Femtoscopic study of flavored hadron interactions and ExHIC

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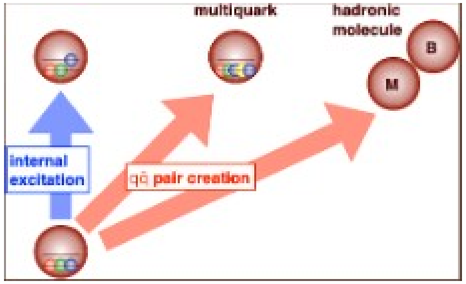
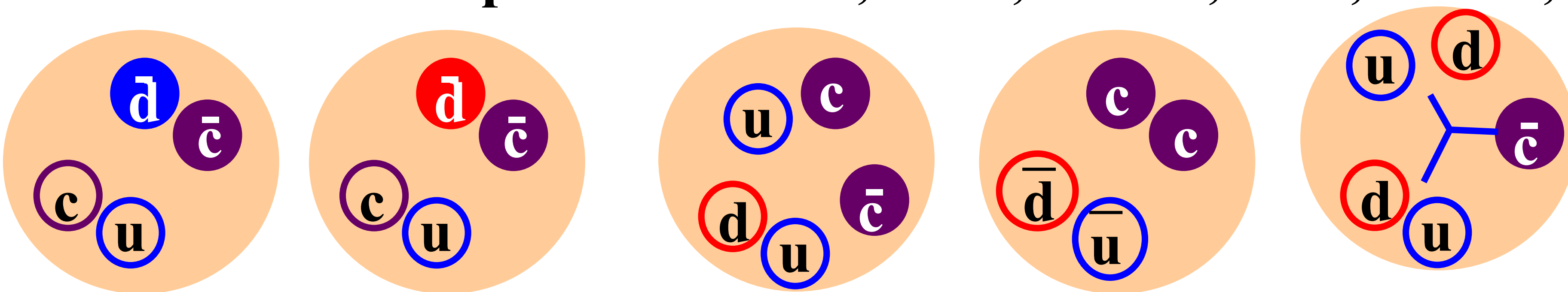
Exotics and Exotic Phenomena in Heavy-Ion Collisions, Sep.29-Oct.1, 2022, APCTP, Korea



- **Introduction – ExHIC to Femtoscopy –**
- **Femtoscopic study of DD* and D$\bar{D}$* interactions**
- **Femtoscopic guess on the existence of a bound state**
- **Femtoscopy to ExHIC**
- **Summary**

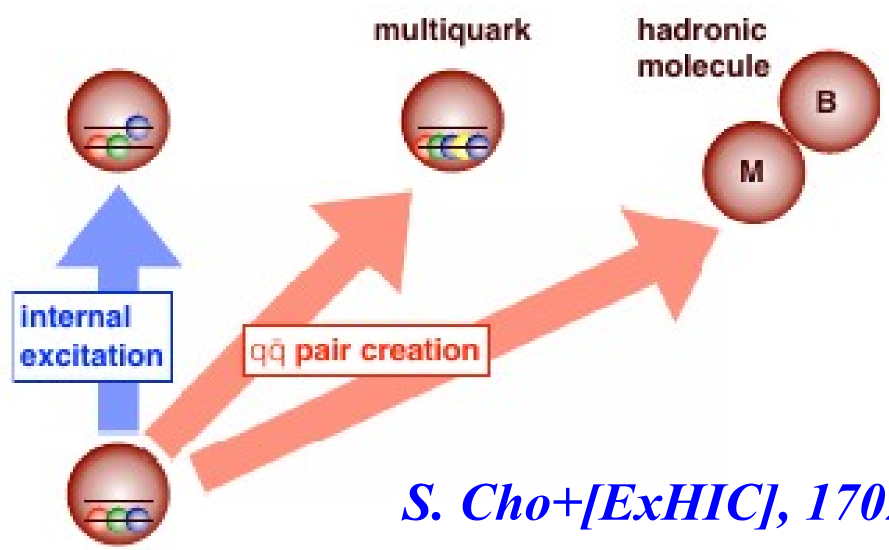
Exotic Hadrons

- Exotic hadrons (Θ^+, X, Y, Z, Pc ….)
  → Discovered/Proposed at LEPS, Belle, BaBar, BES, LHCb, ...



X(3872) Z(4430) Pc T_{cc} Θ_c

- Various pictures
  - Compact multiquark state with di-quark component
  - Hadronic molecule
  - (Triangle) Singularity
  - $Q\bar{Q}$ couples with $Q\bar{Q}\, q\bar{q}$
  - …



S. Cho+[ExHIC], 1702.00486.

Let's Categorize exotic hadrons by quark models (w/ diquarks) and molecules!

ExHIC 2010 (5/17-30)

List of registered participants

Su Houng Lee (YITP/Yonsei)(5/17-30),
Che-ming Ko (Texas A & M)(5/17-29),
Marina Nielsen (Sao Paulo)(5/16-23),
Huan Z. Huang (UCLA)(5/17-21),
In-Kwon Yoo (Pusan)(5/19-21),
Sungtae Cho (Yonsei)(5/17-29),
Youngjoon Kwon (Yonsei) (5/19-21),
A. I. Titov (Dubna, JINR),
Atsushi Hosaka (RCNP),
Tetsuo Hyodo (TITech)(5/23-28),
Chiho Nonaka (Nagoya)(5/17-18,5/21,5/26-28),
Maya Shimomura (Tsukuba),
Shigehiro Yasui (KEK)(5/17-30),
Koichi Yazaki (RIKEN/YITP)(5/19-21),
C.J. Yoon (SPring8)(5/19-22),
Ken'ichi Imai (JAEA)(5/19-21),
Shunzo Kumano (KEK)(5/19-20),
Makoto Oka (TITech)(5/20),
Masayuki Niiyama (RIKEN)(5/20+...),
Masayuki Asakawa (Osaka)(5/20),
Tetsuo Hatsuda (Tokyo)(5/21),
Yasuo Miake (Tsukuba)(*),
Takayuki Matsuki (Tokyo Kasei)(*),
Kenji Fukushima (YITP),
Alberto Martinez Torres (YITP),
Teiji Kunihiro (Kyoto)(*),
Hideo Suganuma (Kyoto),
Yoshiko Kanada-En'yo (Kyoto)(5/20),
Hiroyuki Fujioka (Kyoto)(*),
Tomofumi Nagae (Kyoto)(*),
Daisuke Jido (YITP),
Akira Ohnishi (YITP)

YIPQS workshop on
Exotics from Heavy Ion Collisions
May 17-30, 2010, Yukawa Inst., Kyoto, Japan

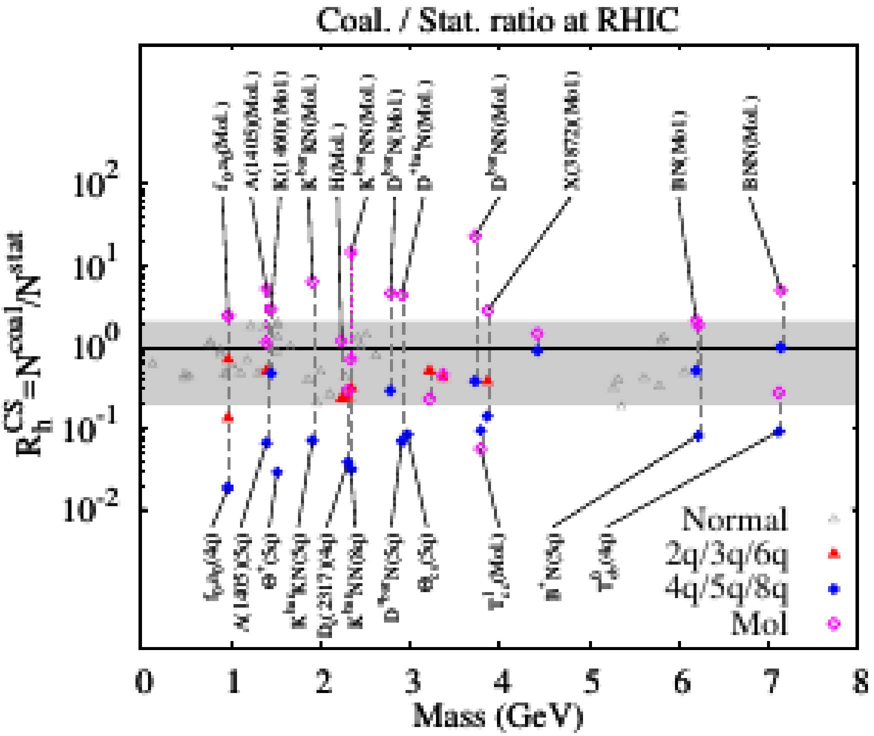
- **Identifying Multiquark hadrons from Heavy Ion Collisions, PRL106 ('11), 212001 [1011.0852] (140 times cited)**
- **Exotic Hadrons in Heavy Ion Collisions, PRC84 ('11), 064910[1107.1302] (127 times cited)**

S.Cho, T.Furumoto, T.Hyodo, D.Jido, C.M.Ko, S.H.Lee, M.Nielsen, A.Ohnishi, T.Sekihara, S.Yasui, K.Yazaki [ExHIC collaboration (2010-2011)]

Coalescence / Statistical Ratio

- **If the coalescence is the underlying hadronization mechanism, hadron yields will deviate from statistical model estimate depending on the number of constituents, spin, and size.**

ExHIC (2011)



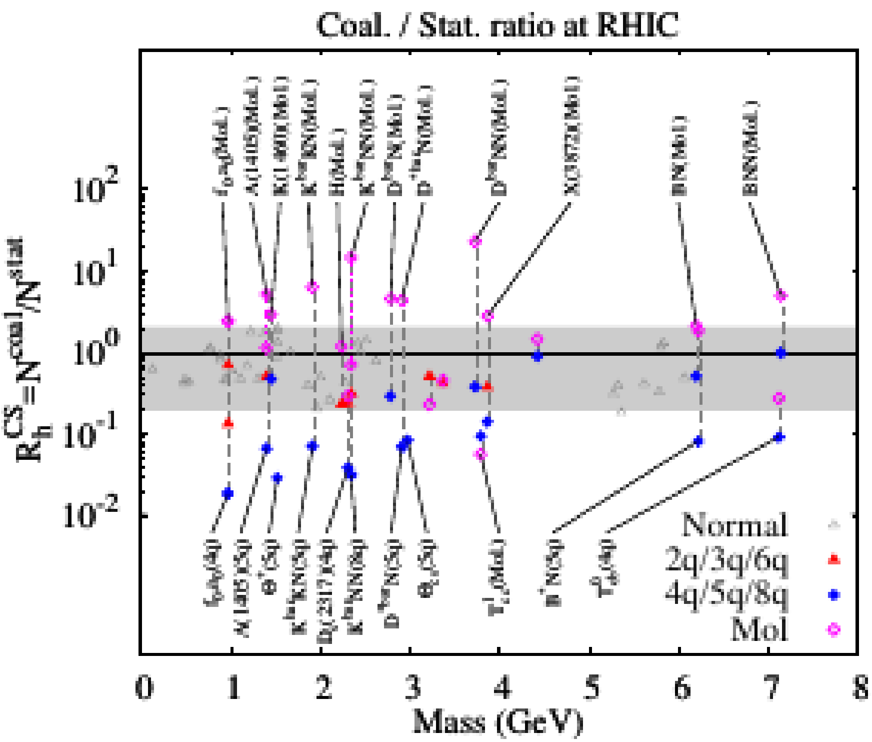
ExHIC2016(3/23-4/6@YITP+9/29-10/2@Yonsei)

- **Exotic Hadrons from Heavy Ion Collisions, S.Cho, T.Hyodo, D.Jido, C.M.Ko, S.H.Lee, S.Maeda, K.Miyahara, K.Morita, M.Nielsen, A.Ohnishi, T.Sekihara, T.Song, S.Yasui, K.Yazaki [ExHIC collaboration (2016-2017)], PPNP 95 (2017), 279-322[1702.00486] (103 times cited)**

Coalescence / Statistical Ratio

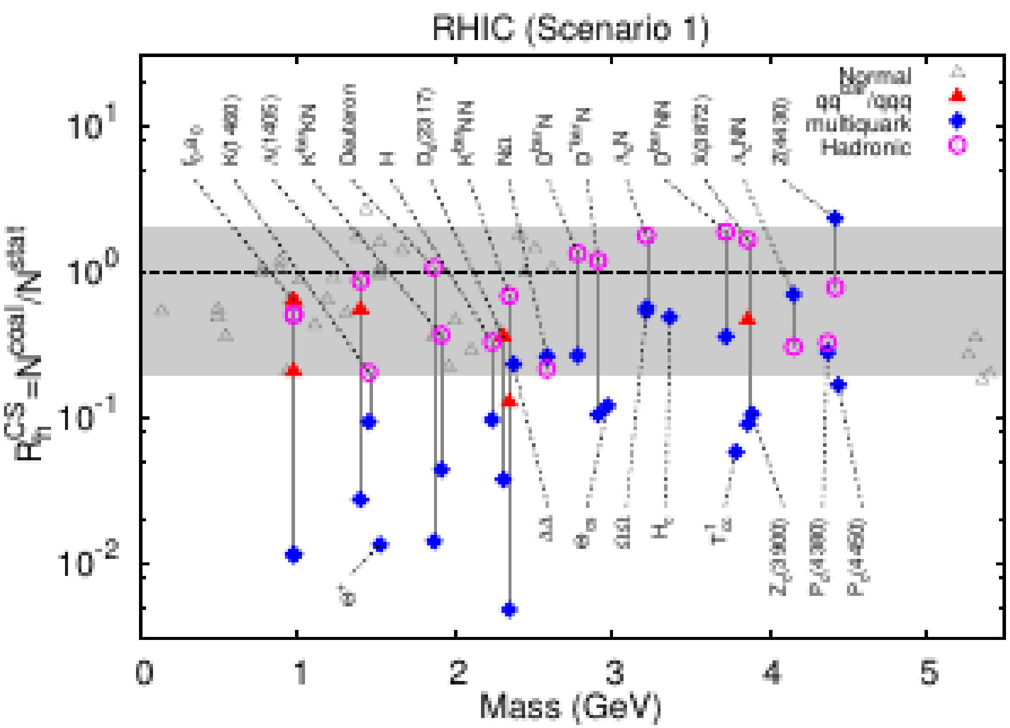
- **If the coalescence is the underlying hadronization mechanism, hadron yields will deviate from statistical model estimate depending on the number of constituents, spin, and size.**

ExHIC (2011)



Coalescence deuteron yield is larger than stat. model. ($R^{CS}_d \sim 1$ in data.)

ExHIC (2017)

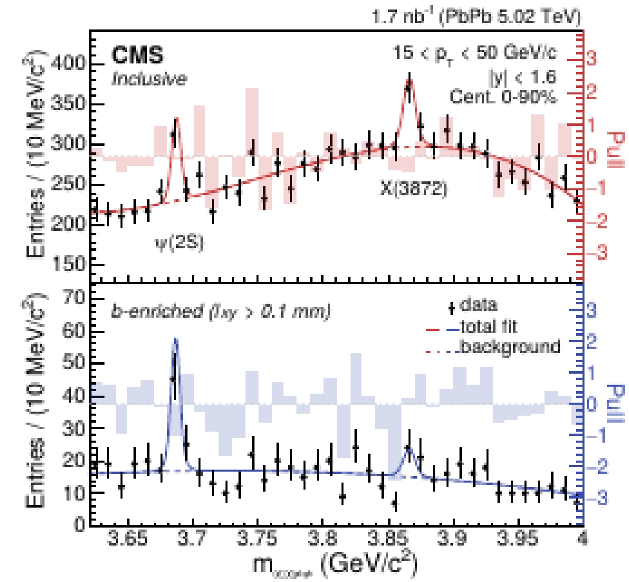


Freeze-out T is carefully chosen to give $R^{CS}_d \sim 1$.

A New Insight from CMS: Exotic/Normal Ratio

- **ExHIC index = Coalescence / Statistical Ratio**

$$R_h^{\mathrm{CS}} = \frac{\text{Yields in Coalescence}}{\text{Yields in Statistical model}}$$



- **CMS index = Exotic / Normal Ratio**

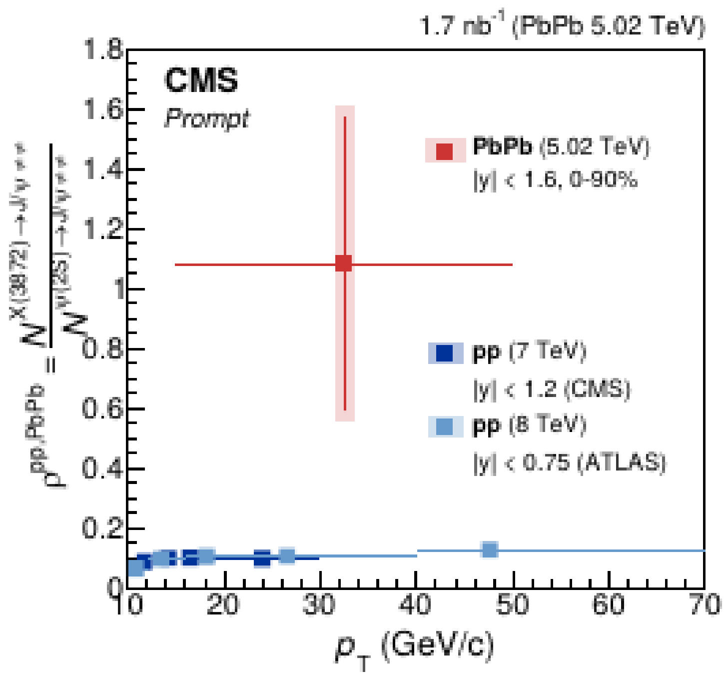
Sirunyan+ [CMS], arXiv:2102.13048

$$\rho_{\mathrm{exo/nor}} = \frac{N(\text{Exotic hadron candidate})}{N(\text{Normal hadron})}$$

- **X(3872) / ψ(2S) ratio in pp and PbPb collisions.**

$$\rho_{X/\psi}(\mathrm{PbPb}) = 1.08 \pm 0.49(\mathrm{stat.}) \pm 0.52(\mathrm{syst.})$$

$$\rho_{X/\psi}(pp) \simeq 0.1$$



ExHIC prediction is found to be (qualitatively) true !

Femtoscopy from ExHIC

ExHIC2010

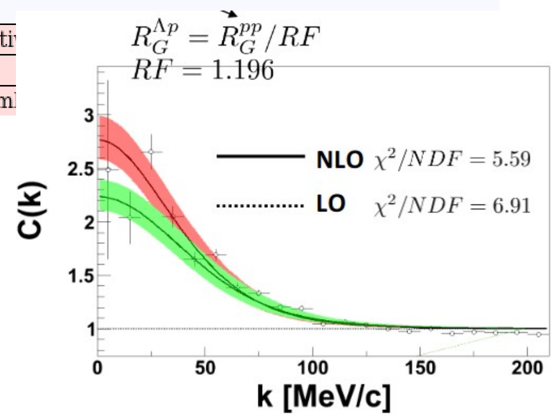
- Huan Z. Huang: Exotic Particle Searches with STAR at RHIC (45 min.) (14:00-14:45)
- Akira Ohnishi: Lambda-Lambda correlation in (K-,K+) reaction and in heavy-ion collisions (16:00-16:30) (AO's PC was broken, and the final slide is lost. A little different version will be prepared later.)

Huan Z. Huang hired a postdoc (Neha Shah), and she started to analyze ΛΛ correlation function at RHIC → STAR('15) paper on ΛΛ corr. func.

ExHIC2016

Time	Speaker	Affiliation	Title	Material
13:00-13:30	Registration			
"Overview I" Chair: T. Hyodo				
13:30-14:10	Su Houng Lee	Yonsei University	Exotics from a constituent quark model and its implication to ExHIC	abstract slide
"Overview II" Chair: L. Fabbietti				
14:40-15:20	Yuji Kato	KMI, Nagoya University	Exotic hadron spectroscopy at Belle and Belle II	abstract slide
15:20-16:00	Che-Ming Ko	Texas A&M University	Light nuclei production in relativistic heavy ion collisions	abstract slide
"Overview III" Chair: A. Ohnishi				
16:30-17:10	Neha Shah	SINAP, CAS	Hyperon interactions from heavy ion collisions	abstract slide
17:10-17:50	Shigehiro Yasui	Tokyo Institute of Technology	Charm nuclei and related topics	abstract slide

Time	Speaker	Affiliation	Title	Material
"Hadron correlation" Chair: C.M. Ko				
10:00-10:30	Kenji Morita	YITP	Probing Omega-Nucleon interaction in relati	
10:30-11:00	Laura Fabbietti	Technische Universität München	Lambda-proton femtoscopy	
11:00-11:30	Tetsuo Hyodo	YITP	Quark mass dependence of the Lambda-Lam	



Valid alternative to scattering experiments

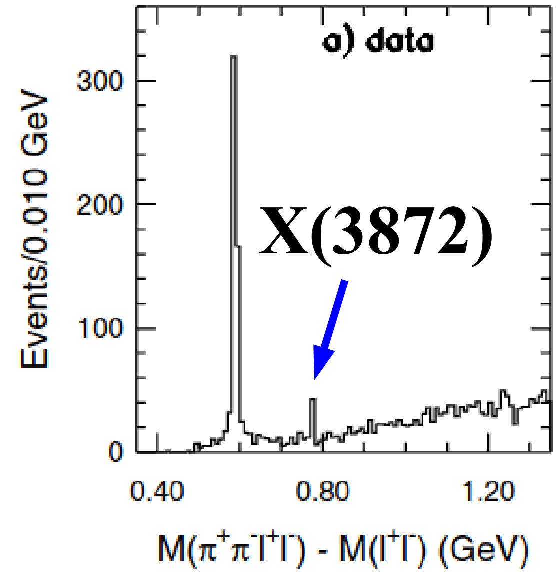
Laura Fabbietti joined the game.

ExHIC triggered Femtoscopic study of hadron-hadron interaction!

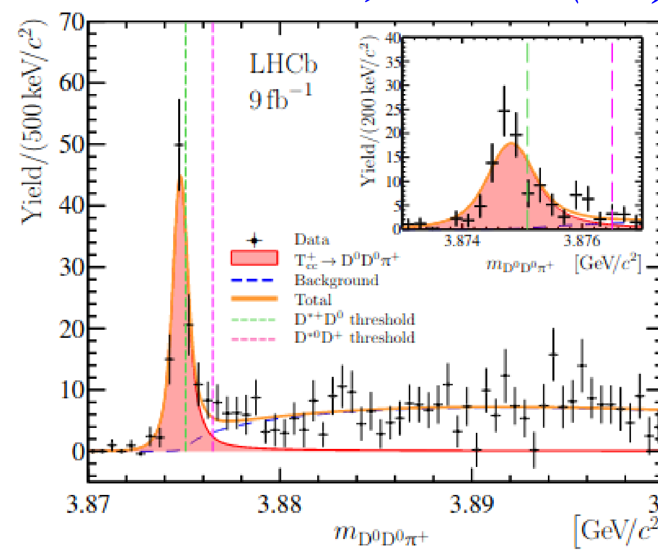
Femtoscopic study of DD^* and $D\bar{D}^*$ interaction

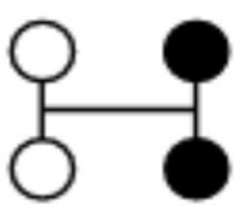
Exotic Hadrons including $c\bar{c}$ / cc / $\bar{c}\bar{c}$

- **Main play ground of exotic hadron physics**
  - **X(3872)** ***Belle ('03)*** $c\bar{c}q\bar{q}$ **Beijing Spectrometer**
  - **Many X,Y,Z states**
    Belle, CDF, BaBar, LHCb, CMS, BESIII, ...
  - **Charmed pentaquark Pc** ***LHCb ('15, '19)***
  - **Doubly charmed tetraquark state Tcc**
    LHCb ('21) $cc\bar{q}\bar{q}$
- **Structure of exotic hadrons**
  - **Compact multiquark states**
    → "good" [ud] diquark gains energy
  - **Hadronic molecules**
    → Many exotic states around thresholds
  - **Their mixture...**

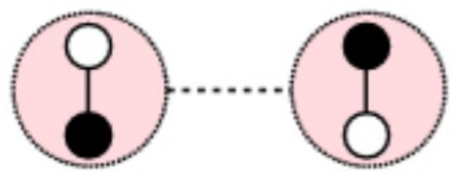


S.K.Choi+[Belle], PRL91, 262001 ('03)





Tetraquarks



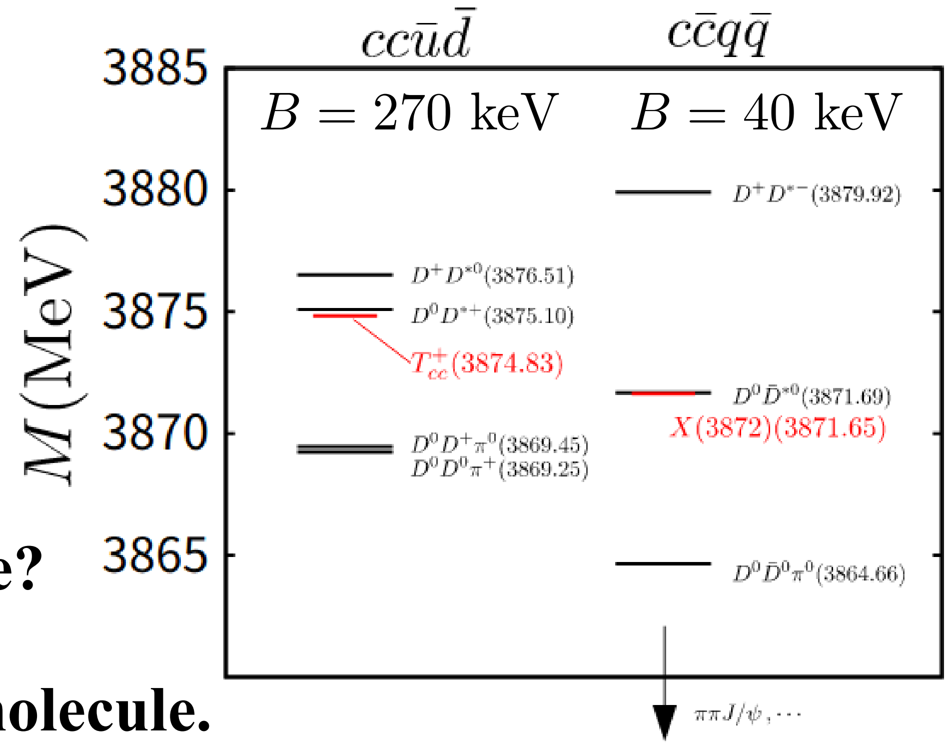
Hadronic Molecules

R. Aaji+ [LHCb], 2109.01038, 2109.01056

Compact Tetraquarks or Hadronic Molecules

- **Tcc = Compact Tetraquark ?**
  Good $[\bar{u}\bar{d}]$ diquark gains energy
  S. Zouzou+('86), ZPC30,457.

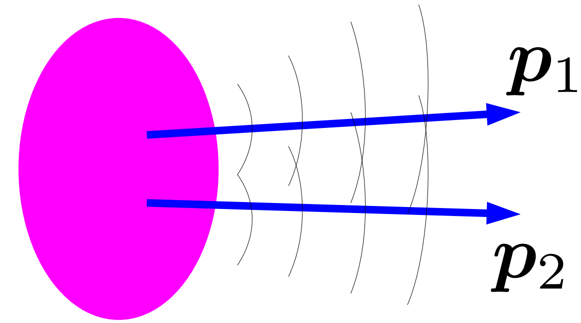
- **X(3872)**
  - **$c\bar{c}$ component ? production cross section** *Bignamini+ (0906.0882)*
  - **Large yield in Pb+Pb → Molecule?**
    Sirunyan+ [CMS] (2102.13048)
    c.f. Δr/Δp is similar in HIC and molecule.
    ExHIC ('11,'11,'17)



- **Hadronic Molecule Conditions**
  - **Appears around the threshold → OK**
  - **Have large size $R \simeq 1/\sqrt{2\mu B}$ → Yield**
  - **Described by the *hh* interaction**

How can we access hh int. with charm ?
→ Femtoscopy

Two particle momentum correlation function



- **Single particle emission function**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x, \boldsymbol{p})$$

- **Two-particle momentum correlation function**
  - **Two particles are produced independently, and correlation is generated in the final state. (Koonin-Pratt formula)**

Koonin('77), Pratt+('86), Lednicky+('82)

2 body w.f.

$$C(\boldsymbol{q}) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} \simeq \frac{\int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(y, \boldsymbol{p}_2) |\Phi_{\boldsymbol{p}_1, \boldsymbol{p}_2}(x, y)|^2}{\int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(x, \boldsymbol{p}_2)}$$

$$= \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi(\boldsymbol{r}; \boldsymbol{q})|^2 = 1 + \int d\boldsymbol{r} S(r) \left[|\varphi_0(r; q)|^2 - |j_0(qr)|^2 \right]$$

CM var. int. **Source fn.** **relative w.f. (q=relative momentum)** **s-wave**

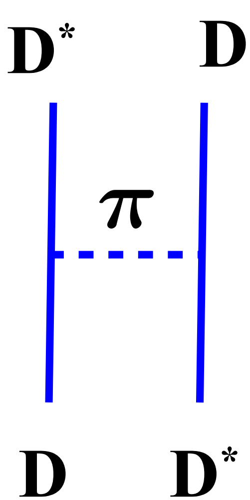
Spherical static source, non-identical particles, s-wave, No Coulomb

Note: k* is more popular instead of q in experiment papers.

Femtoscopic study of charmed hadron int.

- DD^* and $D\bar{D}^*$ correlation functions. *Kamiya, Hyodo, AO (2203.13814)*
  - Related with Tcc and X(3872)
  - *ALICE3 (2034~) can measure the correlation functions.*
- Model interaction
  - Range = one pion exchange *Yasui, Sudoh (0906.1452)*
  - Strength is fitted to the pole mass.
  - Isospin dep.
    - I=0: One range gaussian, strength fitted to the mass
    - I=1: ignored

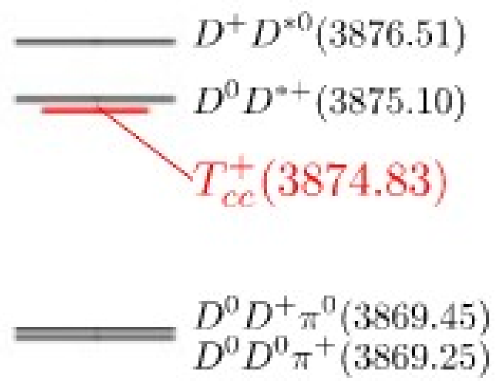
$$\{D^0\bar{D}^{*0}\} = (D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})/\sqrt{2}\ (C=+1)$$
$$\{D^+D^{*-}\} = (D^+D^{*-} + D^-D^{*+})/\sqrt{2}\ (C=+1)$$

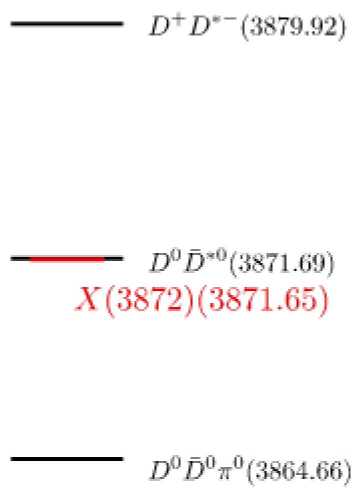


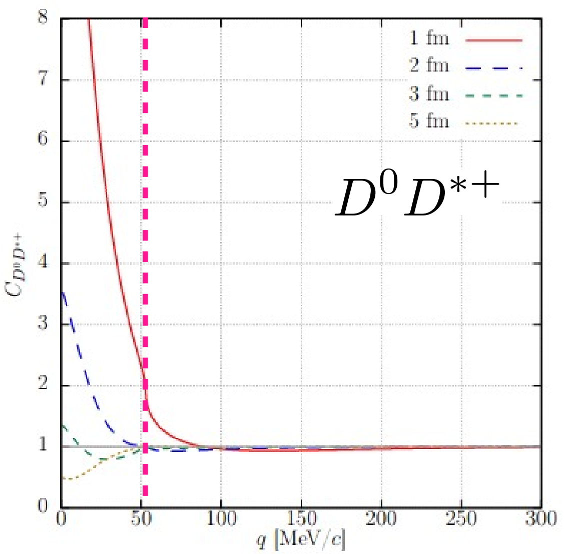
DD^*	V_0 [MeV]	$a_0^{D^0D^{*+}}$ [fm]	$a_0^{D^+D^{*0}}$ [fm]
	$-36.569 - i1.243$	$-7.16 + i1.85$	$-1.75 + i1.82$
$\{D\bar{D}^*\}$	V_0 [MeV]	$a_0^{\{D^0\bar{D}^{*0}\}}$ [fm]	$a_0^{\{D^+D^{*-}\}}$ [fm]
	$-43.265 - i6.091$	$-4.23 + i3.95$	$-0.41 + i1.47$

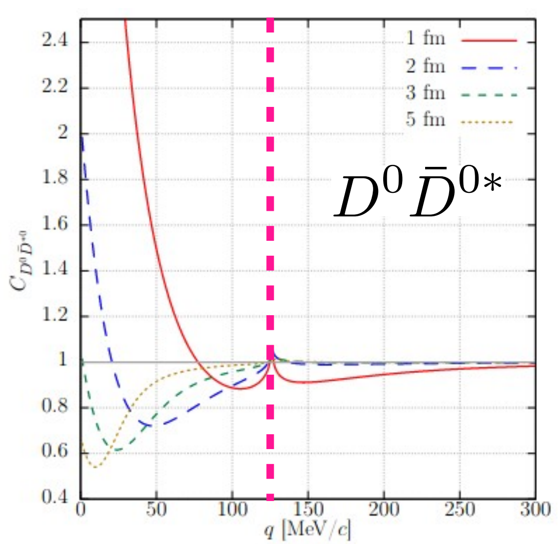
$D^0 D^{*+}$ and $D^+ \bar{D}^{*0}$ Correlation Functions

- Features of C(q) with a bound state
  - Enhancement at small source, Dip at large source.
  - Modification of potential (Changing the range, V(I=1)=0 or ± V(I=0)/3) does not change C(q) significantly. (dominated by the pole)
  - Measurement in ALICE3 (2034~) is awaited.



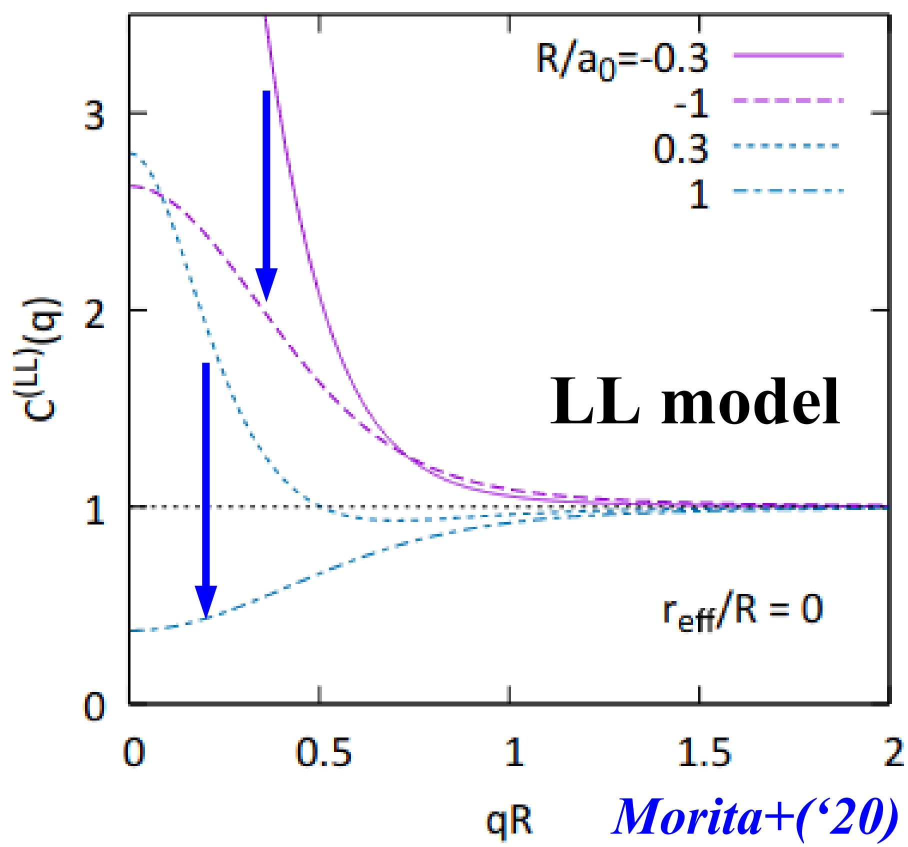






Interaction Dependence of C(q)

- Repulsive interaction → C(q) is suppressed.
- Attractive interaction
  - Wave function grows rapidly at small r with attraction.
    → C(q) is enhanced for small source.
  - Without a bound state ($a_0 < 0$)
    → C(q) > 1
  - With a bound state ($a_0 > 0$)
    → Region with C(q) < 1 appears



Morita+('20)

Source size dependence of DD* and $D\bar{D}$* will judge the nature of Tcc and X(3872), hadronic molecules or others.

Femtoscopic guess on the existence of a bound state

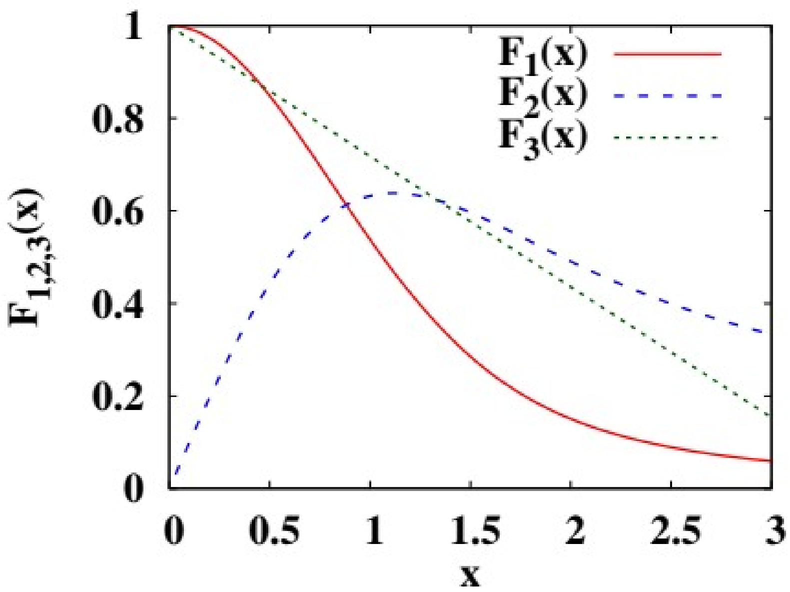
Analytic model of correlation function

- **Correlation function in Lednicky-Lyuboshits (LL) formula**
(asymptotic w.f., non-identical particle pair, short range int. (only s-wave is modified), single channel, no Coulomb pot., static Gaussian source, real δ) *(Lednickey, Lyuboshits ('82))*

$$\varphi_0^{(-)}(r;q) \simeq \frac{e^{-i\delta}\sin(qr+\delta)}{qr}$$

$$C_{\rm LL}(q) = 1 + \frac{2{\rm Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{{\rm Im}\, f(q)}{R} F_2(2qR) + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\rm eff}}{R}\right)$$

$$\left[f(q) = (q\cot\delta - iq)^{-1},\ F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2-x^2},\ F_2(x) = (1-e^{-x^2})/x,\ F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}\right]$$



If you have a_0, r_{eff} and R, you can draw C(q) !

$$F_1(x) \simeq \frac{1 + c_1x^2 + c_2x^4 + c_3x^6}{1 + (c_1+2/3)x^2 + c_4x^4 + c_5x^6 + c_3x^8} \quad (0 \le x < 20)$$

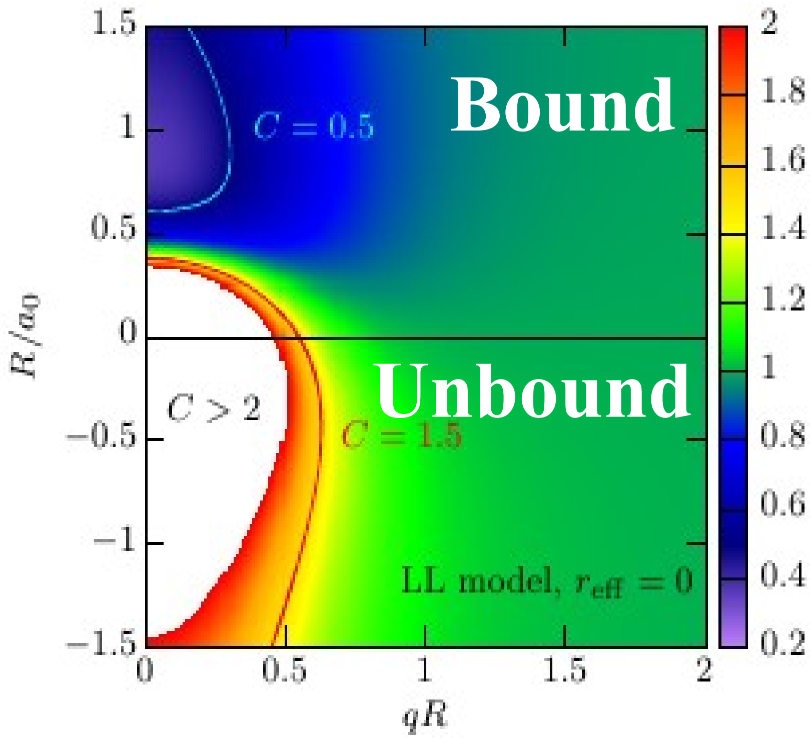
$$(c_1, c_2, c_3, c_4, c_5) = (0.123, 0.0376, 0.0107, 0.304, 0.0617)$$

AO,Morita,Mihayara,Hyodo, NPA 954 ('16)294.

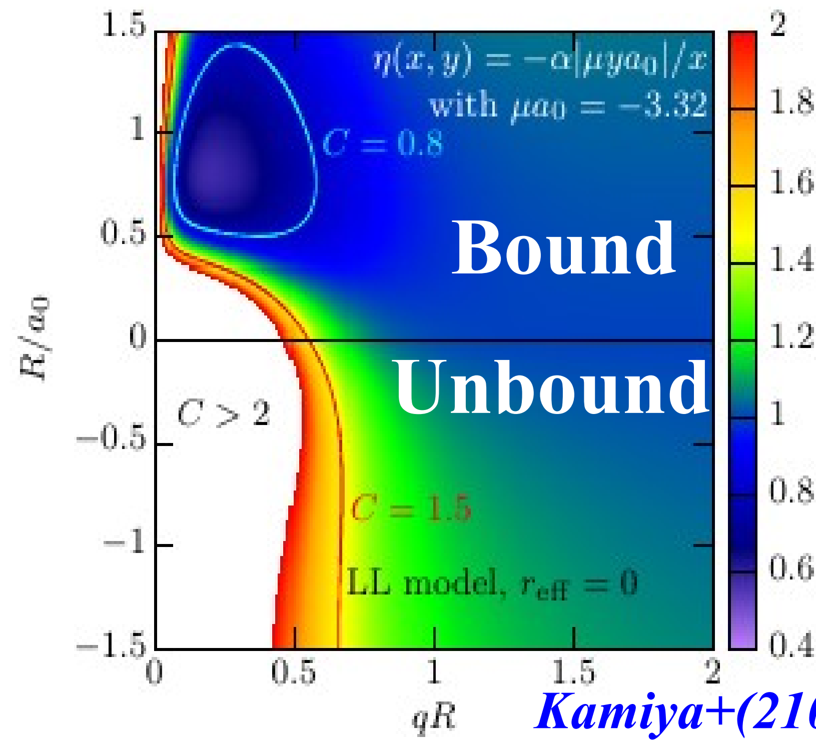
R Dependence of Correlation Function

- **Source size (R) dependence of C(q) is helpful to deduce the existence of a bound state.**
  Morita+('16, '20), Kamiya+('20), Kamiya+(2108.09644)
- **Bird's-eye view of C(q) using the Lednicky-Lyuboshits formula with the zero range approx. (r_{eff}=0)** *[Lednickey, Lyuboshits ('82)]*
  - **Universal function, C(q)=C(qR, R/a_0) (r_{eff}=0, w/o Coulomb)**

w/o Coulomb



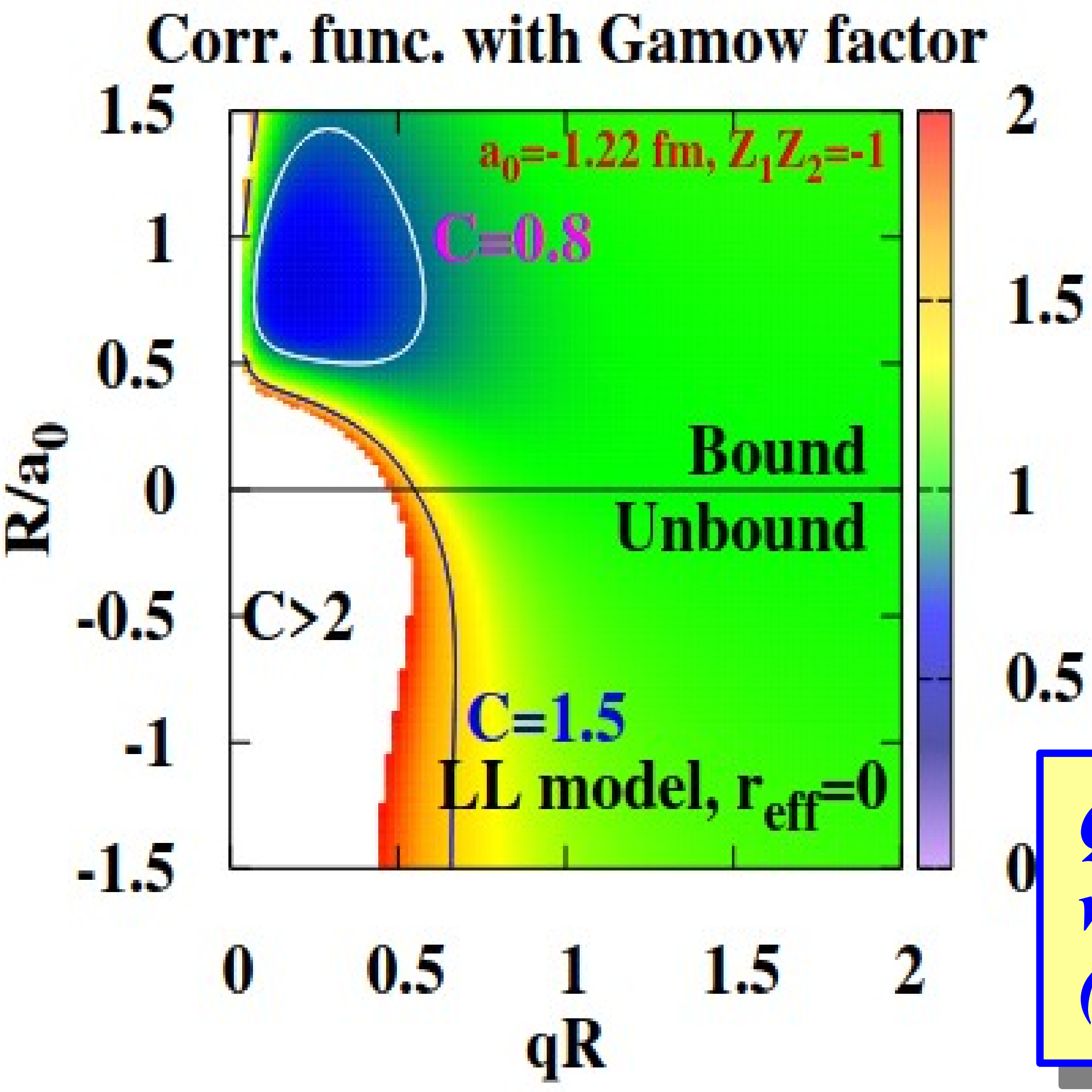
With Coulomb



Kamiya+(2108.09644)

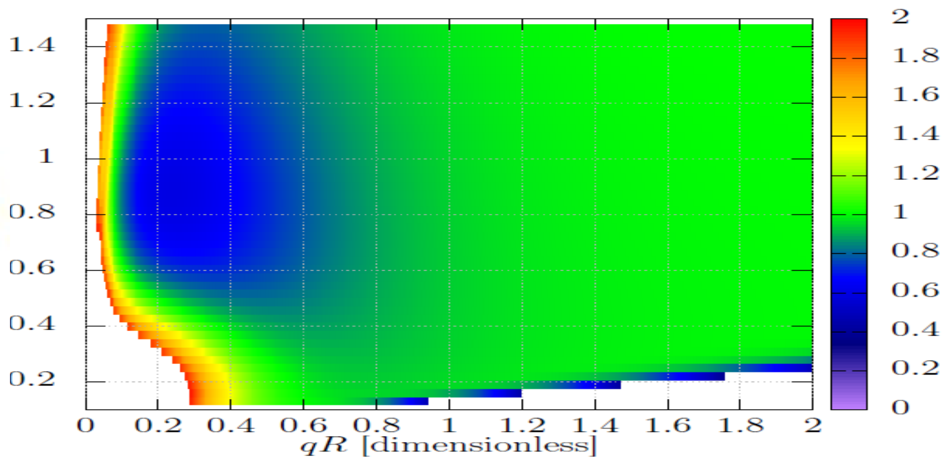
R Dependence of Correlation Function

LL model with Coulomb (r_{eff}=0)



Realistic NΩ potential (J=2, HAL QCD, a_0=3.4 fm) + Coulomb, Coupled-channel

Courtesy of Y. Kamiya



Qualitative feature remains with realistic interactions (and coupled-channel effects)

Wave function around threshold (S-wave, attraction)

- **Low energy w.f. and phase shift**

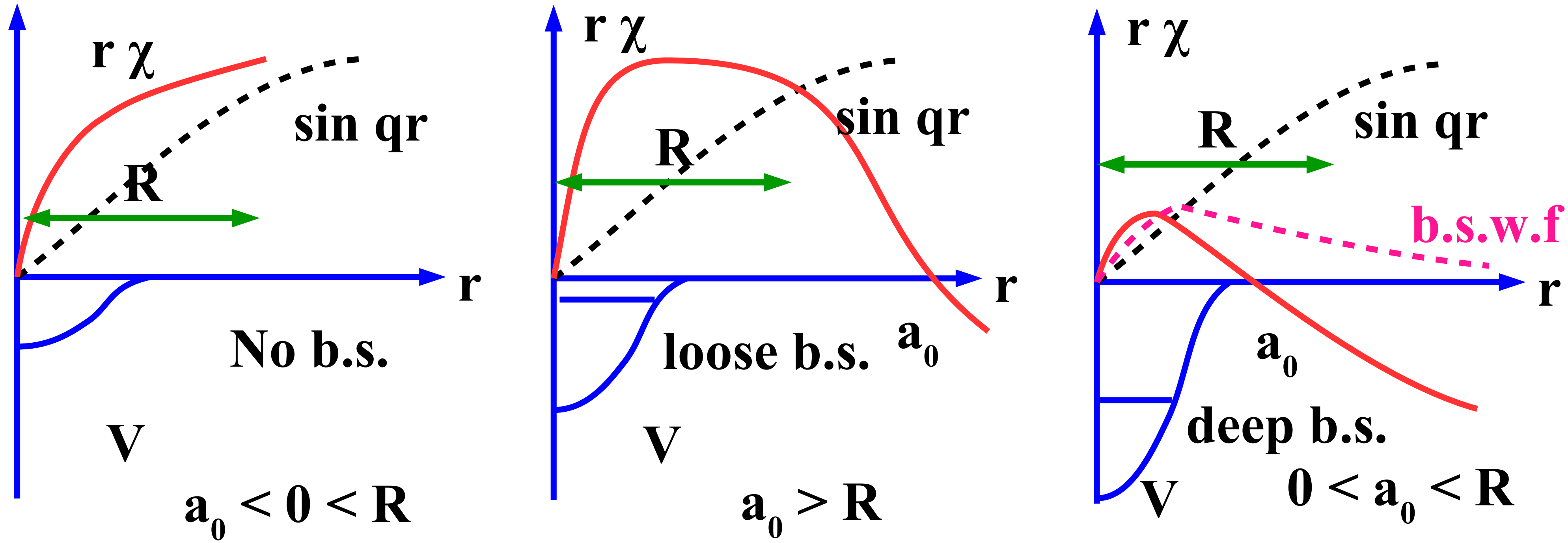
a_0 =scatt. length
r_{eff}=eff. range

$$u(r) = qr\chi_q(r) \to \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4)\ \ (\delta \sim -a_0 q)$$

Nucl. and Atomic Phys. convention

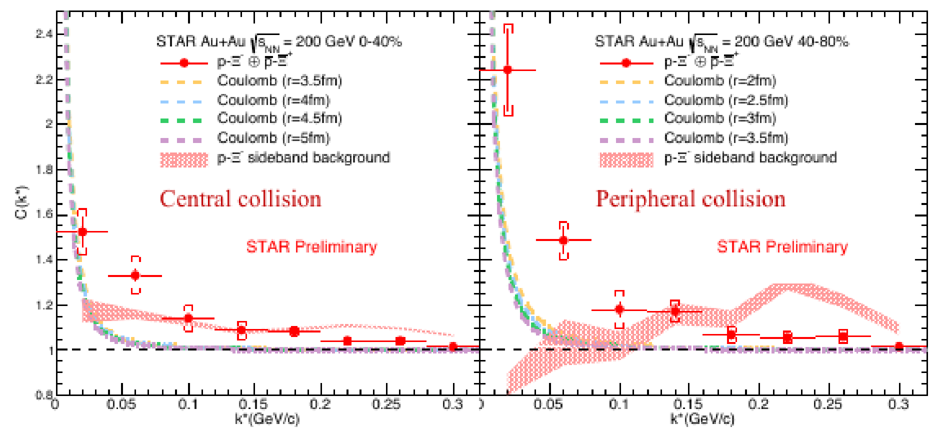
- **Wave function grows rapidly at small r with attraction.**
- **With a bound state (a_0>0), a node appears around r=a_0**
  → Suppressed |w.f.|^2 on average



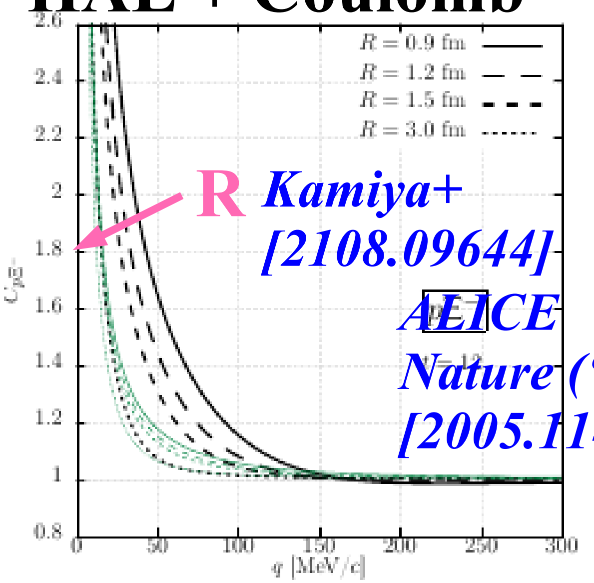
Case without a bound state ($p\Xi^-$)

- **No NΞ bound state from lattice QCD**
  Sasaki+ [HAL], NPA998 ('20)121737 [1912.08630]
- **R dep. of calculated results**
  → Enhanced region shrinks with larger R. No Dip.
- **Larger R data from Au+Au seem to show similar behavior.**

K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.
(No Dip at larger R)

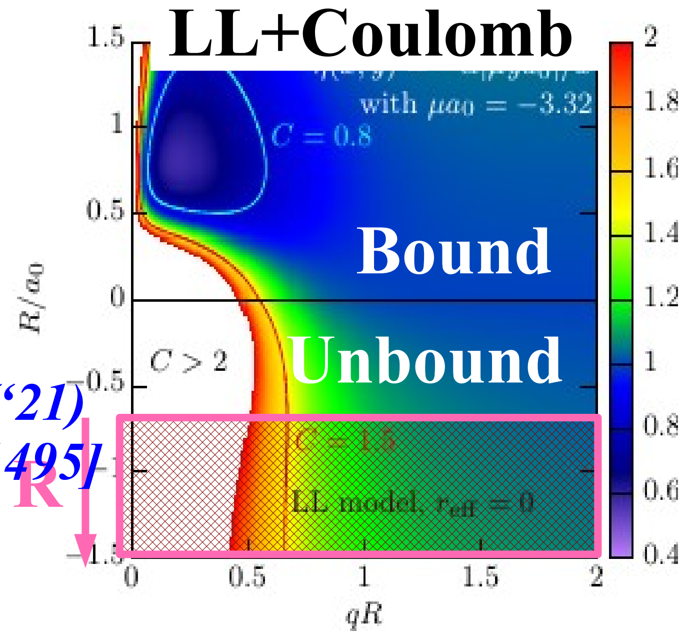


HAL + Coulomb



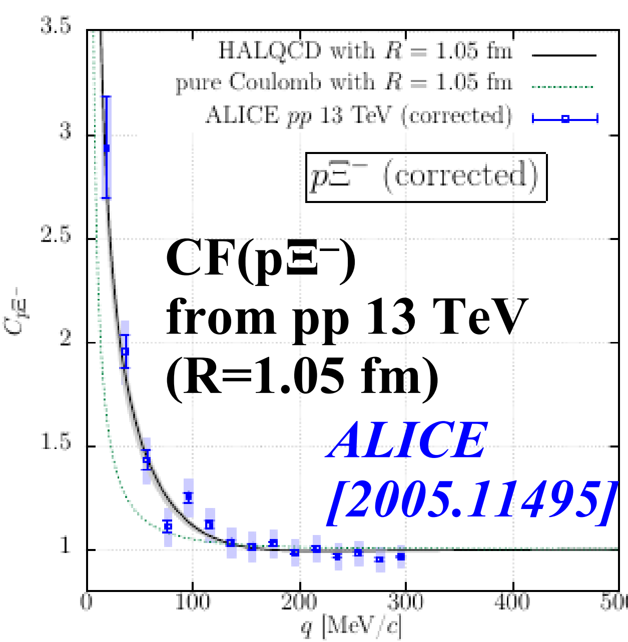
R *Kamiya+ [2108.09644]*

LL+Coulomb



R

ALICE Nature ('21) [2005.11495]

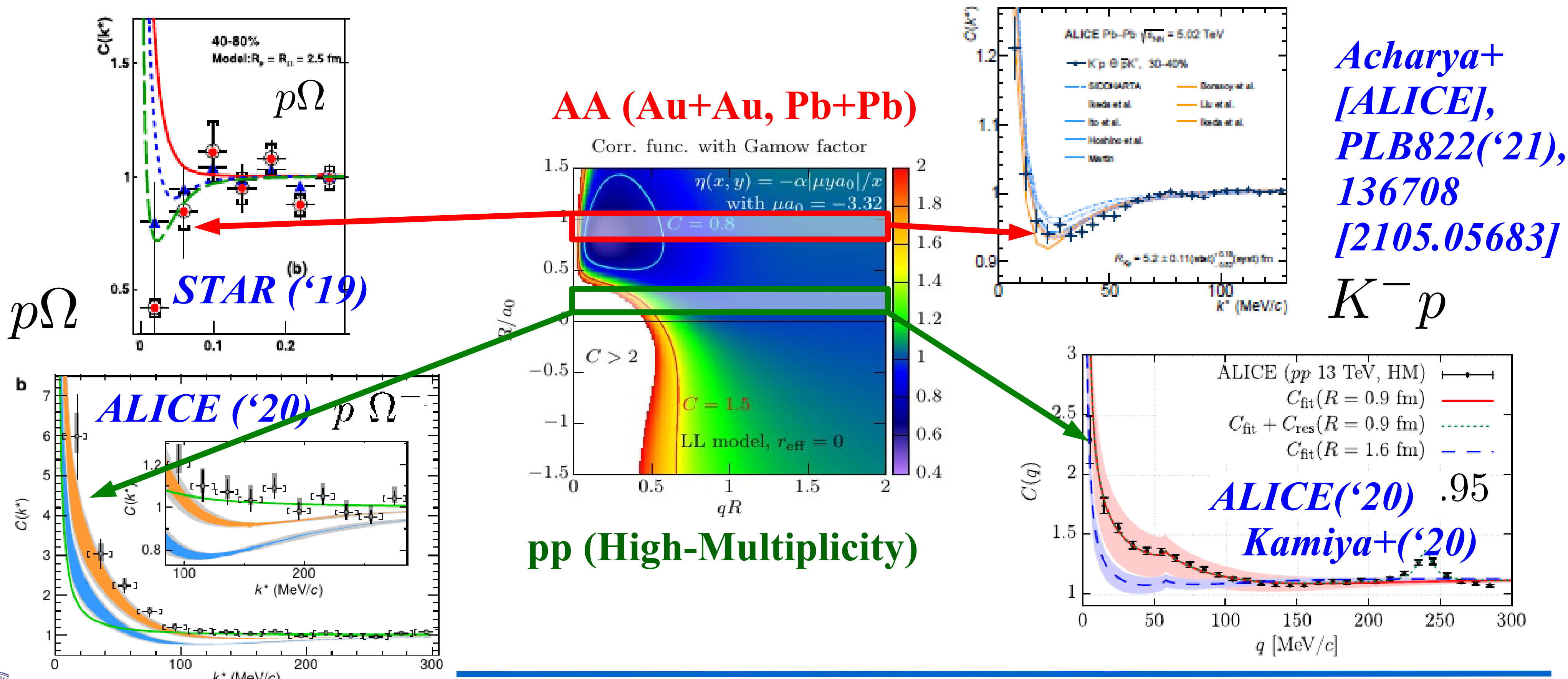


CF(pΞ⁻) from pp 13 TeV (R=1.05 fm)

ALICE [2005.11495]

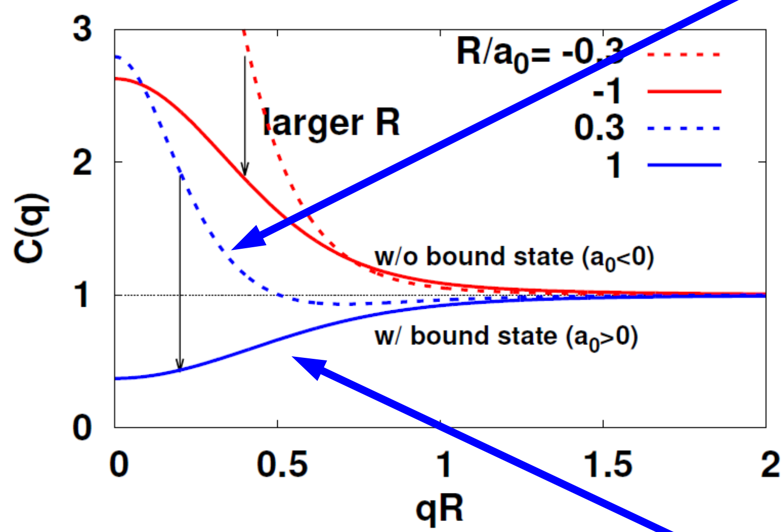
Bound State Dip

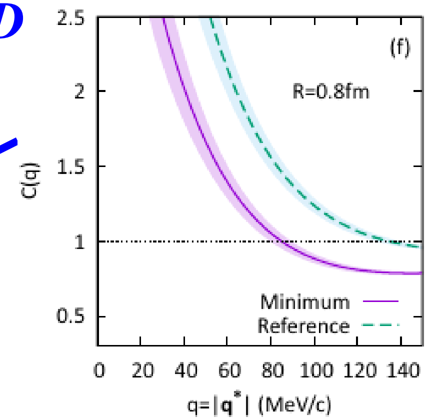
- **With a bound state, C(q) is expected to show a dip for R ~ $|a_0|$.**
- **KN, ΩN → Bound states are expected, and dip is observed in AA**
  Goldman+('87); Oka ('88); Etminan+[HAL QCD] ('14); Iritani+[HAL QCD]('19); Dalitz, Tuan ('59); Akaishi, Yamazamki ('02); Jido+('03); Hyodo, Jido ('12); Morita+('16,'20); Kamiya+('20); Haidenbauer('18).
- **a_0(ΩN)=3.4 fm (Iritani+('19, HAL QCD)), a_0(K^- p)=0.65-0.80i fm (SIDDHARTA)**

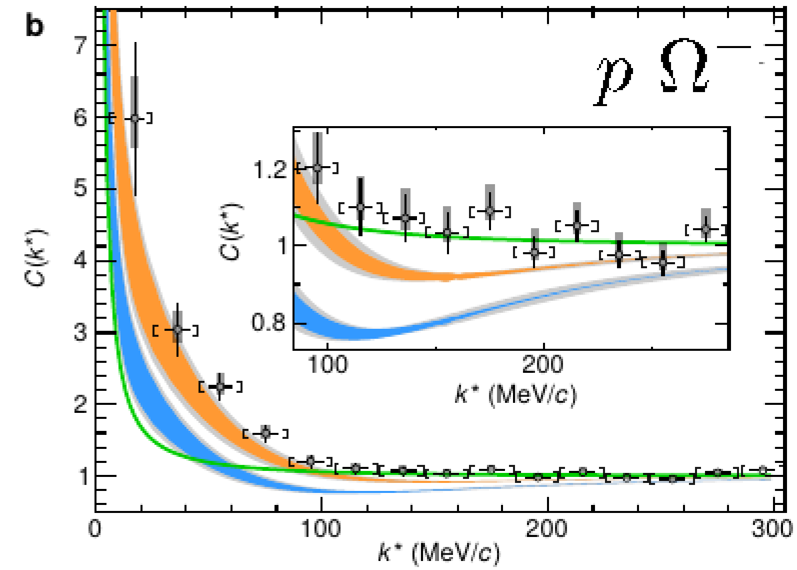


STAR+ALICE suggests a NΩ dibaryon state

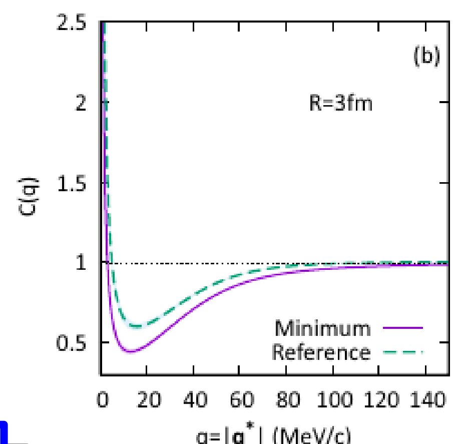
Morita+, PRC101('20)015201 [1908.0414] (Gaussian source)
Lattice BB pot. from HAL QCD [Iritani+('19)]

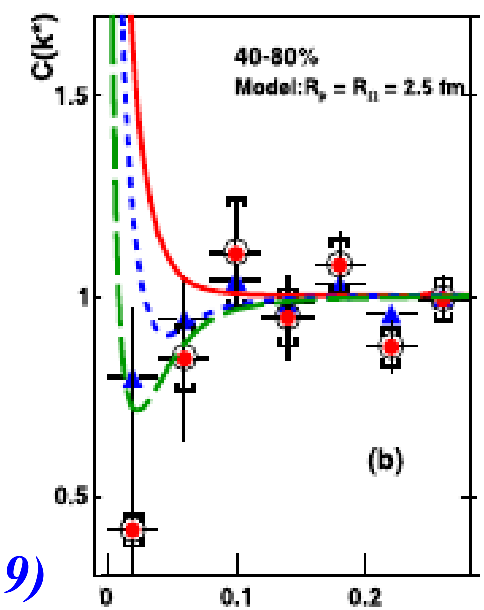






ALICE, Nature588('20)232 [2005.11495]





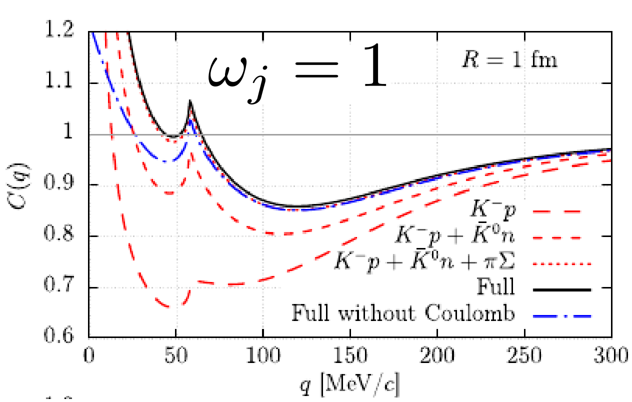
STAR, PLB790 ('19) 490 [1808.02511].

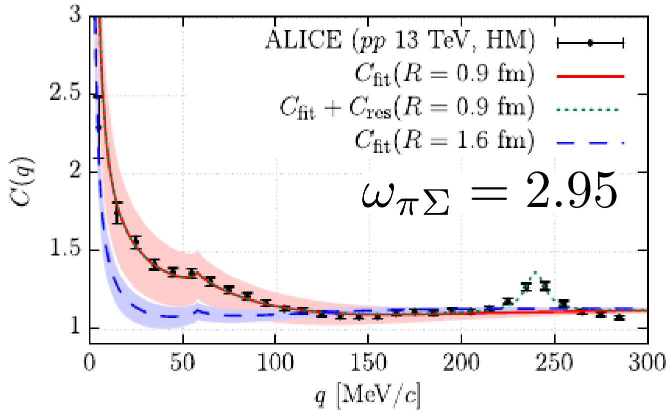
Reference: $V_{J=1}=V_{J=2}$
Minimum: $\varphi_{J=1}=0$

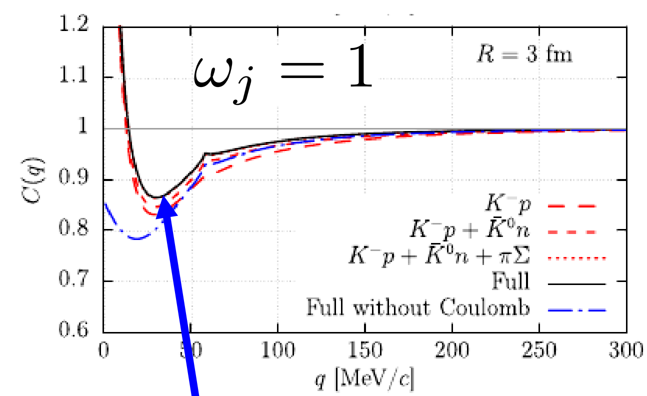
Dip from a bound state survives Coulomb.

Source Size Dependence of C(pK $^-$)

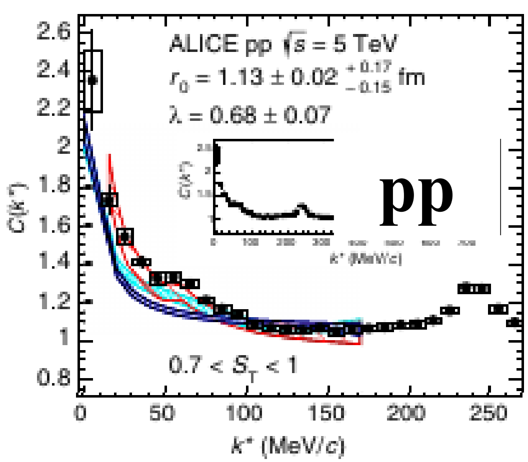
- **Coupled-channel effects are suppressed when R is large, and "pure" pK$^-$ wave function may be observed in HIC.**



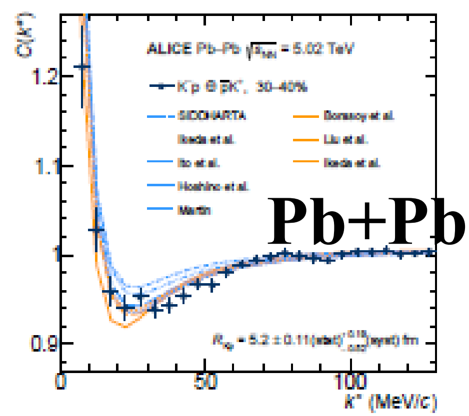




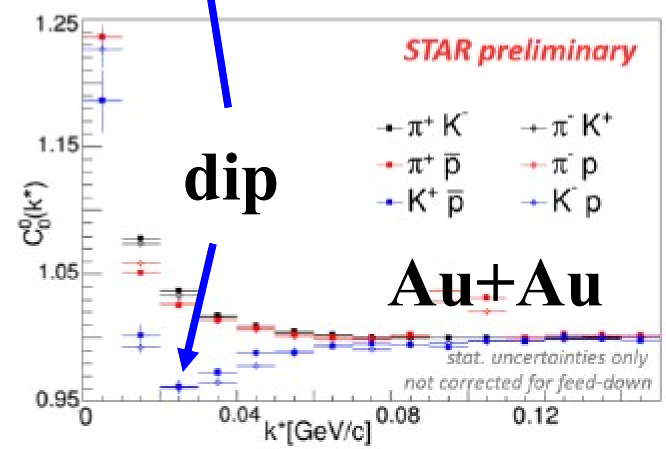
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.



S. Acharya+[ALICE], PRL124('20)092301



S. Acharya+[ALICE], 2105.05683



Siejka+[STAR, preliminary], NPA982 ('19)359.

> ***STAR(prel.) & new ALICE data show a dip at small q.***

Scattering length from K$^-$p correlation function

- **LL model fit (w/ Coulomb) to the correlation function data**

  S. Acharya+[ALICE], PLB 822 ('21) 136708 [2105.05683] ($\delta \sim +a_0 q$, HEP convention)

$$a_0 = -0.91 \pm 0.03(\mathrm{stat})^{+0.17}_{-0.03}(\mathrm{syst}) + i[0.92 \pm 0.05(\mathrm{stat})^{+0.12}_{-0.33}\ (\mathrm{syst})]\ \mathrm{fm}$$

- **Consistent with SIDDHARTA (kaonic atom) data, and errors are comparable to previous dedicated experiments.**

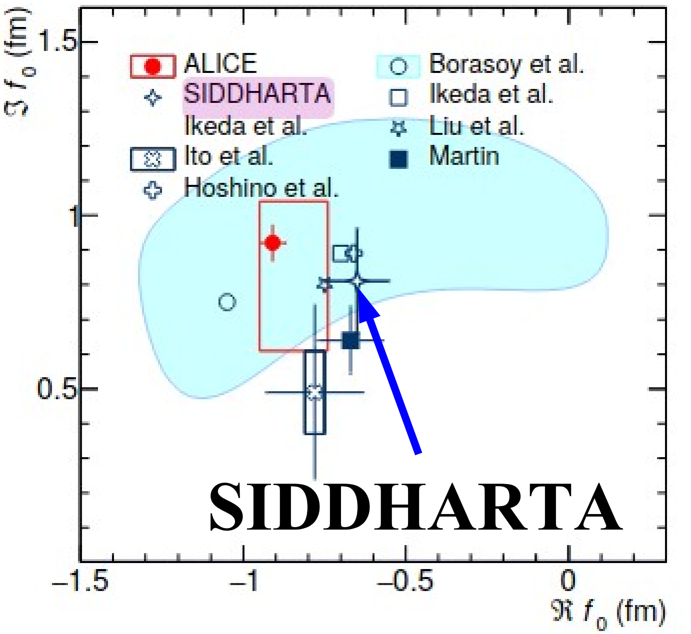
  M. Bassi et al. [SIDDHARTA], NPA 881 ('12) 88 [1201.4635]

$$a_0 = -0.65 \pm 0.10 + i[0.81 \pm 0.15]\ \mathrm{fm}$$

- **Femtoscopy reconfirmed $\bar{K}N$ bound state nature of Λ(1405)**

Table 4: Values of the scattering parameters and the χ^2/ndf for the deviation between the ALICE data and available model calculations and previous measurements for K$^-$p pairs at low relative momentum.

Model calculation:	$\Re f_0$ (fm)	$\Im f_0$ (fm)	χ^2/ndf
Lednický–Lyuboshitz fit to data	$-0.91 \pm 0.03(\mathrm{stat})^{+0.17}_{-0.03}(\mathrm{syst})$	$0.92 \pm 0.05(\mathrm{stat})^{+0.12}_{-0.33}(\mathrm{syst})$	1.4
Kyoto [39, 80]	–	–	2.8
Lednický–Lyuboshitz with fixed parameters from:			
Kaonic deuterium (Hoshino et al.) [78]	−0.66	0.89	2.0
Scattering experiments (Martin) [75]	−0.67±0.1	0.64±0.1	3.3
Chiral SU(3) (Ikeda et al.) [17, 18]	−0.7	0.89	1.9
SIDDHARTA chiral SU(3) [17, 18]	−0.65±0.1	0.81±0.15	2.3
Hamiltonian EFT (Liu et al.) [77]	−0.75	0.80	1.9
Kaonic hydrogen (Ito et al.) [76]	−0.78±0.15	0.49±0.25	4.2
Chiral SU(3) (Borasoy et al.) [79]	−1.05±0.5	0.75±0.4	1.6

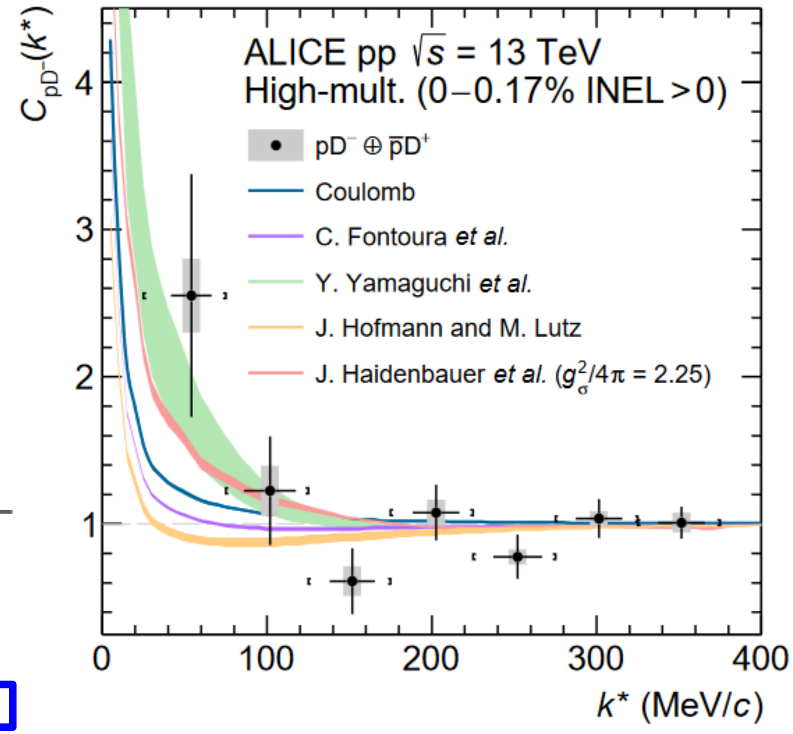


Marginal case: D^- p correlation function

- **"First study of the two-body scattering involving charm hadrons"**

  Acharya+[ALICE] (2201.05352, PRD106 ('22), 052010)

  - **D^- p corr. func. is measured.**
  - **Enhanced CF from Coulomb.**
  - **One range gaussian potential with strength fitted to the I=0 scattering length of the model → attractive potentials are favored**



Model	f_0 (I = 0)	f_0 (I = 1)	n_σ
Coulomb			(1.1–1.5)
Haidenbauer et al. [21]			
$- g_\sigma^2/4\pi = 1$	0.14	−0.28	(1.2–1.5)
$- g_\sigma^2/4\pi = 2.25$	0.67	0.04	(0.8–1.3)
Hofmann and Lutz [22]	−0.16	−0.26	(1.3–1.6)
Yamaguchi et al. [24]	−4.38	−0.07	(0.6–1.1)
Fontoura et al. [23]	0.16	−0.25	(1.1–1.5)

→ *Yasui's talk*

[21] Haidenbauer+(0704.3668) (weakly / mildly attractive (I=0), no bound state)
[22] Hofmann, Lutz (hep-ph/0507071) (repulsive (I=0))
[23] Fontoura+(1208.4058) (weakly attractive (I=0))
[24] Yamaguchi, Ohkoda, Yasui, Hosaka (1105.0734) (att., w/ bound state (I=0))

To be bound or not to be bound

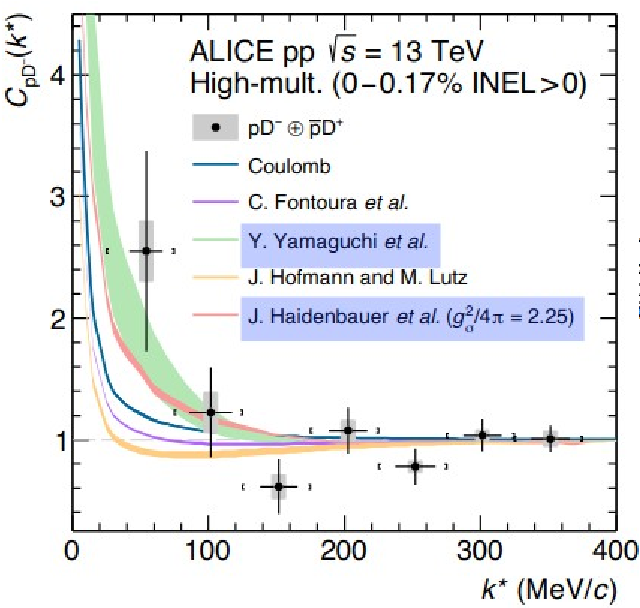
- When there is a bound state, CF shows interesting dependence on the source size and relative momentum.
- D^-p corr. func. shows the behavior with a bound state, and the best fit parameter set (R, a_0) is in the bound region. (If bound, it is the first weakly decaying pentaquark state.)

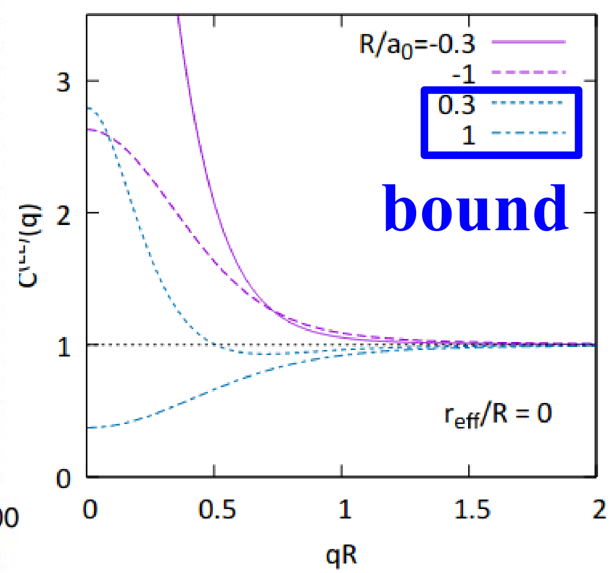
$$k\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}k^2 + \mathcal{O}(k^3)$$

(Nuclear and atomic phys. convention.)

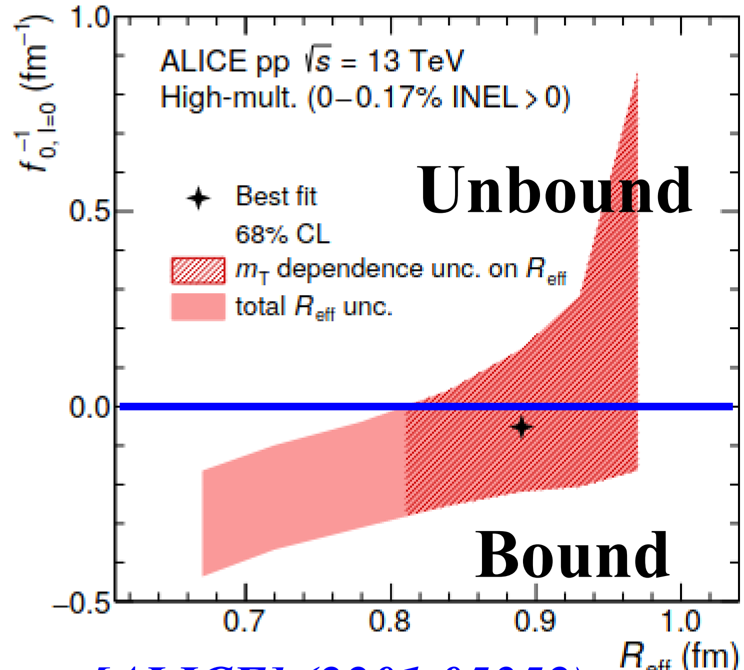
$$k\cot\delta = +\frac{1}{f_0} + \frac{1}{2}r_{\rm eff}k^2 + \mathcal{O}(k^3)$$

(High-E. phys. convention.)





Morita+(1908.05414)



[ALICE] (2201.05352)

From Femtoscopy
to ExHIC
(Exotic hadron structure
from HIgh-energy Collisions)

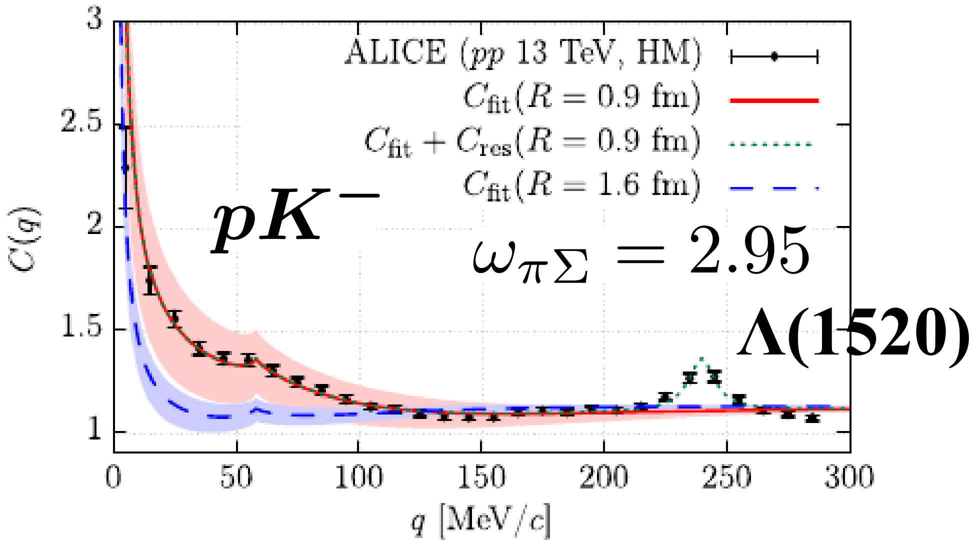
ExHIC → Femtoscopy → ExHIC

- ExHIC → Femtoscopy
  Femtoscopy has been applied to pairs relevant to exotics
  ($\Lambda\Lambda$, pΞ^-, pΩ, K^-p, DD*, D$\bar{D}$*, …)
  - C(q) of some of the pairs show bound state dip in pp, pA or AA.
    → K^-p(Λ(1405)), pΩ, DD*(Tcc), D$\bar{D}$* (X(3872)), pD^- (Θ_c)

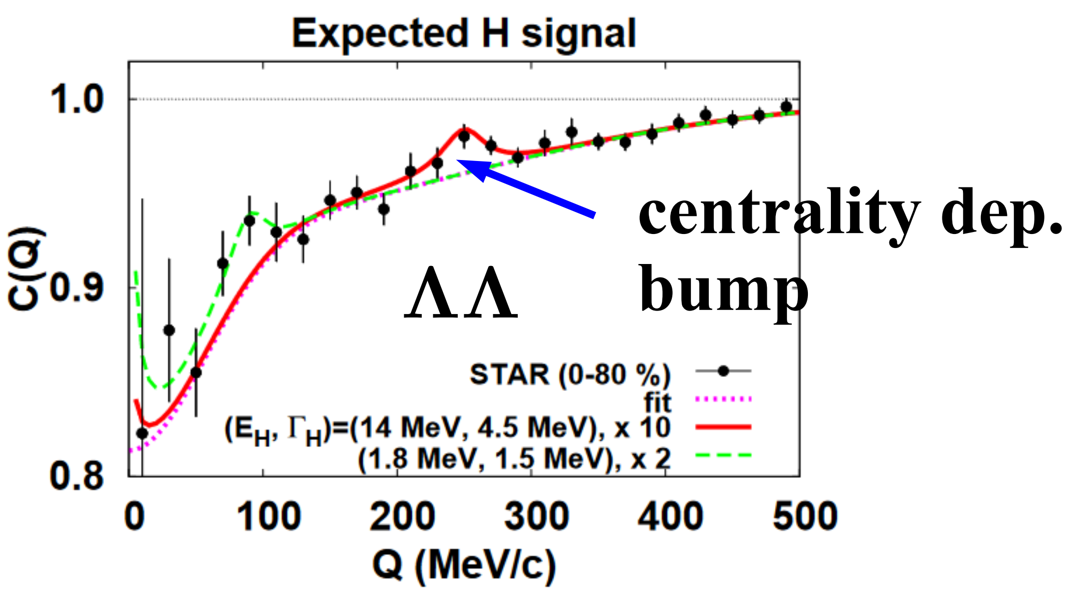
    Established! Not established!

- Femtoscopy → ExHIC
  How can we establish the hadronic molecule nature of these ?
  - ExHIC+CMS method
    Coal/Stat ratio → Exotic/Normal ratio in pp and AA collisions
    (E.g. X(3872) from CMS)
  - Compositeness (c.f. Hyodo's talk)
    Deviation from the weak binding relation (Scattering length ~ $1/\sqrt{2\mu B}$) may tell us the compositeness
    (E.g. DD*(Tcc), D$\bar{D}$* (X(3872)) will be measured in ALICE3(2034~)
  - Peak in the invariant mass spectra will be also seen in C(q)

Resonance seen in C(q)



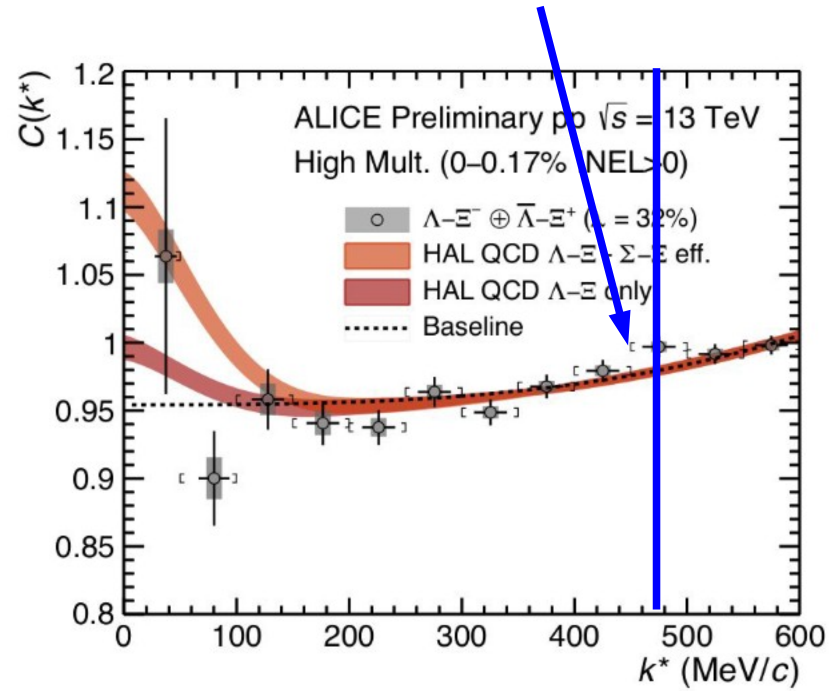
Kamiya+('20)/ALICE



Morta+('15)/ STAR ('15)

What is this?

- *** Statistical fluc.**
- *** Threshold cusp**
- *** Resonance above NΩ**
- *** Quasibound state of NΩ**



Georgios Mantzaridis [ALICE] (FemTUM2022)

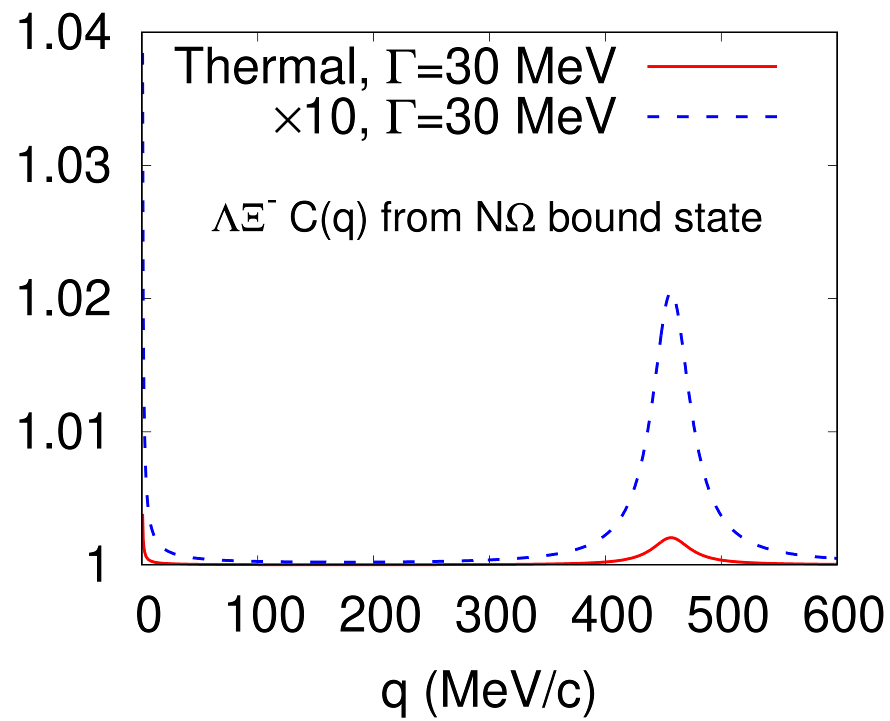
Resonance seen in C(q)

- **Resonance contribution in C(q)**

$$\Delta C_{ij}(q) = \frac{dN_R/dq}{d(N_iN_j)/dq}\frac{\Delta y}{(\Delta y)^2}$$

$$\frac{dN_R}{dq} = N_R f_{BW}(E_q)\frac{dE_q}{dq}$$

$$\frac{d(N_iN_j)}{dq} = N_iN_j 4\pi q^2 \frac{\exp(-q^2/2\mu T)}{(2\pi\mu T)^{3/2}}$$



- **ExHIC estimate**

$$N_\Lambda = 6.5, N_\Xi = 4.4/2$$

$$N_{N\Omega} = (6.4 - 7.0)\times 10^{-3}/2$$

- **Thermal contribution, ΔC~ 0.002 << Stat. Err. 0.01**
  - **Factor 50 statistics in Run3 → Err. x 1/7, Reachable**
  - **Loosely bound molecule may be suppressed → Need more or measure in AA.**

Summary

- **ExHIC collaboration (since 2010) has claimed that hadronic molecules can be formed as frequently as normal hadrons, and one example seems to be found, X(3872).**
  - **But ExHIC did not give predictions from pp collisions.**
    Does someone volunteers to work ?
    (Gaussian source would be reasonable.)
  - **Exotic/Normal ratio would be nice.**
- **Femtoscopy is a good tool to constrain the scattering length and to guess the existence of a bound state for pairs whose scattering experiments are not available.**
  - **One can study the interactions involving charm hadrons!**
  - **In some pairs (K^-p, φp), quantitative discussions on the scattering length and coupled-channel effects have started. Hadron physics side may need to update the interactions.**
  - **Some pairs are suggested to have a bound state. Confirmation is needed.**

Thank you for attention !

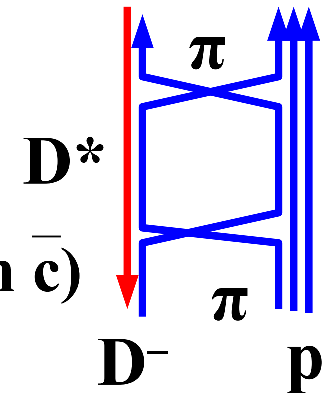
Charmed Hadron Interactions

- C(q) including a charmed hadron
  - Extremely important in recent hadron physics.
  - $D^-(\bar{c}d)$-p(uud) correlation
    - Probes $\Theta_c(\bar{c}$-ud-ud) state (replace $\bar{s}$ in $\Theta(\bar{s}$-ud-ud) with $\bar{c}$)

      D. O. Riska, N. N. Scoccola, PLB299('93)338 (pred.);
      A. Aktaset+ [H1], PLB588('04)17 (positive);
      J. M. Linket+ [FOCUS], PLB622('05)229 (negative).
    - Attraction from two pion exchange

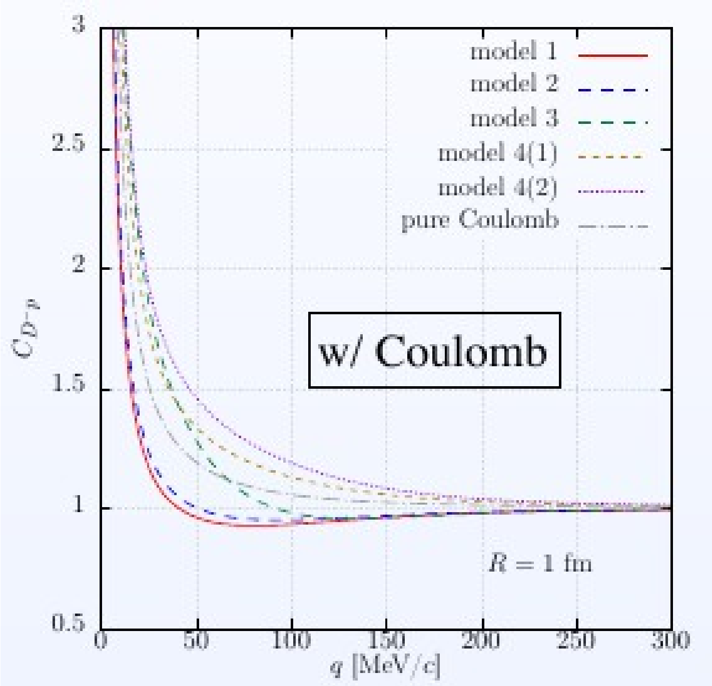
      S. Yasui, K. Sudoh, PRD80('09)034008.
    - Easy to calculate the potential in LQCD.

      Y. Ikeda et al. (private communication)



- $D^-(\bar{c}d)$-p(uud) CFs from proposed potentials

  Hofmann, Lutz ('05) (repulsive);
  Haidenbauer+('07) (repulsive);
  Yamaguchi+('11) (att., w/ bs); Fontoura+('13) (repulsive)



Kamiya, Hyodo, AO

Data will discriminate these potentials !

Tcc and X(3872) structure

- **Hadronic molecule structure is assumed**
  → Eigenmomentum $k \simeq -i/a_0\,,\ a_0 \simeq R = 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  → Deviation from weak binding relation (X=compositeness)
  Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524), Kunigawa, Hyodo (2112.00249)

$$a_0 = R\left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ})$$

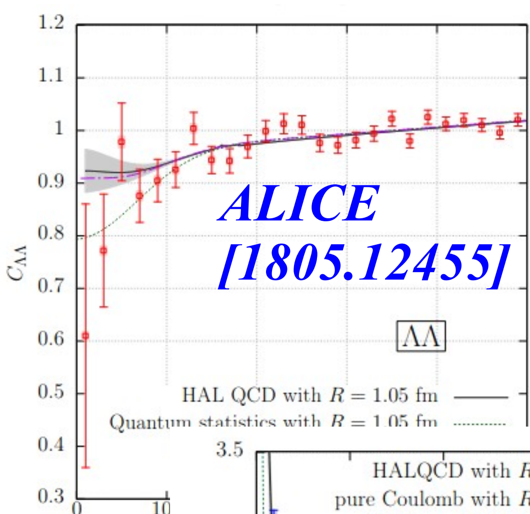
$$\left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff}), R = 1/\sqrt{2\mu B}\right]$$

  - **Hadronic molecule assumption → X=1**
    Pure multiquark state → X=0
  - **Smaller scattering length in DD* may signal the *genuine* tetraquark nature of Tcc.**

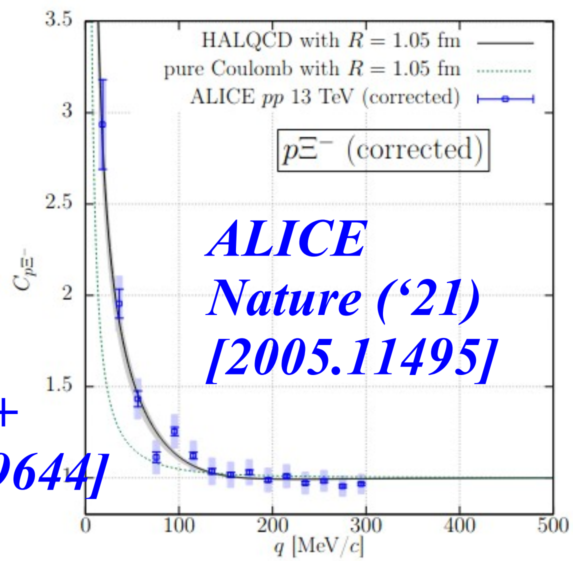
Cases without a bound state

- **ΛΛ and NΞ seem to be unbound from lattice QCD calculation !**
  Sasaki+ [HAL], NPA998 ('20)121737 [1912.08630]
- **Source size dependence of ΛΛ and pΞ⁻ correlation functions**
  → No dip or suppressed behavior in AA collisions.

pp



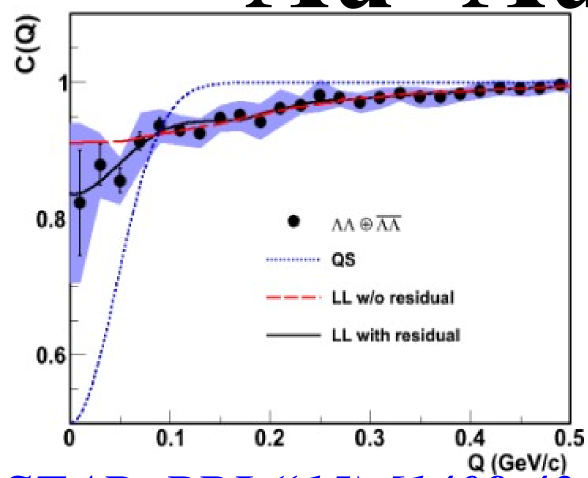
ALICE [1805.12455]



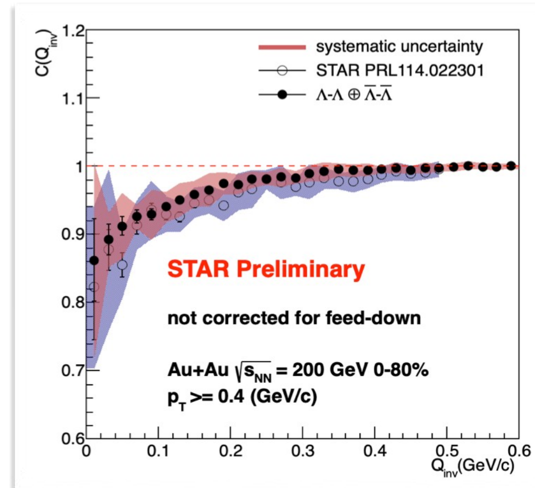
ALICE Nature ('21) [2005.11495]

Kamiya+ [2108.09644]

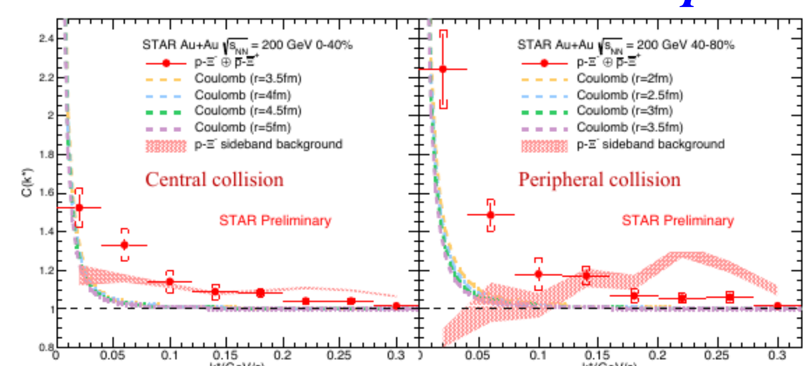
Au+Au



STAR, PRL('15) [1408.4360]



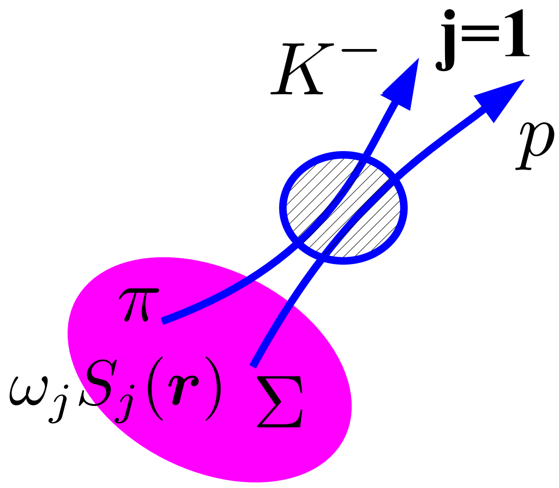
Moe Isshiki+ (STAR, prelim., 2109.10953).



K. Mi+ (STAR, preliminary), Au+Au, APS2021.

Correlation function with coupled-channel effects

- **KPLLL formula = CC Schrodinger eq. under $\Psi^{(-)}$ boundary cond. + channel source**
  Koonin('77), Pratt+('86), Lednicky-Lyuboshits-Lyuboshits ('98), Heidenbauer ('19), Kamiya, Hyodo, Morita, AO, Weise ('20).



$$\Psi^{(-)}(\boldsymbol{q};\boldsymbol{r}) = [\phi(\boldsymbol{q};\boldsymbol{r}) - \phi_0(q;r)]\,\delta_{1j} + \psi^{(-)}(q;r)$$

$$\psi_j^{(-)}(q;r) \to \frac{1}{2iq_j}\left[\frac{u_j^{(+)}(q_j r)}{r}\delta_{1j} - A_j(q)\frac{u_j^{(-)}(q_j r)}{r}\right]$$

$$C(q) = \int d\boldsymbol{r} S_1(r)\left[|\phi(\boldsymbol{q};\boldsymbol{r})|^2 - |\phi_0(q;r)|^2\right] + \sum_j \int d\boldsymbol{r}\omega_j S_j(r)|\psi_j^{(-)}(q;r)|^2$$

- **No Coulomb** $\phi(\boldsymbol{q};\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}}, \phi_0(q;r) = j_0(qr), u_j^{(\pm)}(qr) = e^{\pm iqr},$

$$A_j(q) = \sqrt{(\mu_j q_j)/(\mu_1 q_1)}S_{1j}^\dagger(q_1)\ \ (S_{ji} = i \to j \text{ S-matrix})$$

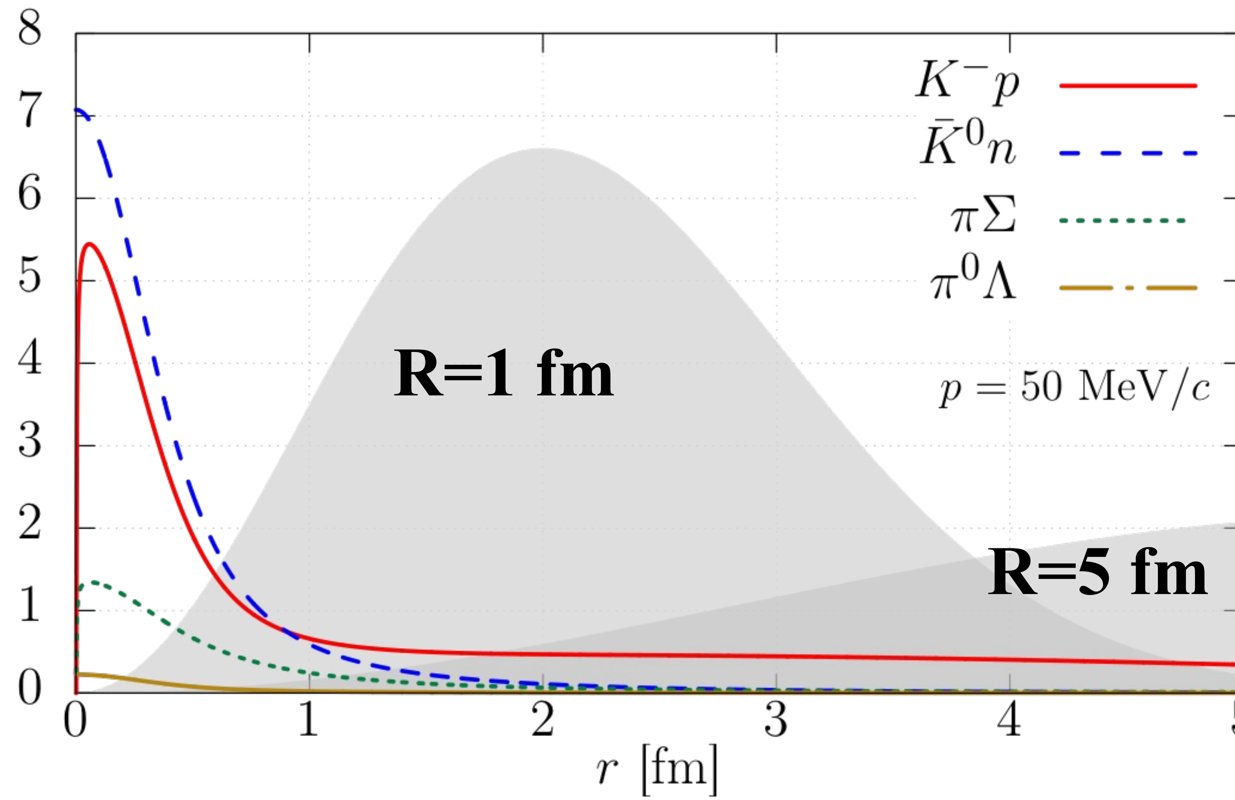
- **With Coulomb**

$$\phi(\boldsymbol{q};\boldsymbol{r}) = \text{Full Coulomb w.f.}, \phi_0(q;r) = \text{s-wave Coulomb w.f.},$$

$$u_j^{(\pm)}(qr) = \pm e^{\mp i\sigma_j}\left[iF(qr) \pm G(qr)\right] (F, G = \text{regular (irregular) Coulomb fn.})$$

R-dep. of coupled-channel contribution

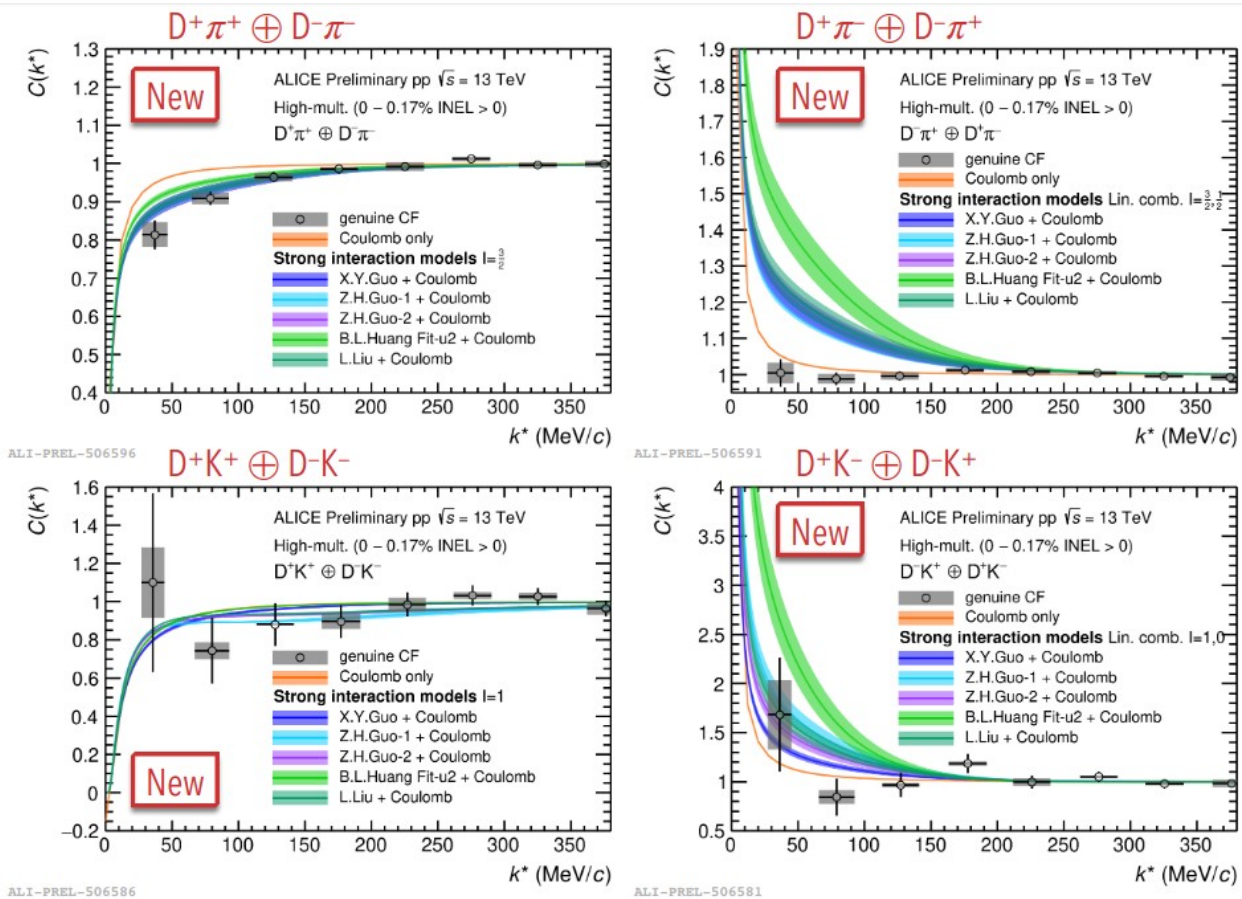
- **Wave functions of coupled-channels (other than the observed channel) are localized in the small r region.**
- **With a large source, C(q) is dominated by the wave functions in the observed channel.**
- **With a small source, C(q) is modified by coupled-channel source.**

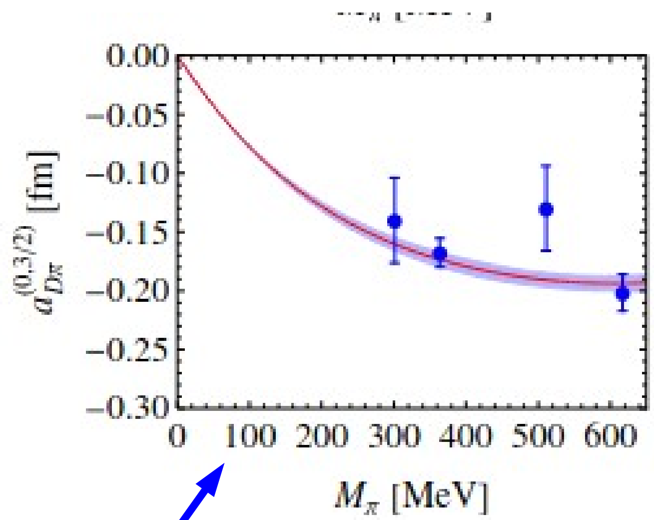


w.f. Kamiya+,arXiv:1911.01041v1

Homework to Hadron Physics (1)

- **Present chiral models do not explain $D\pi$ and $D\bar{K}$ correlation.**
  - **Overestimate C(D⁺π⁻) → Mystery ? Extrapolation to phys. mass ?**
    Leading order = Weinberg-Tomozawa (vector exch., repulsive)
    Further repulsive interaction ?
  - **Overestimate C(D⁺K⁻) → Further repulsion or bound state ?**





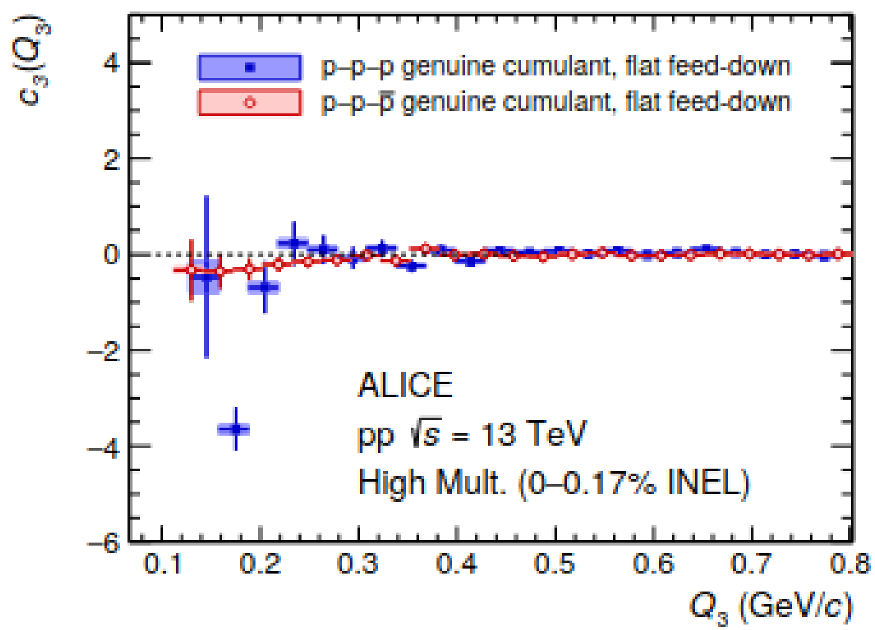
L. Liu et al, Phys. Rev. D87 (2013) 014508
X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510
B.-L. Huang et al, Phys. Rev. D 105 (2022) 036016
Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13

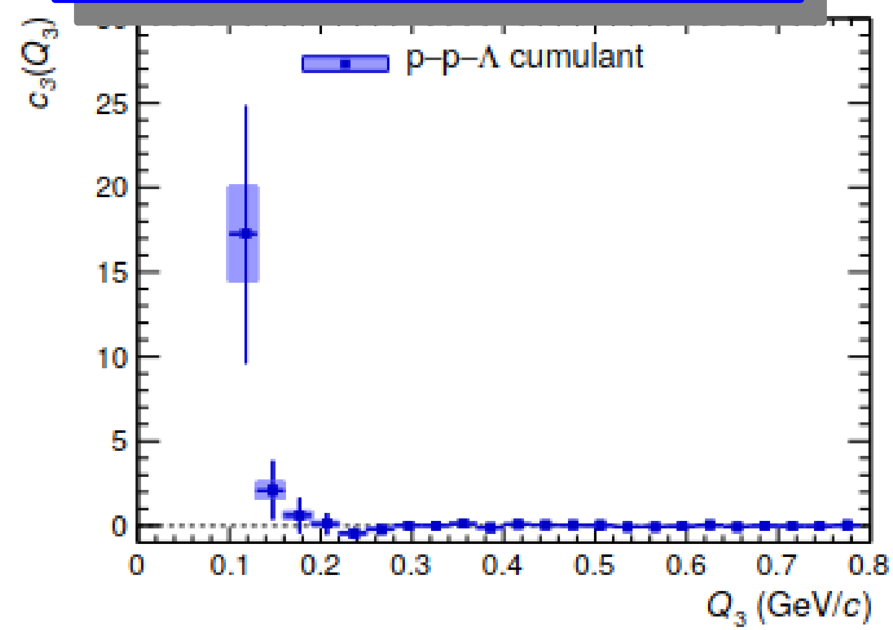
Fabrizio Grosa@QM2022

Homework to Hadron (Nuclear) Physics (2)

- **Three-body correlation function (ppp, ppΛ)**
  - **Cumulant** $$c_3 = C_{123} - C_{12} - C_{23} - C_{31} + 2$$
  - **Can we extract three-baryon repulsion ?**
    (important to solve the hyperon puzzle)
    → One needs to solve continuum three-body w.f.
    with Coulmb potential.

Theoretical challenge





ALICE [2206.03344] (Raffaele Del Grande @QM2022)

Homework to Hadron (Nuclear) Physics (3)

- **Correlation function including vector mesons**
  - **Femtoscopy** *ALICE (PRL, 2105.05578)*

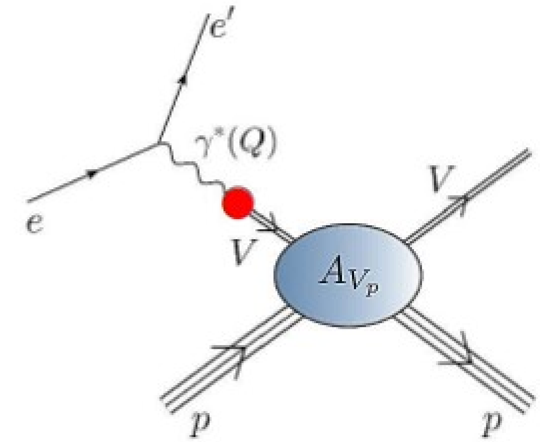
$$a_0(\phi p) = 0.85 + i0.16 \text{ fm}$$

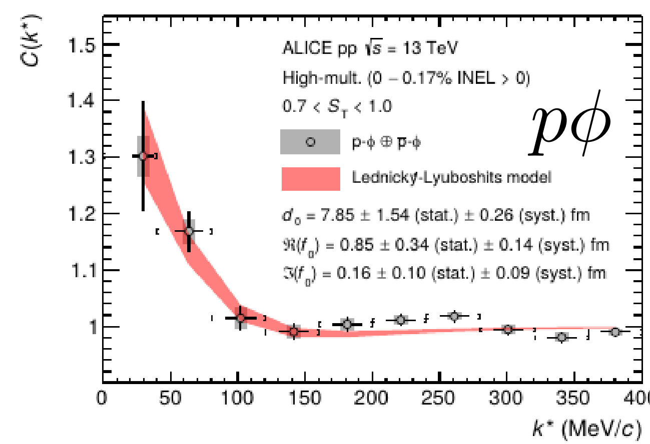
  - **Contradiction with the photo production ? scattering length is O(0.1 fm)**
    E.g. Strakovsky, Pentchev, Titov (2001.08851)

$$|a_0(\phi p)| = (0.063 \pm 0.010) \text{ fm}$$

  - **Smaller than lattice QCD result (J=3/2) ?**
    Lyu, Doi, Hatsuda, Ikeda (2205.10544)

$$a_0(\phi p, J = 3/2) = 1.43 \text{ fm}$$





ALICE, 2105.05578

What's wrong ?

Toward dynamical source

- **Calculating HBT radius in dynamical models is not easy (HBT puzzle).**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014];

  choices then tends to exceed the number of experimental constraints. In fact, all the model results that we review in the current subsection remain unsatisfactory with this respect: They either deviate significantly from femtoscopic data, or they reproduce these data at the price of missing other important experimental information. In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

- **But carefully constructed hydrodynamic model may answer.**
  S. Pratt, PRL102('09)232301 [0811.3363].

  Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term "HBT puzzle," where HBT refers to femtoscopic studies which were originally based on Hanbury Brown–Twiss interferometry. Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects: mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

- **How about afterburner effects ?**