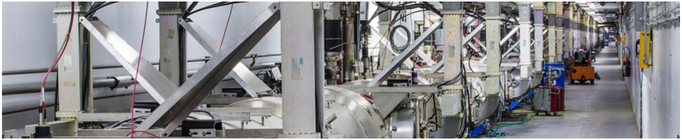
Extraction of TMD distributions from the SIDIS data: looking towards JLab22

Alexey Vladimirov



APCTP Focus Program
July 20, 2022

I follow the request of Harut, and will present the description of

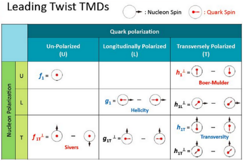
- how modern TMD phenomenology works.
- how impact studies for future colliders are done.

Also I will present some superficial (due to lack of time) studies of pseudo-data from JLab22.

Hope to have live-full discussion afterwards!

Outline

- Present state-of-the-art in TMD phenomenology
- Features and problems of TMD phenomenology
- TMDs at JLab22 (first look)

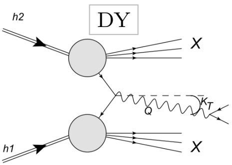


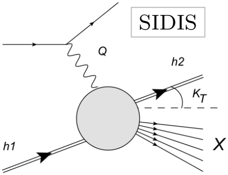
Leading Twist TMDs

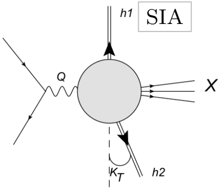
: Nucleon Spin
: Quark Spin

		Quark polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 =		$h_1^\perp$ = − Boer-Mulder
	L		g_1 = − Helicity	$h_{1L}^\perp$ = −
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_{1T} = − Transversity $h_{1T}^\perp$ = −

$$\frac{d\sigma}{dp_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bp_T)} C\left(\frac{Q}{\mu}\right) F_1(x_1, b; \mu, \zeta) F_2(x_2, b; \mu, \zeta)$$

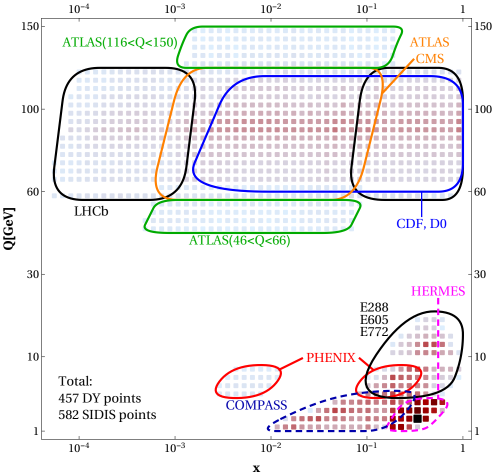






Main scales:
The invariant mass of photon: $|q^2| = Q^2$
Transverse component of photon momentum: q_T

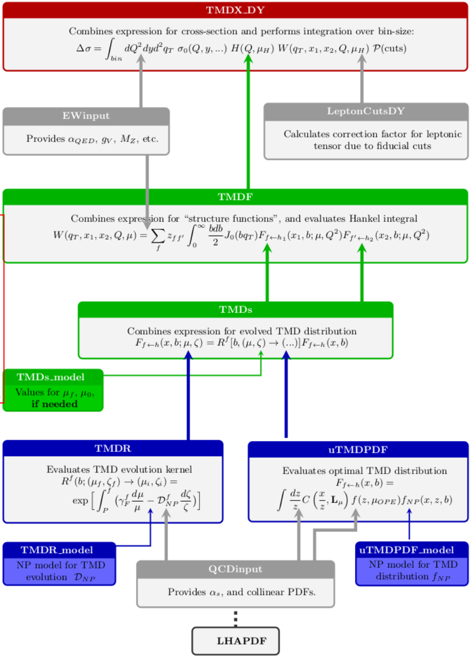
Present state-of-the-art (unpolarized)



Unpolarized sector

- Data
    - High-energy=DY
    - Mid-energy = fix-target DY
    - Low-energy = SIDIS
- Joined fits
    - SV19 [Scimemi, AV, 1912.06532]
    - MAP22 [Bacchetta, at al, 2206.07598]
- Theory
    - N^4LO evolution (!)
    - N^3LO coeff.functions

Present unpol.fit are LHC-driven.
There are issues with SIDIS description.



TMDX_DY

Combines expression for cross-section and performs integration over bin-size:

$$\Delta\sigma = \int_{bin} dQ^2 dy d^2 q_T\ \sigma_0(Q,y,...)\ H(Q,\mu_H)\ W(q_T,x_1,x_2,Q,\mu_H)\ \mathcal{P}(\text{cuts})$$

EWinput

Provides α_{QED}, g_V, M_Z, etc.

LeptonCutsDY

Calculates correction factor for leptonic tensor due to fiducial cuts

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Combines expression for “structure functions”, and evaluates Hankel integral

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TMDs_model

Values for μ_f, μ_0, if needed

TMDR

Evaluates TMD evolution kernel

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Provides α_s, and collinear PDFs.

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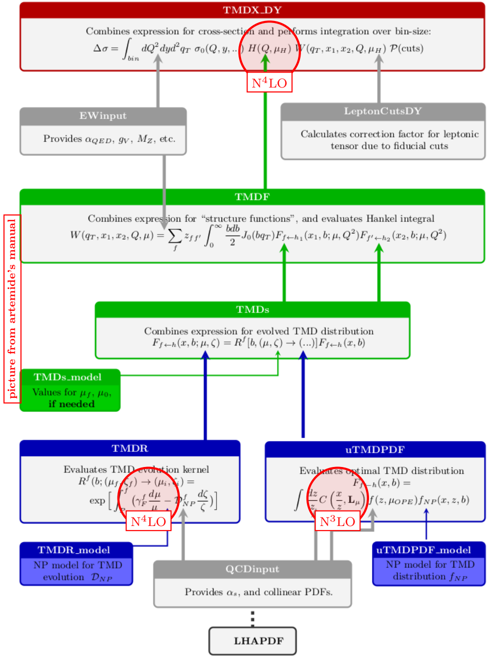
LHAPDF

picture from artemide's manual

TMD theory is complex mixture of elements of different origins



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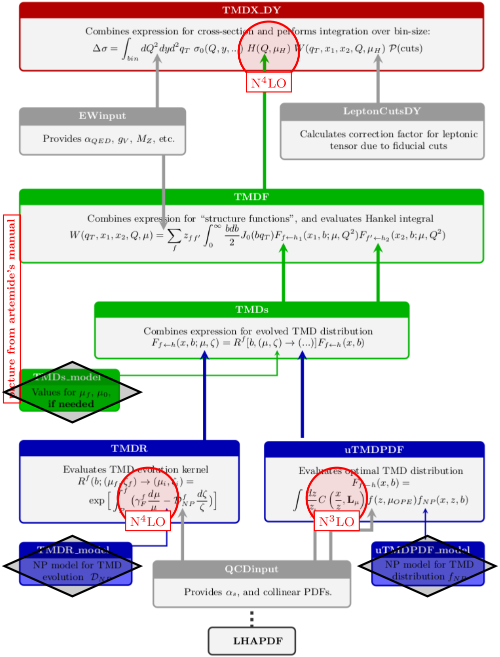

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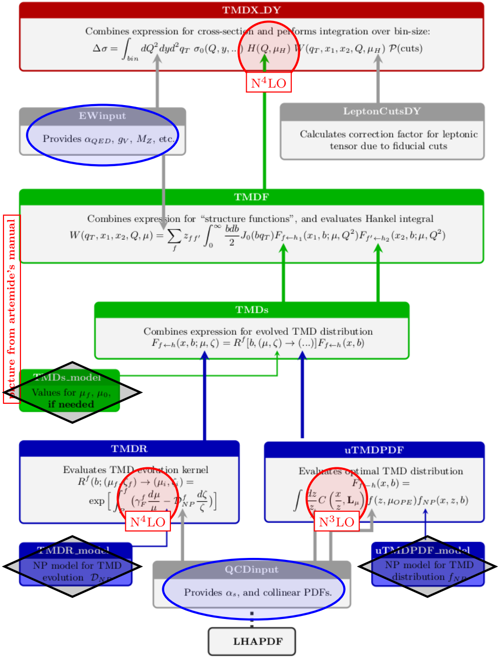

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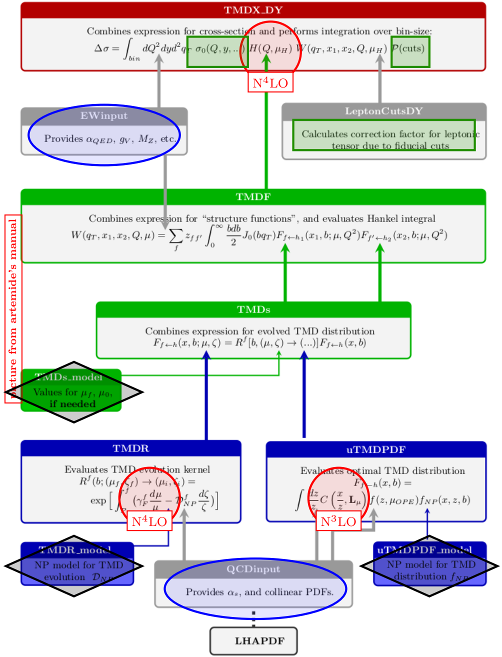

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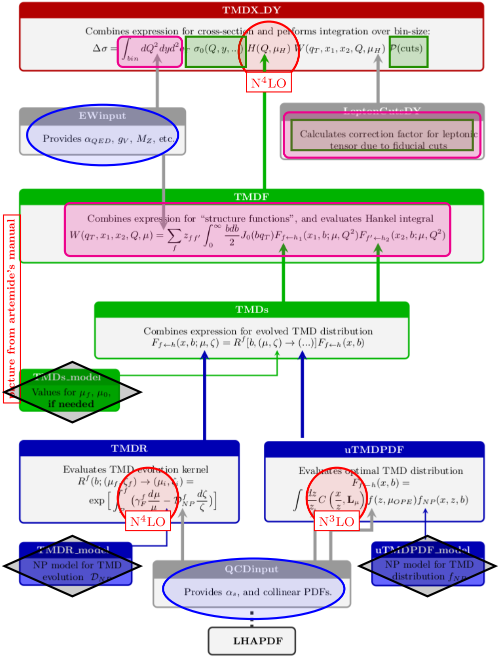

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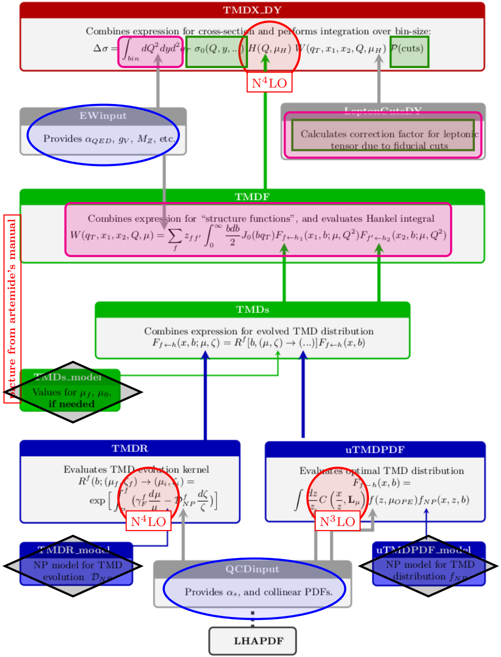

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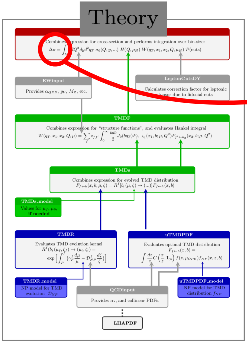
1) Each element is a product of hundreds of investigations/papers and could not be simply changed.
2) A tiny modification in any of elements could lead to significant difference in the output.
3) You are free to modify **only** **models/parametrizatons** but also within certain limitations



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minimization

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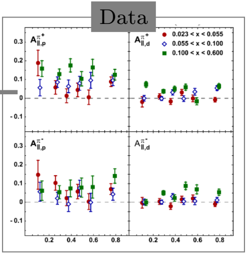
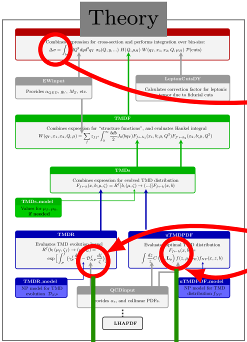
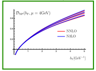
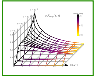

Data

Interaction with data only through χ^2.







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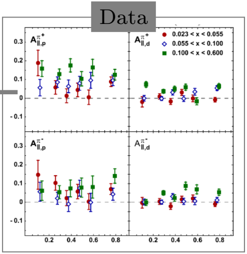

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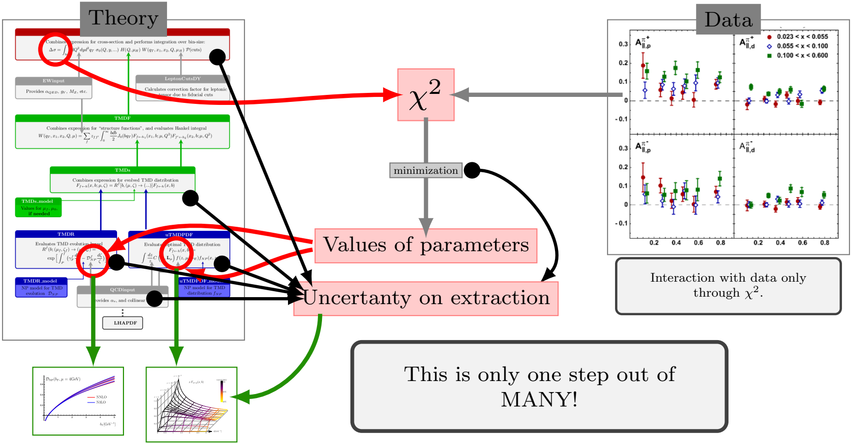

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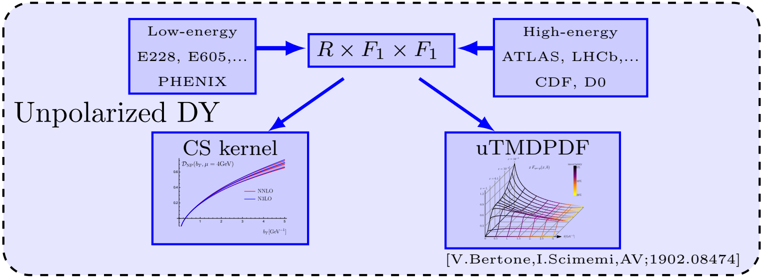
Uncertanty on extraction

Interaction with data only through χ^2.

This is only one step out of MANY!

Universality & the chain of extractions



Low-energy
E228, E605,...
PHENIX

$R \times F_1 \times F_1$

High-energy
ATLAS, LHCb,...
CDF, D0

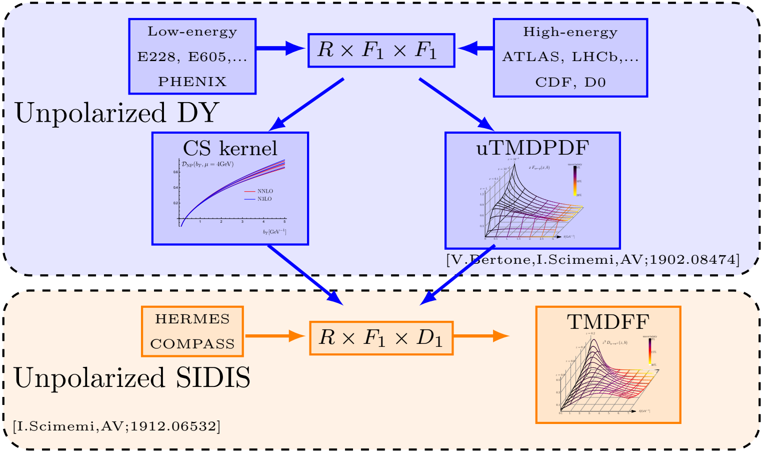
Unpolarized DY

CS kernel

uTMDPDF

[V.Bertone,I.Scimemi,AV;1902.08474]

Universality & the chain of extractions



Low-energy
E228, E605,...
PHENIX

$R \times F_1 \times F_1$

High-energy
ATLAS, LHCb,...
CDF, D0

Unpolarized DY

CS kernel

uTMDPDF

[V.Bertone,I.Scimemi,AV;1902.08474]

HERMES
COMPASS

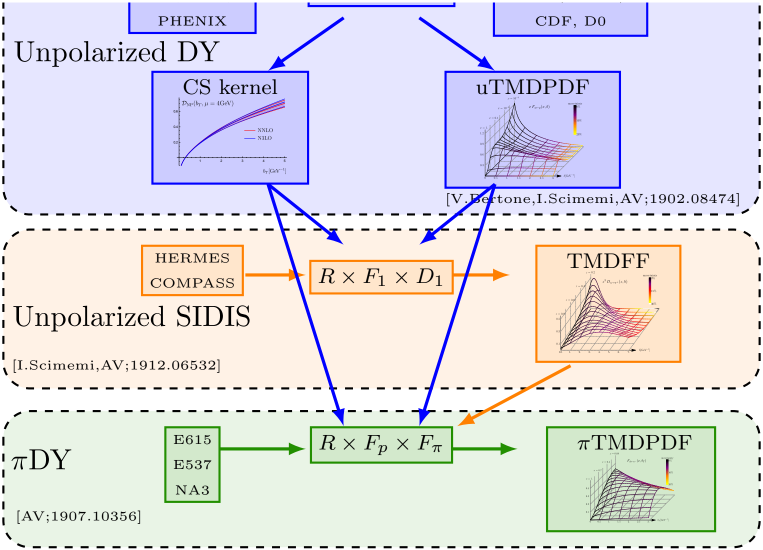
$R \times F_1 \times D_1$

TMDFF

Unpolarized SIDIS

[I.Scimemi,AV;1912.06532]

Universality & the chain of extractions



PHENIX

CDF, D0

Unpolarized DY

CS kernel

uTMDPDF

[V.Bertone,I.Scimemi,AV;1902.08474]

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COMPASS

$R \times F_1 \times D_1$

TMDFF

Unpolarized SIDIS

[I.Scimemi,AV;1912.06532]

πDY

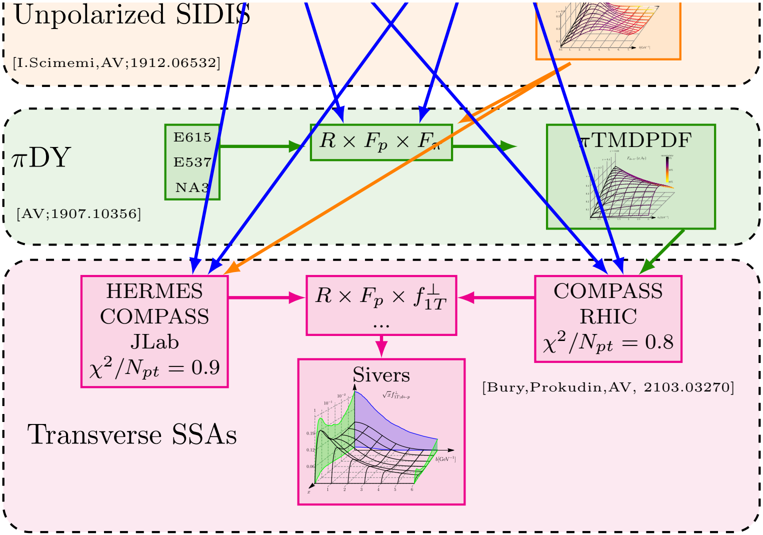
E615
E537
NA3

$R \times F_p \times F_\pi$

πTMDPDF

[AV;1907.10356]

Universality & the chain of extractions



Unpolarized SIDIS

[I.Scimemi,AV;1912.06532]

πDY

E615
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πTMDPDF

[AV;1907.10356]

HERMES
COMPASS
JLab
$\chi^2/N_{pt} = 0.9$

$R \times F_p \times f_{1T}^{\perp}$
...

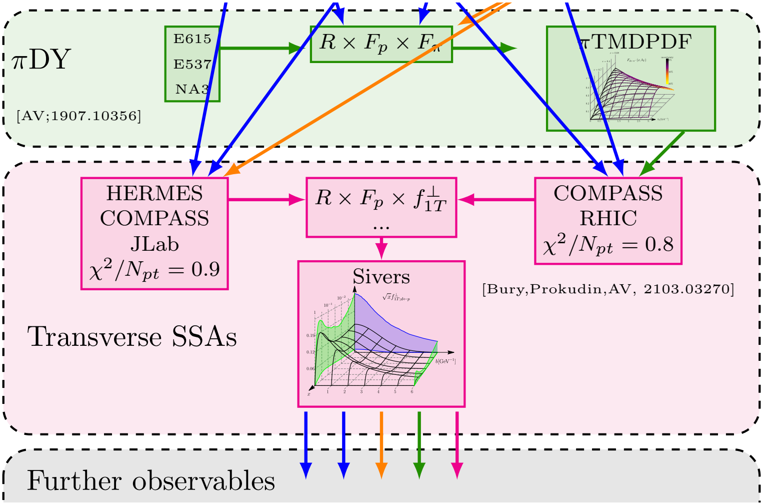
COMPASS
RHIC
$\chi^2/N_{pt} = 0.8$

Sivers

[Bury,Prokudin,AV, 2103.03270]

Transverse SSAs

Universality & the chain of extractions



πDY

E615
E537
NA3

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[AV;1907.10356]

HERMES
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JLab
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...

COMPASS
RHIC
$\chi^2/N_{pt} = 0.8$

Sivers

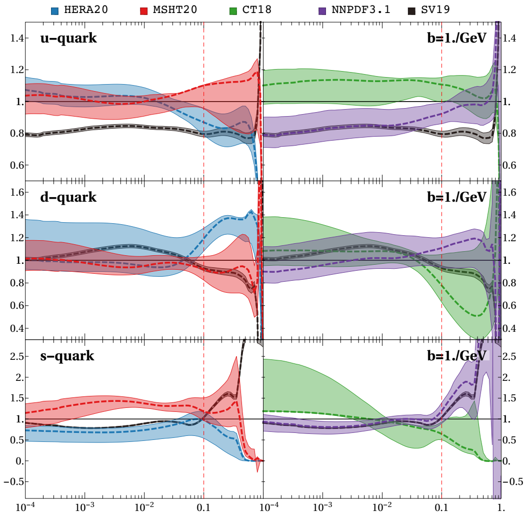
[Bury,Prokudin,AV, 2103.03270]

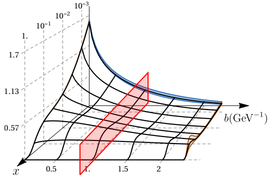
Transverse SSAs

Further observables

All the structure is rather shaky.
Modification in any element (**could**) lead to significant modification in the output

LATEST EXAMPLE: PDF-bias and flavor dependence

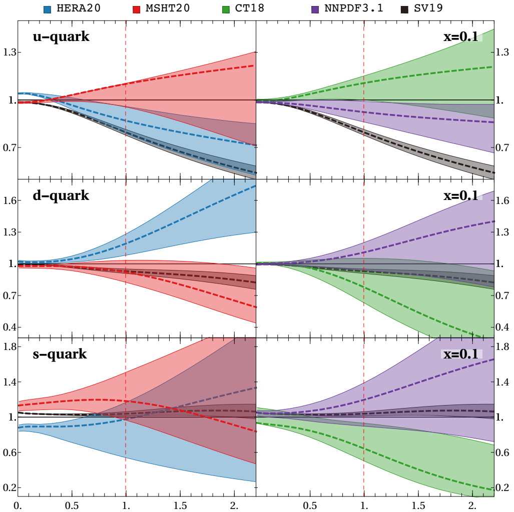


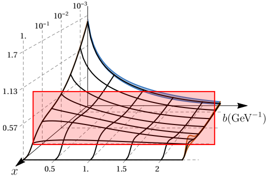


[2201.07114]

All the structure is rather shaky.
Modification in any element (**could**) lead to significant modification in the output

LATEST EXAMPLE: PDF-bias and flavor dependence





[2201.07114]

Unpolarized TMDPDF.
It is the foundation of
all other extractions.

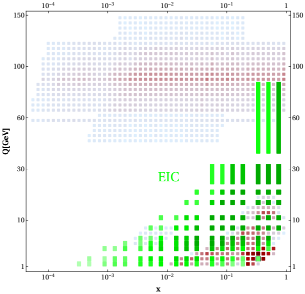
☹

- No single group can explain SIDIS-data plainly
- TMD factorization is valid in a small corner of phase space
- Many sources of uncertainty which accumulate fast
- Many hidden problems, which we are not aware yet
- Data are imperfect
- Cannot confirm sing-change (need better DY data)
- ...

☺

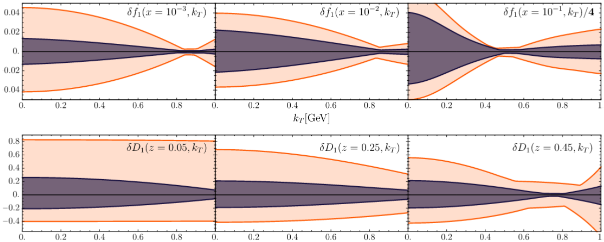
- Perturbative parts are very well known (most simple problem)
- There is some agreement in-between extraction in the unpolarized sector
- Predictions of TMD factorization work quite well (in the range of applicability)

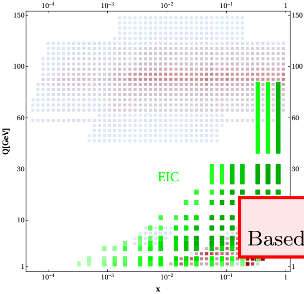
Still I am very optimistic regarding the future of TMDs



EIC:

- SIDIS (!)
- Large number of data-points in **totally unexplored region**.
- High precision.
- Extreme pt-resolution

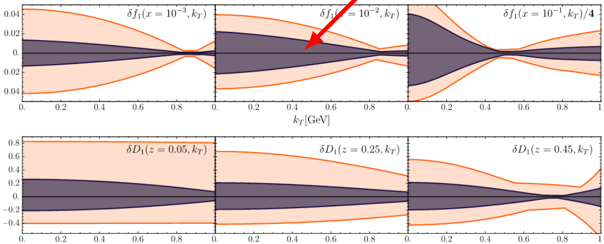




EIC:

- SIDIS (!)
- Large number of data-points in **totally unexplored region**.
- High precision.

Do not **trust** it!
Based on the 2-years-ago understanding...

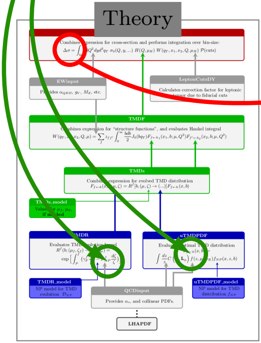


I want the same picture but for JLab22

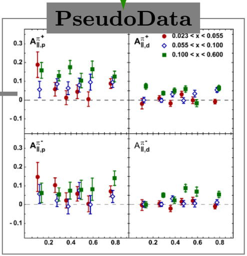
OK... Let's try

Values of parameters
determined in previous fit

MC-generator
+ ???

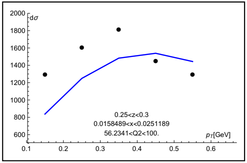


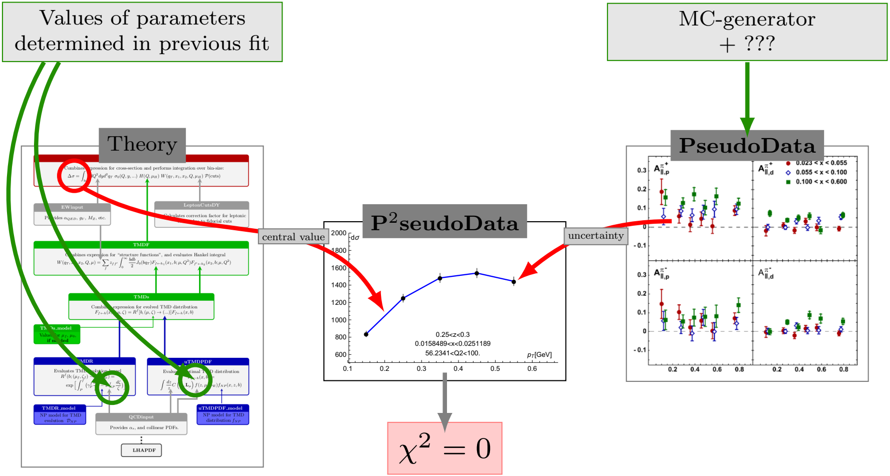
$$\chi^2$$



Does not work!

$$\chi^2/N_{pt} \gg 1$$





Values of parameters determined in previous fit

MC-generator + ???

Theory

P^{2}seudoData

PseudoData

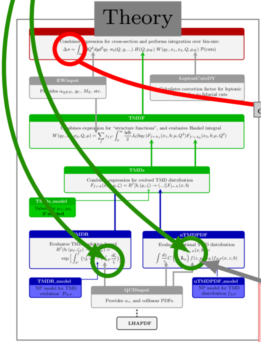
central value

uncertainty

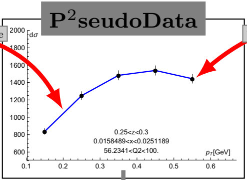
$$\chi^2 = 0$$

Values of parameters determined in previous fit

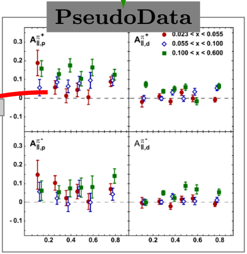
MC-generator
+ ???



central value



uncertainty



$\delta\chi^2 = 1$ variation of parameters

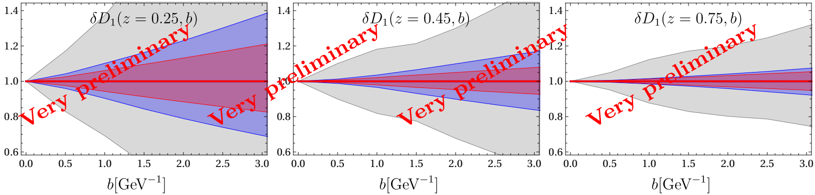
$\chi^2 = 0$

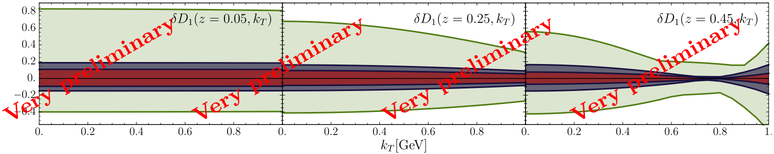
Estimation of the impact

This is minimal scheme

- +extra d.o.f.
- +existing data
- +other sources of uncertanties
- ...

First look at JLab22





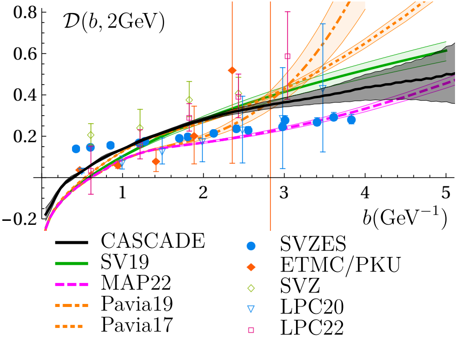
This is a minimal study. **What must be included**

- Variation of other parameters (so far only 4 parameters of TMDFF)
- Extra terms to be sensitive for different regions
- More systematics (correlated uncertainties...)

The modern TMD phenomenology is cumbersome and indirect.
Is there a way to directly probe TMDs?

Direct extraction of Collins-Soper kernel
and
direct tests of TMD factorization

[A.B.Martinez, AV, 2206.01105]



CASCADE does not have CS-kernel. It is build on different principles, but nicely predicts TMD cross-sections. Still one can extract CS-kernel and text agreement with TMD factorization.

- No parametrization
- No need for large coverage in Q
- Ultimate test of universality
- Works with SIDIS better than with DY

How does it work? (theory)

TMD fac. $$\frac{d\sigma}{dQ^2dxdzdk_\perp^2}=\frac{\pi\alpha_{\text{em}}^2(Q)}{Q^4}\frac{y^2}{1-\varepsilon}W(Q,x,z,k_\perp)$$

$$W(Q,x,z,k_\perp)=\int_0^\infty \frac{bdb}{(2\pi)^2}J_0\left(\frac{k_\perp b}{z}\right)R[b,Q\to\mu]|C_V(Q)|^2\sum_f e_f^2 f_1(x,b;\mu)d_1(z,b;\mu)$$

Evol.factor
our goal!

TMDs
trash

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Evol.factor
our goal!

TMDs
trash

1) inv. Fourier $\quad \Sigma(Q,x,z,b)=\int dq_Tq_TJ_0(q_Tb)\frac{d\sigma}{dQ^2dxdzdk_\perp^2}, \qquad q_T=\frac{k_\perp}{z}$

How does it work? (theory)

TMD fac.
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Evol.factor
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TMDs
trash

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$$\Sigma(Q,x,z,b)=\int dq_Tq_TJ_0(q_Tb)\frac{d\sigma}{dQ^2dxdzdk_\perp^2},\qquad q_T=\frac{k_\perp}{z}$$

2) ratio
$$\frac{\Sigma(Q_1,x,z,b)}{\Sigma(Q_2,x,z,b)}=\left(\frac{Q_2}{Q_1}\right)^4\frac{\alpha_{\text{em}}^2|C_V(Q_1)|^2}{\alpha_{\text{em}}^2|C_V(Q_2)|^2}\frac{R[b,Q_1\to\mu]\cancel{\sum_f f_1(x,b,\mu)d_1(x,b,\mu)}}{R[b,Q_2\to\mu]\cancel{\sum_f f_1(x,b,\mu)d_1(x,b,\mu)}}$$

How does it work? (theory)

TMD fac.

$$\frac{d\sigma}{dQ^2dxdzdk_\perp^2}=\frac{\pi\alpha_{\text{em}}^2(Q)}{Q^4}\frac{y^2}{1-\varepsilon}W(Q,x,z,k_\perp)$$

$$W(Q,x,z,k_\perp)=\int_0^\infty\frac{bdb}{(2\pi)^2}J_0\left(\frac{k_\perp b}{z}\right)R[b,Q\to\mu]|C_V(Q)|^2\sum_f e_f^2 f_1(x,b;\mu)d_1(z,b;\mu)$$

Evol.factor
our goal!

TMDs
trash

1) inv. Fourier $$\Sigma(Q,x,z,b)=\int dq_Tq_TJ_0(q_Tb)\frac{d\sigma}{dQ^2dxdzdk_\perp^2},\qquad q_T=\frac{k_\perp}{z}$$

2) ratio $$\frac{\Sigma(Q_1,x,z,b)}{\Sigma(Q_2,x,z,b)}=\left(\frac{Q_2}{Q_1}\right)^4\frac{\alpha_{\text{em}}^2|C_V(Q_1)|^2}{\alpha_{\text{em}}^2|C_V(Q_2)|^2}\frac{R[b,Q_1\to\mu]\cancel{\sum_f f_1(x,b,\mu)d_1(x,b,\mu)}}{R[b,Q_2\to\mu]\cancel{\sum_f f_1(x,b,\mu)d_1(x,b,\mu)}}$$

3) recall that $$R[b,Q_1\to\mu]=\exp\left(2\int_{P(Q_1\to\mu)}\left(\gamma_F(\mu,\zeta)\frac{d\mu}{\mu}-\mathcal{D}(b,\mu)\frac{d\zeta}{\zeta}\right)\right)$$

How does it work? (theory)

TMD fac.

$$\frac{d\sigma}{dQ^2dxdzdk_\perp^2}=\frac{\pi\alpha_{\text{em}}^2(Q)}{Q^4}\frac{y^2}{1-\varepsilon}W(Q,x,z,k_\perp)$$

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Evol.factor
our goal!

TMDs
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1) inv. Fourier

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2) ratio

$$\frac{\Sigma(Q_1,x,z,b)}{\Sigma(Q_2,x,z,b)}=\left(\frac{Q_2}{Q_1}\right)^4\boxed{\frac{\alpha_{\text{em}}^2|C_V(Q_1)|^2}{\alpha_{\text{em}}^2|C_V(Q_2)|^2}}\frac{R[b,Q_1\to\mu]\cancel{\sum_f f_1(x,b,\mu)d_1(x,b,\mu)}}{R[b,Q_2\to\mu]\cancel{\sum_f f_1(x,b,\mu)d_1(x,b,\mu)}}$$

3) recall that

$$R[b,Q_1\to\mu]=\exp\left(2\int_{P(Q_1\to\mu)}\left(\boxed{\gamma_F(\mu,\zeta)\frac{d\mu}{\mu}}-\mathcal{D}(b,\mu)\frac{d\zeta}{\zeta}\right)\right)$$

4) After some manipulations

$$\mathcal{D}(b,\mu)=\frac{\ln\left(\frac{\Sigma(Q_1)}{\Sigma(Q_2)}\right)-\ln Z(Q_1,Q_2)-2\Delta_R(Q_1,Q_2,\mu)}{4\ln(Q_2/Q_1)}-1$$

accurate expressions see in [2206.01105]

How does it work? (practice)

(must have!)

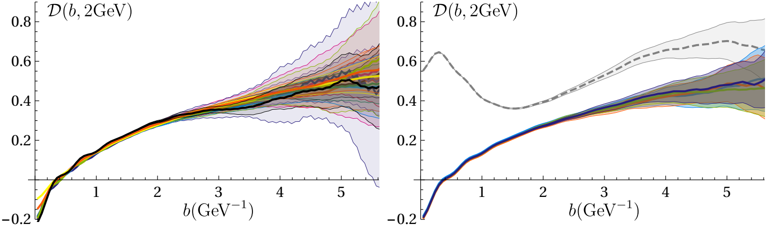
▶ Cross-section in the photon frame	$q_T = p_\perp / z$
▶ Fine binning in q_T	smaller bins larger-b
▶ As small as possible uncertanties	Fourier is uncertainty-hungry
▶ (At least) two narrow bins in Q	Large Q-bin = large systematic

(what helps)

▶ Integrate over x	such that ranges for Q_1 and Q_2 coincides
▶ Integrate over z	in photon frame
▶ Large-q_T tail is not interesting	no TMD-fac.

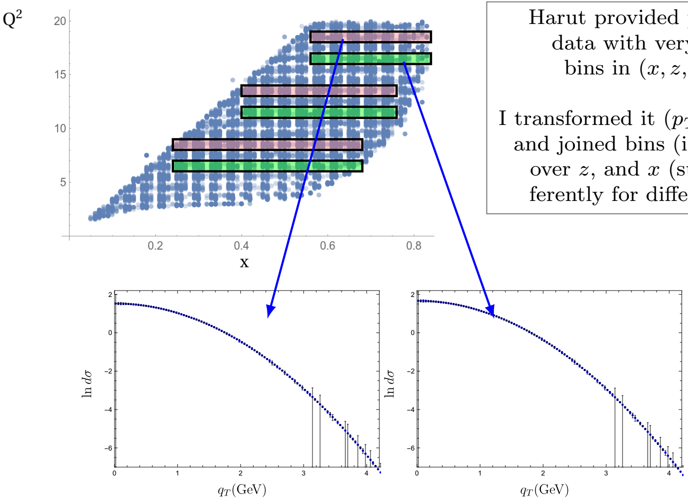
Why is it interesting?

- Direct extraction of Collins-Soper kernel
  - CS kernel is one of the most fundamental QCD functions
- **Ultimate test of factorization hypothesis**
  - Different (Q, x, z) MUST result into the same curve
  - Different final states $(\pi^{\pm}, K^{\pm})$ MUST result into the same curve



It is a very precise test!

How does it works? (JLab22)



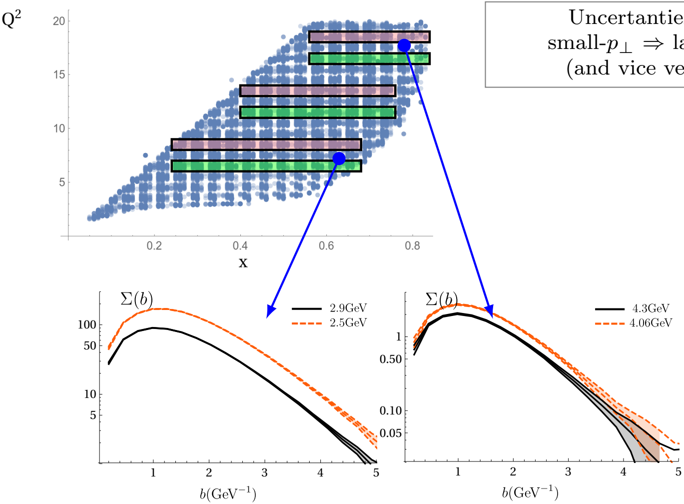
Harut provided pseudo-data with very fine bins in (x, z, p_T)

I transformed it ($p_T/z \to q_T$) and joined bins (integrate) over z, and x (such differently for different Q)

Huge statistical precision due to wide covarage in (x,z). Systematics?
Empty bins are replaced by 200%uncertanty

How does it works? (JLab22)

Uncertanties at
small-$p_\perp$ $\Rightarrow$ large-b
(and vice versa)



How does it works? (JLab22)

Uncertanties at
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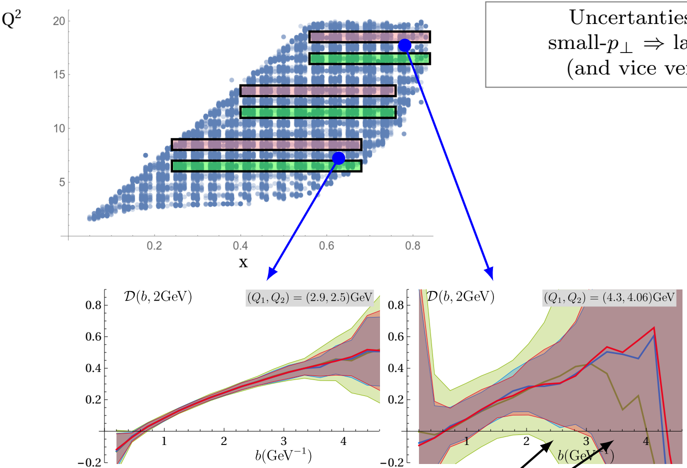
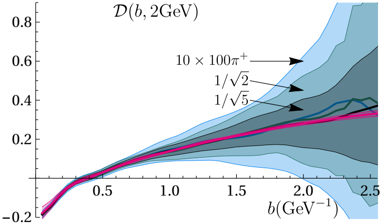


1 day

20 days

Comparison with EIC

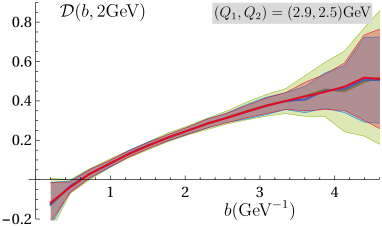


In the EIC case I included (estimation) of systematics, which is larger than statistics

EIC → Much better small-b

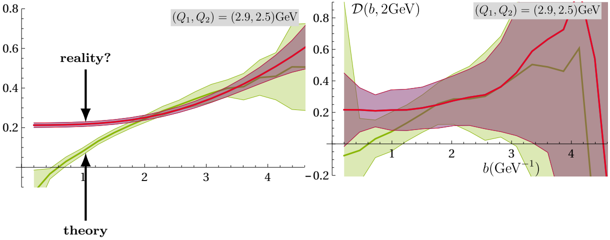
JLab → Much better large-b

definite complementarity

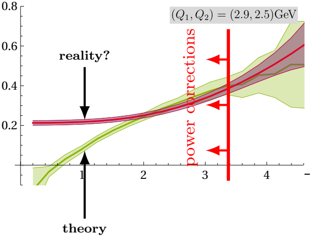


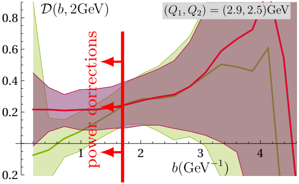
WARNING!
This estimation uses exact TMD factorization
In reality it will
look VERY different
(the most interesting part)

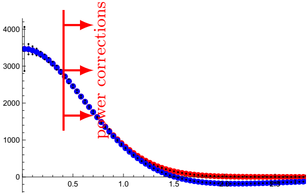
More realistic picture

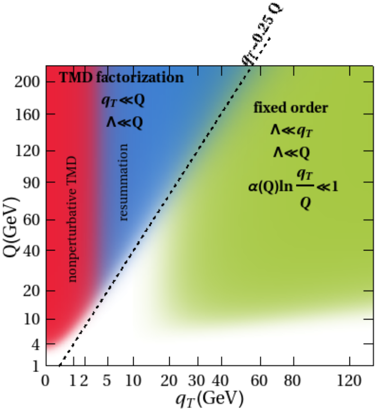


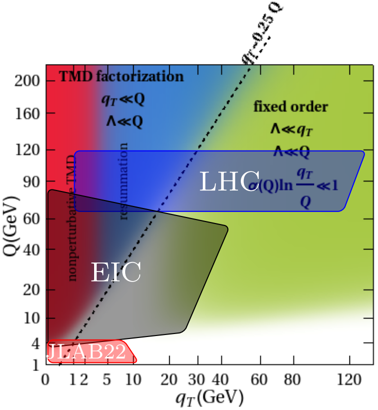
More realistic picture

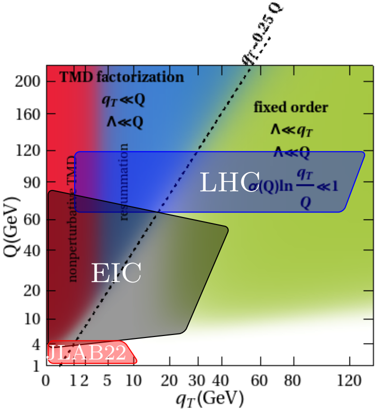




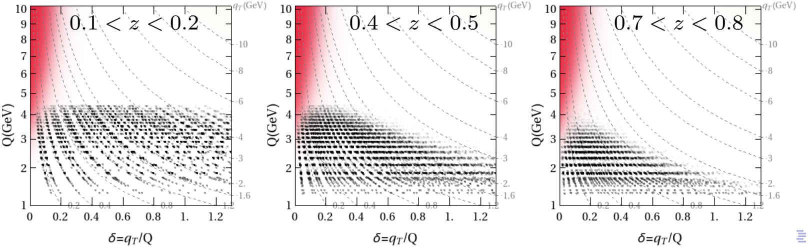



q_T~0.25 Q
TMD factorization
$q_T \ll Q$
$\Lambda \ll Q$
nonperturbative TMD
resummation
fixed order
$\Lambda \ll q_T$
$\Lambda \ll Q$
$\alpha(Q)\ln\frac{q_T}{Q} \ll 1$
Q(GeV)
200
160
120
90
60
40
20
10
4
1
0
1 2
5
10
20
30 40
60
80
120
q_T(GeV)


$q_T \sim 0.25\,Q$
TMD factorization
$q_T \ll Q$
$\Lambda \ll Q$
fixed order
$\Lambda \ll q_T$
$\Lambda \ll Q$
$\alpha(Q) \ln \frac{q_T}{Q} \ll 1$
nonperturbative TMD
resummation
LHC
EIC
JLAB22
Q(GeV)
200
160
120
90
60
40
20
10
4
1
0
1 2
5
10
20
30
40
60
80
120
q_T(GeV)



The MOST part of
JLab(22) data is in the
power corection region.
It is not bad!
It is an opportunity to
study true QCD, not just
a perturbative component



Conclusions

- TMD factorization for SIDIS and/or low-energy **is in a very badly shape**
    - All extractions (MAP22, SV19, ...) contains some explicit/implicit feature which make them suspicious
    - The problem is (most probably) due to power corrections
- Impact studies (for SIDIS and/or low-energies) **are schematic**
    - They estimate the uncertainty on the ideal theory.
    - Miss many elements (also because they are time consuming, but not interesting)
- JLab22 looks very good
    - It will zoom-in different regions in comparison to EIC
      Example: small-b vs. large-b in Collins-Soper
    - If we will tame power corrections