Spin Polarization in Heavy Ion Collisions

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Fudan University, Shanghai

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APCTP Pohang, Korea & Online

ExHIC and spin polarization

- From Su Houng's talk:

Origin of Exotics from Heavy Ion Collision (ExHIC): Theory Collaboration

- **2007.12-3 -12-14: APCTP Focus program on Hadron Physics at RHIC**
- **2010.5.17-30: Yukawa Institute (YITP) workshop on ExHIC**
- **2019.3-25 -4.5: YITP workshop on "Hadron Interactions and Polarization from High energy Collisions"**

ExHIC-P Collaboration

PHYSICAL REVIEW C **102**, 021901(R) (2020)

Rapid Communications

Signatures of the vortical quark-gluon plasma in hadron yields

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(ExHIC-P Collaboration)

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- Global spin polarization
  - Experimental results
  - Theoretical interpretation
- Local spin polarization
  - Experimental results
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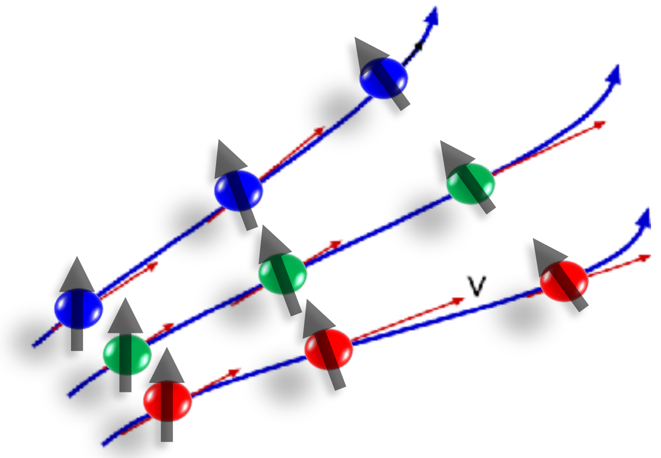
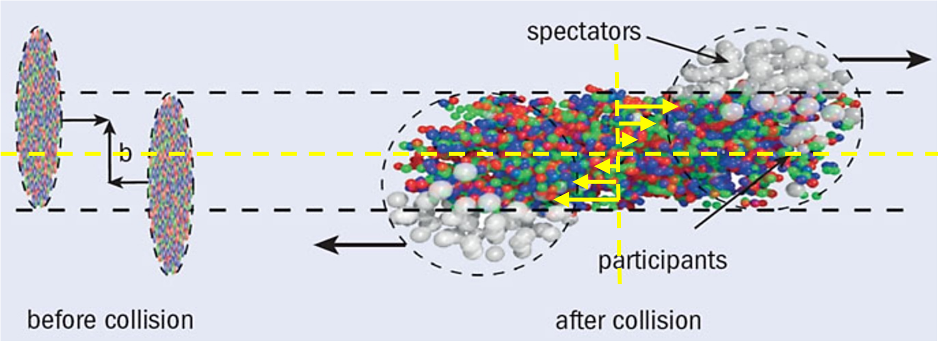
- Spin alignment of vector mesons
- Summary

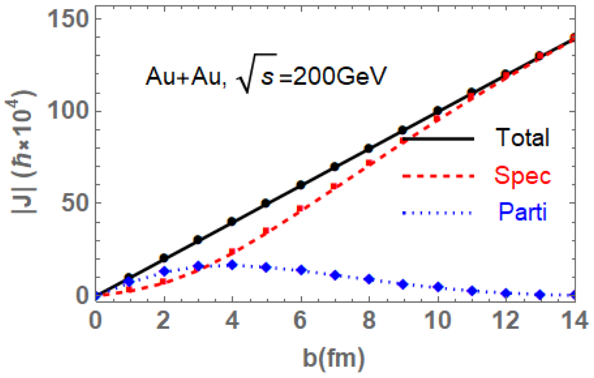


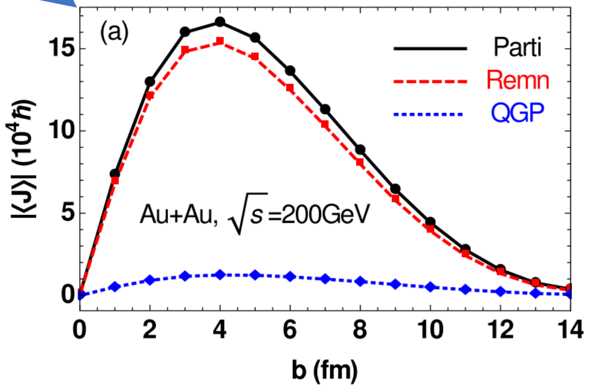
Introduction

Vorticity by global angular momentum

- Angular momentum conservation

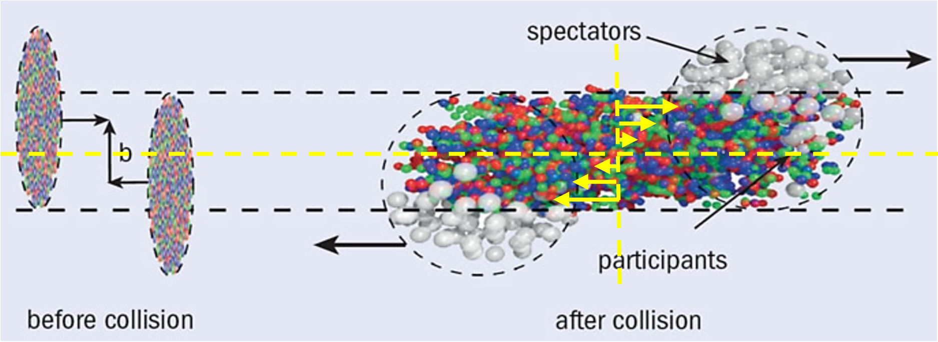






Vorticity by global angular momentum

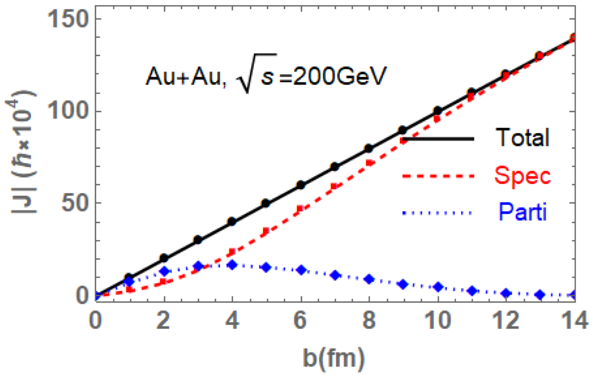
- Angular momentum conservation

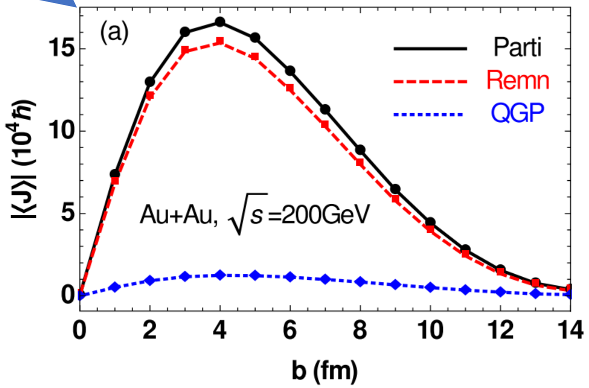


No rigid rotation*, but local fluid vorticity

$$\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{v}$$

(Angular velocity of fluid cell)

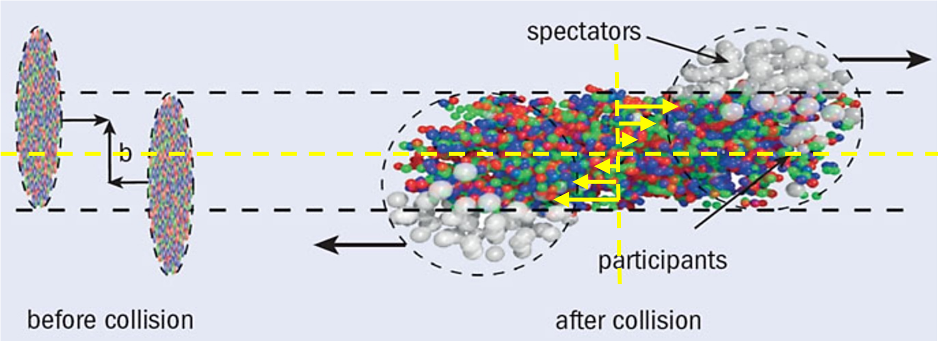




* At low energy, there is a possibility that two colliding nuclei fuse into a compound high-spin nucleus

Vorticity by global angular momentum

- Angular momentum conservation



No rigid rotation*, but local fluid vorticity

$$\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{v}$$

(Angular velocity of fluid cell)

- Estimation at low energy $\sqrt{s} \gtrsim 2m_N$

part of $J_0 \sim Ab(\sqrt{s} - 2m_N)$ **retained in the produced matter**:

$$J = \int d^3\boldsymbol{x}\, I(\boldsymbol{x})\omega(\boldsymbol{x}) \approx \int d^3\boldsymbol{x}\, \varepsilon(\boldsymbol{x}) x_\perp^2 \bar{\omega} \sim 2m_N\, A\, R_A^2\, \bar{\omega} \quad \textbf{for } b < 2R_A$$

$$\bar{\omega} \sim \frac{b}{R_A^2}\frac{\sqrt{s} - 2m_N}{2m_N} \sim 10^{22} s^{-1}$$

$(b = R_A, \sqrt{s} = 3\text{ GeV})$

- Estimation at high energy $\sqrt{s} \gg 2m_N$

part of $J_0 \sim Ab\sqrt{s}$ **retained in the produced matter**:

$$J \approx \int d^3\boldsymbol{x}\, \gamma^2(\boldsymbol{x})\varepsilon(\boldsymbol{x}) x_\perp^2 \bar{\omega} \sim s\, A\sqrt{s}\, R_A^2\, \bar{\omega}/(2m_N)^2 \quad \textbf{for } b < 2R_A$$

$$\bar{\omega} \sim \frac{b}{R_A^2}\left(\frac{2m_N}{\sqrt{s}}\right)^2 \sim 10^{19} s^{-1}$$

$(b = R_A, \sqrt{s} = 200\text{ GeV})$

* At low energy, there is a possibility that two colliding nuclei fuse into a compound high-spin nucleus

Vorticity by global angular momentum

- Relativistic vorticities

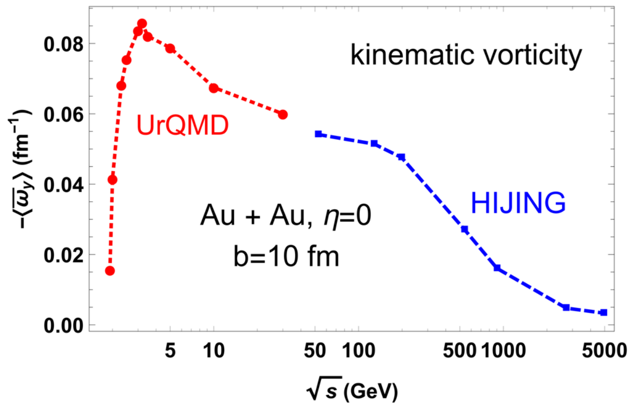
Kinematic vorticity $$\omega^{\mathrm{K}}_{\mu\nu} = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu) \quad \Rightarrow \quad \omega^\mu_{\mathrm{K}} = -(1/2)\epsilon^{\mu\nu\rho\sigma} u_\nu \omega^{\mathrm{K}}_{\rho\sigma}$$

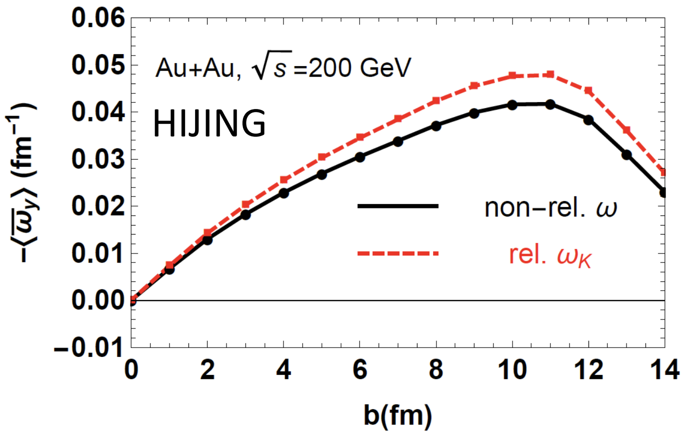
Temperature vorticity $$\omega^{\mathrm{T}}_{\mu\nu} = -\frac{1}{2}[\partial_\mu(Tu_\nu) - \partial_\nu(Tu_\mu)]$$

Thermal vorticity $$\omega^{\beta}_{\mu\nu} = -\frac{1}{2}[\partial_\mu(\beta u_\nu) - \partial_\nu(\beta u_\mu)]$$

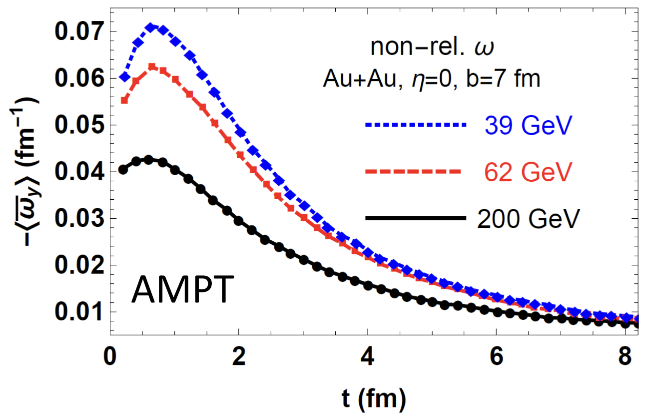
… …

- Numerical results for various vorticities





(Deng-XGH 2016; Deng-XGH-Ma-Zhang 2020)



(Jiang-Lin-Liao 2016)

Vorticity by global angular momentum

- Relativistic vorticities

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Temperature vorticity $\omega^{\mathrm{T}}_{\mu\nu} = -\frac{1}{2}[\partial_\mu(Tu_\nu) - \partial_\nu(Tu_\mu)]$

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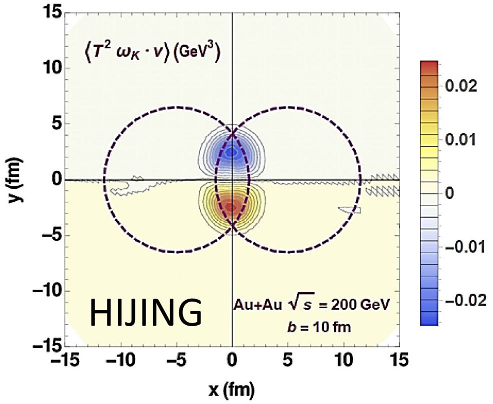
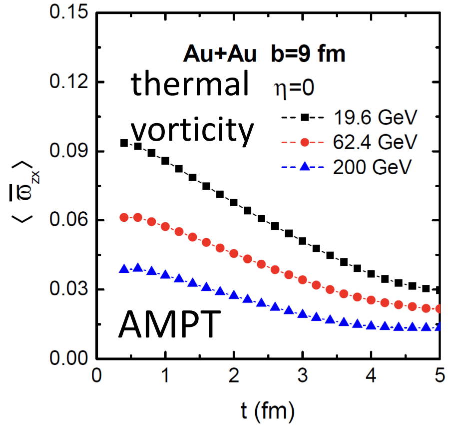
… …

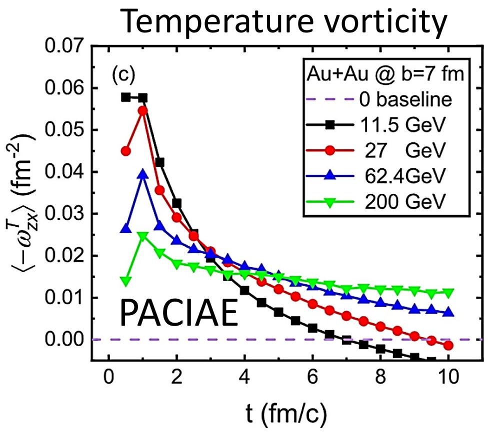
- Numerical results for various vorticities



(Deng-XGH 2016)



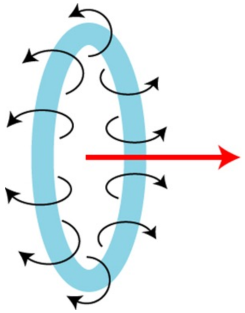
(Wei-Deng-XGH 2018)

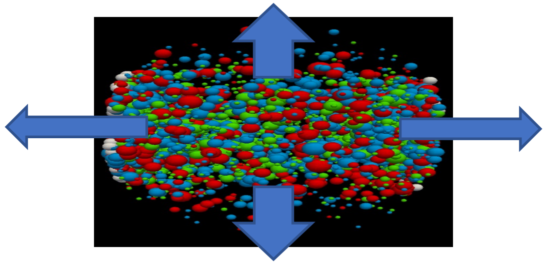


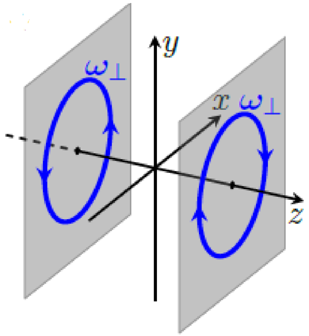
(Lei etal 2021)

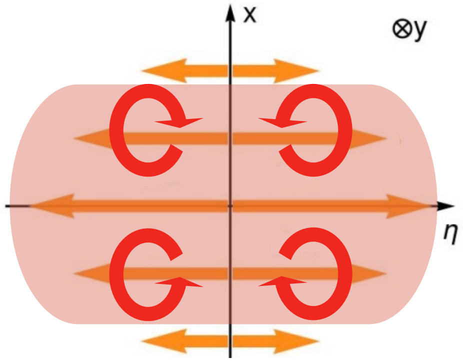
(See also: Becattini-Karpenko etal 2015,2016; Xie-Csernai etal 2014,2016,2019; Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017; Ivanov etal 2017-2020; … …)

Vorticity by inhomogeneous expansion

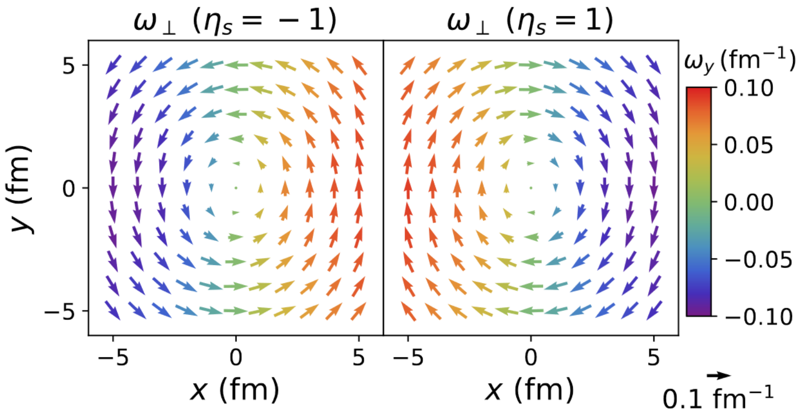




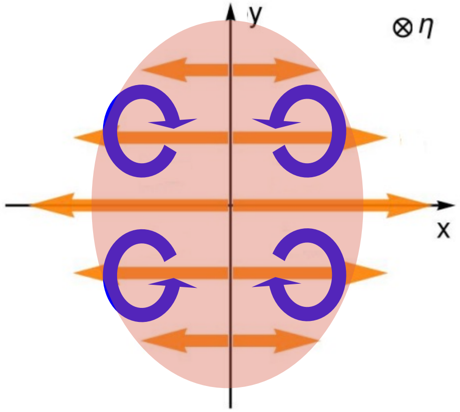




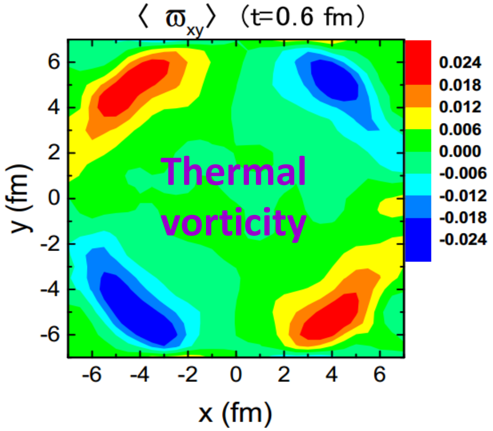
Transverse



(Xia-Li-Wang 2017)



Longitudinal

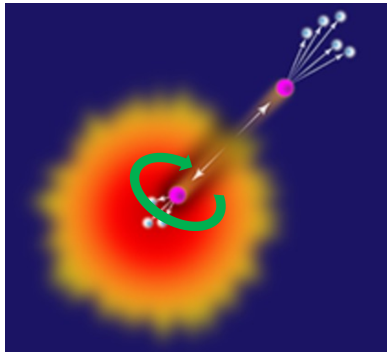


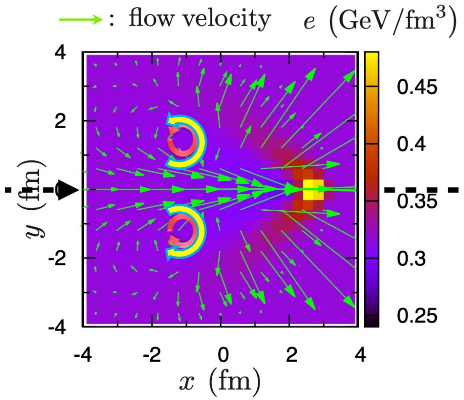
(Wei-Deng-XGH 2019)

(See also: Karpenko-Becattini 2017; Csernai etal 2014; Teryaev-Usubov 2015; Ivanov-Soldatov 2018; Fu etal 2020; Lei etal 2021; … …)

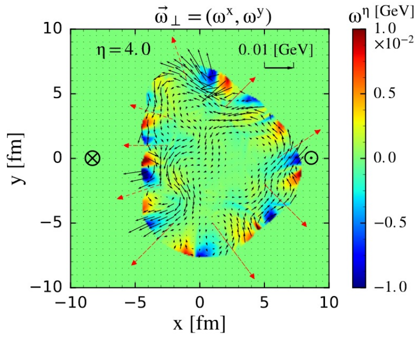
Other sources of vorticity

1) Jet

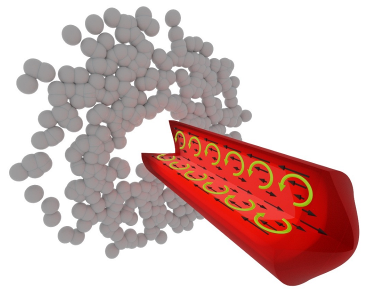




(Betz-Gyulassy-Torrieri 2007)

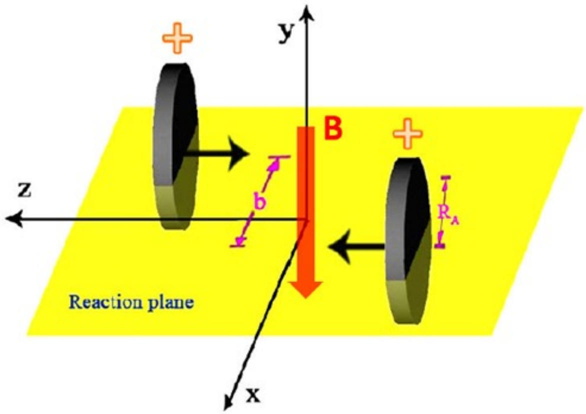


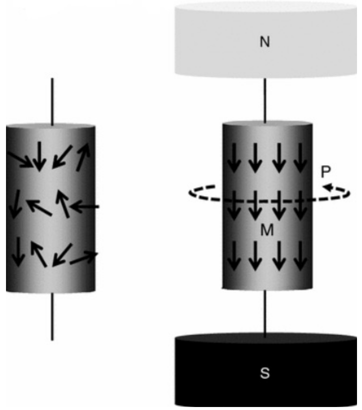
(Pang-Peterson-Wang-Wang 2016)



(Voloshin 2018; Lisa etal 2021)

2) Magnetic field

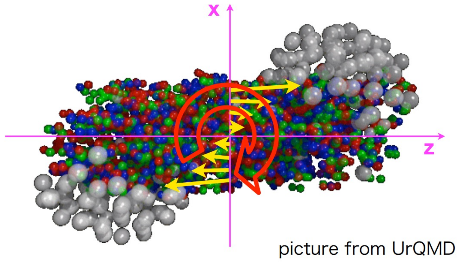




Einstein-de-Haas effect

Summary of this part

1. Global AM induces strong vorticity in HICs

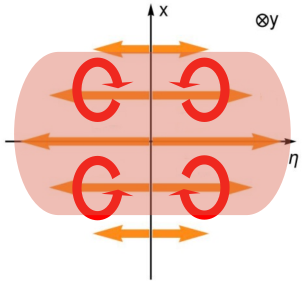


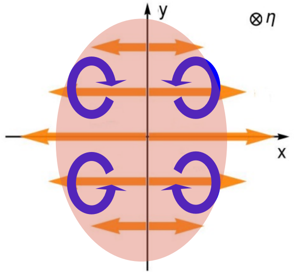
picture from UrQMD

$: \boldsymbol{\omega \approx 10^{19} - 10^{22}\ s^{-1}}$

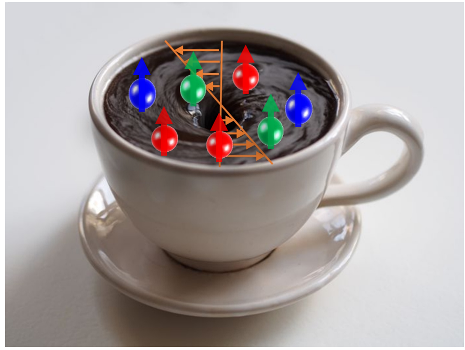
(QGP: The most vortical fluid)

2. Inhomogeneous expansion: quadrupoles in both xy and xz planes





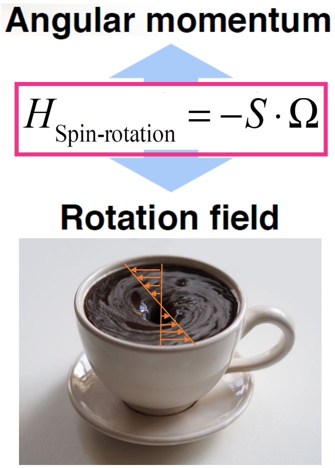
3. Vorticity (i.e., rotation of local fluid cell) can polarize spin



Global spin polarization

Global spin polarization

- Spin can thus be polarized by vorticity

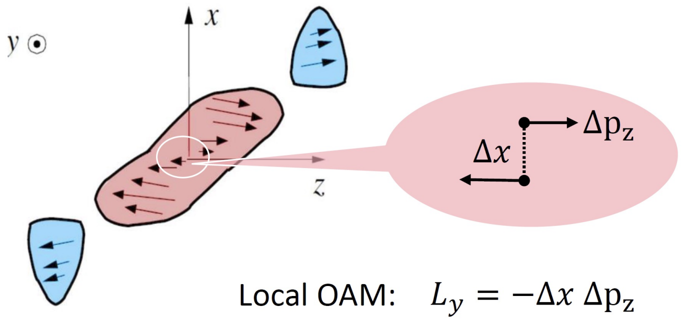


(at thermal equilibrium)

$$\frac{dN_s}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{S})/T}$$

$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

- The original idea was proposed by Liang and Wang



(Figure by J. H. Gao)

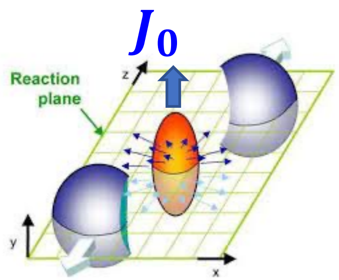
Quark polarization:

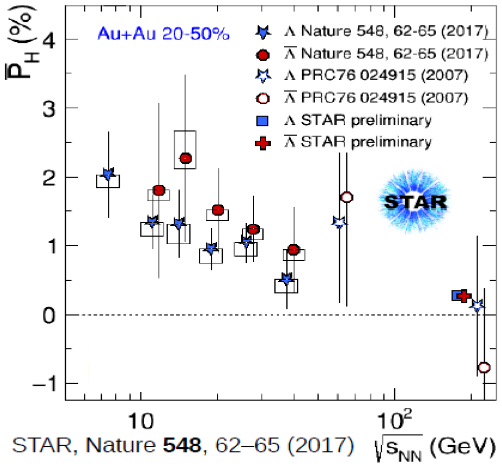
$$\mathrm{P} = -\frac{\pi \mu p}{2E(E+m)}$$

(Liang-Wang 2004)

Global spin polarization: Experiments

- First measurement of Λ global polarization (in rest frame) by STAR@RHIC





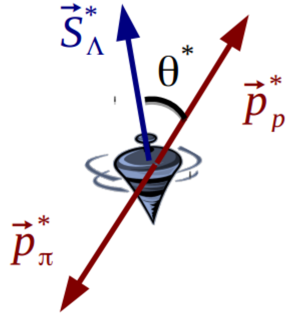
STAR, Nature **548**, 62–65 (2017)

parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \hat{\mathbf{p}}^*_\mathbf{p})$$

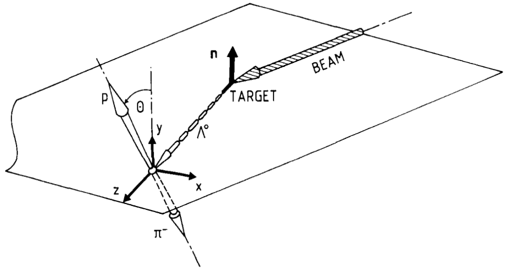
α: Λ decay parameter (α_Λ=0.732)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



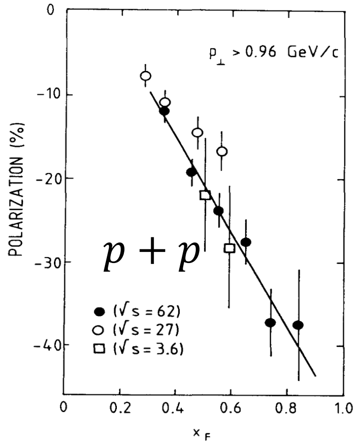
$\Lambda \to p + \pi^-$
(BR: 63.9%, cτ ~7.9 cm)

- Using Λ to study spin physics in p+p, e+p, e+e collisions has a long history

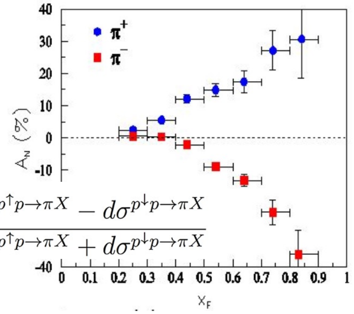
Λ polarization w.r.t production plane



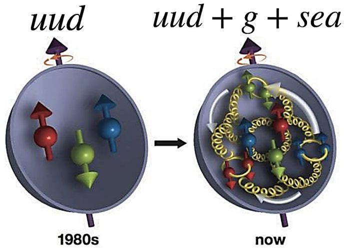
(Lesnik etal 1975; Bunce etal 1976; see e.g. review: Panagiotou 1990)



Useful for understanding e.g. single spin asymmetry, proton spin puzzle, …

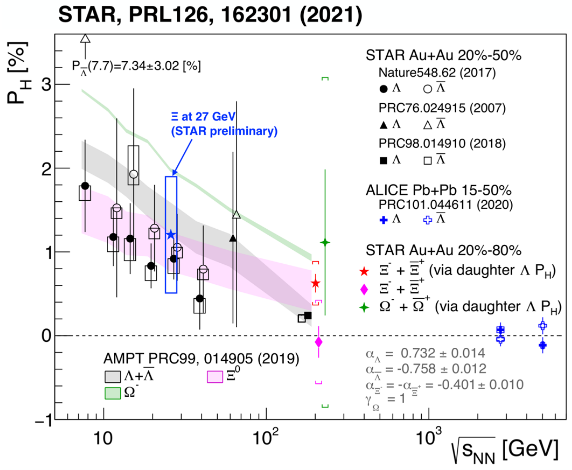


$$A_N = \frac{d\sigma^{p^\uparrow p \to \pi X} - d\sigma^{p^\downarrow p \to \pi X}}{d\sigma^{p^\uparrow p \to \pi X} + d\sigma^{p^\downarrow p \to \pi X}}$$



Global spin polarization: Experiments

- Ξ^-, Ω^- global polarization by STAR@RHIC, Λ global polarization by ALICE@LHC



Theory curves: Wei-Deng-XGH 2019

Exp. method 1:

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha_H \mathbf{P}_H^* \cdot \hat{\boldsymbol{p}}_B^*\right)$$

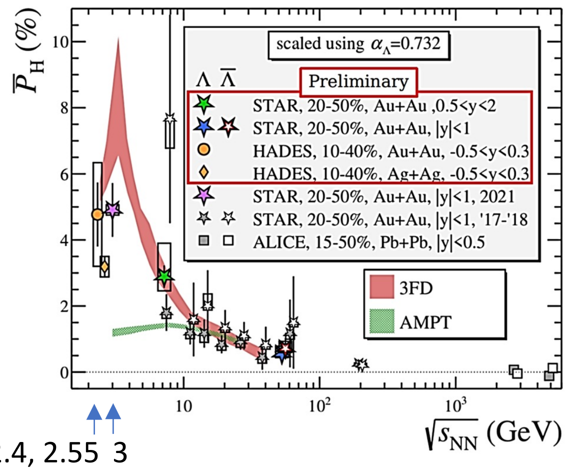
Exp. method 2:

$$\mathbf{P}_\Lambda^* = C_{\Omega^-\Lambda}\mathbf{P}_\Omega^* \qquad \mathbf{P}_\Lambda^* = C_{\Xi^-\Lambda}\mathbf{P}_\Xi^*$$

hyperon	decay mode	α_H	magnetic moment μ_H	spin
Λ (uds)	$\Lambda \to p\pi^-$ (BR: 63.9%)	0.732	-0.613	1/2
Ξ^- (dss)	$\Xi^- \to \Lambda\pi^-$ (BR: 99.9%)	-0.401	-0.6507	1/2
Ω^- (sss)	$\Omega^- \to \Lambda K^-$ (BR: 67.8%)	0.0157	-2.02	3/2

Useful to understand B-field, but still big uncertainties

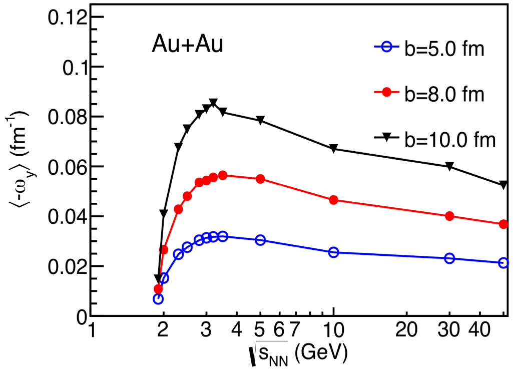
- Global polarization at low energy by STAR@RHIC 2021, HADES@GSI 2021



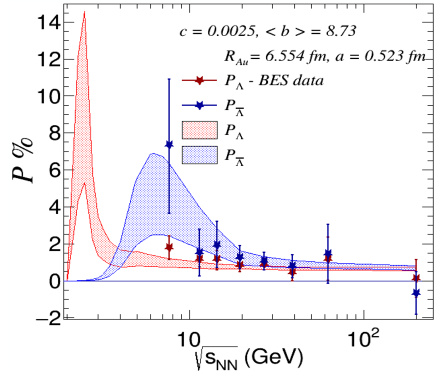
2.4, 2.55 3

Theories suggest peak at low energy

Experiments not see peak till 2.X GeV



(Deng-XGH-Ma-Zhang 2020)



(Ayala etal 2021)

A spin polarization formula

- Global polarization is (mainly) due to global angular momentum (AM)
- Vorticity: a bridge connecting initial AM and final global polarization

An estimate for static spin: $$\boldsymbol{P} = \frac{\langle \boldsymbol{s} \rangle}{s} = \frac{1}{sZ}\mathrm{Tr}\left(\boldsymbol{s}e^{-\beta H + \beta \boldsymbol{s}\cdot\boldsymbol{\omega}}\right) \approx \frac{s+1}{3}\frac{\boldsymbol{\omega}}{T}$$

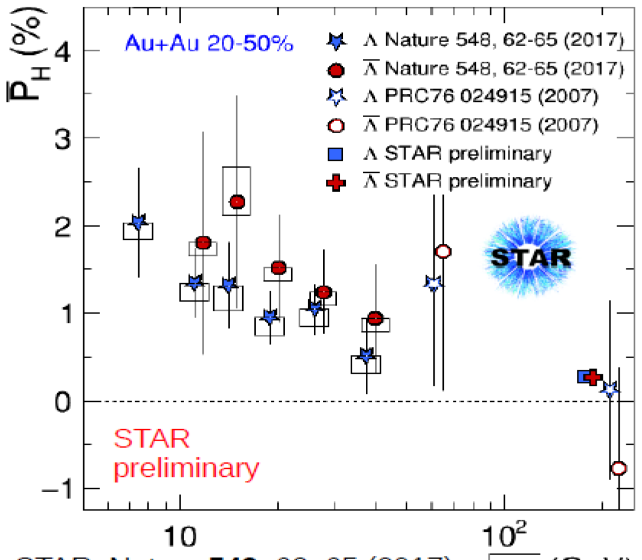
Covariant extension to moving spin-1/2: (Becattini etal 2013, Fang-Pang-Wang-Wang 2016, Liu-Mameda-XGH 2020)

$$P^{\mu}(p) = -\frac{1}{8E_p}\epsilon^{\mu\nu\rho\sigma}p_{\nu}\frac{\int d\Sigma_{\lambda}p^{\lambda}f'(x,p)\varpi_{\rho\sigma}(x)}{\int d\Sigma_{\lambda}p^{\lambda}f(x,p)} + O(\varpi^2)$$

- Valid at global equilibrium in lab frame. $f(x,p)$ is Fermi-Dirac distribution
- Thermal vorticity $\varpi_{\rho\sigma} = \left(\frac{1}{2}\right)\left(\partial_{\sigma}\beta_{\rho} - \partial_{\rho}\beta_{\sigma}\right), \beta_{\mu} = u_{\mu}/T$
- Spin polarization is enslaved to thermal vorticity, not dynamical
- Friendly for numerical simulation (A Cooper-Frye type freeze-out formula)
- When magnetic field is present: $\boldsymbol{\omega} \Rightarrow \boldsymbol{\omega} + s^{-1}\mu_H \boldsymbol{B}$ and $\varpi^{\perp}_{\rho\sigma} \Rightarrow \varpi^{\perp}_{\rho\sigma} - 2\beta\mu_H F^{\perp}_{\rho\sigma}$

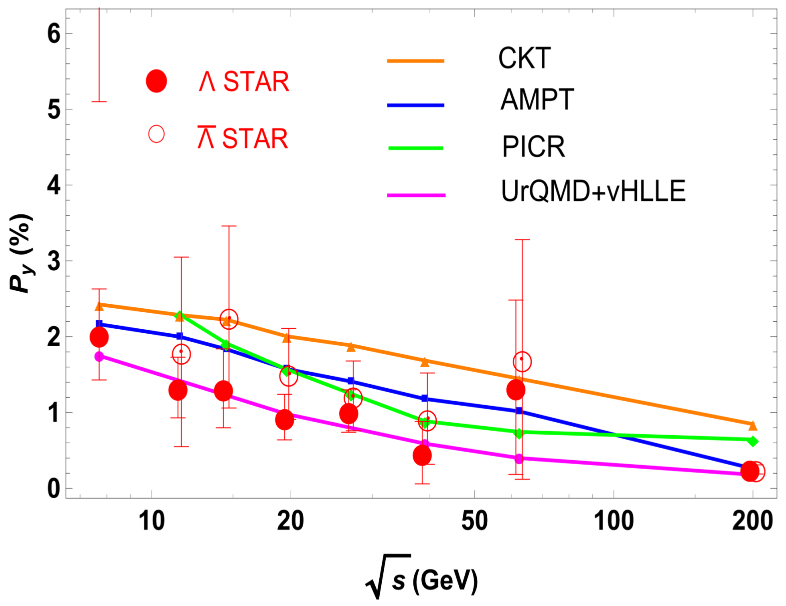
Global spin polarization: Vorticity

Λ hyperons: Experiment = Theory



STAR, Nature **548**, 62–65 (2017)

$$\langle\omega\rangle = \langle T(P_\Lambda + P_{\bar{\Lambda}})\rangle_{\sqrt{s}=7-200\mathrm{GeV}}$$
$$\approx (9 \pm 1) \times 10^{21}\mathrm{s}^{-1}$$



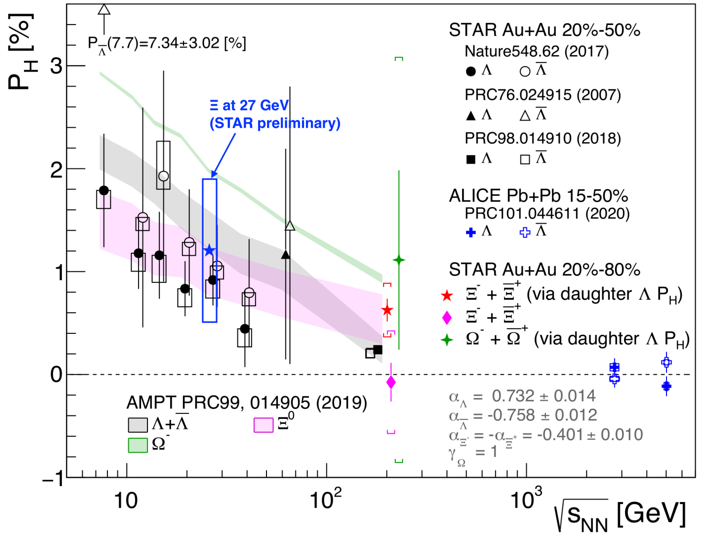
(Li-Pang-Wang-Xia 2017; Sun-Ko 2017; Wei-Deng-XGH 2019; Xie-Wang-Csernai 2017; Karpenko-Becattini 2016)

(See also: Sun-Ko etal 2019; Xie-Wang-Csernai etal 2018-2021; Ivanov etal 2017-2019; Liao etal 2018-2021; Deng-XGH-Ma 2021; ……)

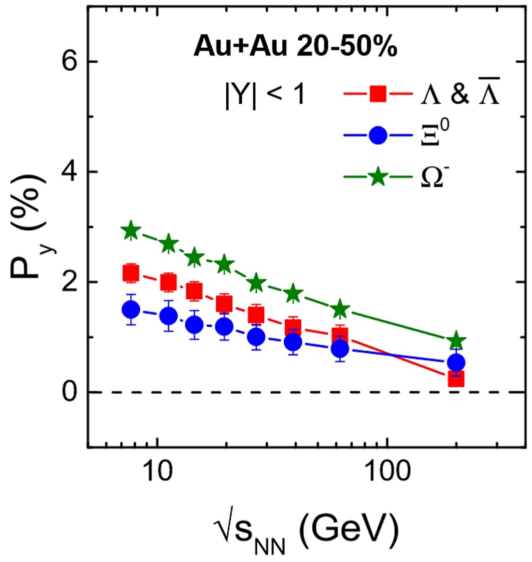
Global spin polarization: Vorticity

Ξ, Ω hyperons: Experiment = Theory

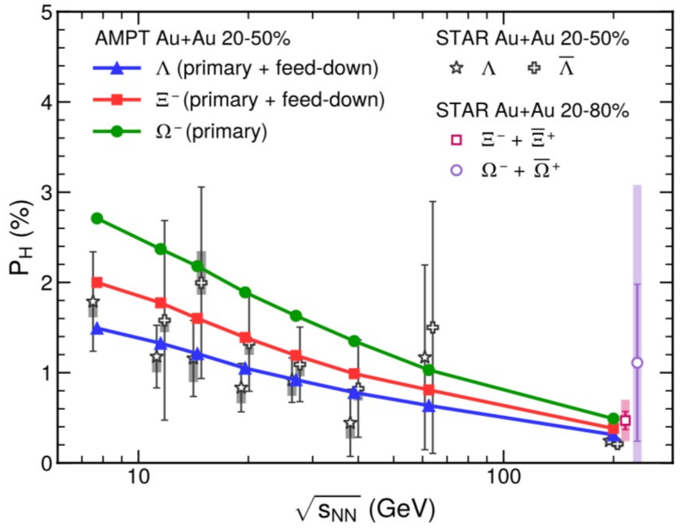
STAR, PRL126, 162301 (2021)



(Wei-Deng-XGH 2019)



(Li-Xia-XGH-Huang 2021)



Vorticity interpretation of global spin polarization works well!

Local spin polarization

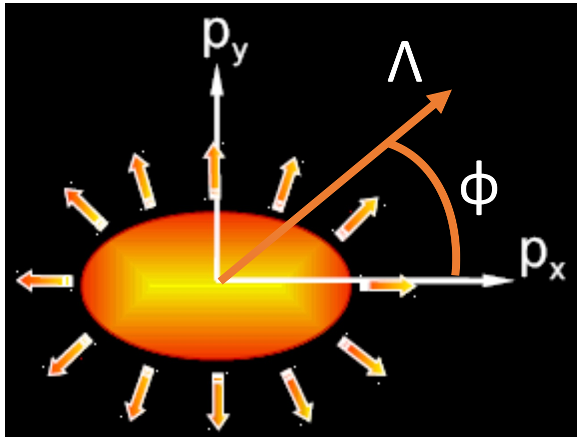
Local Λ spin polarization

The global Λ polarization reflects the total amount of angular momentum retained in the mid-rapidity region. How is it distributed in different ϕ ?

- Spin harmonic flow: $$\frac{dP_{y,z}}{d\phi} = \frac{1}{2\pi}\left[P_{y,z} + 2f_{2y,z}\sin(2\phi) + 2g_{2y,z}\cos(2\phi) + \cdots\right]$$

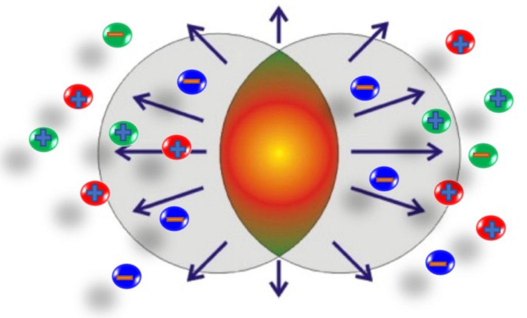
Azimuthal angle ϕ

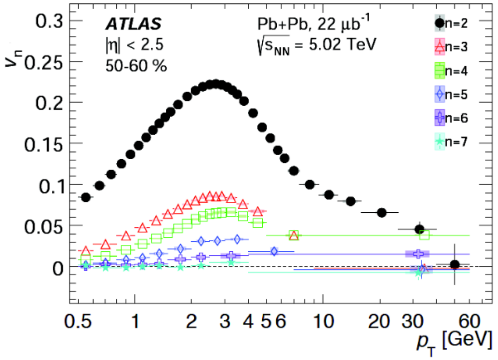
f_2, g_2: Spintronics analogue of elliptic flows



⊙ z direction

$$\frac{dN_{\rm ch}}{d\phi} \propto 1 + 2v_1\cos(\phi - \Psi_1) + 2v_2\cos[2(\phi - \Psi_2)] + \cdots$$





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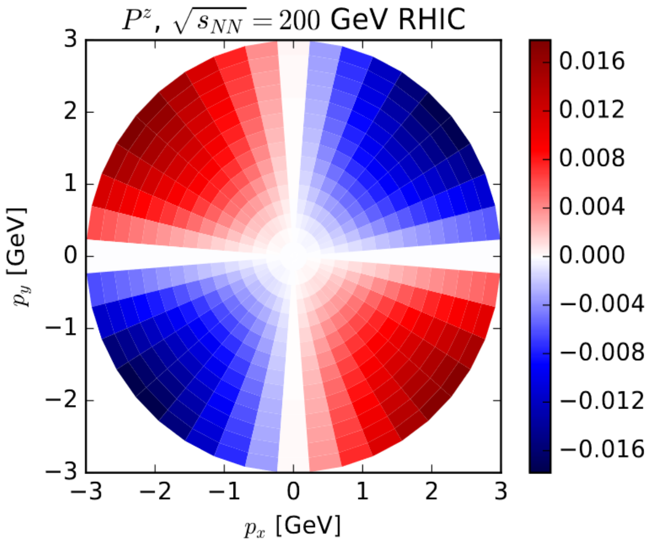
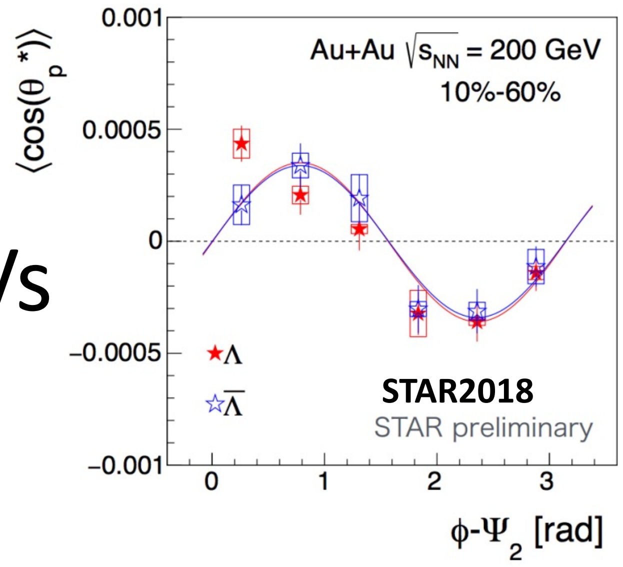
1) longitudinal polarization vs ϕ

2) Transverse polarization vs ϕ

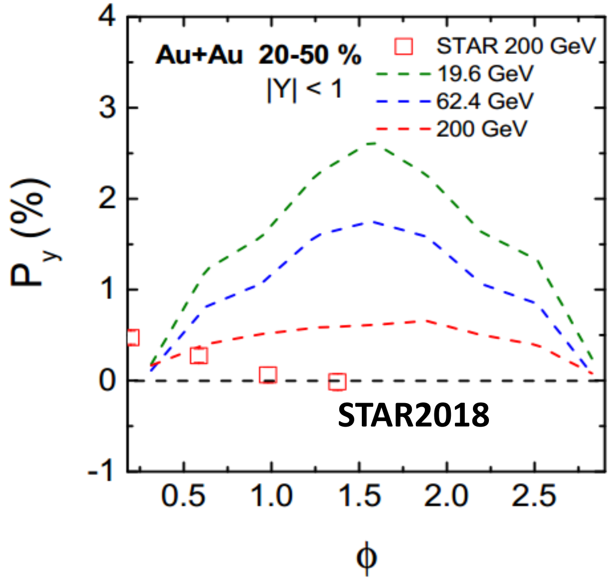
(Becattini-Karpenko 2018)



Vs



(Wei-Deng-XGH 2019)



$f_{2z}^{\mathrm{ther}} < 0$

$f_{2z}^{\mathrm{exp}} > 0$

$g_{2y}^{\mathrm{ther}} < 0, g_{2y}^{\mathrm{exp}} > 0$

How to resolve the local spin polarization puzzles

Attack the spin sign problem from theory side:

- Understand the vorticity (☺)
- Effect of feed-down decays is not enough (☺)
- Go beyond equilibrium treatment (spin as a dynamic d.o.f)

  spin hydrodynamics

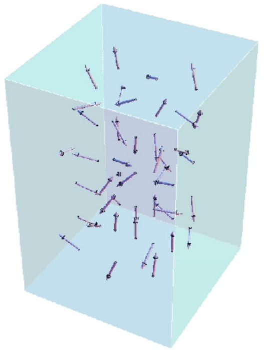
  spin kinetic theory
- Initial condition

  (Initial polarization, initial flow, … …)
- Other possibilities

  (chiral vortical effect (Liu-Sun-Ko 2019), mesonic mean-field(Csernai-Kapusta-Welle 2019), other spin chemical potential (Wu-Pang-XGH-Wang 2019, Florkowski etal 2019), contribution from shear flow (Becattini etal 2021, Fu-Liu-Pang-Song-Yin 2021, Yi-Pu-Yang 2021), contribution from gluons, … …)

Revisit spin polarization formula

- Consider a local Gibbs state for spin-1/2 fermions* (Zubarev etal 1979, Van Weert 1982, Becattini etal 2013)



Canonical stress tensor   Canonical spin tensor

$$\hat{\rho}_{\mathrm{LG}} = \frac{1}{Z_{\mathrm{LG}}} \exp\left\{ - \int_{\Xi} d\Xi_\mu(y) \left[\hat{\Theta}^{\mu\nu}(y)\beta_\nu(y) - \frac{1}{2}\hat{\Sigma}^{\mu\rho\sigma}(y)\mu_{\rho\sigma}(y) \right] \right\}$$

Thermal flow vector   Spin potential

- The corresponding Wigner function

$$W(x,p) = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{LG}} \hat{W}(x,p)\right] = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{LG}} \int d^4 s e^{-ip\cdot s} \hat{\bar{\psi}}\left(x + \frac{s}{2}\right) \otimes \hat{\psi}\left(x - \frac{s}{2}\right)\right]$$

- The canonical spin vector in phase space

$$S^\mu(x,p) = -\frac{1}{6}\epsilon^{\mu\nu\rho\sigma}\Sigma_{\nu\rho\sigma}(x,p) = -\frac{1}{24}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}_{\mathrm{D}}\left[\{\gamma_\nu, \Sigma_{\rho\sigma}\} W(x,p)\right]$$

* Obtained by maximizing Von Neumann entropy under local constraints of stress and angular momentum tensors:

$s = -\mathrm{Tr}\left(\hat{\rho}\ln\hat{\rho}\right)$ with $n_\mu \mathrm{Tr}(\hat{\rho}\,\hat{\Theta}^{\mu\nu}) = n_\mu \Theta^{\mu\nu}$ and $n_\mu \mathrm{Tr}(\hat{\rho}\,\hat{\Sigma}^{\mu\rho\sigma}) = n_\mu \Sigma^{\mu\rho\sigma}$

Revisit spin polarization formula

- For Dirac fermions (on-shell) at local equilibrium (Liu-XGH 2021; Buzzegoli 2021)

$$\bar{S}_\mu(p) = \bar{S}_{5\mu}(p) - \frac{1}{8\int d\Xi \cdot p \;\; n_F} \int d\Xi \cdot p \frac{n_F(1-n_F)}{E_p} \left\{ \epsilon_{\mu\nu\alpha\beta} p^\nu \mu^{\alpha\beta} + 2 \frac{\varepsilon_{\mu\nu\rho\sigma} p^\rho n^\sigma}{p \cdot n} \left[p_\lambda (\xi^{\nu\lambda} + \Delta\mu^{\nu\lambda}) + \partial^\nu \alpha \right] \right\}$$

$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$: Thermal shear tensor $\qquad \alpha = -\beta\mu$: Baryon chemical potential

$\Delta\mu_{\rho\sigma} = \mu_{\rho\sigma} - \varpi_{\rho\sigma}$ with $\varpi_{\rho\sigma} = \frac{1}{2}(\partial_\sigma \beta_\rho - \partial_\rho \beta_\sigma)$ thermal vorticity tensor

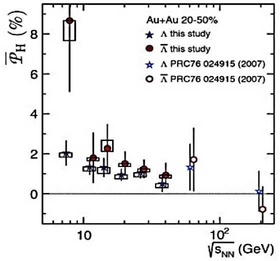
$\bar{S}_5^\mu$ is the polarization induced by finite chirality

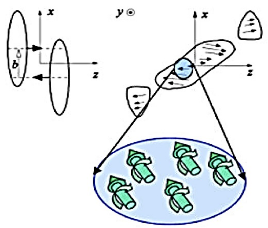
- This is a Cooper-Frye type formula for spin polarization

Recall Cooper-Frye formula for number spectrum:

$$N(p) = \int d\Xi_\mu \frac{p^\mu}{E_p} f(T(x), u^\mu(x), \mu(x))$$

$$\bar{S}^\mu(p) \quad \Leftarrow \quad T(x), u^\alpha(x), \mu(x), \mu_{\alpha\beta}(x)$$





Revisit spin polarization formula

- It is worth writing down different components in non-rel. form in phase space:

$$\boldsymbol{S} = \boldsymbol{S}_{(\mu)} + \boldsymbol{S}_{(\omega)} + \boldsymbol{S}_{(\sigma)} + \boldsymbol{S}_{(T)} + \boldsymbol{S}_{(\alpha)}$$

- Spin potential: (Buzzegoli 2021, Liu-XGH 2021)

$$S^i_{(\mu)}(x,\mathbf{p}) = \left[\frac{\mu^i}{2} - \frac{\mathbf{p}^2\mu^i - \boldsymbol{\mu}\cdot\mathbf{p}\,p^i}{2E_p^2}\right] n_F(1-n_F) \quad \text{with} \quad n_F = n_F(\alpha + \beta E_p)$$

- Vorticity: (Becattini etal 2013, Fang etal 2016, Liu etal 2020)

$$S^i_{(\omega)}(x,\mathbf{p}) = \frac{\mathbf{p}^2\omega^i - \boldsymbol{\omega}\cdot\mathbf{p}\,p^i}{2T\,E_p^2} n_F(1-n_F) \quad \text{with} \quad \boldsymbol{\omega} = (\boldsymbol{\nabla}\times\boldsymbol{v})/2$$

- Shear tensor: (Becattini-Buzzegoli-Palermo 2021, Liu-Yin 2021)

$$S^i_{(\sigma)}(x,\mathbf{p}) = \frac{\epsilon^{ijk}\,p^j p^l\,\sigma_{kl}}{2T\,E_p^2} n_F(1-n_F) \quad \text{with} \quad \sigma_{ij} = (\partial_i v_j + \partial_j v_i + 2\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{v}/3)/2$$

- T gradient: (Becattini etal 2013, Fang etal 2016, Liu etal 2020)

$$S^i_{(T)}(x,\mathbf{p}) = -\frac{(\mathbf{p}\times\boldsymbol{\nabla}T)^i}{2T^2\,E_p} n_F(1-n_F)$$

- Chemical potential: (Fang etal 2016, Liu-XGH 2021, Yi etal 2021, Fu etal 2022)

$$S^i_{(\alpha)}(x,\mathbf{p}) = \frac{(\mathbf{p}\times\boldsymbol{\nabla}\alpha)^i}{2E_p^2} n_F(1-n_F)$$

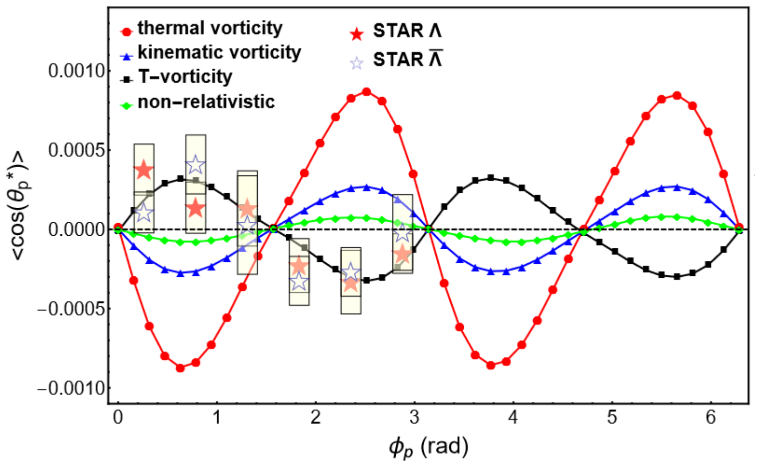
Temperature vorticity as spin chemical potential

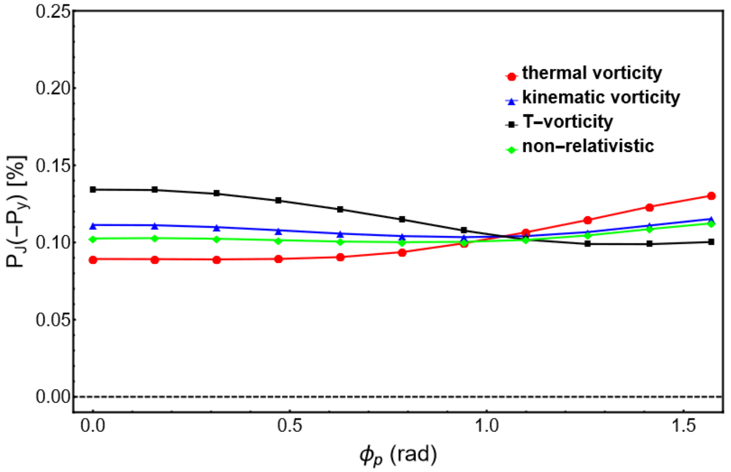
- Recall

$$S^\mu(x,p) = -\frac{1}{E_p}\left[\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}p_\nu\mu_{\alpha\beta} + \frac{1}{2p\cdot n}\epsilon^{\mu\nu\alpha\beta}\left(\xi_{\nu\lambda}+\Delta\mu_{\nu\lambda}\right)n_\beta p_\alpha p^\lambda\right] n_F(1-n_F) + O(\mu^2_{\rho\sigma},\partial^2)$$

- Relax the global equilibrium condition (1)

$$\partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0 \qquad\qquad \mu_{\rho\sigma} = \frac{1}{2T^2}\left[\partial_\sigma(Tu_\rho) - \partial_\rho(Tu_\sigma)\right]$$





(Wu-Pang-XGH-Wang 2019)

Shear tensor contribution

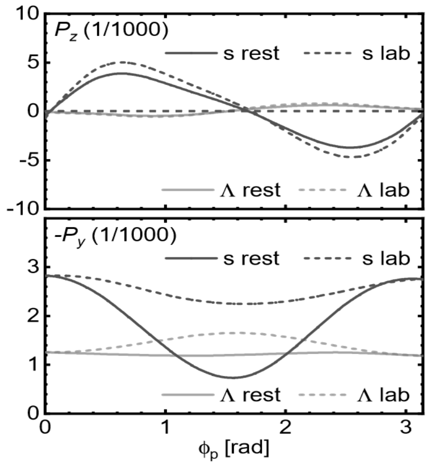
- Recall

$$S^{\mu}(x,p) = -\frac{1}{E_p}\left[\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}p_{\nu}\mu_{\alpha\beta} + \frac{1}{2p\cdot n}\epsilon^{\mu\nu\alpha\beta}\left(\xi_{\nu\lambda}+\Delta\mu_{\nu\lambda}\right)n_{\beta}p_{\alpha}p^{\lambda}\right]n_F(1-n_F) + O(\mu_{\rho\sigma}^2, \partial^2)$$

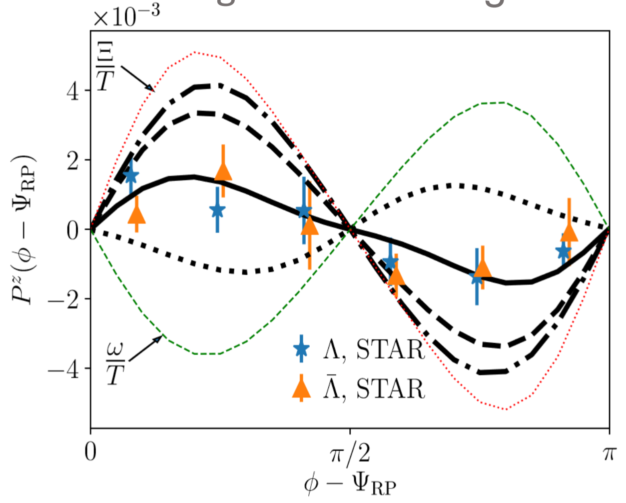
- Relax the global equilibrium condition (2) (Becattini-Buzzegoli-Palermo 2021, Liu-Yin 2021)

$$\partial_{\mu}\beta_{\nu} + \partial_{\nu}\beta_{\mu} \neq 0 \qquad \mu_{\rho\sigma} = \varpi_{\rho\sigma} = \frac{1}{2}\left(\partial_{\sigma}\beta_{\rho} - \partial_{\rho}\beta_{\sigma}\right)$$

(Fu-Liu-Song-Yin 2021)



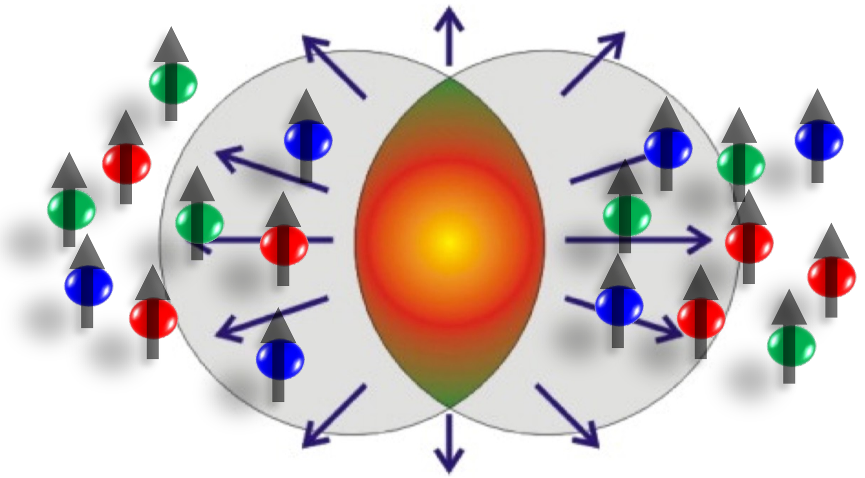
(Becattini-Buzzegoli-Palermo-Inghirami-Karpenko 2021)



(See also Yi-Pu-Yang 2021; Florkowski-Kumar-Mazeliauskas-Ryblewski 2021; Sun-Zhang-Ko-Zhao 2021; Alzhrani-Ryu-Shen 2022)

Dynamic description of spin polarization

- Local spin polarization is still a puzzle.
- Dynamic theories for spin transport and polarization are needed
  - Spin hydrodynamics: Fluid velocity, temperature, and spin density evolve together
  - Spin kinetic theory: Particle and spin phase-space distribution functions evolve together
- A lot of theoretical progress since 2019



Spin alignment of vector mesons

Spin density matrix

- Spin state is conveniently described by spin density matrix (SDM)
- Choose a direction (e.g. y-axis in lab frame) to quantize spin.
- For spin-1/2 particles (e.g. quarks), a state with only polarization in y-direction:

$$\rho^q = \frac{1}{2}\left(1 + P_y^q \sigma_y\right) = \frac{1}{2}\begin{pmatrix} 1 + P_y^q & 0 \\ 0 & 1 - P_y^q \end{pmatrix}$$

- A spin-1 particle (e.g. ϕ meson), SDM is 3×3 matrix in basis $|1\rangle, |0\rangle, |-1\rangle$

$$\rho^V = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix}$$

- Hermiticity and unit-trace constrain that only 8 elements are independent:
  3 form a vector and 5 form a rank-2 spherical tensor

Spin density matrix

- Consider recombination $q + \bar{q} \to \phi$
- ρ^V is obtained by $\rho^q \otimes \rho^{\bar{q}}$ projected onto spin-1 subspace

$$\rho^V = \begin{pmatrix} \frac{(1+P_y^q)(1+P_y^{\bar{q}})}{3+P_y^q p_y^{\bar{q}}} & 0 & 0 \\ 0 & \frac{1-P_y^q P_y^{\bar{q}}}{3+P_y^q p_y^{\bar{q}}} & 0 \\ 0 & 0 & \frac{(1-P_y^q)(1-P_y^{\bar{q}})}{3+P_y^q p_y^{\bar{q}}} \end{pmatrix}$$

- Vorticity dominance: $P_q = P_{\bar{q}}$, $\rho_{00} < \frac{1}{3}$

  Magnetic-field dominance: $P_q = -P_{\bar{q}}$, $\rho_{00} > \frac{1}{3}$
- The results of Λ polarization suggests vorticity dominance

(Liang-Wang 2004)

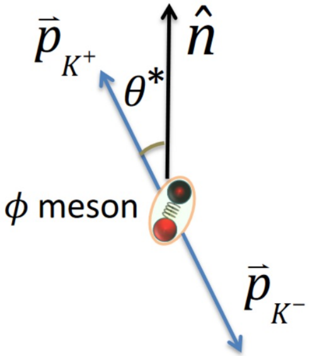
$$\rho_{00} = \frac{1-P_y^2}{3+P_y^2} \approx \frac{1}{3} - \frac{4}{9}P_y^2$$

- Spin alignment: $\rho_{00} - \frac{1}{3}$

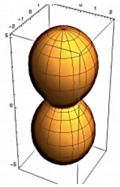
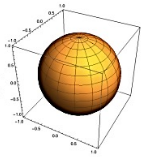
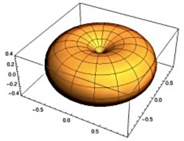
Expectation: spin alignment is a 10^{-4} level phenomenon

Spin alignment

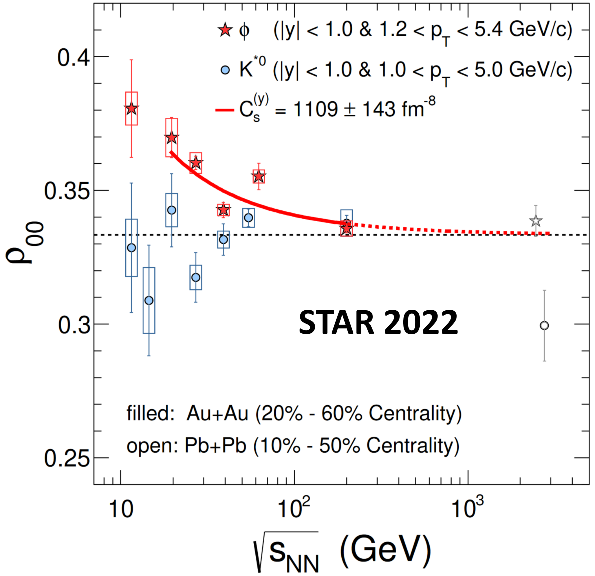
- Main decay channels, parity-even strong decay: $\phi \to KK,\ K^{*0} \to K\pi$



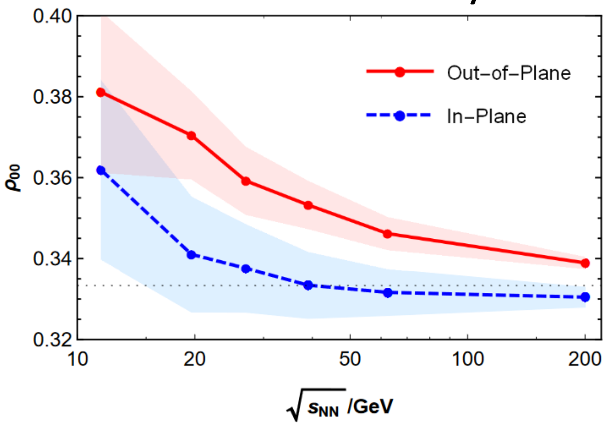
$$\frac{dN}{d(\cos\theta^*)} = N_0 \times \left[(1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta^* \right]$$

$\rho_{00}>1/3$:  $\rho_{00}=1/3$:  $\rho_{00}<1/3$: 

- Global spin alignment **Puzzle: ϕ-meson $\rho_{00} > \frac{1}{3}$ and too big!**



A recent theory

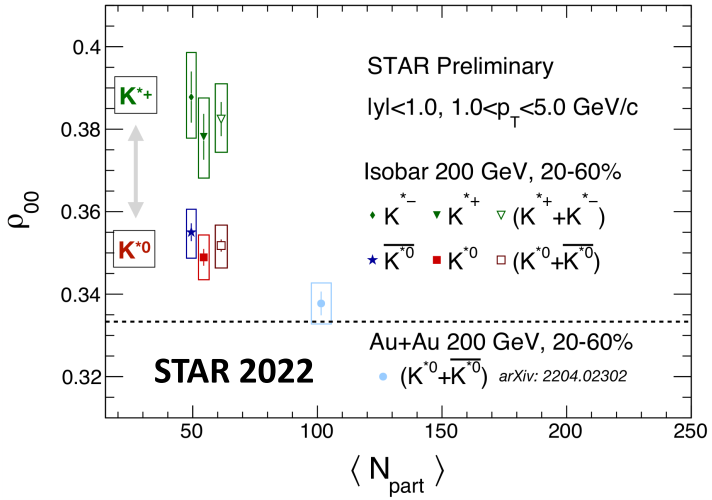


$$\rho_{00}(x,\mathbf{0}) - \frac{1}{3} \propto \left\langle (g_\phi \mathbf{B}^\phi_{x(y)})^2 \right\rangle$$
$$\left\langle (g_\phi \mathbf{E}^\phi_{x(y)})^2 \right\rangle$$

(Sheng-Oliva-Liang-Wang-Wang 2022)

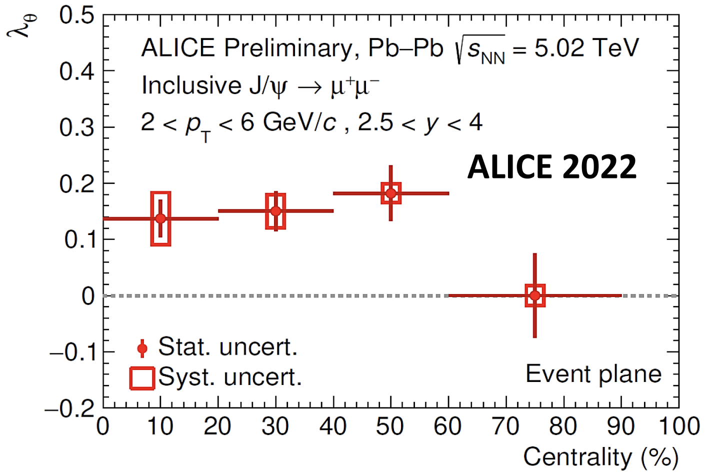
Spin alignment

- Global spin alignment



Puzzle: charge sensitive but $K^{*+}(u\bar{s})$ larger than $K^{*0}(d\bar{s})$, opposite to B-effect

- Global spin alignment of $J/\psi \to l^+l^-$

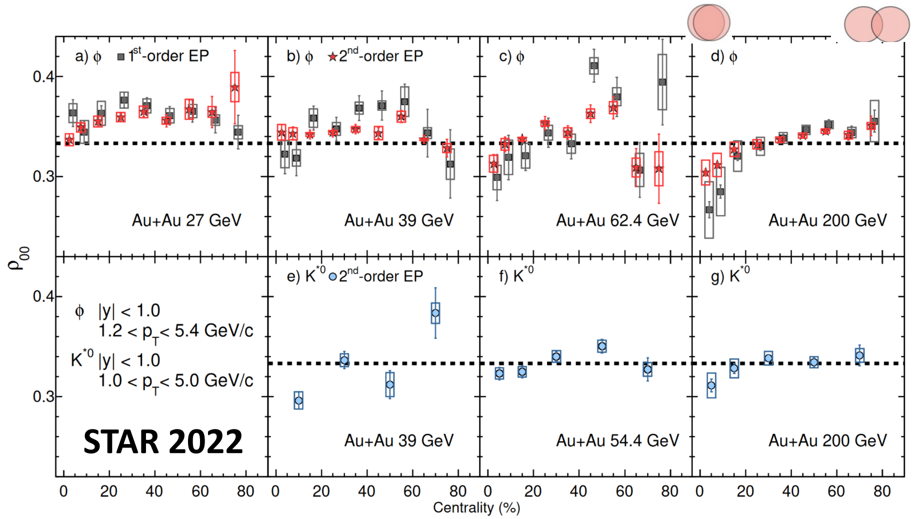


$$W(\theta) \propto \frac{1}{3+\lambda_\theta}\left(1+\lambda_\theta\cos^2\theta\right) \qquad \rho_{00}-\frac{1}{3}=\frac{2}{3}\frac{\lambda_\theta}{3+\lambda_\theta}$$

Puzzle: Λ polarization at LHC is very small but a big J/ψ spin alignment

Local spin alignment

- Centrality dependence

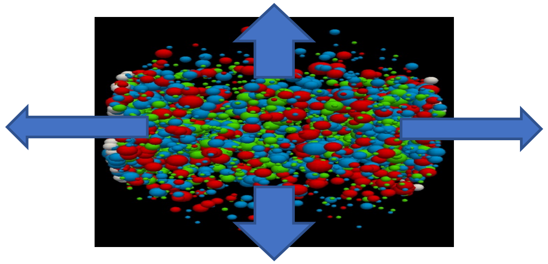


Central: $\rho_{00} < \frac{1}{3}$

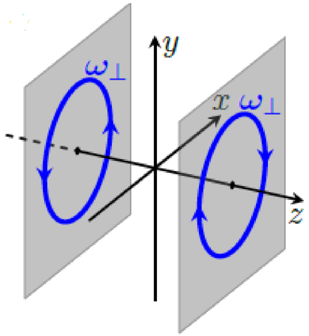
Noncentral: $\rho_{00} > \frac{1}{3}$

More significant at higher energies

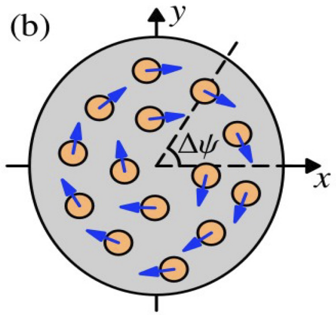
- Local spin alignment



Central collisions



$$\mathbf{P}^{q,\bar{q}} = (P_x^{q,\bar{q}}, P_y^{q,\bar{q}}, P_z^{q,\bar{q}})$$



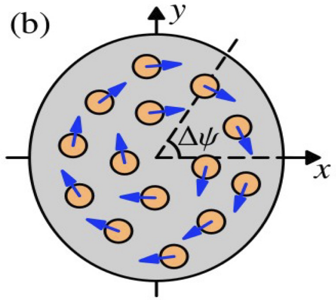
Quark spin density matrix:

$$\rho^{q,\bar{q}} = \frac{1}{2}\begin{pmatrix} 1 + P_y^{q,\bar{q}} & P_z^{q,\bar{q}} - iP_x^{q,\bar{q}} \\ P_z^{q,\bar{q}} + iP_x^{q,\bar{q}} & 1 - P_y^{q,\bar{q}} \end{pmatrix}$$

More significant at higher energies

Local spin alignment

- Vector meson spin density matrix element



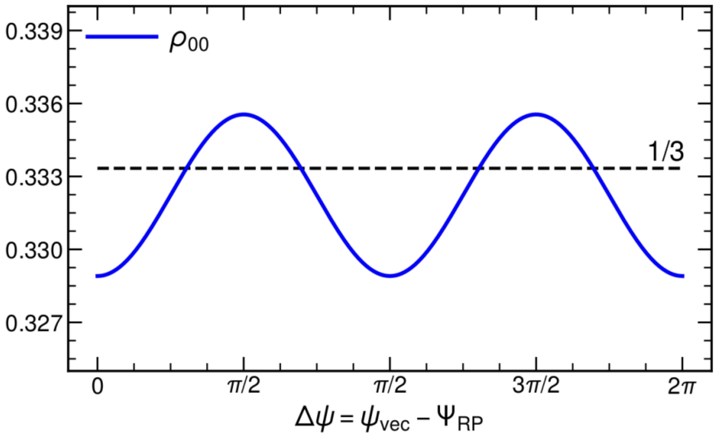
$$P_x^{q,\bar{q}}(\Delta\psi) = F_\perp \sin(\Delta\psi)$$
$$P_y^{q,\bar{q}}(\Delta\psi) = -F_\perp \cos(\Delta\psi)$$
$$P_z^{q,\bar{q}}(\Delta\psi) = 0$$

$$\Rightarrow \quad \rho_{00} = \frac{1 - P_y^q P_y^{\bar{q}} + P_x^q P_x^{\bar{q}} + P_z^q P_z^{\bar{q}}}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}} \approx \frac{1}{3} - \frac{F_\perp^2}{9} - \frac{F_\perp^2}{3}\cos(2\Delta\psi)$$

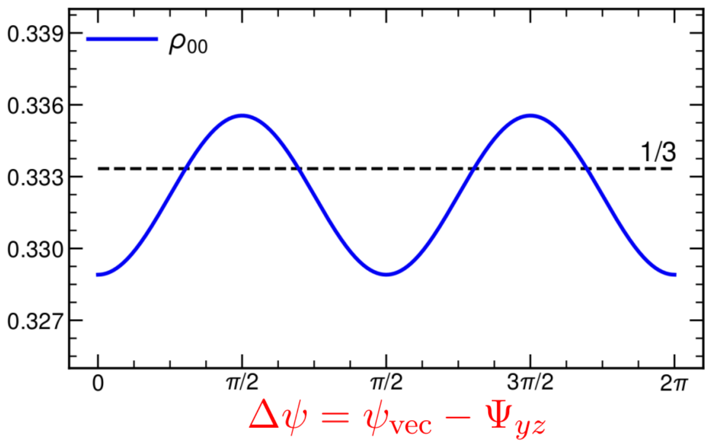
(Xia-Li-XGH-Huang 2020)

- More experimental verification of this scenario is needed

1) Measure azimuthal angle dependence



2) Measure ρ_{00} w.r.t other plane, e.g., yz plane



Local spin alignment unchanged, but global one may change significantly

Summary

Global angular momentum

Inhomogeneous expansion

⋮

→ Vortical strongly interacting matter →

Hyperon spin polarization

Vector meson spin alignment

… …

↓

Global hyperon polarization ☺

Local hyperon polarization ☹

Vector meson spin alignment ☹

↓

Exotics in a vortical fluid?

Thank you

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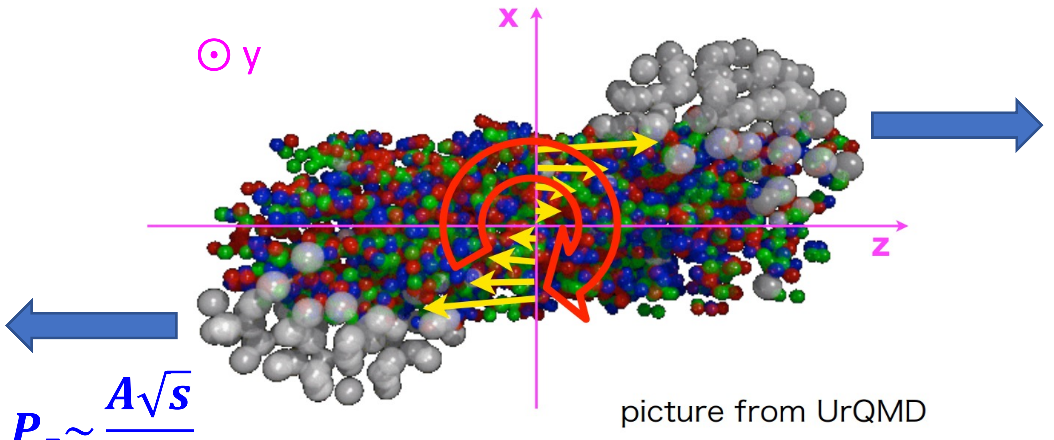
Global angular momentum



$P_z \sim \frac{A\sqrt{s}}{2}$

Global angular momentum

$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

(RHIC Au+Au 200 GeV, b=10 fm)