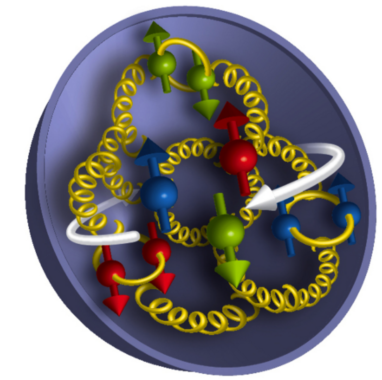
with Prof. Chueng-Ryong Ji, Prof. Ho-Meoyng Choi,
and Prof. Yongseok Oh

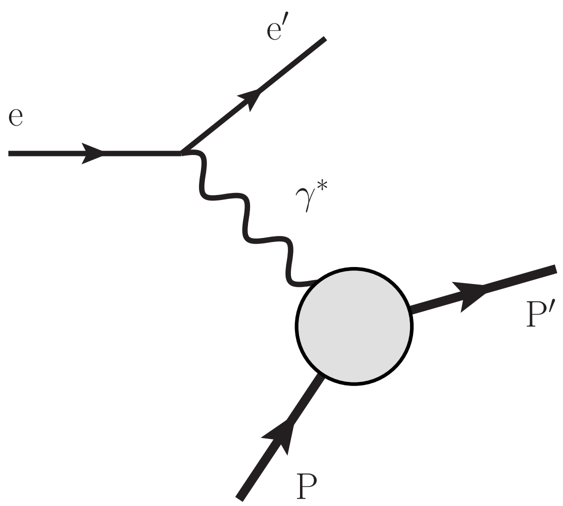


Virtual Meson Production (VMP)

Hadron (meson and baryon) is a composite system made of quarks, anti-quarks, and gluons (parton), held together by non-perturbative QCD interaction.

complicated structure



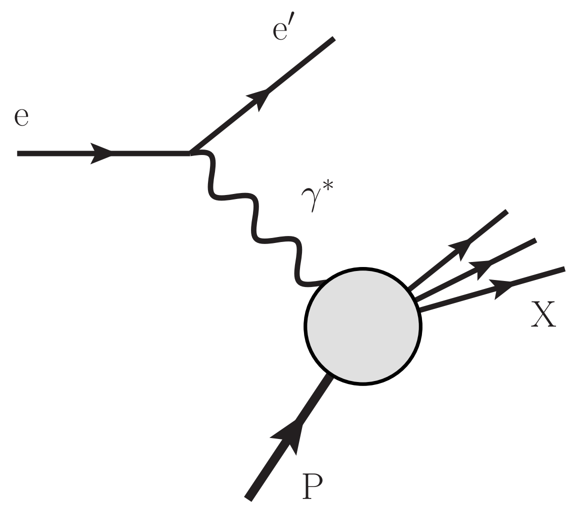


Elastic scattering

density

$$\langle P' \,|\, \bar{\psi}(0)\ \hat{\mathcal{O}}\ \psi(0) \,|\, P \rangle$$

Electro**m**agnetic
Form
Factors
(EM FFs)

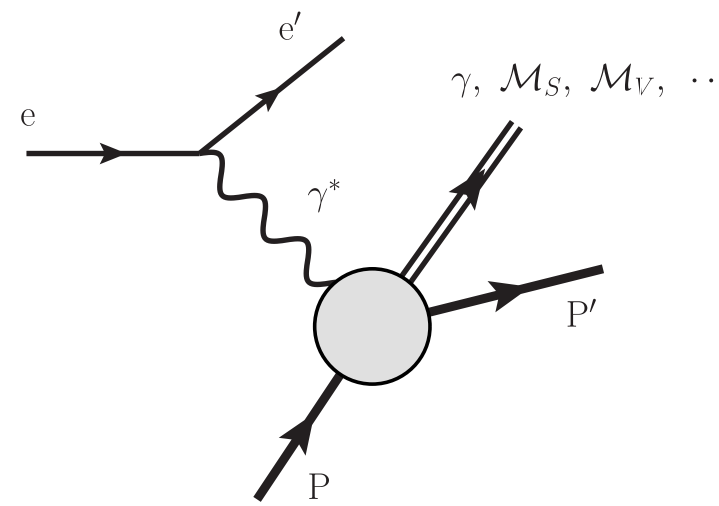


Inclusive / Inelastic

momentum

$$\langle P \,|\, \bar{\psi}(z)\ \hat{\mathcal{O}}\ \psi(0) \,|\, P \rangle$$

Parton
Distribution
Functions
(PDFs)



Exclusive / Inelastic

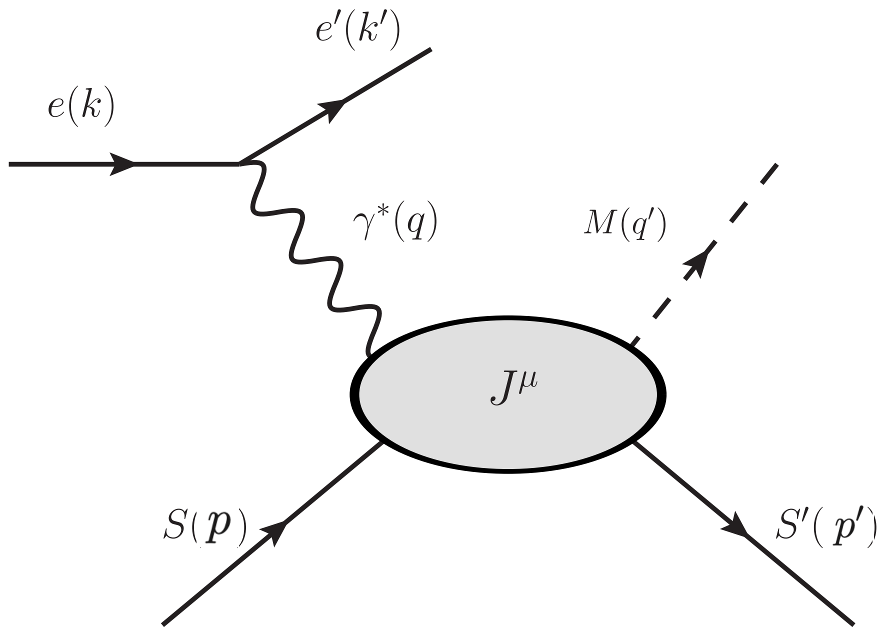
angular momentum

$$\langle P' \,|\, \bar{\psi}(z)\ \hat{\mathcal{O}}\ \psi(0) \,|\, P \rangle$$

Generalized
Parton
Distributions
(GPDs)

Generalized Structure Functions for VMP (GVMP)

Generalized structure functions (Compton Form Factors (CFFs))



Through **symmetries**, construct hadronic currents.

DNA method provides most general hadronic currents,

for the pseudoscalar target ($J^{PC} = 0^{-+}$) : $\epsilon^{\mu\nu\alpha\beta}$

for the **scalar target** (0^{++}) : $d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta}$

$$J^\mu_S = (\mathscr{F}_1(Q^2, x, t)\ q_\alpha + \mathscr{F}_2(Q^2, x, t)\ \mathscr{P}_\alpha)\ \boxed{d^{\mu\nu\alpha\beta}} q_\beta \Delta_\nu$$

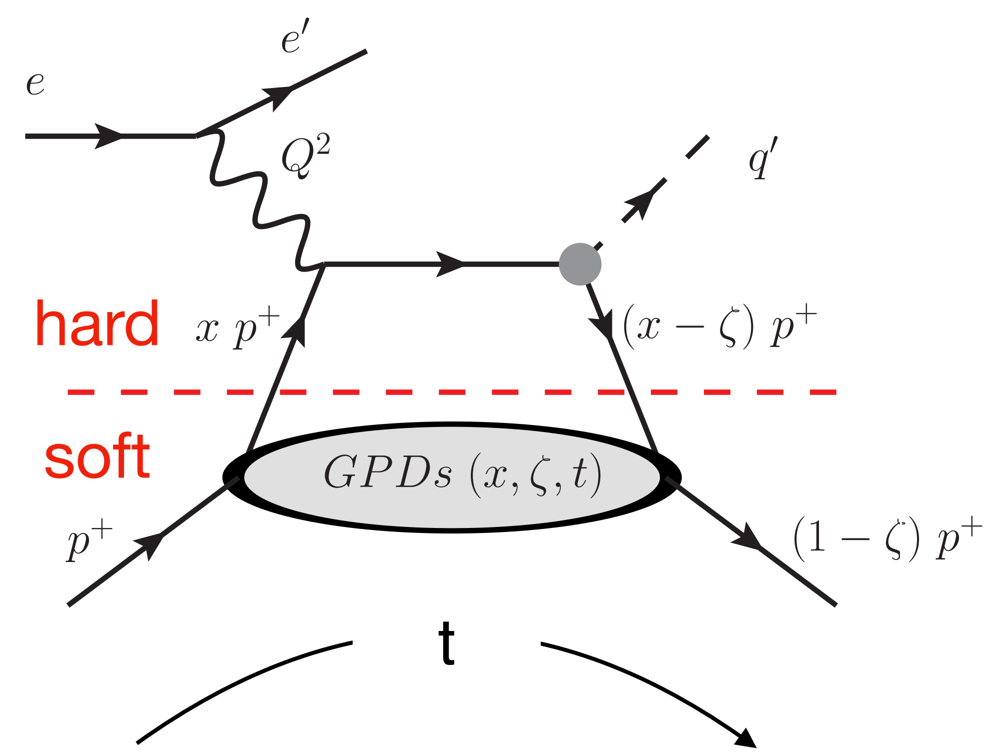
$$= \left\{q^2\ \Delta^\mu - (\Delta \cdot q)\ q^\mu\right\} \boxed{\mathscr{F}_1(Q^2, x, t)} + \left\{(\mathscr{P} \cdot q)\ \Delta^\mu - (\Delta \cdot q)\ \mathscr{P}^\mu\right\} \boxed{\mathscr{F}_2(Q^2, x, t)}$$

where $\Delta = p - p' = q' - q, \quad \mathscr{P} = p + p'$

Ji, Chueng-Ryong and Choi, Ho-Meoyng and Lundeen, Andrew and Bakker, Bernard L. G. Phys. Rev. D **99**, 11 116008 (2019)

Generalized Parton Distributions (GPDs)

High energy approach → twist expansion



Skewed (off-forward) parton distribution :

hard part (OPE) $\otimes$ soft part (GPD)

In the forward limit ($\zeta \to 0$), parton distribution :

hard part (OPE) $\otimes$ soft part (PDF)

$\int dx$, Electromagnetic form factor :

F(t) = $\int dx$ GPD

p^+ : Light-front (LF) longitudinal momentum of incoming target,

Q^2 : Virtuality → factorizable in $Q^2 >> |t|, M_t^2, M_s^2, m_{Q_1}^2, m_{Q_2}^2, \cdots,$

x : Longitudinal momentum fraction,

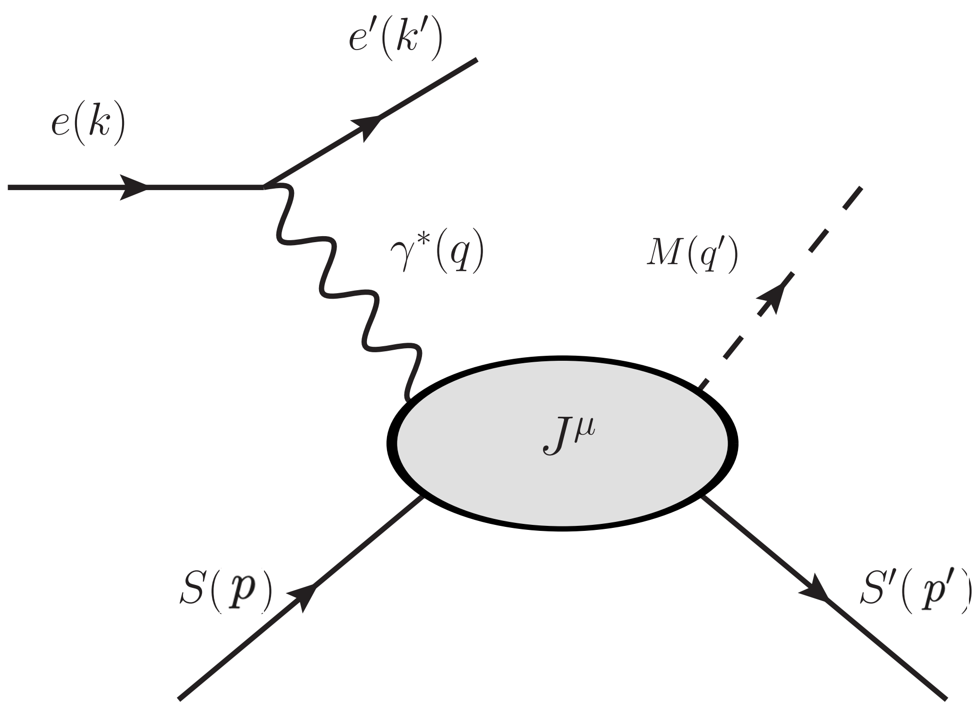
ζ : Skewness, $(p^+ - p'^+)/p^+$, asymmetry of LF longitudinal momentum of target,

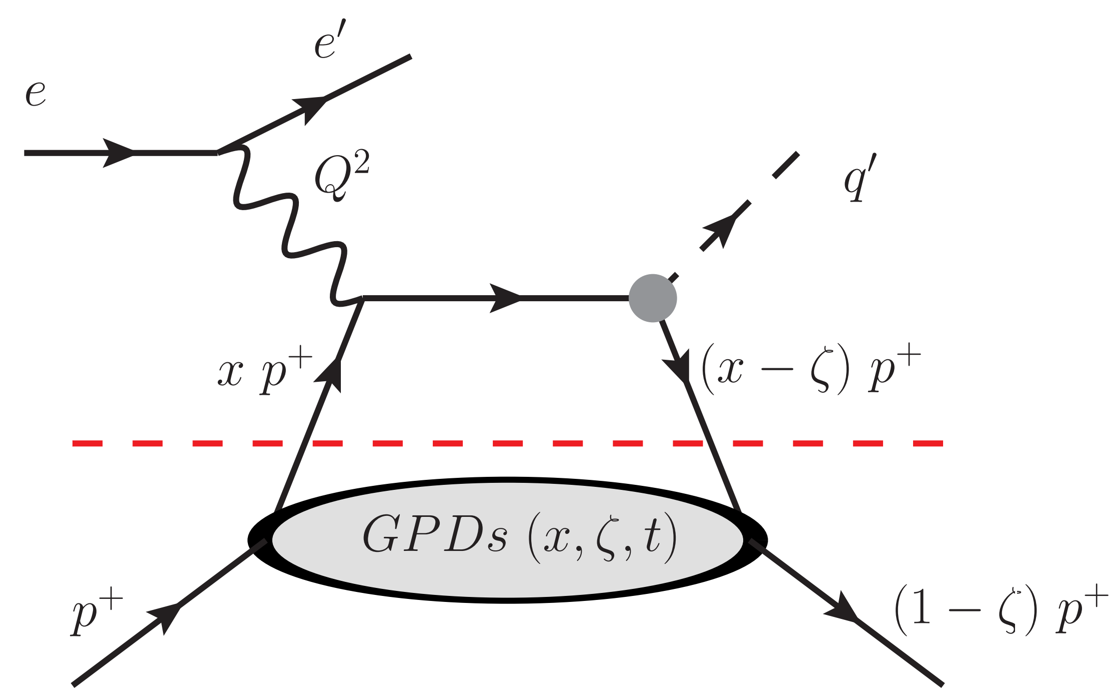
t : Square momentum transfer, $\Delta^2 = (q' - q)^2 = (p - p')^2$,

"kick" transverse momentum depending on scattering angle.

$$t = -\frac{\zeta^2 M_t^2 + \Delta_\perp^2}{1 - \zeta}$$

GVMP with one-loops vs GPD



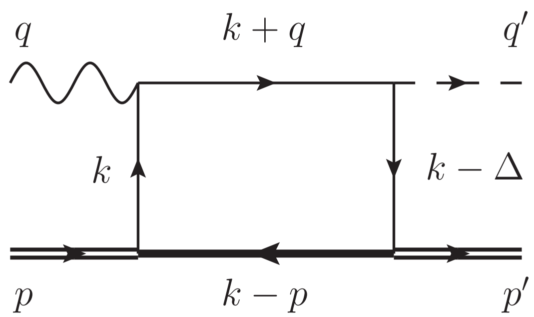


Spin-0 target	GVMP	GPD (Twist-2)
Factorizable	X	O
Diagrams	S-, U- box, and Cat's ears	Parts of S-, U- box
Ward Identity	Satisfied	Not satisfied
Kinematic range	All	Limited
# of CFFs	2	1
Exp.	Compton Form Factors, Beam Spin Asymmetry, …	

Box, Cat's ears, and Effective tree diagrams

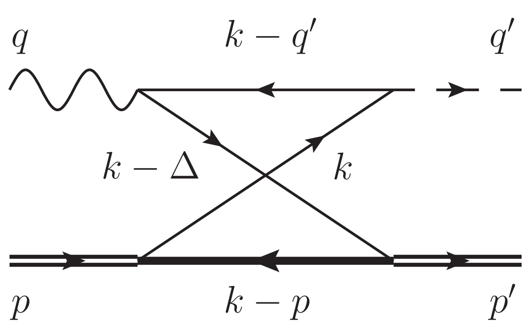
Partons inside hadron can be fermions (quarks and anti-quarks) or spin-1 boson (gluons).
In this work, we introduce **scalar currents** ($J(0) = \phi^{\dagger}(z)\ \phi(0)$) for the theoretical study, and hadron vertex functions are regarded as constants for simplicity.

Box-S



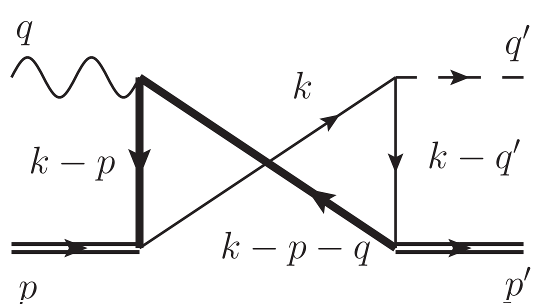
$$\mathcal{M}_s^{\mu} \sim i \int d^2k \frac{2k^{\mu} + q^{\mu}}{(k^2 - m^2)((k+q)^2 - m^2)((k-\Delta)^2 - m^2)((k-p)^2 - M^2)}$$

Box-U



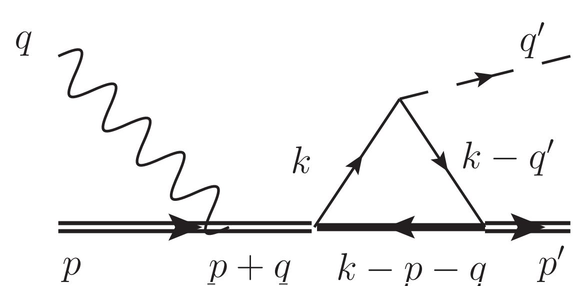
$$\mathcal{M}_u^{\mu} \sim i \int d^2k \frac{2k^{\mu} - \Delta^{\mu} - q'^{\mu}}{(k^2 - m^2)((k-q')^2 - m^2)((k-\Delta)^2 - m^2)((k-p)^2 - M^2)}$$

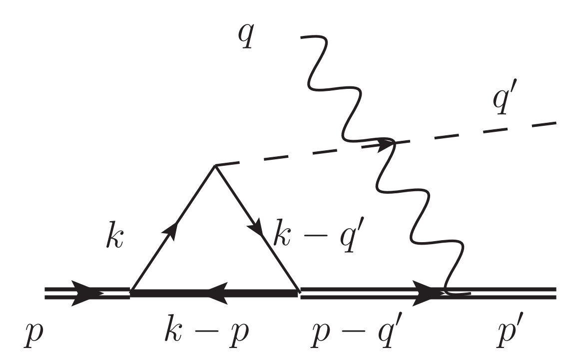
Cat's ears



$$\mathcal{M}_c^{\mu} \sim i \int d^2k \frac{2k^{\mu} - 2p^{\mu} - q^{\mu}}{(k^2 - m^2)((k-q')^2 - m^2)((k-p-q)^2 - M^2)((k-p)^2 - M^2)}$$

For the charged target, adding





Operator Product Expansion (OPE)

- **Twist** expansion : **Bilocal** current = sum of **local** currents.

$$\hat{T}\{J^{\mu}(z)J^{\nu}(0)\} = \sum_{n} C_n(z)\ \hat{\mathcal{O}}_n(0) \qquad \text{for large } Q^2$$

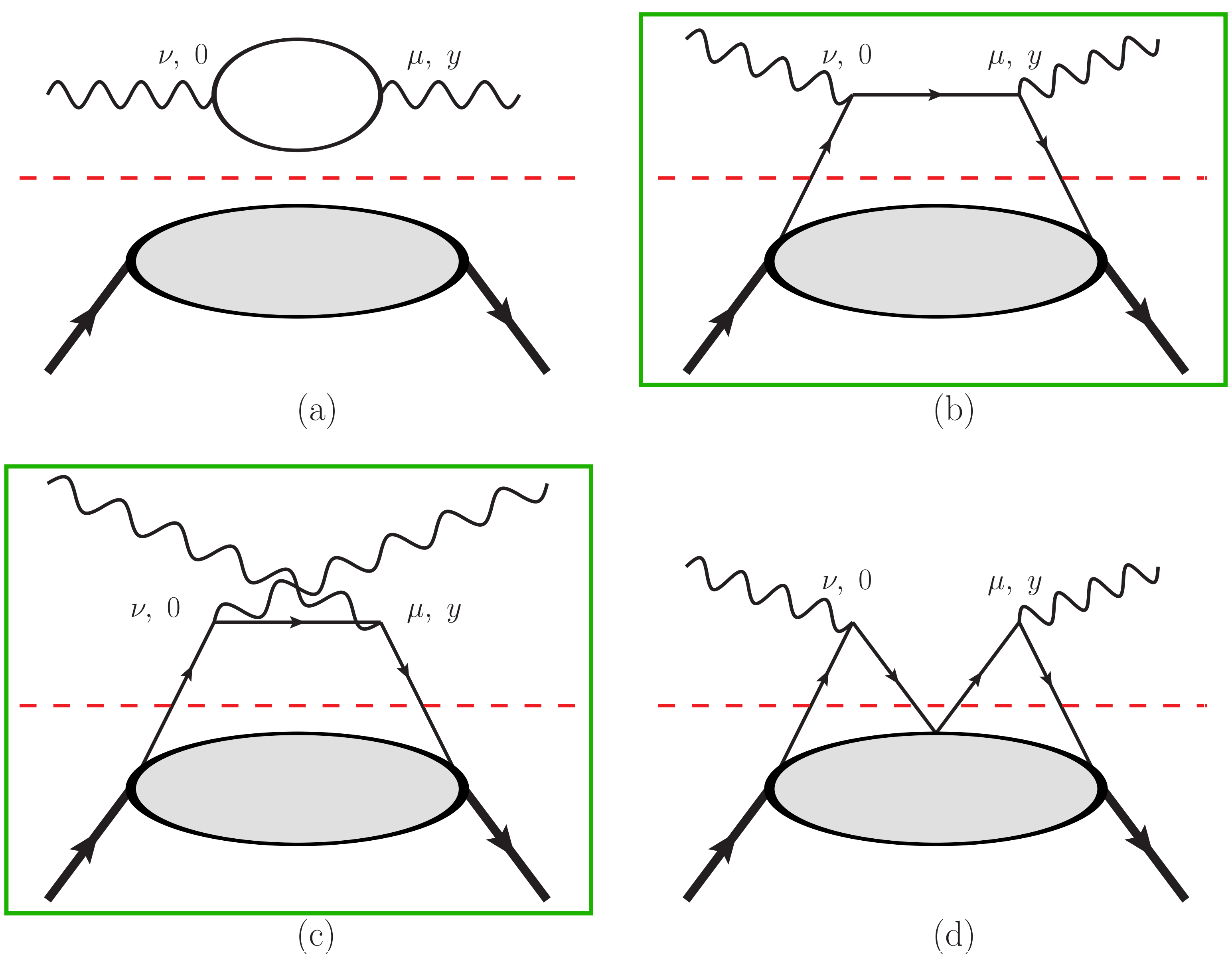
$\hat{T}\{J^{\mu}(z)J^{\nu}(0)\} =$

$-\mathrm{Tr}[\gamma^{\mu}S_F(z)\gamma^{\nu}S_F(-z)]$ (a)

$+ :\bar{\psi}_q(z)\gamma^{\mu}S_F(z)\gamma^{\nu}\psi_q(0):$ (b)

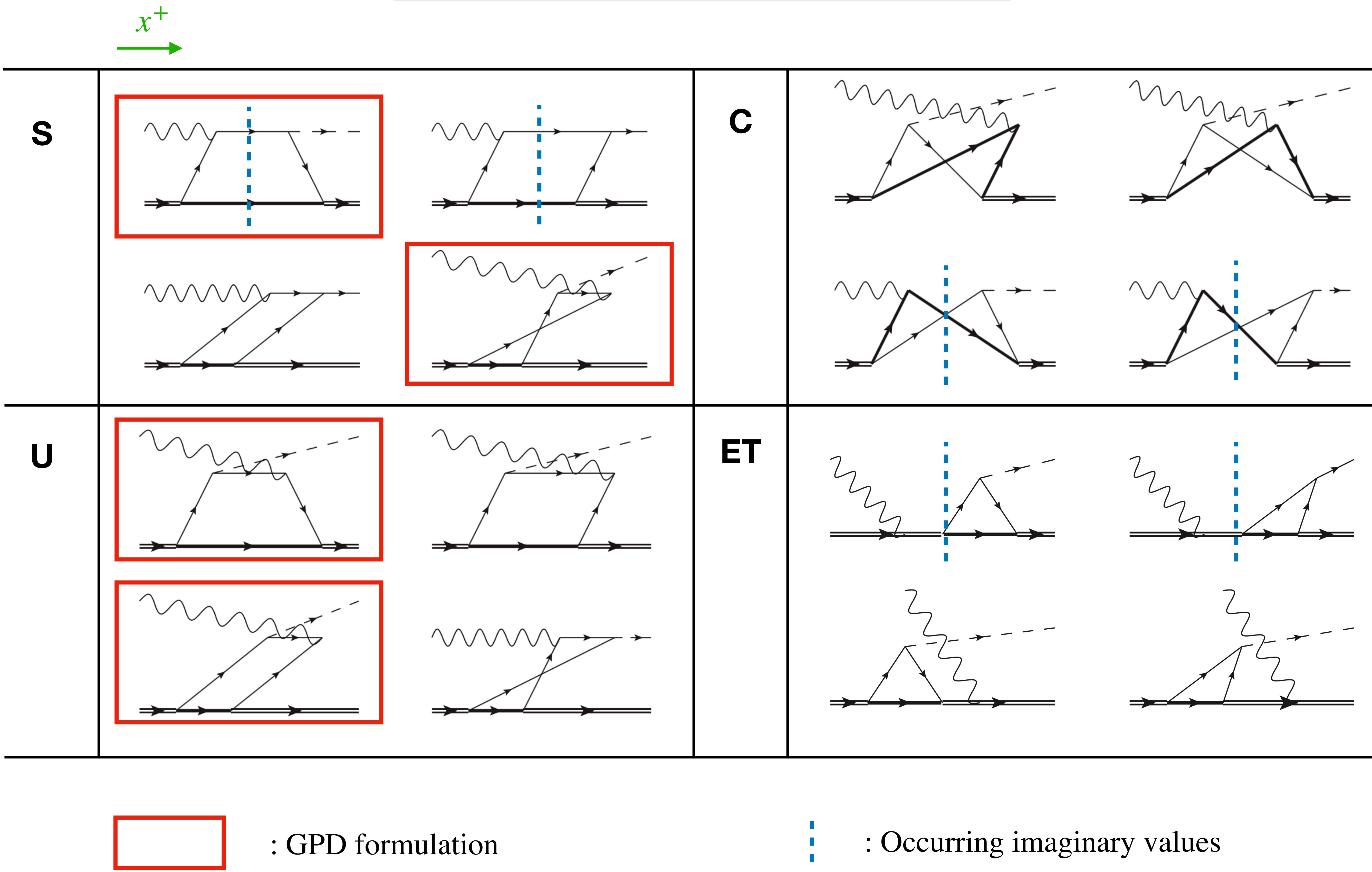
$+ :\bar{\psi}_q(0)\gamma^{\nu}S_F(-z)\gamma^{\mu}\psi_q(z):$ (c)

$+ :\bar{\psi}_q(z)\gamma^{\mu}\psi_q(z)\bar{\psi}_q(0)\gamma^{\nu}\psi_q(0):$ (d)



: Handbag dominance

LF time-ordered amplitudes



Motivation

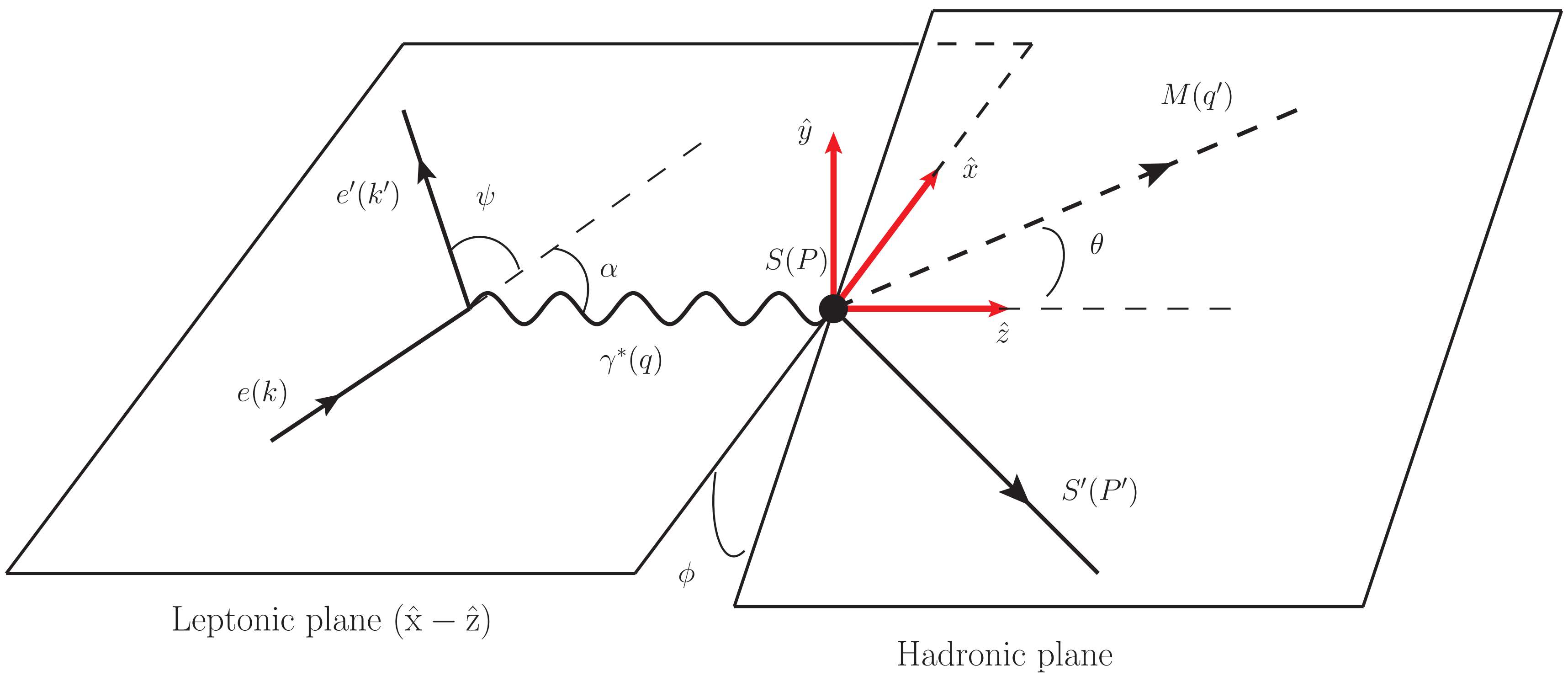
The numbers of Compton Form Factors (CFFs) are different for the **GVMP** (includes box, cat's ears, and effective tree diagrams) and the **leading twist-GPD** (box diagrams only => handbag dominance).

The equivalence of two approaches can be expected in the specific energy region, but a direct relation is not yet identified.

The purpose of this work is

doing the theoretical simulations using two approches in JLab kinematic range. It provides information on the kinematic range in which the use of leading twist can be ensured.

Beam Spin Asymmetry



Beam **S**pin **A**symmetry (BSA) of the scalar meson electroproduction off the **scalar target** :

$$\frac{d\sigma^S_{\lambda=+1} - d\sigma^S_{\lambda=-1}}{d\sigma^S_{\lambda=+1} + d\sigma^S_{\lambda=-1}} = \frac{d\sigma^S_{BSA}}{d\sigma^S_T\ (1+\epsilon\ \cos(2\phi)) + d\sigma^S_L\ \epsilon_L + d\sigma^S_{LT}\ \cos\phi\sqrt{\epsilon_L(1+\epsilon)/2}}$$

$$\sim \mathscr{F}_1\ \mathscr{F}^*_2\ -\ \mathscr{F}_2\ \mathscr{F}^*_1$$

Leading twist GPD does not provide BSA, because it has only one GPD for the spin-0 target.

Compton Form Factors

- **Each Lorentz vector is satisfied with Ward identity : $q \cdot \mathscr{M} = 0$**
- **Two Lorentz vectors are not independent in (1+1)-dimensions.**

$$\mathscr{M}^{\mu}_{tot} = \left\{ (\Delta \cdot q)\, q^{\mu} - q^2\, \Delta^{\mu} \right\} \mathscr{F}_1 + \left\{ (\Delta \cdot q)\, \mathscr{P}^{\mu} - (\mathscr{P} \cdot q)\, \Delta^{\mu} \right\} \mathscr{F}_2$$

$$= A^{\mu}\, \mathscr{F}_1 + B^{\mu}\, \mathscr{F}_2 \qquad \text{in (3+1)-dimensions}$$

$$= A^{\mu}\, (\mathscr{F}_1 + c\mathscr{F}_2) = A^{\mu}\, \mathscr{F}_A \qquad \text{in (1+1)-dimensions}$$

where $\mathscr{P} = p + p'$, $\Delta = p - p' = q' - q$, $c = B^{+}/A^{+} = B^{-}/A^{-}$.

In the large Q^2 ,

$$\mathscr{M}^{\mu}_{tot} \sim \frac{1}{2}\, \zeta\, Q^2\, \mathscr{F}_1 \quad \Rightarrow \quad \mathscr{F}_1 \sim \mathscr{F}_A$$

Kinematics & GPD limit

Kinematics in (1+1)-LFD

$$k = [\ xp^+,\ \ k^-\ ],$$

$$p = \left[\ p^+,\ \frac{M_t^2}{p^+}\ \right],$$

$$q = \left[\ \left(\frac{\zeta' M_s^2}{Q^2} - \zeta\right) p^+,\ \left(\frac{1}{\zeta'} - \frac{t}{\zeta Q^2}\right) \boxed{\frac{Q^2}{p^+}}\ \right],$$

$$\Delta = q' - q = p - p' = \left[\ \zeta p^+,\ \frac{t}{\zeta p^+}\ \right],$$

$$p' = \left[\ (1-\zeta)\, p^+,\ \left(M_t^2 - \frac{t}{\zeta}\right) \frac{1}{p^+}\ \right],$$

$$q' = \left[\ \frac{\zeta' M_s^2 p^+}{Q^2},\ \frac{\boxed{Q^2}}{\zeta' p^+}\ \right],$$

where $\zeta = \dfrac{p^+ - p'^+}{p^+}$.

GPD limit ($Q^2 >> |t|,\ M_s^2,\ \cdots$)

$$k = [\ xp^+,\ \ k^-\ ],$$

$$p = \left[\ p^+,\ \frac{M_t^2}{p^+}\ \right],$$

$$q = \left[-\zeta p^+,\ \frac{Q^2}{\zeta' p^+}\ \right] \simeq \left[-\zeta p^+,\ \boxed{\frac{Q^2}{\zeta p^+}}\right],$$

$$\Delta = \left[\ \zeta p^+,\ \frac{t}{\zeta p^+}\ \right],$$

$$p' = \left[\ (1-\zeta)\, p^+,\ \left(M_t^2 - \frac{t}{\zeta}\right) \frac{1}{p^+}\ \right],$$

$$q' = \left[\ 0,\ \frac{Q^2}{\zeta' p^+}\ \right] \simeq \left[\ 0,\ \boxed{\frac{Q^2}{\zeta p^+}}\right],$$

where there is no M_s dependence, and $\zeta \simeq \zeta'$.

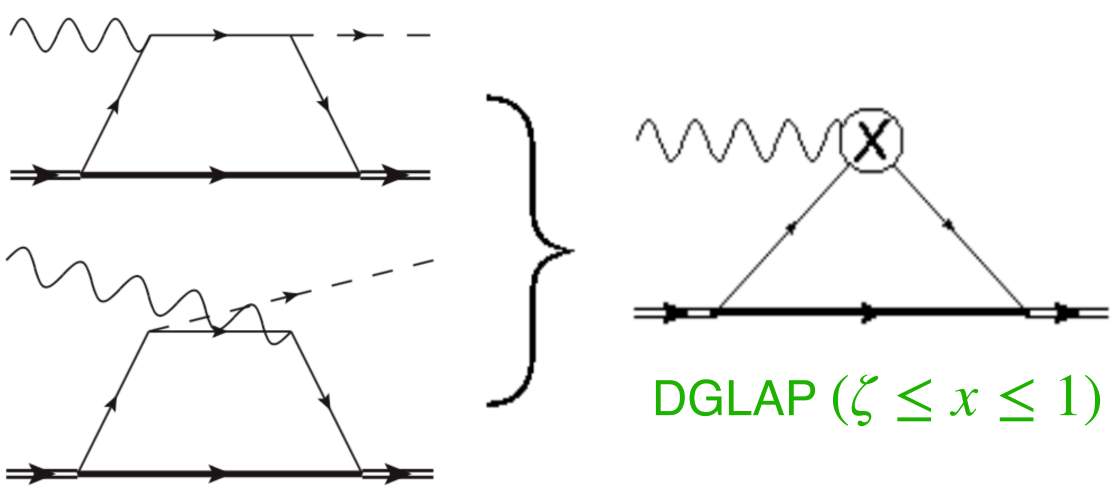
GPD does not distinguish between DVMP & DVCS.

CR Ji, BLG Bakker,
Int. J. Mod. Phys. E 22, 1330002 (2013)

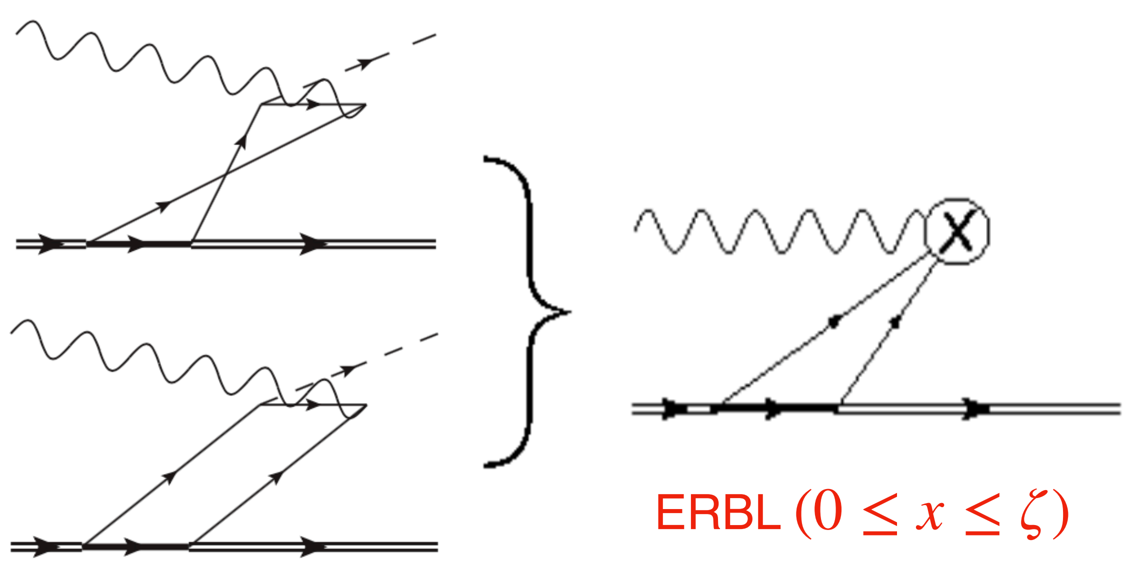
GPD Formulation

- In GPD limit ($Q^2 >> |t|, M_s^2, \cdots$), q^- and q'^- are dominant, it leads to the reduced scattering amplitudes with GPD functions.

s-ch : $\dfrac{1}{(k+q)^2 - m^2} \sim \dfrac{1}{q^-\ (k^+ + q^+)} \sim \dfrac{1}{q^-\, \boxed{(x-\zeta)}\, p^+}$, u-ch : $\dfrac{1}{(k-q)^2 - m^2} \sim -\dfrac{1}{q'^-\ (k^+ - q'^+)} \sim -\dfrac{1}{q^-\, \boxed{x}\, p^+}$



DGLAP ($\zeta \le x \le 1$)



ERBL ($0 \le x \le \zeta$)

Generalized **P**arton **D**istributions (GPDs) : angular momentum

$$\mathscr{M} \sim \int_\zeta^1 dx \left(\frac{1}{\boxed{x-\zeta}} - \frac{1}{\boxed{x}} \right) \boxed{H_{DGLAP}(x,\ t)}$$

$$+ \int_0^\zeta dx \left(\frac{1}{\boxed{x-\zeta}} - \frac{1}{\boxed{x}} \right) \boxed{H_{ERBL}(x,\ t)}$$

Parton **D**istribution **F**unctions (PDFs) : momentum

$$f(x) \sim \lim_{\zeta \to 0} H(x,\ \zeta,\ t) = H_{DGLAP}(x,\ t=0)$$

Electro**m**agnetic **F**orm **F**actors (EM FFs) : density

$$F(t) \sim \frac{1}{2-\zeta} \left[\int_0^\zeta dx\ H_{ERBL}(x,\ t) + \int_\zeta^1 dx\ H_{DGLAP}(x,\ t) \right]$$

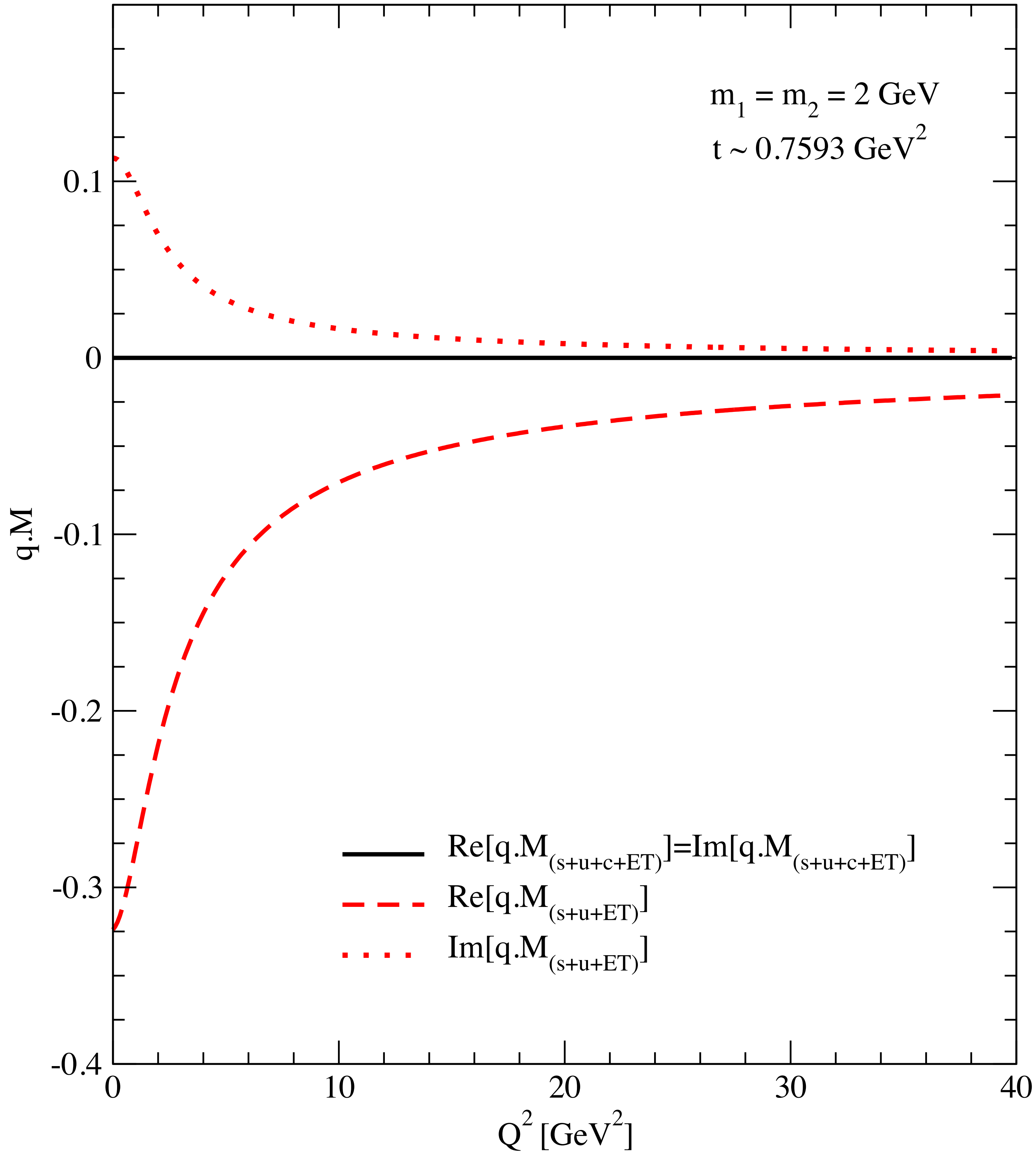
Numerical Result - Ward Identity

- **Ward identity does not hold without cat's ears diagrams.**

$$q \cdot \mathcal{M} = 0 \quad :$$

$$q^+ \, \mathrm{Re}[\mathcal{M}^-] \; = -\, q^- \, \mathrm{Re}[\mathcal{M}^+]$$

$$q^+ \, \mathrm{Im}[\mathcal{M}^-] \; = -\, q^- \, \mathrm{Im}[\mathcal{M}^+]$$

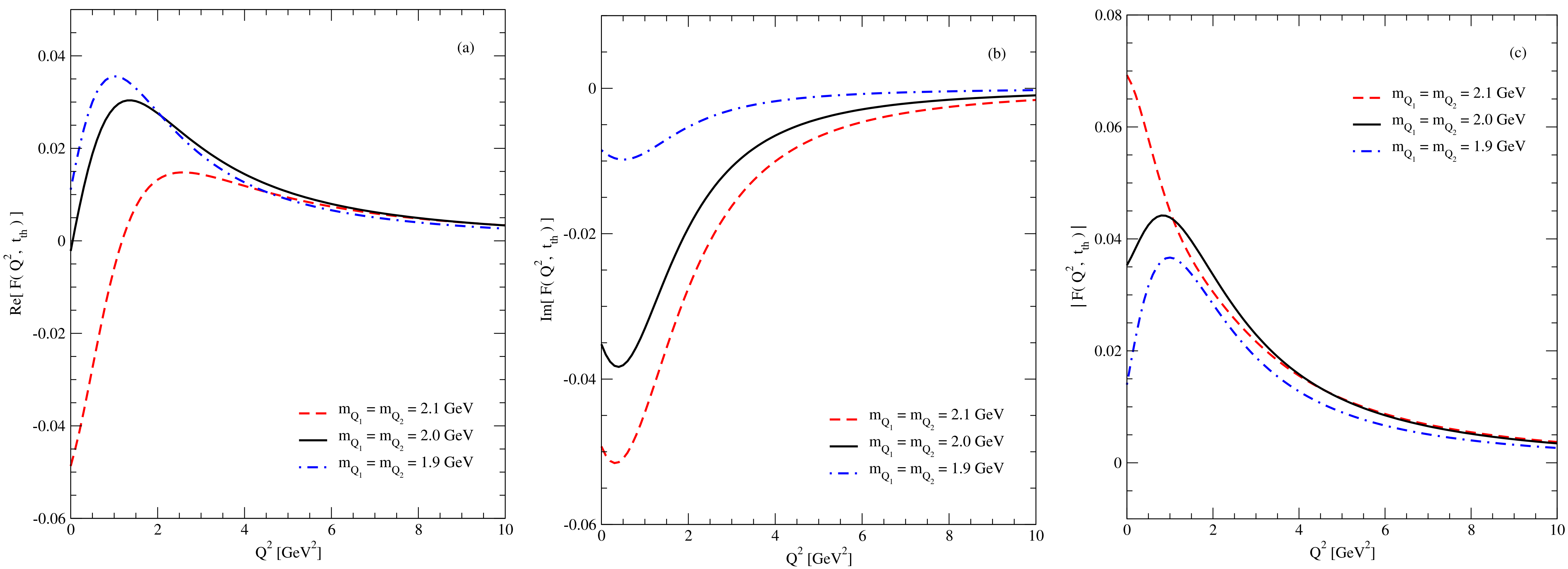


Numerical Result - Compton Form Factors (GVMP)

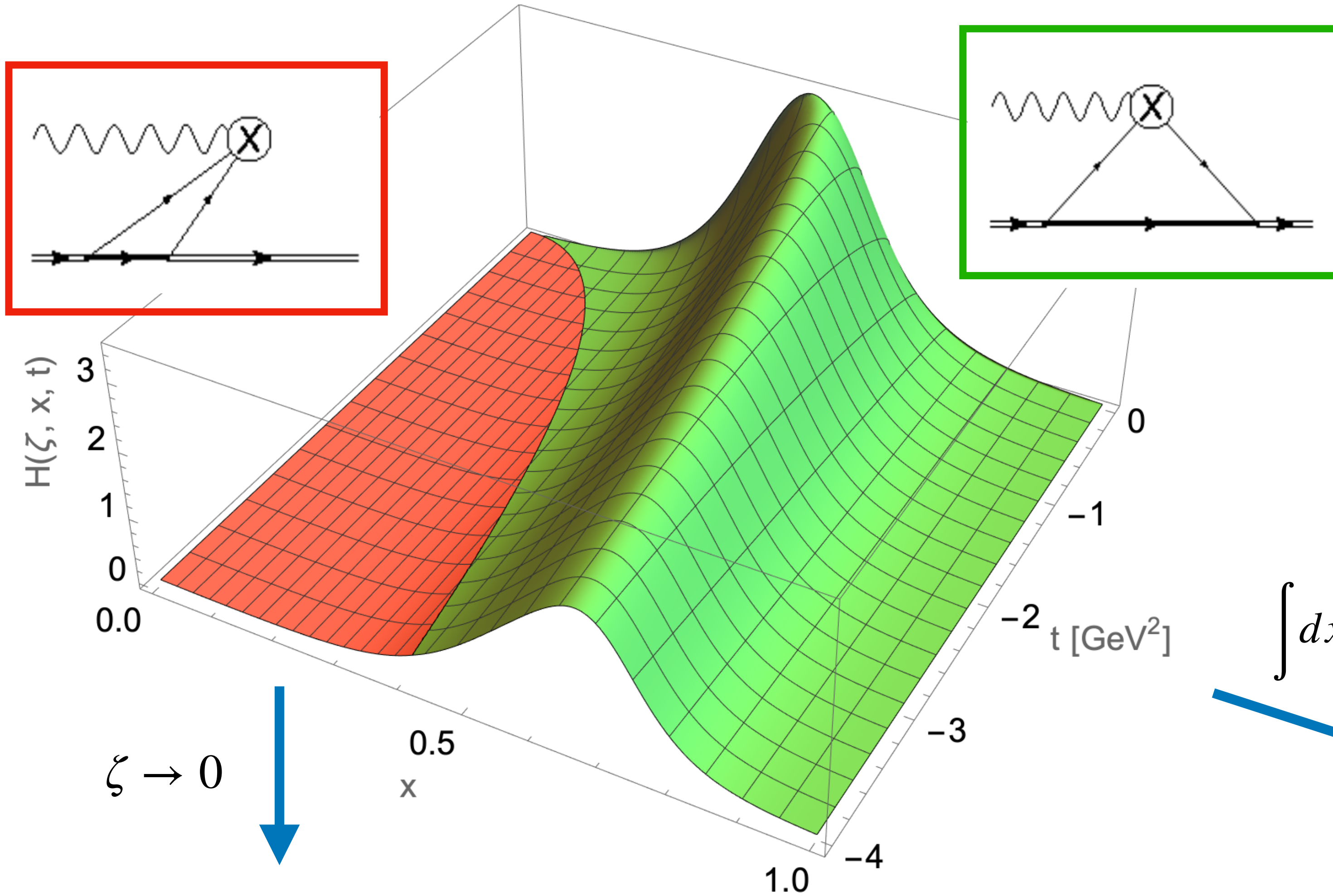
- Parameter set :

$$m_{Q_1} = 2,\ \ m_{Q_2} = 2,\ \ M_t\ (^4He) = 3.7,\ \ M_s\ (f_0) = 0.98\ \text{GeV},\ \ t = -0.7593\ \text{GeV}^2$$

- The imaginary part related to the beam spin asymmetry (BSA) is relatively not small.



Numerical Result - GPDs, PDFs, and FFs

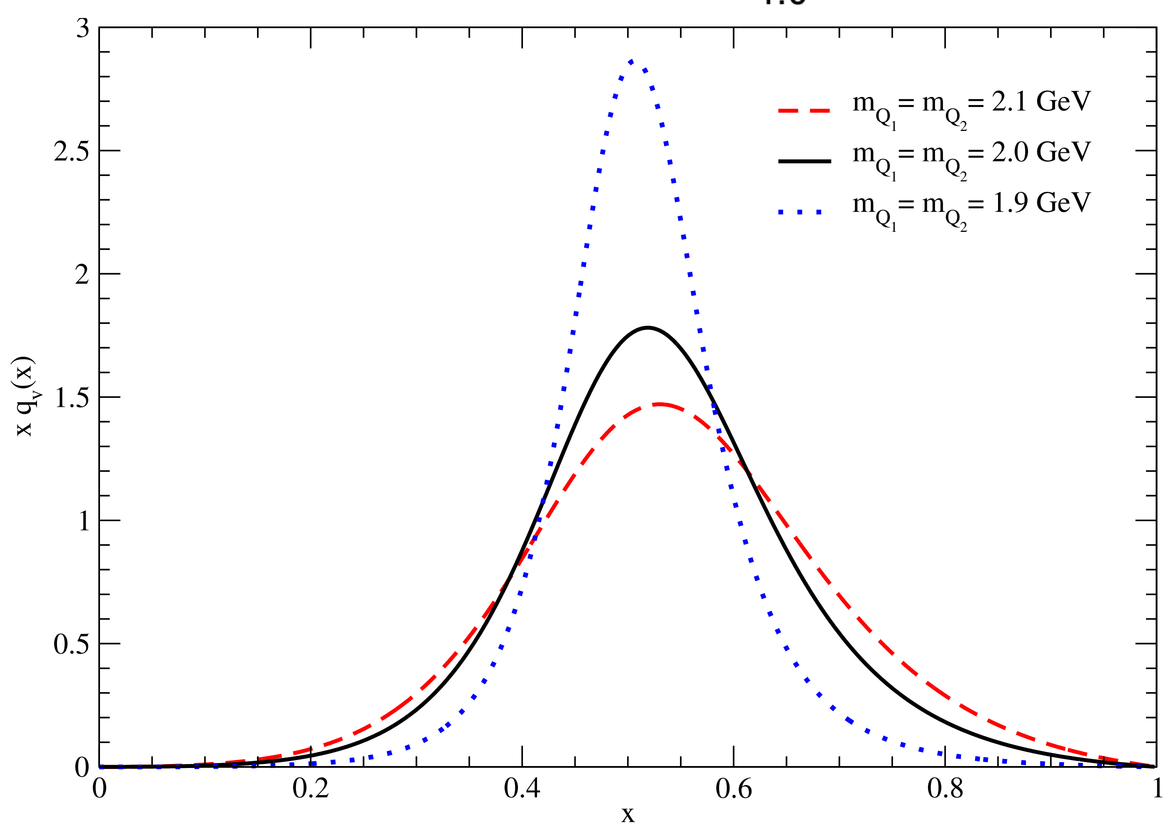


$$\zeta = \frac{1}{2\ M_t^2}\left(t + \sqrt{t^2 - 4\ t\ M_t^2}\right)$$

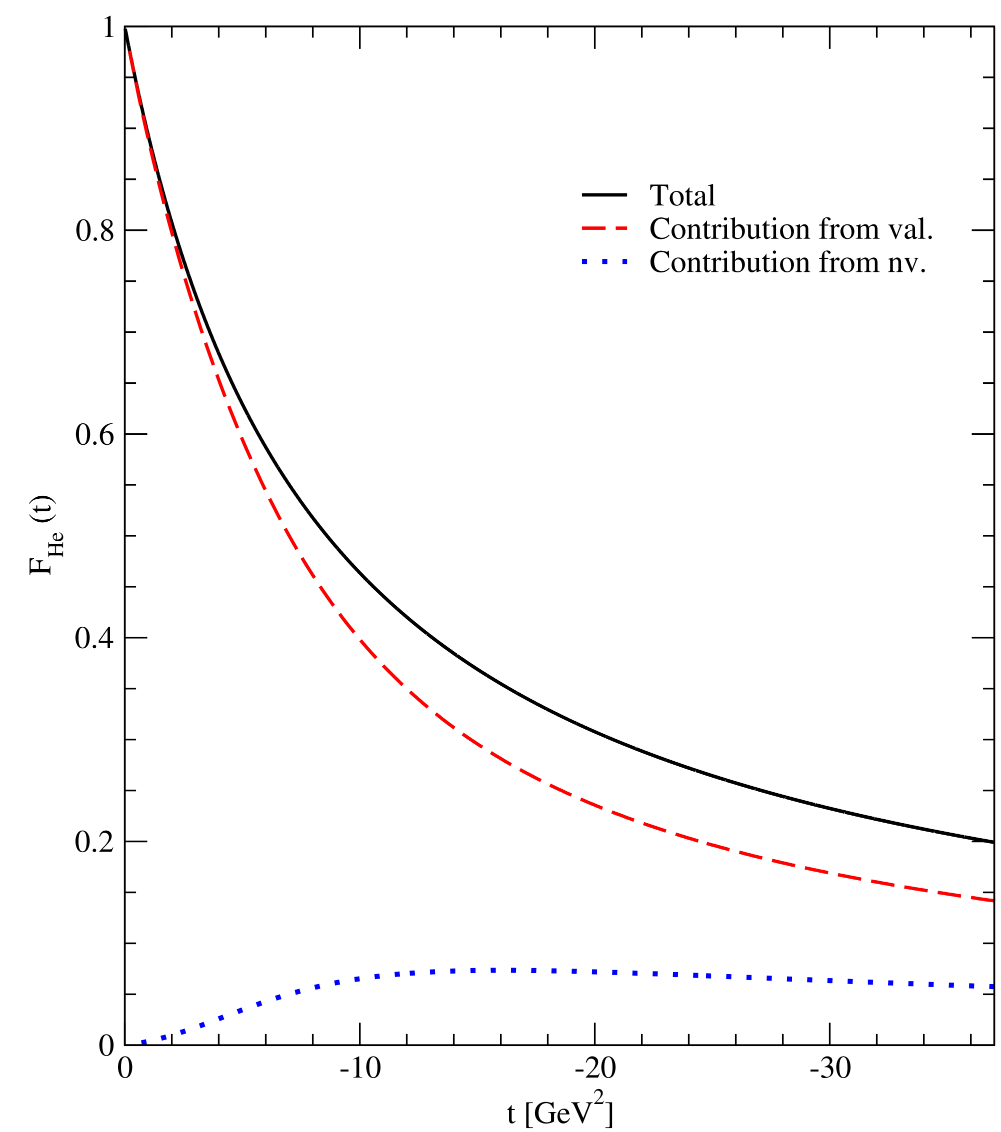
DGLAP $(\zeta \le x \le 1) \rightarrow$ valence

ERBL $(0 \le x \le \zeta) \rightarrow$ non-valence

$\zeta \rightarrow 0$



$\int dx$



Kinematic settings

- In Jefferson laboratory experiments, for $e + {}^4He \to e' + \gamma + {}^4He$ (DVCS) :

$$1.9 < Q^2 < 9.0\ GeV^2, \quad -4.5 < t < -1.0\ GeV^2, \quad -t/Q^2 \sim 0.5$$

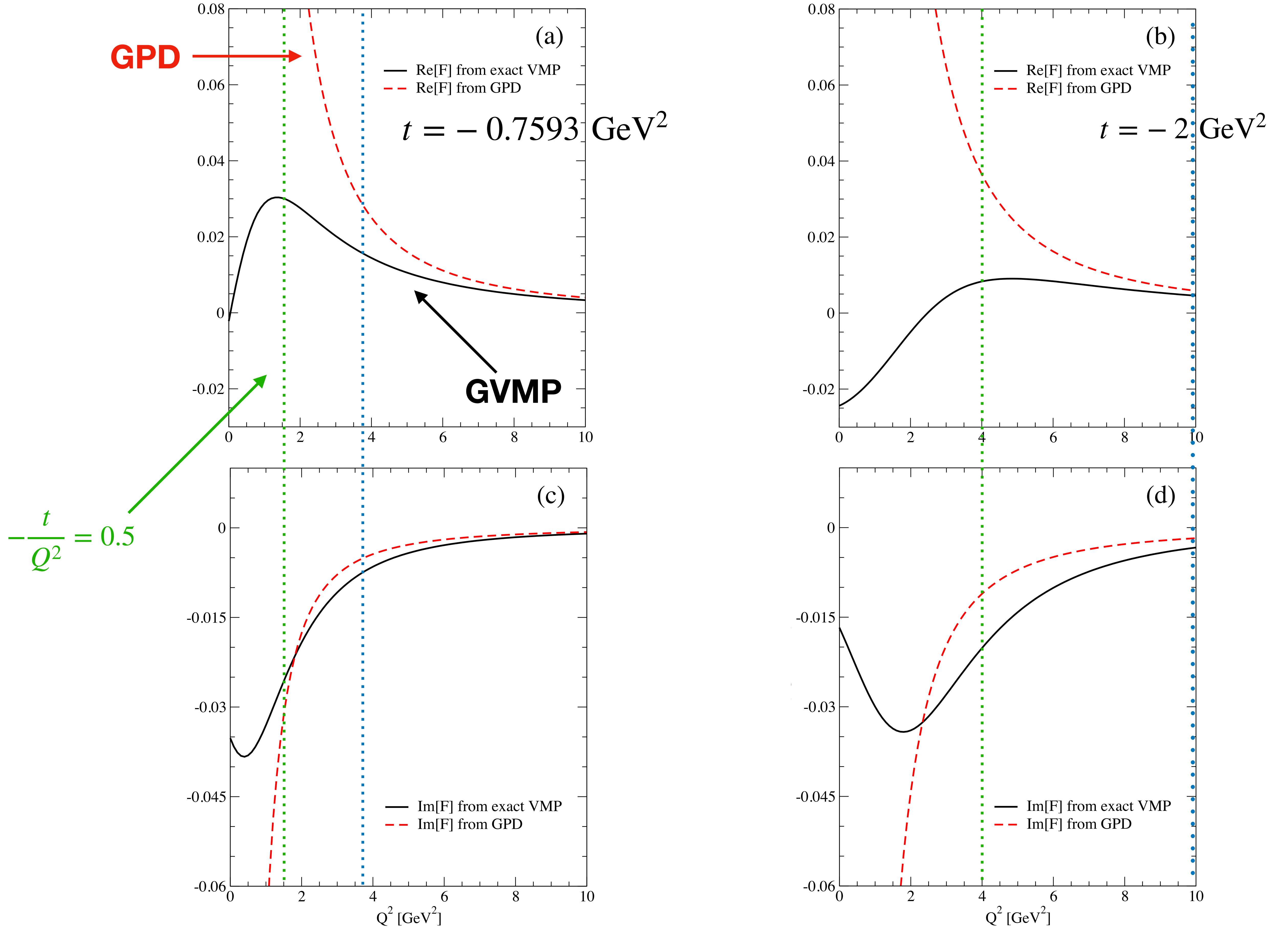
Q^2(GeV^2)	x	k|(GeV)	k' (GeV)	θ_e (°)	θ_q (°)	q'(0°) (GeV)	W^2 (GeV^2)	M (GeV)	t (GeV^2)	t_min (GeV^2)	$-$t/Q^2
1.9	0.36	5.75	2.94	19.3	18.1	2.73	4.2	3.72738	−1.06554	−0.955137	0.560813
3.	0.36	6.6	2.15	26.5	11.7	4.35	6.2	3.72738	−1.32826	−1.22078	0.442755
4.	0.36	8.8	2.88	22.9	10.3	5.83	8.	3.72738	−1.54314	−1.40201	0.385786
4.55	0.36	11.	4.26	17.9	10.8	6.65	9.	3.72738	−1.68065	−1.48463	0.369374
3.1	0.5	6.6	3.2	22.5	18.5	3.11	4.1	3.72738	−1.9983	−1.83768	0.644611
4.8	0.5	8.8	3.68	22.2	14.5	4.91	5.7	3.72738	−2.64071	−2.41298	0.550148
6.3	0.5	11	4.29	21.1	12.4	6.5	7.2	3.72738	−3.09918	−2.81838	0.491934
7.2	0.5	11.	3.32	25.6	10.2	7.46	8.1	3.72738	−3.27475	−3.02728	0.454826
5.1	0.6	8.8	4.27	21.1	17.8	4.18	4.3	3.72738	−3.41689	−3.15331	0.669978
6	0.6	8.8	3.47	25.6	14.1	4.97	4.9	3.72738	−3.74772	−3.51599	0.624621
7.7	0.6	11	4.16	23.6	13.1	6.47	6	3.72738	−4.45326	−4.12602	0.578346
9.	0.6	11	3	30.2	10.2	7.62	6.9	3.72738	−4.81139	−4.53706	0.534599

Experimentally, $|t|$ increases as Q^2 increases,

it is difficult to measure experiments for enough small $-t/Q^2$.

J. Roche *et al.*, Measurements of the electron-helicity dependent cross sec- tions of deeply virtual Compton scattering with CEBAF at 12 GeV, arXiv/nucl- ex/0609015, JLab proposal.

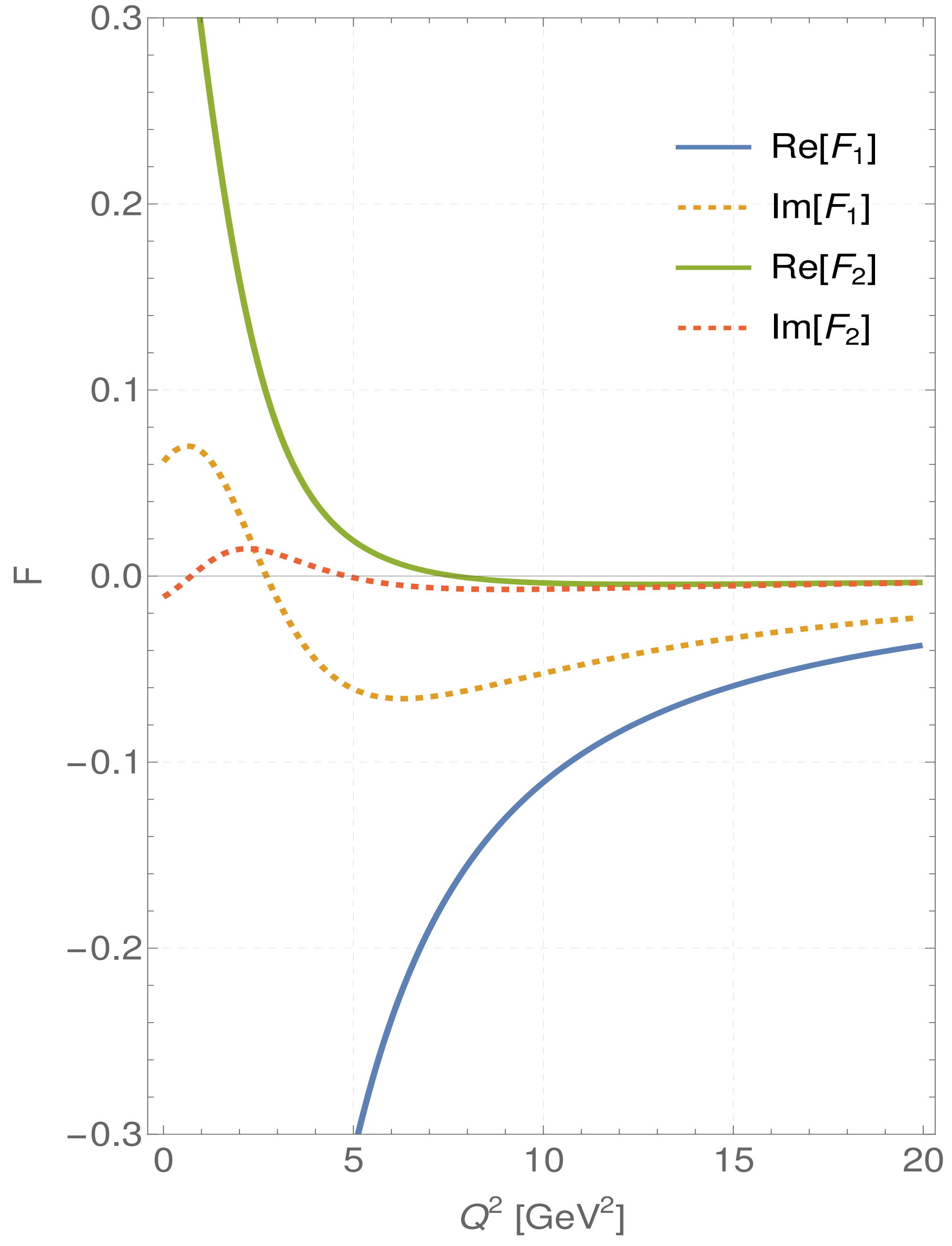
Numerical Result - CFF for GVMP vs GPD



Extension to more realistic models

In (3+1)-dimensions,

$$\left\{(\Delta\cdot q)\, q^\mu - q^2\,\Delta^\mu\right\}\boxed{\mathscr{F}_1} + \left\{\,(\Delta\cdot q)\,\mathscr{P}^\mu - (\mathscr{P}\cdot q)\,\Delta^\mu\right\}\boxed{\mathscr{F}_2}$$



For the vertex functions,

$$|M\rangle = \psi_{q\bar{q}}\,|q\bar{q}\rangle \;+\; \psi_{q\bar{q}g}\,|q\bar{q}g\rangle \;+\; \cdots$$

$$\approx \psi_{Q\bar{Q}}\,|Q\bar{Q}\rangle$$: mock-hadron approx.

where $\psi_{Q\bar{Q}} = \phi(x,\mathbf{k}_\perp)\,\chi(x,\mathbf{k}_\perp,\lambda)$

Fermion loops (quarks and anti-quarks) :

$$J^\mu = \bar{\psi}(z)\;\hat{\mathscr{O}}\;\psi(0)$$

where $\hat{\mathscr{O}}$ can be $\gamma^\mu,\;\gamma^\mu\gamma_5,\;\cdots$.

Conclusion

- We investigate the virtual meson production by using the ϕ^3-scalar field model in (1+1) light-front dynamics.
- The virtual meson production is theoretically accessed by the generalized hadronic current and generalized parton distribution.
- The Compton form factors from the GVMP and twist-2 GPD formulation are quite different in JLab kinematic setting.
- We expect that $-t/Q^2$ should be at least less than 0.2 for twist-2 GPD. But, we have to check the same tendency in the more realistic model.