Generalized Parton distribution of the Goldstone boson

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APCTP Focus Program in Nuclear Physics 2022: Hadron Physics Opportunities with Lab Energy and Luminosity Upgrade

Outline

1. Introduction
2. Meson in BSE-NJL model
3. Generalized Parton distribution in BSE-NJL model
4. Summary and outlook

Introduction-1

- Generalized Parton distribution function: a tool to study three (multi)-dimensional structure of hadrons — connecting with the chiral symmetry breaking—Radyushkin, PRD56 (1997), M.Diehl, PR388(2003), Belitsky&Radyushkin, PR418(2005)
- Gluon content plays a crucial role in pions, in comparison with kaons, in response to the pion's deeply virtual Compton scattering (DVCS) in the Electron-ion collider (EIC) kinematics
- Amongst hadrons, as (pseudo)-Goldstone boson of QCD, pions are expected to play important role for deeply understanding of the mass origin or emergent hadron mass (EHM)—how gluons and quarks give rise mass to the pions

Introduction-2

- Hard exclusive processes: DVCS and deeply virtual meson production (DVMP) — generalized Parton distributions (GPDs)
- Experimentally, the study of GPDs could be interested for the upgraded Jlab-12 — which potentially provide very precise experimental data
- Many attempts/studies have been done for investigating the GPDs: BSE—NJL model—**PTPH**, Ian Cloet, Anthony Thomas, PRC94(2016), Ian Cloet PRC90(2014), Light-front holographic model —Brodsky&Teramond, PRL102 (2009), Chiral—quark model—H.Weigel, Pramana61 (2003), and Dyson—Schwinger equations (DSEs) model—P.Maris&Roberts, IJMPE12 (2003), Roberts, AG.Williams, PPNP33(1994)
- These model studies are very useful for interpreting the data — as we know that the GPDs, PDFs and FFs data for the pion and kaon are very limited and scarce

Introduction-3

- “Tension” on the power counting at asymptotic regime x—>1 or at endpoint features remain uncertain
- JAM Analysis—PC.Barry, et.al, PRL127(2021), PRD105(2022) prediction $(1-x)^{1.0-1.2}$, which is similar as BSE-NJL model—**PTPH**, Ian Cloet, Anthony Thomas, PRC94(2016) after evolving via QCD evolution prediction and DSE yields $(1-x)^{2.0}$
- To resolve this issue, more new data from experiments facilities are needed as well as the Lattice-QCD simulation
- Beside this power counting rule at high-x, the gluon distributions for the pion and kaon for different models yield rather different results

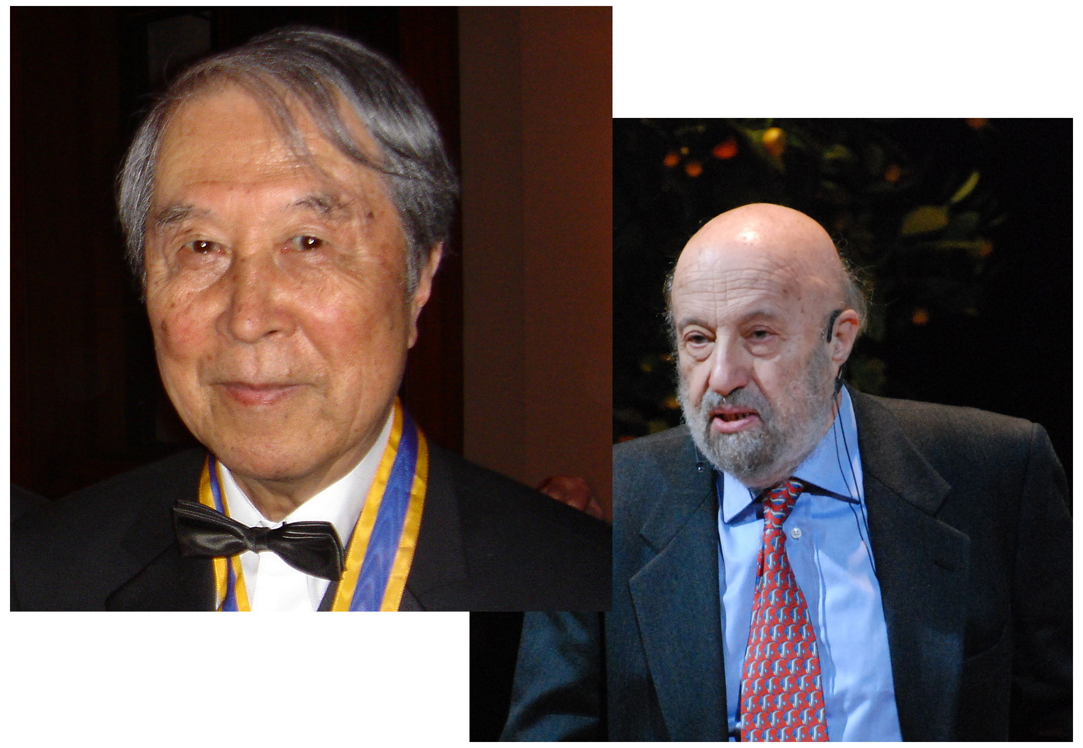
BSE-NJL model

The Lagrangian NJL model—contain local four-fermion interactions—**PTPH**, Ian Cloet, Anthony Thomas, PRC94(2016), Ian Cloet PRC90(2014)

$$\mathscr{L}_{\mathrm{NJL}} = \bar{\psi}[i\partial\!\!\!/ - \hat{m}]\psi + G_\pi \sum_{a=0}^{8} [(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2] + G_\rho \sum_{a=0}^{8} [\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2] - G_\omega(\bar{\psi}\gamma^\mu\psi)^2$$

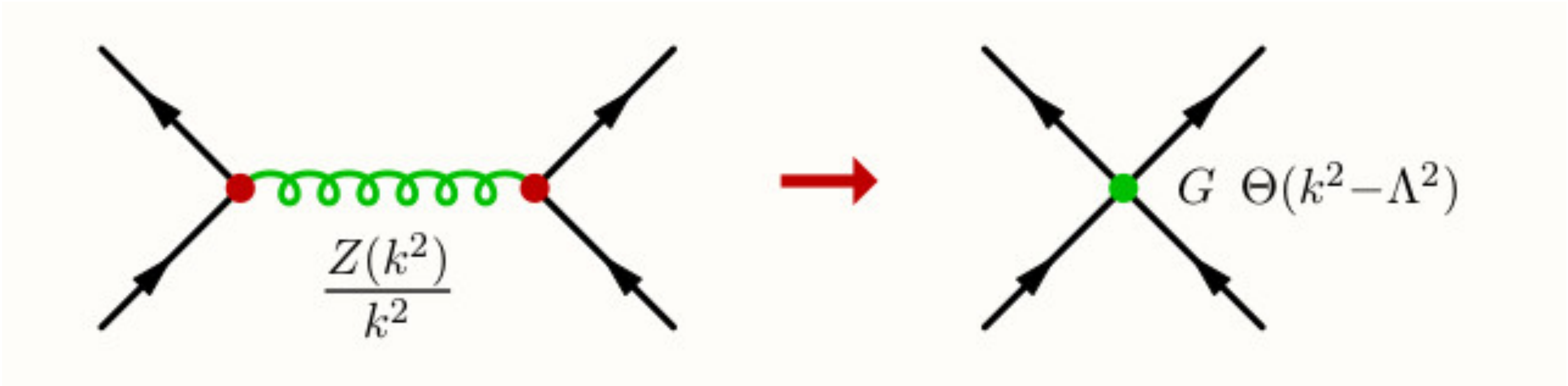
where

- $\psi = (u, d, s)^T$ is the quark field with the flavor components
- $G_\pi, G_\rho,$ and G_ω are local four-fermion coupling constants
- $\hat{m}_q = \mathrm{diag}[m_u,\ m_d,\ m_s]$ is the current quark mass matrix



BSE-NJL model

- In the NJL model, the gluon fields are integrated out and absorbing in the G_π coupling constants—Local four-fermion contact interactions—**PTPH**, Ian Cloet, Anthony Thomas, PRC94(2016), Ian Cloet PRC90(2014), S.Klevansky, RMP64(1992), Vogl &Weise, PPNP27(1991), Hatsuda&Kunihiro, PR247(1994)



- NJL model — lack of the confinement and divergence (pole in quark propagator) — We perform the Proper-time regularization (PTR) scheme — Simulating the confinement of QCD—**PTPH**, Ian Cloet, Anthony Thomas, PRC94(2016), Ian Cloet PRC90(2014)

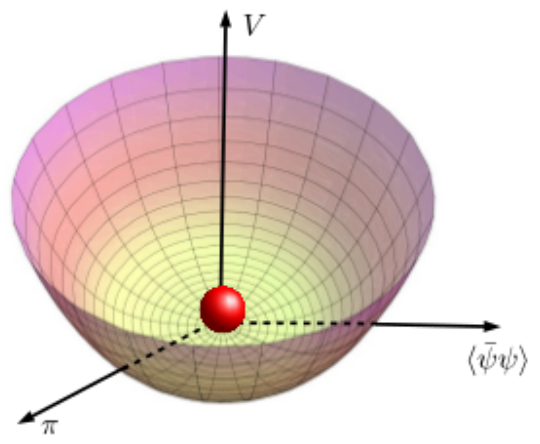
$$\frac{1}{(G)^n} = \frac{1}{[n-1]!}\int_0^\infty d\tau\tau^{[n-1]}\exp[-\tau G] \to \frac{1}{[n-1]}\int_{\tau_{UV}}^{\tau_{IR}} d\tau\tau^{[n-1]}\exp[-\tau G]$$

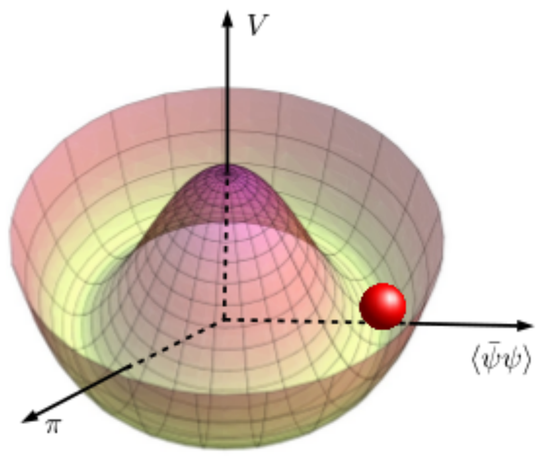
BSE-NJL model

- Where $\tau_{\rm UV} = \dfrac{1}{\Lambda_{\rm UV}^2}$ and $\tau_{\rm IR} = \dfrac{1}{\Lambda_{\rm IR}^2}$ with $\Lambda_{\rm IR} \simeq \Lambda_{\rm QCD} \simeq 240$ MeV and $\Lambda_{\rm UV}$ is determined to fit the pion mass and pion weak decay constant ($m_\pi = 140$ MeV and $f_\pi = 93$ MeV)

- NJL gap equation —dynamical quark mass— is determined through the quark propagator in momentum space

$$M_q = m_q + M_q \frac{3G_\pi}{\pi^2} \int_{\tau_{\rm UV}}^{\tau_{\rm IR}} \frac{d\tau}{\tau^2} \exp[-\tau M_q^2] = m_q - 2G_\pi \langle \bar{\psi}\psi \rangle$$



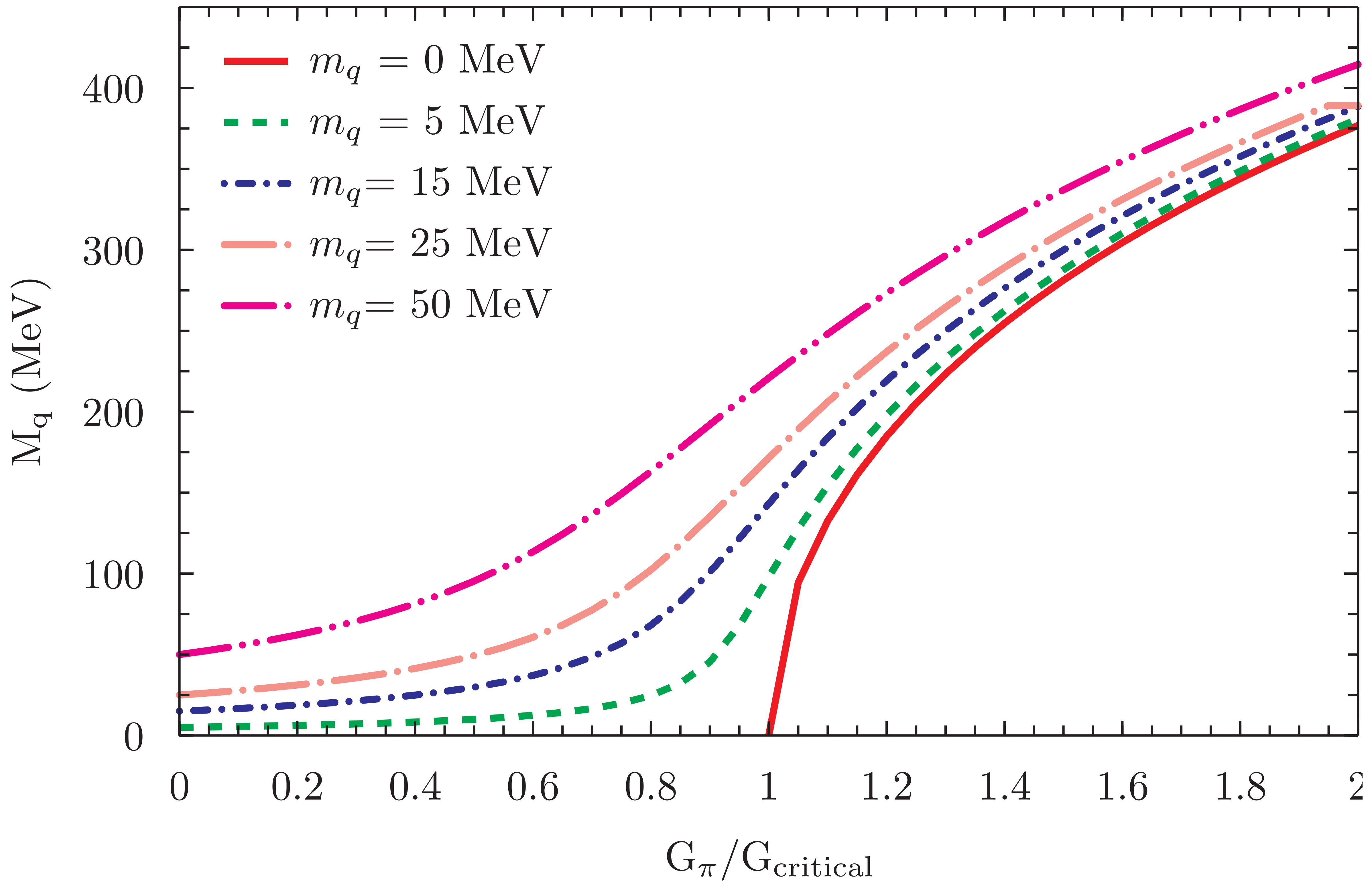


- $\langle \bar{\psi}\psi \rangle \neq 0$ —chiral QCD condensate—order parameter of chiral spontaneously symmetry breaking (CSSB)—generated mass via interaction with vacuum

BSE-NJL model

NJL Gap equation — dynamical quark mass

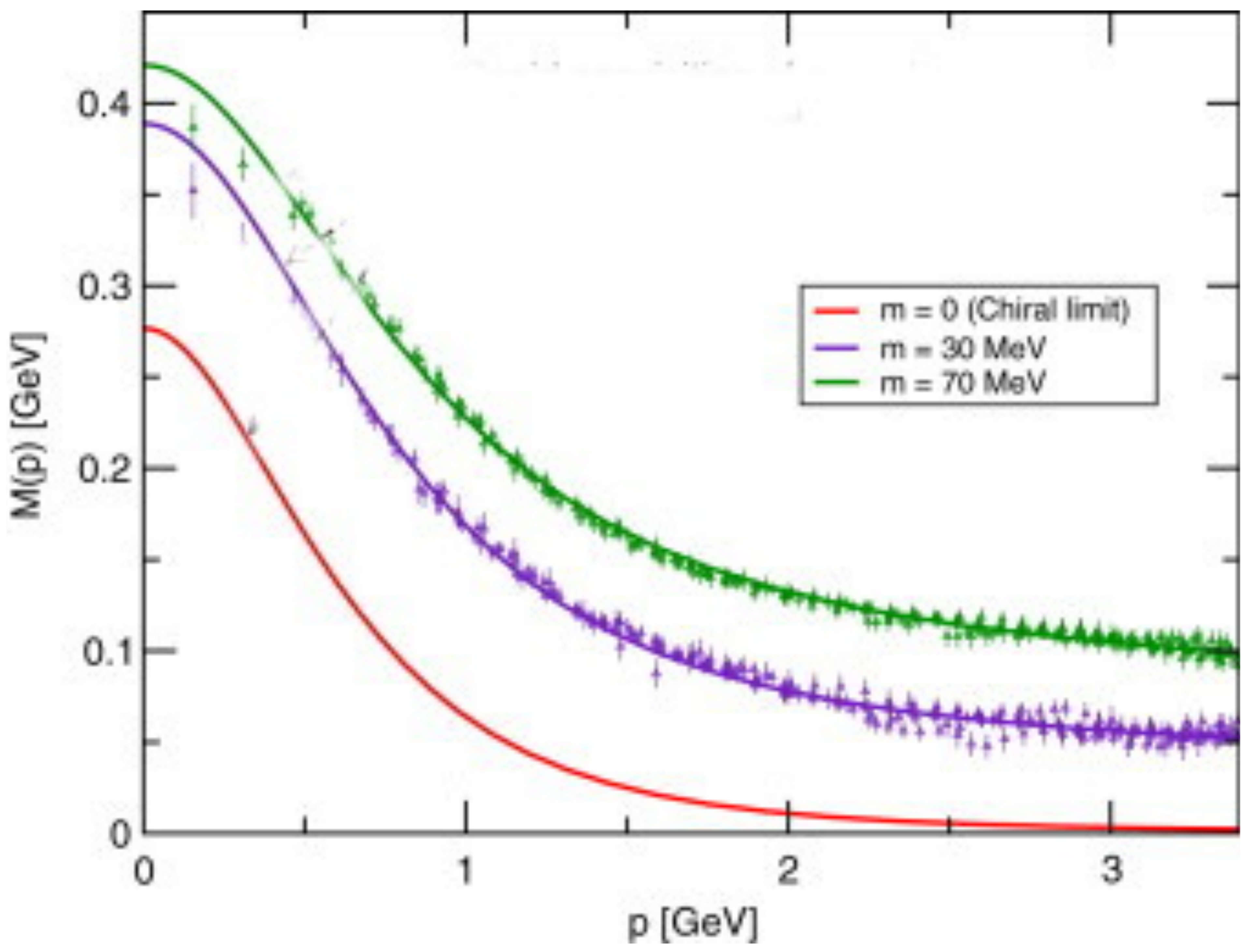
- Result for the NJL dynamical quark mass—without momentum dependent

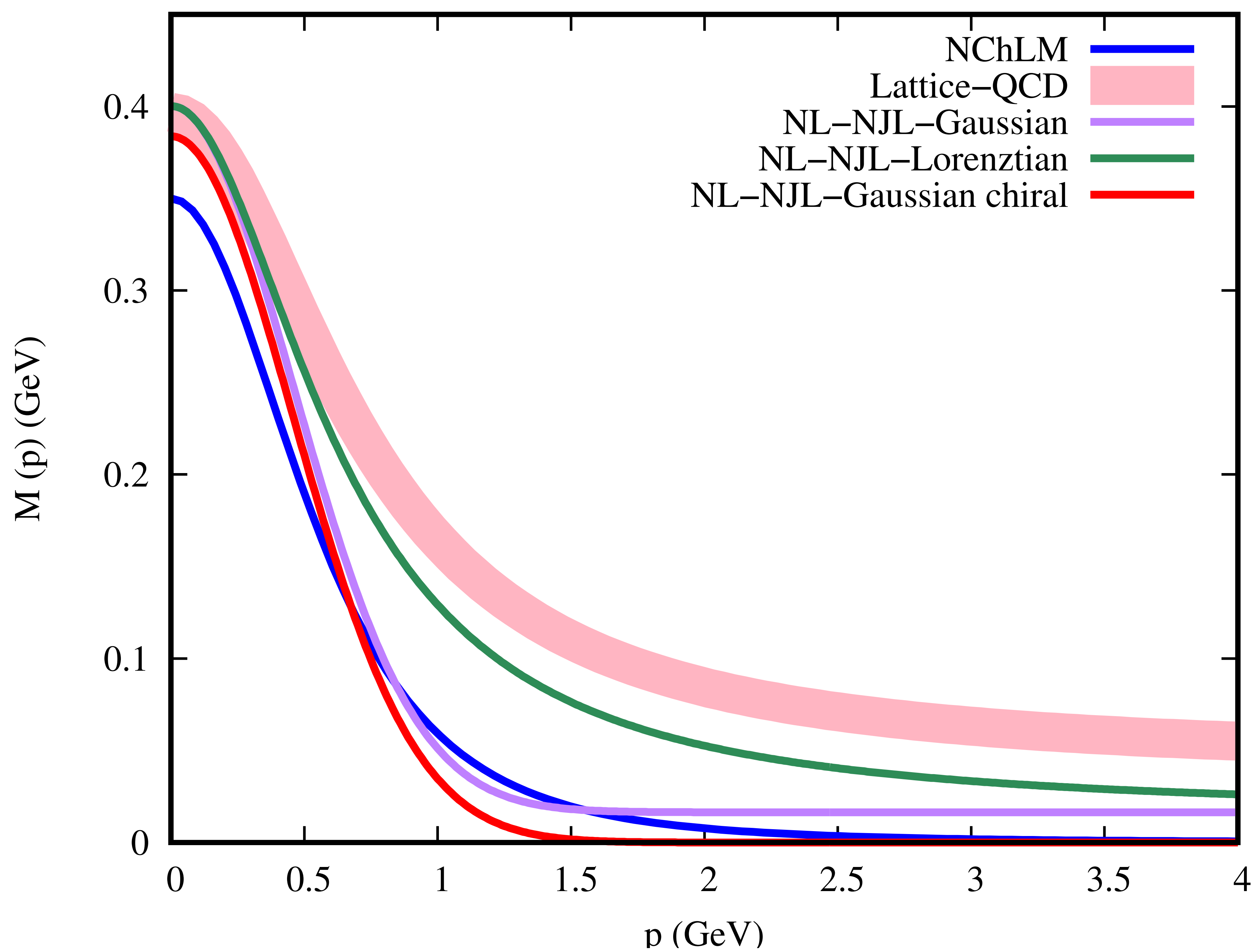


BSE-NJL model

DSE model—comparison with the BSE—NJL model

- Dynamical quark mass in the DSE model



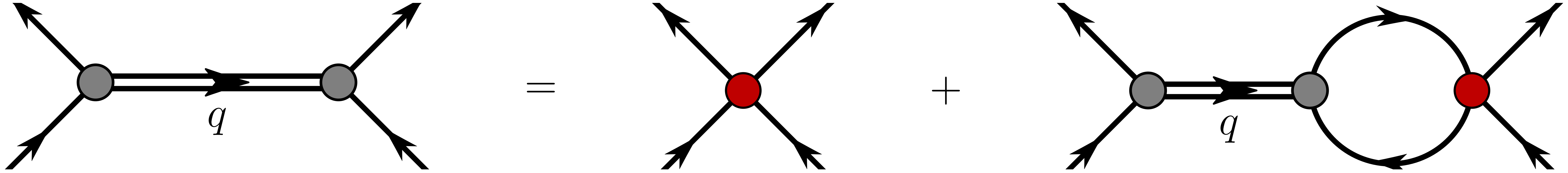


BSE-NJL model

Bethe-Salpeter Equation (BSE)—bound states

- In the BSE-NJL model, the dressed quark and anti-dressed quark bound state whose the properties are determined by solving the BSE:

$$\mathcal{T}(q) = \mathcal{K} + \int \frac{d^4k}{(2\pi)^4} \mathcal{K} S(q+k) \mathcal{T}(q) S(k)$$



- Simply we obtain the reduced t-matrix in the appropriate channel

$$t_\alpha(q) = \frac{-2iG_\pi}{[1 + 2G_\pi \Pi_{(\pi,K)}(q^2)]}$$

BSE-NJL model

Polarization insertion—Bubble diagram

- The polarization insertion for the pion and kaon are given by

$$\Pi_{(\pi,K)} = 6i \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma_5 S_l(k) \gamma_5 S_s(k+q)]$$

- Meson masses can be evaluated via the pole of the t-matrix

$$1 + 2G_\pi \Pi_{(\pi,K)}(k^2 = m^2_{(\pi,K)}) = 0$$

- Analytically, the expression for the pion and kaon masses

$$m_\pi^2 = \frac{m}{M_l} \frac{2}{G_\pi \mathcal{I}_{ll}(m_\pi^2)} \qquad m_K^2 = (\frac{m_s}{M_s} + \frac{m}{M_l}) \frac{1}{G_\pi \mathcal{I}_{ls}(m_K^2)} + (M_s - M_l)^2$$

BSE-NJL model

Meson-quark coupling and meson weak decay constants

- The meson-quark coupling constants are given by

$$g^2_{(\pi,K)q\bar{q}} = -\left[\frac{\partial \Pi_{(\pi,K)}(q^2)}{\partial q^2}\right]\Bigg|_{q^2=m^2_{(\pi,K)}}$$

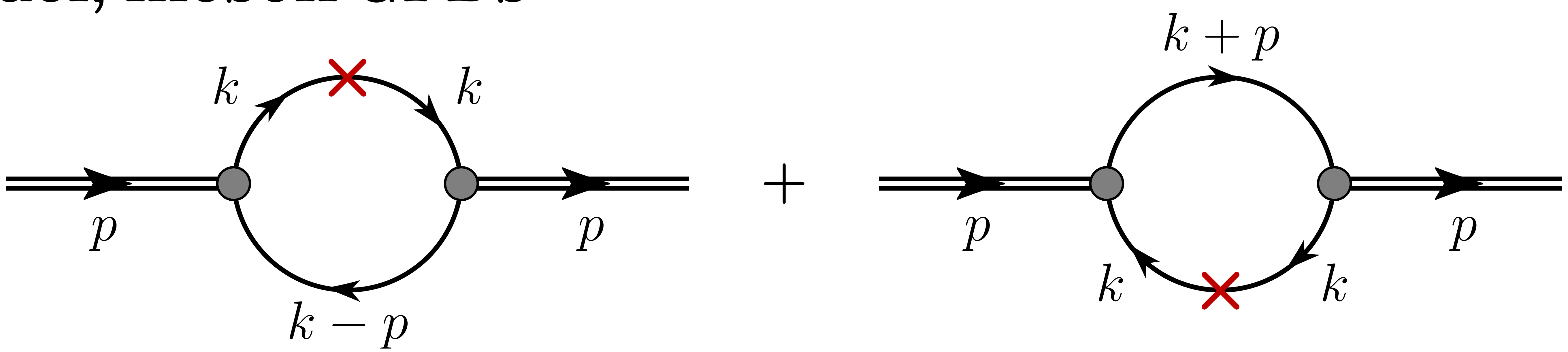
- Meson decay constants

$$f_{(\pi,K)} = \frac{N_c g_{(\pi,K)}}{4\pi^2}[(1-x)M_2 + xM_1]\int_0^1 dx \int_{\tau_{UV}}^{\tau_{IR}} \frac{d\tau}{\tau} \exp[-\tau(k^2(x^2-x) + xM_2^2 + (1-x)M_1^2)]$$

BSE-NJL model

Generalized Parton distributions (GPDs)

- In the NJL model, meson GPDs



- where the initial and final meson momentum are respectively given by p and p’

$$p^2 = p'^2 = m^2_{(\pi,K)}, \qquad t = q^2 = -Q^2 = (p' - p)^2, \qquad P = \frac{p+p'}{2}, \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$

- With ξ stands for **the skewness parameter** and the light-cone four-vector is given as $n = (1,0,0,-1)$

BSE-NJL model

The vector and tensor quark GPDS of the meson — General definition

- The vector (**no spin flip**) and tensor (**spin flip**) quark GPDs of the meson are given by

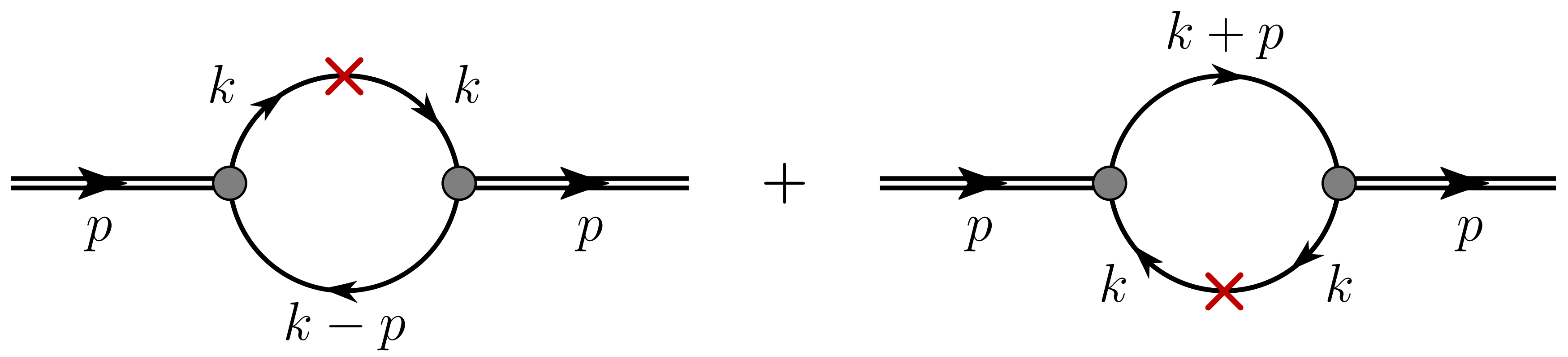
$$H^q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} \exp[ixP^+z^-]\langle p' \mid \bar{\psi}_q\left(-\frac{1}{2}z\right)\gamma^+\bar{\psi}_q\left(\frac{1}{2}z\right) \mid p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

$$E^q(x,\xi,t) = \frac{P^+m_{(\pi,K)}}{2(P^+q^j - P^jq^+)}\int \frac{dz^-}{2\pi} \exp[ixP^+z^-]\langle p' \mid \bar{\psi}_q\left(-\frac{1}{2}z\right)i\sigma^{+j}\psi_q\left(\frac{1}{2}z\right) \mid p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

- Where x is the longitudinal momentum

BSE-NJL model

Up-quark vector and tensor GPDs for the kaon



- In the NJL model, up-quark vector and tensor GPDs for the kaon are given by

$$H^u(x,\xi,t) = 2iN_c g_{Kq\bar{q}}^2 \int \frac{d^4k}{(2\pi)^4} \delta\left(xP^+ - k^+\right) \mathrm{Tr}[\gamma_5 S_u(k+\frac{q}{2})\gamma^+ S_u(k-\frac{q}{2})\gamma_5 S_s(k-P)]$$

$$E^u(x,\xi,t) = 2iN_c g_{Kq\bar{q}}^2 \left(\frac{P^+ m_K}{(P^+q^j - P^j q^+)}\right) \int \frac{d^4k}{(2\pi)^4} \delta\left(xP^+ - k^+\right) \mathrm{Tr}[\gamma_5 S_u(k+\frac{q}{2} i\sigma^{+j} S_u(k-\frac{q}{2})\gamma_5 S_s(k-P)]$$

- Performing the Feynman parametrization, WTI-like, and the proper-time regularization scheme

- Finally, the up-quark vector and tensor GPDs for the kaon are obtained by

BSE-NJL model

NJL up-quark vector and tensor GPDs for the kaon — final expressions

- Vector GPDs for the kaon in the proper-time regularization scheme

$$H^u(x,\xi,t) = \frac{N_c g_{Kq\bar{q}}^2}{8\pi^2}\left[\Theta_{\bar{\xi}_1}\bar{C}_1(\sigma_3) + \Theta_{\xi_1}\bar{C}_1(\sigma_4) + \frac{\Theta_{\bar{\xi}\xi}}{\xi}x\bar{C}_1(\sigma_5)\right] + \frac{N_c g_{Kq\bar{q}}^2}{8\pi^2}\int_0^1 dx\frac{\Theta_{x\xi}}{\xi}\frac{1}{\sigma_6}\bar{C}_2(\sigma_6)((1-x)t + 2x(m_K^2 - (M_u - M_s)^2))$$

- Tensor GPDs for the kaon in the proper-time regularization scheme

$$E^u(x,\xi,t) = \frac{N_c g_{Kq\bar{q}}^2}{4\pi^2}\int_0^1 dx\frac{\Theta_{x\xi}}{\xi}m_K((M_s - M_u)x + M_u)\frac{1}{\sigma_6}\bar{C}_2(\sigma_6)$$

- The Θ is the step function

BSE-NJL model
Properties of the GPDs

1. Forward limit – $\xi = 0$, and $t = 0$, the vector GPDs can be reduced into the kaon PDFs

2. Symmetries properties

$$H^{[I=0]}(x,\xi,t) = H^u(x,\xi,t) - H^u(-x,\xi,t)$$

$$H^{[I=1]}(x,\xi,t) = H^u(x,\xi,t) + H^u(-x,\xi,t)$$

3. The NJL results preserve the time reversal invariance property of GPDs

$$H^u(x,\xi,t) = H^u(x,-\xi,t) \qquad E^u(x,\xi,t) = E^u(x,-\xi,t)$$

BSE-NJL model
Properties of the GPDs

4. Condition of the Polynomiality

$$\int_{-1}^{1} x^n dx H^q(x,\xi,t) = \sum_{i=0}^{((n+1)/2)} \xi^{2i} \mathscr{A}^q_{(n+1),2i}(t)$$

$$\int_{-1}^{1} dx E^q(x,\xi,t) = \sum_{i=0}^{((n+1)/2)} \xi^{2i} \mathscr{B}^q_{(n+1),2i}(t)$$

5. For n = 0, we simply obtain the u-quark vector FFs ($F^u_K(Q^2)$) and tensor FFs ($F^u_T(Q^2)$)

$$\int_{-1}^{1} H^u(x,\xi,t)dx = \mathscr{A}^u_{1,0}(t) = F^u_K(Q^2)$$

$$\int_{-1}^{1} E^u(x,\xi,t)dx = \mathscr{B}^u_{1,0}(t) = F^u_T(Q^2)$$

BSE-NJL model

Properties of the GPDs

6. For n= 1, the GPDs will preserve the sum rule:

$$\int_{-1}^{1} xH^u(x,\xi,t)dx = \mathscr{A}^u_{2,0}(t) + \xi^2\mathscr{A}^u_{2,2}(t) = \Theta^u_2(t) - \xi^2\Theta^u_1(t)$$

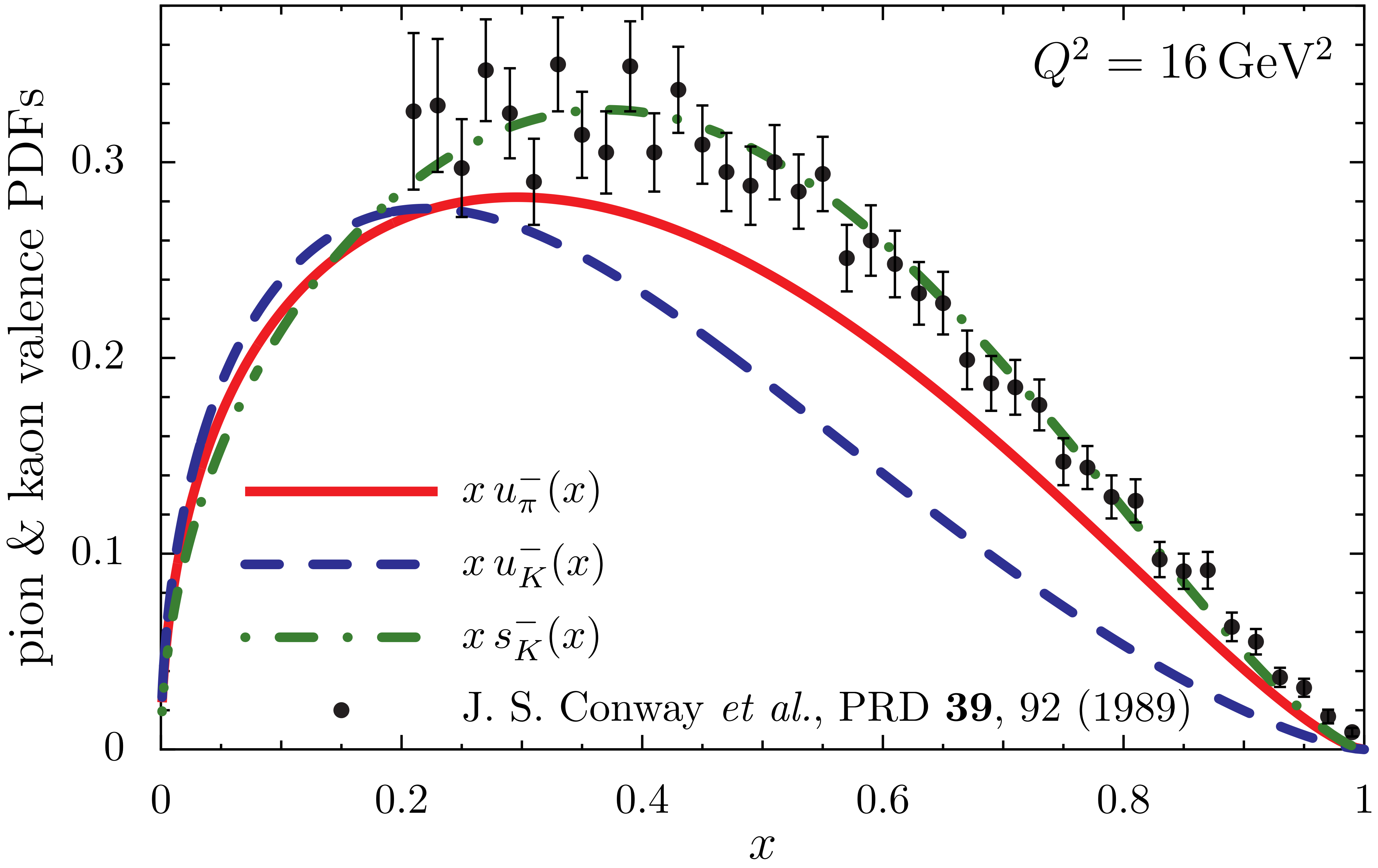
- $\Theta^u_2(t)$ and $\Theta^u_1(t)$ – the u-quark distribution for the kaon and pressure distribution
- $\mathscr{A}^u_{2,0}(Q^2)$ and $\mathscr{A}^u_{2,2}(Q^2)$ are the generalized FFs for n=1 in the BSE-NJL model
- The first derivation of $\mathscr{A}^u_{2,0}(Q^2)$ in respect with Q^2 at around $Q^2 = 0$ will give the light-cone energy radius
- $\mathscr{B}^u_{2,0}(Q^2)$ and $\mathscr{B}^u_{2,2}(Q^2) = 0$ are the u-quark tensor GPD for the kaon in the BSE-NJL model

$$\int_{-1}^{1} xE^u(x,\xi,t)dx = \mathscr{B}^u_{2,0}(t) + \xi^2\mathscr{B}^u_{2,2}(t)$$

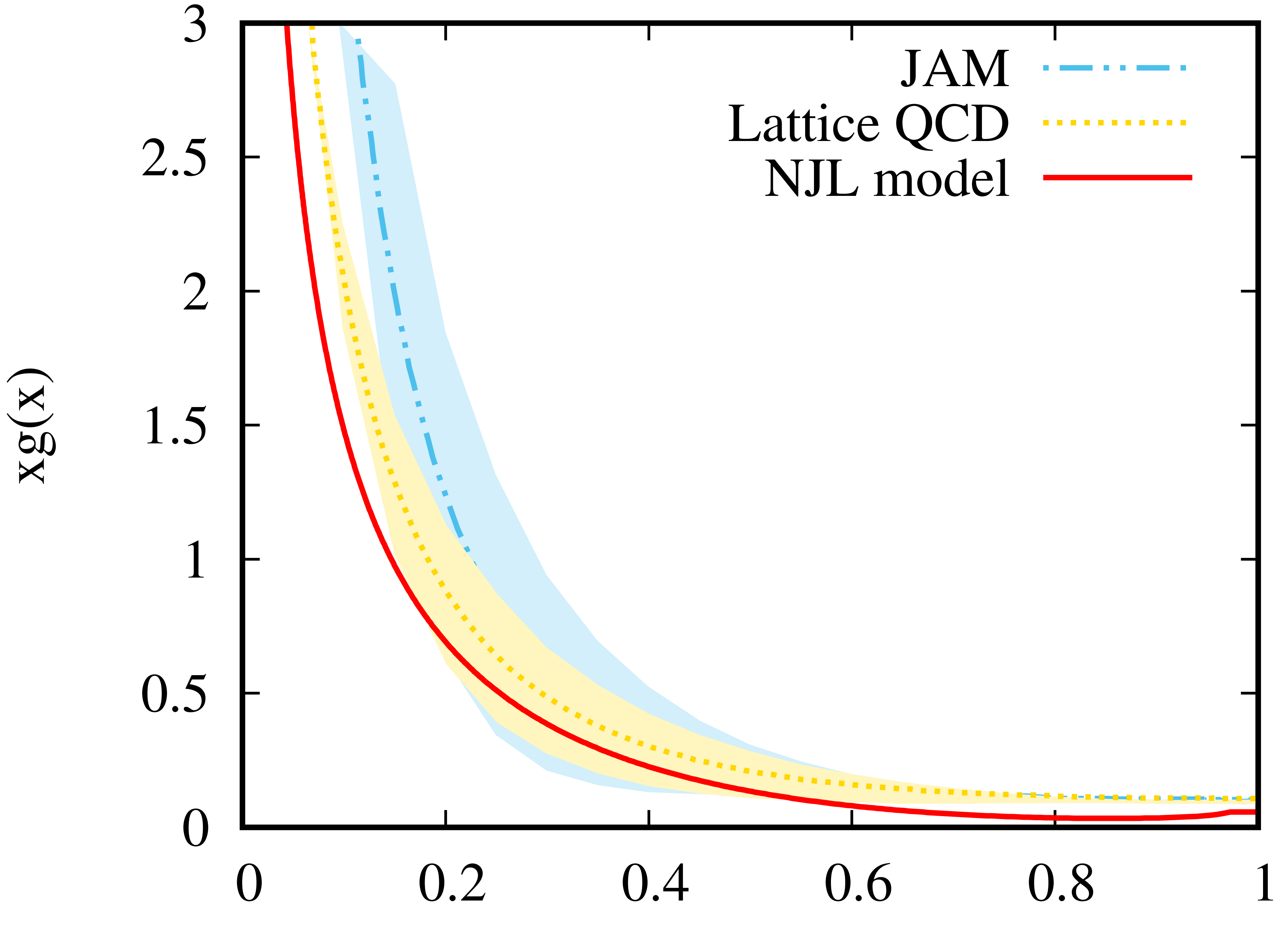
BSE-NJL model

Parton distribution functions for the meson—Forward limit $\xi = 0$ and $t = 0$

- Parton distribution functions for the pion and kaon after evolving at Q^2 =16 GeV² using NLO—DGLAP QCD evolution



PTPH, Ian Cloet & Anthony Thomas, PRC94(2016)

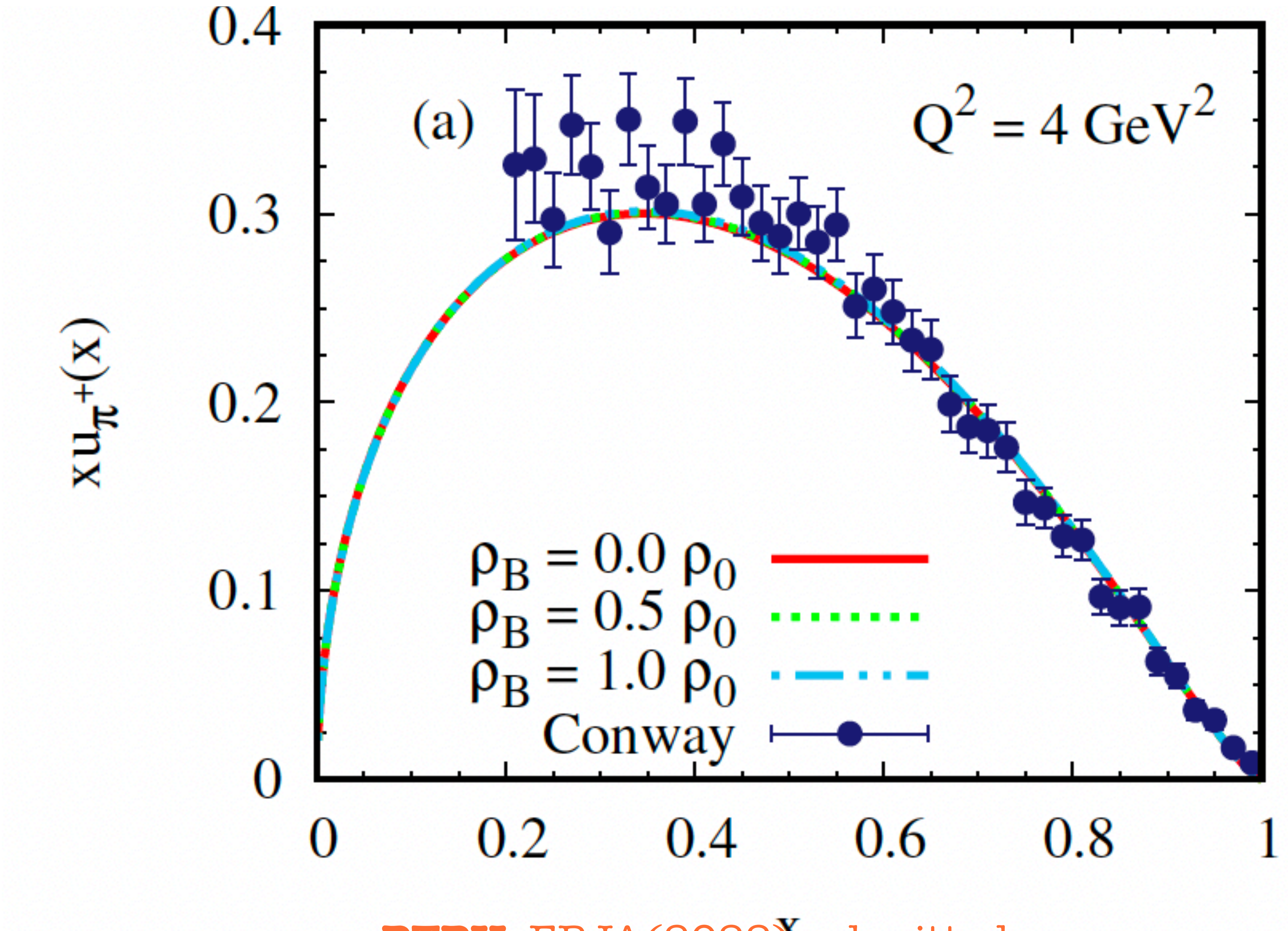


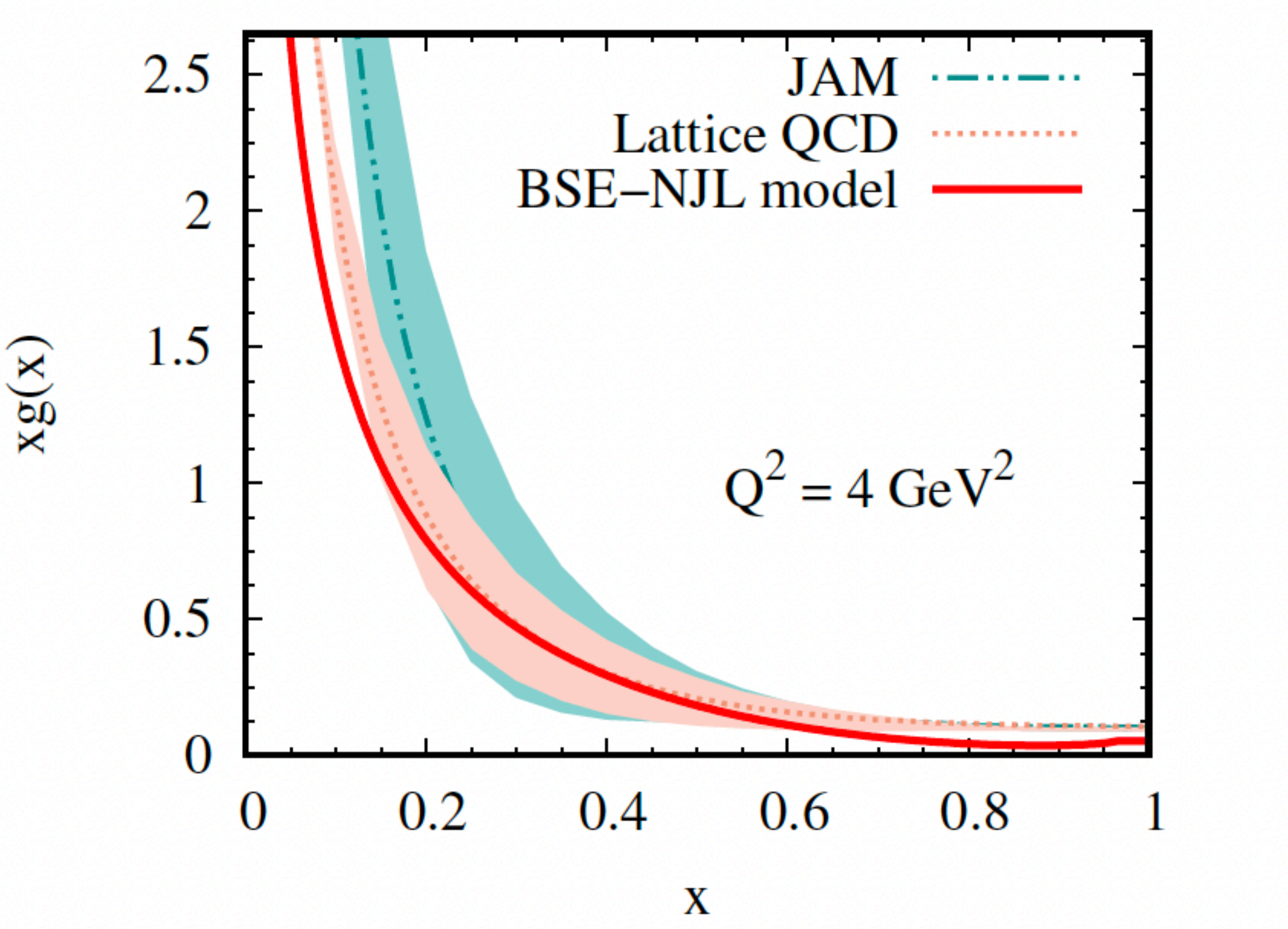
PTPH & Seung-il Nam, PRD105(2022)

BSE-NJL model

Parton distribution functions for the meson—Forward limit $\xi = 0$ and $t = 0$

- Valence and gluon distributions for the pion at Q^2 = 4 GeV²



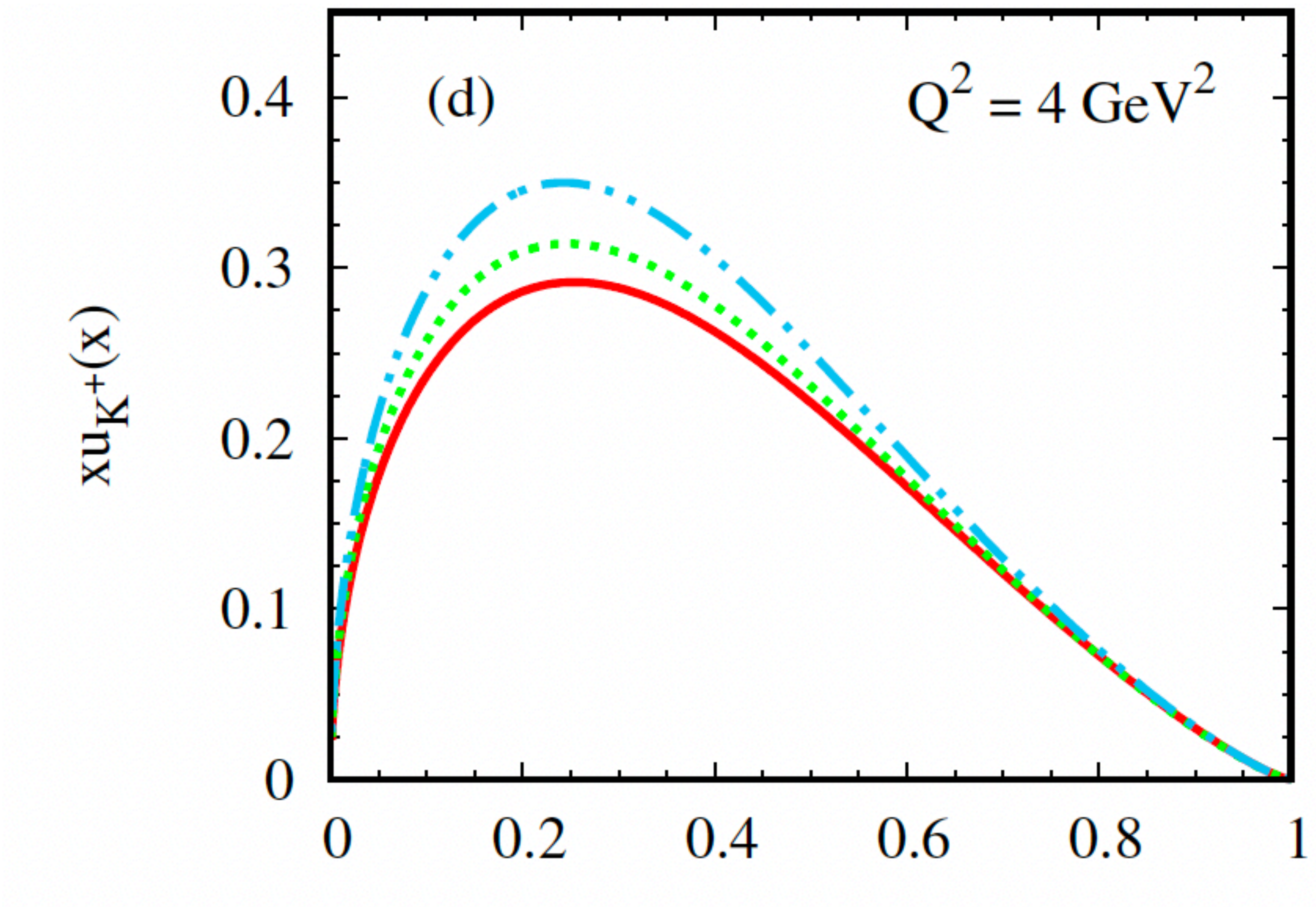


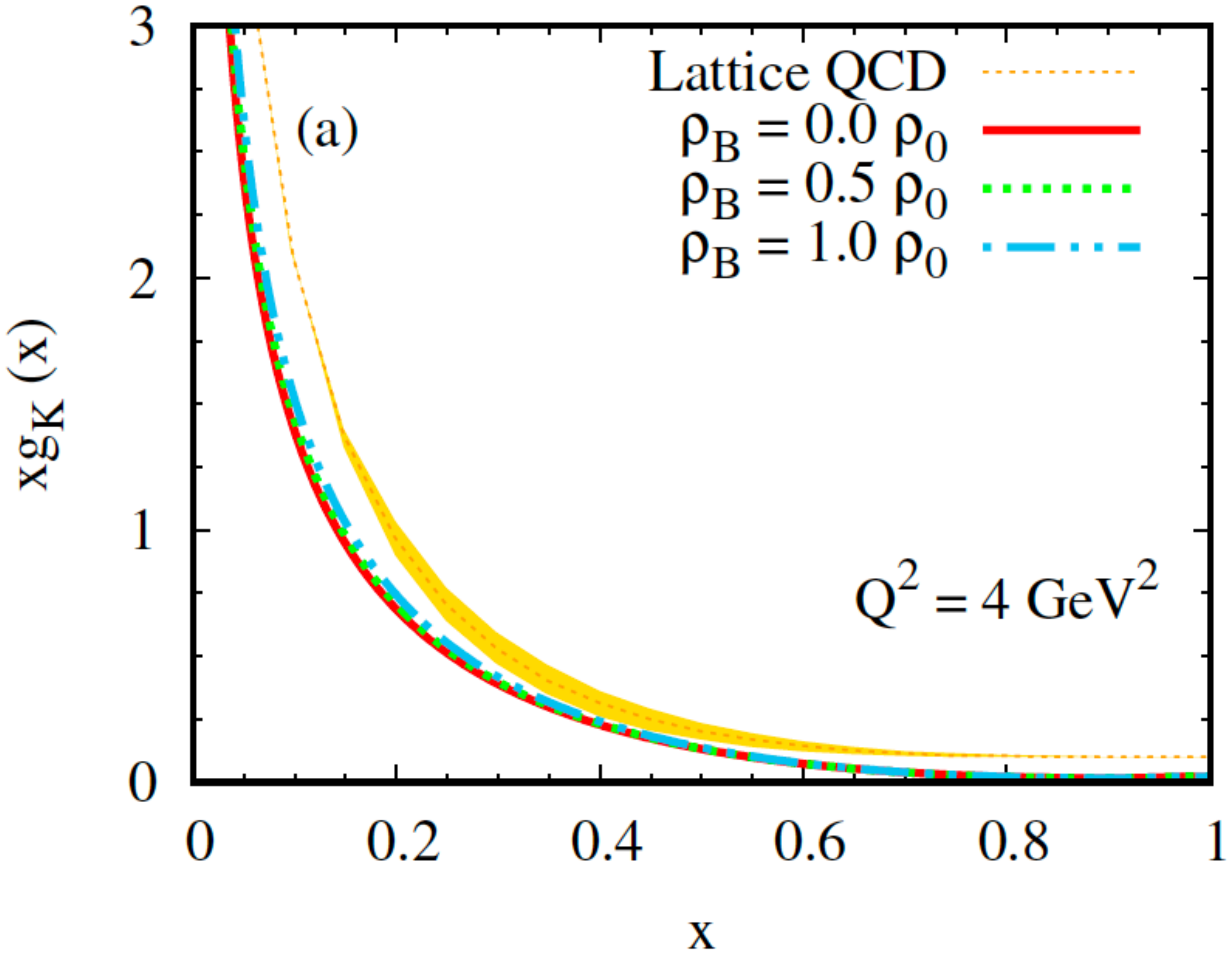
PTPH, EPJA(2022) submitted

BSE-NJL model

Parton distribution functions of the meson—Forward limit $\xi = 0$ and $t = 0$

- The valence and gluon distributions for the kaon at Q^2 = 4 GeV²

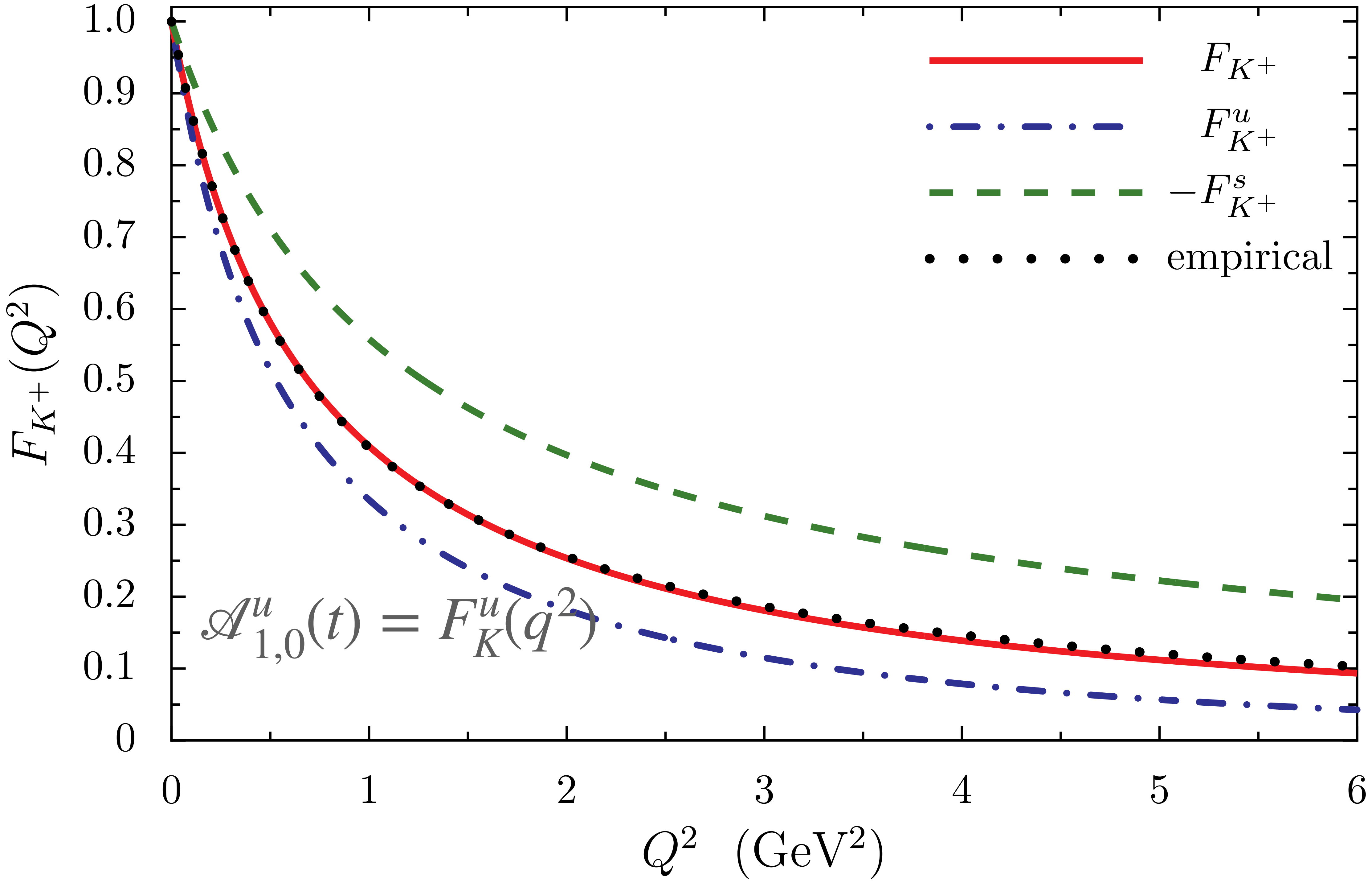


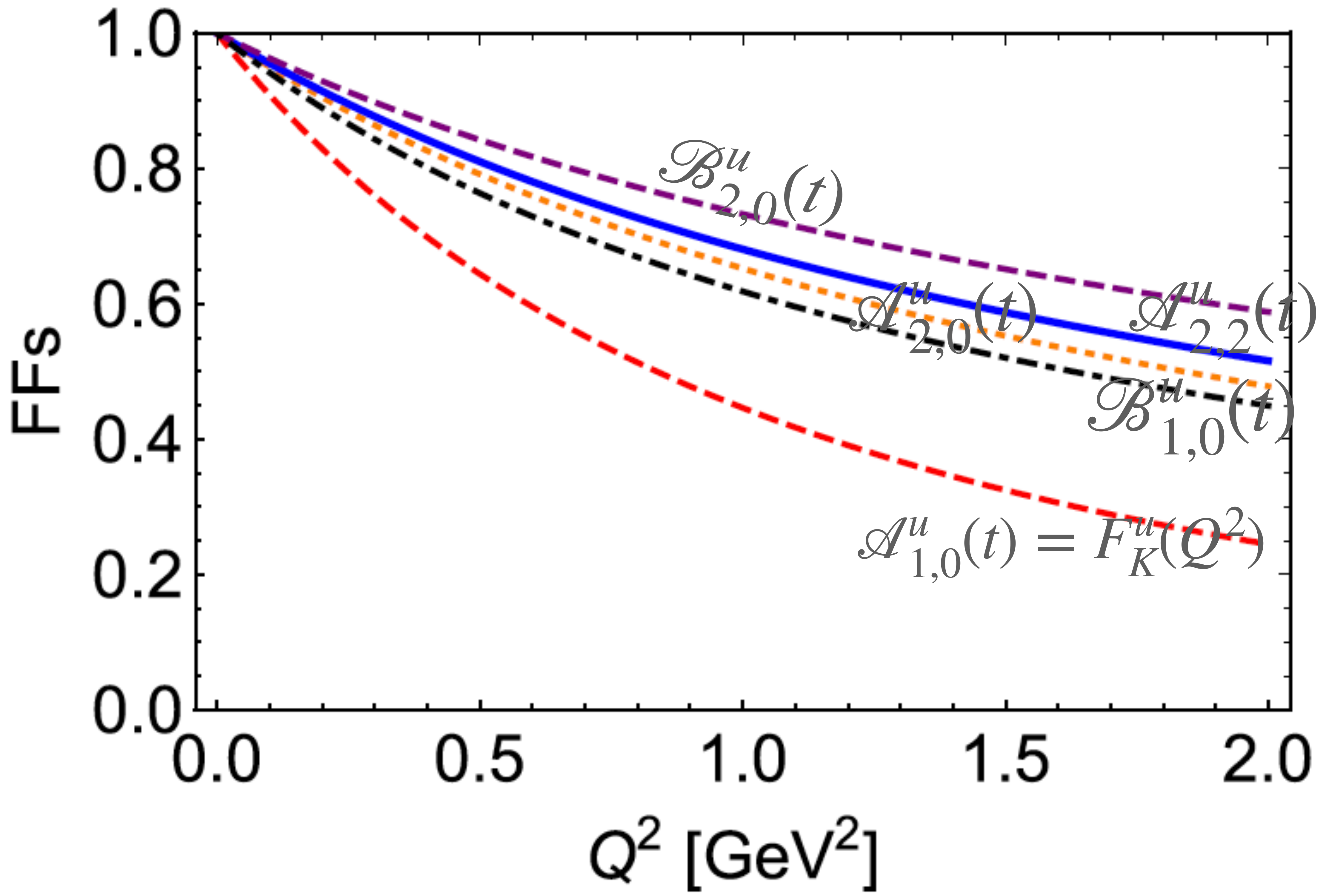


PTPH, EPJC(2022) submitted

BSE-NJL model
Form Factors for the meson

- Form factor for the pion and kaon $\mathscr{A}^u_{1,0}(t) = F^u_K(Q^2)$ and $\mathscr{B}^u_{1,0}(t) = F^u_T(Q^2)$



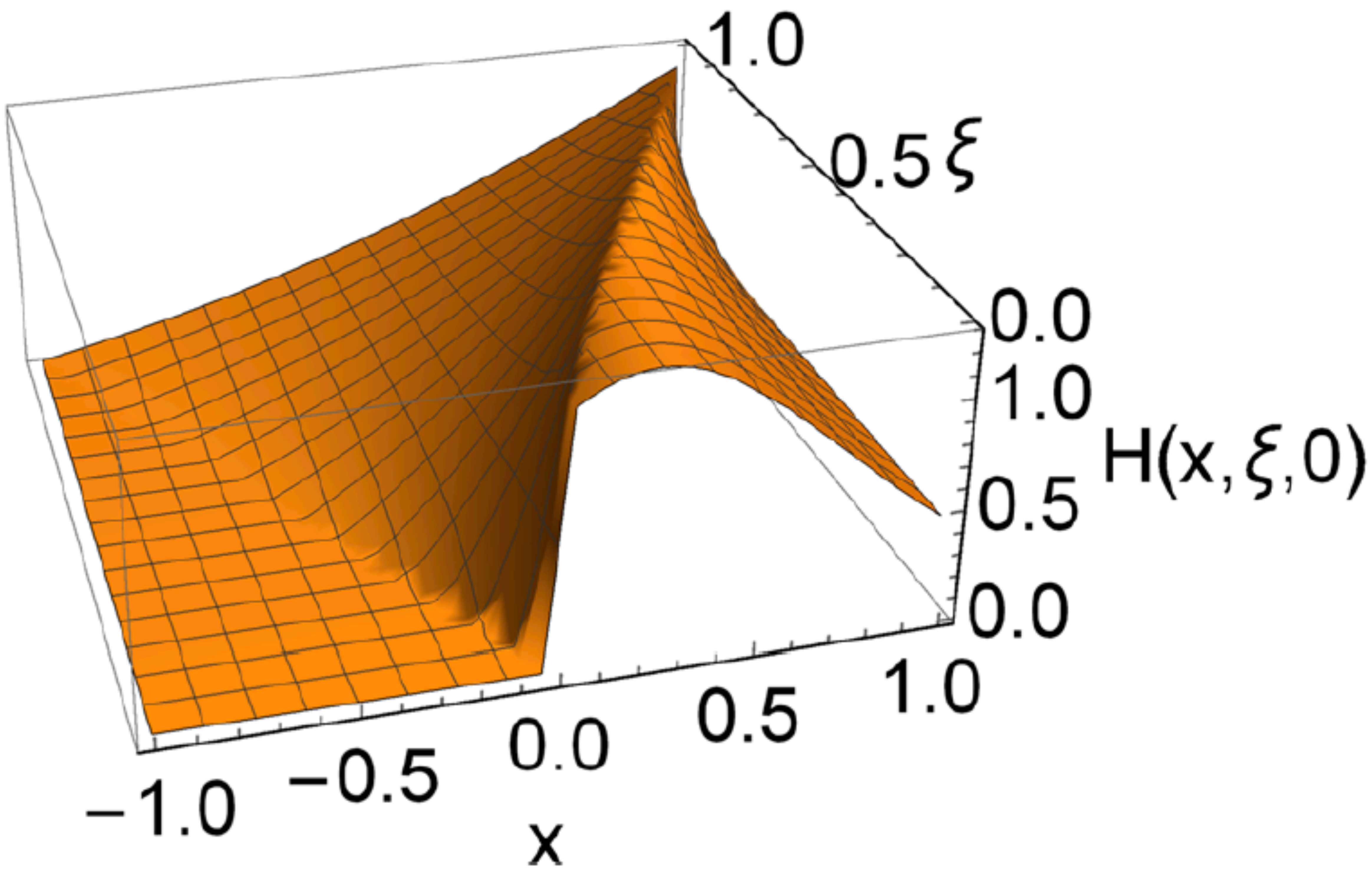


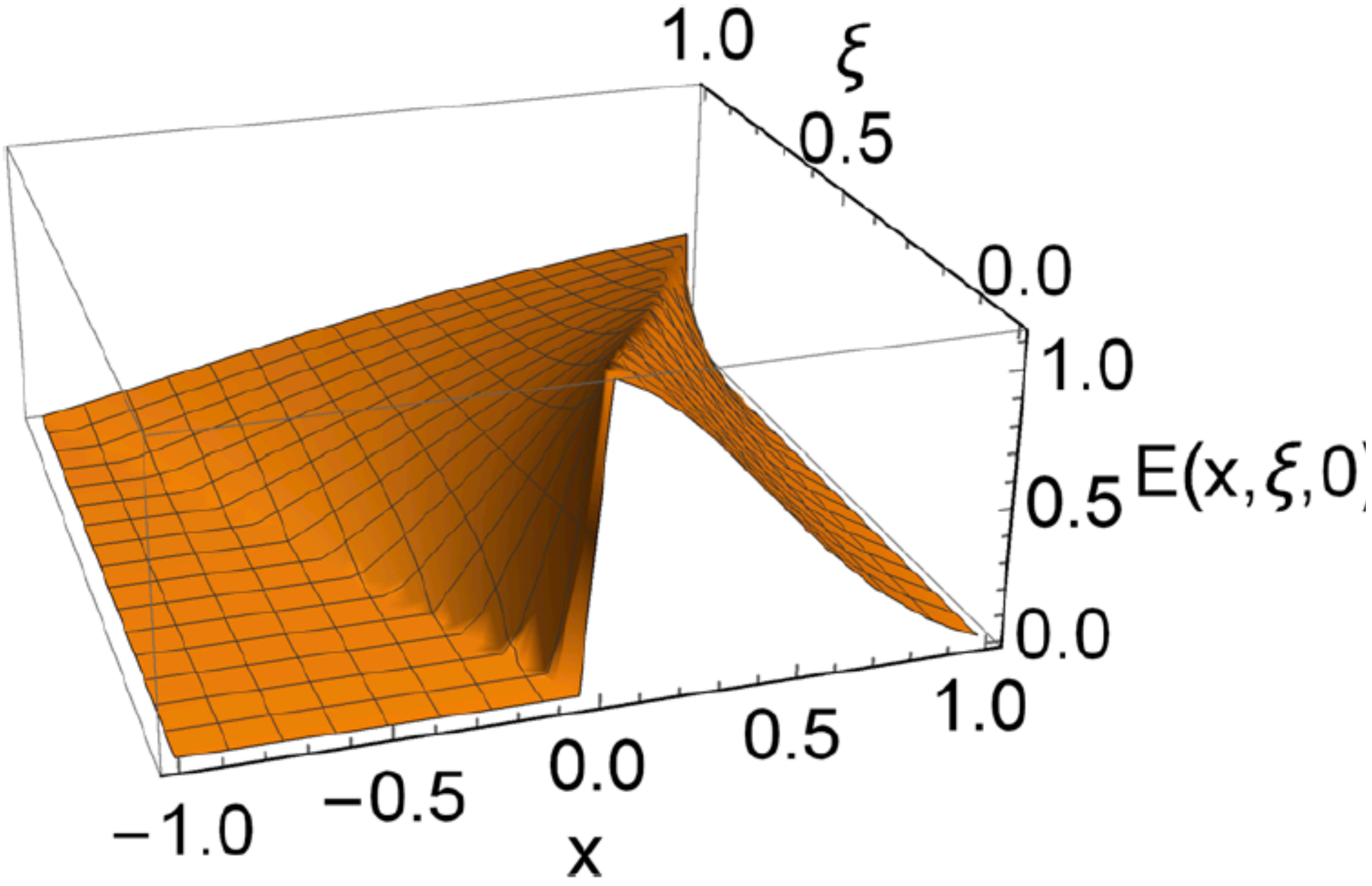
PTPH, Ian Cloet & Anthony Thomas, PRC94(2016)

BSE-NJL model

Kaon vector GPD—$H^u(x, \xi, 0)$ and tensor GPD—$E^u(x, \xi, 0)$

- Kaon vector and tensor GPDs for the kaon for $\xi > 0$





Summary and outlook

- We have calculated the GPDs of the meson in the BSE—NJL model and the prediction results are shown
- New data from the EIC, EIcC, AMBER COMPASS, and upgrade JLab-12 are really required to resolve the "tension" on the Parton distribution functions for the meson at high—x [power counting rule or the endpoint behavior] and to understand the gluon distributions for the meson
- Besides the PDFs for the meson, the data for the form factor of the meson are also needed to firmly understand the structure of the meson as the Goldstone boson
- Understanding of the meson structure in the BSE—NJL model will pave a way to understand the nucleon structure (more complex structure) using the more sophisticated model and lattice QCD

Thank you for attention!

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