

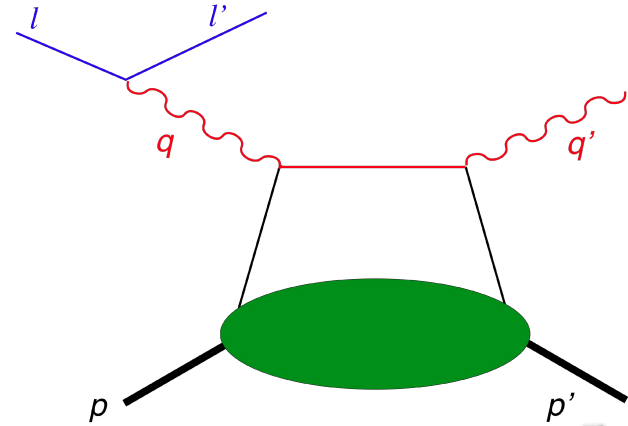
Theoretical Simulation of the Virtual Meson Production in the Forward Direction

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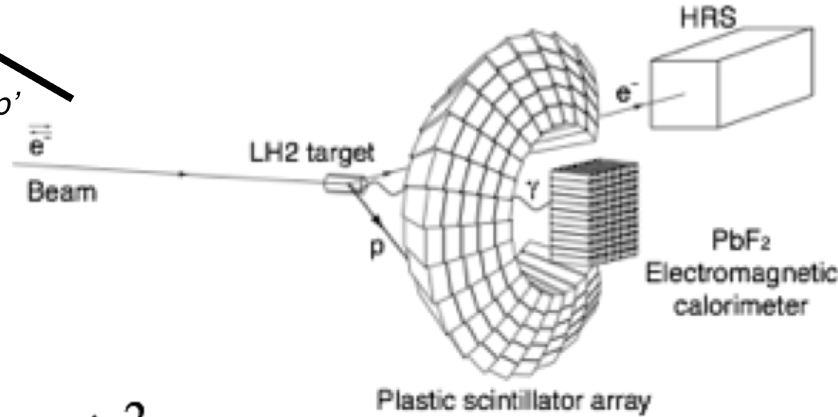


July 21, 2022

Better Work in Forward Direction



GPD



LFD

$$t = \Delta^2 = -\frac{\xi^2 M^2 + \Delta_{\perp}^2}{1 - \xi}; \Delta^+(\equiv \Delta^0 + \Delta^3) = \xi P^+; \Delta_{\perp}^2 > \Delta_{\perp \min}^2 \neq 0$$

Analysis of virtual meson production in a $(1+1)$ -dimensional scalar field model

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Light-front dynamic analysis of the longitudinal charge density using the solvable scalar field model in $(1+1)$ dimensions

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Outline

- Dirac's Proposition for Relativistic Dynamics

Instant Form Dynamics(IFD) vs. Light-Front Dynamics(LFD)

- Link between IFD and LFD : **IMF \neq LFD**
- Virtual Meson Production off a Scalar Target
- Benchmarking GPD Applicability in Forward Direction
- GPD Sum Rule and Valence/Nonvalence Decomposition
- Conclusion and Outlook

Dirac's Proposition for Relativistic Dynamics

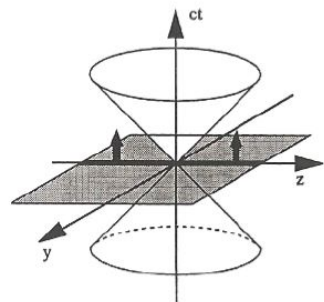


1949

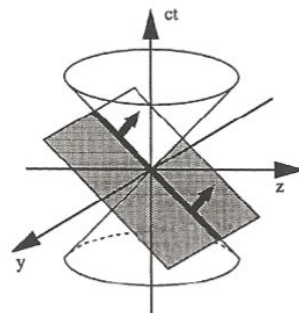
Equal t

Equal τ

$$\begin{aligned} p^0 &\leftrightarrow p^- = p^0 - p^3 \\ (p^1, p^2) &\leftrightarrow \vec{p}_\perp \\ p^3 &\leftrightarrow p^+ = p^0 + p^3 \end{aligned}$$



The instant form

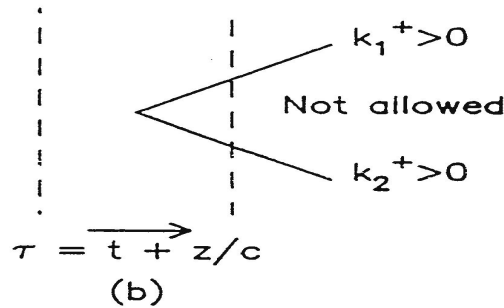
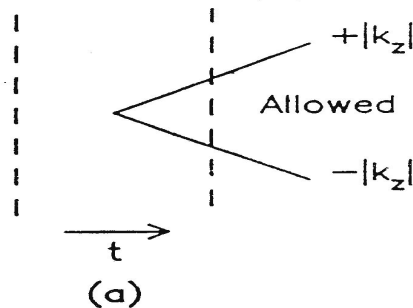


The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^2 + m^2}$$

$$p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}$$



Except zero-modes

$$k_1^+ = k_2^+ = 0$$

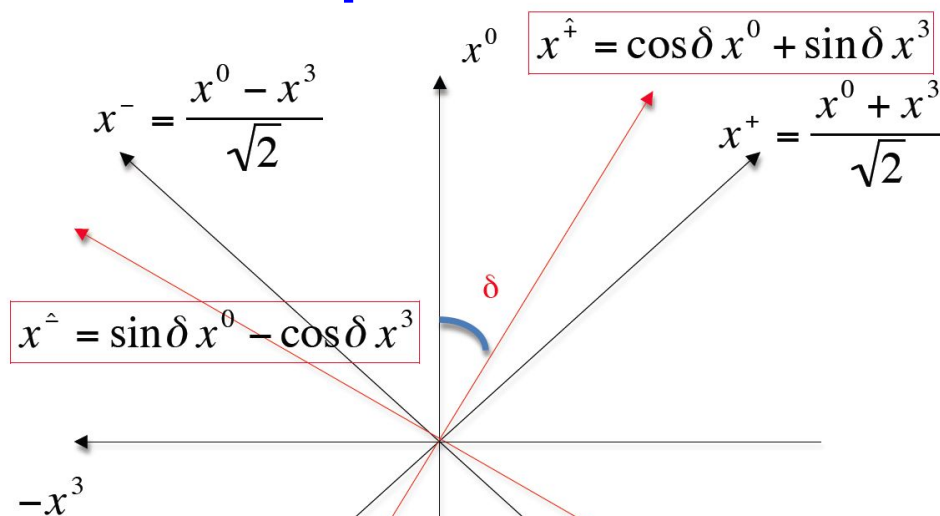
IFD

Instant Form Dynamics

LFD

Light-Front Dynamics

Interpolation between IFD and LFD



$$(IFD) \quad 0 \leq \delta \leq \frac{\pi}{4} \quad (LFD)$$

$$1 \geq C \equiv \cos(2\delta) \geq 0$$

K. Hornbostel, PRD45, 3781 (1992) – RQFT

C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly

C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra

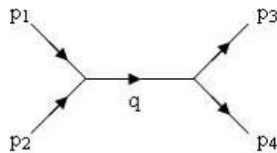
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps

C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges

Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors

C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED

B.Ma and C.Ji, PRD104, 036004(2021) – QCD₁₊₁



$$\delta = 0$$

$$p_0 = p^0$$

$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$

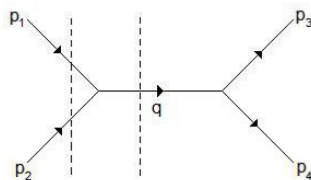
$$p_{\hat{+}} = p^0 \cos \delta - p^3 \sin \delta$$

$$p_{\hat{-}} = p^0 \sin \delta + p^3 \cos \delta$$

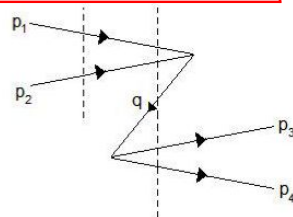
$$\delta = \pi/4$$

$$p_+ = p^-$$

$$p_- = p^+$$



(a)



(b)

$$\frac{1}{2q^0} \left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0} \right)$$

$$\frac{1}{2\omega_q} \left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}} \right)$$

$$\frac{1}{P^+} \frac{1}{\left\{ P^- - \frac{(\vec{p}_{\perp}^2 + m^2)}{2P^+} \right\}}$$

$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}(\vec{q}_{\perp}^2 + m^2)}$$

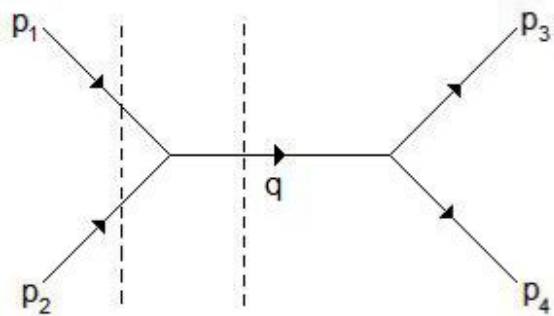
$$\mathbb{C} = \cos 2\delta$$

$$\mathbb{S} = \sin 2\delta$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \rightarrow \frac{2}{\mathbb{C}} - \frac{\vec{q}_{\perp}^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$

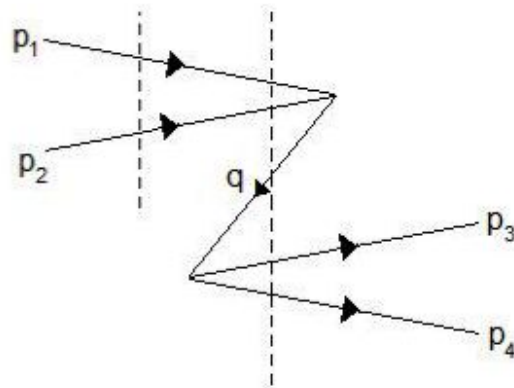
$$\rightarrow \infty \quad \text{as } \mathbb{C} \rightarrow 0$$

Infinite Momentum Frame (IMF) Approach



(a)

$$\frac{1}{E_1 + E_2 - Eq}$$

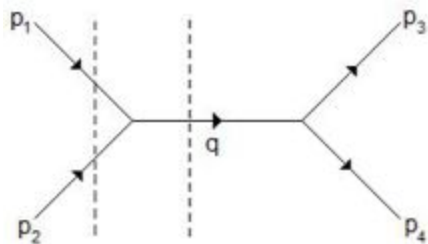
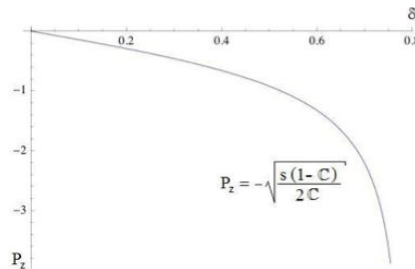
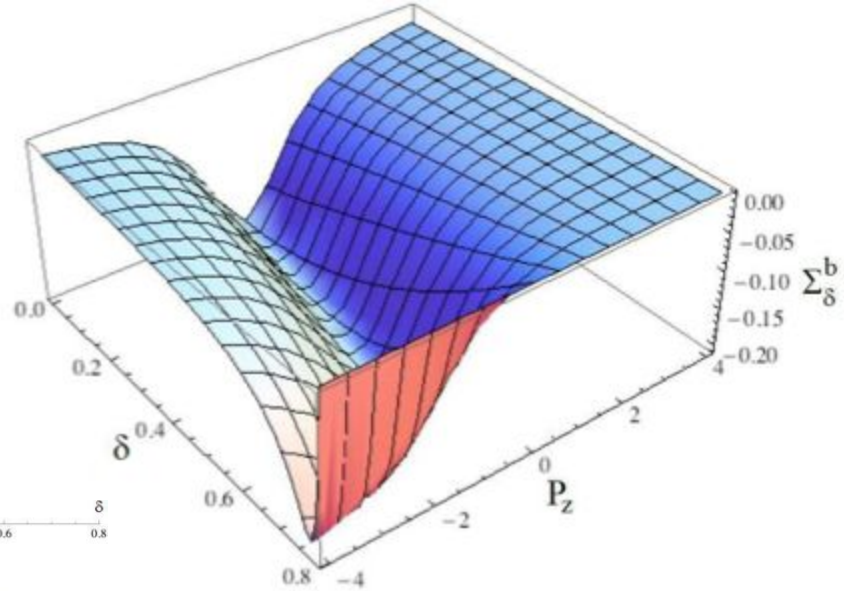
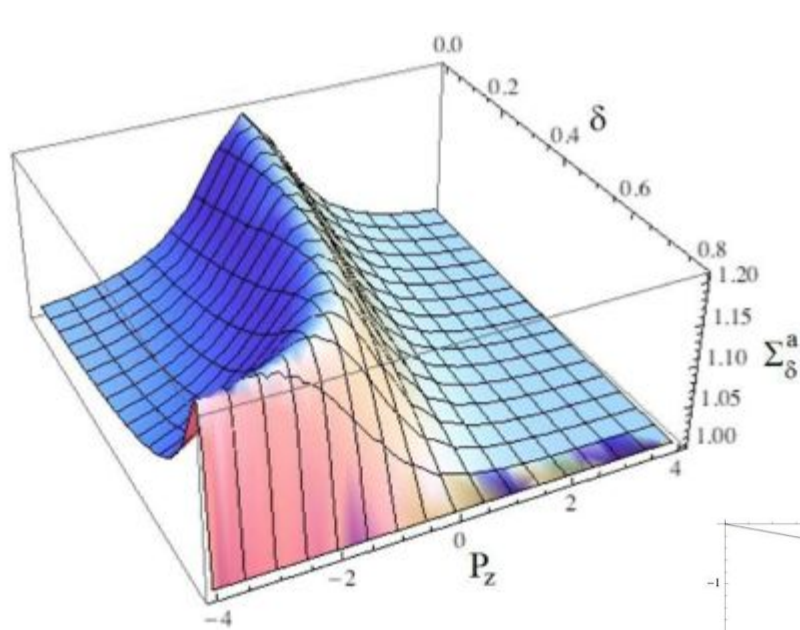


(b)

$$\begin{aligned} & -\frac{1}{Eq + E_3 + E_4} \\ &= -\frac{1}{Eq + E_1 + E_2} \\ &\rightarrow 0 \end{aligned}$$

S.Weinberg, PR158,1638(1967)
“Dynamics at Infinite Momentum”

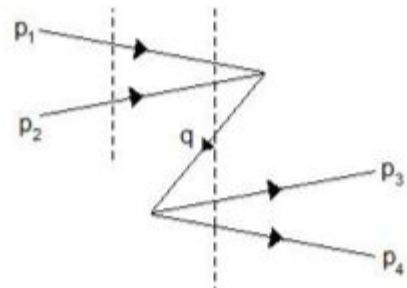
Note that this is still in the instant form (IFD).



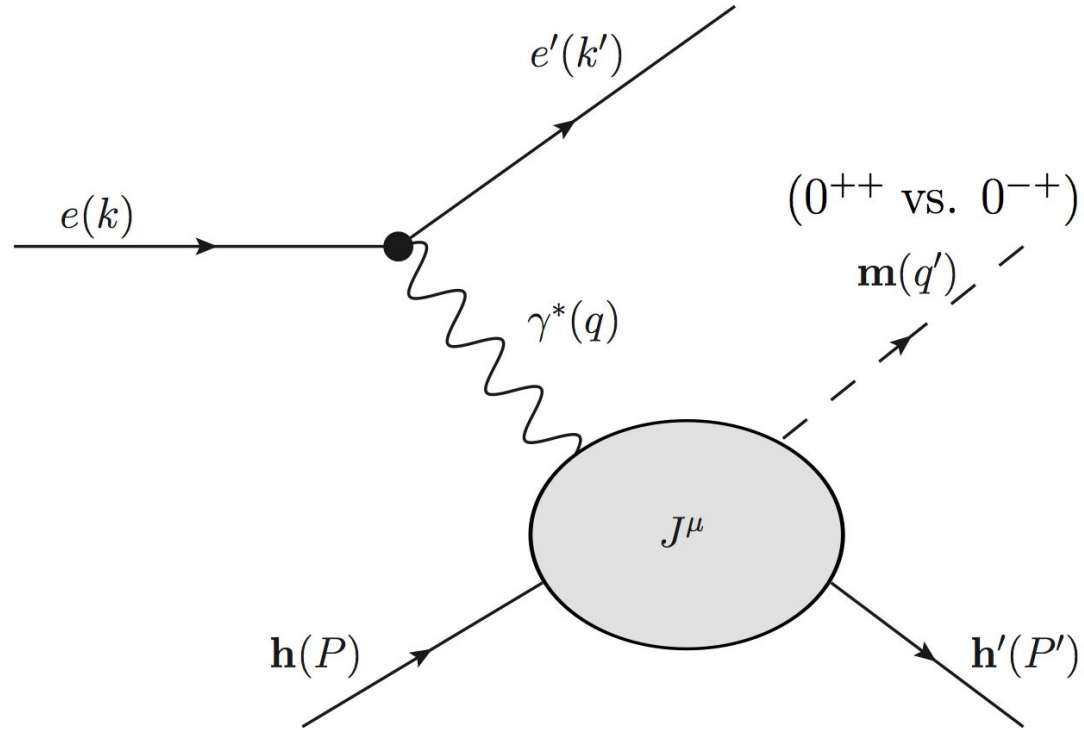
$$\Sigma(a) + \Sigma(b) = 1/(s - m^2) ; s = 2 \text{ GeV}^2, m = 1 \text{ GeV}$$

$$\text{J-shape peak \& valley : } P_z = -\sqrt{\frac{s(1-C)}{2C}} ; C = \cos(2\delta)$$

As $C \rightarrow 0$, $P^+ = P^0 + P_z \rightarrow 0$ leads to LF Zero-modes.



Virtual Meson Production off a Scalar Target



C.Ji, H.-M.Choi, A.Lundeen, B.Bakker, PRD99,116008(2019)

Salient Features

- No interference with the Bethe-Heitler process
- Consistency between our benchmark BSA prediction for 0^{-+} meson production off the scalar target with the data of the exclusive coherent electroproduction of the π^0 off ^4He measured at JLab Hall B
- General formulation of hadronic amplitudes in Meson Production off the Scalar Target (0^{++} vs. 0^{-+})
- Comparison/Contrast with the leading twist GPD formulation.

Beam-Spin Asymmetry of Exclusive Coherent

Electroproduction of the π^0 Off ^4He

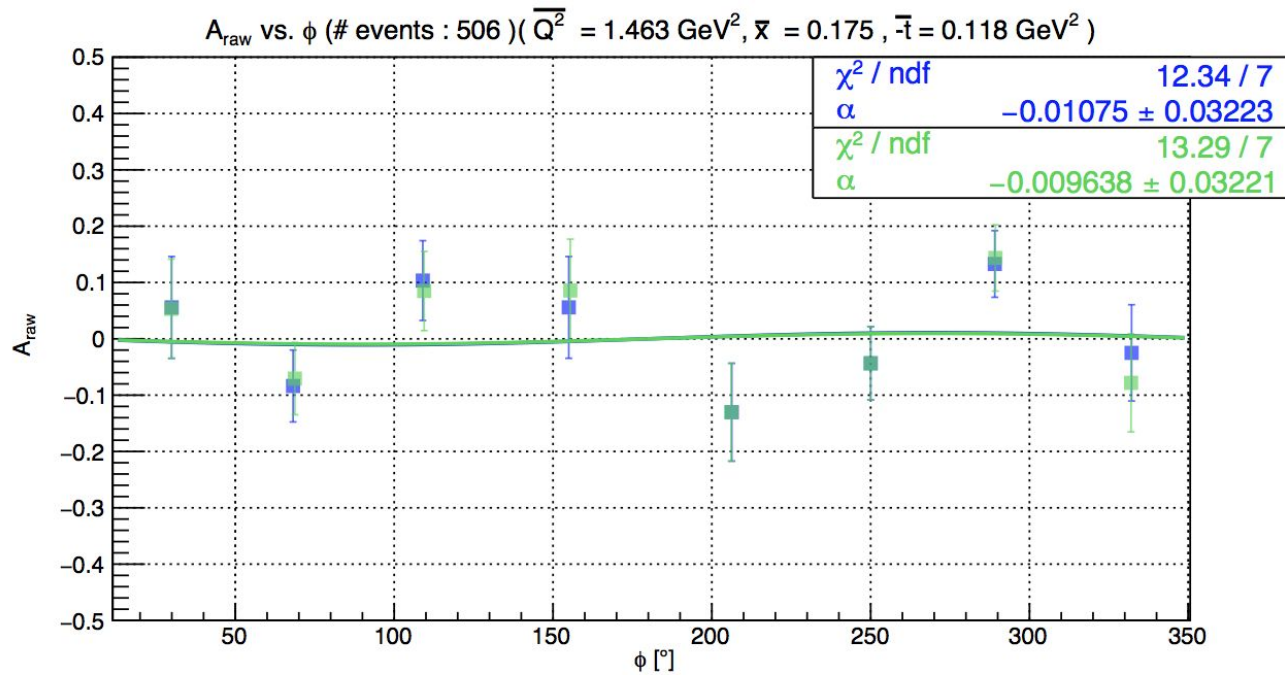
Frank Thanh Cao, Ph.D.

University of Connecticut, 2019

To understand the partonic structure of nucleons in nuclei, extracting the beam spin asymmetry (BSA) from exclusive processes is an important measurement to get at the so-called Generalized Parton Distributions (GPDs) that describe the partons behavior inside the nucleon. In particular, BSA in Deeply Virtual Meson Production (DVMP) can offer valuable constraints on the transverse GPDs which are not accessible through Deeply Virtual Compton Scattering (DVCS).

.....

This benchmark measurement is in agreement with symmetry arguments presented in a recent theoretical formulation [2] that offers a framework complementary to that of the GPDs and gives confidence in the assumptions made for future studies of exclusive nuclear processes.



$$A_{LU}(\phi) = A_{LU}^{90^\circ} \sin \phi$$

$$A_{LU}^{90^\circ} = -1.08 \pm 3.22 \text{ (stat.)} \pm 2.83 \text{ (sys.) } \%$$

4-26-2019

Beam-Spin Asymmetry of Exclusive Coherent
Electroproduction of the π^0 Off ^4He

Frank Thanh Cao

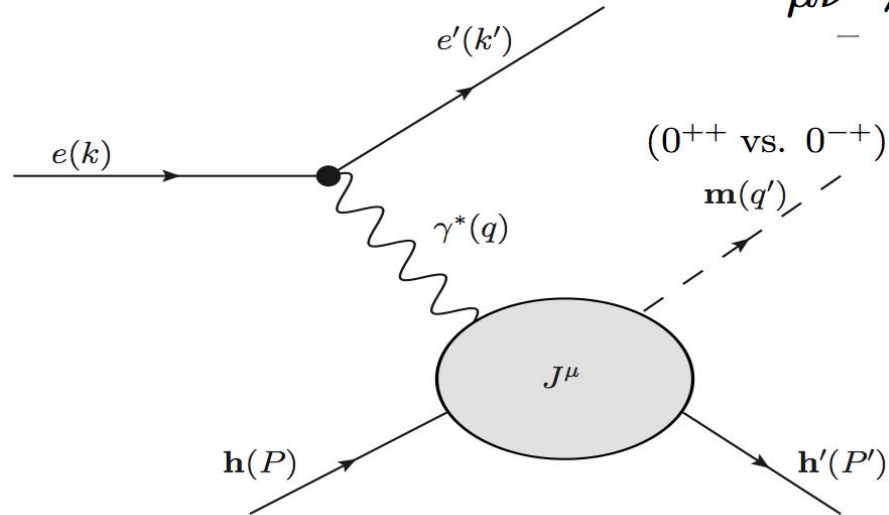
University of Connecticut - Storrs, franktcao@gmail.com

$$\langle |\mathcal{M}|^2 \rangle = \left(\frac{e^2}{q^2} \right)^2 \mathcal{L}^{\mu\nu} \mathcal{H}_{\mu\nu}$$

$$\mathcal{L}^{\mu\nu} = q^2 \left[g^{\mu\nu} + \frac{2}{q^2} (k^\mu k'^\nu + k'^\mu k^\nu) \right] + 2i h \epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta$$

$$\mathcal{H}_{\mu\nu} = J_\mu^\dagger J_\nu$$

$$\mathcal{H}_{\mu\nu} \neq \mathcal{H}_{\nu\mu}$$



$$J_{PS}^\mu = F_{PS} \epsilon^{\mu\nu\alpha\beta} q_\nu \bar{P}_\alpha \Delta_\beta$$

$$\mathcal{H}_{\mu\nu} = J_\mu^\dagger J_\nu$$

$$= |F_{PS}|^2 \epsilon_{\mu\alpha\beta\gamma} \epsilon_{\nu\alpha'\beta'\gamma'} q^\alpha \bar{P}^\beta \Delta^\gamma q^{\alpha'} \bar{P}^{\beta'} \Delta^{\gamma'}$$

$$= \mathcal{H}_{\nu\mu}$$

$$\epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \mathcal{H}_{\mu\nu} = 0$$

$$\frac{d\sigma_{h=+1}^{PS} - d\sigma_{h=-1}^{PS}}{d\sigma_{h=+1}^{PS} + d\sigma_{h=-1}^{PS}} = 0$$

Pseudoscalar(0⁻) Meson **vs.** Scalar(0⁺⁺) Meson

$$\epsilon^{\mu\nu\alpha\beta} \quad \text{vs.} \quad d^{\mu\nu\alpha\beta} = g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta}$$

C.Ji & B.Bakker, PoS QCDEV2017,038(2017);
B.Bakker & C.Ji, Few Body Syst. 58,no.1,8(2017)

$$F_{PS}(Q^2, t, x) \quad J_S^\mu = (S_q q_\alpha + S_{\bar{P}} \bar{P}_\alpha) d^{\mu\nu\alpha\beta} q_\beta \Delta_\nu$$

$$J_{PS}^\mu = F_{PS} \epsilon^{\mu\nu\alpha\beta} q_\nu \bar{P}_\alpha \Delta_\beta$$

$$F_1 = S_q - S_{\bar{P}} \\ F_2 = S_{\bar{P}}$$

$$J_S^\mu = F_1(Q^2, t, x) (q^2 \Delta^\mu - q^\mu q \cdot \Delta) + F_2(Q^2, t, x) [(\bar{P} \cdot q + q^2) \Delta^\mu - (\bar{P}^\mu + q^\mu) q \cdot \Delta]$$

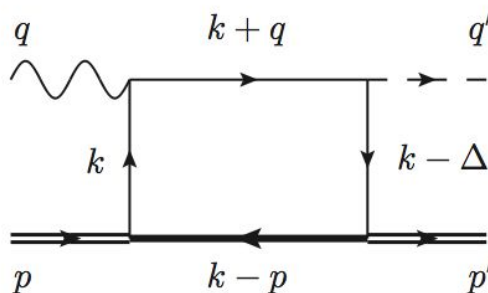
$$q \quad ; \quad \bar{P} = P + P' \quad ; \quad \Delta = P - P' = q' - q$$

Scalar Field Model Simulation of VMP in Forward Direction

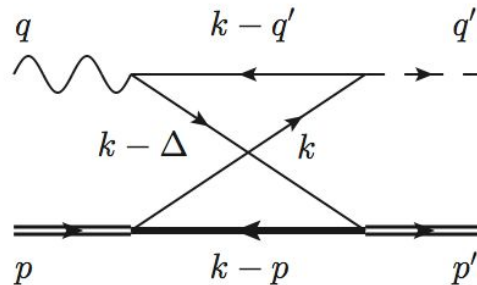
$$\gamma^* + {}^4\text{He} \rightarrow f_0(980) + {}^4\text{He}$$

$$\mathcal{M}_{\text{tot}}^{\mu(1+1)} = [(\Delta \cdot q)q^\mu - q^2 \Delta^\mu] \mathcal{F}$$

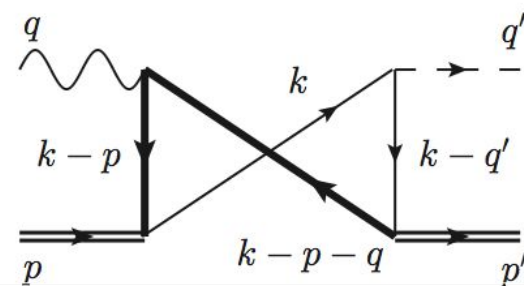
(a)



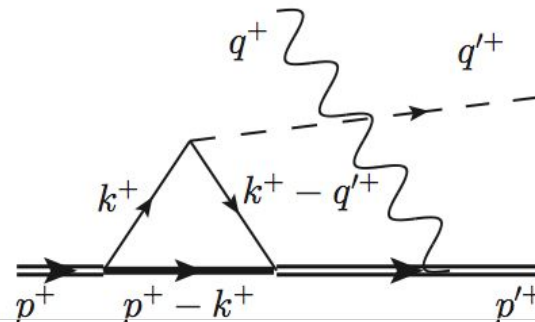
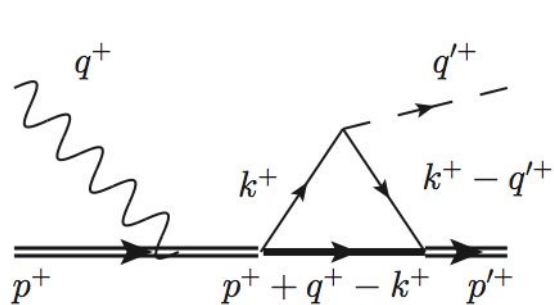
(b)



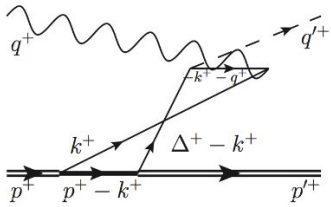
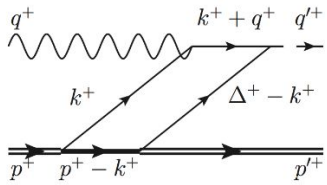
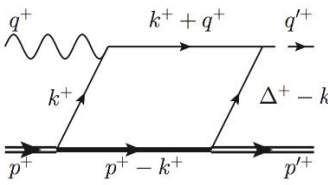
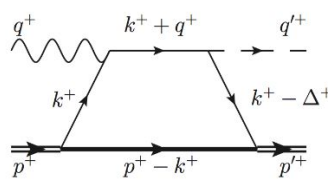
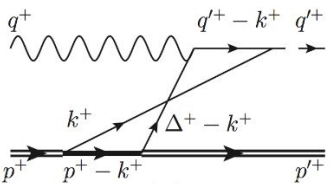
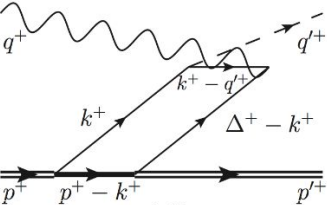
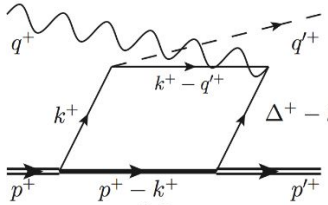
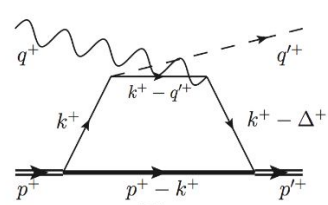
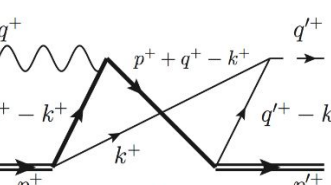
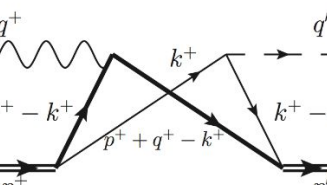
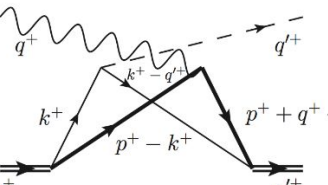
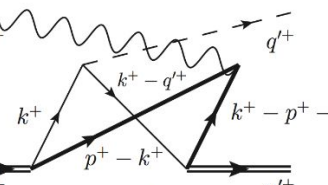
(c)



Two more amplitudes for the charged target, but not for the neutral target



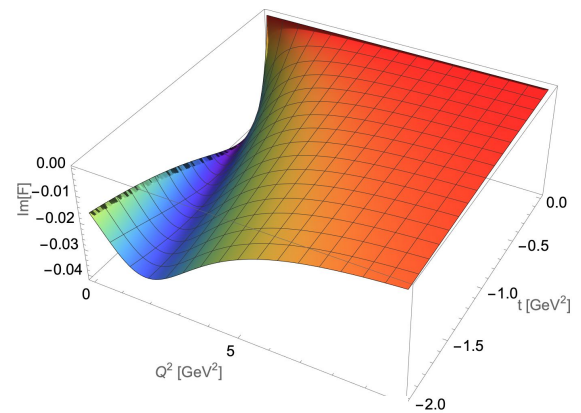
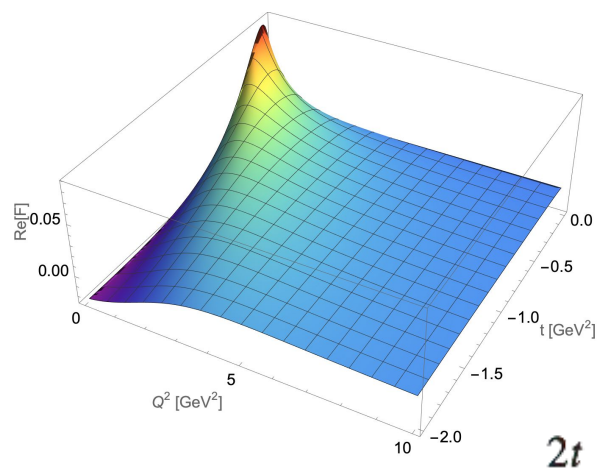
Light-Front Time-Ordered Amplitudes

| | | | | |
|-----|---|---|---|---|
| S | $0 < k^+ < -q^+$ | $-q^+ < k^+ < \Delta^+$ | | $\Delta^+ < k^+ < p^+$ |
| |  <p style="text-align: center;">(a)</p> |  <p style="text-align: center;">(b)</p> |  <p style="text-align: center;">(c)</p> |  <p style="text-align: center;">(d)</p> |
| U | $0 < k^+ < q'^+$ | $q'^+ < k^+ < \Delta^+$ | | $\Delta^+ < k^+ < p^+$ |
| |  <p style="text-align: center;">(e)</p> |  <p style="text-align: center;">(f)</p> |  <p style="text-align: center;">(g)</p> |  <p style="text-align: center;">(h)</p> |
| C | $0 < k^+ < q'^+$ | $q'^+ < k^+ < p^+ + q^+$ | | $p^+ + q^+ < k^+ < p^+$ |
| |  <p style="text-align: center;">(i)</p> |  <p style="text-align: center;">(j)</p> |  <p style="text-align: center;">(k)</p> |  <p style="text-align: center;">(l)</p> |

Compton Form Factor

$$\mathcal{M}_{\text{tot}}^{\mu(1+1)} = [(\Delta \cdot q)q^\mu - q^2\Delta^\mu]\mathcal{F}$$

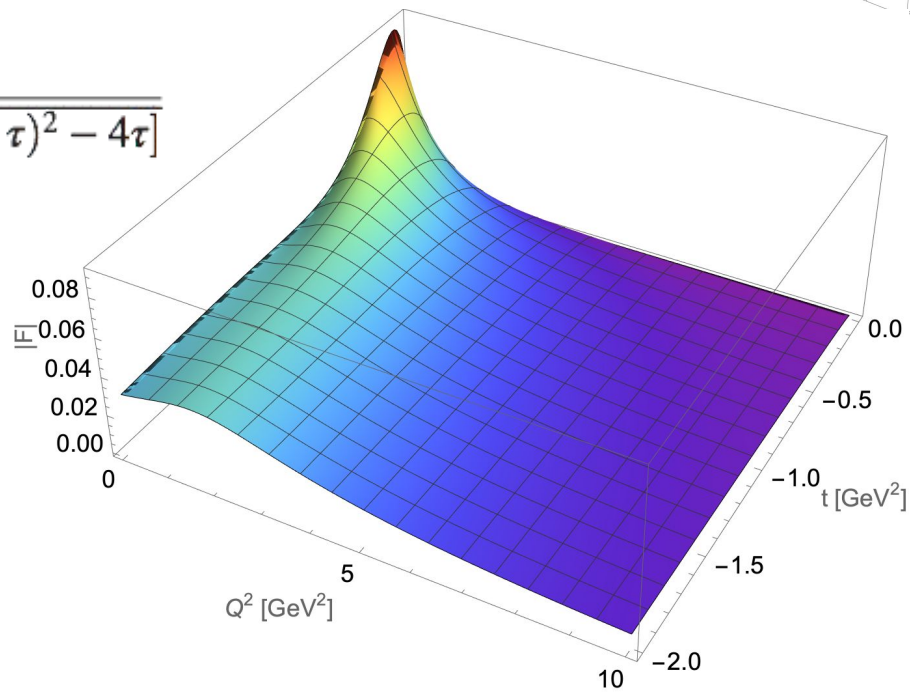
$$\mathcal{F}_c^{\text{VMP}}(Q^2, t) = \frac{\mathcal{M}_{\text{charged}}^\mu}{(\Delta \cdot q)q^\mu - q^2\Delta^\mu}$$

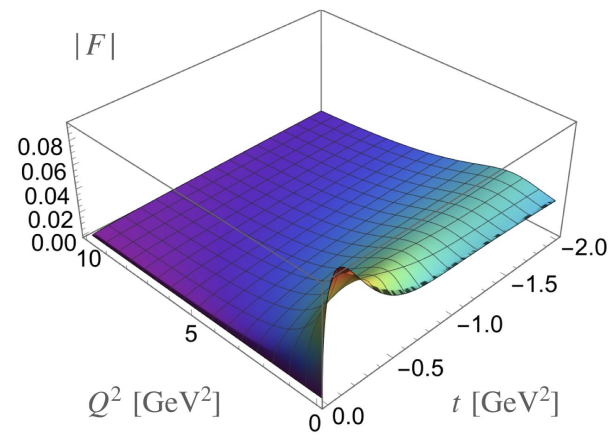
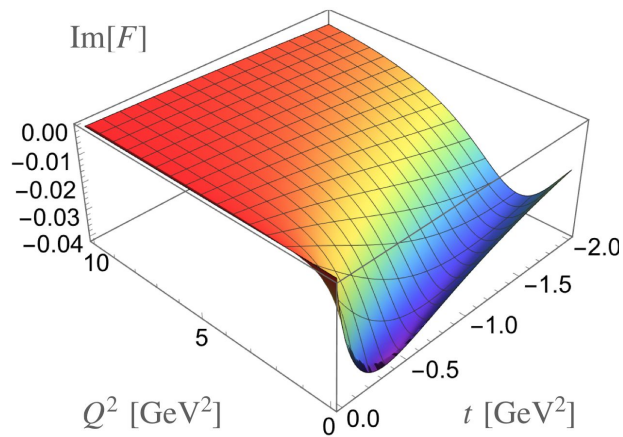
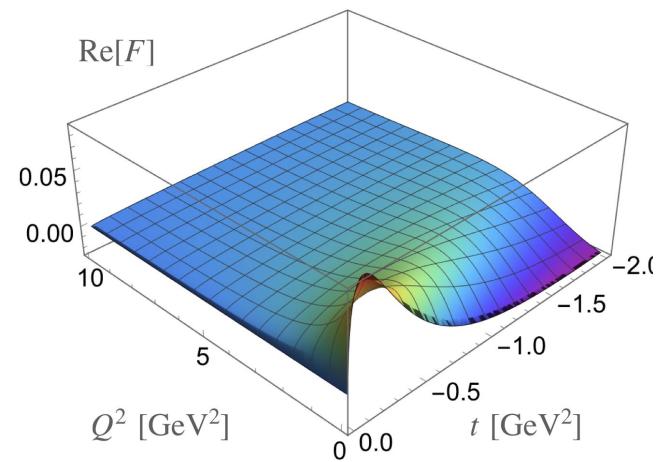
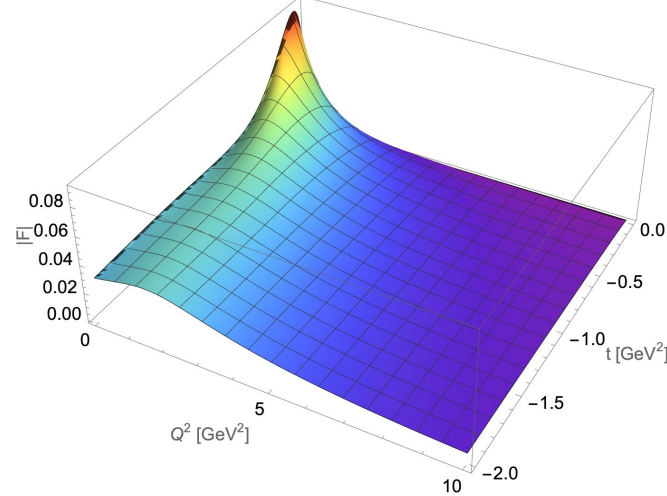
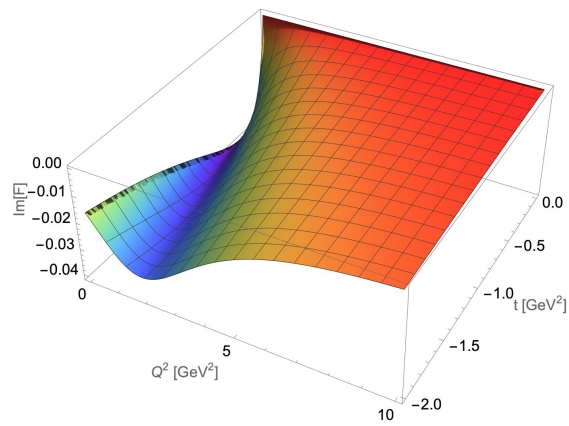
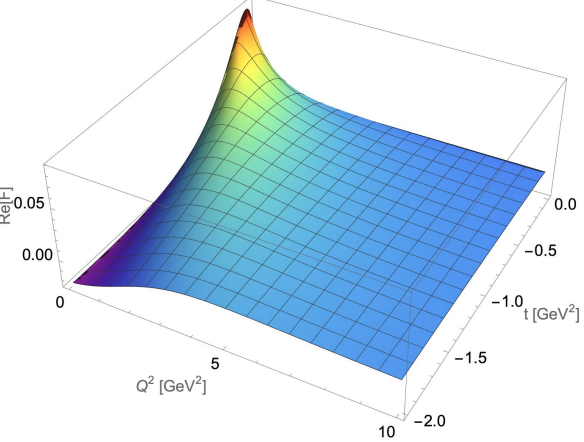


$$x_{\text{Bj}} = \frac{2t}{t(1 + \mu_s + \tau) - \sqrt{t(t - 4M_T^2)[(1 + \mu_s + \tau)^2 - 4\tau]}}$$

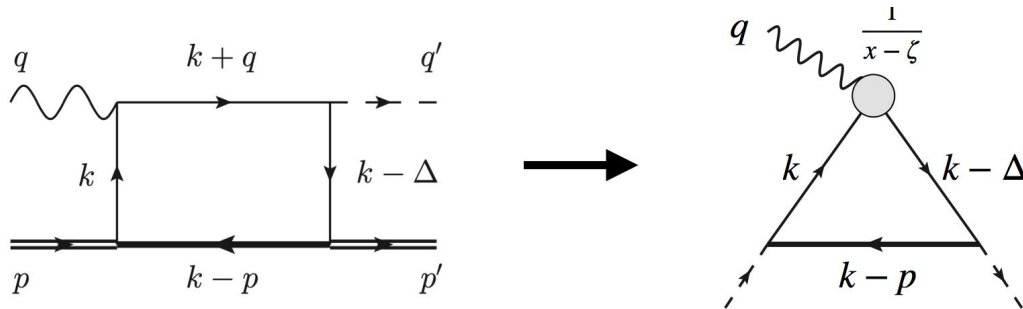
$$\zeta = \frac{1}{2M_T^2} \left(t + \sqrt{t^2 - 4tM_T^2} \right)$$

$$\mu_s = M_S^2/Q^2 \quad \text{and} \quad \tau = -t/Q^2$$





DVMP Reduction to GPD in S-channel with + Current



$$\mathcal{M}_s^\mu \sim \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - m^2} \boxed{\frac{2k^\mu + q^\mu}{(k+q)^2 - m^2}} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}$$

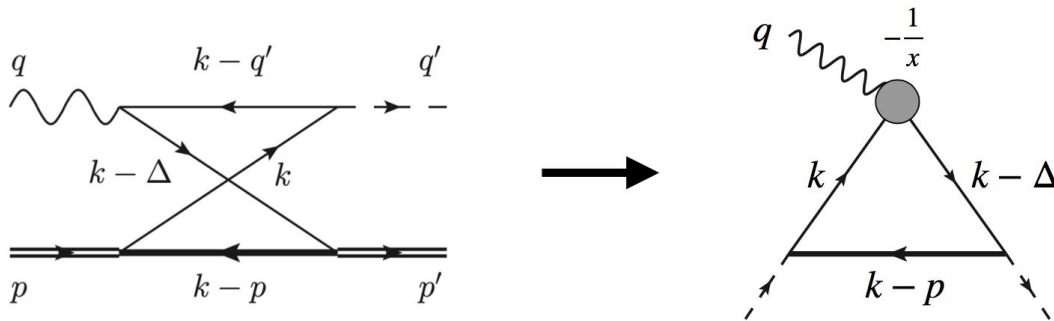
For a plus current of a virtual photon,

$$\boxed{\frac{2k^+ + q^+}{(k^+ + q^+)(k^- - k_t^-)}} \frac{1}{k^2 - m^2} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2} \quad \text{where} \quad k_t^- = -q^- + \frac{m_{Q_1}^2}{k^+ + q^+} - i \frac{\epsilon}{k^+ + q^+}$$

For large Q^2 : $\boxed{} \simeq \frac{1}{(x-\zeta)} \frac{\zeta'}{Q^2} (2x-\zeta) + \mathcal{O}\left(\frac{1}{Q^4}\right)$

$$= \boxed{\frac{1}{x-\zeta}} \frac{\zeta'}{Q^2} \boxed{\frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}}$$

DVMP Reduction to GPD in U-channel with + Current



$$\mathcal{M}_u^\mu \sim \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - m^2} \boxed{\frac{2k^\mu - \Delta^\mu - q'^\mu}{(k - q')^2 - m^2}} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2}$$

For a plus current of a virtual photon,

$$\boxed{\frac{2k^+ - \Delta^+ - q'^+}{(k^+ - q'^+)(k^- - k_u^-)}} \frac{1}{k^2 - m^2} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2} \quad \text{where} \quad k_u^- = q'^- + \frac{m_{Q_1}^2}{k^+ - q'^+} - i \frac{\epsilon}{k^+ - q'^+}$$

For large Q^2 : $\boxed{\phantom{\frac{2k^+ - \Delta^+ - q'^+}{(k^+ - q'^+)(k^- - k_u^-)}}} \simeq -\frac{1}{x} \frac{\zeta'}{Q^2} (2x - \zeta) + \mathcal{O}\left(\frac{1}{Q^4}\right)$

$$= \boxed{-\frac{1}{x}} \frac{\zeta'}{Q^2} \boxed{\frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2}}$$

$$\mathcal{M}_{Leading}^+ = \frac{1}{4\pi} \frac{\zeta'}{Q^2} \left(\frac{1}{x-\zeta} - \frac{1}{x} \right) \frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2}$$

$$A^+ = (\Delta \cdot q) q^+ - q^2 \Delta^+ = \frac{1}{2} Q^2 \zeta p^+ \left[1 + \frac{t}{Q^2} + \dots \right]$$

$$\frac{\mathcal{M}_{Leading}^+}{A_{Leading}^+} = \frac{1}{2\pi} \frac{1}{Q^4} \left(\frac{1}{x-\zeta} - \frac{1}{x} \right) \frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2}$$

DVMP Reduction to GPD with - Current works as well.

$$\frac{\mathcal{M}_{Leading}^-}{A_{Leading}^-} = \frac{1}{2\pi} \frac{1}{Q^4} \left(\frac{1}{x-\zeta} - \frac{1}{x} \right) (2x - \zeta) \frac{1}{k^2 - m^2} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2}$$

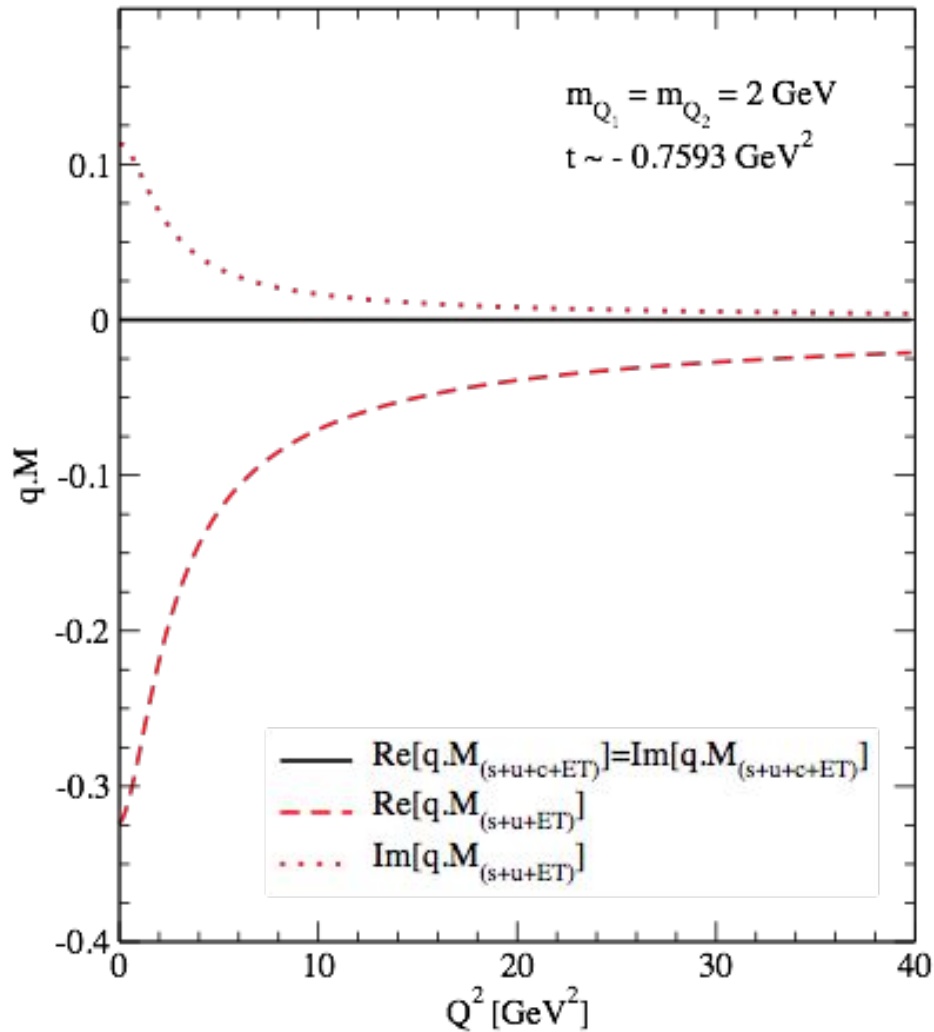
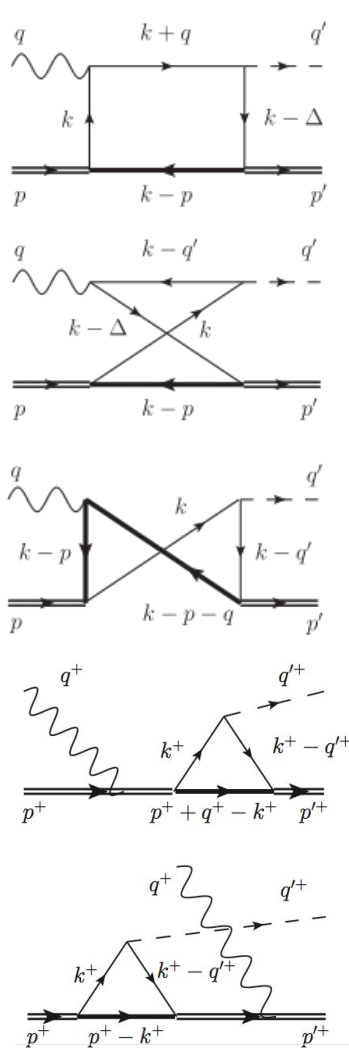
$$M^\mu = A^\mu F$$

$$\mathcal{M}_{\text{Leading}}^\mu \sim \frac{q^\mu - 2q'^\mu}{q^2} \left(\frac{1}{x - \zeta} - \frac{1}{x} \right) \frac{1}{k^2 - m^2} (2x - \zeta) \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2}$$

$$(\Delta \cdot q) q^\mu - q^2 \Delta^\mu \longrightarrow \frac{Q^2}{2} (2 q'^\mu - q^\mu)$$

Gauge Invariance
works asymptotically:

$$q' \cdot q \rightarrow q^2 / 2$$

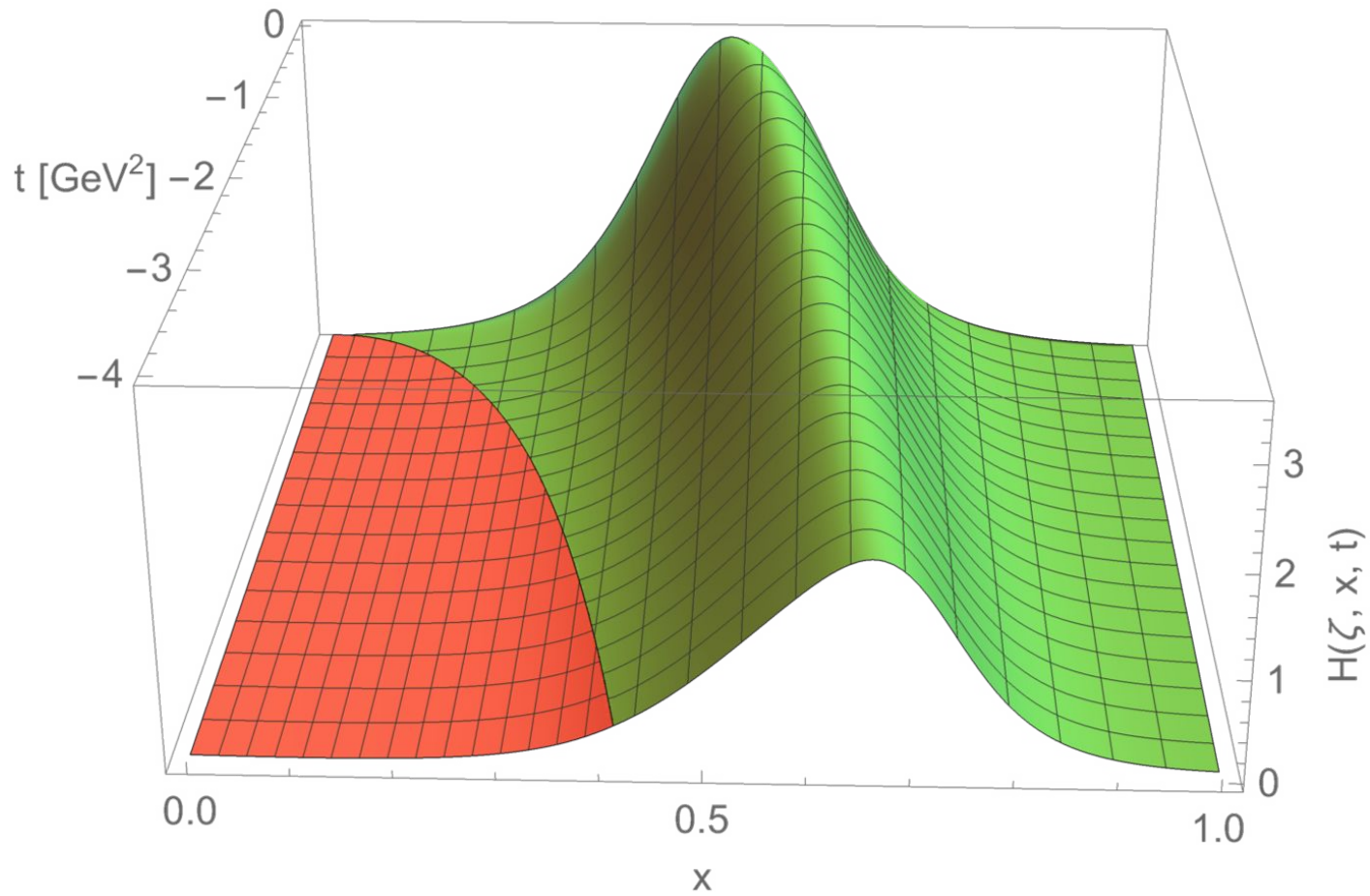


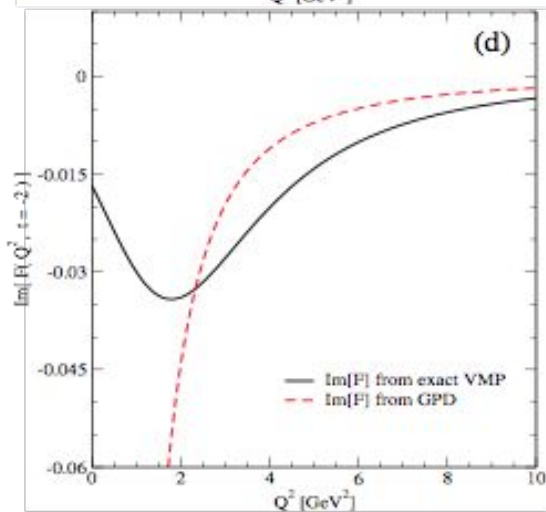
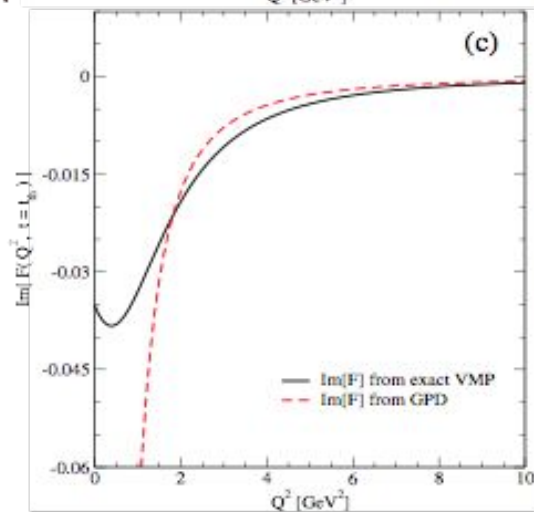
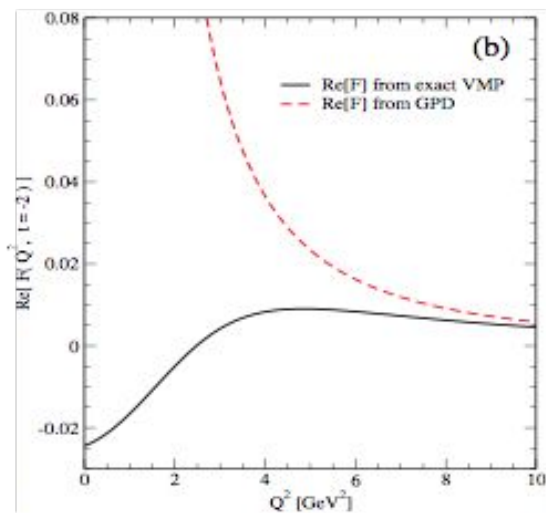
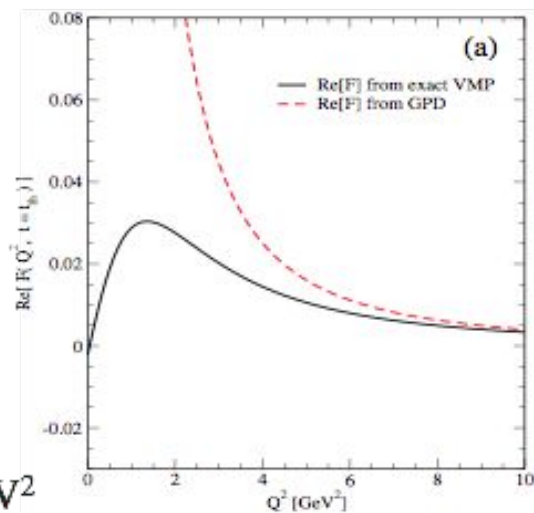
$$(\Delta \cdot q) q^\mu - q^2 \Delta^\mu$$



$$\frac{Q^2}{2} (2 q'^\mu - q^\mu)$$

$$\mathcal{M}_{s+u}^{\pm \text{DVMP}} = \frac{e_{Q_1} \zeta}{4\pi Q^2} \int_0^1 dx \left(\frac{1}{x - \zeta} - \frac{1}{x} \right) H(\zeta, x, t)$$





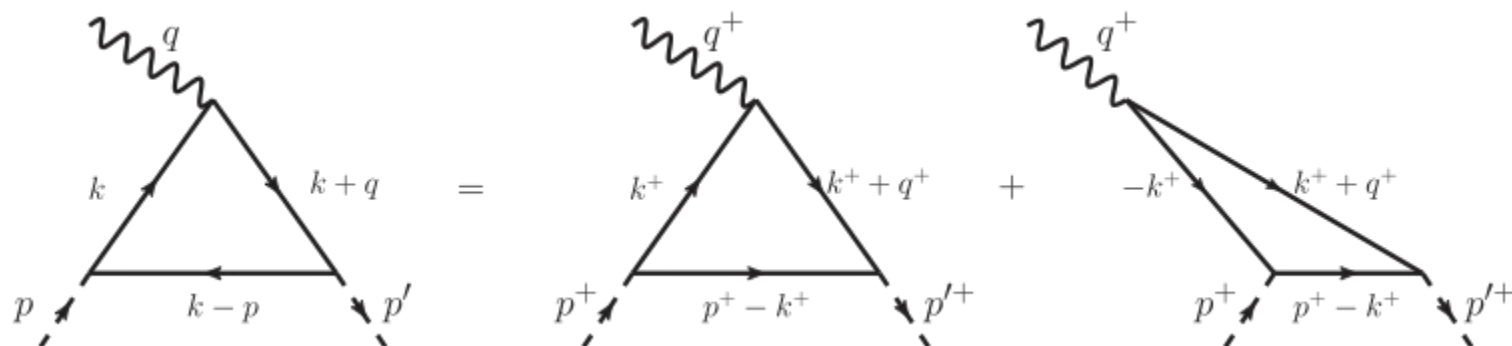
$$t = t_{\text{th}} \simeq -0.7593 \text{ GeV}^2$$

$$t = -2 \text{ GeV}^2$$

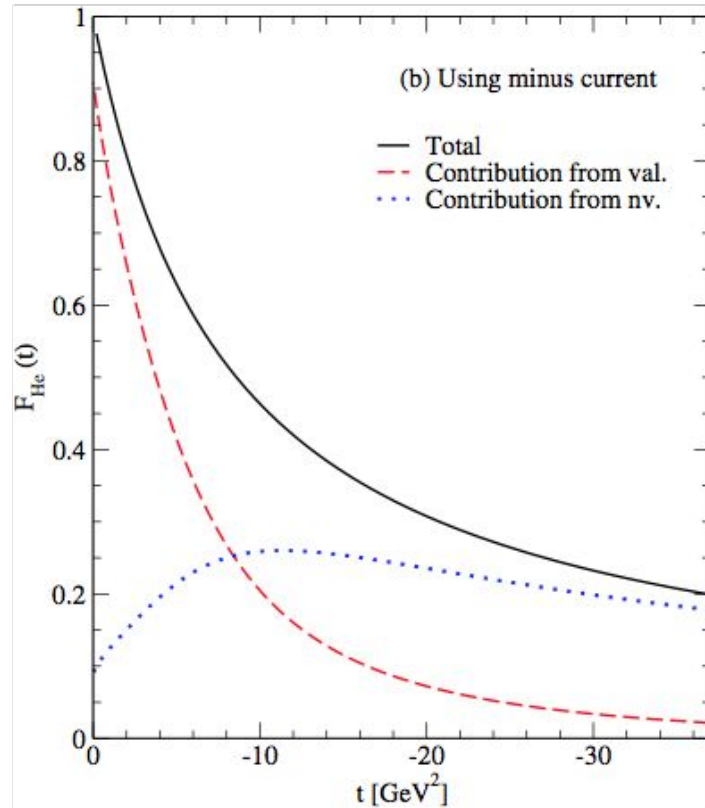
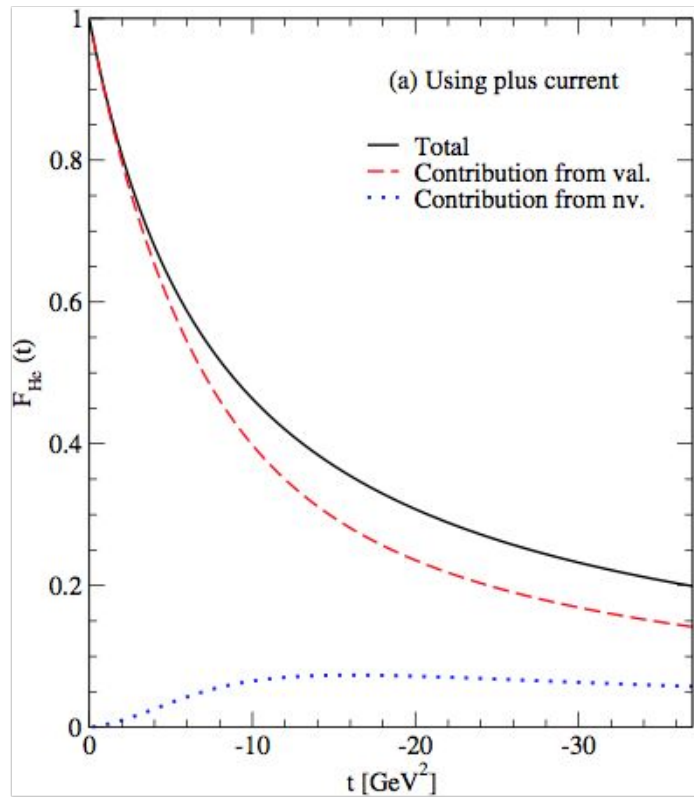
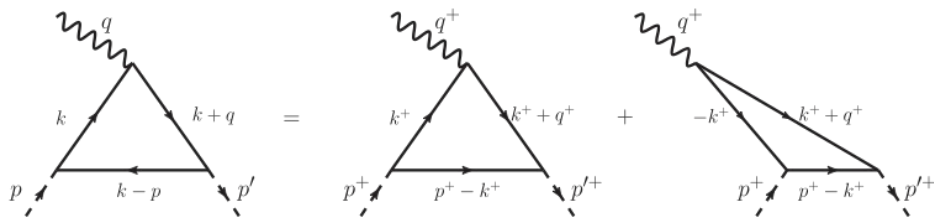
$$-t/Q^2 \lesssim 0.1$$

$$F_{\mathcal{M}}(t) = \int_0^1 \frac{dx}{1 - \zeta/2} H(\zeta, x, t).$$

$$H(\zeta, x, t) = \begin{cases} H_{\text{ERBL}}(\zeta, x, t), & \text{for } 0 \leq x \leq \zeta, \\ H_{\text{DGLAP}}(\zeta, x, t), & \text{for } \zeta \leq x \leq 1 \end{cases}$$



$$J_S^\mu(0) = (p + p')^\mu F_{\mathcal{M}}^S(q^2)$$



Decomposition of the Form Factor

$$\sum_{i=V,NV} J_i^\mu(t) = (P + P')^\mu \sum_{i=V,NV} F_i(t) = (2P^\mu - \Delta^\mu) \sum_{i=V,NV} F_i(t)$$

$$J_V^\mu(t) = \int_{\Delta^+}^{P^+} dk^+ \int dk^- \frac{2k^\mu - \Delta^\mu}{D_V}$$

$$J_{NV}^\mu(t) = \int_0^{\Delta^+} dk^+ \int dk^- \frac{2k^\mu - \Delta^\mu}{D_{NV}}$$

$$\frac{2k^\mu - \Delta^\mu}{2P^\mu - \Delta^\mu} = \frac{2x - \xi}{2 - \xi} \quad \text{only if} \quad \Delta^\mu = \xi P^\mu \quad \text{or} \quad q^\mu = q'^\mu + \xi P^\mu$$

Note here that $(q - q')^2 = \Delta^2 = t = \xi^2 M^2 > 0$
while $t < 0$ in DVMP.

Conclusion and Outlook

- Unless small $|t|/Q^2$, “Cat’s ears” contribution should not be neglected.
- Sum rule correspondence between DGLAP/ERBL GPDs and Valence/Nonvalence contributions to the form factor works only for a certain current component.
- Form factor decomposition depends on the current component although the form factor itself is independent of the choice of the current component. (Democracy in current components)
- 3+1 D extension with BSA investigation is underway.
- Application to the energy-momentum tensor decomposition appears feasible.

Backup Slides

QCD Evolution Workshop Preparation
April 29, 2022

Conceptual Issues of GPDs

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CONCEPTUAL ISSUES CONCERNING GENERALIZED PARTON DISTRIBUTIONS

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Nonforward parton distributions

A. V. Radyushkin*

$$q = q' - \xi p$$

Note that here $(q - q')^2 = \Delta^2 = t = \xi^2 M^2 > 0$

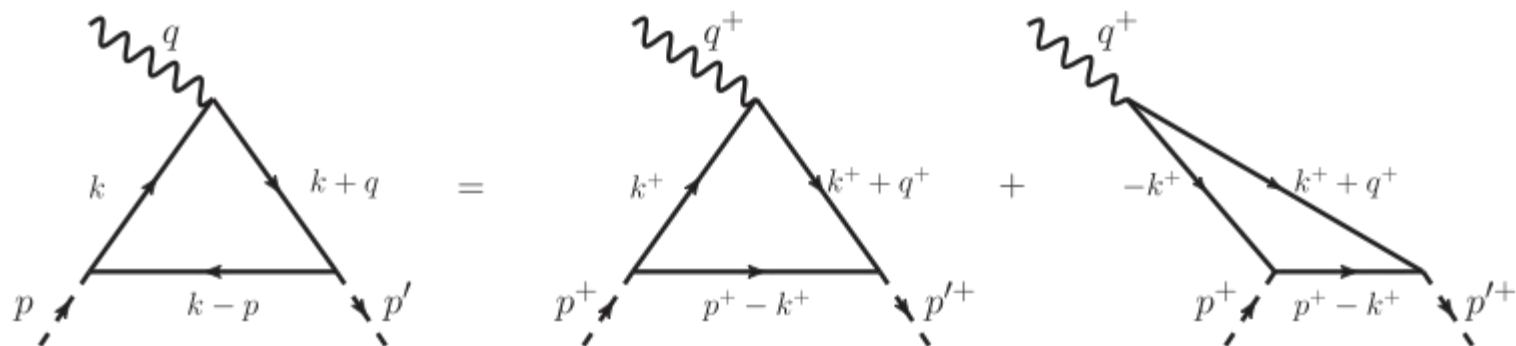
while $t < 0$ in DVCS.

$$(\Delta \cdot q) q^\mu - q^2 \Delta^\mu \longrightarrow \frac{Q^2}{2} (2 q'^\mu - q^\mu)$$

Key Finding

Link between + and - Currents in GPD Coefficient Functions

$$\left(\frac{1}{x - \zeta} + \frac{1}{x} \right) = \frac{2x - \zeta}{(x - \zeta)x} = \left(\frac{1}{x - \zeta} - \frac{1}{x} \right) \frac{2x - \zeta}{\zeta}$$



$$F_{\mathcal{M}}(t) = \int_0^1 \frac{dx}{1 - \zeta/2} H(\zeta, x, t).$$

$$\frac{2k^\mu - \Delta^\mu}{2p^\mu - \Delta^\mu} = \frac{2x - \zeta}{2 - \zeta}$$