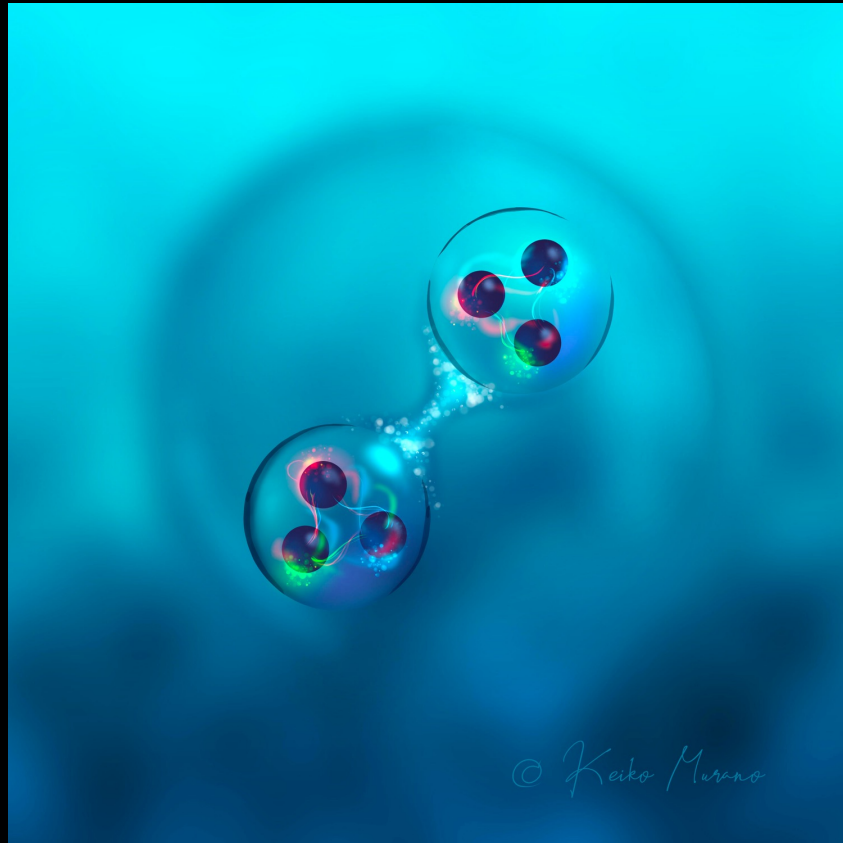


Hadron Interactions from Lattice QCD



Tetsuo Hatsuda
(RIKEN iTHEMS)

iTHEMS
RIKEN interdisciplinary
Theoretical & Mathematical
Sciences

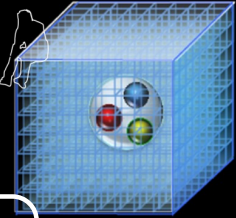
Hadrons to Atomic nuclei
HAL
from Lattice QCD

The HAL logo features the letters 'HAL' in a large, blue, sans-serif font. Below the letters is a blue grid representing a lattice. Two small spheres, each containing three quarks (red, green, blue), are positioned on the grid. The text 'Hadrons to Atomic nuclei' is at the top, and 'from Lattice QCD' is at the bottom.

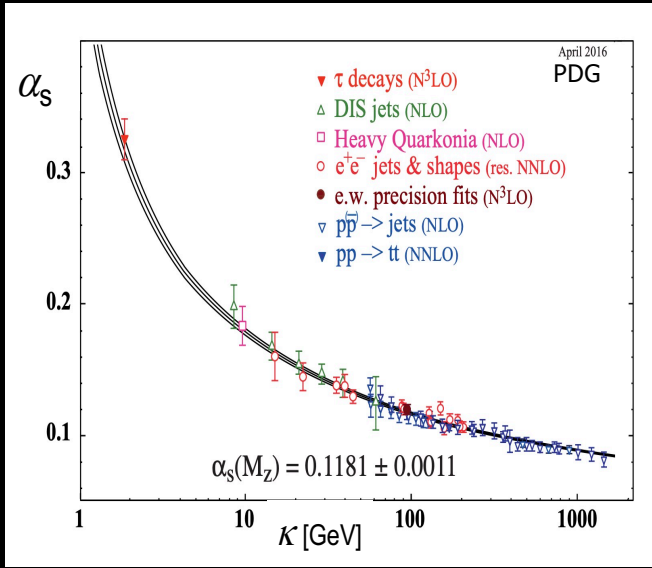
ExHIC WS (Sept. 29, 2022)

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q} \gamma^\mu (i\partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$

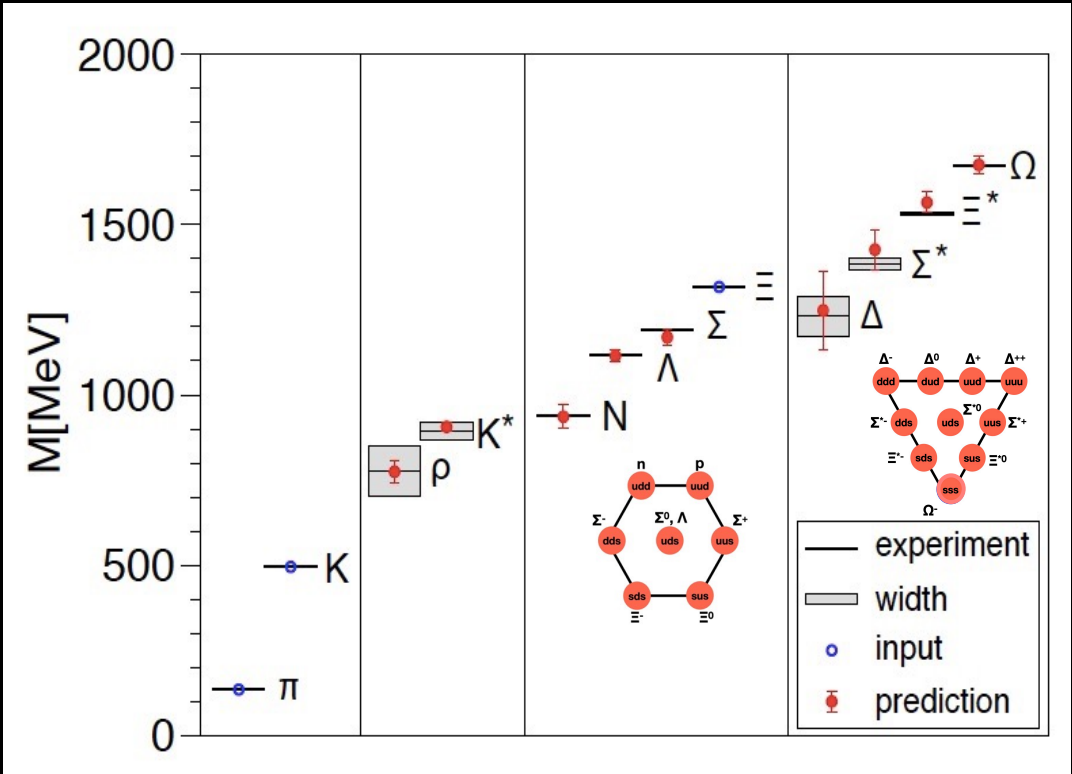
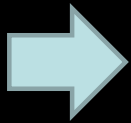
Lattice QCD



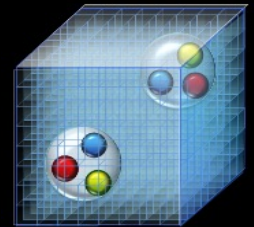
$$Z_{\text{QCD}} = \int [dU] [dq d\bar{q}] e^{-[S_{\text{glue}}(U) + \bar{q} F(U) q]}$$



light quarks	MS-bar mass at 2GeV
m_u	2.27(9) MeV
m_d	4.67(9) MeV
m_s	92.0(1.1) MeV



Interaction between composite particles in QFT

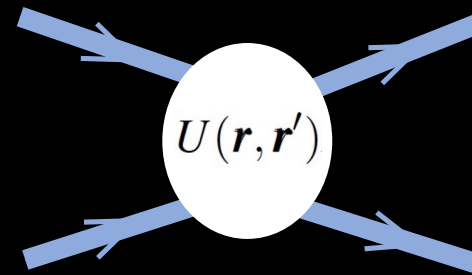


Foundation:	Haag, Nishijima, Zimmermann reduction formula (1957) Borchers' theorem (1961)
Luscher's Method:	Luscher, Nucl. Phys. B354 (1991) 531
HAL QCD Method:	Ishii, Aoki, Hatsuda, Phys. Rev. Lett. 99 (2007) 022001 HAL QCD Coll., Phys. Lett. B712 (2012) 437
Review:	Aoki & Doi, Front. Phys. 8 (2020) 307

Time-dependent HAL QCD Equation

$$(\nabla^2 + \partial_{2t}^2 - m^2)F^J(\mathbf{r}, t) = m \int d^3r' U(\mathbf{r}, \mathbf{r}')F^J(\mathbf{r}', t)$$

1. Derived from QCD
2. Fully relativistic equation
3. Faithful to S-matrix
4. Insensitive to lattice volume
5. Applicable to BB, MM, MB, BBB etc
6. Applicable to coupled channel systems

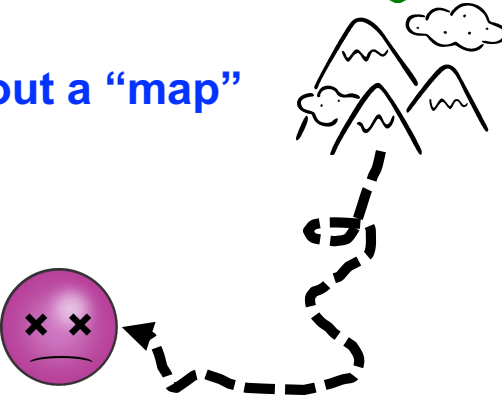


Lattice QCD

Naïve Method
by NPL QCD

$$F(\tau)$$

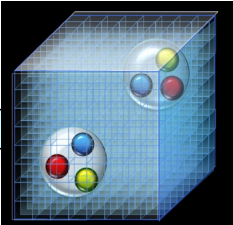
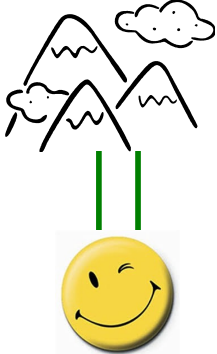
without a “map”



HAL QCD Method

$$F(\mathbf{r}, \tau)$$

with a “map”



[HAL QCD Coll.]

- JHEP1610 (2016) 101
- PRD96 (2017) 034521
- PRD99 (2019) 014514
- JHEP1903 (2019) 007

[CALAT Coll.]

- PRC103 (2021)014003

[sLapHnn Coll.]

Nicholson et al., Lattice 2022

Observables (phase shift, binding energy)

HAL QCD Lattice data



CP-PACS @Tsukuba
0.6 Tflops
 (1996-2005)



PACS-CS @Tsukuba
14 TFlops
 (2006-2011)

3-flavor & (2+1)-flavor
 $V \sim (3 \text{ fm})^3, m_\pi > 400 \text{ MeV}$

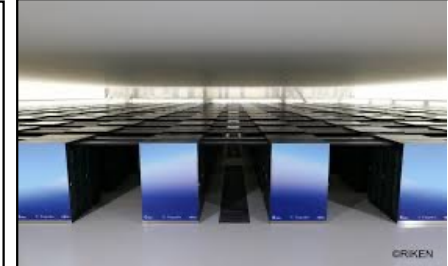
BB	Inoue+, PTP 124 ('10)
H	Inoue+, PRL 106 ('11) NPA 881 ('12)
BB	Sasaki+, PTEP 2015 ('15)
NΩ	Etminan+, NPA 928 ('14)
ΩΩ	Yamada+, PTEP 2015 ('15)
KN, πΣ	Ikeda+, PoS Lat ('11)
KN	Murakami+, PTEP 2020 ('20)
T_{cc}	Ikeda+, PLB 729 ('14)
Z_c	Ikeda+, PRL 117 ('16)
D^{bar}-N	Ikeda+, HAL internal rep. ('16)
J/ψ-N	Sugiura+, PoS Lat ('18)
Λ_c-N	Miyamoto+, NPA 971 ('18)



K computer
 @ RIKEN
10 PFlops
 (2011-2019)

(2+1)-flavor
 $V = (8.1 \text{ fm})^3, m_\pi = 146 \text{ MeV}$

S=-1 (ΛN, ΣN)	Nemura+, EPJ conf. 175 ('18)
S=-3 (ΞΣ, ΞΛ - ΞΣ)	Ishii+, EPJ conf. 175 ('18)
S=-4 (ΞΞ)	Doi+, EPJ conf, 175 ('18)
SU(3) basis	Inoue+, AIP conf. 2130 ('19)
S=-2 (ΛΛ, NΞ)	Sasaki+, NPA 998 ('20)
S=-3 (NΩ)	Iritani+, PLB 792 ('19)
S=-6 (ΩΩ)	Gongyo+, PRL 120 ('18)
C=+6 (Ω_{ccc}Ω_{ccc})	Tong+, PRL 127 ('21)
φN	Lyu+, 2205.10544 ('22)



Fugaku
 @RIKEN
440 PFlops
 (2020-)

(2+1)-flavor
 $V = (8.1 \text{ fm})^3, m_\pi = 138 \text{ MeV}$

BB: Octet x Octet
BB: Octet x Decuplet
BB: Octet x Charmed
BB: LS force
MB: KN, φN, DN, J/ψ-N, ...
MM: ππ, πK, DD*, J/ψ-J/ψ, ...
BBB

Phase I: exploratory studies → **Phase II: almost physical point** → **Phase III: physical point**

Phase II (almost physical point) Highlights

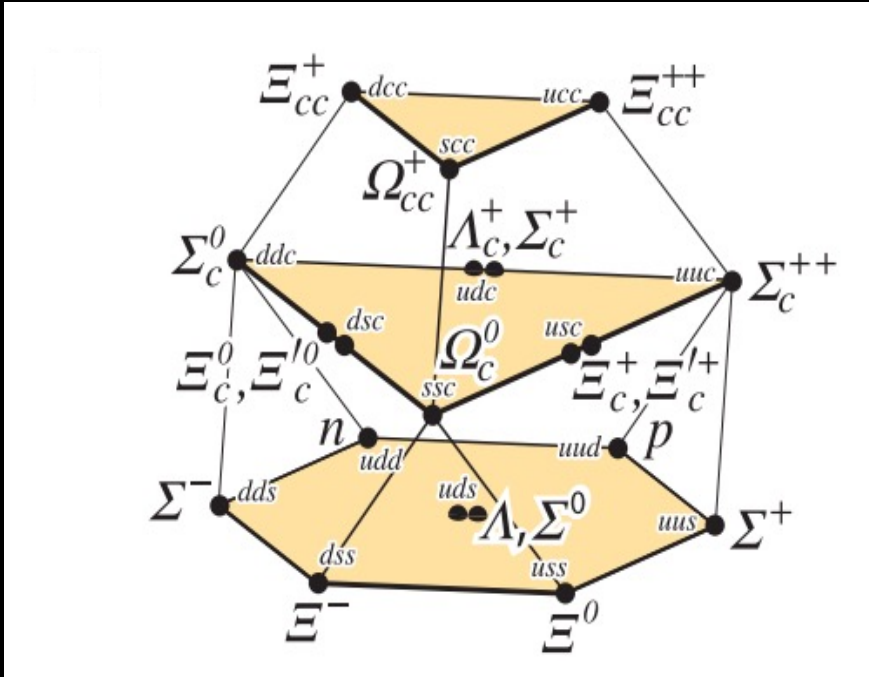


K computer
@ RIKEN
10 PFlops
(2011-2019)

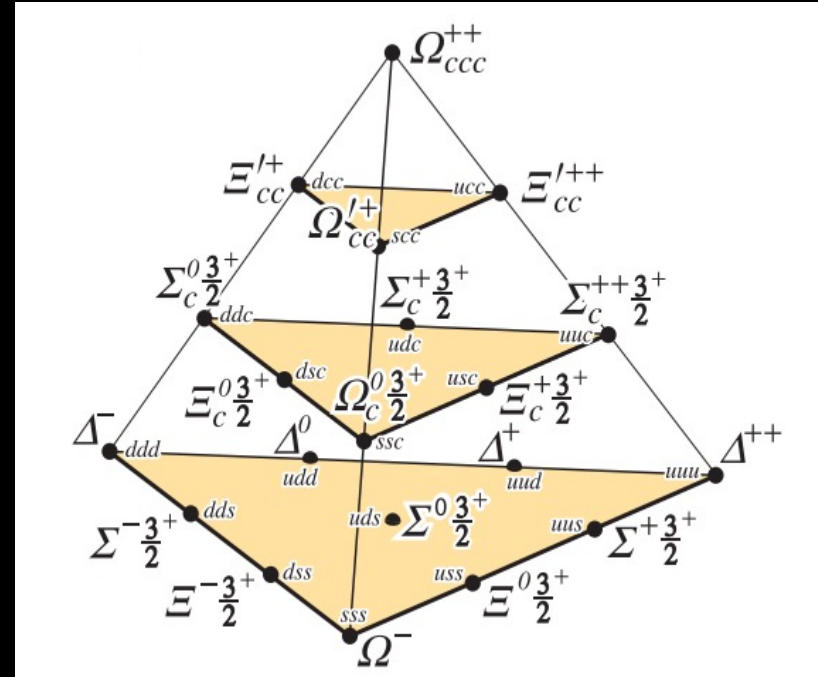
(2+1)-flavor
 $V=(8.1 \text{ fm})^3$, **$m_\pi=146 \text{ MeV}$**

Baryons multiplet

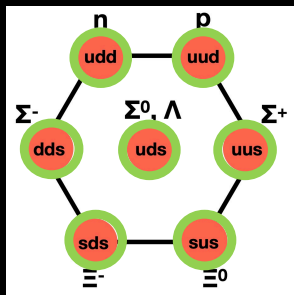
spin 1/2 **20-plet**



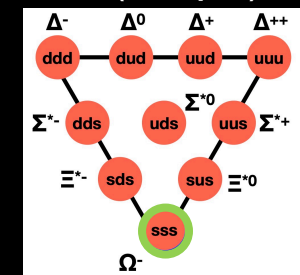
spin 3/2 **20-plet**



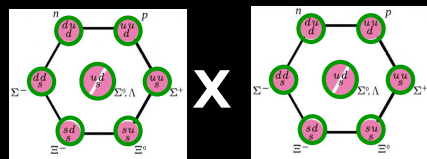
8 (Octet)



10 (Decuplet)



Baryon pairs in flavor SU(3)



x

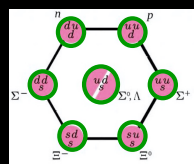
$$8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a$$

$NN(^1S_0)$

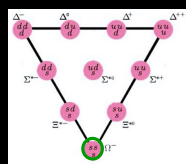
$H_{\Lambda\Lambda-NE-\Lambda\Sigma} (^1S_0)$

$NN(^3S_1)$

Jaffe (1977)

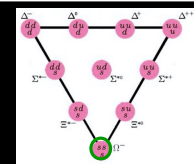


x

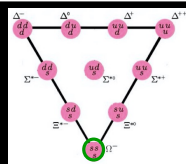


$$8 \times 10 = 35 + 8 + 10 + 27$$

$N\Omega (^5S_2)$ Goldman+ (1987), Oka (1988)



x

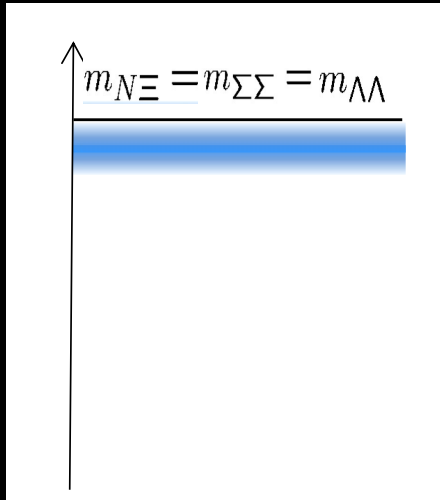


$$10 \times 10 = 28 + 27 + 35 + 10^*$$

Kopeliovich+ (1990) $\Omega\Omega (^1S_0)$

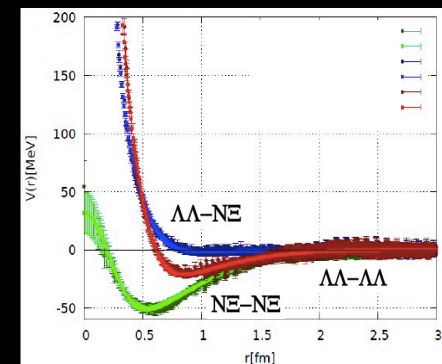
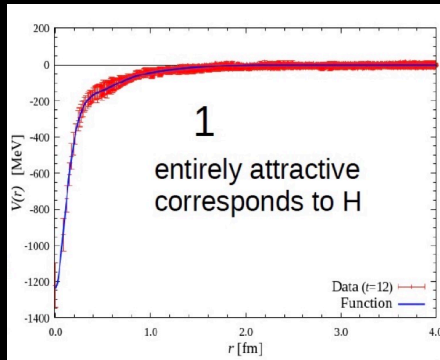
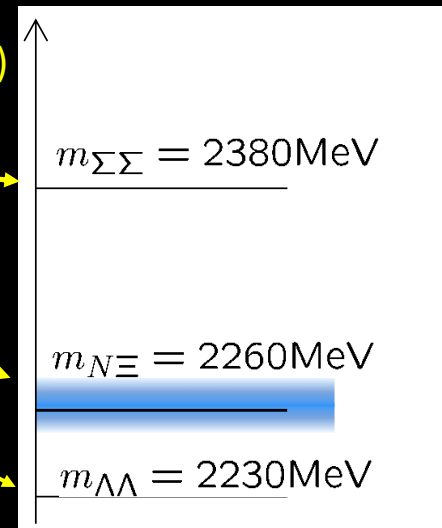
$\Delta\Delta (^7S_3)$ Dyson+ (1964)

Fate of “H (uuddss)” dibaryon from LQCD



$m_{ud} = m_s$
(~ 100 MeV)

$m_{ud} < m_s$
(~10MeV) (~100MeV)



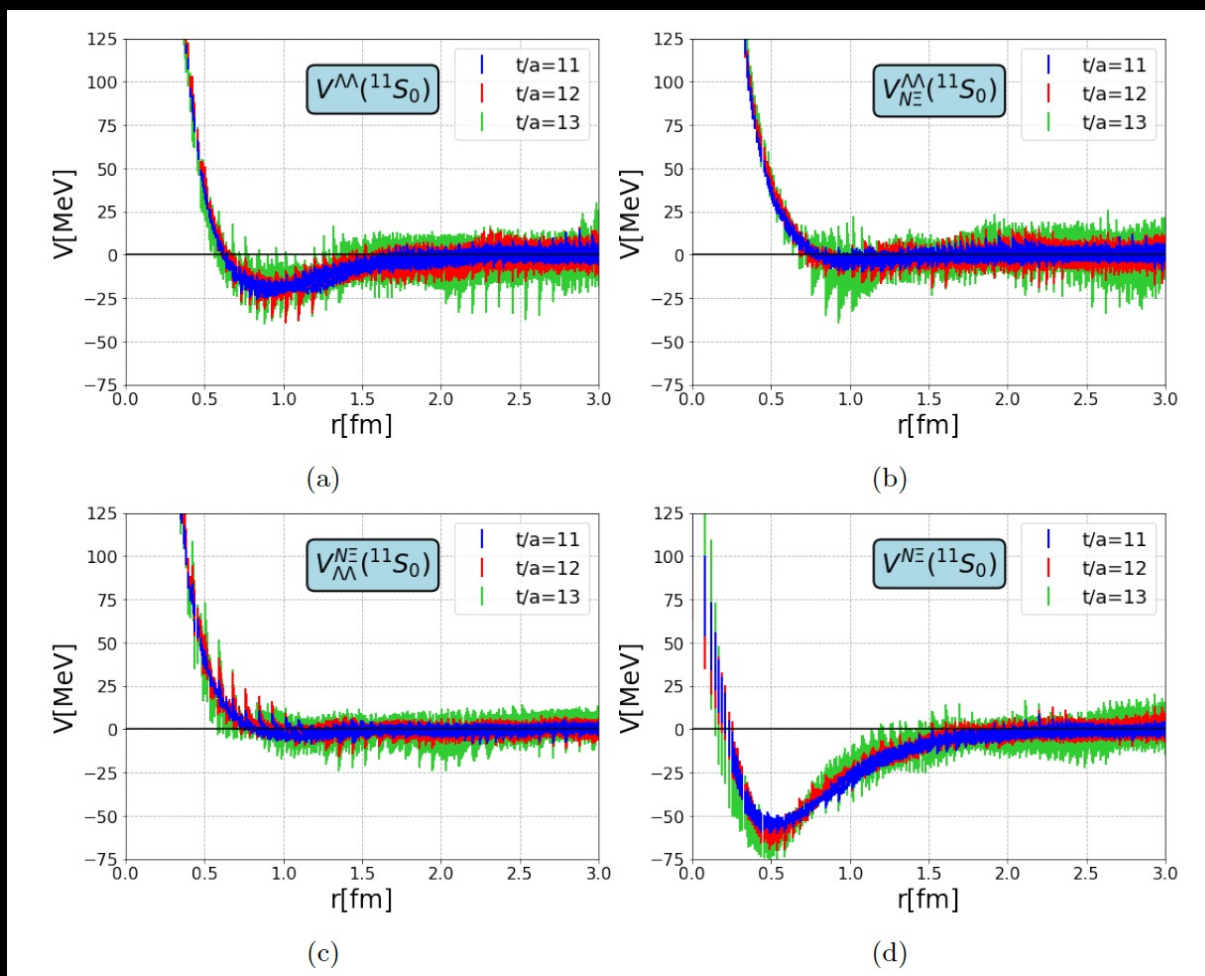
Inoue et al., [HAL QCD Coll.]
Nucl. Phys. A881 (2012) 28

Sasaki et al., [HAL QCD Coll.]
Nucl. Phys. A998 (2020) 121737

Coupled Channel S=-2 system ($^{11}S_0$)

K. Sasaki+ [HAL QCD Coll.]
Nucl. Phys. A998 (2020)

Weak
 $\Lambda\Lambda$ attraction



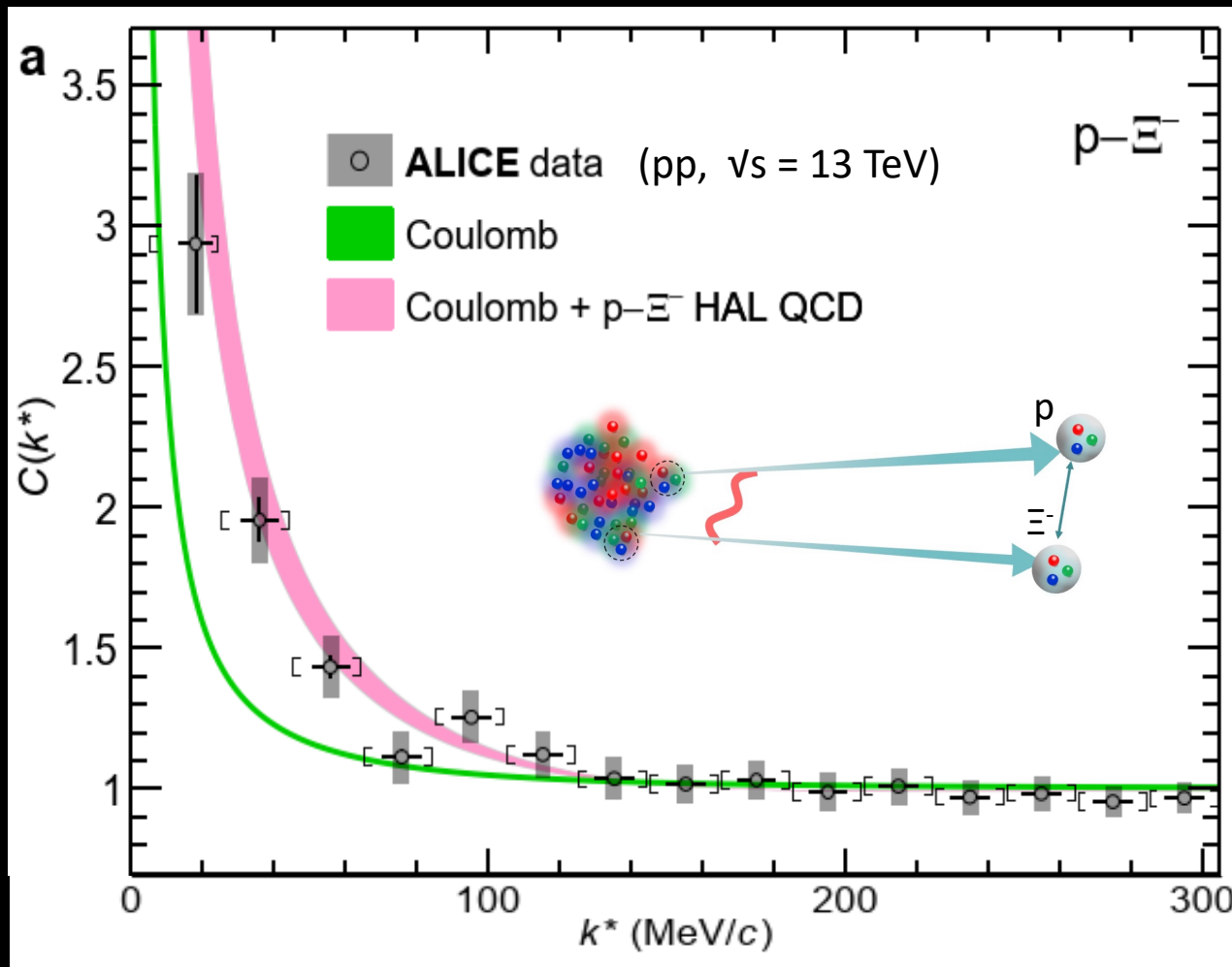
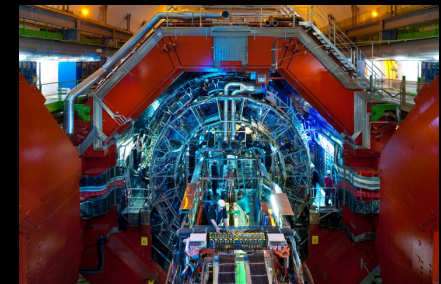
Short-range
(weak)
 $N\Xi$ - $\Lambda\Lambda$ coupling

Short-range
(weak)
 $N\Xi$ - $\Lambda\Lambda$ coupling

Strong
 $N\Xi$ attraction

Femtoscopy: $N\Xi$ pair in pp collisions

LHC ALICE Coll., Nature 588 (2020) 232

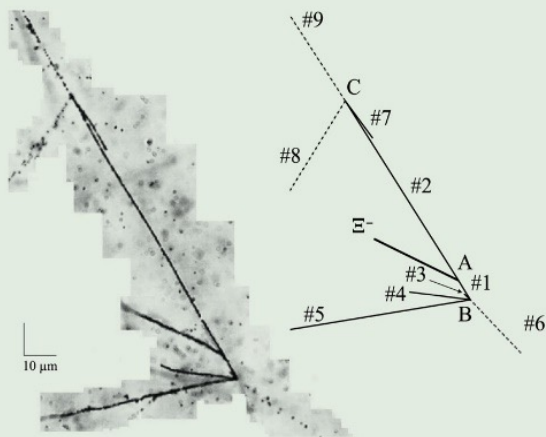


Discovery of Ξ hypernuclei (J-PARC)



PHYSICAL
REVIEW
LETTERS

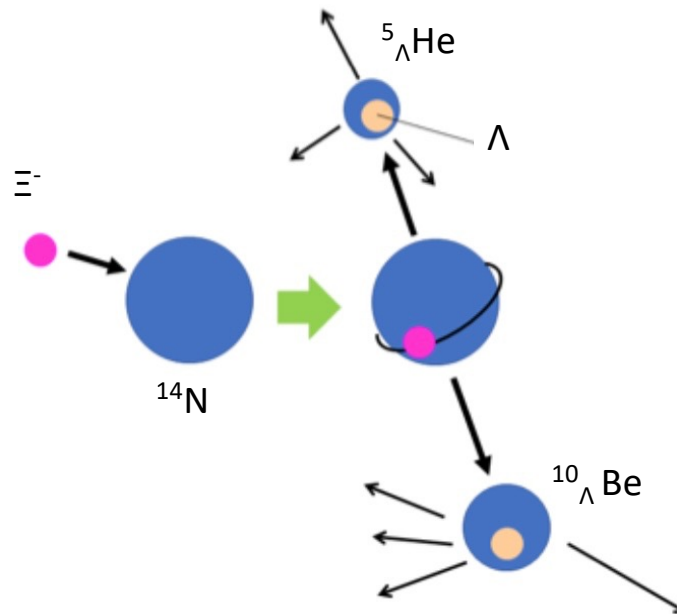
Published week ending 12 FEBRUARY 2021



Published by
American Physical Society



Volume 126, Number 6

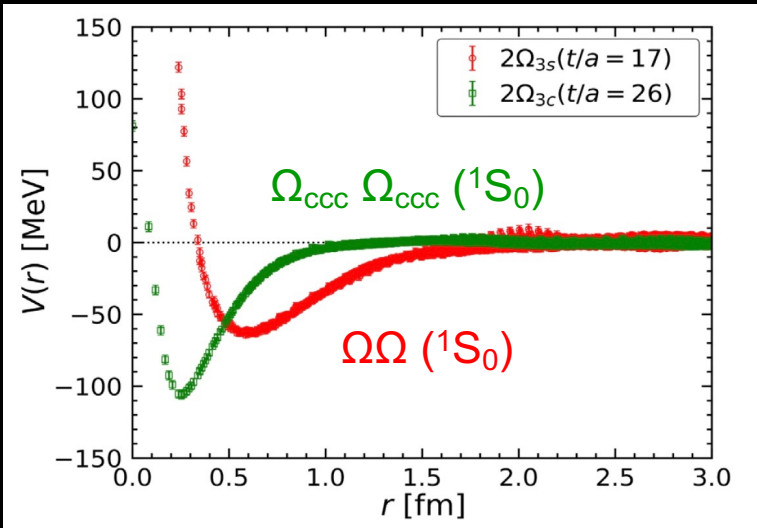
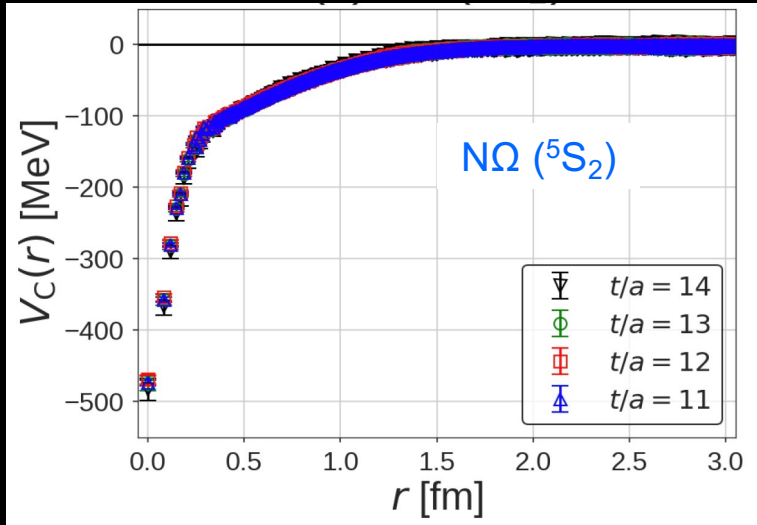
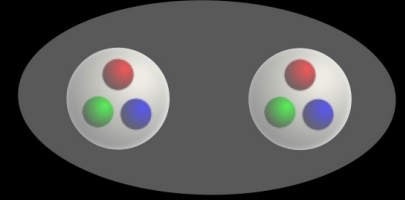


J-PARC E07 Coll.,
Phys.Rev.Lett.
126 (2021) 062501

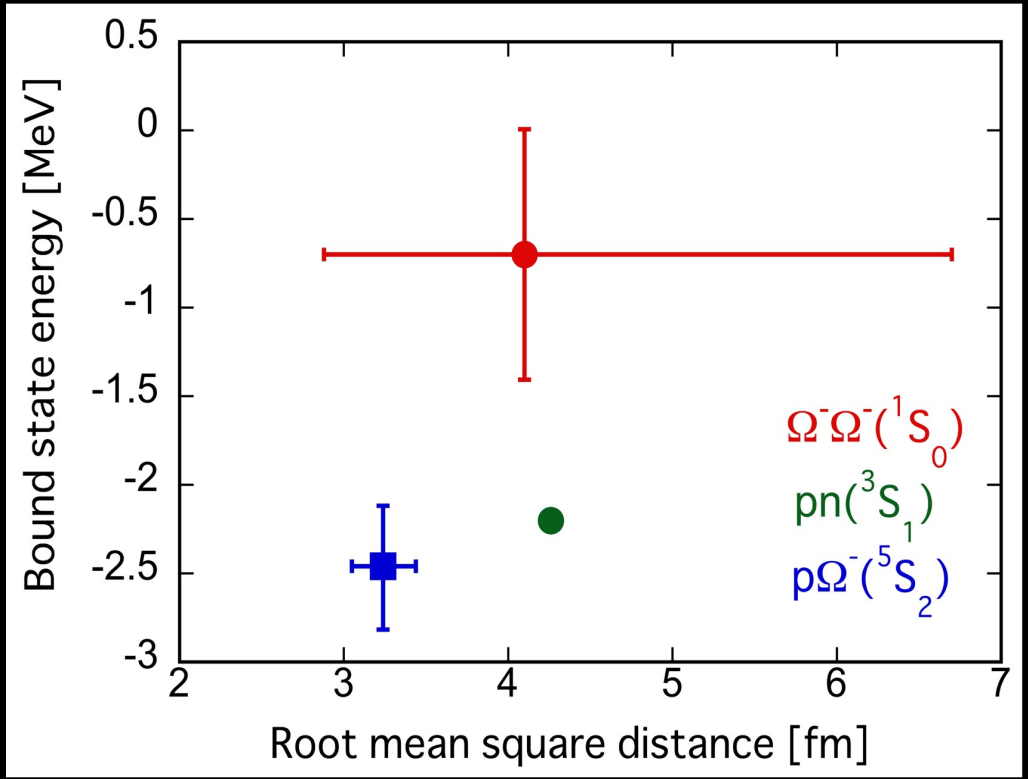
Event	Target	Decay mode	B_{Ξ^-} [MeV]	
KISO [9,10]	^{14}N	^{10}Be	5He	3.87 ± 0.21
	^{14}N	$^{10}\text{Be}^*$	5He	1.03 ± 0.18
IBUKI (present data)	^{14}N	^{10}Be	5He	1.27 ± 0.21

➔ Attraction in $N\Xi$, Weak $N\Xi - \Lambda\Lambda$ coupling

Baryon pair with $S=-3$, $S=-6$, $C=+6$

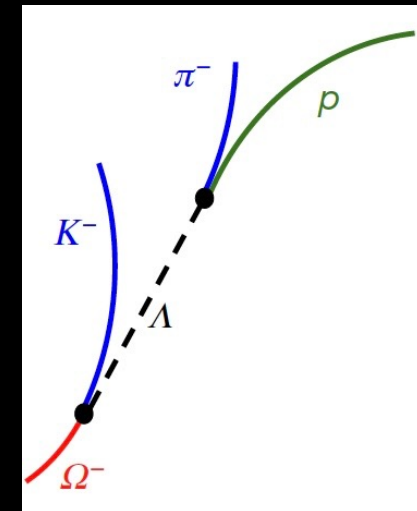
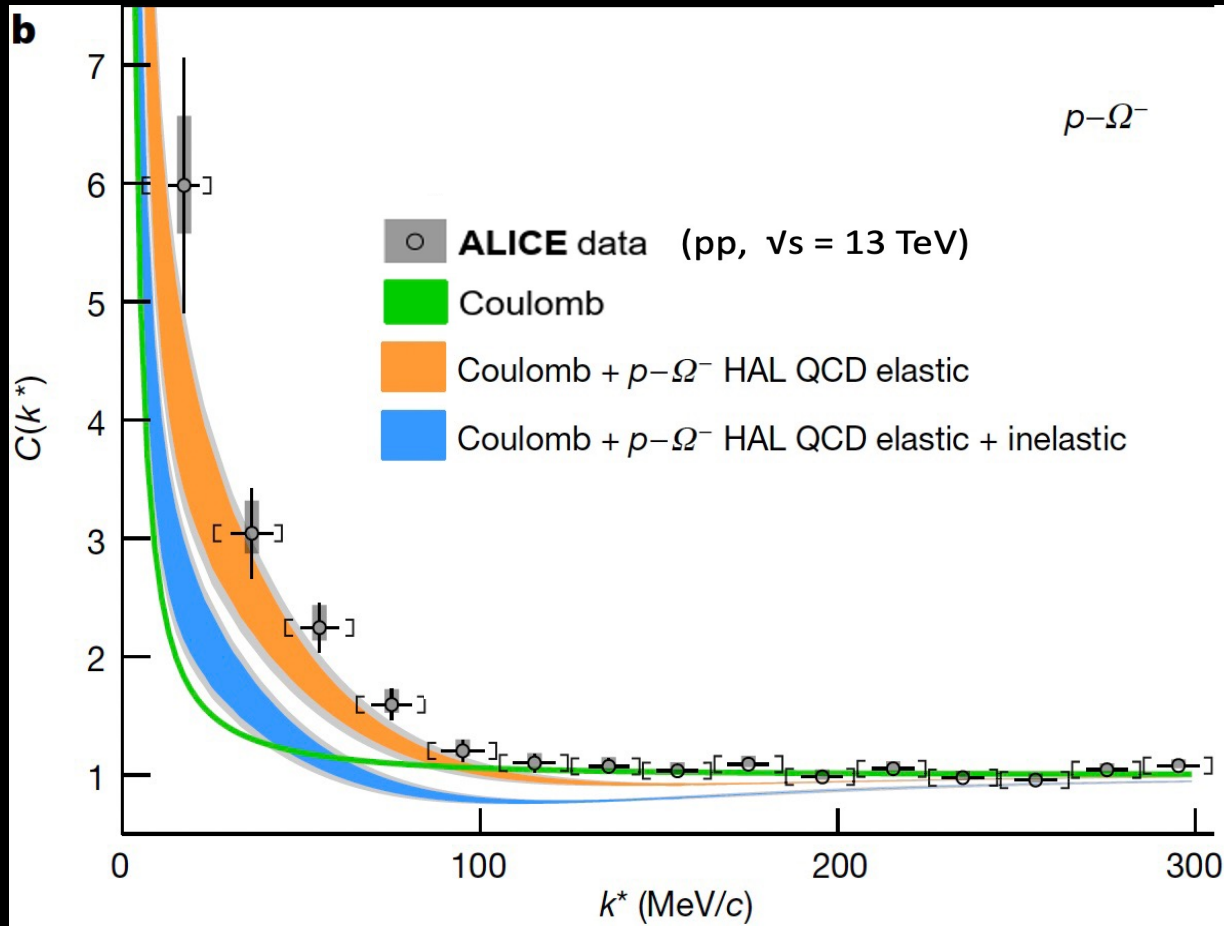
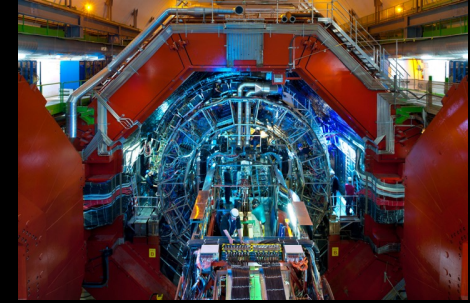


Gongyo+ [HAL QCD Coll.], PRL 120 (2018) 212001
 Iritani+ [HAL QCD Coll.], PLB 792 (2019) 284
 Tong+ [HAL QCD Coll.], PRL 127 (2021) 072003



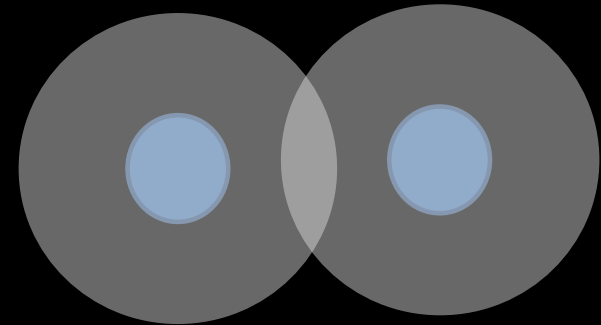
Femtoscscopy: $N\Omega$ pair in pp collisions

LHC ALICE Coll., Nature [588](#) (2020) 232



Question:

What is the force between
“neutral particles” at long range ?



Answer:

Atoms

2-photon exchange force

= van der Waals (Casimir-Polder) force $\rightarrow -1/r^7$

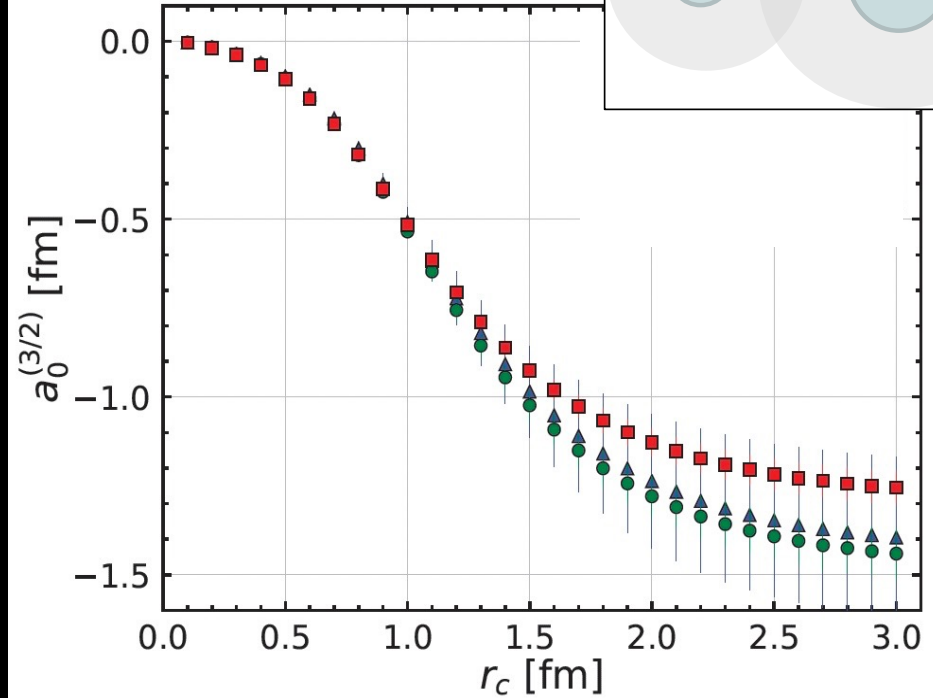
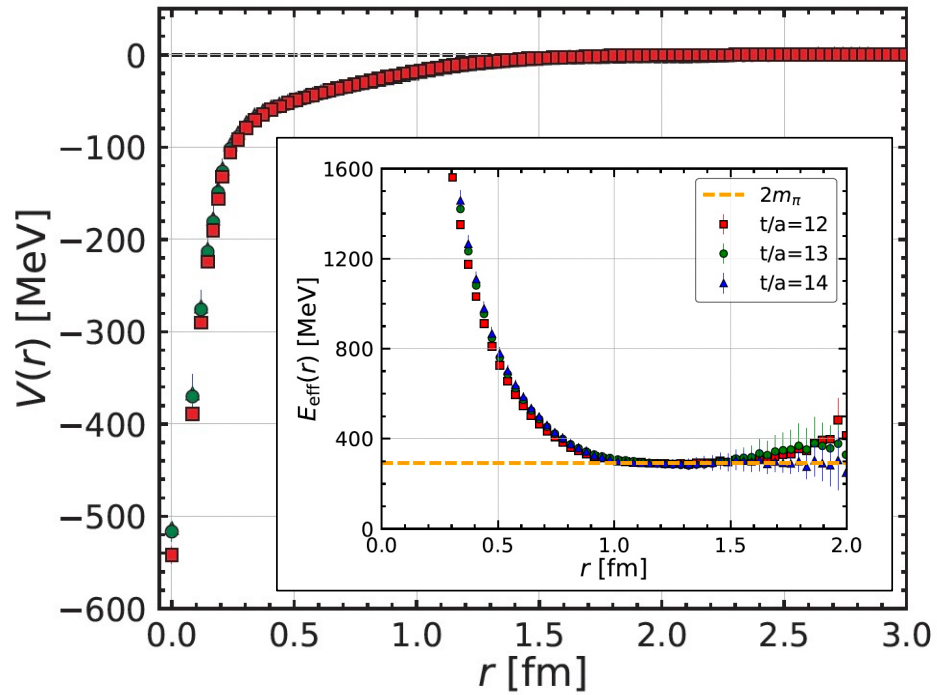
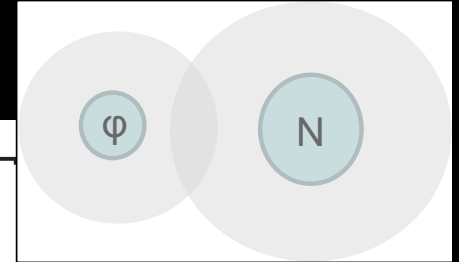
QCD

2-pion exchange force

Bhanot and Peskin, Nucl. Phys. B156 (1979) 391
Fujii and Kharzeev, Phys. Rev. D60 (1999) 114039
Brambilla et al., Phys. Rev. D93 (2006) 054002

First evidence of two-pion exchange in LQCD

Lyu+ [HAL QCD Coll.],
2205.10544 [hep-lat]



Two-pion Tail at $r > (2m_\pi)^{-1}$

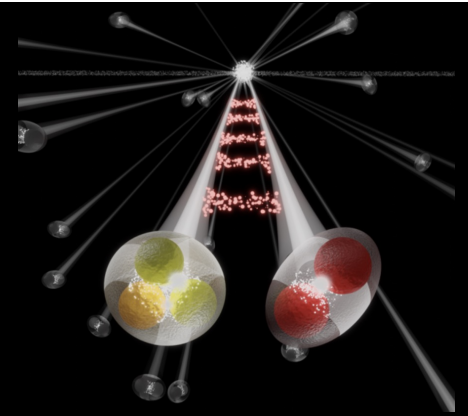
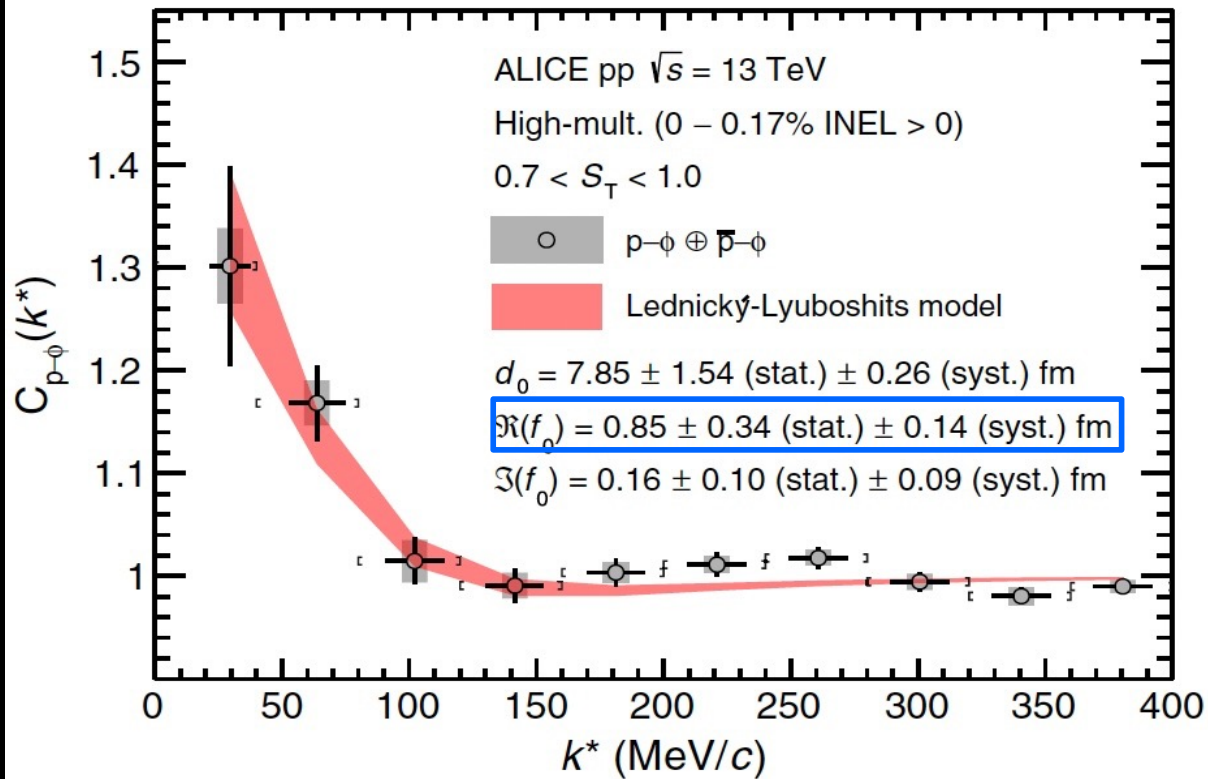
A diagram showing two horizontal lines representing nucleons. A dashed arc between them represents the exchange of two pions. A yellow arrow points to the right, leading to the potential formula:

$$V(r) = \frac{3g_A^2 m_\pi^4 (c_{di} + c_m)}{128\pi^2 F^2} \frac{e^{-2m_\pi r}}{r^2}.$$

Krein and Castella, Phys. Rev. D98 (2018) 0140289.

Experimental Evidence for an Attractive p - ϕ Interaction

S. Acharya *et al.**
(ALICE Collaboration)



<https://home.cern/news/news/physics/alice-takes-next-step-understanding-interaction-between-hadrons>

HAL QCD Lattice data



CP-PACS @Tsukuba
0.6 Tflops
 (1996-2005)



PACS-CS @Tsukuba
14 TFlops
 (2006-2011)

3-flavor & (2+1)-flavor
 $V \sim (3 \text{ fm})^3, m_\pi > 400 \text{ MeV}$

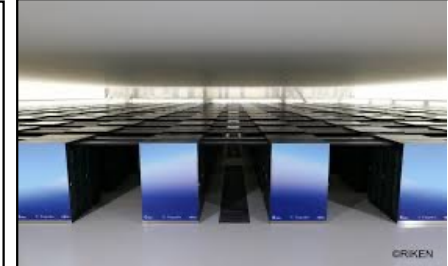
BB	Inoue+, PTP 124 ('10)
H	Inoue+, PRL 106 ('11) NPA 881 ('12)
BB	Sasaki+, PTEP 2015 ('15)
N Ω	Etminan+, NPA 928 ('14)
$\Omega\Omega$	Yamada+, PTEP 2015 ('15)
KN, $\pi\Sigma$	Ikeda+, PoS Lat ('11)
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T_{cc}	Ikeda+, PLB 729 ('14)
Z_c	Ikeda+, PRL 117 ('16)
$D^{\text{bar}}-N$	Ikeda+, HAL internal rep. ('16)
$J/\psi-N$	Sugiura+, PoS Lat ('18)
Λ_c-N	Miyamoto+, NPA 971 ('18)



K computer
 @ RIKEN
10 PFlops
 (2011-2019)

(2+1)-flavor
 $V = (8.1 \text{ fm})^3, m_\pi = 146 \text{ MeV}$

S=-1 ($\Lambda N, \Sigma N$)	Nemura+, EPJ conf. 175 ('18)
S=-3 ($\Xi\Sigma, \Xi\Lambda - \Xi\Sigma$)	Ishii+, EPJ conf. 175 ('18)
S=-4 ($\Xi\Xi$)	Doi+, EPJ conf, 175 ('18)
SU(3) basis	Inoue+, AIP conf. 2130 ('19)
S=-2 ($\Lambda\Lambda, N\Xi$)	Sasaki+, NPA 998 ('20)
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C=+6 ($\Omega_{ccc}\Omega_{ccc}$)	Tong+, PRL 127 ('21)
ϕN	Lyu+, 2205.10544 ('22)



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 @RIKEN
440 PFlops
 (2020-)

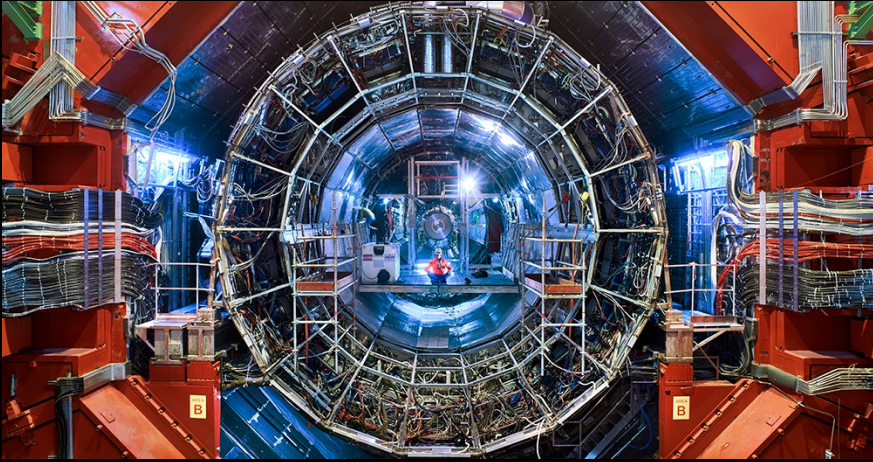
(2+1)-flavor
 $V = (8.1 \text{ fm})^3, m_\pi = 138 \text{ MeV}$

BB: Octet x Octet
BB: Octet x Decuplet
BB: Octet x Charmed
BB: LS force
MB: KN, ϕN , DN, $J/\psi-N$, ...
MM: $\pi\pi$, πK , DD*, $J/\psi-J/\psi$, ...
BBB

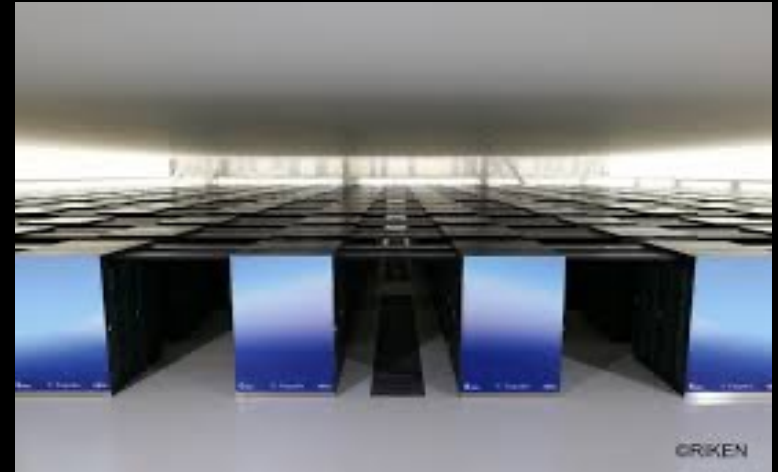
WE ARE HERE !

Summary

Laboratory Data



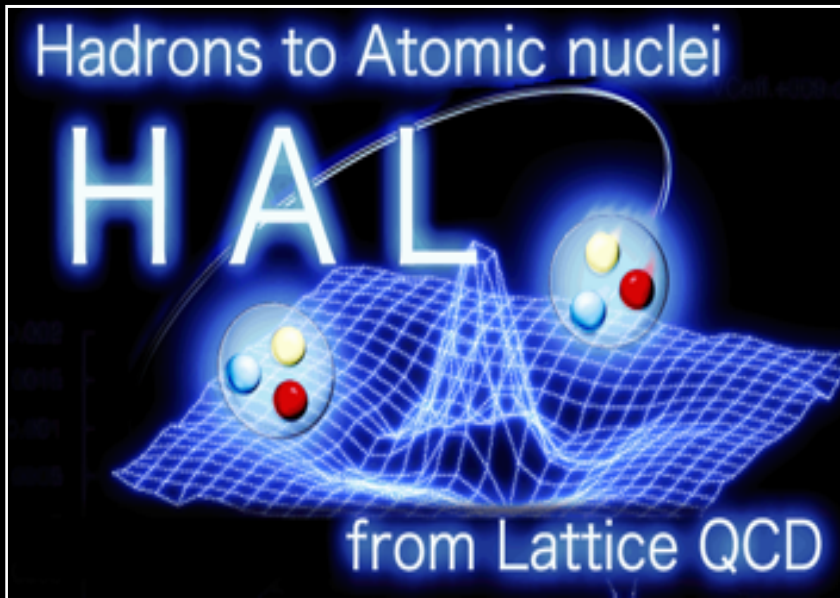
Lattice Data



HAL QCD
Method

HAL QCD Method applied to charm sector.
→ Takuya Sugiura's talk (Sept. 30, 17:30-)

Collaborators are very much welcome!
- lattice analysis, applications -



(KEK) T. Aoyama

(RIKEN) T. Doi, T. Hatsuda, T. Sugiura

(Nihon) T. Inoue

(YITP) Y. Akahoshi, S. Aoki, K. Murakami

(RCNP) T. M. Doi, N. Ishii, K. Murano,
H. Nemura

(Osaka) Y. Ikeda, K. Sasaki

(Birjand) F. Etminan

(Beijing) Yan Liu, Hui Tong