

Exotic hadrons and light nuclei in HIC

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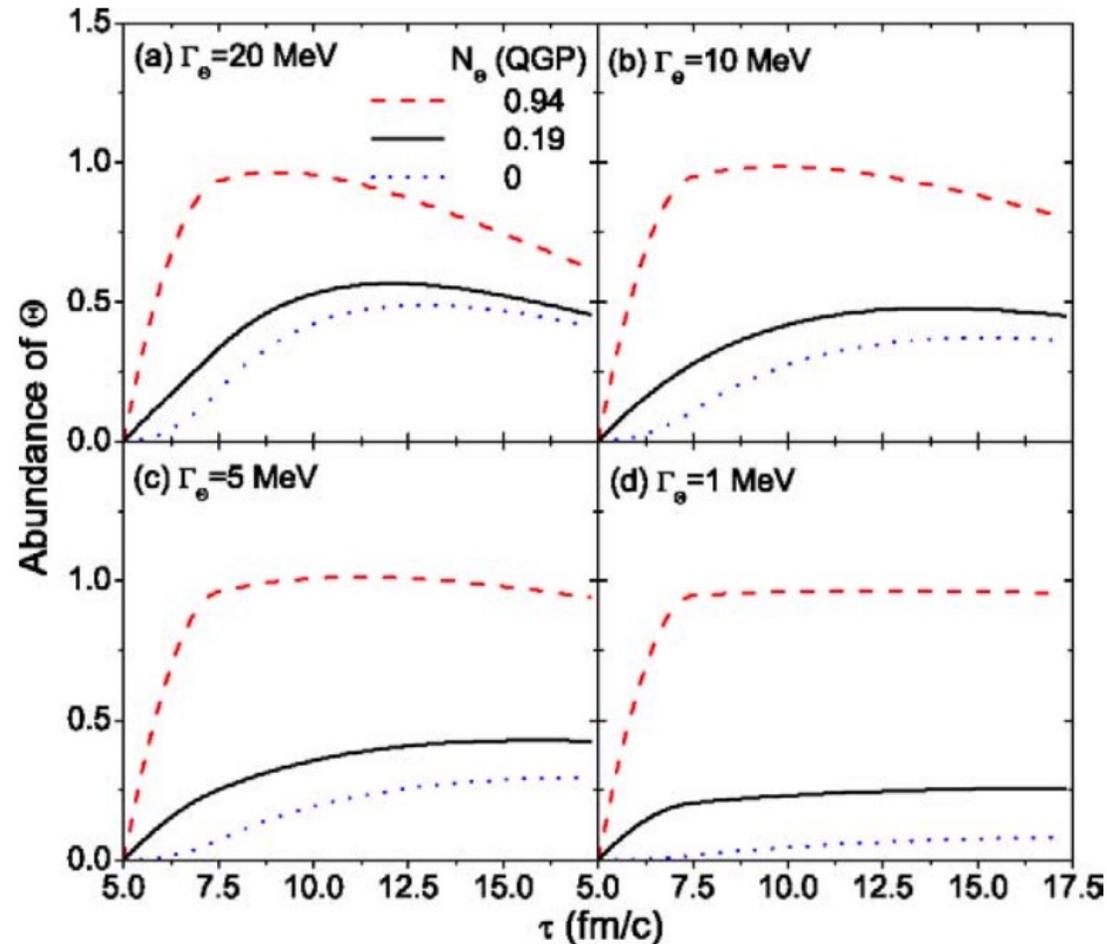
- Introduction
- X(3872)
- Light nuclei
- Summary

In collaboration with Sungtae Cho and Su Hounng Lee,
as well as with Kaijia Sun and Rui Wang

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Θ^+ (1530) production at RHIC

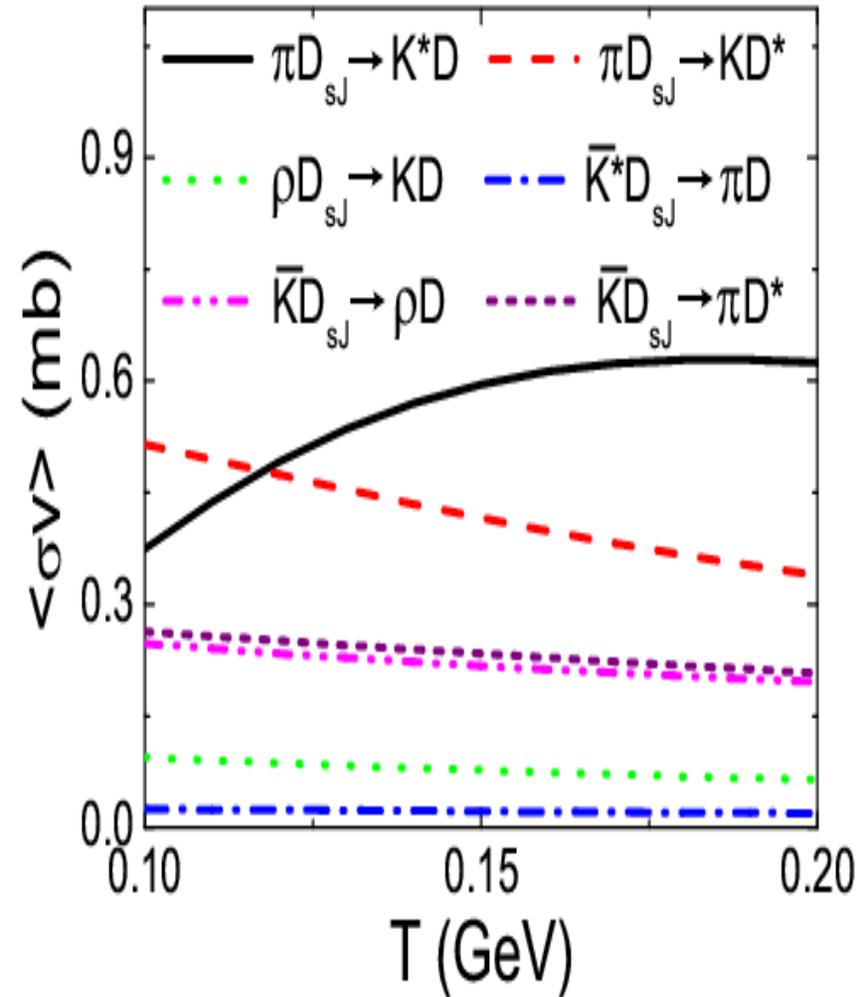
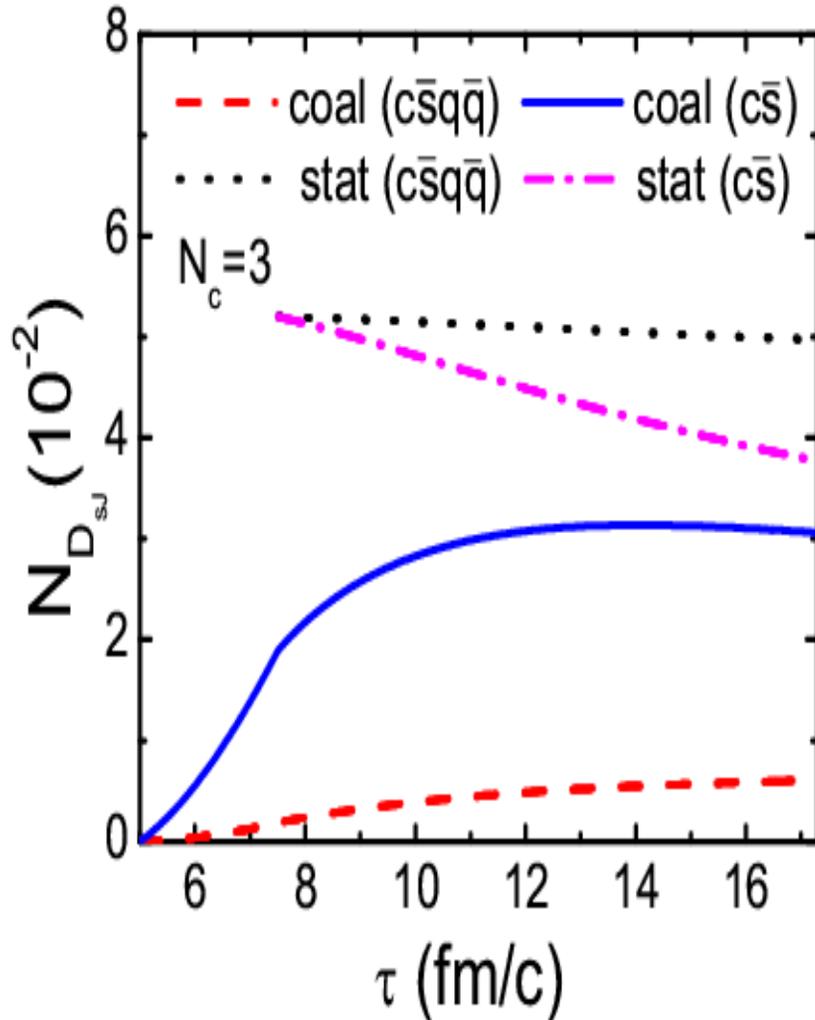
Chen, Greco, Ko, Lee & Liu, PLB 601, 34 (2004)



- Include reactions $KN \leftrightarrow \Theta$ (~ 1 mb), $KN \leftrightarrow \pi\Theta$ (~ 0.3 mb), and $\pi N \leftrightarrow \bar{K}\Theta$ ($1-3 \mu\text{b}$) during hadronic evolution.
- Final Θ yield is sensitive to its width and also its initial number produced from QGP by quark coalescence.

$D_{sJ}(2317)$ production at RHIC

Chen, Liu, Nielsen & Ko, PRC 76, 064903 (2007)



- Cross sections for strangeness-exchange reactions are for four-quark state and are larger by ~ 9 for two-quark state.
- Final yield is sensitive to the quark structure of $D_{sJ}(2317)$.

Charmed exotics in heavy ion collisions

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Abstract. Based on the color–spin interaction in diquarks, we argue that charmed multiquark hadrons are likely to exist. Because of the appreciable number of charm quarks produced in central nucleus–nucleus collisions at ultrarelativistic energies, the production of charmed multiquark hadrons is expected to be enhanced in these collisions. Using both the quark coalescence model and the statistical hadronization model, we estimate the yield of charmed tetraquark mesons, T_{cc} , and pentaquark baryons, Θ_{cs} , in heavy ion collisions at RHIC and LHC. We further discuss the decay modes of these charmed exotic hadrons in order to facilitate their detections in experiments.

Exotic mesons, baryons and dibaryons

Cho, Furumoto, Hyodo, Jido, Ko, Lee, Nielsen, Ohnishi, Sekihara, Yasui, and Yazaki [ExHIC Collaboration], PRL 106, 212001 (2011); PRC 84, 064910 (2011); PPNP 95, 279 (2017)

Particle	m (MeV)	g	I	J^P	$2q/3q/6q$	$4q/5q/8q$	Mol.	$\omega_{\text{Mol.}}$ (MeV)	Decay mode
Mesons									
$f_0(980)$	980	1	0	0^+	$q\bar{q}, s\bar{s}(L=1)$	$q\bar{q}s\bar{s}$	$\bar{K}K$	67.8(B)	$\pi\pi$ (Strong decay)
$a_0(980)$	980	3	1	0^+	$q\bar{q}(L=1)$	$q\bar{q}s\bar{s}$	$\bar{K}K$	67.8(B)	$\eta\pi$ (Strong decay)
$K(1460)$	1460	2	$1/2$	0^-	$q\bar{s}$	$q\bar{q}q\bar{s}$	$\bar{K}KK$	69.0(R)	$K\pi\pi$ (Strong decay)
$D_s(2317)$	2317	1	0	0^+	$c\bar{s}(L=1)$	$q\bar{q}c\bar{s}$	DK	273(B)	$D_s\pi$ (Strong decay)
T_{cc}^{1a}	3797	3	0	1^+	—	$qqc\bar{c}$	$\bar{D}\bar{D}^*$	476(B)	$K^+\pi^- + K^+\pi^- + \pi^-$
$X(3872)$	3872	3	0	$1^+, 2^-^c$	$c\bar{c}(L=2)$	$q\bar{q}c\bar{c}$	$\bar{D}\bar{D}^*$	3.6(B)	$J/\psi\pi\pi$ (Strong decay)
$Z^+(4430)^b$	4430	3	1	0^-^c	—	$q\bar{q}c\bar{c}(L=1)$	$D_1\bar{D}^*$	13.5(B)	$J/\psi\pi$ (Strong decay)
T_{cb}^{0a}	7123	1	0	0^+	—	$qqc\bar{b}$	$\bar{D}B$	128(B)	$K^+\pi^- + K^+\pi^-$
Baryons									
$\Lambda(1405)$	1405	2	0	$1/2^-$	$qqqs(L=1)$	$qqqs\bar{q}$	$\bar{K}N$	20.5(R)–174(B)	$\pi\Sigma$ (Strong decay)
$\Theta^+(1530)^b$	1530	2	0	$1/2^+^c$	—	$qqqq\bar{s}(L=1)$	—	—	KN (Strong decay)
$\bar{K}KN^a$	1920	4	$1/2$	$1/2^+$	—	$qqqs\bar{s}(L=1)$	$\bar{K}KN$	42(R)	$K\pi\Sigma, \pi\eta N$ (Strong decay)
$\bar{D}N^a$	2790	2	0	$1/2^-$	—	$qqqq\bar{c}$	$\bar{D}N$	6.48(R)	$K^+\pi^-\pi^- + p$
\bar{D}^*N^a	2919	4	0	$3/2^-$	—	$qqqq\bar{c}(L=2)$	\bar{D}^*N	6.48(R)	$\bar{D} + N$ (Strong decay)
Θ_{cs}^a	2980	4	$1/2$	$1/2^+$	—	$qqqs\bar{c}(L=1)$	—	—	$\Lambda + K^+\pi^-$
BN^a	6200	2	0	$1/2^-$	—	$qqqq\bar{b}$	BN	25.4(R)	$K^+\pi^-\pi^- + \pi^+ + p$
B^*N^a	6226	4	0	$3/2^-$	—	$qqqq\bar{b}(L=2)$	B^*N	25.4(R)	$B + N$ (Strong decay)
Dibaryons									
H^a	2245	1	0	0^+	$qqqqss$	—	ΞN	73.2(B)	$\Lambda\Lambda$ (Strong decay)
$\bar{K}NN^b$	2352	2	$1/2$	0^-^c	$qqqqqs(L=1)$	$qqqqq\bar{s}$	$\bar{K}NN$	20.5(T)–174(T)	ΛN (Strong decay)
$\Omega\Omega^a$	3228	1	0	0^+	$ssssss$	—	$\Omega\Omega$	98.8(R)	$\Lambda K^- + \Lambda K^-$
H_c^{++a}	3377	3	1	0^+	$qqqqsc$	—	$\Xi_c N$	187(B)	$\Lambda K^-\pi^+\pi^+ + p$
$\bar{D}NN^a$	3734	2	$1/2$	0^-	—	$qqqqq\bar{q}\bar{c}$	$\bar{D}NN$	6.48(T)	$K^+\pi^- + d, K^+\pi^-\pi^- + p + p$
BNN^a	7147	2	$1/2$	0^-	—	$qqqqq\bar{q}\bar{b}$	BNN	25.4(T)	$K^+\pi^- + d, K^+\pi^- + p + p$

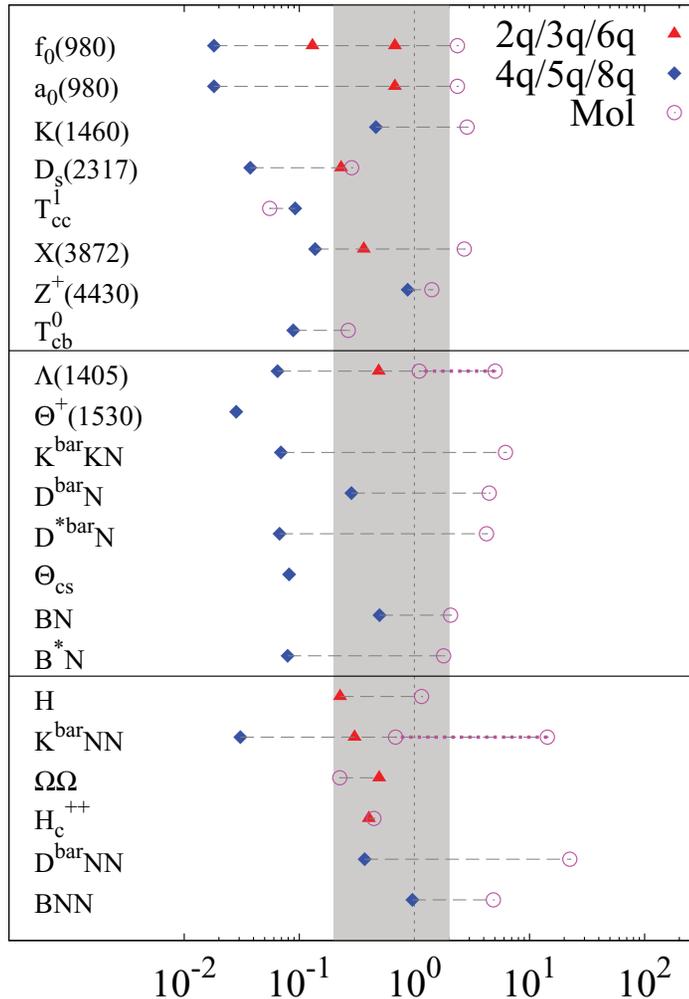
^aParticles that are newly predicted by theoretical models.

^bParticles that are not yet established.

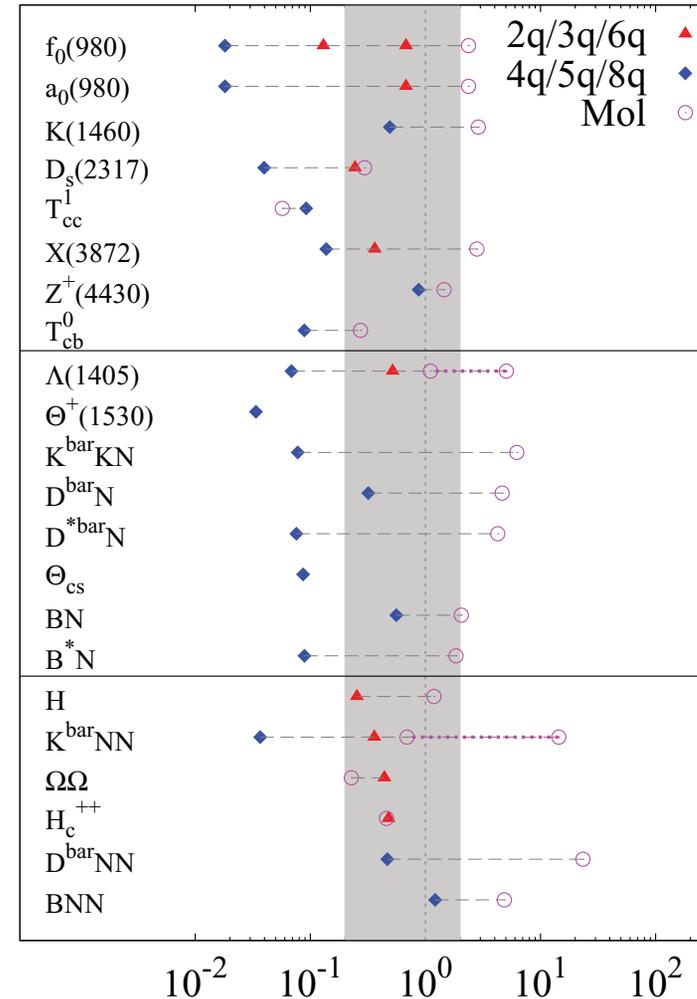
^cUndetermined quantum numbers of existing particles.

Ratio of exotic hadron yields from coalescence and statistical models

Coalescence / Statistical model ratio at RHIC



Coalescence / Statistical model ratio at LHC



Multiquark hadrons are suppressed while hadronic molecules are enhanced in coalescence model, compared to the statistical model predictions.

Coalescence model for hadron production

$$N_h^{\text{coal}} = g_h \int \left[\prod_{i=1}^n \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{d^3 \mathbf{p}_i}{E_i} f(x_i, p_i) \right] f^W(x_1, \dots, x_n; p_1, \dots, p_n)$$

with the constituent number N_i given by $\int p_i \cdot d\sigma_i \frac{d^3 \mathbf{p}_i}{(2\pi)^3 E_i} f(x_i, p_i) = N_i$

Wigner functions of hadrons are approximated by

$$f_s^W(y_i, k_i) = 8 \exp\left(-\frac{y_i^2}{\sigma_i^2} - k_i^2 \sigma_i^2\right),$$

$$f_p^W(y_i, k_i) = \left(\frac{16}{3} \frac{y_i^2}{\sigma_i^2} - 8 + \frac{16}{3} \sigma_i^2 k_i^2\right) \exp\left(-\frac{y_i^2}{\sigma_i^2} - k_i^2 \sigma_i^2\right)$$

$$W_{\psi_{10}}(\vec{r}, \vec{k}) = \frac{16}{3} \left(\frac{r^4}{\sigma^4} - 2\frac{r^2}{\sigma^2} + \frac{3}{2} - 2\sigma^2 k^2 + \sigma^4 k^4 - 2r^2 k^2 + 4(\vec{r} \cdot \vec{k})^2\right) e^{-\frac{r^2}{\sigma^2} - k^2 \sigma^2}.$$

For uniform source of size much larger than hadron size, i.e., $R \gg \sigma$, only spatially integrated Wigner function is needed, i.e.,

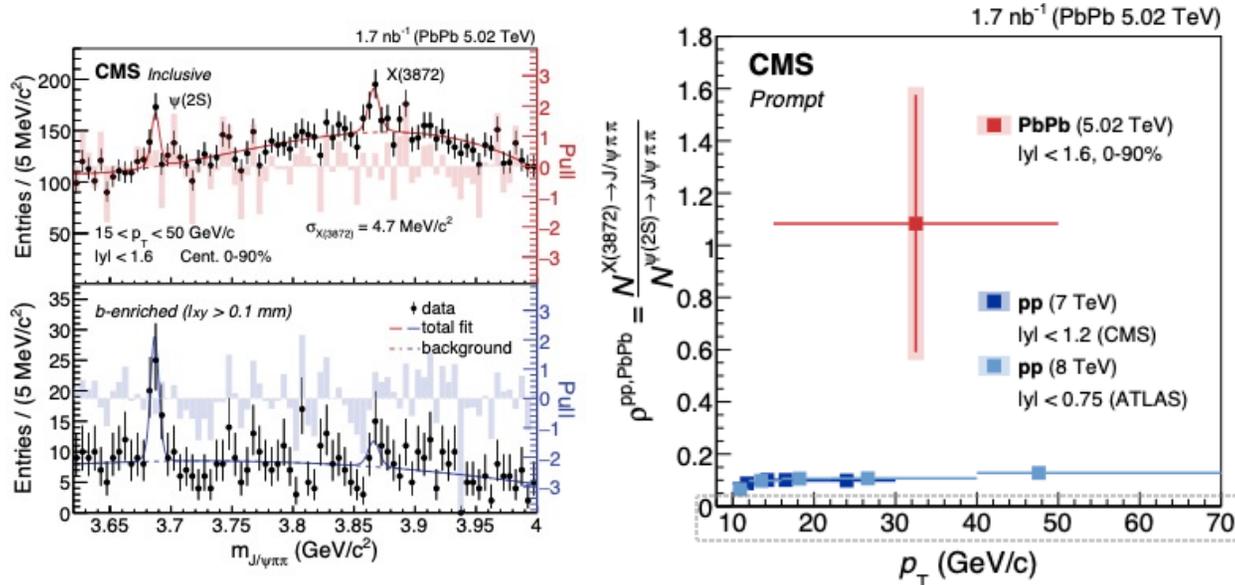
$$f_s^W(\mathbf{k}) = (2\sqrt{\pi}\sigma)^3 e^{-k^2 \sigma^2}, \quad f_p^W(\mathbf{k}) = \frac{2}{3} (2\sqrt{\pi}\sigma)^3 e^{-k^2 \sigma^2} k^2 \sigma^2$$

$$f_{10}^W(\mathbf{k}) = \frac{2}{3} (2\sqrt{\pi}\sigma)^3 e^{-k^2 \sigma^2} \left(k^2 \sigma^2 - \frac{3}{2}\right)^2$$

Evidence for X(3872) in Pb-Pb Collisions and Studies of its Prompt Production at $\sqrt{s_{NN}} = 5.02$ TeV

A. M. Sirunyan *et al.**
 CMS Collaboration

The first evidence for X(3872) production in relativistic heavy ion collisions is reported. The X(3872) production is studied in lead-lead (Pb-Pb) collisions at a center-of-mass energy of $\sqrt{s_{NN}} = 5.02$ TeV per nucleon pair, using the decay chain $X(3872) \rightarrow J/\psi \pi^+ \pi^- \rightarrow \mu^+ \mu^- \pi^+ \pi^-$. The data were recorded with the CMS detector in 2018 and correspond to an integrated luminosity of 1.7 nb^{-1} . The measurement is performed in the rapidity and transverse momentum ranges $|y| < 1.6$ and $15 < p_T < 50 \text{ GeV}/c$. The significance of the inclusive X(3872) signal is 4.2 standard deviations. The prompt X(3872) to $\psi 2S$ yield ratio is found to be $\rho^{\text{Pb-Pb}} = 1.08 \pm 0.49(\text{stat}) \pm 0.52(\text{syst})$, to be compared with typical values of 0.1 for pp collisions. This result provides a unique experimental input to theoretical models of the X(3872) production mechanism, and of the nature of this exotic state.

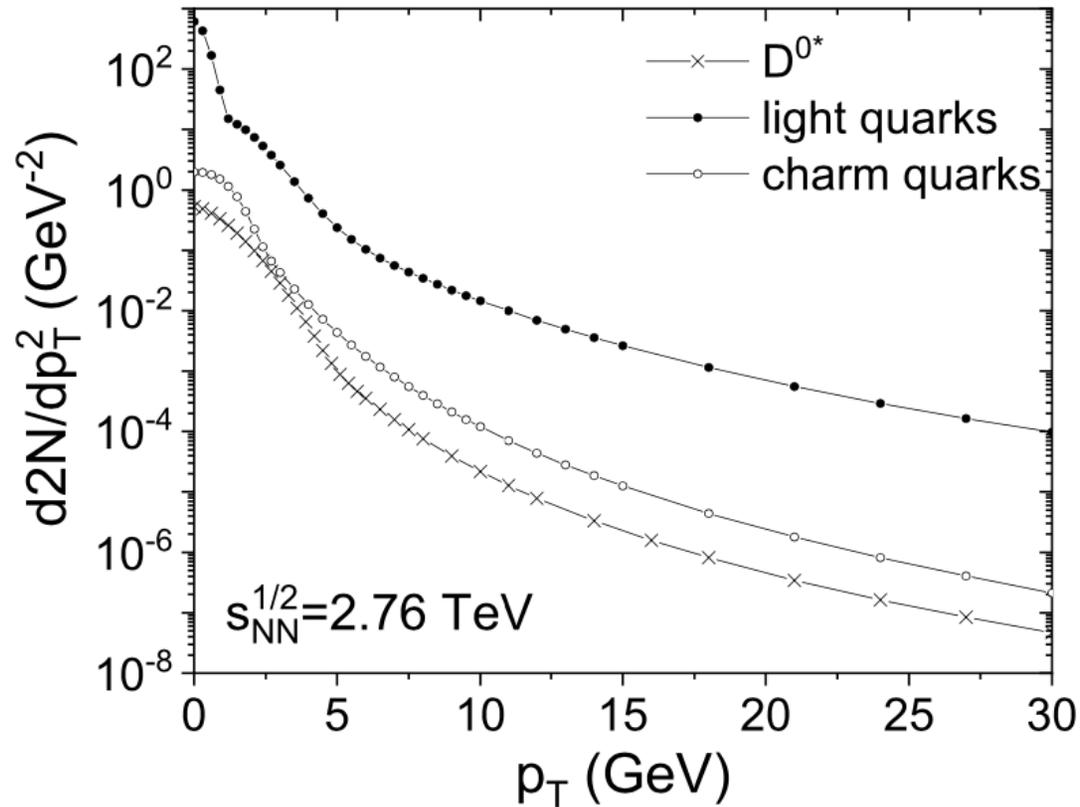


$$\frac{X(3872)}{\psi(2S)} \approx 1$$

which is a factor of ten larger than in p+p collisions

Charm quark and meson spectra

S. Plumari et al., Eur. Phys. J. C 78, 348 (2018); ALICE Collaboration, 1509.06888



Can be well parametrized by

At $T_T \gg P_c \sim 5 \text{ GeV}$, it can be approximated by

$$\frac{d^2 N}{d^2 p_T} = a e^{-b p_T^c} \quad (p_T < p_c),$$

$$= d \frac{p_T}{[1 + (p_T/p_0)^2]^e} \quad (p_T > p_c),$$

$$\frac{d^2 N}{d^2 p_T} \approx A \left(\frac{p_T}{p_0} \right)^{-B}$$

Coalescence production of high momentum hadrons

$$N = g \int d^3 \mathbf{x}_1 \int \frac{d^3 \mathbf{k}_1}{(2\pi)^3} \int d^3 \mathbf{x}_2 \int \frac{d^3 \mathbf{k}_2}{(2\pi)^3} f_1(\mathbf{x}_1, \mathbf{k}_1) f_1(\mathbf{x}_2, \mathbf{k}_2) f^W(\mathbf{x}_1 - \mathbf{x}_2, (\mathbf{k}_1 - \mathbf{k}_2)/2)$$

For uniform constituent distributions and in the center-of-mass of produced hadron,

$$\begin{aligned} N &= g \int \frac{d^3 \mathbf{k}_1}{(2\pi)^3} \int \frac{d^3 \mathbf{k}_2}{(2\pi)^3} f_1(\mathbf{k}_1) f_2(\mathbf{k}_2) \int d^3 \mathbf{x}_1 \int d^3 \mathbf{x}_2 f^W(\mathbf{x}_1 - \mathbf{x}_2, (\mathbf{k}_1 - \mathbf{k}_2)/2) \\ &= g \int \frac{d^3 \mathbf{K}}{(2\pi)^3} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} f_1(\mathbf{K}/2 + \mathbf{k}) f_2(\mathbf{K}/2 - \mathbf{k}) \int d^3 \mathbf{X} \int d^3 \mathbf{x} f^W(\mathbf{x}, \mathbf{k}) \\ &\approx g \int \frac{d^3 \mathbf{K}}{(2\pi)^3} f_1(\mathbf{K}/2) f_2(\mathbf{K}/2) \int \frac{d^3 \mathbf{k}}{(2\pi)^3} V \int d^3 \mathbf{x} f^W(\mathbf{x}, \mathbf{k}) \\ &= g \frac{V}{(2\pi)^3} \int d^3 \mathbf{K} f_1(\mathbf{K}/2) f_2(\mathbf{K}/2) \end{aligned}$$

Since $\frac{d^3 N}{d^3 \mathbf{p}} = \frac{V}{(2\pi)^3} f(\mathbf{p})$, so $\frac{d^3 N}{d^3 \mathbf{K}} = g \frac{(2\pi)^3}{V} \frac{d^3 N_1}{d^3 \mathbf{p}_1} \Big|_{p_1=K/2} \frac{d^3 N_2}{d^3 \mathbf{p}_2} \Big|_{p_2=K/2}$ or

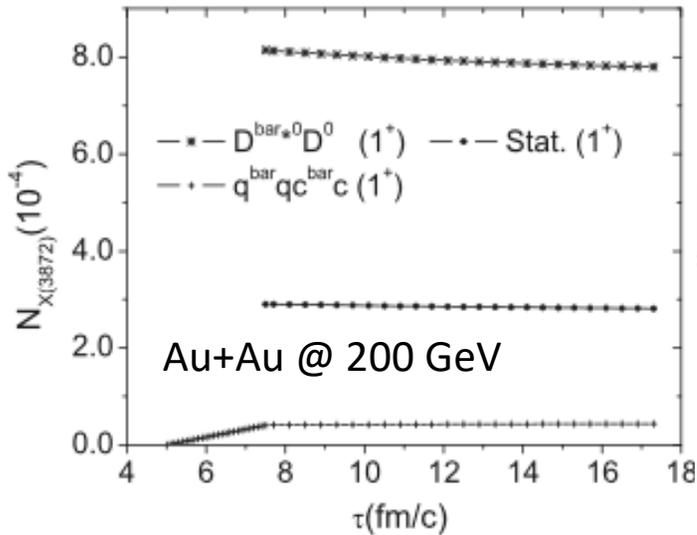
$$\frac{d^3 N}{E dy d^2 \mathbf{K}_T} = g \frac{(2\pi)^3}{V} \frac{d^3 N_1}{E_1 dy_1 d^2 \mathbf{p}_{1T}} \Big|_{p_1=K/2} \frac{d^3 N_2}{E_2 dy_2 d^2 \mathbf{p}_{2T}} \Big|_{p_2=K/2}$$

Hence $\frac{N_X}{N_{\psi(2S)}} = \frac{g_X}{g_{\psi(2S)}} \frac{N_{D^0} N_{D^*}}{N_c N_{\bar{c}}}$.

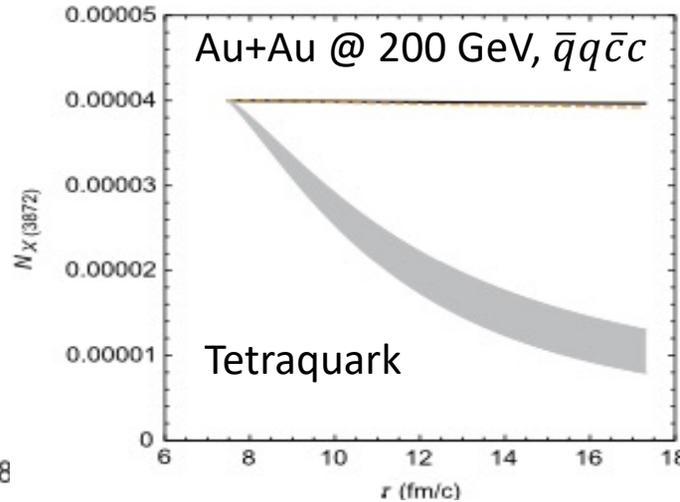
With $g_X = 1$ and $g_{\psi(2S)} = 1/12$, a ratio of $N_X/N_{\psi(2S)} \approx 1$ requires $\frac{N_{D^0} N_{D^*}}{N_c N_{\bar{c}}} \approx 1/12$ or $\frac{N_{D^0}}{N_c} \approx \frac{1}{3.5}$, which is a factor of 2 to 3 larger than known value. Understanding the observed $N_X/N_{\psi(2S)} \approx 1$ at $p_T \approx 30$ GeV is thus challenging and interesting.

X(3872) production in HIC

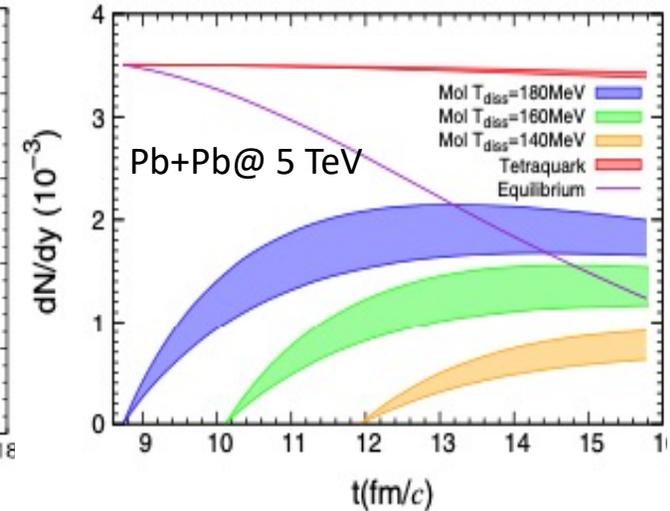
Cho & Lee, PRC 88, 054901 (2013)



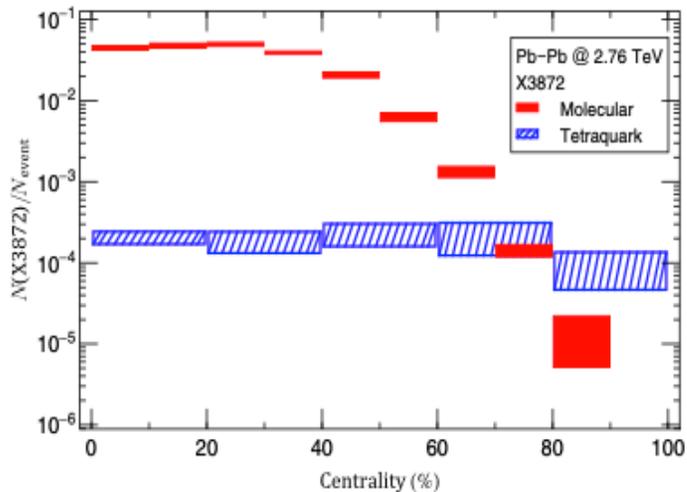
Abreu et al., PLB 761, 303 (2016)



Wu, Du, Sibila & Rapp, EPJA 57, 122 (2021)



Hui Zhang et al, PRL 136, 012301 (2021)



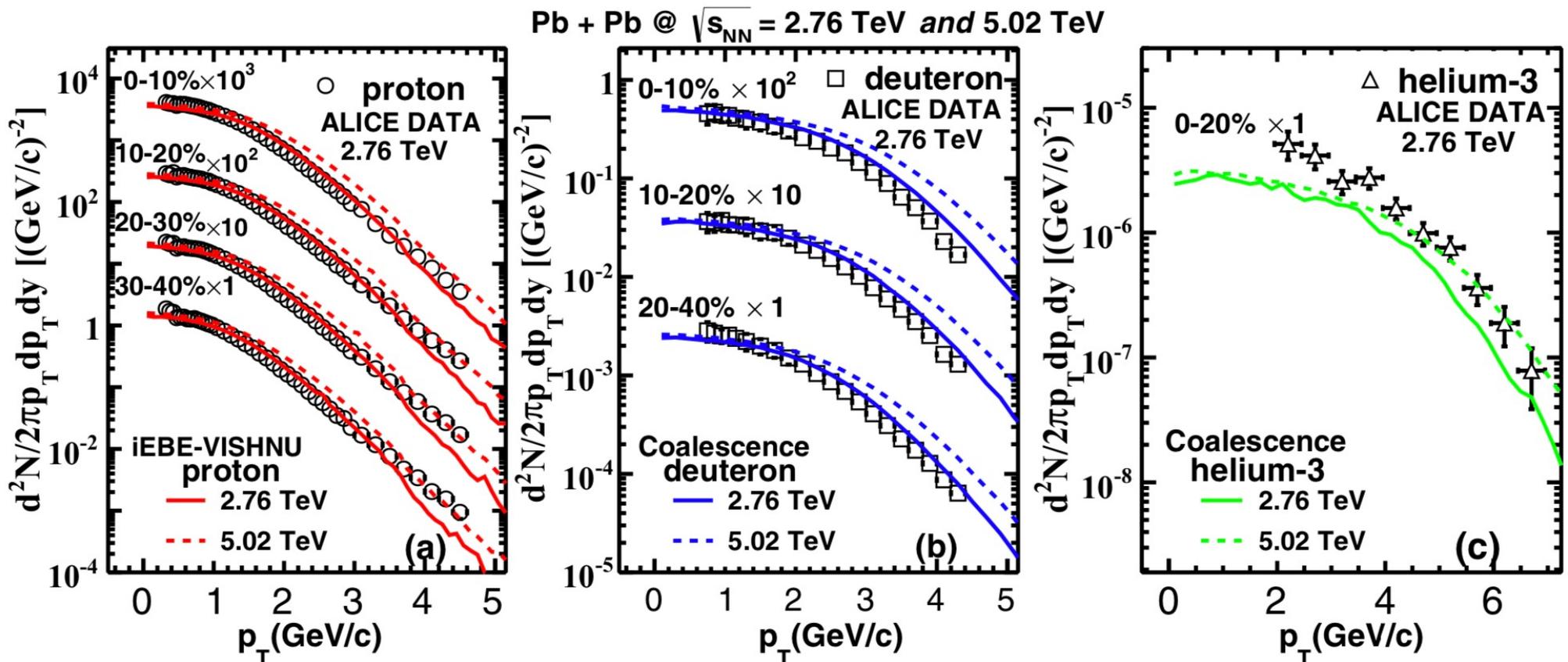
- Cho & Lee use kinetic approach with initial numbers from coalescence model and include $\pi(\rho)X \leftrightarrow D\bar{D}, D^*\bar{D}, D^*\bar{D}^*$ reactions.
- Abreu et al. include anomalous vertices in $\pi X \leftrightarrow D\bar{D}, D^*\bar{D}, D^*\bar{D}^*$, resulting in larger cross sections.
- Wu et al. use thermal model for initial number and assume smaller cross sections for tetraquark scenario. Molecular X(3872) is produced from hadronic reactions.
- Zhang et al. use $D\bar{D}$ coalescence for molecular scenario and diquark-diquark coalescence for tetraquark scenario based only on their spatial distributions from AMPT.

Coalescence production of light nuclei in HIC

Spectra and flow of light nuclei in relativistic heavy ion collisions at energies available at the BNL Relativistic Heavy Ion Collider and at the CERN Large Hadron Collider

Wenbin Zhao,^{1,2} Lilin Zhu,³ Hua Zheng,^{4,5} Che Ming Ko,⁶ and Huichao Song^{1,2,7}

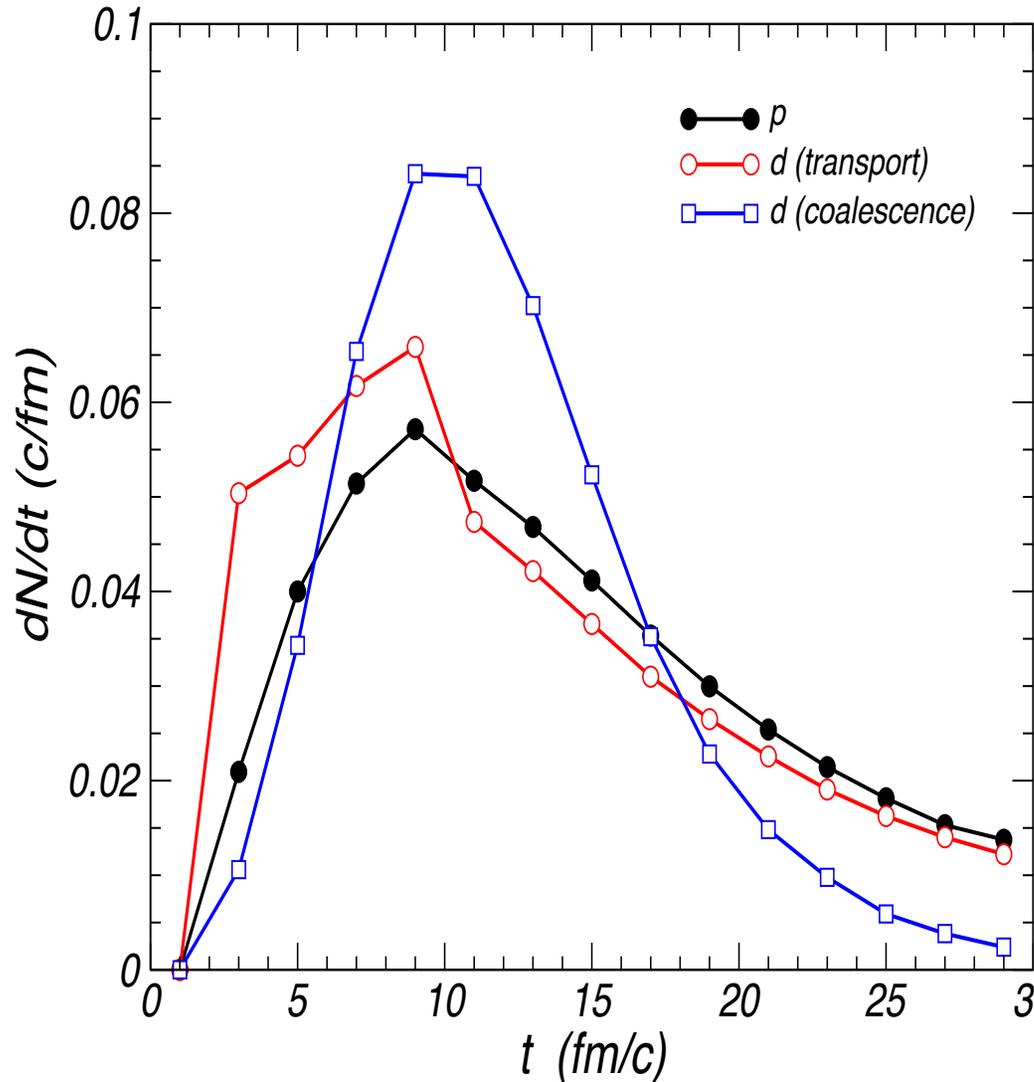
IEBE-VISHNU hybrid model with AMPT initial conditions PRC 98, 054905 (2018)



Elliptic flow of deuteron measured by ALICE is also satisfactorily described. ¹²

Deuteron production from an extended ART model

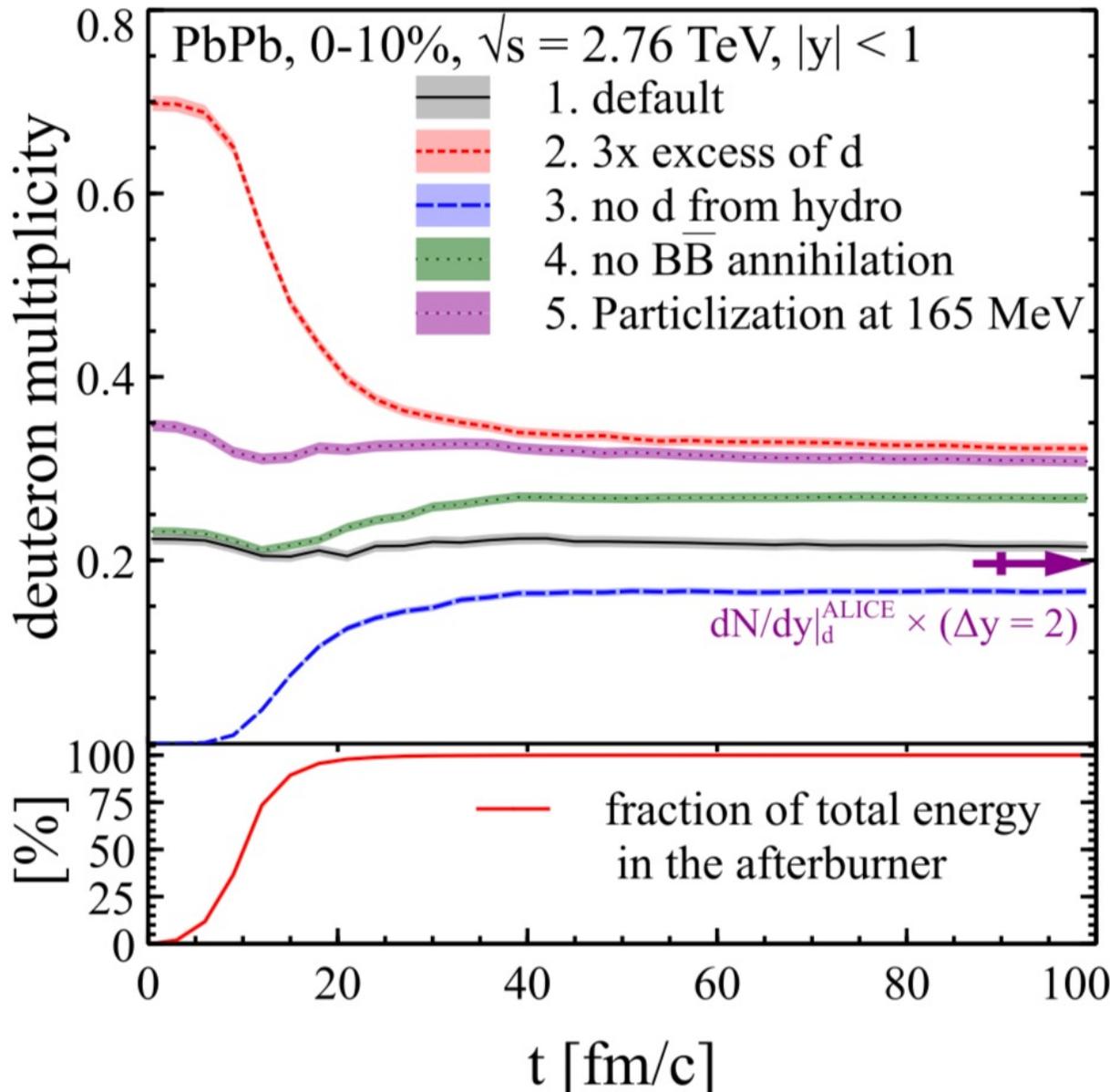
Oh & Ko, PRC 76, 054910 (2007); Oh, Lin & Ko, PRC 80, 064902 (2009)



- Include deuteron production ($n+p \rightarrow d+\pi$) and annihilation ($d+\pi \rightarrow n+p$) as well as its elastic scattering
- Similar emission time distributions for protons and deuterons in coalescence model
- Slightly different deuteron emission time distribution in transport and coalescence models

Deuteron production in SMASH

Oliinychkov, Pang, Elfner & Koch,
PRC 99, 044907 (2019)



- Using a large $\pi NN \leftrightarrow \pi d$ cross section of about 100 mb.
- Deuteron number essentially remains unchanged during hadronic evolution

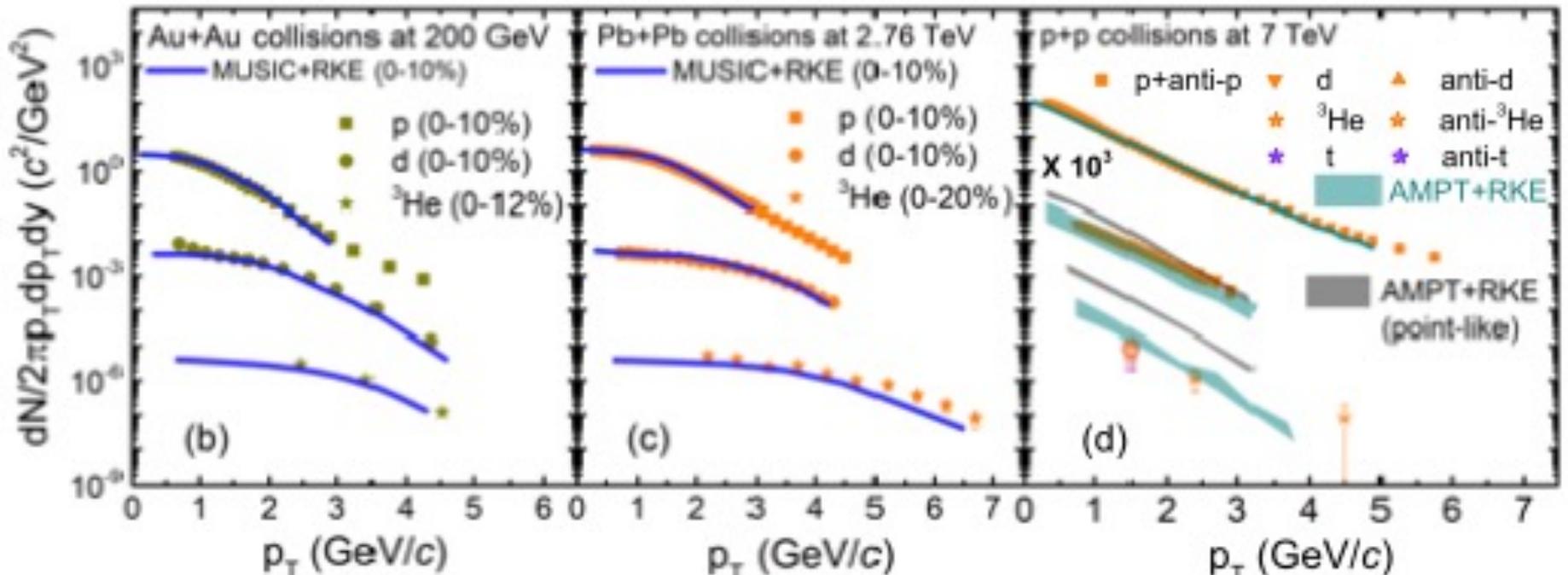
Light nuclei production from non-local many-body scattering

Sun, Wang, Ko, Ma & Shen, arXiv:2106.12742 [nucl-th]

$$\frac{\partial f_d}{\partial t} + \frac{\mathbf{P}}{E_d} \cdot \frac{\partial f_d}{\partial \mathbf{R}} = \frac{1}{2g_d E_d} \int \prod_{i=1}^3 \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \frac{d^3 \mathbf{p}_\pi}{(2\pi)^3 2E_\pi} \frac{E_d d^3 \mathbf{r}}{m_d} 2m_d W_d(\tilde{\mathbf{r}}, \tilde{\mathbf{p}})$$

$$\left(|\overline{\mathcal{M}_{\pi+n \rightarrow \pi+n}}|^2 + n \leftrightarrow p \right) \left[-g_\pi f_\pi g_d f_d \prod_i^3 (1 \pm f_i) + \frac{3}{4} (1 + f_\pi)(1 + f_d) \prod_{i=1}^3 g_i f_i \right]$$

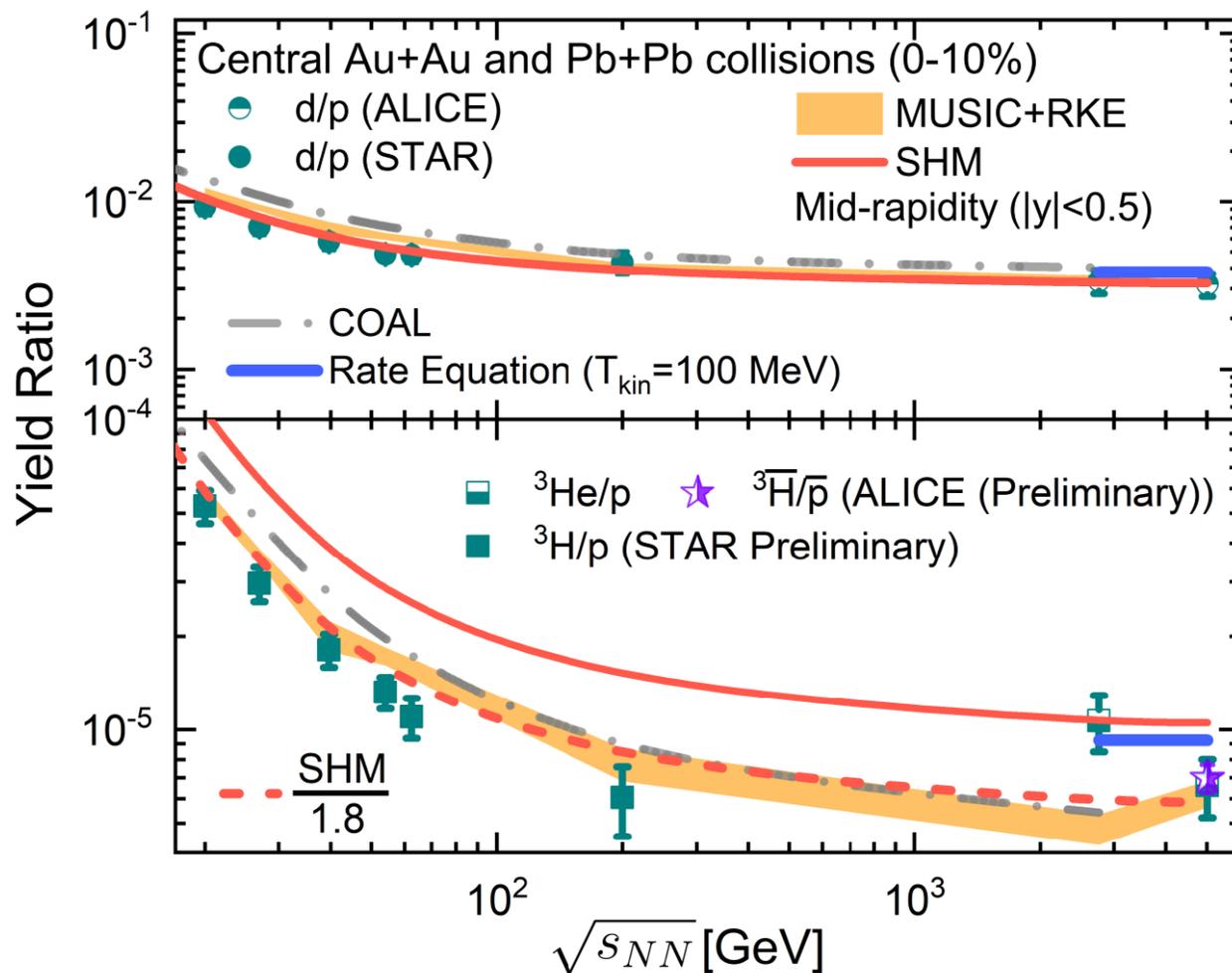
$$\times (2\pi)^4 \delta^4(p_1 + p_2 + p_3 - p_\pi - p_d)$$



Good description of data with $\pi NN \leftrightarrow \pi d$, $\pi NNN \leftrightarrow \pi t$, and $\pi NNN \leftrightarrow \pi^3\text{He}$ ¹⁵

Collision energy dependence of light nuclei production

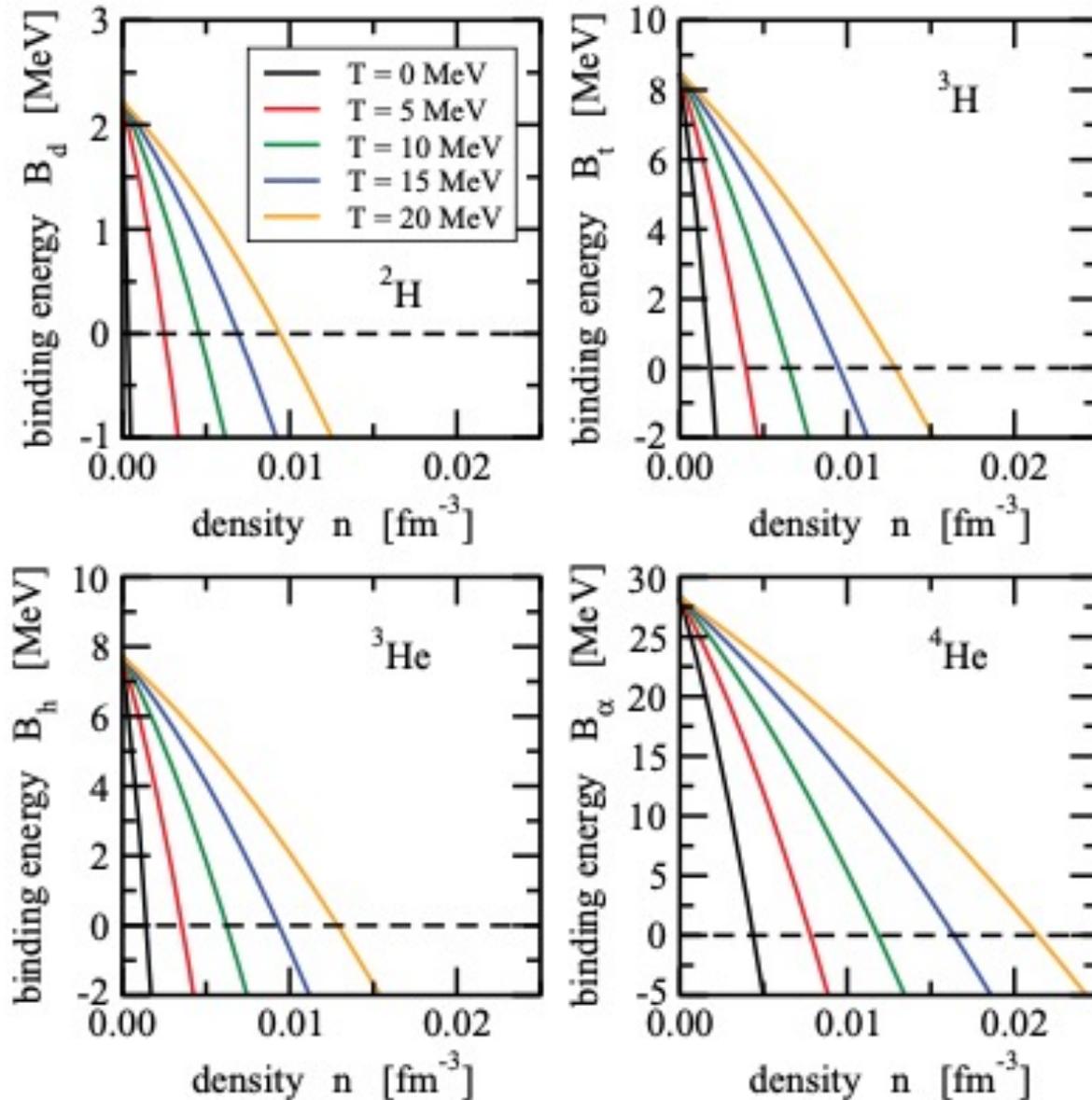
Kaijia Sun et al., arXiv:2207.12532 [nucl-th]



Hadronic rescatterings reduce the triton yields by about a factor of 2.

Binding energies of light nuclei in hot nuclear matter

Typel, Roepke, Klahn, Blaschke & Wolter, PRC 81, 015803 (2010)



- Microscopic quantum statistical approach with relativistic mean-field model.
- Mott effect due to Pauli blocking can lead to bound light nuclei in denser nuclear matter as temperature increases.
- Are light nuclei bounded in hot pion gas?

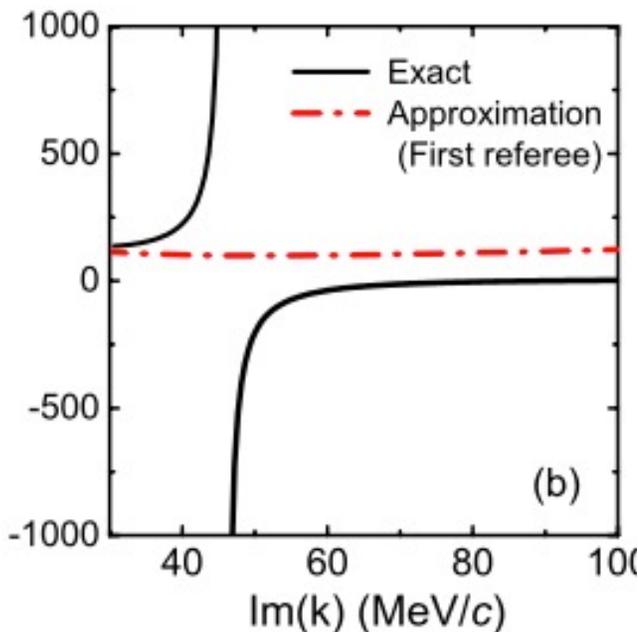
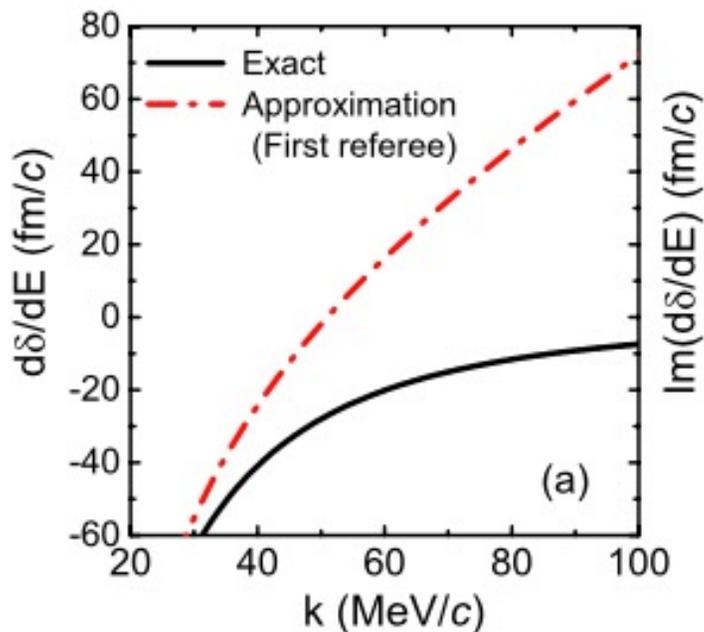
Deuteron formation time in $NN \rightarrow \pi d$ and $\pi NN \rightarrow \pi d$

- Energy-time uncertainty: Danielewicz & Bertsch, NPA 531, 712 (1991)

1) NN at rest: $\tau \approx \frac{\hbar}{2B_d} \approx \frac{197 \text{ MeV}\cdot\text{fm}}{2 \times 2.2 \text{ MeV}\cdot c} \approx 50 \text{ fm}/c$

2) In hot medium: $\tau \approx \frac{\hbar}{2\left(\frac{3T}{2} + B_d\right)} \approx 4.5 \text{ fm}/c$ ($T \approx 20 \text{ MeV}$)
 $\approx 0.5 \text{ fm}/c$ ($T \approx 140 \text{ MeV}$)

- Time delay: Danielewicz & Pratt, PRC 53, 249 (1996)



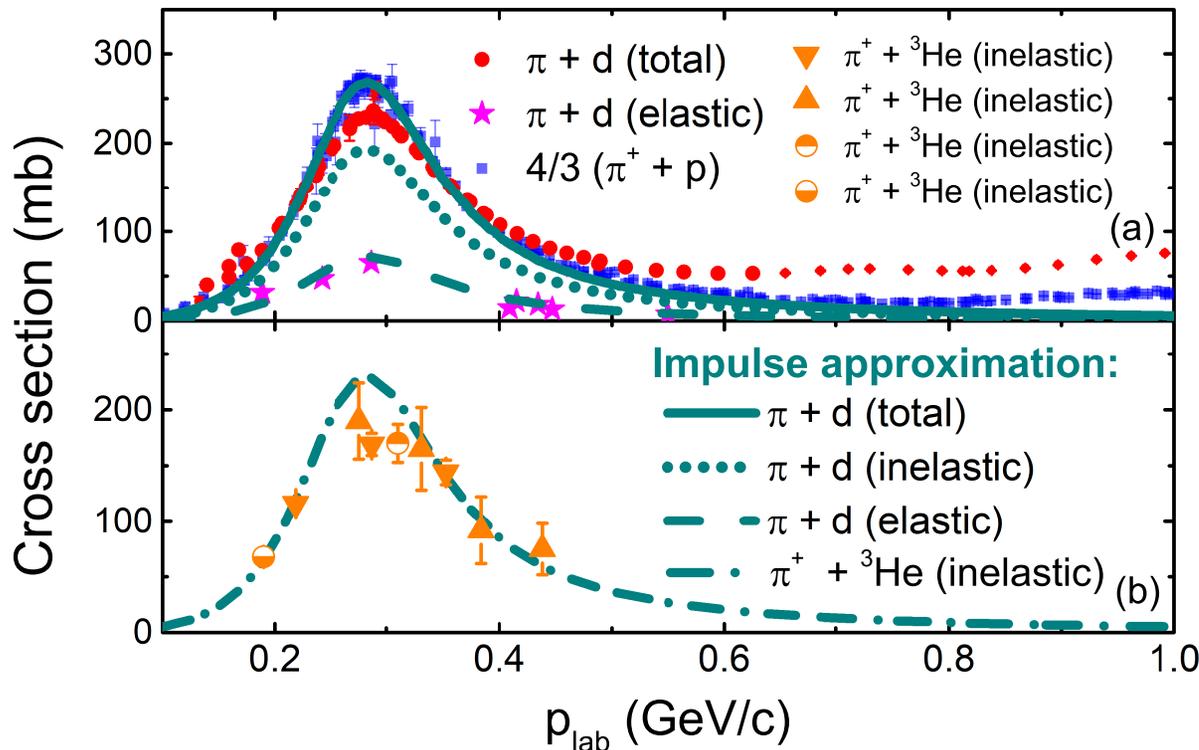
$$\tau \approx \frac{R}{v} + \frac{d\delta_l}{dE}$$

- 1) $\tau \approx 100 \text{ fm}/c$
 (anonymous), more likely ∞ because of forming a bound state

2) $\tau \approx \frac{1}{p_\pi} + \frac{1}{\Gamma_\Delta} \approx 3 \frac{\text{fm}}{c}$
 (K. J. Sun & R. Wang)

■ Time-dependent perturbation theory

$$\begin{aligned} \frac{1}{\tau_{\pi NN \rightarrow \pi d}} &= \frac{2\pi}{4} \int_0^{\sqrt{s}} dE_\pi |T_{\pi NN \rightarrow \pi d}|^2 n_\pi(E_\pi) n_d(\sqrt{s} - E_\pi) \\ &\approx \frac{3\sigma_{\pi d \rightarrow \pi NN} v_{\pi d}}{4V} \frac{n_{\pi d}}{n_{\pi NN}} \\ &\approx \frac{3\sigma_{\pi d \rightarrow \pi NN}}{V} \frac{(k_{\pi d}^*)^2}{k_{NN}^* \sqrt{s}} \approx 7 \text{ fm}/c \end{aligned}$$



Take the interaction volume to have a radius of the inverse of pion mass

$$\begin{aligned} V &\approx \left(\frac{4\pi}{3}\right) \left(\frac{1}{m_\pi}\right)^3 \\ &\approx 12 \text{ fm}^3 \end{aligned}$$

$$\begin{aligned} \sigma_{\pi d \rightarrow \pi NN} &\approx 200 \text{ mb} \\ \text{at } \sqrt{s} &= 2.1 \text{ GeV} \end{aligned}$$

Summary

- Relativistic heavy ion collisions provide a unique opportunity to study exotic hadrons because of the many produced heavy quark pairs.
- The large $X(3872)/\psi(2S) \sim 1$ at $P_T \sim 30$ GeV measured by CMS can only be described by the coalescence model for a ratio $N_D/N_C \sim 1/3.5$.
- Predictions for the $X(3872)$ yield in relativistic HIC vary appreciably among models. Measuring $X(3872)$ at lower momentum is needed to constrain models and to understand the structure of $X(3872)$.
- All kinetic descriptions of $X(3872)$ evolution in the hadronic stage of HIC are at present schematic.
- Realistic transport approach to light nuclei production has recently been developed and successfully used to describe experimental data.
- Besides detecting known exotic hadrons in HIC, discovering new exotic hadrons and developing realistic transport approaches to study their production opens a new page in the physics of heavy ion collisions.