Investigating large momentum fraction behavior of pion parton distributions through global analysis

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Background and Motivation

What do we want?

To study the makeup of nuclear matter

Building blocks of nature are quarks and gluons

What’s the problem?

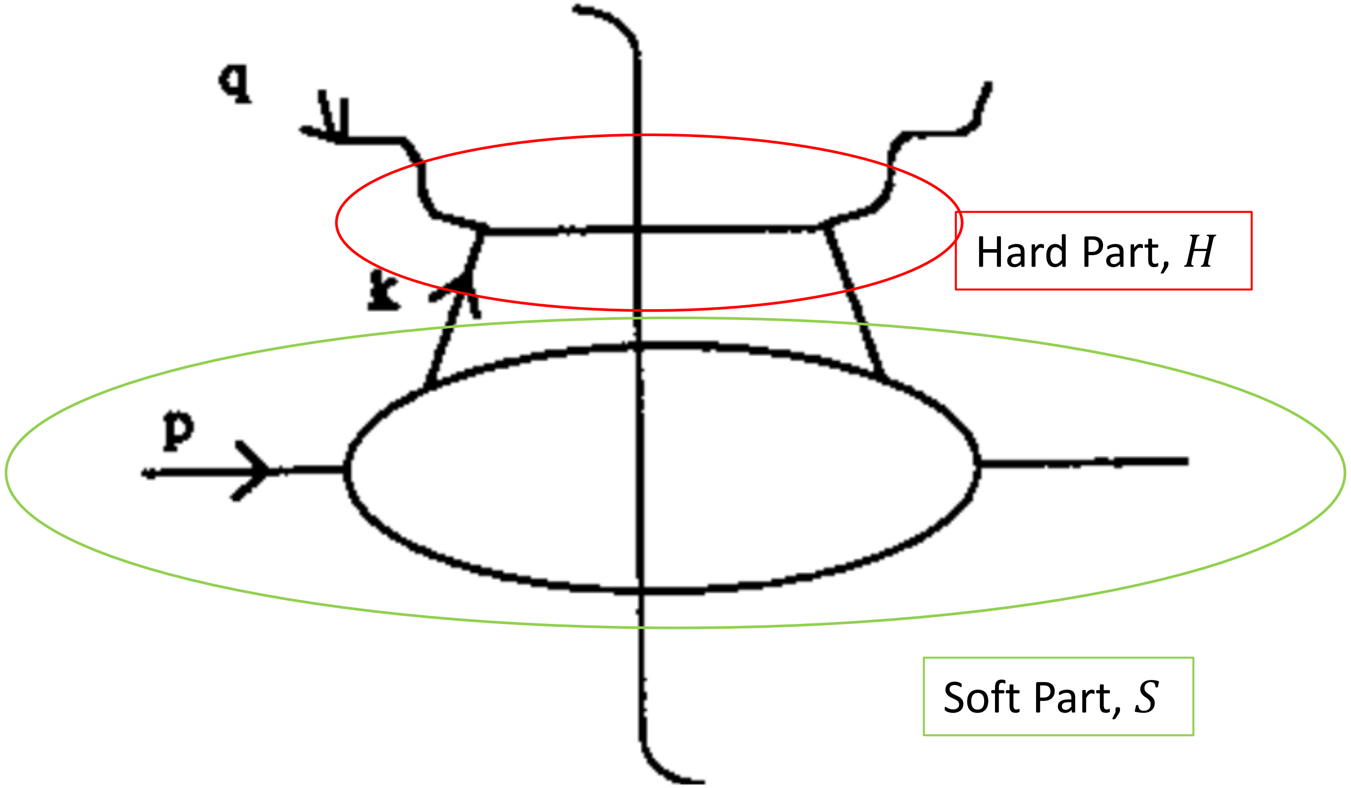
Quarks and gluons are not directly measurable!

Motivation

- QCD allows us to study the structure of hadrons in terms of partons (quarks, antiquarks, and gluons)
- Use factorization theorems to separate hard partonic physics out of soft, non-perturbative objects to quantify structure

Factorization Theorems

- Deep Inelastic Scattering (DIS) $|\mathcal{M}|^2$ at Leading Order shown to the right
- At large $Q^2 = -q^2$, can decouple the soft part from the hard part
- At short distances, virtual photon picks out individual parton



$$W^{\mu\nu} \propto S \otimes H$$

Game plan

What to do:

- Define a structure of hadrons in terms of quantum field theories
- Identify physical observables that can be factorized in theory with controllable approximations, or factorizable lattice QCD observables
- Perform global QCD analysis as structures are universal and are the same in all processes

Complicated Inverse Problem

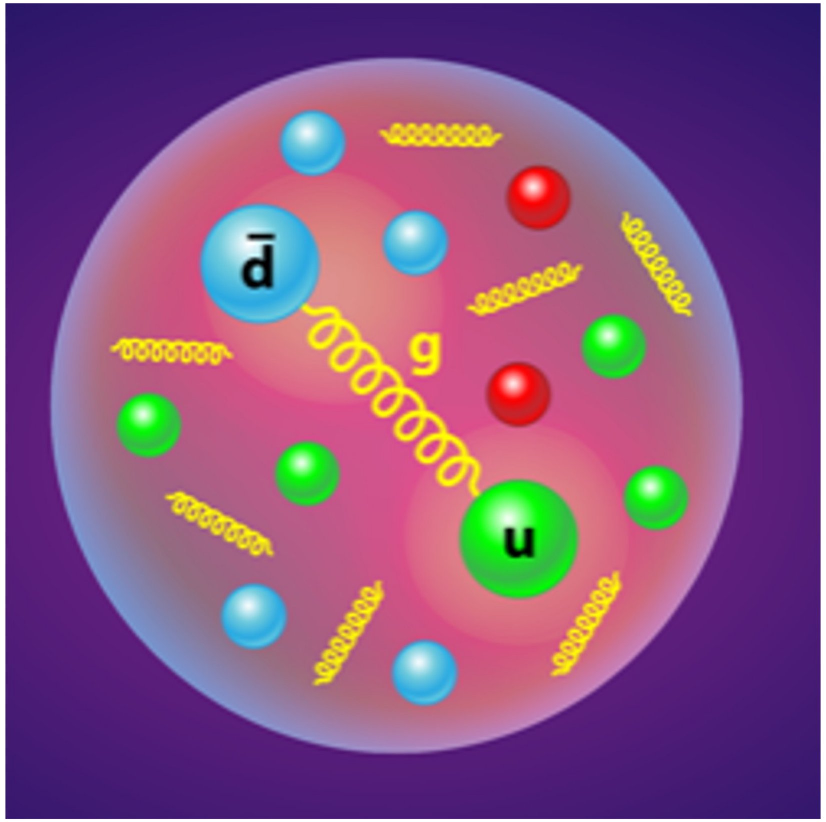
- Factorization theorems involve convolutions of hard perturbatively calculable physics and non-perturbative objects

$$\frac{d\sigma}{d\Omega} \propto \mathcal{H} \otimes f = \int_x^1 \frac{d\xi}{\xi} \mathcal{H}\left(\frac{x}{\xi}\right) f(\xi)$$

- Parametrize the non-perturbative objects and perform global fit

Pions

- Pion is the Goldstone boson associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
- Lightest hadron
- Made up of q and $\bar{q}$ constituents



Large momentum fraction behavior

- Many theoretical papers have studied the behavior of the valence quark distribution as $x \to 1$ and
- Debate whether $q_v^\pi(x \to 1) \sim (1-x)$ or $(1-x)^2$

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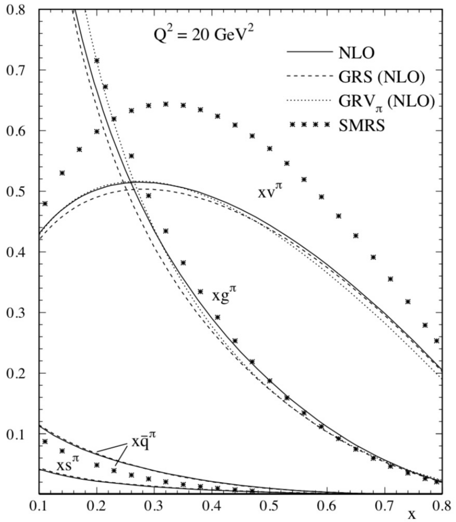
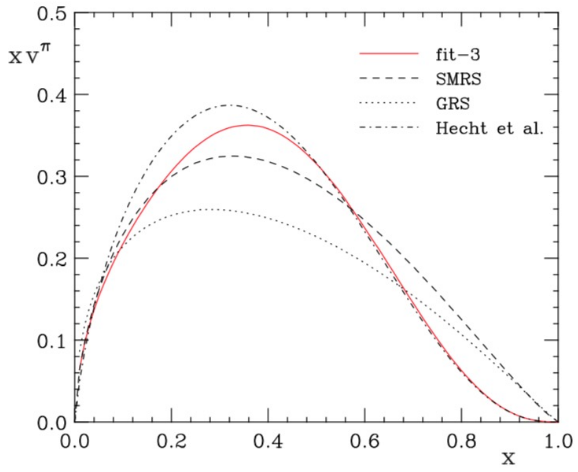
Pion structure in phenomenology

- Historically, pion distributions have been extracted from fixed target πA data
  - Drell-Yan (DY) $\pi A \to \mu^+\mu^- X$
  - Prompt photon $\pi A \to \gamma X$

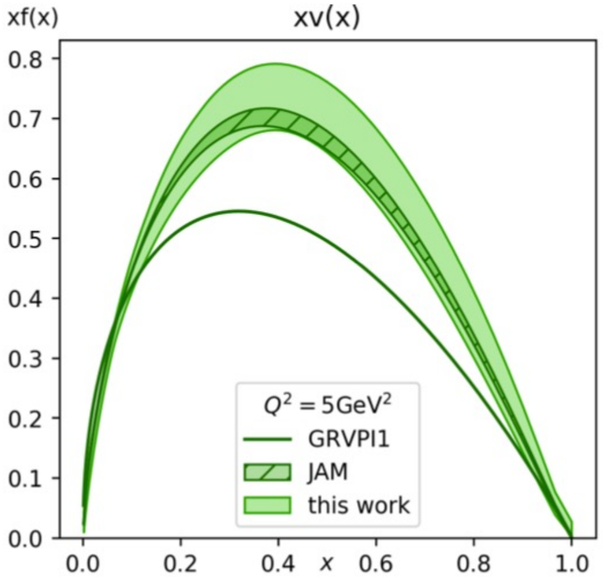
Owens attempted to use J/ψ production



GRS, GRV, and SMRS



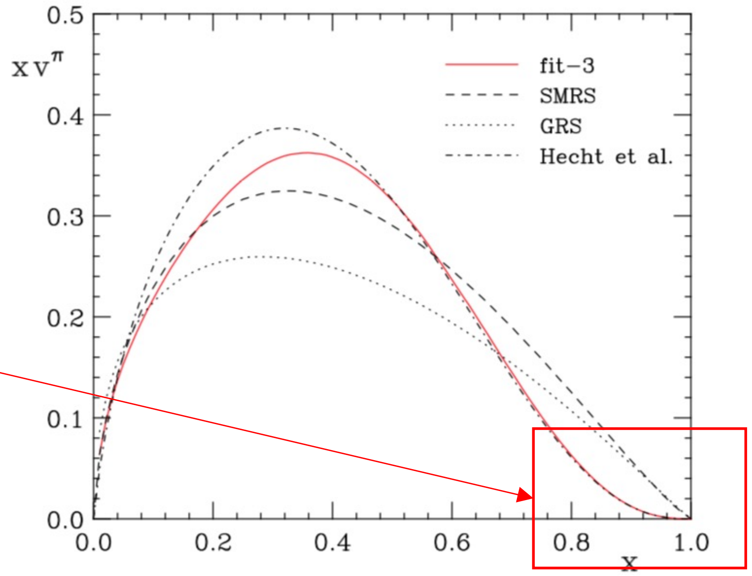
ASV valence PDF



xFitter

Large-x_π behavior

- Generally, the parametrization lends a behavior as $x \to 1$ of the valence quark PDF of $q_v(x) \propto (1-x)^\beta$
- For a fixed order analysis, analyses find $\beta \approx 1$
- Aicher, Schaefer Vogelsang (ASV) found $\beta = 2$ with threshold resummation



ASV valence PDF

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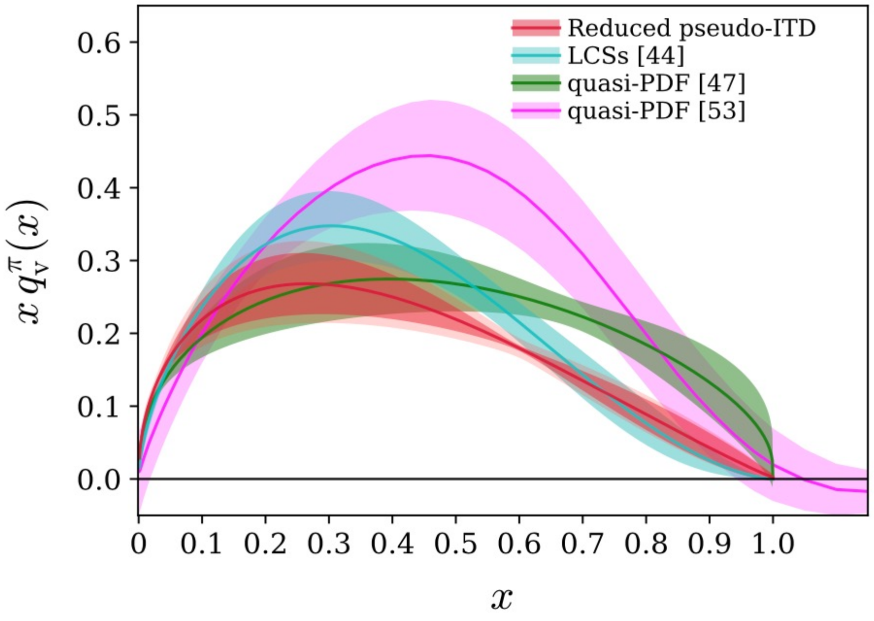
Lattice QCD Activity

- Simulations on the lattice have been done to investigate this structure



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Subset of pion lattice QCD analyses

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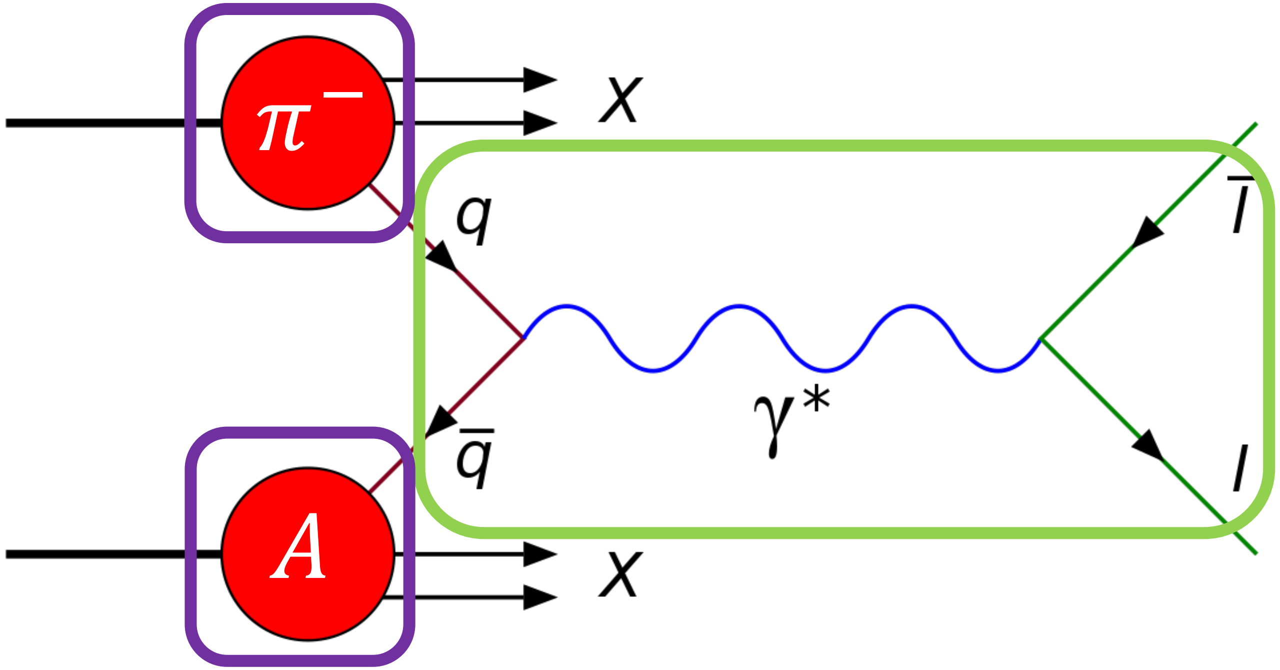
Goals

- Use the available experimental data with modern theoretical calculations in perturbative QCD such as threshold resummation to perform global QCD analysis
- Include lattice QCD data in analysis on the same footing as the experimental data
- Study the large-x_π behavior of the valence quark distribution in the pion

Theoretical Input

Drell-Yan (DY)



$$\sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi, \mu) \otimes f_j^{A}(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

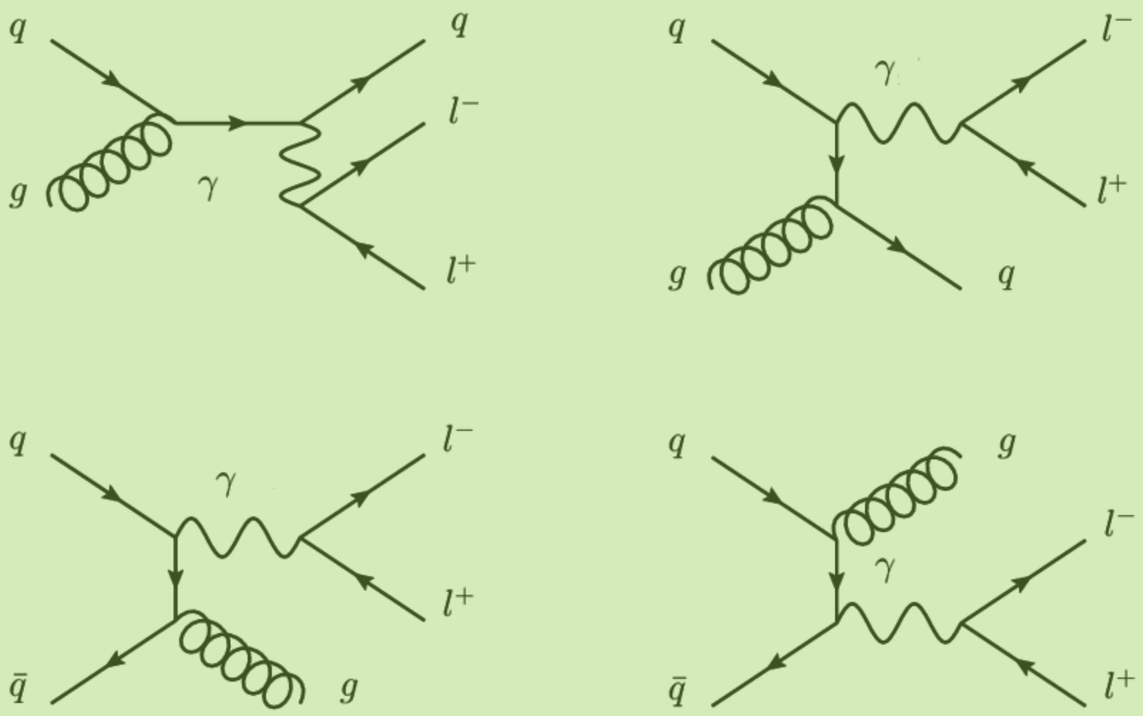
Fixed Order Up to NLO

LO: $\mathcal{O}(1)$

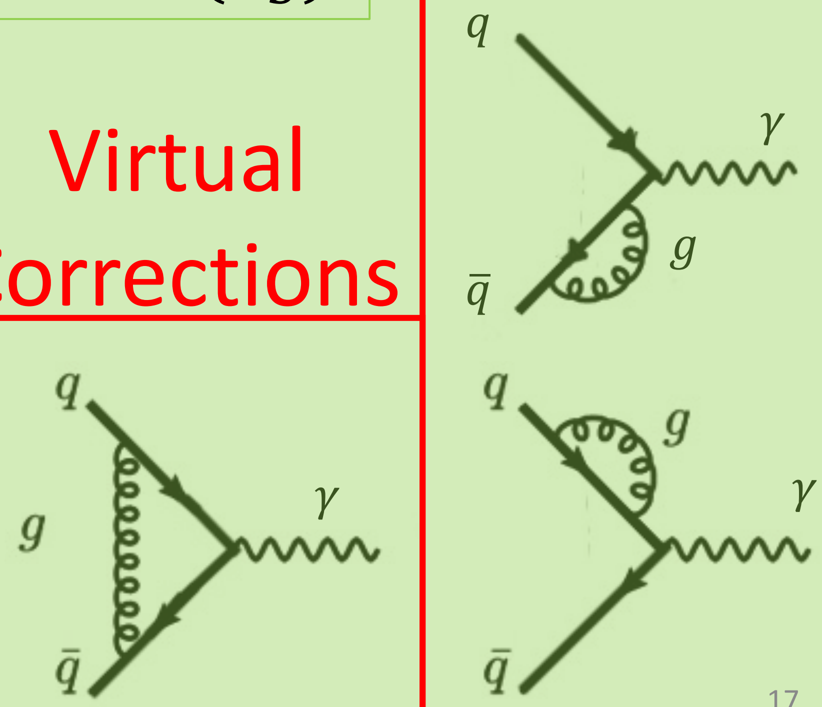
Feynman diagrams for DY amplitudes in collinear factorization

NLO: $\mathcal{O}(\alpha_S)$

Real emissions



Virtual Corrections



Issues with Perturbative Calculations

$$\hat{\sigma} \sim \delta(1-z) + \alpha_S(\log(1-z))_+$$

↓

$$\hat{\sigma} \sim \delta(1-z)[1 + \alpha_S \log(1-\tau)]$$

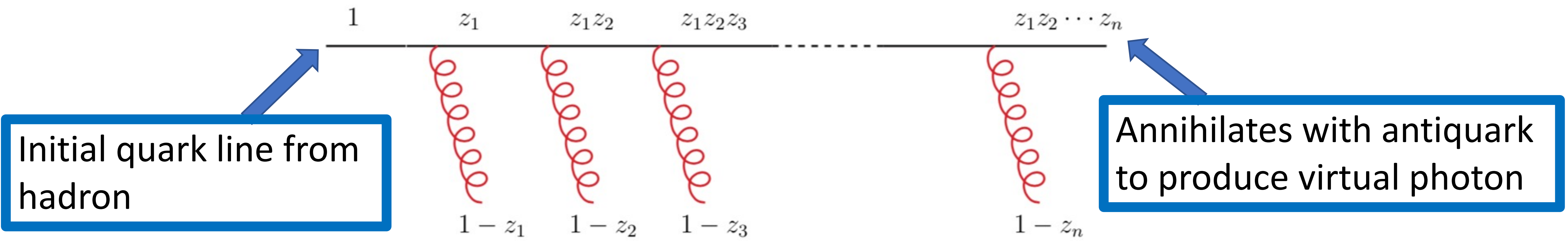
$$\tau = \frac{Q^2}{S}$$

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{\hat{x}_\pi \hat{x}_A}$$

$\hat{S}$ is the center of mass momentum squared of incoming partons

- If τ is large, can potentially spoil the perturbative calculation
- Improvements can be made by resumming $\log(1-z)_+$ terms

Threshold Resummation



Significant contributions to cross section occur in soft gluon emissions and follow the pattern

$$d\hat{\sigma}^{q\bar{q}}_{N^k LO} \propto \alpha_S^k \frac{\ln^{2k-1}(1-z)}{1-z} + \ldots$$

Methods of resummation – Mellin-Fourier

- Threshold resummation is done in conjugate space

$$\sigma_{\rm MF}(N,M) \equiv \int_0^1 {\rm d}\tau\, \tau^{N-1} \int_{\log\sqrt{\tau}}^{\log\frac{1}{\sqrt{\tau}}} {\rm d}Y\, e^{iMY} \frac{{\rm d}^2\sigma}{{\rm d}\tau {\rm d}Y},$$

Two choices occur when isolating the hard part

$$\hat{\sigma}_{\rm MF}(N,M) = \int_0^1 {\rm d}z\, z^{N-1} \cos\left(\frac{M}{2}\log z\right) \frac{{\rm d}^2\hat{\sigma}}{{\rm d}\tau {\rm d}Y}(z)$$

Keep cosine intact – "cosine" method

Keep the first order term in the expansion – $\cos\left(\frac{M}{2}\log z\right) \approx 1$ "expansion" method

Method of resummation – double Mellin

- Alternatively, perform a double Mellin transform

$$\sigma_{\rm DM}(N,M) \equiv \int_0^1 {\rm d}x_\pi^0\,(x_\pi^0)^{N-1} \int_0^1 {\rm d}x_A^0\,(x_A^0)^{M-1} \frac{{\rm d}^2\sigma}{{\rm d}\tau{\rm d}Y}.$$

where $x_\pi^0 = \sqrt{\tau} e^{Y}, \quad x_A^0 = \sqrt{\tau} e^{-Y}$

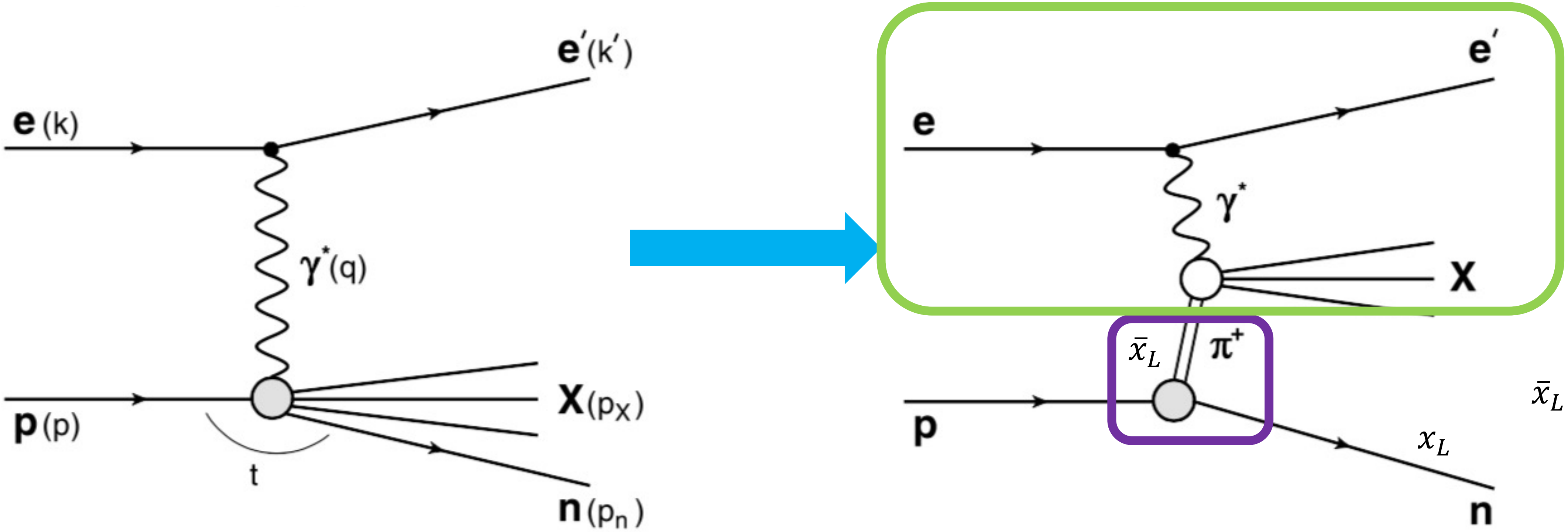
- Double Mellin transform is theoretically cleaner and sums up terms appropriately

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	…	N^pLL
LO	1	--	…	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	…	--
NNLO	$\alpha_s^2 \log(N)^4$	$\alpha_s^2 (\log(N)^2, \log(N)^3)$	…	--
…	…	…	…	…
N^kLO	$\alpha_s^k \log(N)^{2k}$	$\alpha_s^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_s^k \log(N)^{2k-2p} + \cdots$

Leading Neutron (LN)

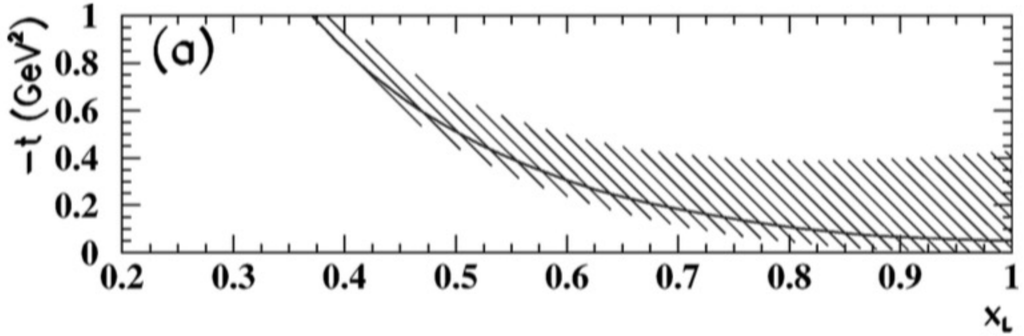


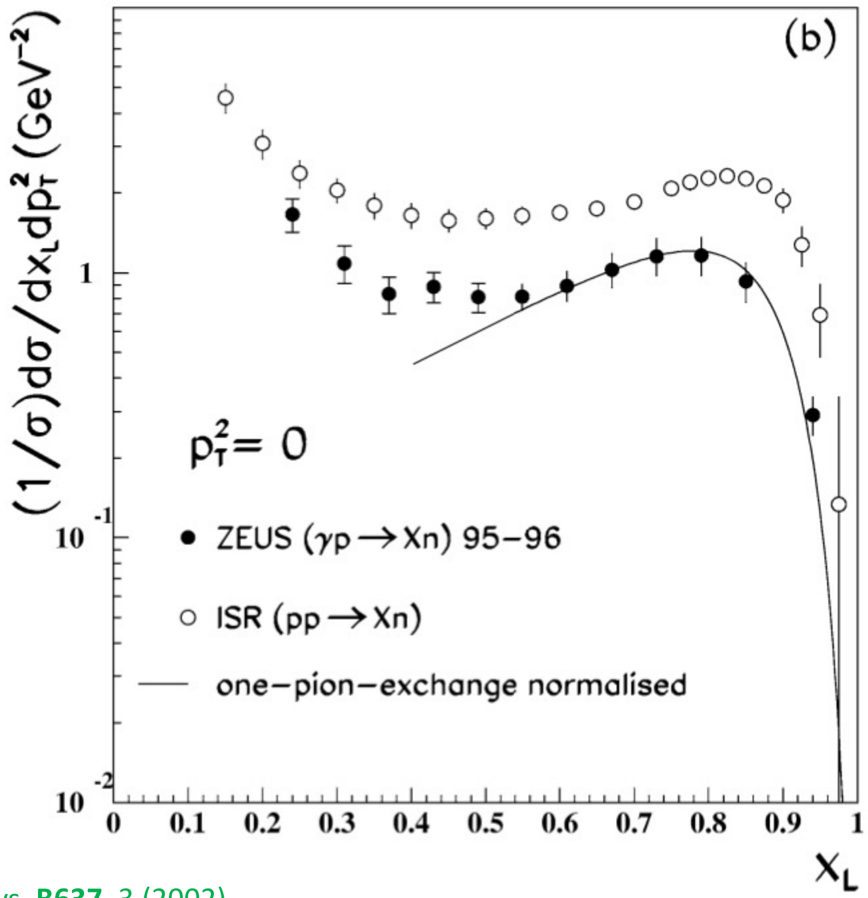
$\bar{x}_L = 1 - x_L$

$$\frac{d\sigma}{dx\,dQ^2\,d\bar{x}_L} \propto f_{\pi N}(\bar{x}_L) \sum_i \int_{x/\bar{x}_L}^{1} \frac{d\xi}{\xi} C\left(\frac{x/\bar{x}_L}{\xi}\right) f_i(\xi,\mu^2)$$

Large x_L

- x_L is fraction of longitudinal momentum carried by neutron relative to initial proton
- For t to be close to pion pole, has to go near 0 – happens at large x_L
- In this region, one pion exchange dominates





Nucl. Phys. **B637**, 3 (2002).

How to relate PDFs with lattice observables?

- Make use of good lattice cross sections and appropriate matching coefficients

$$\Sigma_{n/h}(\nu, z^2) \equiv \langle h(p)|T\{\mathcal{O}_n(z)\}|h(p)\rangle$$
$$= \sum_i f_{i/h}(x, \mu^2) \otimes \mathcal{C}_{n/i}(x\nu, z^2, \mu^2)$$
$$+\mathcal{O}(z^2\Lambda^2_{\rm QCD})$$

- Structure just like experimental cross sections – good for global analysis

Reduced Ioffe time pseudo-distribution (Rp-ITD)

- Lorentz-invariant Ioffe time pseudo-distribution:

$$\mathcal{M}(\nu, z^2) = \frac{1}{2p^0} \langle p| \bar{\psi}(0) \gamma^0 \mathcal{W}(z;0) \psi(z) |p\rangle$$

Quark and antiquark fields

Gauge link

"Ioffe time"

$$\nu = p \cdot z$$

$$z = (0,0,0,z_3)$$

Observable is the *reduced* Ioffe time pseudo-distribution (Rp-ITD)

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

Ratio cancels UV divergences

Fitting the Data and Systematic Corrections

Valence quark distribution in pion

Wilson coefficients for matching

$$\mathrm{Re}\,\mathfrak{M}(\nu, z^2) = \int_0^1 \mathrm{d}x\, q_v(x, \mu_{\mathrm{lat}})\, \mathcal{C}^{\mathrm{Rp-ITD}}\left(x\nu, z^2, \mu_{\mathrm{lat}}\right) + z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu) + \ldots$$

Integration lower bound is 0

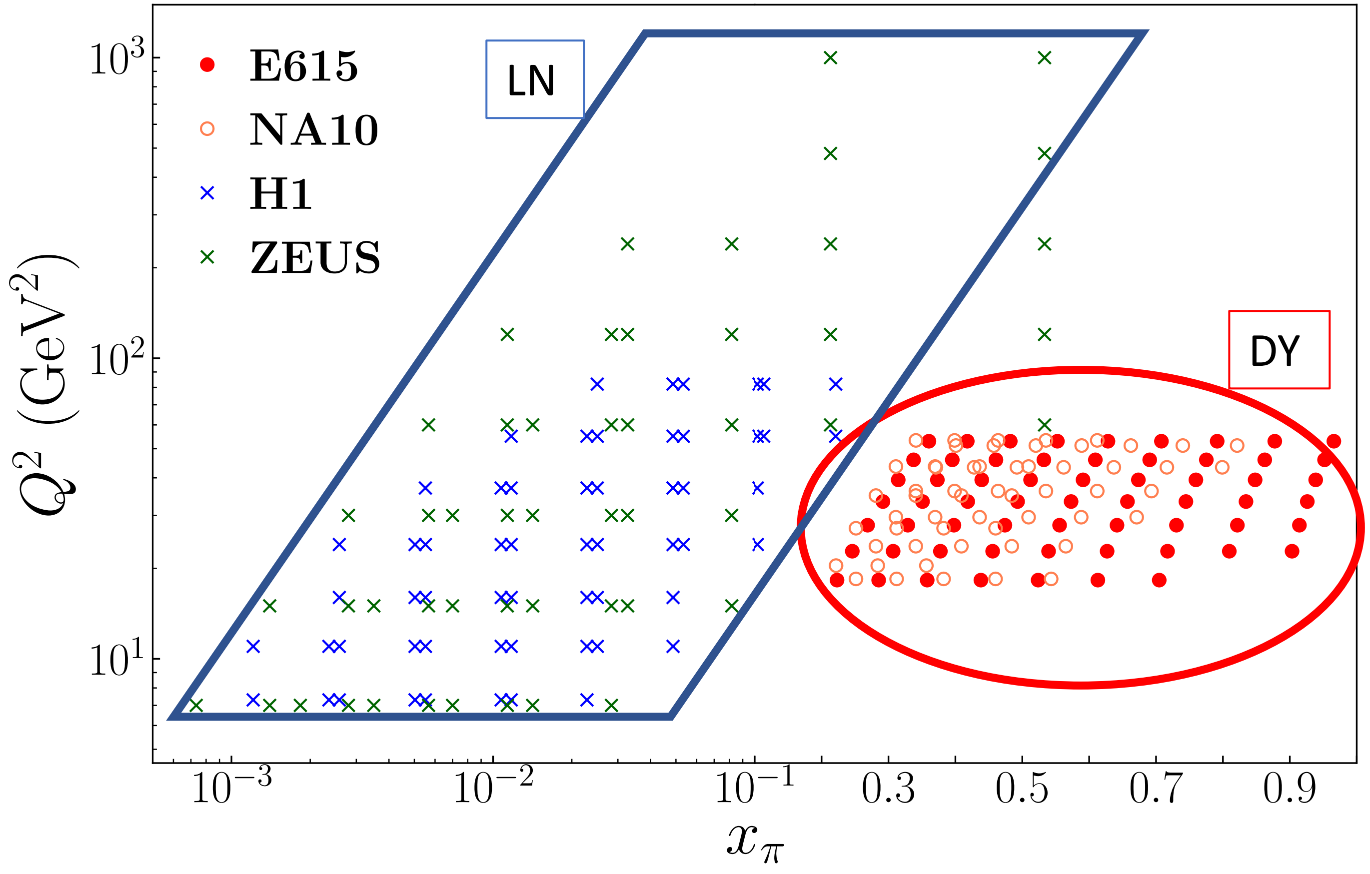
Other potential systematic corrections the data is not sensitive to

Systematic corrections to parametrize

- $z^2 B_1(\nu)$: power corrections
- $\frac{a}{|z|} P_1(\nu)$: lattice spacing errors
- $e^{-m_\pi (L-z)} F_1(\nu)$: finite volume corrections

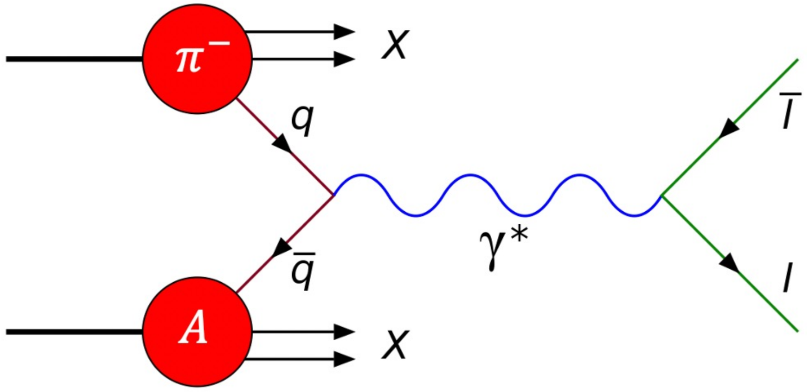
Global Analyses

Datasets -- Kinematics

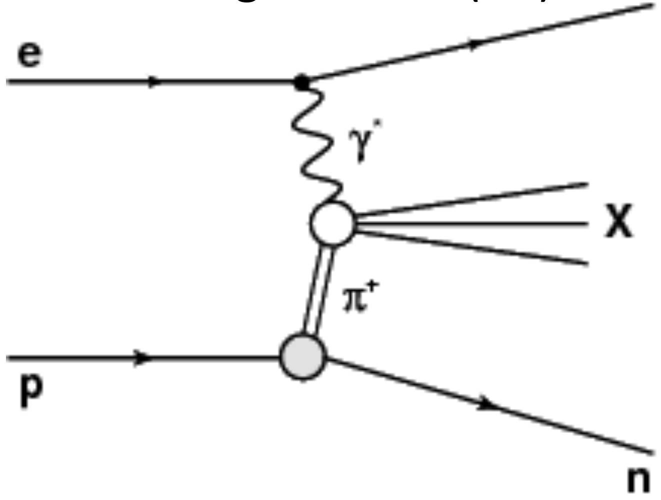


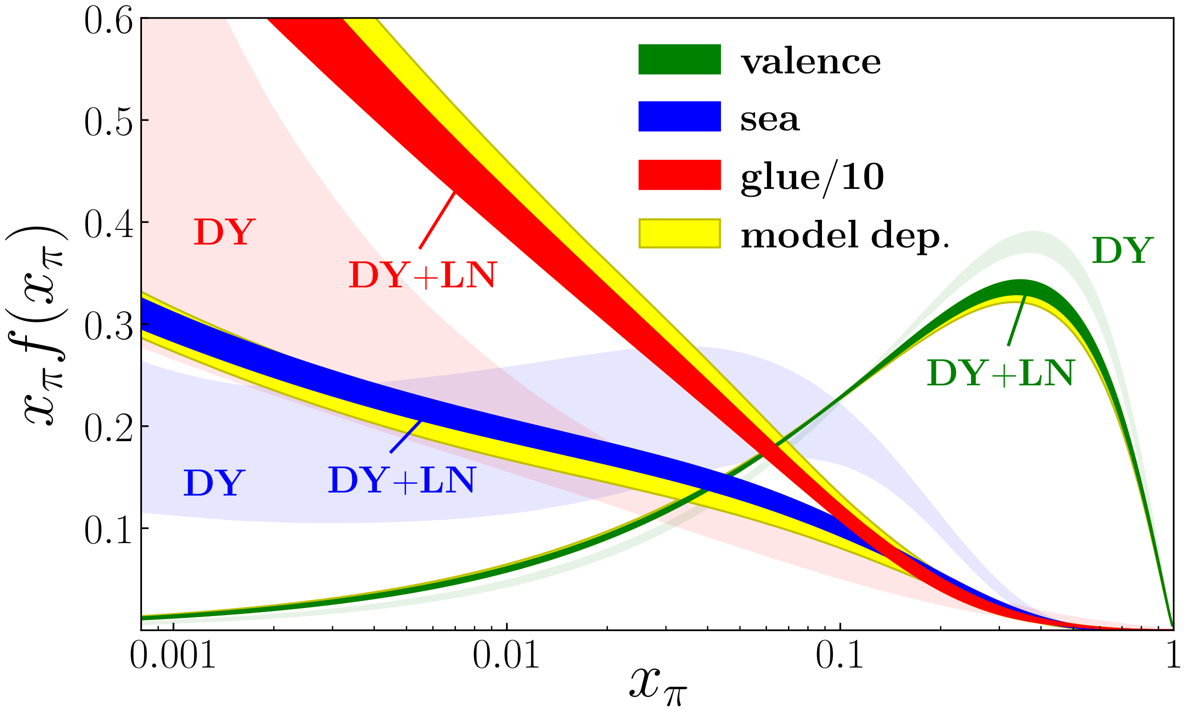
Experiments to probe pion structure

Drell-Yan (DY)



Leading Neutron (LN)





PHYSICAL REVIEW LETTERS **121,** 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

Include Threshold Resummation in DY

- ASV analysis got $(1-x)^2$ behavior using threshold resummation, while all NLO analyses follow $(1-x)$

PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

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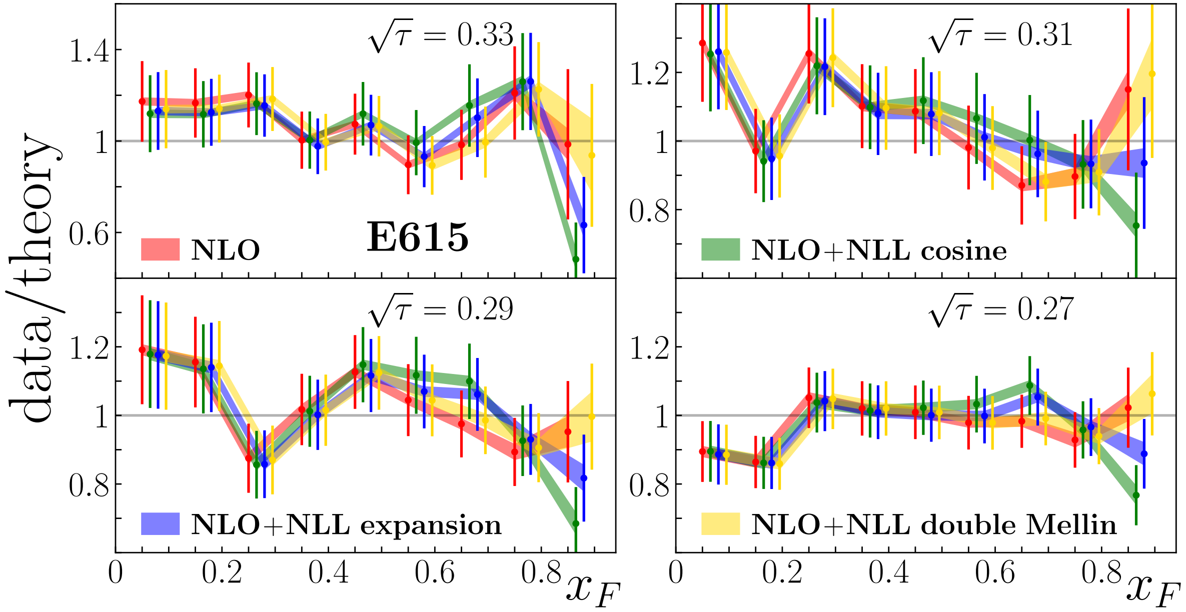
Data and theory comparison

- Cosine method tends to overpredict the data at very large x_F
- Double Mellin method is qualitatively very similar to NLO

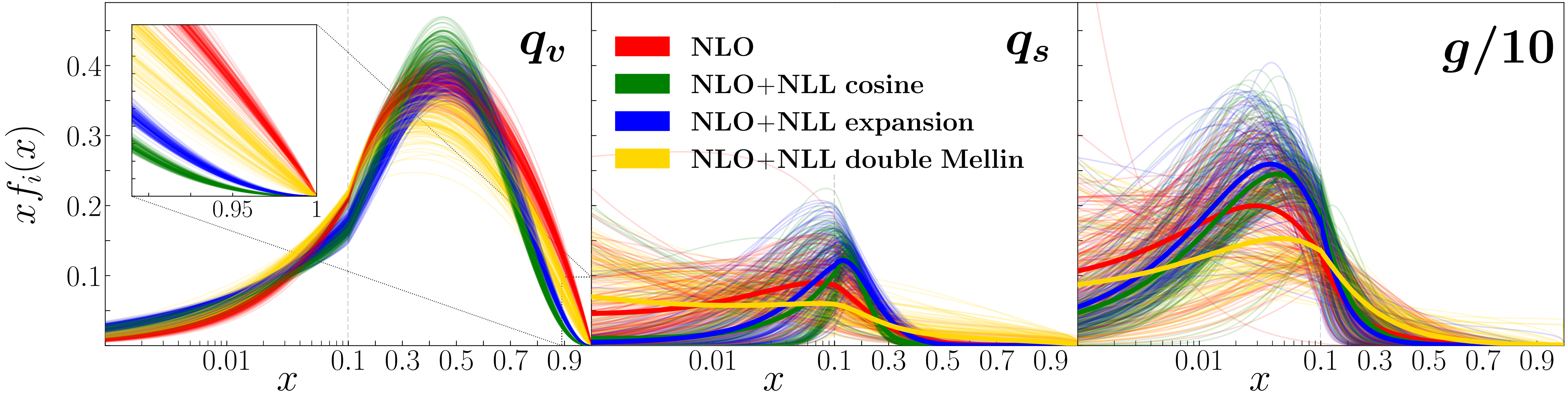


Current data do not distinguish between NLO and NLO+NLL

Method	χ^2/npts
NLO	0.85
NLO+NLL cosine	1.29
NLO+NLL expansion	0.95
NLO+NLL double Mellin	0.80

Slightly disfavored

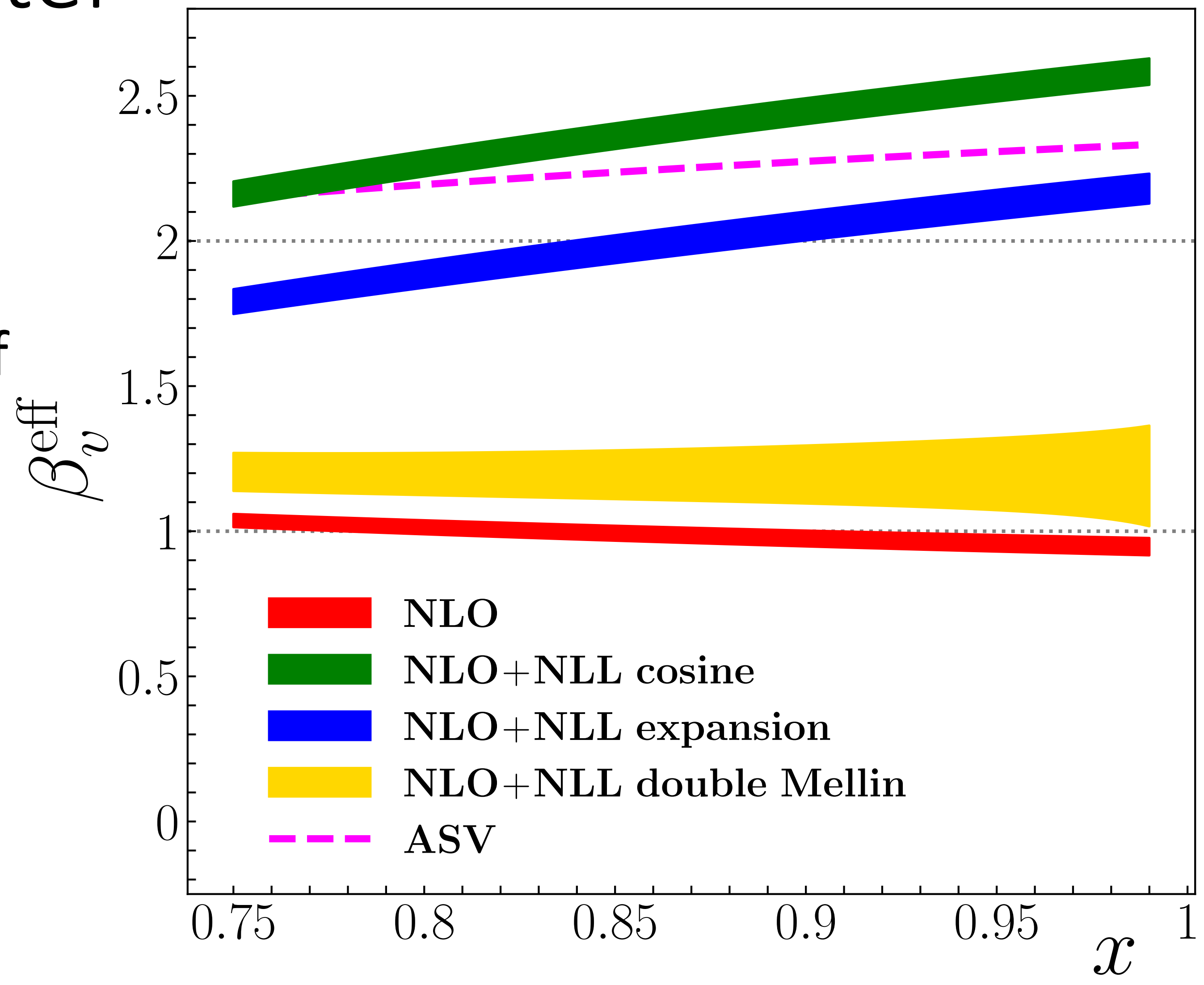
Resulting PDFs



- Large x behavior of q_v **highly sensitive** to method of resummation

Effective β_v parameter

- $q_v(x) \sim (1-x)^{\beta_v^{\text{eff}}}$ as $x \to 1$
- Threshold resummation does not give universal behavior of β_v^{eff}
- NLO and double Mellin give $\beta_v^{\text{eff}} \approx 1$
- Cosine and Expansion give $\beta_v^{\text{eff}} > 2$



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Deriving resummation expressions – MF

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{\hat{x}_\pi \hat{x}_A}$$

Claim: yellow terms give rise to the resummation expressions

$$y = \frac{\frac{\hat{x}_\pi}{\hat{x}_A} e^{-2Y} - z}{(1-z)(1+\frac{\hat{x}_\pi}{\hat{x}_A} e^{-2Y})}$$

$$\begin{aligned}\frac{C_{q\bar{q}}}{e_q^2} &= \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right] \\ &+ \frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ + 1 - z\right]\right. \\ &\left. + \frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right) - 2(1-z)\right]\right\}\end{aligned}$$

Claim: Red terms are power suppressed in $(1-z)$ and wouldn't contribute to the same order as the yellow terms

Generalized Threshold resummation

G. Lustermans, J. K. L. Michel, and F. J. Tackmann, arXiv:1908.00985 [hep-ph].

- Write the (z, y) coefficients in terms of (z_a, z_b), and for the red terms, you get:

$$z_a = \frac{x_\pi^0}{\hat{x}_\pi}$$

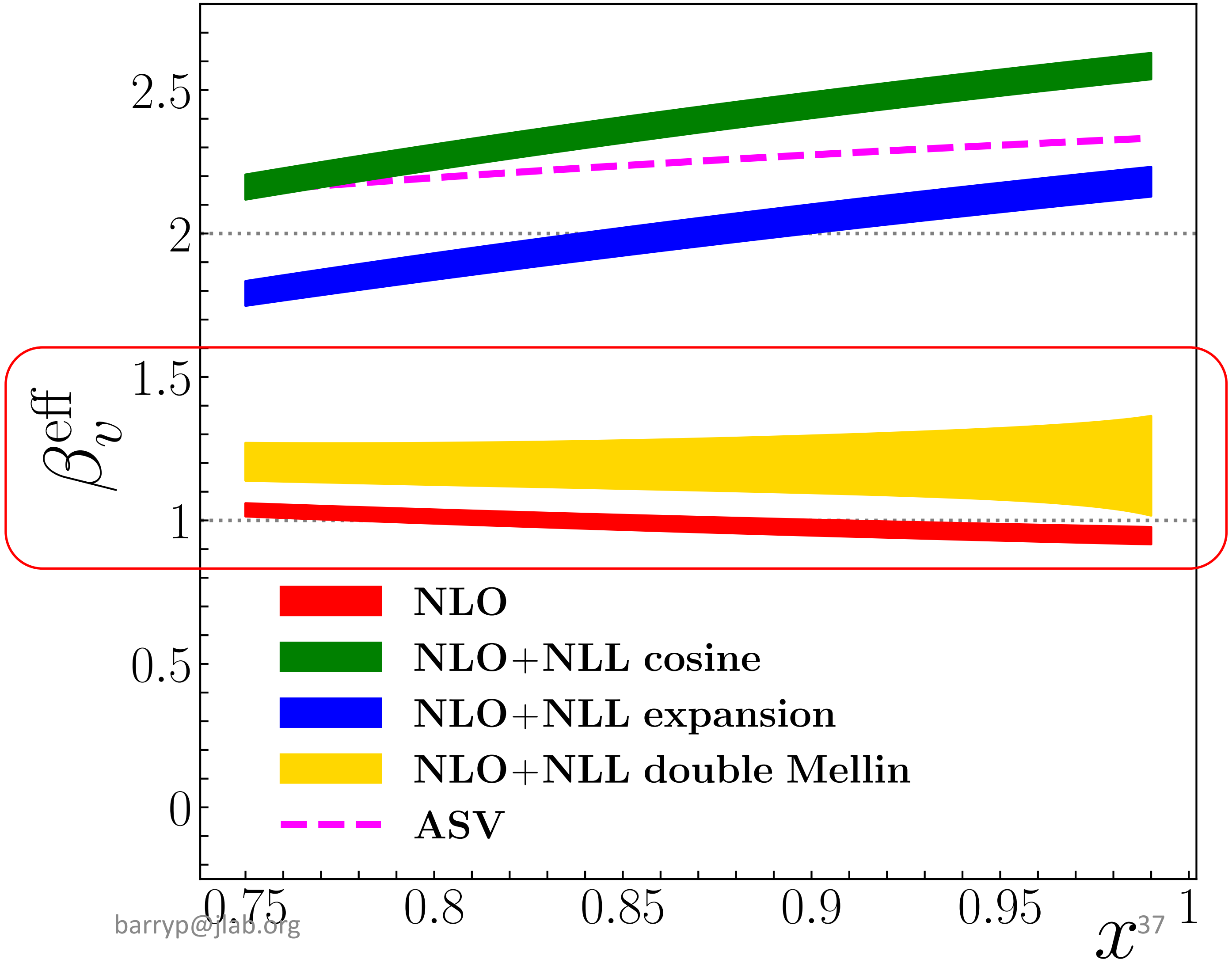
$$\mathrm{d}z\,\mathrm{d}y\,\frac{1}{1-z}\left(\frac{1}{y}+\frac{1}{1-y}\right) = \mathrm{d}z_a\mathrm{d}z_b\,\frac{1}{(1-z_a)(1-z_b)}\left[1+\mathcal{O}(1-z_a, 1-z_b)\right].$$

$$z_b = \frac{x_A^0}{\hat{x}_A}$$

- This is *not* power suppressed in $(1 - z_a)$ or $(1 - z_b)$ but instead the same order as the leading power in the soft limit
- Generalized threshold resummation in the soft limit does not agree with the MF methods

What we believe to be theoretically better

- Take more seriously the red and yellow
- $\beta_v^{\text{eff}} \sim 1 - 1.2$, much closer to 1 than 2



Including lattice QCD data from HadStruc

- Can lattice QCD simulations further discriminate between NLO and the double Mellin methods?

JLAB-THY-22-3592

Complementarity of experimental and lattice QCD data on pion parton distributions

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Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations

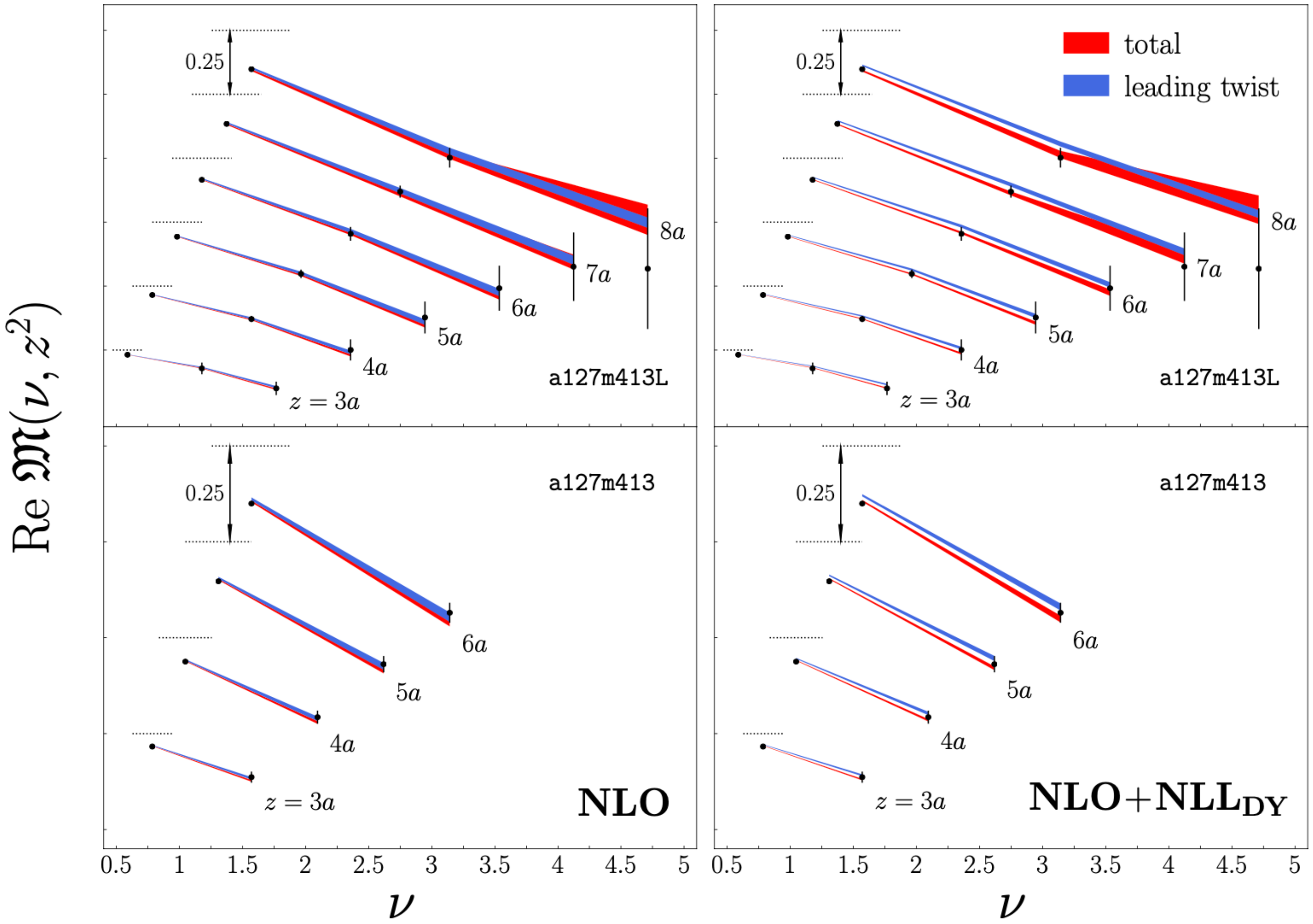
Goodness of fit

- Scenario A: experimental data alone
- Scenario B: experimental + lattice, no systematics
- Scenario C: experimental + lattice, with systematics

			Scenario A		Scenario B		**Scenario C**	
			NLO	+NLL$_{DY}$	NLO	+NLL$_{DY}$	NLO	+NLL$_{DY}$
Process	Experiment	N_{dat}	$\overline{\chi}^2$		$\overline{\chi}^2$		$\overline{\chi}^2$	
DY	E615	61	0.84	0.82	0.83	0.82	**0.84**	**0.82**
	NA10 (194 GeV)	36	0.53	0.53	0.52	0.54	**0.52**	**0.55**
	NA10 (286 GeV)	20	0.80	0.81	0.78	0.79	**0.78**	**0.87**
LN	H1	58	0.36	0.35	0.39	0.39	**0.37**	**0.37**
	ZEUS	50	1.56	1.48	1.62	1.69	**1.58**	**1.60**
Rp-ITD	a127m413L	18	–	–	1.04	1.06	**1.04**	**1.06**
	a127m413	8	–	–	1.98	2.63	**1.14**	**1.42**
Total		**251**	**0.82**	**0.80**	**0.89**	**0.92**	**0.85**	**0.87**

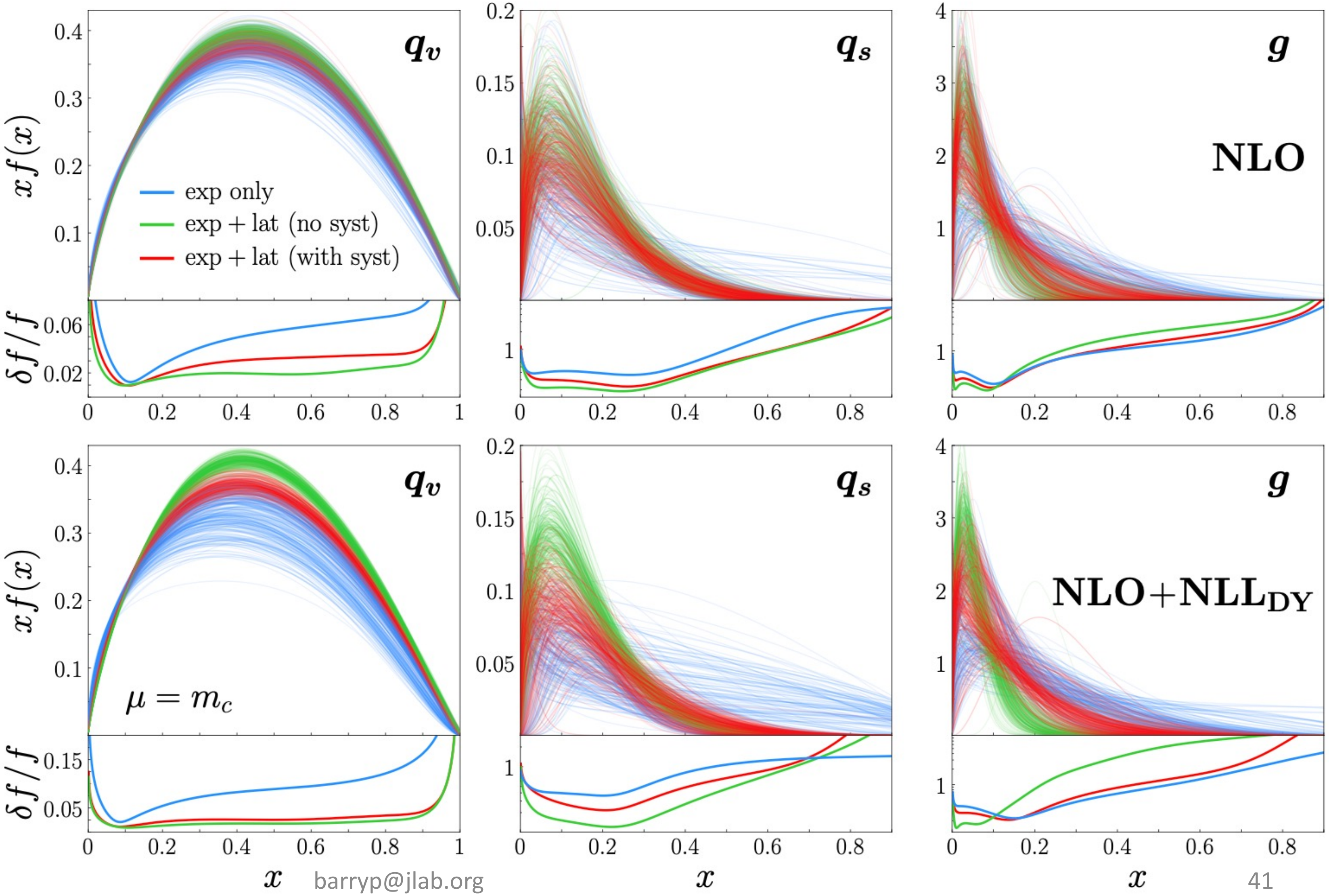
Agreement with the data

- Results from the full fit and isolating the leading twist term
- Difference between bands is the systematic correction



Resulting PDFs

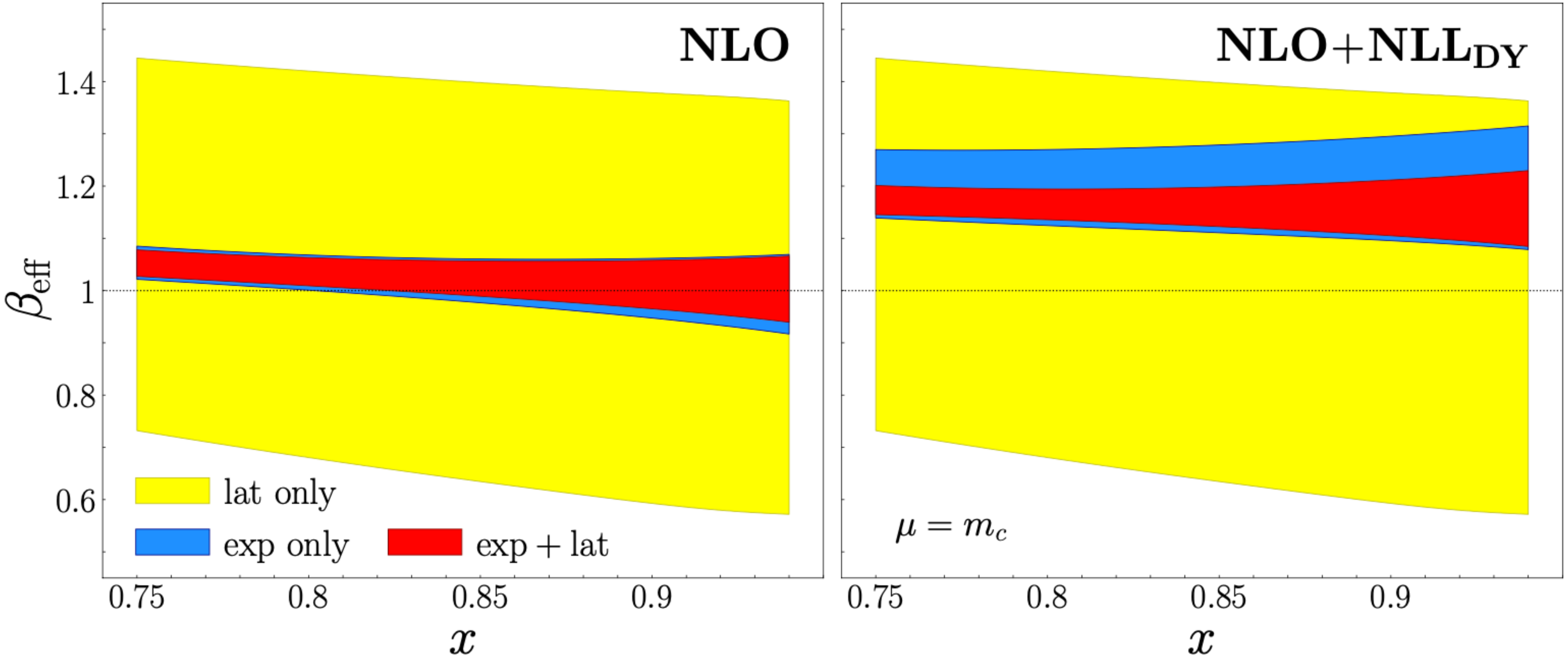
- PDFs and relative uncertainties
- Including lattice reduces uncertainties
- NLO+NLL_{DY} changes a lot – unstable under new data



Effective β from $(1-x)^{\beta_{\text{eff}}}$

$$\beta_{\text{eff}}(x,\mu) = \frac{\partial \log |q_v(x,\mu)|}{\partial \log(1-x)}$$

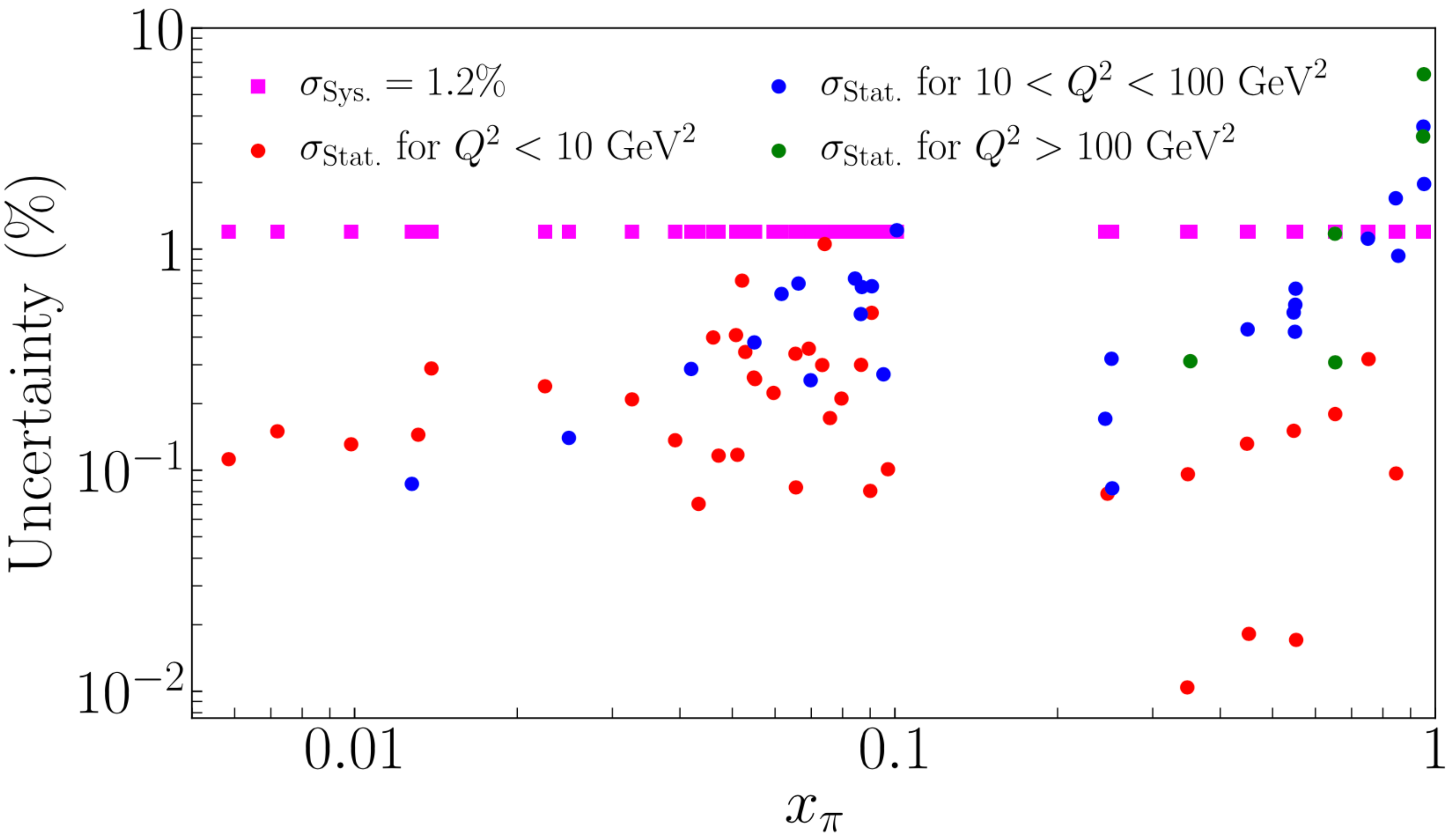
Calculations from QCD do not predict $\beta_{\text{eff}} = 2$



Future Work and Conclusions

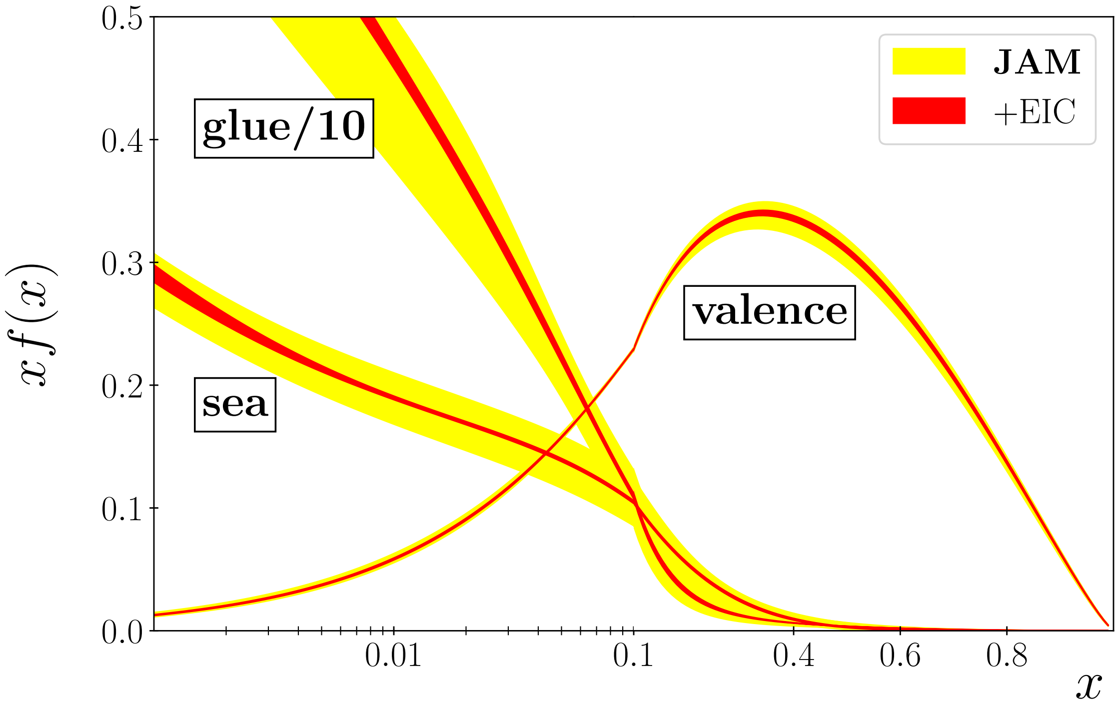
EIC kinematics and uncertainties

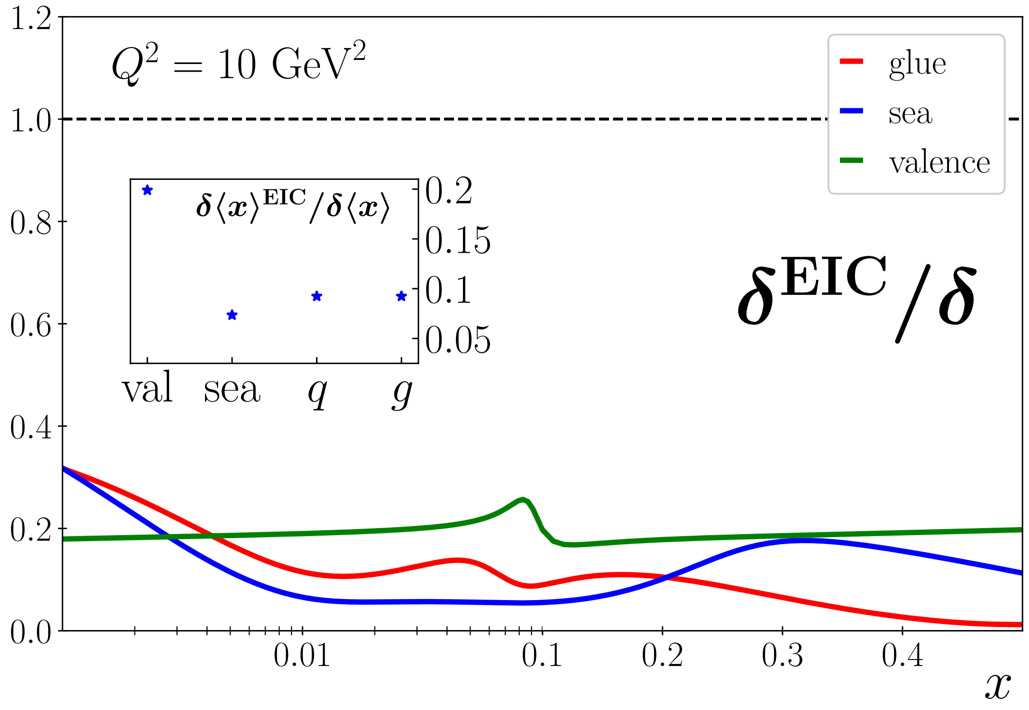
- Uncertainties are dominated by systematics
- Large range in x_π, Q^2 to overlap Drell-Yan and leading neutron regions



EIC Impact on Pion PDFs

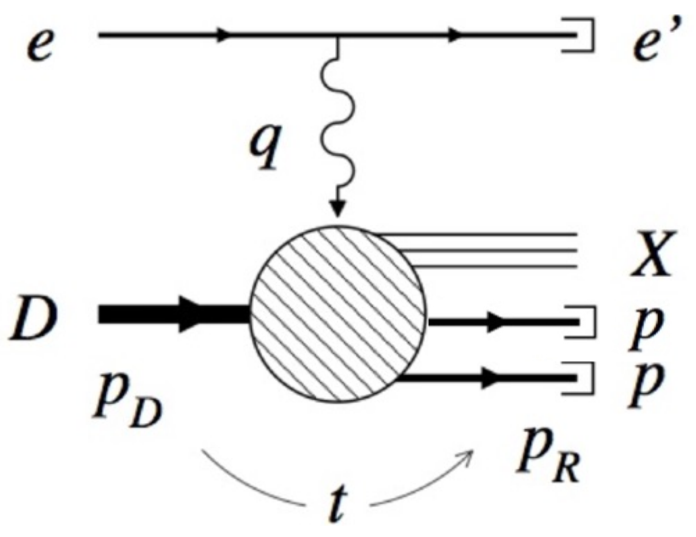
- Statistical uncertainties are small compared with HERA because of larger luminosity – systematics dominate
- $s = 5400$ GeV^2, 1.2% systematic uncertainty, integrated $\mathcal{L} = 100$fb^{-1}





Future Experiments

- TDIS experiment at 12 GeV upgrade from JLab, which will tag a proton in coincidence with a spectator proton
- Gives leading proton observable, complementary to LN, but with a fixed target experiment instead of collider



- Proposed COMPASS++/AMBER also give π-induced DY data
- Both π^+ and π^- beams on carbon and tungsten targets

TMD factorization in Drell-Yan

- In small-p_T region, Use the CSS formalism for TMD evolution

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_\mathrm{T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H_{j\bar{j}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}$$

$$\times\, e^{-g_{j/A}(x_A,b_\mathrm{T};b_\mathrm{max})} \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\, e^{-g_{\bar{j}/B}(x_B,b_\mathrm{T};b_\mathrm{max})} \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times \exp\left\{-g_K(b_\mathrm{T};b_\mathrm{max})\ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Collinear pion PDF

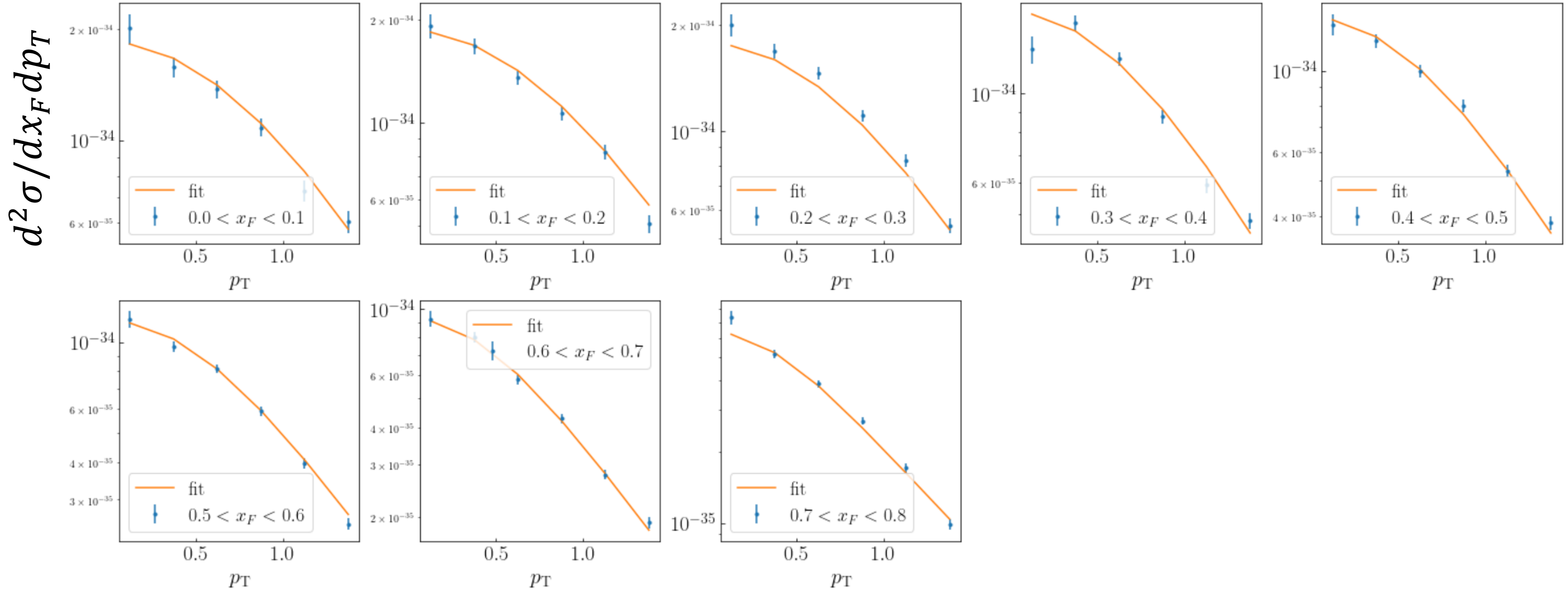
Non-perturbative TMDs to extract

- Fit non-perturbative TMDs to pion-induced E615 data

Low energy Drell-Yan

- Can achieve good description of the data in a single fit
- Will perform global analysis to extract pion TMDPDFs and collinear PDFs

E615 πA



Conclusions

- Behavior of large-x valence distribution with double Mellin threshold resummation $q_v(x \to 1) \propto (1-x)^{\sim 1.2}$
- The complementarity between lattice and experimental data sheds light on the pion PDF itself as well as systematics associated with the lattice
- Future experimental and lattice data are needed to further pin down large-x behavior of the valence quark distribution
- Successfully have performed single fits to low- p_{T} of both pion TMD and collinear PDFs and Monte Carlo is underway

Backup Slides

Critiques suggested $(1-x)^2$ is a fact of QCD

$$u^{\pi}(x;\zeta) \overset{x\simeq 1}{\approx} (1-x)^{\beta = 2+\gamma(\zeta)}$$

T1: If QCD describes the pion, then at any scale for which an analysis of data using known techniques is valid, the form extracted for the pion's valence-quark DF must behave as $(1-x)^{\beta}$, $\beta > 2$, on $x \gtrsim 0.9$ [10, 59, 73, 74].

the associated disagreement with Eq. (27) requires explanation; and these are the only possibilities: [a] the dM scheme is incomplete, omitting or misrepresenting some aspect or aspects of the hard processes involved; [b] (some of) the data being considered in the analysis are not a true expression of a quality intrinsic to the pion; or [c] QCD, as it is currently understood, is not the theory of strong interactions.

- T1: There is no proof of this in QCD
- [a] The double Mellin method is more rigorous than Mellin-Fourier
- [b] We carefully apply factorization; lattice QCD data prefer a linear falloff; there is no evidence to suggest these data are wrong
- [c] There is no indication to insinuate QCD is not the theory of strong interactions

Ezawa

Wide-Angle Scattering in Softened Field Theory.

Not QCD

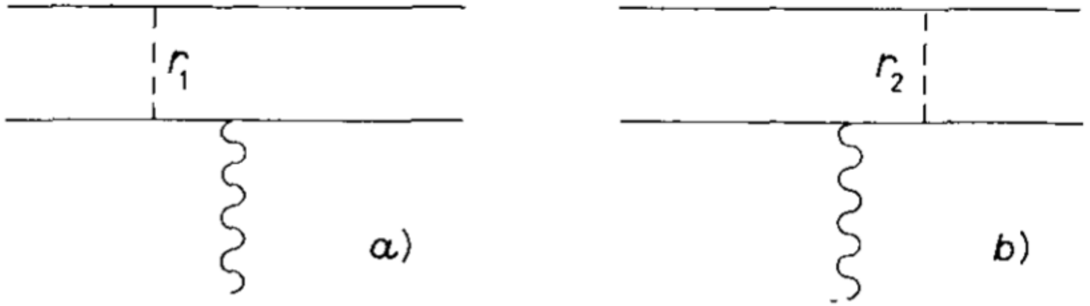
Z. F. Ezawa

Department of Applied Mathematics and Theoretical Physics
University of Cambridge - Cambridge

(ricevuto il 25 Marzo 1974)

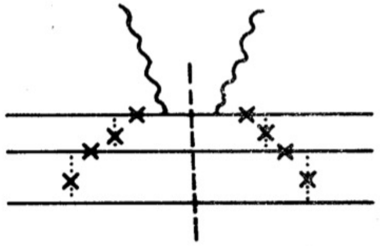
Summary. — The picture of Brodsky and Farrar for scattering processes at large transverse momentum is formulated in softened field theory. A modest softening of the quark-quark-gluon vertex is introduced to suppress unwanted logarithms in the formalism. It is shown that the electromagnetic form factors of the proton and the pion yield asymptotically behaviours which agree with the result of simple dimensional counting. The threshold behaviours of the deep inelastic structure functions are calculated for the proton and the pion to give $\sim(1-\omega)^3$ and $\sim(1-\omega)^2$, respectively. Thus the Drell-Yan-West relation holds in the case of the proton target but is violated in the case of the pion target. It is also proved that the asymptotic behaviours of wide-angle elastic $\pi\pi$ and pp scattering naively predicted by dimensional counting and conjectured by Brodsky and Farrar on the basis of simple Born diagrams are actually the next-to-leading-order terms. The highest-order terms come from a certain set of diagrams that Landshoff studied.

- No explicit proof of nonperturbative $q_v^\pi(x \to 1) \sim (1-x)^2$
- Assumes one hard gluon exchange dominance



Farrar and Jackson

- Assumption made that the below diagram dominates the structure



(a)

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Pion and Nucleon Structure Functions near $x = 1$*

Glennys R. Farrar† and Darrell R. Jackson
California Insitute of Technology, Pasadena, California 91125
(Received 4 August 1975)

In a colored-quark and vector-gluon model of hadrons we show that a quark carrying nearly all the momentum of a nucleon ($x \approx 1$) must have the same helicity as the nucleon; consequently $\nu W_2^n/\nu W_2^p \to \frac{3}{7}$ as $x \to 1$, not $\frac{2}{3}$ as might naively have been expected. Furthermore as $x \to 1$, $\nu W_2^\pi \sim (1-x)^2$ and $(\sigma_L/\sigma_T)^\pi \sim \mu^2 Q^{-2}(1-x)^{-2} + O(g^2)$; the resulting angular dependence for $e^+e^- \to h^\pm + X$ is consistent with present data and has a distinctive form which can be easily tested when better data are available.

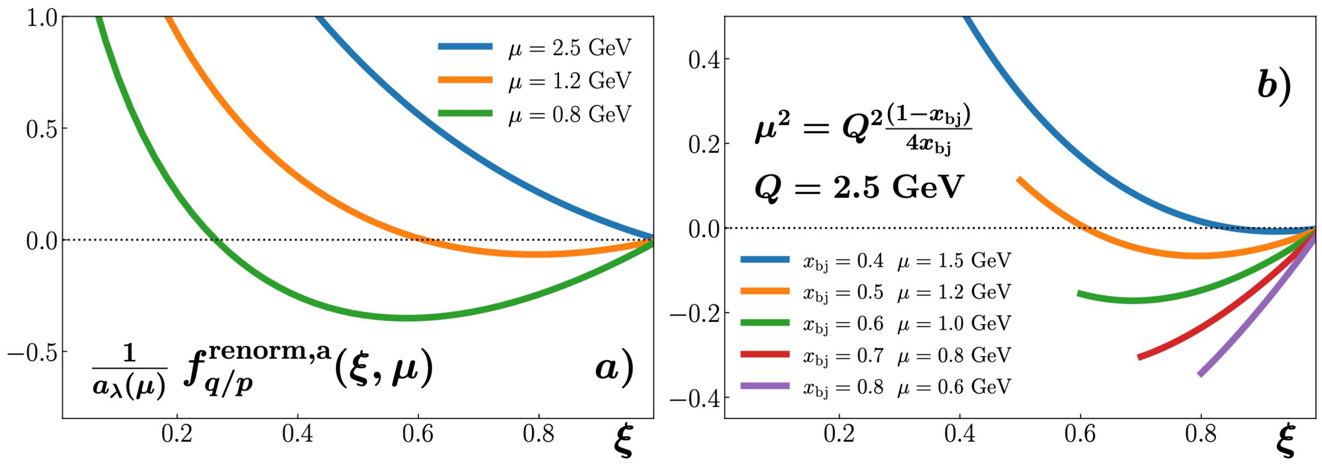
Assumption

go from the normal to "exceptional" (one quark having large p^2) wave functions. We assume that (a) the normal wave function is sufficiently damped at large p^2's that the convolution is dominated by the region in which the p^2's of the incoming quarks are finite, and (b) the spin and

- This is a *perturbative* assumption – we cannot say that higher order terms or soft gluons do not contribute to the *nonperturbative* structure of the hadron in QCD
- First principles QCD does not prove this behavior for PDF

Not necessary to have $(1-x)^{\beta}$ behavior

- A recent work by Collins, Rogers, and Sato proved that $\overline{\mathrm{MS}}$ PDFs were not necessarily positive as long as *cross section was positive.*



Phys. Rev. D **105**, 076010 (2022).

- PDFs do not have to have a large-x behavior associated with the counting rules

QCD does not fail if $\beta_v^\pi \neq 2$

- The perturbative expansion performed in Ezawa and Farrar & Jackson does not capture nonperturbative effects
- Like in threshold resummation, the buildup of very soft gluon exchanges between quark states may be non-negligible contributions to the perturbation
- When $(1-x) \to 0$, the light front zero mode could play a non-trivial role, which cannot be calculated perturbatively

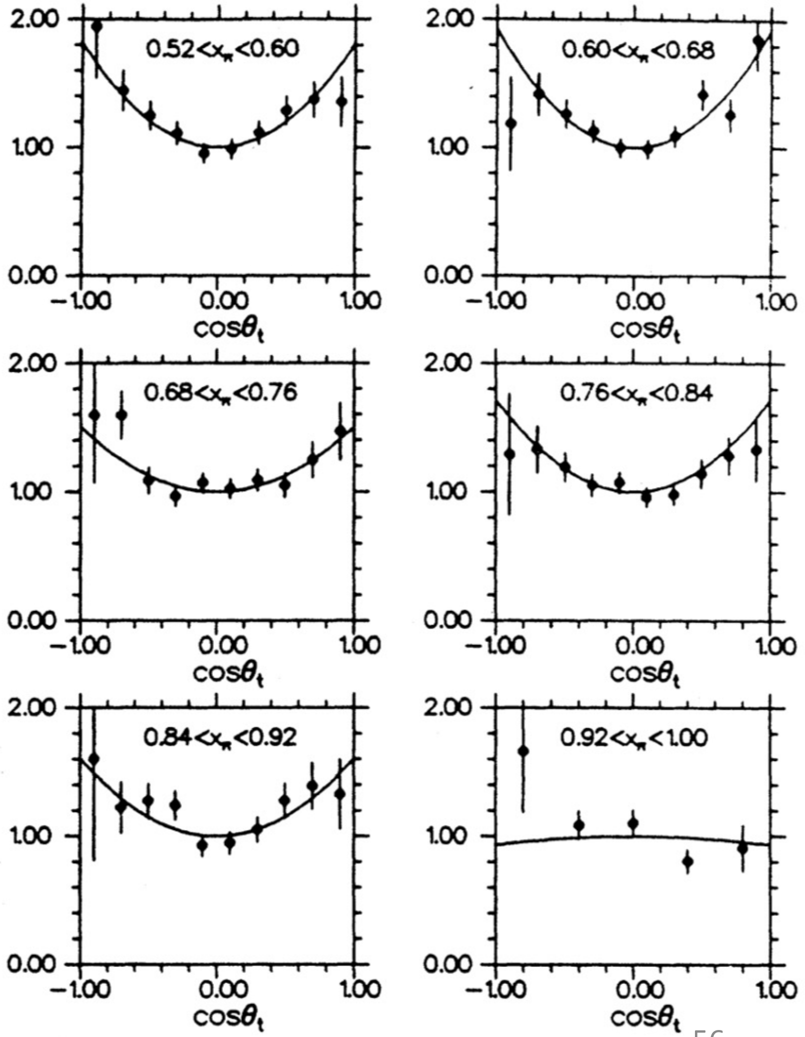
Angular dependence in E615 DY data

- Expected behavior of the cross section

$$d\sigma \propto (1-x_\pi)^2(1+\cos^2\theta)+\boxed{\frac{4x_\pi^2\langle k_T^2\rangle}{9m_{\mu\mu}^2}\sin^2\theta}\,.$$

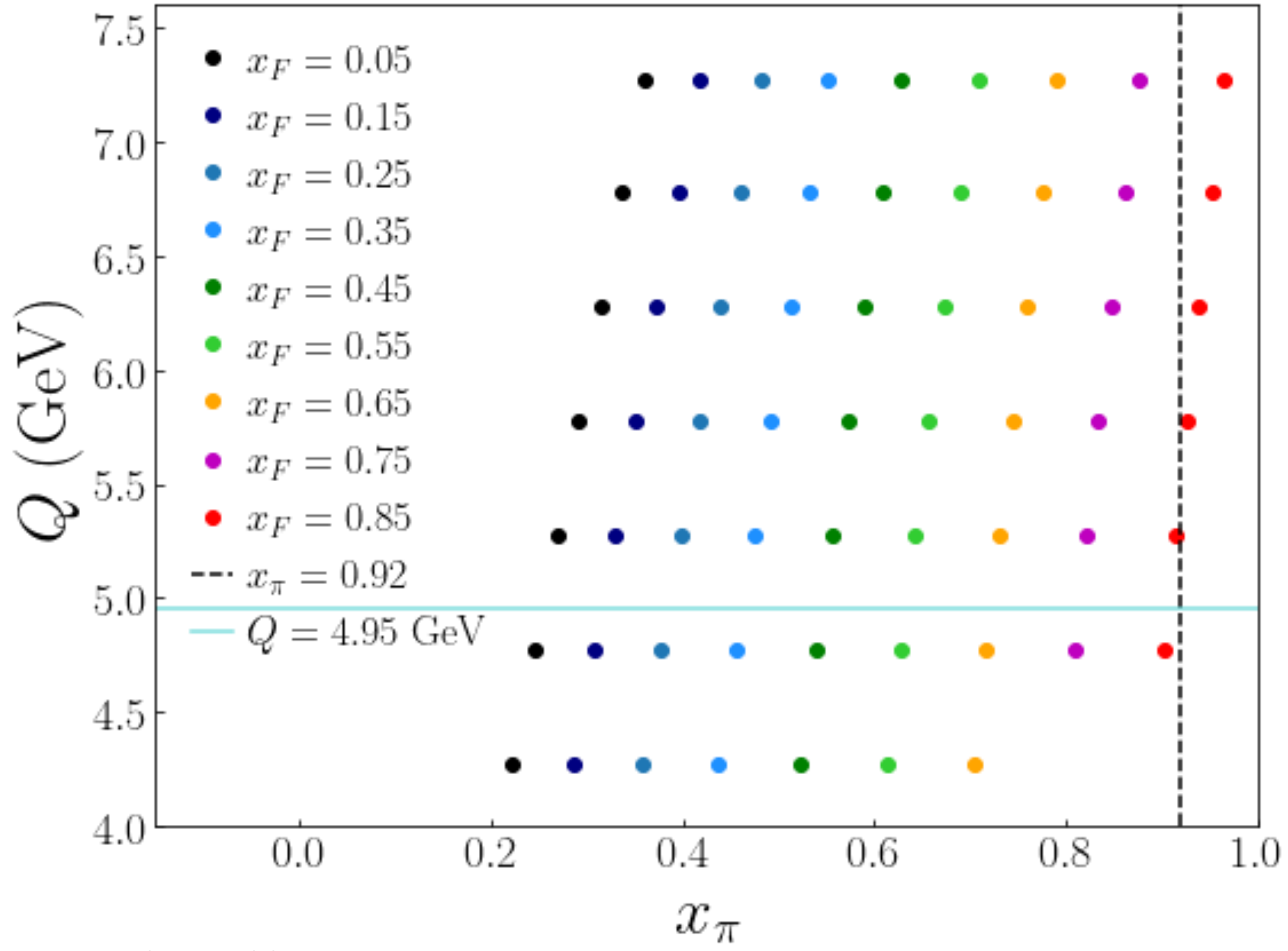
higher twist

- Parabolic = leading twist
- Each range of x_π follows the parabolic behavior except $0.92 < x_\pi < 1$ for shown $4.05 < M_{\mu^+\mu^-} < 4.95$ GeV where higher twist is expected to be most dominant



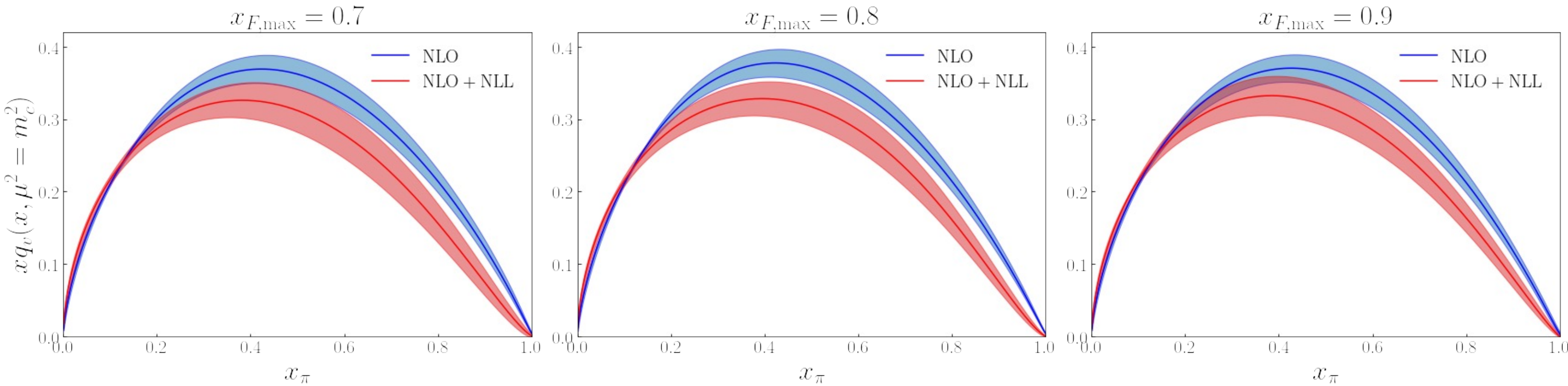
Kinematics of E615

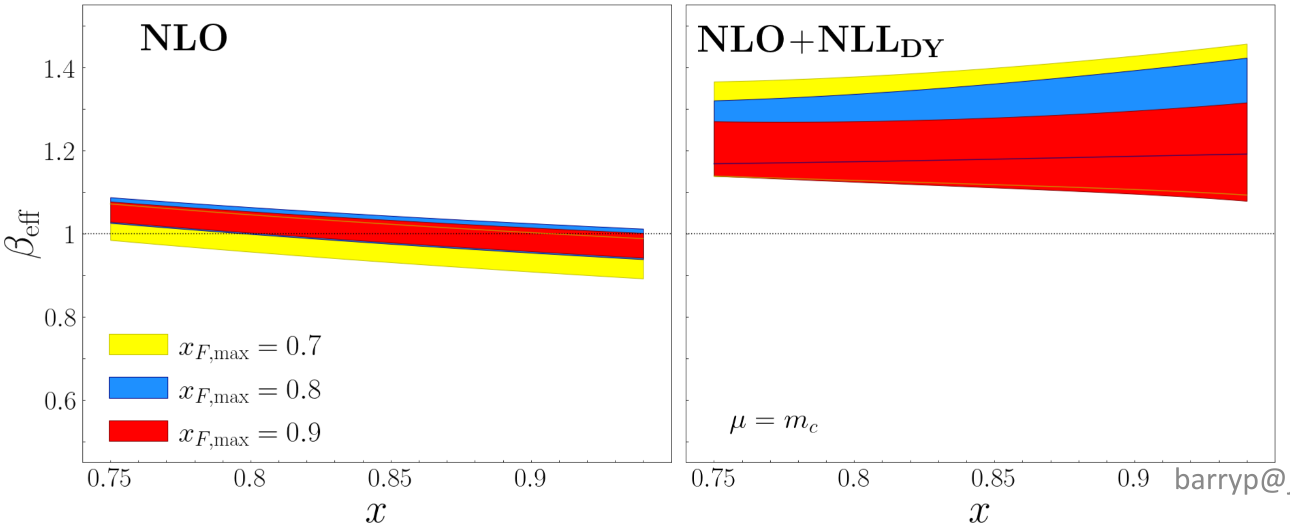
- Each of these points is included in the global analysis
- For small Q, we only have $x_\pi < 0.92$ points



Studying cuts in x_F

- To ensure the leading twist formalism, we also modify the $x_{F,\max}$





Large x behavior is conserved, albeit with larger uncertainties

Quantifying individual systematic corrections on the lattice

- Breaking down by the 3 systematics

$$z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu)$$

- Dominance of power or spacing corrections depends on z
- Finite volume corrections don't matter