Fracture functions formalism for hadron production from the target remnant in hard processes

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Outlook

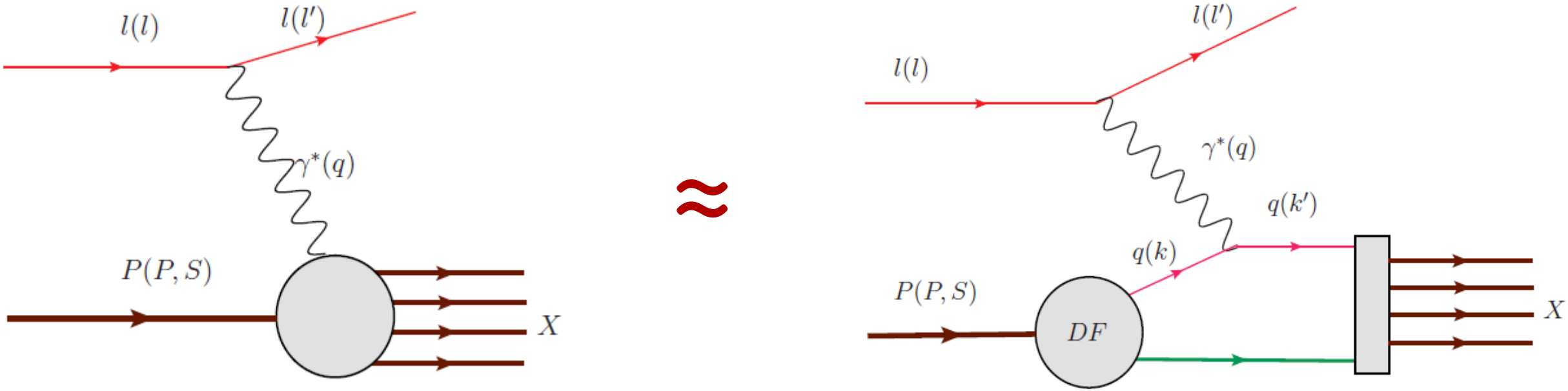
- **Processes to access the non-perturbative inputs Spin and Transverse Momentum Dependent (STMDs)**
  - Parton Distribution Functions in nucleon.
    - STMD PDF: SIDIS, DY
  - Parton Fragmentation Functions STMD FF: Hadron production in e^+e^- annihilation (SIA), SIDIS, high p_T hadron production in pp collisions
  - STMD Fracture Functions: SIDIS, DY
  - String Fragmentation: LEPTO, PYTHIA

Twist-2 TMD PDFs

Nucleon Polarization \ Quark polarization	U	L	T
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij} k_T^{\ j}}{M} h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{[\mathbf{k}_T \times \mathbf{S}_T]_3}{M} f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2) + \frac{\mathbf{k}_T}{M} \frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x,k_T^2)$

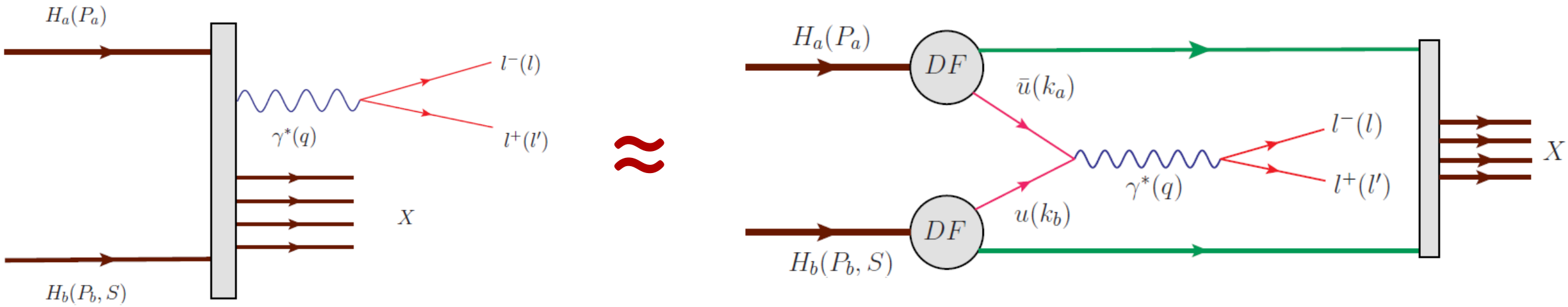
All azimuthal dependences are in prefactors. TMDs do not depend on them

QCD factorization: DIS



Access to nucleon unpolarized $f_1^{q+\bar{q}}(x)$ and longitudinally polarized $g_1^{q+\bar{q}}(x)$ leading twist colinear (transverse momentum integrated) PDFs

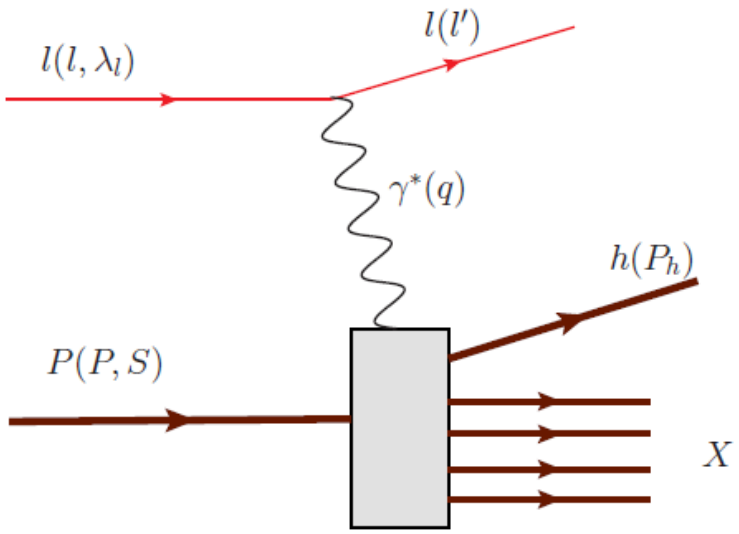
QCD TMD factorization: DY processes



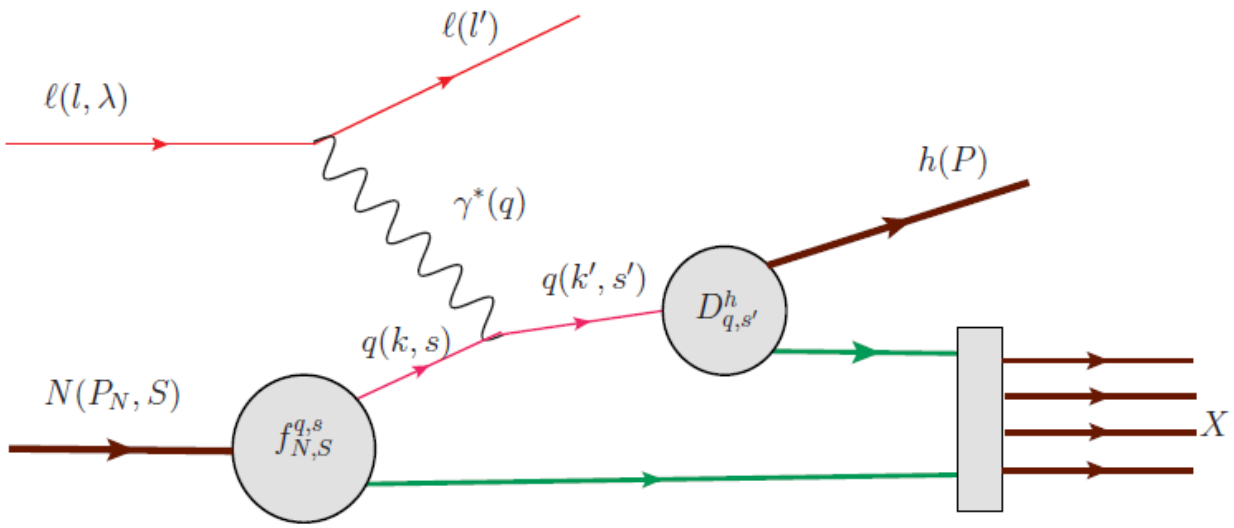
Access to nucleon, pion and kaon $f_1(x, \mathrm{k}_T^2)$, $\mathrm{g}_1(x, \mathrm{k}_T^2), \mathrm{h}_1(x, \mathrm{k}_T^2)$

and $\mathrm{h}_1^{\perp}(x, \mathrm{k}_T^2)$ leading twist PDFs

QCD TMD factorization: SIDIS in CFR



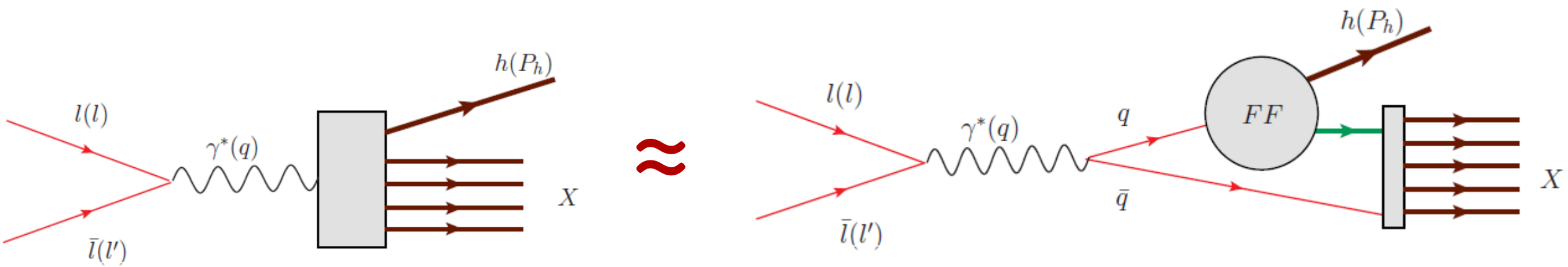
$\approx$



Access to nucleon $f_1^q(x,\mathrm{k}_T^2)$, $\mathrm{g}_1^q(x,\mathrm{k}_T^2)$ and $\mathrm{h}_1^q(x,\mathrm{k}_T^2)$,... leading twist TMD PDFs.

Unpolarized TMD FFs $\mathrm{D}_q^h(\mathrm{z},\mathrm{p}_T^2)$ and Collins TMD FF $\mathrm{H}_{1q}^h(\mathrm{z},\mathrm{p}_T^2)$

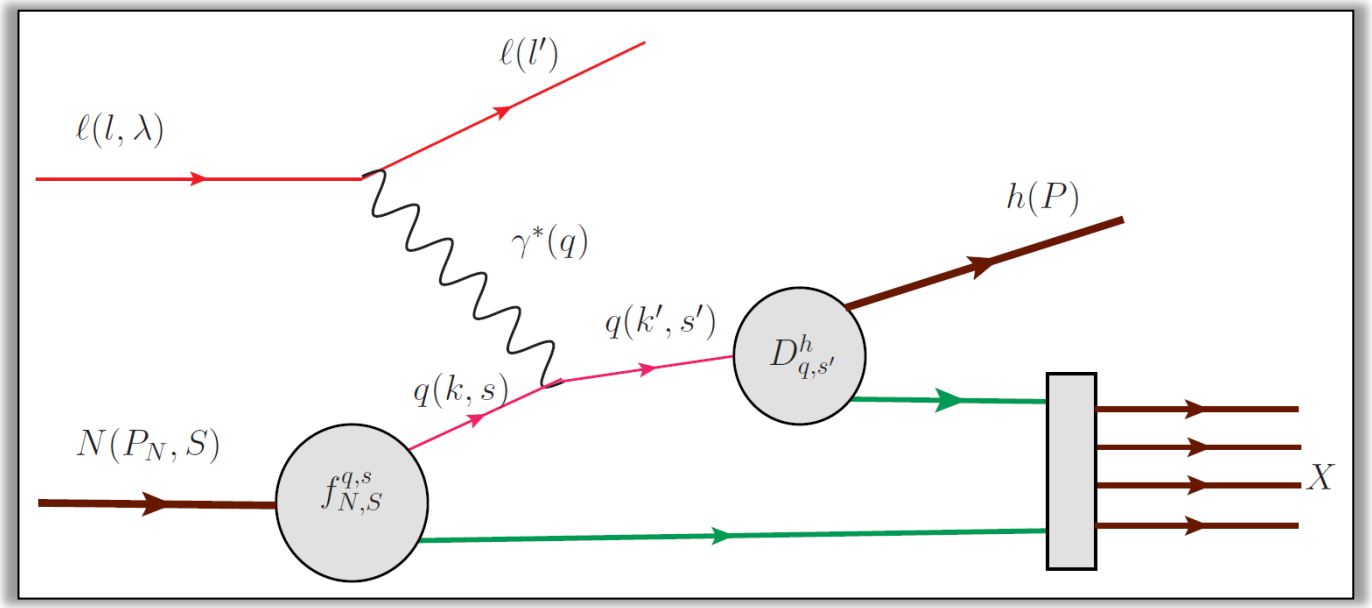
QCD TMD factorization: SIA



Access to $q+\bar{q}$ fragmentation finctions $D^h_{q+\bar{q}}(z, p_\perp^2)$

Two hadron production in opposite hemispheres: acces to Collins FF $H^h_{1q}(z, p_\perp^2)$

SIDIS: CFR



$x_F > 0,\ \lambda -$ beam longitudinal polarization

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S}\otimes\frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}\otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T\times\mathbf{s}'_T}{m_h}H_1(z,p_T^2)$$

Measured in e^+e^- semi inclusive annihilation (SIA) to 2 back-to-back jets $e^+e^-\to h_1h_2 + X$

LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

$$\times\Big[F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+$$

$$S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+$$

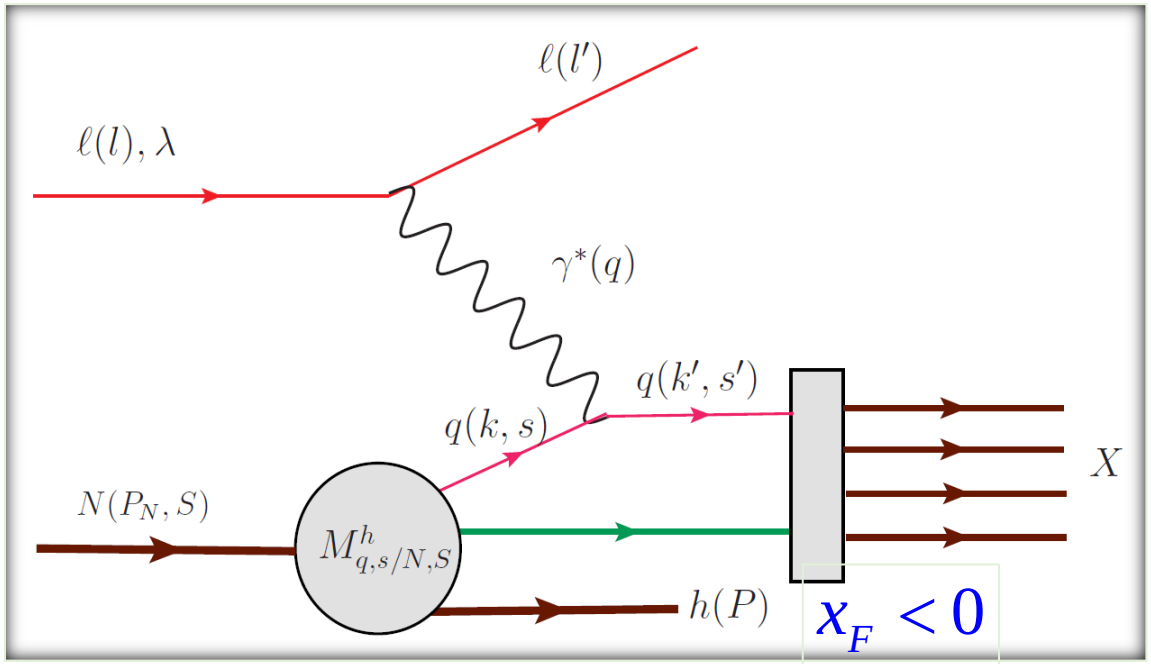
$$S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\sin(\phi_h-\phi_S)+D_{nn}(y)\left(F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\right)\right)+$$

$$\lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\Big]$$

$$D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2},\qquad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}$$

At LO only 8 terms out of 18 Structure Functions entering in the general expression of SIDIS cross section
6 azimuthal modulations, 4 terms are generated by Collins effect in fragmentation

SIDIS: TFR



Trentadue, Veneziano 1994
Graudenz 1994
Collins 1998, 2000, 2002
de Florian, Sassot 1997, 1998
Grazzini, Trentadue, Veneziano 1998
Ceccopieri, Trentadue 2006, 2007, 2008
Sivers 2009
Ceccopieri , Mancusi 2013
Ceccopieri 2013
……..

$$\frac{d\sigma^{\ell(l)+N(P_N)\to\ell(l')+h(P)+X}}{dxdQ^2d\zeta} = M^h_{q/N}\left(x,Q^2,\zeta\right)\otimes\frac{d\sigma^{\ell(l)+q(k)\to\ell(l')+q(k')}}{dQ^2}$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Fracture function *M* is a Conditional Probability Distribution Function (CPDF) to observe the hadron h produced in target nucleon momentum direction in γ^*P CMS when hard probe interacts with parton carrying fraction x of nucleon momentum.

Collinear Frac.Func.: application to HERA data, 1

D. de Florian, R. Sassot, Leading Proton Structure Function. PRD 58, 054003 (1998)

$$\frac{d^3\sigma^p_{target}}{d\beta dQ^2 dx_P} = \frac{4\pi\alpha^2}{\beta Q^4}\left(1-y+\frac{y^2}{2}\right)M_p^h\left(\beta,Q^2,x_P\right),\quad \beta=\frac{x}{1-\zeta},\quad \zeta=\frac{p_h^+}{p_N^+} \qquad x_P=\zeta$$

$$xM_q^{p/p}(\beta,Q_0^2,x_{\mathbb{P}})=N_s\beta^{a_s}(1-\beta)^{b_s}\{C_{\mathbb{P}}\beta x_{\mathbb{P}}^{\alpha_{\mathbb{P}}}+C_{LP}(1-\beta)^{\gamma_{LP}}[1+a_{LP}(1-x_{\mathbb{P}})^{\beta_{LP}}]\}$$

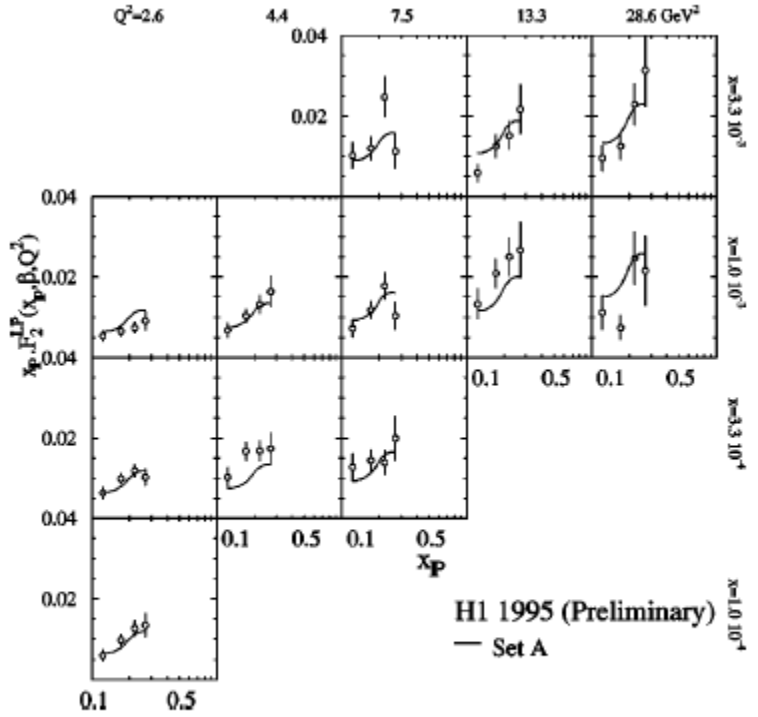


FIG. 2. H1 leading-proton data against the outcome of the fracture function parametrization (solid lines).

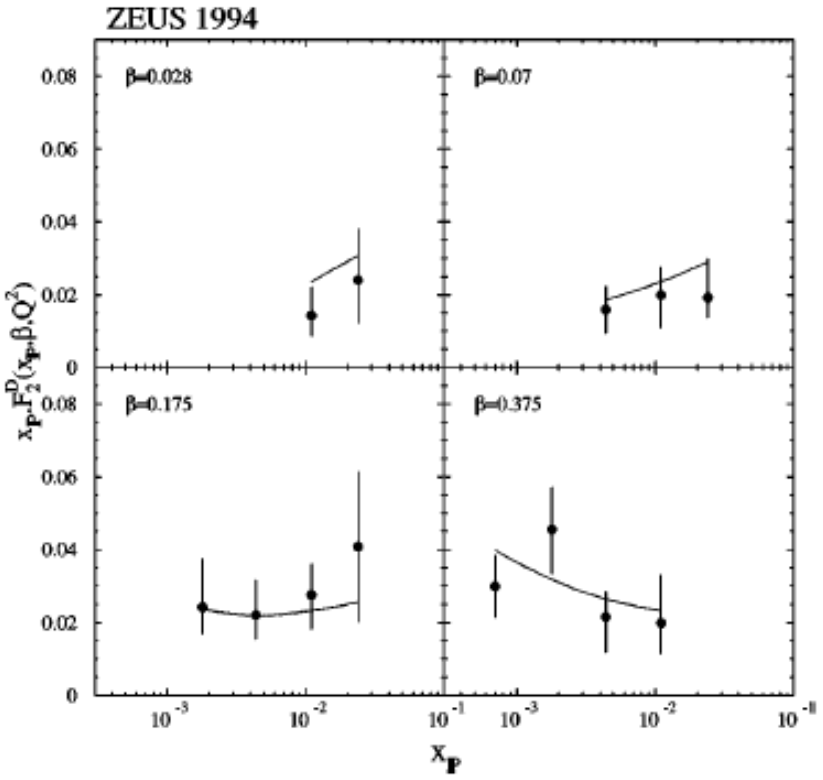
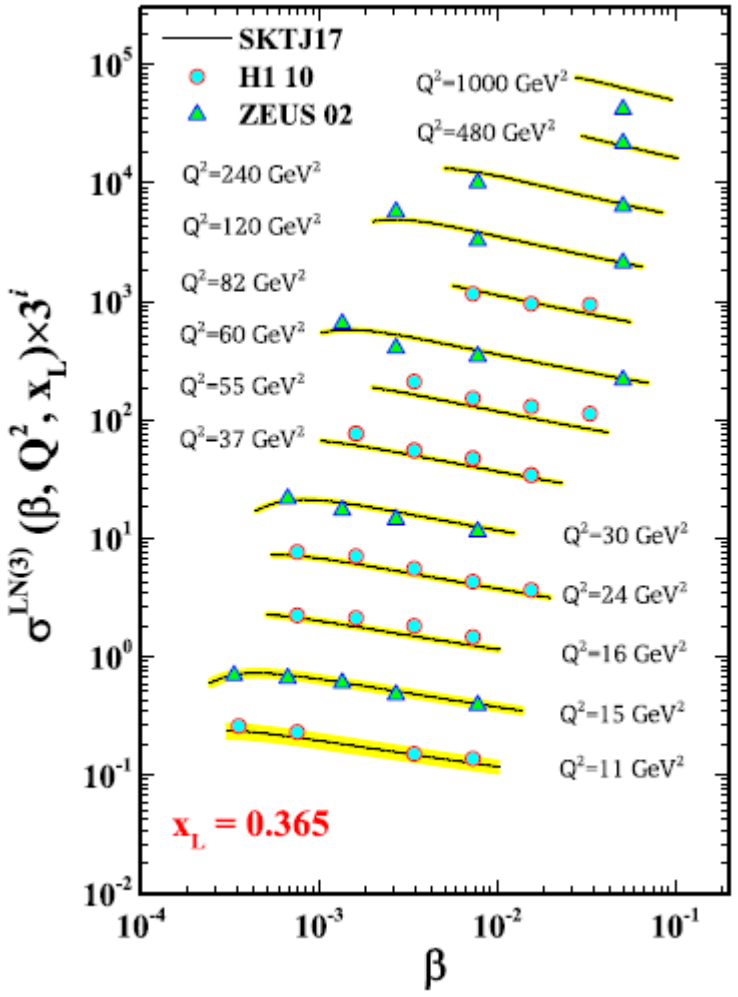


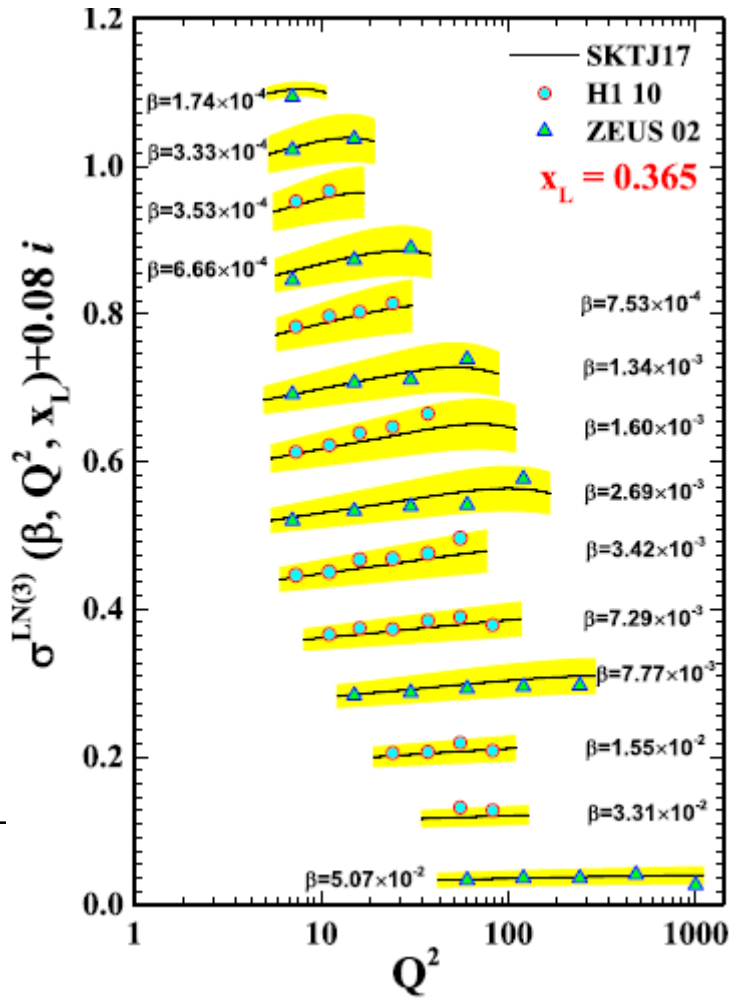
FIG. 8. ZEUS diffractive data, against the expectation coming from the fracture function parametrization (fit A).

Collinear Frac.Func.: application to HERA data, 2

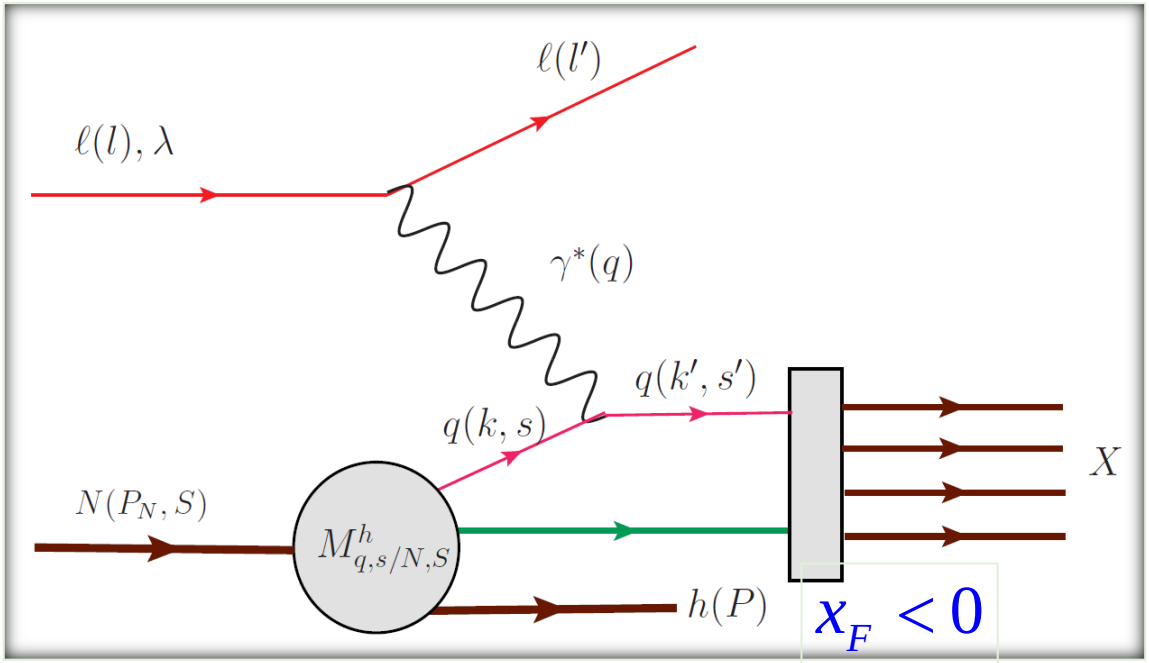
Shoeibi *et al*, Neutron fracture functions. PRD 95, 074011 (2017)



$$x_L \simeq \frac{E_h}{E_P}, \quad \beta = \frac{x}{1-x_L}$$



SIDIS TFR: Spin & TMD (STMD) Fracture Functions



Anselmino, Barone and AK, **PL B 699** (2011)108**; 706** (2011)46**; 713** (2012)317

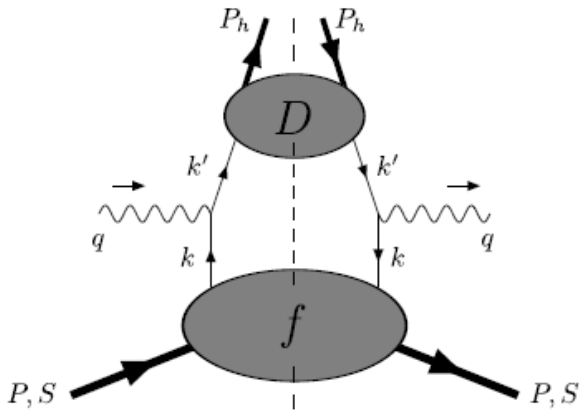
Nucleon and quark polarization are included, produced hadron and quark transverse momentum are not integrated over. Classification of twist-two Fracture Functions and cross sections expressions.

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

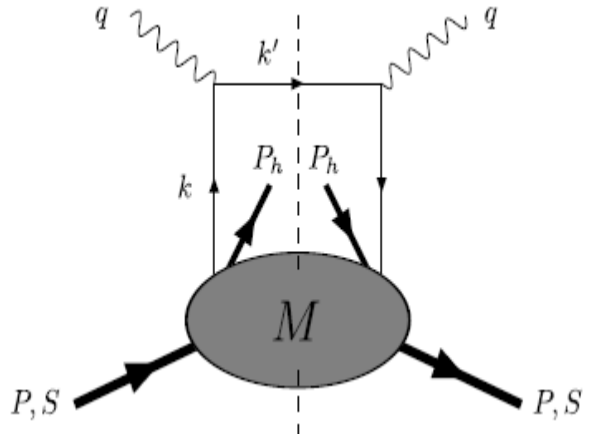
$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Quark correlator

SIDIS



CFR



TFR

$$\mathcal{M}^{[\Gamma]}(x_B,\vec{k}_\perp,\zeta,\vec{P}_{h\perp}) = \frac{1}{4\zeta}\int \frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6} e^{i(x_B P^-\xi^+ - \vec{k}_\perp\cdot\vec{\xi}_\perp)} \sum_X \int \frac{d^3P_X}{(2\pi)^3 2E_X} \times$$

$$\times \langle P,S|\bar{\psi}(0)\Gamma|P_h,S_h;X\rangle\langle P_h,S_h;X|\psi(\xi^+,0,\vec{\xi}_\perp)|P,S\rangle$$

$$\Gamma = \gamma^-,\quad \gamma^-\gamma_5,\quad i\sigma^{i-}\gamma_5$$

Probabilistic interpretation at LO:

the conditional probabilities to find an unpolarized ($\Gamma = \gamma^-$), a longitudinally polarized ($\Gamma = \gamma^-\gamma_5$) or a transversely polarized ($\Gamma = \sigma^{i-}\gamma_5$) quark with longitudinal momentum fraction x_{Bj} and transverse momentum $\boldsymbol{k}_\perp$ inside a nucleon fragmenting into a hadron carrying a fraction ζ of the nucleon longitudinal momentum and a transverse momentum $\boldsymbol{P}_{h\perp}$.

STMD Fracture Functions for spinless hadron production

Nucleon Polarization \ Quark polarization	U	L	T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^{\,j}}{m_h} \hat{t}_1^{\,h} + \frac{\epsilon_T^{ij} k_T^{\,j}}{m_N} \hat{t}_1^{\perp}$
L	$\frac{S_L (\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^{\,h} + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^{\perp}$
T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^{h} + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^{\perp}$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^{h} + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^{\perp}$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T (\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T (\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

For STMD Fracture Functions I was expecting 32 (Trentadue) independent structures. Fortunately, we end up with only 16 of them at twist-2

STMD fracture functions depend on

$$x,\ k_T^2,\ \zeta,\ P_T^2,\ \mathbf{k}_T \cdot \mathbf{P}_T$$

$$\mathbf{k}_T \cdot \mathbf{P}_T = k_T P_T \cos(\phi_h - \phi_q)$$

Azimuthal dependence in fracture functions

The contributions in CFR are marked in red

Sum Rules connecting Fracture Functions to TMD PDFs

Integrating the fracture function over the longitudinal and transverse momentum of hadron in TFR and summing over all possible hadrons recover the corresponding quark TMD DF

$$\sum_h \int \zeta d\zeta \int d^2 P_T \, \hat{u}_1 = (1-x) f_1(x, k_T^2)$$

Similar TMD Sum Rules for all TMD PDFs

$$\sum_h \int \zeta d\zeta \int d^2P_T \, \hat{u}_1 = (1-x)\, f_1(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{u}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{u}_{1T}^{h} \right) = -(1-x)\, f_{1T}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \, \hat{l}_{1L} = (1-x) g_{1L}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{l}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{l}_{1T}^{h} \right) = (1-x) g_{1T}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{t}_{1L}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1L}^{h} \right) = (1-x) h_{1L}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{t}_{1}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1}^{h} \right) = -(1-x) h_{1}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{t}_{1T}^{\perp\perp} + \frac{m_N^2}{m_h^2} \frac{2\left(\mathbf{k}_T \cdot \mathbf{P}\right)^2 - k_T^2 P_T^2}{k_T^4} \hat{t}_{1T}^{hh} \right) = (1-x) h_{1T}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left(\hat{t}_{1T} + \frac{k_T^2}{2m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{P_T^2}{2m_h^2} \hat{t}_{1T}^{hh} \right) = (1-x) h_1(x,k_T^2)$$

LO cross-section in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T}=\frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2\times$$

$$\times\left[\begin{array}{l}\tilde{u}_1\left(x,\zeta,P_T^2\right)-S_T\,\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)+\\ \lambda y(2-y)\left(S_L\tilde{l}_{1L}\left(x,\zeta,P_T^2\right)+S_T\,\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\end{array}\right]$$

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{u}_1$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{u}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^\perp\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{l}_{1L}$$

$$\tilde{l}_{1T}^h(x_B,\zeta_2,\ P_{T2}^2)=\int d^2k_T\left\{\hat{l}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^\perp\right\}$$

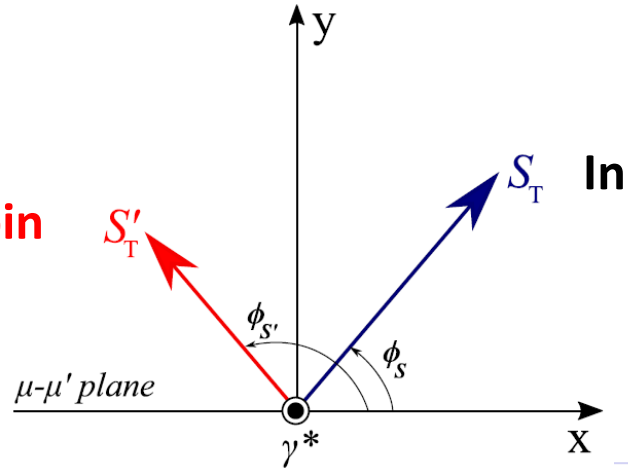
At LO (twist 2) only 4 terms out of 18 Structure Functions in SIDIS,
Only 2 azimuthal modulations

In single hadron production in TFR NO access to final (struck) quark transverse polarization

→ No Collins-like $\sin(\phi_h+\phi_S)$ modulation

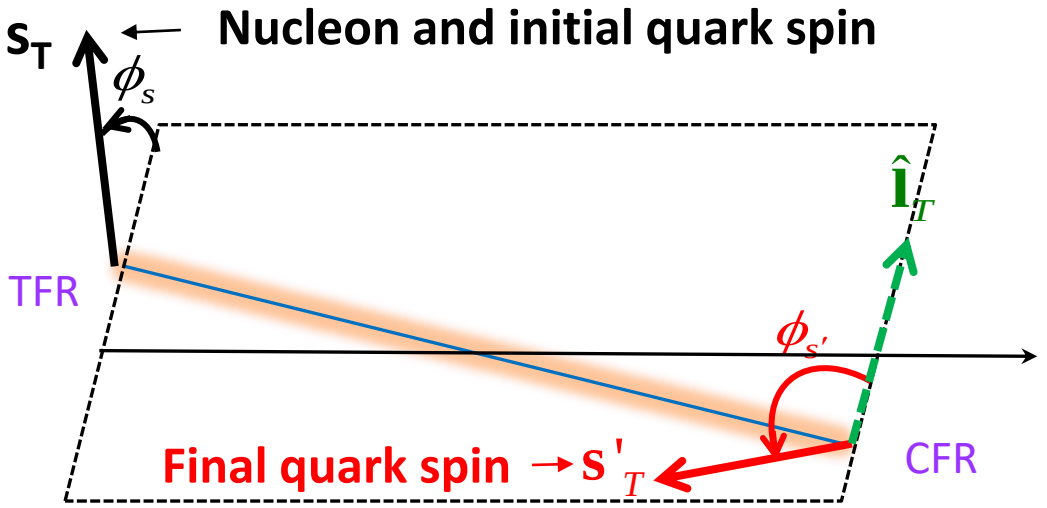
Quark transverse spin in hard *l-q* scattering

AK, Transversity workshop, Yerevan, 2009



Initial quark spin

Final quark spin



$$\text{QED: } lq \to l'q' \;\Rightarrow\; s'_T = D_{nn}(y)\,s_T, \quad D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}, \quad \phi_{s'} = \pi - \phi_s$$

$$\text{CFR: } \left[\mathbf{s}'_T \times \mathbf{p}_T\right] \propto \sin\left(\phi_h - \phi_{s'}\right) = -\sin\left(\phi_h + \phi_s\right) \text{ Collins-like azimuthal modulation}$$

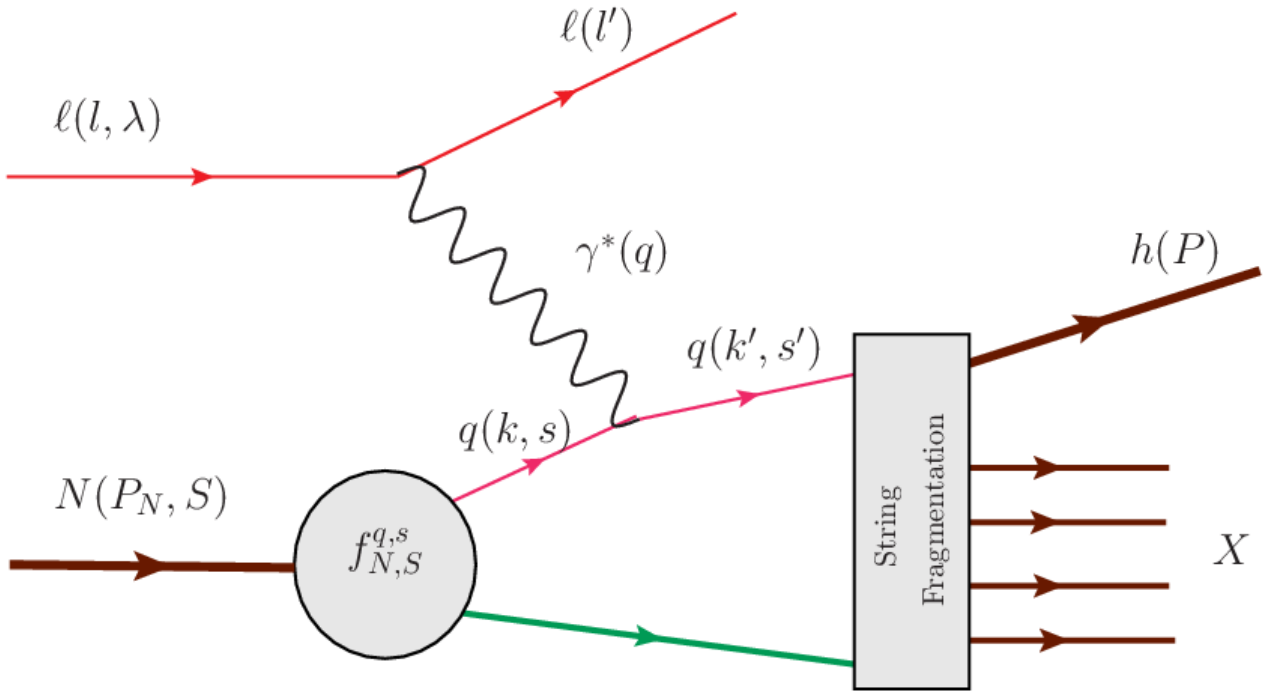
$$\text{TFR: } \left[\mathbf{s}_T \times \mathbf{p}_T\right] \propto \sin\left(\phi_h - \phi_s\right) \text{ Sivers-like azimuthal modulation}$$

If only one hadron in TFR of SIDIS is detected there is no final quark polarimetry.

→ STMD fracture functions depends on initial quark transverse polarization dependent fracture functions.

No Collins like modulation in TFR.

Hadronization Function in MC event generators (LEPTO, PYTHIA)



$$d\sigma^{lN\to lhX} = \sum_q f_q\left(x,\mathrm{k}_T^2\right)\otimes d\sigma^{lq\to lq}\otimes H_{h/N}^{q}\left(x,\mathbf{k_T};x_F,\mathbf{p_T^h}\right)$$

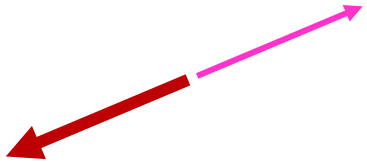
Hadronization Function modeled by Lund String Fragmentation

Quark dynamics in MC even generators

- Before

- After hard scattering

- Include k_T with isotropic azimuth



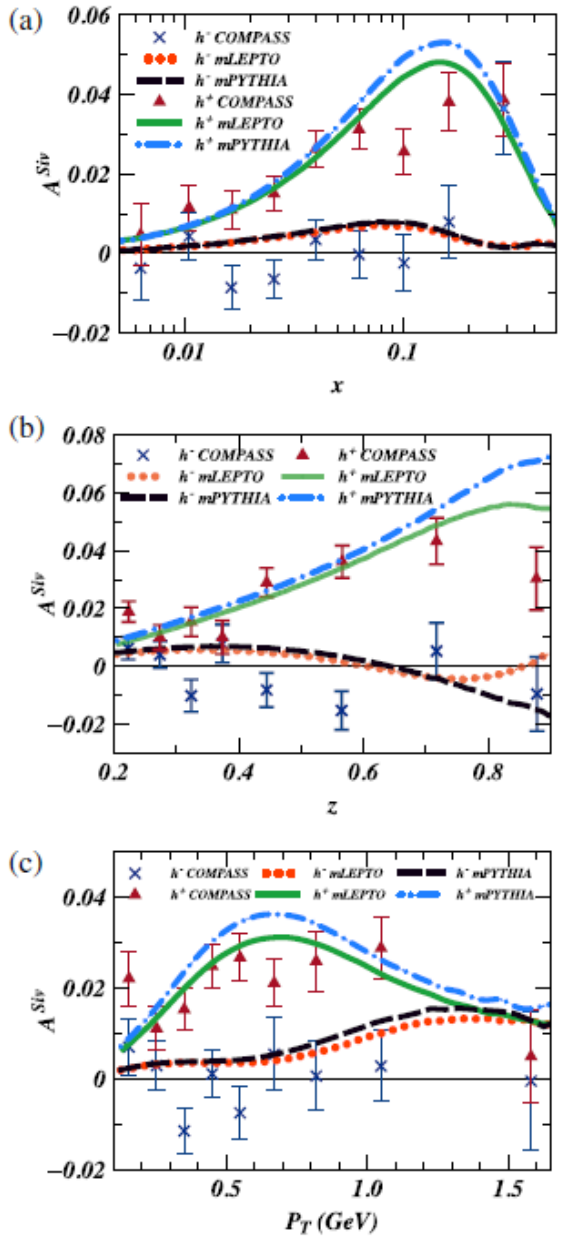
Modified, mLEPTO and mPYTHIA

Include k_T with anisotropic azimuthal modulation according Sivers function

$$d\sigma^{lN\to lhX} = \sum_q \left(f_q(x,\mathrm{k}_T^2) + \frac{\mathbf{k_T}\times\mathbf{S_T}}{M} f_{qT}^{\perp}(x,\mathrm{k}_T^2) \right) \otimes d\sigma^{lq\to lq} \otimes H_{h/N}^{q}(x,\mathbf{k_T};x_F,\mathbf{p_T}^h)$$

Sivers effect in the event generators

Matevosyan, AK, Aschenauer, Avakian, Thomas, PRD 92, 054028 (2015)



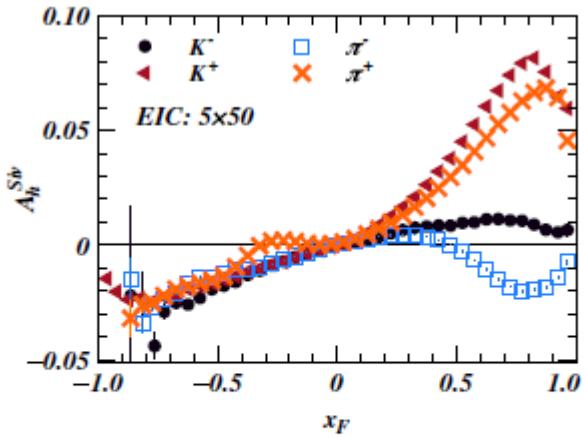


FIG. 13 (color online). EIC model SSAs for 5×50 SIDIS kinematics for charged pions and kaons versus x_F. The Sivers asymmetry is present both in the current and target fragmentation regions.

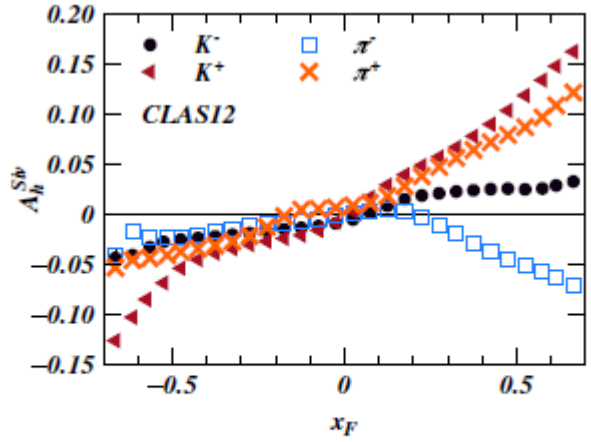
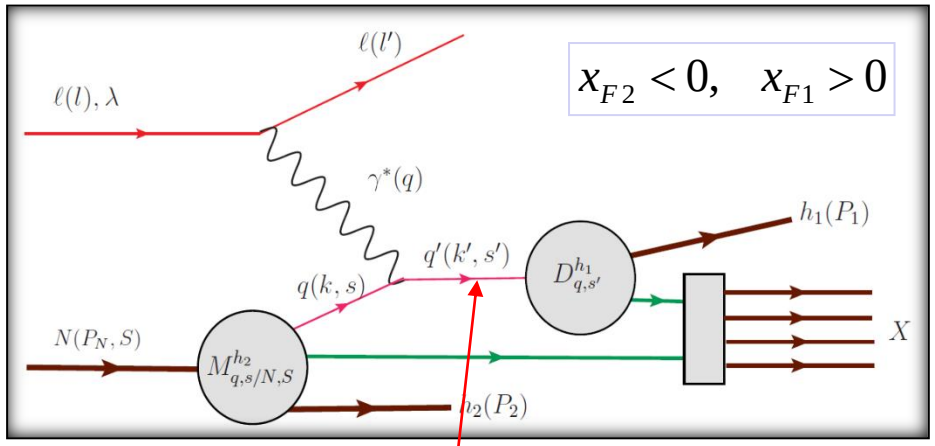


FIG. 17 (color online). Predictions for SSAs for charged pions and kaons versus x_F at CLAS12. The Sivers asymmetry is present both in the current and target fragmentation regions.

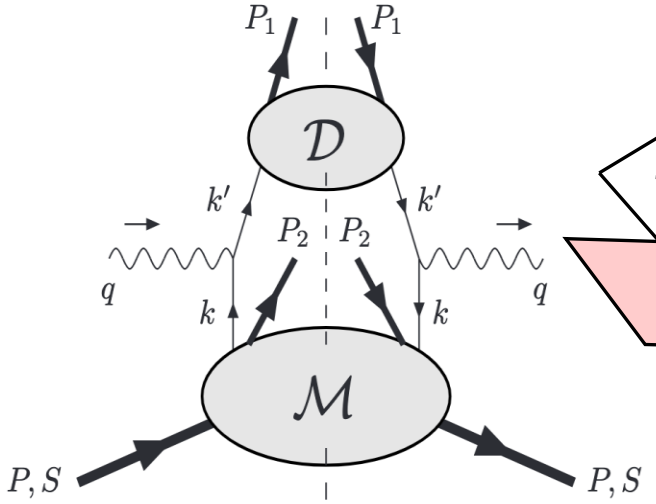
Only correlation of target $\mathbf{S}_T$ and struck quark $\mathbf{k}_T$ is explicitly parametrized using Sivers PDFs. Then this correlation is transferred to produced hadrons via unpolarized string fragmentation .

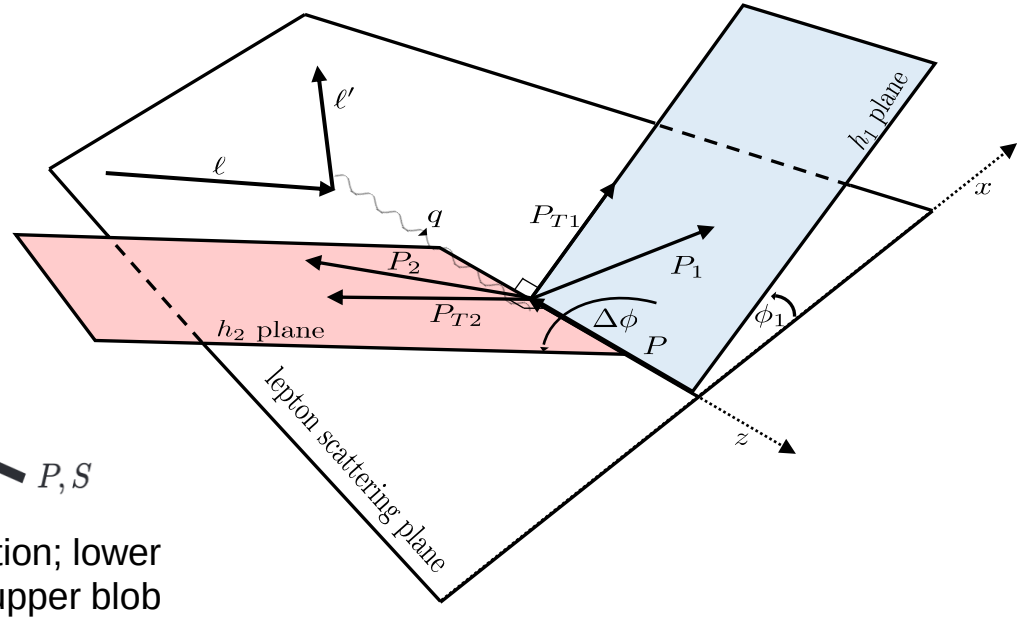
$$\tilde{u}^h_{1T}(x_B, \zeta_2, P^2_{T2}) = \int d^2k_T \left\{ \hat{u}^h_{1T} + \frac{m_2}{m_N} \frac{\mathbf{k}_T \cdot \mathbf{P}_{T2}}{P^2_{T2}} \hat{u}^{\perp}_{1T} \right\}$$

Double hadron production in DIS (DSIDIS): TFR & CFR



Access to final quark transverse momentum and polarization





Handbag diagram for dihadron production; lower blob contains Fracture Functions and upper blob contains the FFs.

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx dQ^2 d\phi_S dz d^2P_{T1} d\zeta d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

Unintegrated DSIDIS LO cross-section: accessing quark polarization

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\begin{array}{l}\hat{u}^{h_2}\otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{l}^{h_2}\otimes D_1^{h_1} \\ +\hat{t}^{h_2}\otimes \dfrac{\mathbf{p}_T\times\mathbf{s}'_T}{m_{h_1}} H_1^{h_1}\end{array}\right)$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\begin{array}{l}\sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \\ \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{LT}\right)\end{array}\right)$$

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms as in 1h SIDIS cross section.

$$D_{ll}(y) = \frac{y(2-y)}{1+(1-y)^2}$$

DSIDIS azimuthal modulations

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\begin{array}{l} \dfrac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1}\cos(2\phi_1) \\ +\dfrac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1}\cos(\phi_1+\phi_2) \\ +\left(\dfrac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \dfrac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1}\right)\cos(2\phi_2) \end{array}\right)$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$C[\hat{M}\cdot Dw] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1})\hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2})D_a(z,p_T^2)w$$

Structure functions $F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi),\ \text{with}\ \Delta\phi = \phi_1 - \phi_2$$

σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$+D_{nn}\left(\begin{array}{l} \dfrac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) \\ +\dfrac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) \\ +\left(\dfrac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \dfrac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1}\right)\sin(2\phi_2) \end{array}\right)$$

σ_{UT}

$$\sigma_{UT} = -\frac{P_{T1}}{m_N} F_{k1}^{\hat{u}_{1T}^{\perp}\cdot D_1} \sin(\phi_1 - \phi_S) - \left(\frac{P_{T2}}{m_2} F_0^{\hat{u}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{u}_{1T}^{\perp}\cdot D_1} \right) \sin(\phi_2 - \phi_S)$$

$$+D_{nn}(y) \Bigg[\left(\frac{P_{T1}}{m_1} F_{p1}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1m_2m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T1}^3}{2m_1m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1m_N^2} F_{kkp5}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + \phi_S)$$

$$+ \left(\frac{P_{T2}}{m_1} F_{p2}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}^2P_{T2}}{2m_1m_2m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1m_2m_N} F_{kp4}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T1}^2P_{T2}}{2m_1m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}}{m_1m_N^2} F_{kkp6}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_2 + \phi_S)$$

$$+ \frac{P_{T1}^3}{2m_1m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(3\phi_1 - \phi_S)$$

$$+ \left(\frac{P_{T2}^3}{2m_1m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(3\phi_2 - \phi_S)$$

$$+ \left(\frac{P_{T1}P_{T2}^2}{2m_1m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + 2\phi_2 - \phi_S)$$

$$- \frac{P_{T1}^2P_{T2}}{2m_1m_2m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(2\phi_1 - \phi_2 + \phi_S)$$

$$- \frac{P_{T1}P_{T2}^2}{2m_1m_2m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(\phi_1 - 2\phi_2 - \phi_S)$$

$$+ \frac{P_{T1}^2P_{T2}}{2m_1m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(2\phi_1 + \phi_2 - \phi_S) \Bigg]$$

σ_{LU}, σ_{LL}, σ_{LT}

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{LL} = F_0^{\hat{l}_1\cdot D_1}$$

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\hat{l}_{1T}^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\hat{l}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{l}_{1T}^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

A_{LU} asymmetry, 1

Anselmino, Barone and AK, PLB **713** (2012) 317

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1} \cos(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right) \cos(2\phi_2) \right)$$

$F_{\ldots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi)$, with $\Delta\phi = \phi_1 - \phi_2$

Choosing as independent angles $\Delta\phi$ and ϕ_2 $(\phi_1 = \Delta\phi + \phi_2)$ and integrating σ_{UU} over ϕ_2 we eliminate terms proportional to $D_{NN} \Rightarrow$

A_{LU} asymmetry, 2

$$\mathcal{A}_{LU} = -\frac{y(1-\frac{y}{2})}{(1-y+\frac{y^2}{2})}\frac{\mathcal{F}_{LU}^{\sin\Delta\phi}}{\mathcal{F}_{UU}}\sin\Delta\phi$$

$$= -\frac{|\boldsymbol{P}_{1\perp}||\boldsymbol{P}_{2\perp}|}{m_N m_2}\frac{y(1-\frac{y}{2})}{(1-y+\frac{y^2}{2})}\frac{\mathcal{C}[w_5\hat{l}_1^{\perp h}D_1]}{\mathcal{C}[\hat{u}_1 D_1]}\sin\Delta\phi.$$

$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = \frac{-\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)\sin(\Delta\phi)}{F_0^{\hat{u}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)}$$

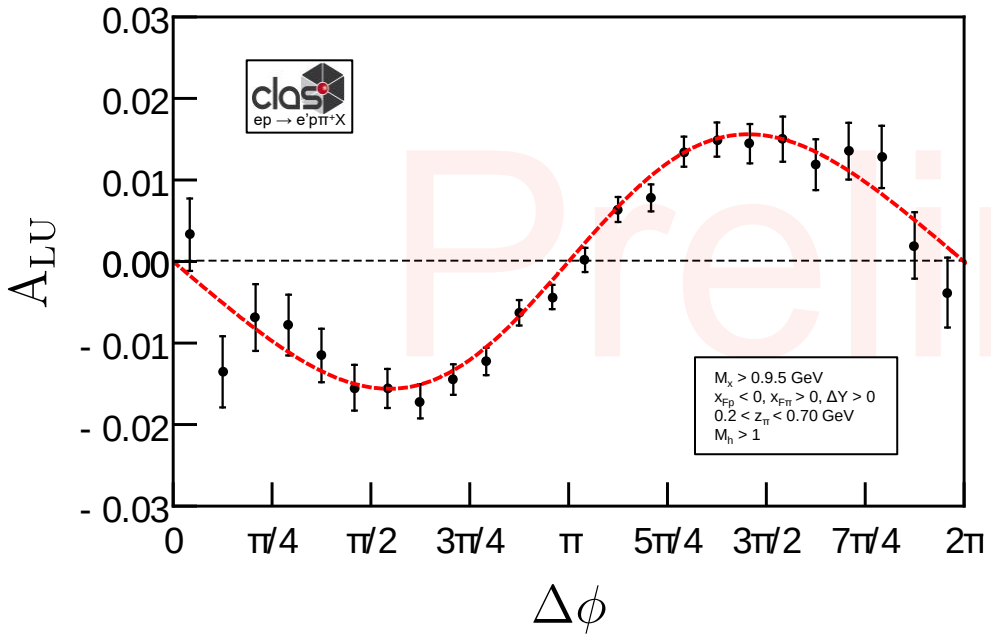
Expected leading-twist asymmetry is proportional to *sin(Δϕ)*

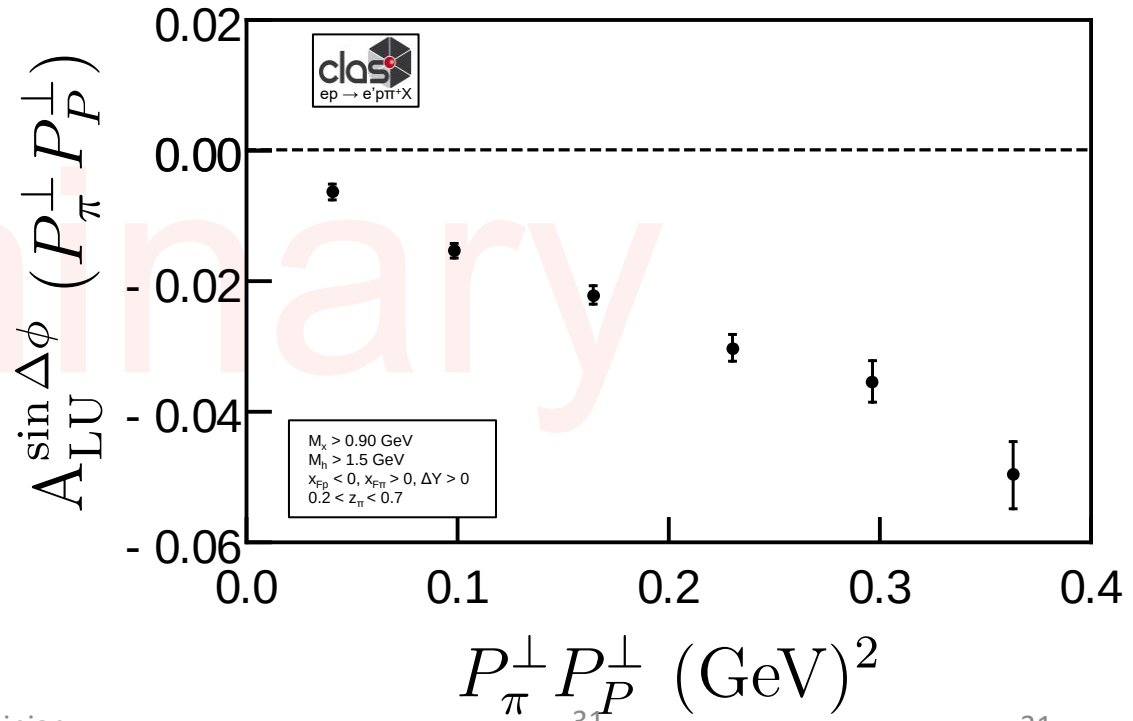
A_{LU} @ CLAS12, (1)

Timothy B. Hayward, H. Avakian and A.Kotzinian. (More details in T. Hayward's talk)

- Observed non-zero asymmetries are the first experimental confirmation of possible spin-orbit correlations between hadrons produced simultaneously in the CFR and TFR.
- Observed linear dependence on the product of transverse momenta is consistent with expectations.

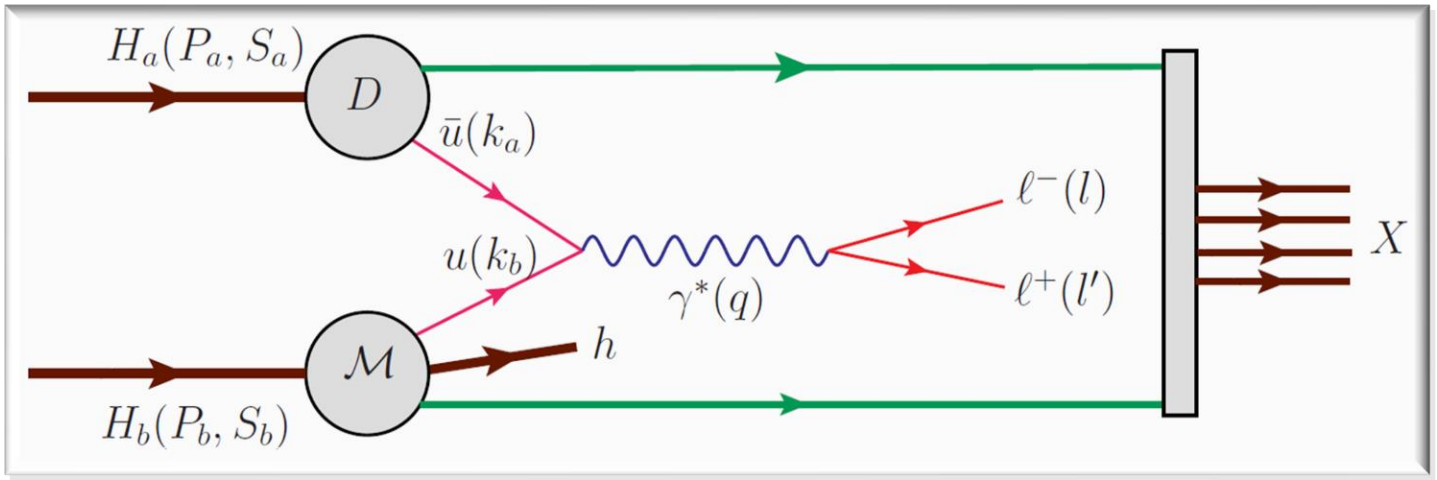
$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = -\frac{P_{T1} P_{T2}}{m_2 m_N} \frac{F_{k1}^{\hat{l}_1^{\perp h} \cdot D_1}}{F_0^{\hat{u} \cdot D_1}} \sin(\Delta\phi)$$





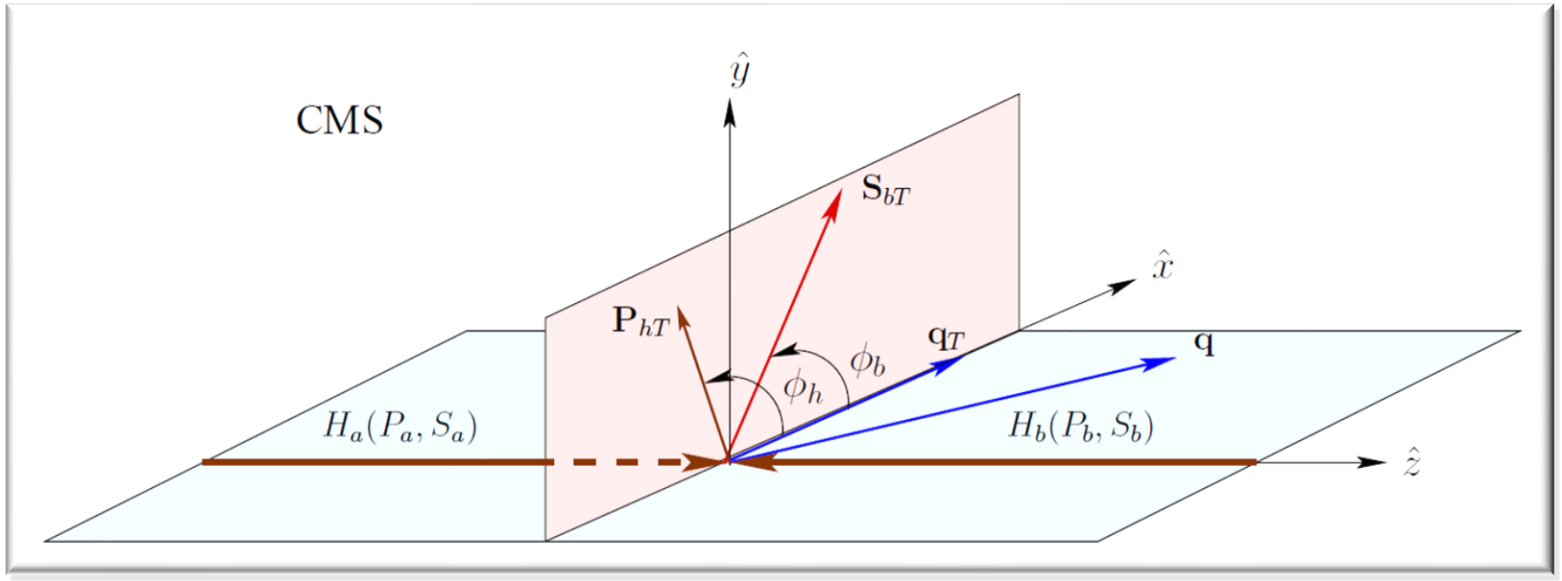
Preliminary

Polarized SIDY

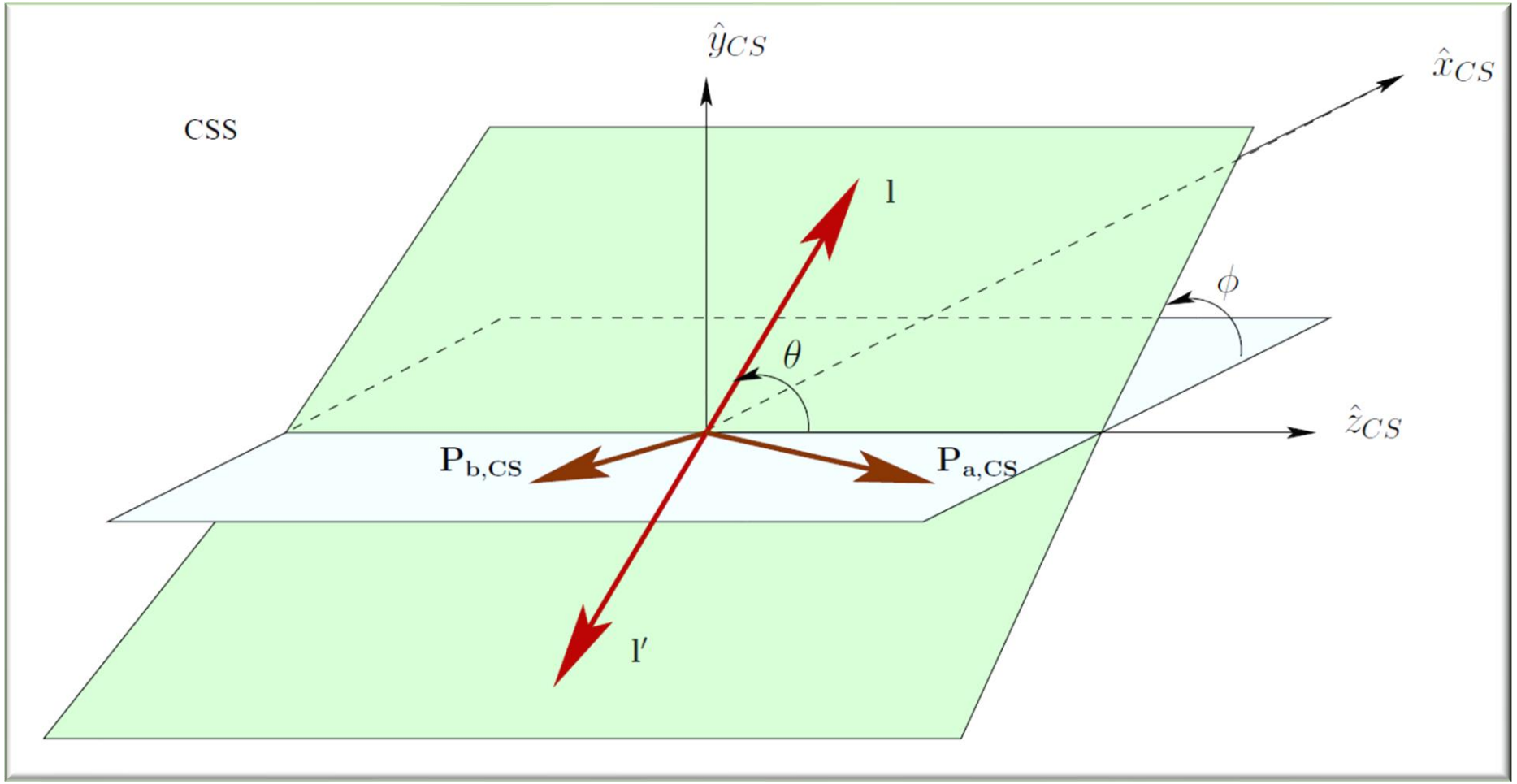


STAR@RHIC?

Kinematics as in Arnold.Metz.Schlegel, PhysRevD.79.034005



Collins-Soper system



SIDY cross section

$$\frac{d\sigma}{d^4q d\Omega d\zeta d^2P_T} = \frac{\alpha_{em}^2 x_a x_b}{2q^4}\frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{aT}\, d^2\vec{k}_{bT}\, \delta^{(2)}\left(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT}\right)\times$$

$$\times\left(\begin{array}{l} \left(1+\cos^2\theta\right)\left(\Phi^{q[\gamma^+]}\overline{\mathcal{M}}^{q[\gamma^-]} + \Phi^{q[\gamma^+\gamma_5]}\overline{\mathcal{M}}^{q[\gamma^-\gamma_5]}\right) \\ +\sin^2\theta\begin{pmatrix}\cos 2\phi\left(\delta^{i1}\delta^{j1} - \delta^{i2}\delta^{j2}\right) \\ +\sin 2\phi\left(\delta^{i1}\delta^{j2} + \delta^{i2}\delta^{j1}\right)\end{pmatrix}\Phi^{q[\mathrm{i}\sigma^{i+}\gamma_5]}\overline{\mathcal{M}}^{q[\mathrm{i}\sigma^{j-}\gamma_5]} \\ +\{\Phi\leftrightarrow\overline{\Phi}, \overline{\mathcal{M}}\leftrightarrow\mathcal{M}\} + \mathcal{O}(1/q) \end{array}\right)$$

$$= \frac{\alpha_{em}^2 x_a x_b}{2q^4}\left(\begin{array}{l} \sigma_{UU} + S_{bL}\sigma_{UL} + S_{bT}\sigma_{UT} \\ +S_{aL}\sigma_{LU} + S_{aL}S_{bL}\sigma_{LL} + S_{aL}S_{bT}\sigma_{LT} \\ +S_{aT}\sigma_{TU} + S_{aT}S_{bL}\sigma_{TL} + S_{aT}S_{bT}\sigma_{TT} \end{array}\right)$$

σ_{UU}

$$\sigma_{UU} = \left(1+\cos^2\theta\right) F_{UU}$$

$$-\sin^2\theta \left[\begin{array}{l} F_{UU}^{\cos(2\phi)} \cos\left(2\phi\right) \\ +F_{UU}^{\cos(2\phi-\phi_h)} \cos\left(2\phi-\phi_h\right) \\ +F_{UU}^{\cos(2\phi-2\phi_h)} \cos\left(2\phi-2\phi_h\right) \end{array} \right]$$

$$F_{UU} = F_0^{f_1\cdot\hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{u}_{1T}^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\hat{u}_{1T}^\perp} \hat{u}_{1T}^\perp$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U	*		*
	L			
	T			

σ_{UL}

Detected hadron originates from longitudinally polarized nucleon remnant (hadron b in reaction diagram).

$$\sigma_{UL} = \left(1+\cos^2\theta\right) F_{LU}^{\sin(\phi_h)} \sin\left(\phi_h\right)$$
$$-\sin^2\theta \left[\begin{array}{l} F_{LU}^{\sin(2\phi-2\phi_h)} \sin\left(2\phi-2\phi_h\right) \\ +F_{LU}^{\sin(2\phi-\phi_h)} \sin\left(2\phi-\phi_h\right) \\ +F_{LU}^{\sin(2\phi)} \sin(2\phi) \end{array} \right]$$

$$F_{UL}^{\sin(\phi_h)} = \frac{P_T q_T}{mM_b} F_{b2}^{f_1 \cdot \hat{l}_1^{\perp h}}$$

Nucleon Polarization \ Quark polarization	U	L	T
U		*	*
L			
T			

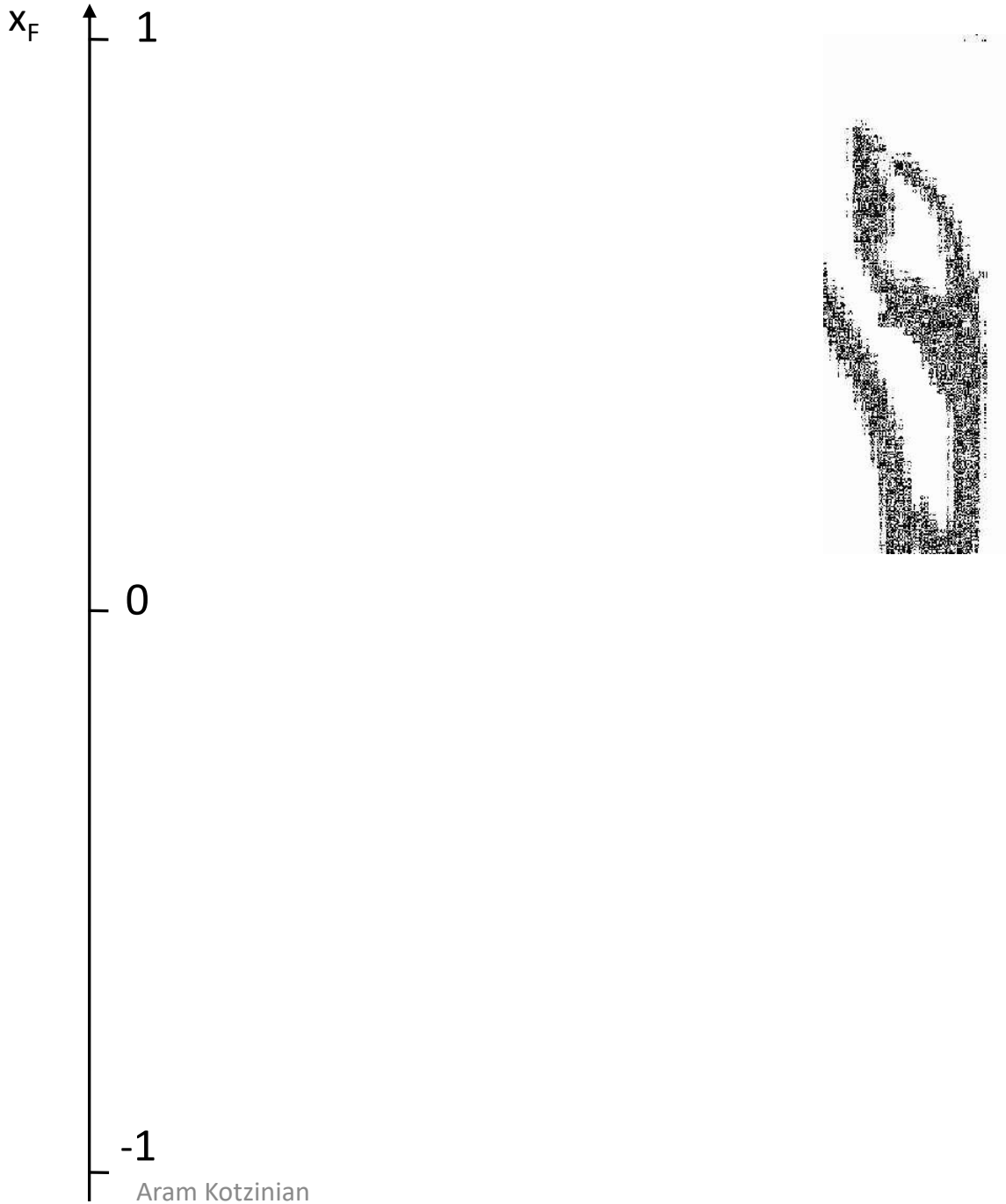
After integration over lepton scattering plane azimuth alangle ϕ

only first term survives and gives access to the fracture function $\hat{l}_1^{\perp h}$

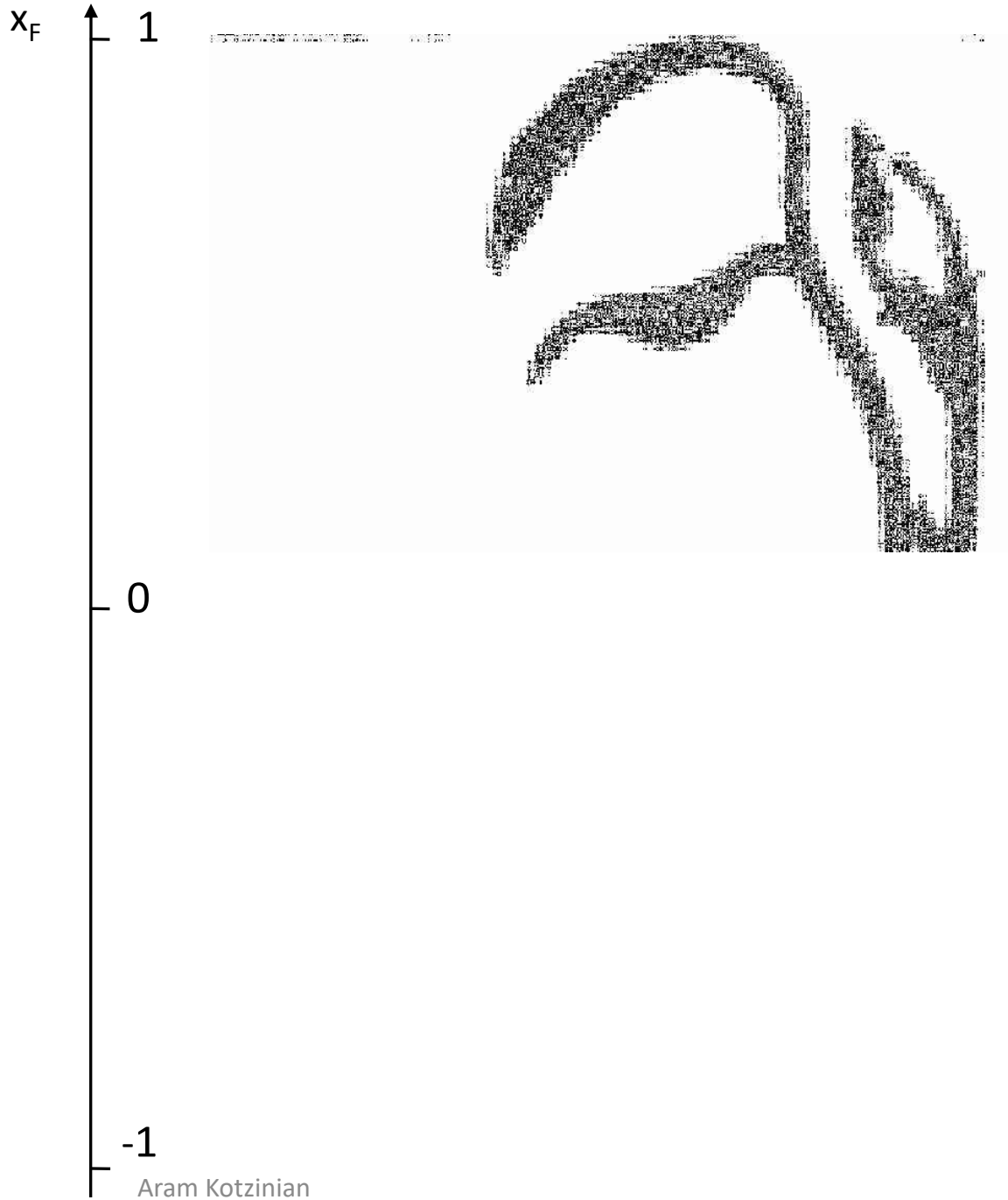
Conclusions

- Frac. Funs: A new members of the polarized TMDs family -- 16 LO STMD fracture functions
- For hadron produced in the TFR of SIDIS, only 4 k_T-integrated fracture functions of unpolarized and longitudinally polarized quarks are accessible at twist-two
  - SSA contains only a Sivers-type modulation $\sin(\phi_h-\phi_S)$ but no Collins-type $\sin(\phi_h+\phi_S)$ or $\sin(3\phi_h-\phi_S)$. The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long-range correlations between the struck quark polarization and P_T of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms. Access to all fracture functions. Access to all 16 STMD fracture functions.
  - The first b2b σ_{LU} asymmetry measurement at JLAB12 shows significant effect
- Polarized SIDY cross section ($p + p \rightarrow l^+l^- + h + X$) at LO contains 2 azimuthal independent, 20 lepton plane azimuthal angle independent and 52 lepton plane azimuthal angle dependent terms. In total – 74 terms. Access to all 16 STMD fracture functions.
- The ideal place to test the fracture functions formalism and measure these new nonperturbative objects are JLab12 (24), EIC facilities and COMPAS using Camera detector

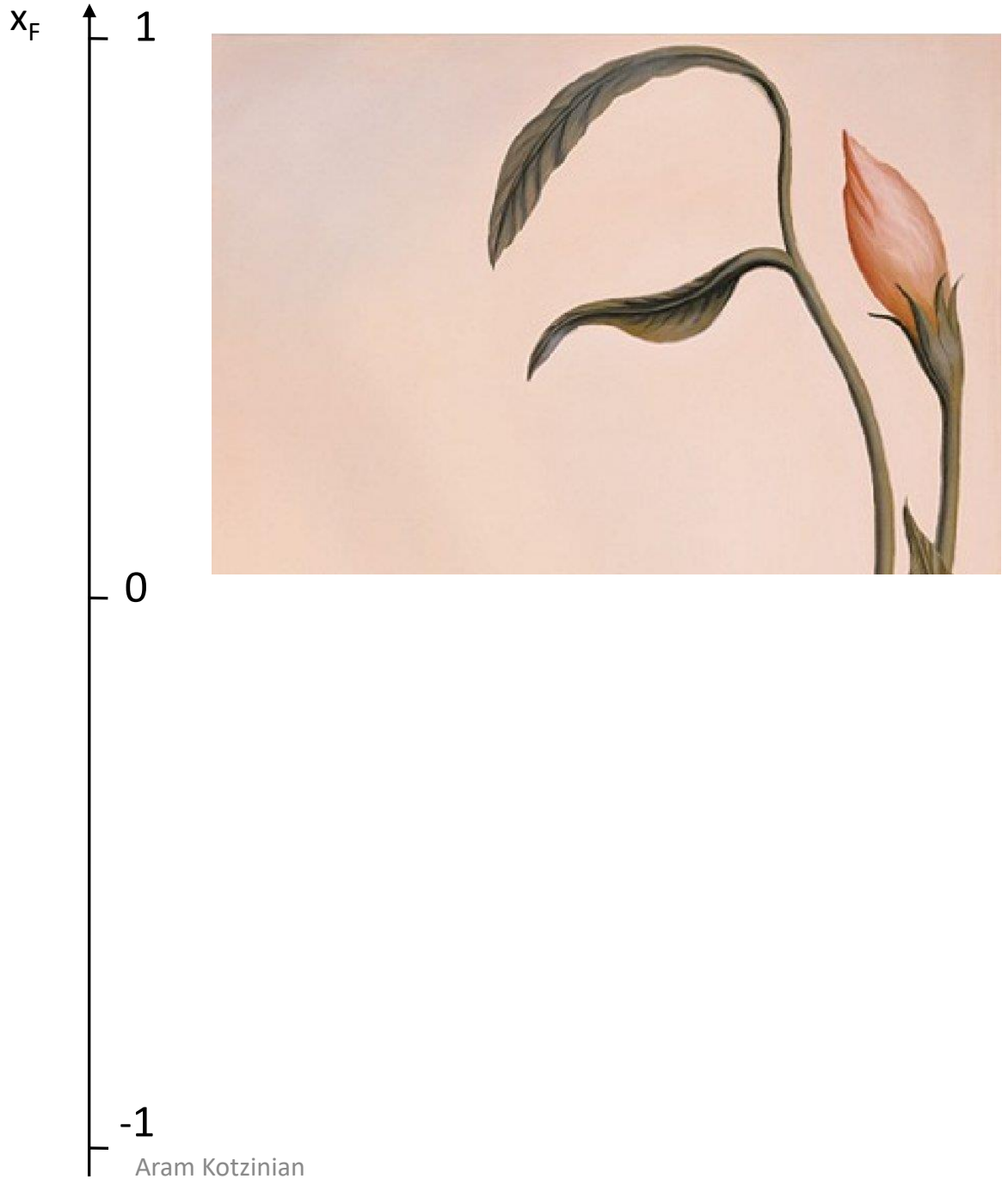
CFR



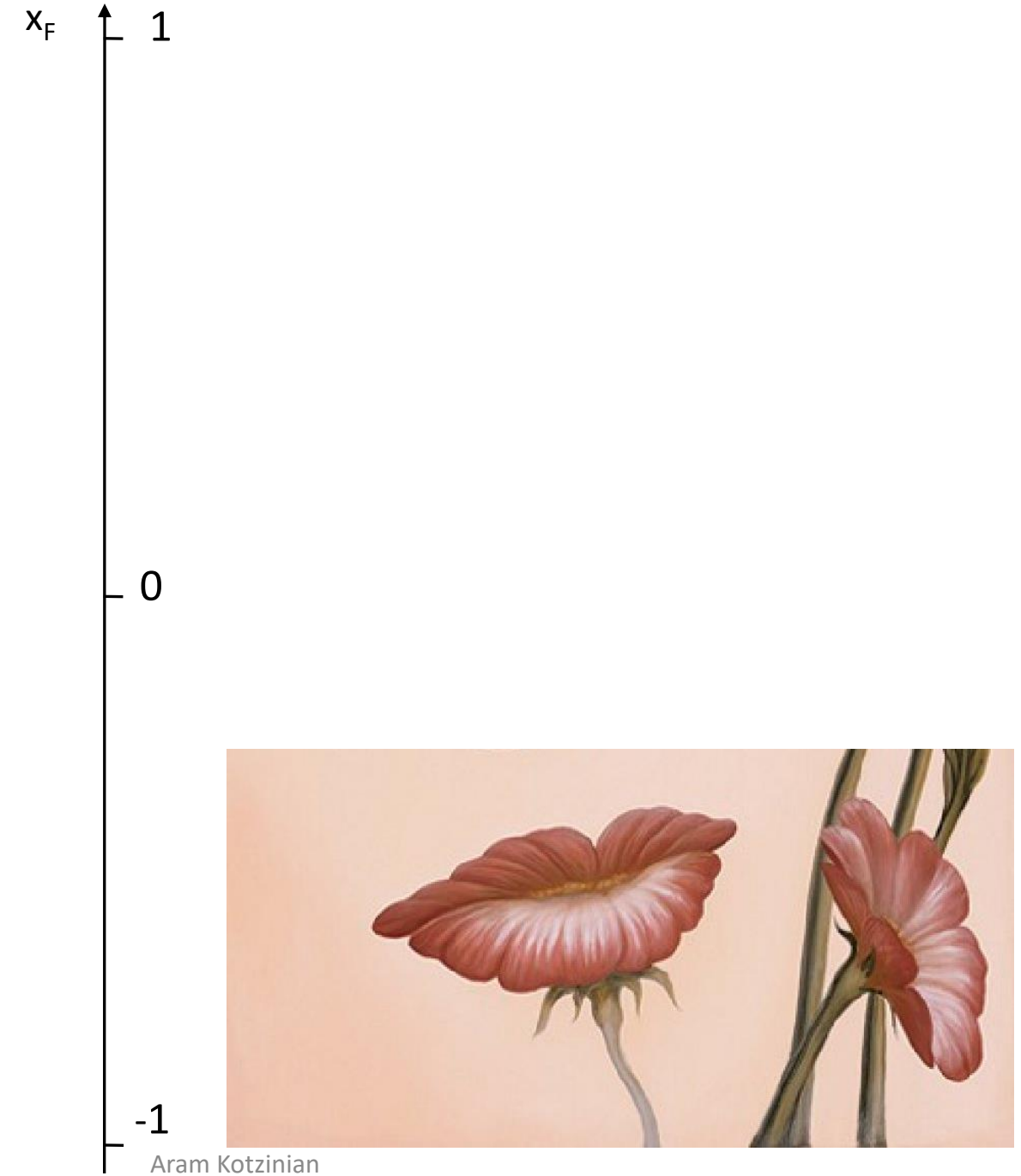
Larger phase space



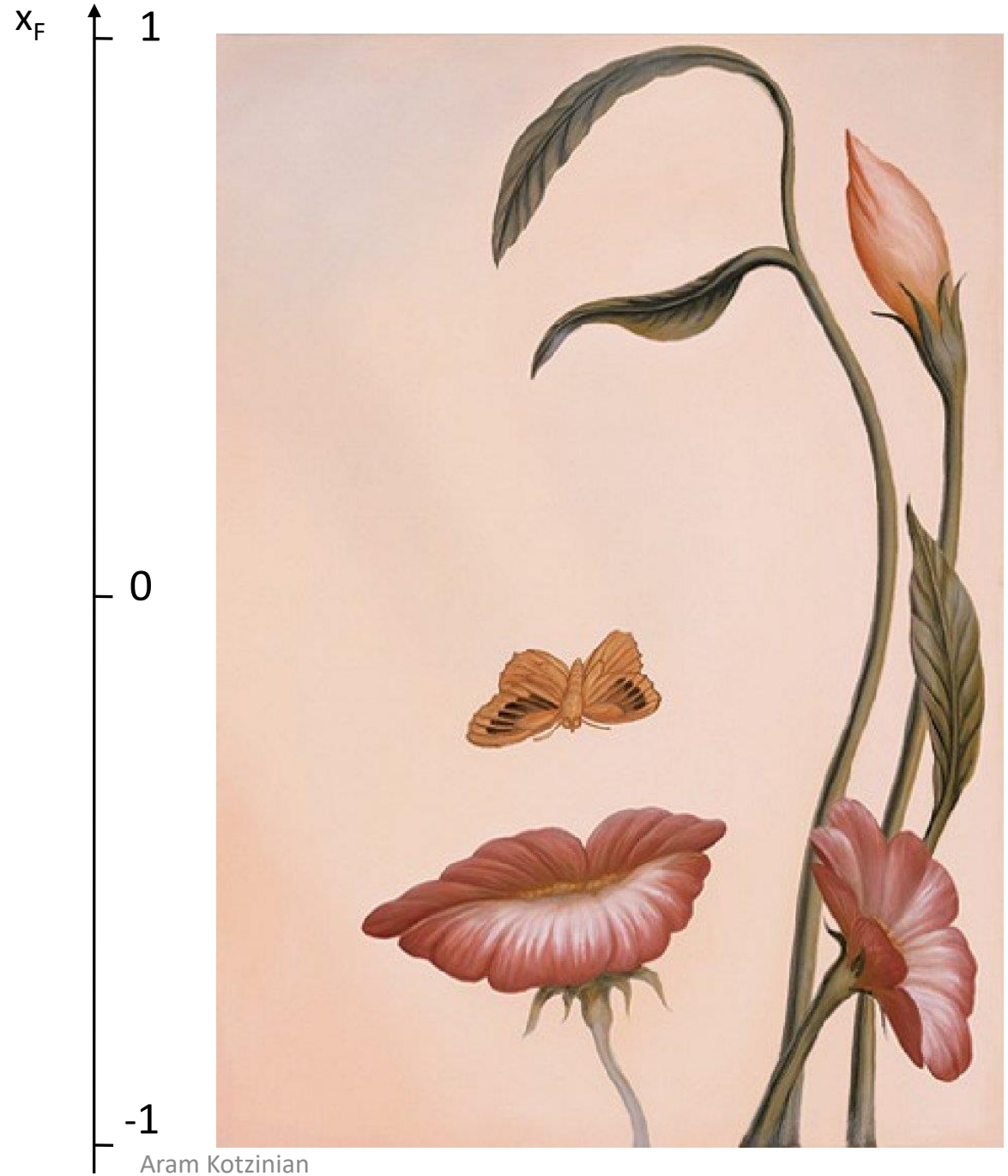
Better statistics



TFR with good statistics



Full picture



Additional slides : LO SIDY cross section

Convolutions & tensorial decomposition

$$C[\hat{M}\cdot Dw]=\sum_a e_a^2\int d^2k_T\,d^2p_T\,\delta^{(2)}(z\mathbf{k}_T+\mathbf{p}_T-\mathbf{P}_{T1})\hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2})D_a(z,p_T^2)w$$

$$C[\hat{M}\cdot D]=F_0^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i]=P_{T1}^iF_{k1}^{\hat{M}\cdot D}+P_{T2}^iF_{k2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dp^i]=P_{T1}^iF_{p1}^{\hat{M}\cdot D}+P_{T2}^iF_{p2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ik^j]=P_{T1}^iP_{T1}^jF_{kk1}^{\hat{M}\cdot D}+P_{T2}^iP_{T2}^jF_{kk2}^{\hat{M}\cdot D}+\delta^{ij}F_{kk3}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ip^j]=P_{T1}^iP_{T1}^jF_{kp1}^{\hat{M}\cdot D}+P_{T2}^iP_{T2}^jF_{kp2}^{\hat{M}\cdot D}+\left(P_{T1}^iP_{T2}^j-P_{T1}^jP_{T2}^i\right)F_{kp3}^{\hat{M}\cdot D}+\delta^{ij}F_{kp4}^{\hat{M}\cdot D}$$

$$\begin{aligned}C[\hat{M}\cdot Dk^ik^jp^k]=&P_{T1}^iP_{T1}^jP_{T1}^kF_{kkp1}^{\hat{M}\cdot D}+P_{T1}^iP_{T1}^jP_{T2}^kF_{kkp2}^{\hat{M}\cdot D}+P_{T2}^iP_{T2}^jP_{T2}^kF_{kkp3}^{\hat{M}\cdot D}\\&+P_{T2}^iP_{T2}^jP_{T1}^kF_{kkp4}^{\hat{M}\cdot D}+P_{T1}^k\delta^{ij}F_{kkp5}^{\hat{M}\cdot D}+P_{T2}^k\delta^{ij}F_{kkp6}^{\hat{M}\cdot D}\end{aligned}$$

where $F_{\dots}^{\hat{M}\cdot D}$ depend on $x,z,\zeta,P_{T1}^2,P_{T2}^2,\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)$

Structure functions

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k})-(\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{k2}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T1}\cdot\mathbf{k})-(\mathbf{P}_{T2}\cdot\mathbf{k})\mathbf{P}_{T1}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{kk1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{\left(-2(\mathbf{P}_{T1}\cdot\mathbf{k})^2+\mathbf{k}^2\mathbf{P}_{T1}^2\right)\mathbf{P}_{T2}^4+\left(2(\mathbf{P}_{T2}\cdot\mathbf{k})^2-\mathbf{k}^2\mathbf{P}_{T2}^2\right)\left(2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}{4(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk2}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{\left(2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)(\mathbf{P}_{T1}\cdot\mathbf{k})^2+\mathbf{P}_{T1}^2\left(\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2-(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\right)\mathbf{k}^2-(\mathbf{P}_{T2}\cdot\mathbf{k})^2\mathbf{P}_{T1}^4}{2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk3}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\,\frac{\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2+\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)\mathbf{k}^2-(\mathbf{P}_{T2}\cdot\mathbf{k})^2\mathbf{P}_{T1}^2-(\mathbf{P}_{T1}\cdot\mathbf{k})^2\mathbf{P}_{T2}^2}{2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2}\right]$$

$$F_{kp1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\left(\frac{\left(-2(\mathbf{P}_{T1}\cdot\mathbf{k})(\mathbf{P}_{T1}\cdot\mathbf{p})+(\mathbf{k}\cdot\mathbf{p})\mathbf{P}_{T1}^2\right)\mathbf{P}_{T2}^4}{4(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^4-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}+\frac{\left(2(\mathbf{P}_{T2}\cdot\mathbf{k})(\mathbf{P}_{T2}\cdot\mathbf{p})-(\mathbf{k}\cdot\mathbf{p})\mathbf{P}_{T2}^2\right)\left(2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}{4(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^4-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right)\right]$$

σ_{UU}

$$\sigma_{UU} = \left(1+\cos^2\theta\right) F_{UU}$$

$$-\sin^2\theta \left[\begin{array}{l} F_{UU}^{\cos(2\phi)} \cos\left(2\phi\right) \\ +F_{UU}^{\cos(2\phi-\phi_h)} \cos\left(2\phi-\phi_h\right) \\ +F_{UU}^{\cos(2\phi-2\phi_h)} \cos\left(2\phi-2\phi_h\right) \end{array} \right]$$

$$F_{UU} = F_0^{f_1\cdot\hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{u}_{1T}^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\hat{u}_{1T}^\perp} \hat{u}_{1T}^\perp$$

Nucleon Polarization \ Quark polarization	U	L	T
U	*		*
L			
T			

σ_{UL}

$$\sigma_{UL} = \left(1+\cos^2\theta\right) F_{UL}^{\sin(\phi_h)} \sin\left(\phi_h\right)$$
$$-\sin^2\theta \left[\begin{array}{l} F_{UL}^{\sin(2\phi)} \sin\left(2\phi\right) \\ +F_{UL}^{\sin(2\phi-\phi_h)} \sin\left(2\phi-\phi_h\right) \\ +F_{UL}^{\sin(2\phi-2\phi_h)} \sin\left(2\phi-2\phi_h\right) \end{array} \right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L	*		*
	T			

σ_{UT}

$$\sigma_{UT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{UT}^{\sin(\phi_b-\phi_h)}\sin\left(\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(\phi_b)}\sin\left(\phi_b\right) \end{array}\right]$$

$$-\sin^2\theta\left[\begin{array}{l} F_{UT}^{\sin(2\phi+\phi_b)}\sin\left(2\phi+\phi_b\right) \\ +F_{UT}^{\sin(2\phi-\phi_b+\phi_h)}\sin\left(2\phi-\phi_b+\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-\phi_h)}\sin\left(2\phi+\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)}\sin\left(2\phi+\phi_b-3\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)}\sin\left(2\phi-\phi_b-2\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)}\sin\left(2\phi+\phi_b-2\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b-\phi_h)}\sin\left(2\phi-\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b)}\sin\left(2\phi-\phi_b\right) \end{array}\right]$$

Nucleon Polarization \ Quark polarization	U	L	T
U			
L			
T	*		*

σ_{LU}

$$\sigma_{LU} = \left(1+\cos^2\theta\right) F_{LU}^{\sin(\phi_h)} \sin\left(\phi_h\right)$$
$$-\sin^2\theta \left[\begin{array}{l} F_{LU}^{\sin(2\phi-2\phi_h)} \sin\left(2\phi-2\phi_h\right) \\ +F_{LU}^{\sin(2\phi-\phi_h)} \sin\left(2\phi-\phi_h\right) \\ +F_{LU}^{\sin(2\phi)} \sin(2\phi) \end{array} \right]$$

$$F_{LU}^{\sin(\phi_h)} = \frac{P_T q_T}{mM_b} F_{b2}^{f_1 \cdot \hat{l}_1^{\perp h}}$$

Nucleon Polarization \ Quark polarization	U	L	T
U		*	*
L			
T			

After integration over azimuthal angle of lepton scattering plane only first term survives and gives access to $\hat{l}_1^{\perp h}$ fracture function

σ_{LL}

$$\sigma_{LL} = \left(1+\cos^2\theta\right) F_{LL}$$

$$+\sin^2\theta \left[\begin{array}{l} F_{LL}^{\cos\left(2\phi-2\phi_h\right)} \cos\left(2\phi-2\phi_h\right) \\ +F_{LL}^{\cos\left(2\phi-\phi_h\right)} \cos\left(2\phi-\phi_h\right) \\ +F_{LL}^{\cos\left(2\phi\right)} \cos(2\phi) \end{array} \right]$$

Nucleon Polarization	Quark polarization: U	L	T
U			
L		*	*
T			

σ_{LT}

$$\sigma_{LT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{LT}^{\cos\left(\phi_b-\phi_h\right)}\cos\left(\phi_b-\phi_h\right) \\ +F_{LT}^{\cos\left(\phi_b\right)}\cos\left(\phi_b\right) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{LT}^{\cos\left(2\phi-\phi_b+\phi_h\right)}\cos\left(2\phi-\phi_b+\phi_h\right) \\ +F_{LT}^{\cos\left(2\phi+\phi_b\right)}\cos\left(2\phi+\phi_b\right)+ \\ F_{LT}^{\cos\left(2\phi+\phi_b-\phi_h\right)}\cos\left(2\phi+\phi_b-\phi_h\right) \\ +F_{LT}^{\cos\left(2\phi+\phi_b-3\phi_h\right)}\cos\left(2\phi+\phi_b-3\phi_h\right) \\ +F_{LT}^{\cos\left(2\phi-\phi_b-2\phi_h\right)}\cos\left(2\phi-\phi_b-2\phi_h\right) \\ +F_{LT}^{\cos\left(2\phi+\phi_b-2\phi_h\right)}\cos\left(2\phi+\phi_b-2\phi_h\right) \\ +F_{LT}^{\cos\left(2\phi-\phi_b-\phi_h\right)}\cos\left(2\phi-\phi_b-\phi_h\right) \\ +F_{LT}^{\cos\left(2\phi-\phi_b\right)}\cos\left(2\phi-\phi_b\right) \end{array}\right]$$

Nucleon Polarization \ Quark polarization	U	L	T
U			
L			
T		*	*

σ_{TU}

$$\sigma_{TU} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{TU}^{\sin\left(\phi_a-\phi_h\right)}\sin\left(\phi_a-\phi_h\right) \\ +F_{TU}^{\sin\left(\phi_a\right)}\sin\left(\phi_a\right) \\ +F_{TU}^{\sin\left(\phi_a+\phi_h\right)}\sin\left(\phi_a+\phi_h\right) \\ +F_{TU}^{\sin\left(\phi_a-2\phi_h\right)}\sin\left(\phi_a-2\phi_h\right) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{TU}^{\sin\left(2\phi-\phi_a-\phi_h\right)}\sin\left(2\phi-\phi_a-\phi_h\right) \\ +F_{TU}^{\sin\left(2\phi-\phi_a\right)}\sin\left(2\phi-\phi_a\right) \\ +F_{TU}^{\sin\left(2\phi+\phi_a-3\phi_h\right)}\sin\left(2\phi+\phi_a-3\phi_h\right)+ \\ F_{TU}^{\sin\left(2\phi+\phi_a-\phi_h\right)}\sin\left(2\phi+\phi_a-\phi_h\right) \\ +F_{TU}^{\sin\left(2\phi+\phi_a-2\phi_h\right)}\sin\left(2\phi+\phi_a-2\phi_h\right) \\ +F_{TU}^{\sin\left(2\phi+\phi_a\right)}\sin\left(2\phi+\phi_a\right) \end{array}\right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U	*	*	*
	L			
	T			

σ_{TL}

$$\sigma_{TL} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{TL}^{\cos\left(\phi_a-\phi_h\right)}\cos\left(\phi_a-\phi_h\right) \\ +F_{TL}^{\cos\left(\phi_a\right)}\cos\left(\phi_a\right) \\ +F_{TL}^{\cos\left(\phi_a+\phi_h\right)}\cos\left(\phi_a+\phi_h\right) \\ +F_{TL}^{\cos\left(\phi_a-2\phi_h\right)}\cos\left(\phi_a-2\phi_h\right) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{TL}^{\cos\left(2\phi+\phi_a-3\phi_h\right)}\cos\left(2\phi+\phi_a-3\phi_h\right) \\ +F_{TL}^{\cos\left(2\phi+\phi_a-2\phi_h\right)}\cos\left(2\phi+\phi_a-2\phi_h\right) \\ +F_{TL}^{\cos\left(2\phi+\phi_a\right)}\cos\left(2\phi+\phi_a\right) \\ +F_{TL}^{\cos\left(2\phi-\phi_a-\phi_h\right)}\cos\left(2\phi-\phi_a-\phi_h\right) \\ +F_{TL}^{\cos\left(2\phi-\phi_a\right)}\cos\left(2\phi-\phi_a\right) \end{array}\right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L		*	*
	T			

σ_{TT}

$$\sigma_{TT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{TT}^{\cos(\phi_a-\phi_b)}\cos\left(\phi_a-\phi_b\right) \\ +F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)}\cos\left(\phi_a+\phi_b-2\phi_h\right) \\ +F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)}\cos\left(\phi_a-\phi_b+\phi_h\right) \\ +F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)}\cos\left(\phi_a+\phi_b-\phi_h\right) \\ +F_{TT}^{\cos(\phi_a+\phi_b)}\cos\left(\phi_a+\phi_b\right) \\ +F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)}\cos\left(\phi_a-\phi_b-\phi_h\right) \end{array}\right] + \sin^2\theta \left[\begin{array}{l} F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)}\cos\left(2\phi-\phi_a-\phi_b-\phi_h\right) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)}\cos\left(2\phi-\phi_a-\phi_b+\phi_h\right) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b)}\cos\left(2\phi+\phi_a+\phi_b\right) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)}\cos\left(2\phi+\phi_a-\phi_b+\phi_h\right)+ \\ F_{TT}^{\cos(2\phi-\phi_a+\phi_b)}\cos\left(2\phi-\phi_a+\phi_b\right) \\ +F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)}\cos\left(2\phi-\phi_a+\phi_b-2\phi_h\right) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)}\cos\left(2\phi+\phi_a+\phi_b-4\phi_h\right) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)}\cos\left(2\phi+\phi_a-\phi_b-3\phi_h\right) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b)}\cos\left(2\phi-\phi_a-\phi_b\right) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)}\cos\left(2\phi+\phi_a-\phi_b-2\phi_h\right) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)}\cos\left(2\phi+\phi_a+\phi_b-2\phi_h\right) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)}\cos\left(2\phi+\phi_a-\phi_b-\phi_h\right) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b)}\cos\left(2\phi+\phi_a-\phi_b\right) \end{array}\right]$$

Nucleon Polarization	Quark polarization: U	L	T
U			
L			
T	*	*	*

Structure functions of σ_{UU}, σ_{UL}, σ_{UT}

$$F_{UU} = F_0^{f_1\cdot\hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\hat{u}_{1T}^\perp}$$

$$F_{UL}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{f_1\cdot\hat{l}_1^{\perp h}}$$

$$F_{UL}^{\sin(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{l}_{1T}^\perp}$$

$$F_{UL}^{\sin(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\hat{l}_{1T}^h}$$

$$F_{UL}^{\sin(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\hat{l}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\hat{l}_{1T}^\perp}$$

$$F_{UT}^{\sin(\phi_b-\phi_h)} = \frac{P_T}{m} F_0^{f_1\cdot\hat{t}_1^h} + \frac{P_T}{M_b} F_{b1}^{f_1\cdot\hat{t}_1^\perp}, \qquad F_{UT}^{\sin(\phi_b)} = \frac{q_T}{M_b} F_{b2}^{f_1\cdot\hat{t}_1^\perp}$$

$$F_{UT}^{\sin(2\phi-\phi_b+\phi_h)} = -\frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{t}_1^{\perp h}}, \qquad F_{UT}^{\sin(2\phi+\phi_b)} = \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}, \qquad F_{UT}^{\sin(2\phi+\phi_b-\phi_h)} = \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)} = \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}, \qquad F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{UT}^{\sin(2\phi-\phi_b-\phi_h)} = \left(\begin{array}{l} \frac{P_T}{M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}} + \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} \\ + \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T}{mM_a M_b} F_{ab4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T}{M_a M_b^2} F_{abb5}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} \end{array} \right)$$

$$F_{UT}^{\sin(2\phi-\phi_b)} = \left(\begin{array}{l} \frac{q_T}{M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}} + \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} \\ - \frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{q_T}{M_a M_b^2} F_{abb6}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} \end{array} \right)$$

Structure functions of σ_{LU}, σ_{LL}

$$F_{LU}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{g_{1L}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{LU}^{\sin(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_{1L}^{\perp}\cdot\hat{u}_{1T}^{h}} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_{1L}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

$$F_{LU}^{\sin(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_{1L}^{\perp}\cdot\hat{u}_{1T}^{h}}$$

$$F_{LU}^{\sin(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

$$F_{LL} = F_0^{g_{1L}\cdot\hat{l}_{1L}}$$

$$F_{LL}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_{1L}^{\perp}\cdot\hat{l}_{1T}^{h}} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_{1L}^{\perp}\cdot\hat{l}_{1T}^{\perp}}$$

$$F_{LL}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_{1L}^{\perp}\cdot\hat{l}_{1T}^{h}}$$

$$F_{LL}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot\hat{l}_{1T}^{\perp}}$$

Structure functions of σ_{LT}

$$F_{LT}^{\cos(\phi_b-\phi_h)} = \frac{P_T}{m} F_0^{g_{1L}\cdot\hat{t}_{1L}^{h}} + \frac{P_T}{M_b} F_{b1}^{g_{1L}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{LT}^{\cos(\phi_b)} = \frac{q_T}{M_b} F_{b2}^{g_{1L}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b+\phi_h)} = -\frac{P_T q_T^2}{2mM_aM_b} F_{ab2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{LT}^{\cos(2\phi+\phi_b)} = \frac{q_T^3}{2M_aM_b^2} F_{abb3}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-\phi_h)} = \frac{P_T q_T^2}{2M_aM_b^2} F_{abb4}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-3\phi_h)} = \frac{P_T^3}{2m^2M_a} F_{a1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_aM_b^2} F_{abb1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m^2M_a} F_{a2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^2 q_T}{2M_aM_b^2} F_{abb2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b-\phi_h)} = \left(\begin{array}{l} \frac{P_T}{M_a} F_{a1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}} + \frac{P_T^3}{2m^2M_a} F_{a1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_aM_b^2} F_{abb1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T q_T^2}{2mM_aM_b} F_{ab2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} \\ + \frac{P_T q_T^2}{2M_aM_b^2} F_{abb4}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T}{mM_aM_b} F_{ab4}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T}{M_aM_b^2} F_{abb5}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} \end{array} \right)$$

$$F_{LT}^{\cos(2\phi-\phi_b)} = \left(\begin{array}{l} \frac{q_T}{M_a} F_{a2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}} + \frac{P_T^2 q_T}{2m^2M_a} F_{a2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{q_T^3}{2M_aM_b^2} F_{abb3}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} - \frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} \\ + \frac{P_T^2 q_T}{2M_aM_b^2} F_{abb2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{q_T}{M_aM_b^2} F_{abb6}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} \end{array} \right)$$

Structure functions of σ_{TU}

$$F_{TU}^{\sin(\phi_a-\phi_h)} = -\frac{P_T}{M_a} F_{a1}\, f_{1T}^{\perp}\cdot\hat{u}_1 - \frac{P_T q_T^2}{2mM_aM_b} F_{ab2}\, g_{1T}\cdot\hat{u}_{1L}^{\perp h} - \frac{P_T}{mM_aM_b} F_{ab4}\, g_{1T}\cdot\hat{u}_{1L}^{\perp h}$$

$$F_{TU}^{\sin(\phi_a)} = -\frac{q_T}{M_a} F_{a2}\, f_{1T}^{\perp}\cdot\hat{u}_1 + \frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}\, g_{1T}\cdot\hat{u}_{1L}^{\perp h}$$

$$F_{TU}^{\sin(\phi_a+\phi_h)} = \frac{P_T q_T^2}{2mM_aM_b} F_{ab2}\, g_{1T}\cdot\hat{u}_{1L}^{\perp h}$$

$$F_{TU}^{\sin(\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}\, g_{1T}\cdot\hat{u}_{1L}^{\perp h}$$

$$F_{TU}^{\sin(2\phi-\phi_a-\phi_h)} = -\frac{P_T}{M_b} F_{b1}\, h_1\cdot\hat{u}_{1T}^{\perp} - \frac{P_T}{m} F_0\, h_1\cdot\hat{u}_{1T}^{h}$$

$$F_{TU}^{\sin(2\phi-\phi_a)} = -\frac{q_T F_{b2}\, h_1\cdot\hat{u}_{1T}^{\perp}}{M_b}$$

$$F_{TU}^{\sin(2\phi+\phi_a-3\phi_h)} = -\frac{P_T^3}{2mM_a^2} F_{aa1}\, h_{1T}^{\perp}\cdot\hat{u}_{1T}^{h} - \frac{P_T^3}{2M_a^2M_b} F_{aab1}\, h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}$$

$$F_{TU}^{\sin(2\phi+\phi_a-\phi_h)} = -\frac{P_T q_T^2}{2mM_a^2} F_{aa2}\, h_{1T}^{\perp}\cdot\hat{u}_{1T}^{h} - \frac{P_T q_T^2}{2M_a^2M_b} F_{aab4}\, h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}$$

$$F_{TU}^{\sin(2\phi+\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2M_a^2M_b} F_{aab2}\, h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}$$

$$F_{TU}^{\sin(2\phi+\phi_a)} = -\frac{q_T^3}{2M_a^2M_b} F_{aab3}\, h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}$$

Structure functions of σ_{TL}

$$F_{TL}^{\cos(\phi_a-\phi_h)} = \frac{P_T}{M_a} F_{a1}\, g_{1T}\cdot \hat{l}_{1L} - \frac{P_T q_T^2}{2mM_aM_b} F_{ab2}\, f_{1T}^{\perp}\cdot \hat{u}_{1L}^{\perp h} - \frac{P_T}{mM_aM_b} F_{ab4}\, f_{1T}^{\perp}\cdot \hat{u}_{1L}^{\perp h}$$

$$F_{TL}^{\cos(\phi_a)} = \frac{q_T}{M_a} F_{a2}\, g_{1T}\cdot \hat{l}_{1L} + \frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}\, f_{1T}^{\perp}\cdot \hat{u}_{1L}^{\perp h}$$

$$F_{TL}^{\cos(\phi_a+\phi_h)} = \frac{P_T q_T^2}{2mM_aM_b} F_{ab2}\, f_{1T}^{\perp}\cdot \hat{u}_{1L}^{\perp h}$$

$$F_{TL}^{\cos(\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2mM_aM_b} F_{ab3}\, f_{1T}^{\perp}\cdot \hat{u}_{1L}^{\perp h}$$

$$F_{TL}^{\cos(2\phi+\phi_a-3\phi_h)} = \frac{P_T^3}{2mM_a^2} F_{aa1}\, h_{1T}^{\perp}\cdot \hat{l}_{1T}^{h} + \frac{P_T^3}{2M_a^2M_b} F_{aab1}\, h_{1T}^{\perp}\cdot \hat{l}_{1T}^{\perp}$$

$$F_{TL}^{\cos(2\phi+\phi_a-2\phi_h)} = \frac{P_T^2 q_T}{2M_a^2M_b} F_{aab2}\, h_{1T}^{\perp}\cdot \hat{l}_{1T}^{\perp}$$

$$F_{TL}^{\cos(2\phi+\phi_a)} = \frac{q_T^3}{2M_a^2M_b} F_{aab3}\, h_{1T}^{\perp}\cdot \hat{l}_{1T}^{\perp}$$

$$F_{TL}^{\cos(2\phi-\phi_a-\phi_h)} = \frac{P_T}{M_b} F_{b1}\, h_1\cdot \hat{l}_{1T}^{\perp} + \frac{P_T}{m} F_0\, h_1\cdot \hat{l}_{1T}^{h}$$

$$F_{TL}^{\cos(2\phi-\phi_a)} = \frac{q_T}{M_b} F_{b2}\, h_1\cdot \hat{l}_{1T}^{\perp}$$

Structure functions σ_{TT}

$$F_{TT}^{\cos(\phi_a-\phi_b)} = \begin{pmatrix} -\frac{P_T^2}{2mM_a} F_{a1}^{f_{1T}^{\perp}\cdot\hat{t}_1^h} + \frac{P_T^2}{2mM_a} F_{a1}^{g_{1T}\cdot\hat{t}_{1L}^h} - \frac{P_T^2}{2M_aM_b} F_{ab1}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{P_T^2}{2M_aM_b} F_{ab1}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}} \\ -\frac{q_T^2}{2M_aM_b} F_{ab2}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{q_T^2}{2M_aM_b} F_{ab2}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}} - \frac{1}{M_aM_b} F_{ab4}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{1}{M_aM_b} F_{ab4}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}} \end{pmatrix}$$

$$F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2}{2mM_a} F_{a1}^{f_{1T}^{\perp}\cdot\hat{t}_1^h} + \frac{P_T^2}{2mM_a} F_{a1}^{g_{1T}\cdot\hat{t}_{1L}^h} + \frac{P_T^2}{2M_aM_b} F_{ab1}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{P_T^2}{2M_aM_b} F_{ab1}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T}{2mM_a} F_{a2}^{f_{1T}^{\perp}\cdot\hat{t}_1^h} + \frac{P_Tq_T}{2mM_a} F_{a2}^{g_{1T}\cdot\hat{t}_{1L}^h} + \frac{P_Tq_T}{2M_aM_b} F_{ab3}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} - \frac{P_Tq_T}{2M_aM_b} F_{ab3}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)} = \frac{P_Tq_T}{2mM_a} F_{a2}^{f_{1T}^{\perp}\cdot\hat{t}_1^h} + \frac{P_Tq_T}{2mM_a} F_{a2}^{g_{1T}\cdot\hat{t}_{1L}^h}, \quad F_{TT}^{\cos(\phi_a+\phi_b)} = \frac{q_T^2}{2M_aM_b} F_{ab2}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{q_T^2}{2M_aM_b} F_{ab2}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T}{2M_aM_b} F_{ab3}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}} - \frac{P_Tq_T}{2M_aM_b} F_{ab3}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}}, \quad F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T}{2mM_b} F_{b2}^{h_1\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T}{2mM_b} F_{b2}^{h_1\cdot\hat{t}_{1T}^{\perp h}}, \quad F_{TT}^{\cos(2\phi+\phi_a+\phi_b)} = \frac{q_T^4}{4M_a^2M_b^2} F_{aabb3}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T^3}{4mM_a^2M_b} F_{aab3}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a+\phi_b)} = \frac{q_T^2}{2M_b^2} F_{bb2}^{h_1\cdot\hat{t}_{1T}^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2}{2m^2} F_0^{h_1\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2}{2M_b^2} F_{bb1}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)} = \frac{P_T^4}{4m^2M_a^2} F_{aa1}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T^4}{4M_a^2M_b^2} F_{aabb1}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)} = \frac{P_T^3q_T}{4mM_a^2M_b} F_{aab2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} = F_0^{h_1\cdot\Delta_T\hat{M}_T} + \frac{P_T^2}{2m^2} F_0^{h_1\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2}{2M_b^2} F_{bb1}^{h_1\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{q_T^2}{2M_b^2} F_{bb2}^{h_1\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{1}{M_b^2} F_{bb3}^{h_1\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)} = \frac{P_T^2}{2M_a^2} F_{aa1}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}} + \frac{P_T^4}{4m^2M_a^2} F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^4}{4M_a^2M_b^2} F_{aabb1}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2}{2M_a^2M_b^2} F_{aabb5}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2} F_{aabb2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2q_T^2}{4m^2M_a^2} F_{aa2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2} F_{aabb2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2} F_{aabb4}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T^3}{4mM_a^2M_b} F_{aab3}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} - \frac{P_T^3q_T}{4mM_a^2M_b} F_{aab2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b)} = \frac{q_T^2}{2M_a^2} F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T} + \frac{P_T^2q_T^2}{4m^2M_a^2} F_{aa2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{q_T^4}{4M_a^2M_b^2} F_{aabb3}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2} F_{aabb4}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{q_T^2}{2M_a^2M_b^2} F_{aabb6}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$