Hadron mass in chiral symmetry restored vacuum and in nuclear matter

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1. General remarks on hadron mass
2. Chiral partners
3. Vector meson mass and chiral symmetry breaking effect
4. Summary

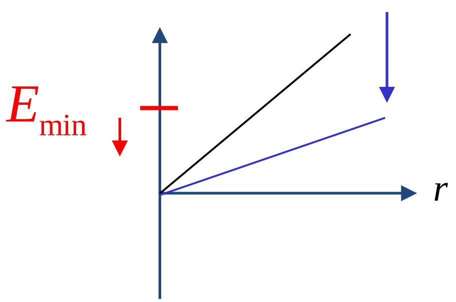
Previous work +
- T. Song, T. Hatsuda, Su Houng Lee, PLB792 (2019) 160
- Jisu Kim and Su Houng Lee, PRD103(21) L051501,PRD105 (2022)014014
- Jisu Kim, Philipp Gubler, Su Houng Lee in preparation

Confinement and Chiral symmetry breaking in quark model

☞ Consider a meson $$H = 2m_q + \frac{p^2}{2m_q} + \sigma\, r$$

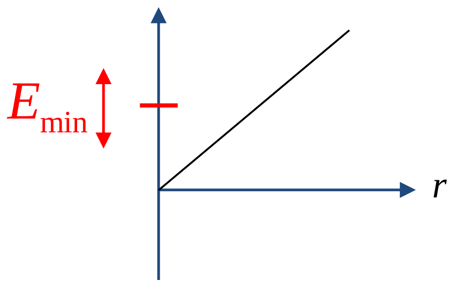
Deconfinement

$\sigma \rightarrow$ decrease



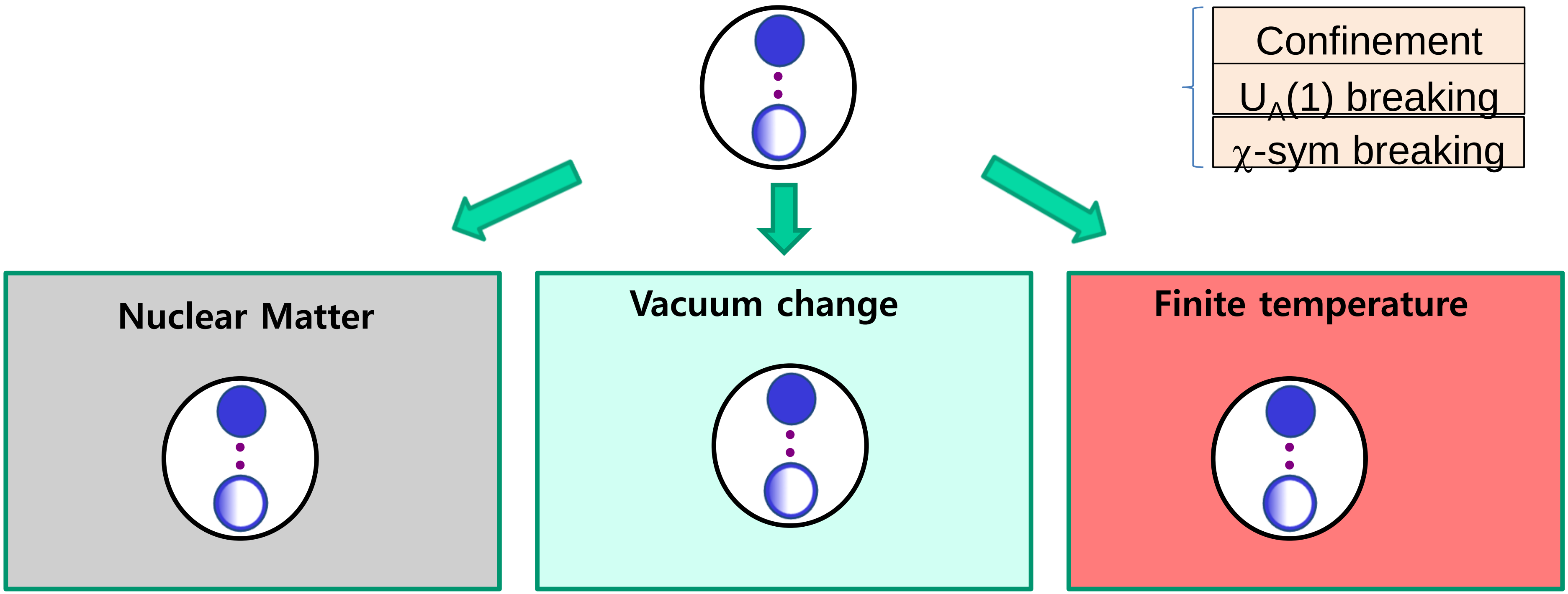
Chiral symmetry restoration

$m_q \rightarrow$ decrease



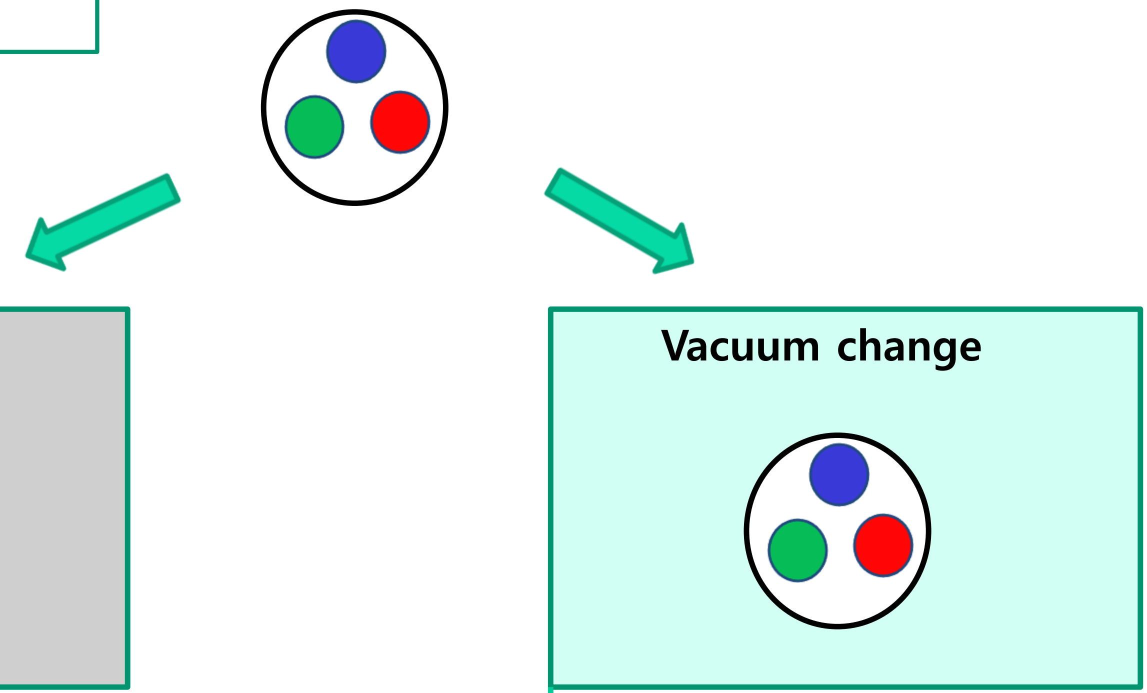
Separation in QCD ?
χ –symmetry ?
Confinement ?

To understand the origin of Hadron mass



1. *Mass changes are in general* *different*

Example



$$\left[E\gamma^0 - \vec{\gamma}\,\vec{p} - M - \left(S + \gamma^0 V\right)\right]\psi = 0$$

For proton at rest inside nuclear matter

$$E = M + \left(S + V\right) \approx M + \left(-400 + 300\right)\mathrm{MeV}$$

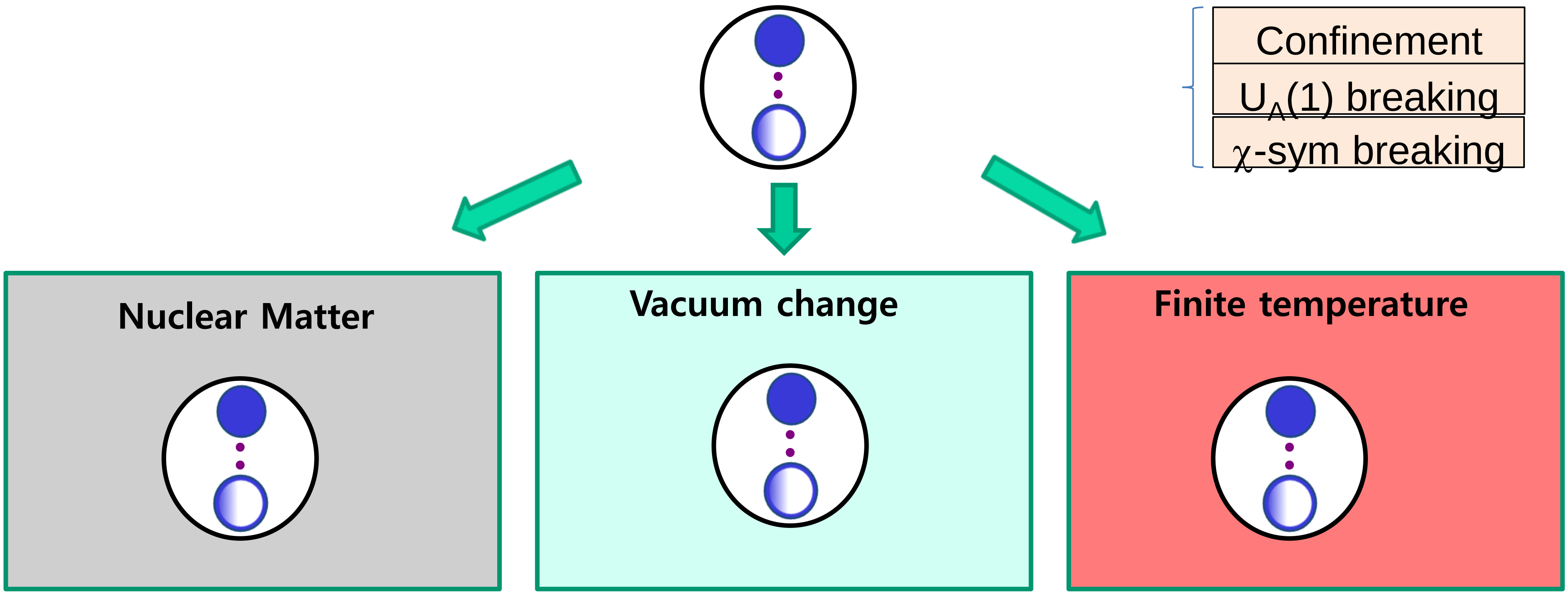
$$\left[E\gamma^0 - \vec{\gamma}\,\vec{p} - M^*\right]\psi = 0$$

But is

$$E = M^* \xrightarrow{?} M + \left(S\right) \quad ?$$

Understanding origin of mass involves
understanding relation between $M + \left(S\right)$ and M^*

To understand the origin of Hadron mass



But common in all cases

1. *Chiral symmetry is restored in all cases*
2. *Mass difference between chiral partners only depends on chiral symmetry breaking*

Fortunately, WE Can identify chiral symmetry breaking effects in individual hadron mass

KEK E325, J-PARC E16

Vacuum values	Mass	Width
ϕ	1020 MeV	4.266 MeV

CBELSA/TAPS coll (V. Metag, M. Nanova et al)

Vacuum values	Mass	Width
ω	782.65 MeV	8.49 MeV
η‘	957.78 MeV	0.198 MeV

☞ Lesson from experiment

1. Look at small width hadrons (<100 MeV)
2. Look at spectral change
3. Can look at excitation energy → mass shift
4. Look at transparency → Width

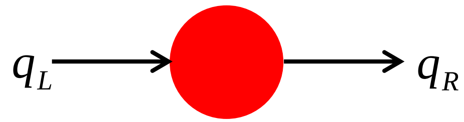
Chiral symmetry breaking

and chiral partners

Chiral symmetry breaking (m→0) : $\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R \rightarrow \mathrm{SU}(N_F)_V$

- *Chiral order parameter: Quark condensate*

☞ $$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle = -\lim_{x\to 0} \langle \mathrm{Tr}[S(x,0)] \rangle = -\lim_{x\to 0} \left\langle \frac{1}{2} \mathrm{Tr}\left[S(x,0) - i\gamma^5 S(x,0) i\gamma^5 \right] \right\rangle$$



Chiral rotation $q \rightarrow \exp(i\gamma^5 \tau^a \alpha^a) q$

☞ Casher Banks formula: nontrivial zero mode (λ =0) contribution

$$\langle \bar{q}q \rangle = -\left\langle \mathrm{Tr}\left[\left(0 \,|\, \frac{1}{\not{D} + m} \,|\, 0 \right) \right] \right\rangle = -\frac{1}{2} \left\langle \sum_\lambda \psi_\lambda^+ \left(\frac{1}{i\lambda + m} + \frac{1}{-i\lambda + m} \right) \psi_\lambda \right\rangle = -\frac{1}{2} \left\langle \sum_\lambda \psi_\lambda^+ \left(\frac{2m}{\lambda^2 + m^2} \right) \psi_\lambda \right\rangle$$

$$\xrightarrow{m=0} \langle \pi\rho(\lambda = 0) \rangle$$ where $i\not{D}\psi_\lambda = \lambda \psi_\lambda$

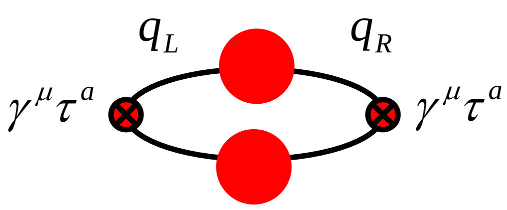
cf. $\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = 12 \left\langle \sum_\lambda \rho(\lambda) \right\rangle$

(SHL, S.Cho, arXiv:1302.0642)

• *Chiral order parameter: $\rho\rho - a_1 a_1$ correlator*

☞ $$\Pi^{\rho\rho} - \Pi^{a_1 a_1} = \frac{1}{V}\int d^4x \left[\left\langle \bar{q}\gamma^\mu\tau^a q(x), \bar{q}\gamma^\mu\tau^a q(0) \right\rangle - \left\langle \bar{q}\tau^a i\gamma^5\gamma^\mu q(x), \bar{q}\tau^a i\gamma^5\gamma^\mu q(0) \right\rangle \right]$$

$$= -\frac{1}{2}\mathrm{Tr}\left[\gamma^\mu \left(S(x,0) - i\gamma^5 S(x,0) i\gamma^5 \right) \gamma^\mu \left(S(0,x) - i\gamma^5 S(0,x) i\gamma^5 \right) \right] \propto \left\langle \rho^2(\lambda=0) \right\rangle$$



$$\propto \left\langle \rho^2(\lambda=0) \right\rangle$$

☞ Weinberg sum rule

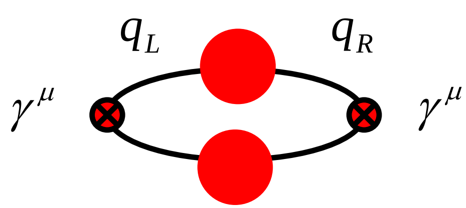
$$\left\{ \begin{array}{l} f_\rho^2 m_\rho^2 - f_{a_1}^2 m_{a_1}^2 = f_\pi^2 \\ f_\rho^2 m_\rho^4 - f_{a_1}^2 m_{a_1}^4 = 0 \end{array} \right\} \quad f_\rho^2 m_\rho^2 \left(1 - \frac{m_\rho^2}{m_{a_1}^2} \right) = f_\pi^2$$

$m_\rho = m_{a_1} = m_0$ when chiral symmetry is restored. What about m_0 ?

• *Not* a Chiral order parameter: $\omega\omega - f_1^{\bar{q}q} f_1^{\bar{q}q}$ correlator

☞ $$\Pi^{\omega\omega} - \Pi^{f_1 f_1} = \frac{1}{V}\int d^4x \left[\left\langle \bar{q}\gamma^\mu q(x), \bar{q}\gamma^\mu q(0) \right\rangle - \left\langle \bar{q} i\gamma^5\gamma^\mu q(x), \bar{q} i\gamma^5\gamma^\mu q(0) \right\rangle \right]$$

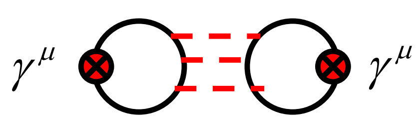
$$= -\frac{1}{2}\mathrm{Tr}\left[\gamma^\mu \left(S(x,0) - i\gamma^5 S(x,0) i\gamma^5 \right) \gamma^\mu \left(S(0,x) - i\gamma^5 S(0,x) i\gamma^5 \right) \right]$$



$\propto \left\langle \rho^2(\lambda = 0) \right\rangle \propto \left\langle \bar{q}q \right\rangle^n$

$$+ \frac{1}{4}\mathrm{Tr}\left[\gamma^\mu \left(S(x) + i\gamma^5 S(x) i\gamma^5 \right) \right] \mathrm{Tr}\left[\gamma^\mu \left(S(0) + i\gamma^5 S(0) i\gamma^5 \right) \right] + \left(\gamma^\mu \to \gamma^\mu\gamma^5 \right)$$

↳ Invariant under Chiral rotation $q \to \exp\left(i\gamma^5 \tau^a \alpha^a \right) q$



$\propto \left\langle \rho^2(\lambda \neq 0) \right\rangle$

ω has no chiral partner

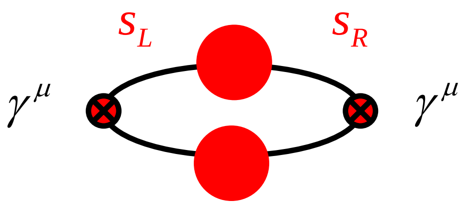
- *Not a Chiral order parameter:* $\phi\phi - f_1^{\bar{s}s} f_1^{\bar{s}s}$ *correlator*

☞

$$\Pi^{\phi\phi} - \Pi^{f_1 f_1} = \frac{1}{V}\int d^4x \left[\left\langle \bar{s}\gamma^\mu s(x), \bar{s}\gamma^\mu s(0) \right\rangle - \left\langle \bar{s} i\gamma^5\gamma^\mu s(x), \bar{s} i\gamma^5\gamma^\mu s(0) \right\rangle \right]$$

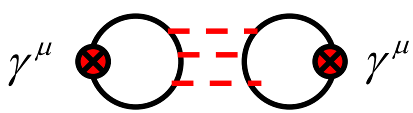
$$= -\frac{1}{2}\mathrm{Tr}\left[\gamma^\mu \left(S^s(x,0) - i\gamma^5 S^s(x,0) i\gamma^5 \right) \gamma^\mu \left(S^s(0,x) - i\gamma^5 S^s(0,x) i\gamma^5 \right) \right]$$

$$\sum_\lambda \psi_\lambda^+ \left(\frac{2m_s}{\lambda^2 + m_s^2} \right) \psi_\lambda$$



$\propto \left\langle \bar{s}s \right\rangle^n$

$$+ \frac{1}{4}\mathrm{Tr}\left[\gamma^\mu \left(S^s(x) + i\gamma^5 S^s(x) i\gamma^5 \right) \right] \mathrm{Tr}\left[\gamma^\mu \left(S^s(0) + i\gamma^5 S^s(0) i\gamma^5 \right) \right] + \left(\gamma^\mu \to \gamma^\mu\gamma^5 \right)$$



$\propto \left\langle \rho^2 (\lambda \neq 0) \right\rangle$

ϕ has no chiral partner

Light vector mesons – chiral partners ?

$J^{PC}=1^{--}$	Mass	Width	$J^{PC}=1^{++}$	Mass	Width
ρ $(\bar{q}\gamma_\mu \tau q)$	770	150.	a_1 $(\bar{q}\gamma_\mu \gamma^5 \tau q)$	1260	250-600
K* $(\bar{s}\gamma_\mu q)$	892	50.3	K_1 $(\bar{s}\gamma_\mu \gamma^5 q)$	1270	90

Are chiral partners

$J^{PC}=1^{--}$	Mass	Width	$J^{PC}=1^{++}$	Mass	Width
ω $(\bar{q}\gamma_\mu q)$	782	8.49	f_1 $(\bar{q}\gamma_\mu \gamma^5 q)$	1285	24.2
ϕ $(\bar{s}\gamma_\mu s)$	1020	4.266	f_1 $(\bar{s}\gamma_\mu \gamma^5 s)$	1420	54.9

Are NOT chiral partners

Light vector mesons – chiral partners ?

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Are chiral partners

Are NOT chiral partners

Studying individual masses are nevertheless important because

Experimentally feasible

Theory can link to chiral symmetry breaking to mass shift

Individual masses and chiral symmetry breaking

Vector meson mass in the chiral symmetry restored vacuum

- *QCD sum rule for ρ and a_1 meson*

Jisu Kim, SHL: arXiv2012.06463 (PRD21)

Jisu Kim, SHL: arXiv2109.12791 (PRD22)

☞
$$\Pi^{\rho\rho} = \dots \frac{1}{Q^6}\left[-2\pi\alpha\left\langle\left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2\right\rangle - \frac{4\pi\alpha}{9}\left\langle\left(\sum_{ud}\bar{q}\gamma_\mu\lambda^a q\right)\left(\sum_{uds}\bar{q}\gamma_\mu\lambda^a q\right)\right\rangle\right]$$

$$\Pi^{a_1 a_1} = \dots \frac{1}{Q^6}\left[-2\pi\alpha\left\langle\left(\bar{q}\gamma_\mu\lambda^a\tau^3 q\right)^2\right\rangle - \frac{4\pi\alpha}{9}\left\langle\left(\sum_{ud}\bar{q}\gamma_\mu\lambda^a q\right)\left(\sum_{uds}\bar{q}\gamma_\mu\lambda^a q\right)\right\rangle\right]$$

$$\left\langle\left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2\right\rangle = \frac{1}{2}\left[\left\langle\left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2 + \left(\bar{q}\gamma_\mu\lambda^a\tau^3 q\right)^2\right\rangle\right]_S + \frac{1}{2}\left[\left\langle\left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2 - \left(\bar{q}\gamma_\mu\lambda^a\tau^3 q\right)^2\right\rangle\right]_B$$

$$\left\langle\left(\bar{q}\gamma_\mu\lambda^a\tau^3 q\right)^2\right\rangle = \frac{1}{2}\left[\left\langle\left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2 + \left(\bar{q}\gamma_\mu\lambda^a\tau^3 q\right)^2\right\rangle\right]_S - \frac{1}{2}\left[\left\langle\left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2 - \left(\bar{q}\gamma_\mu\lambda^a\tau^3 q\right)^2\right\rangle\right]_B$$

$$\propto \left(S(0) + i\gamma^5 S(0) i\gamma^5\right) \quad \pm \quad \left(S(0) - i\gamma^5 S(0) i\gamma^5\right) \propto \langle\bar{q}q\rangle^2$$

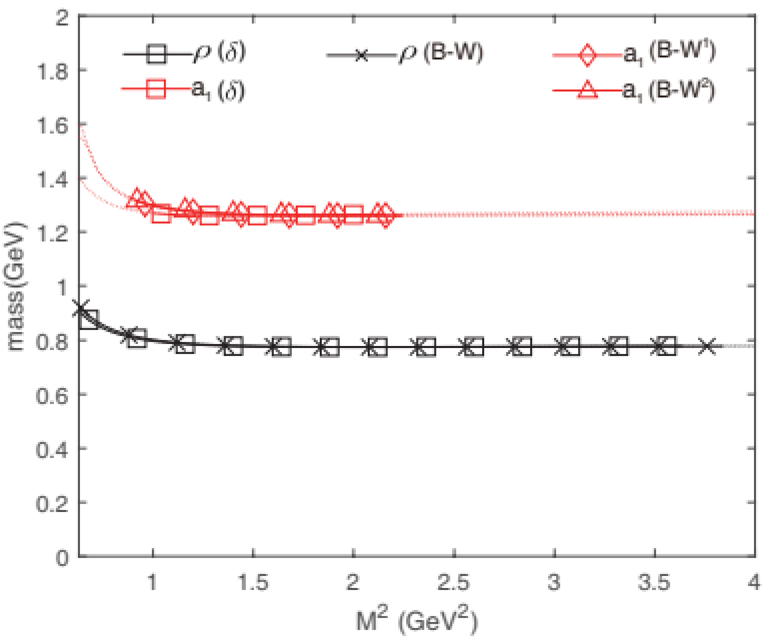
Chiral symmetric operator ± Chiral symmetry breaking operator

☞ $$\Pi^{\rho\rho} = \ldots \frac{1}{Q^6}\left[\frac{14}{9}\langle B\rangle + \langle S\rangle\right], \qquad \Pi^{a_1 a_1} = \ldots \frac{1}{Q^6}\left[-\frac{22}{9}\langle B\rangle + \langle S\rangle\right]$$

$$\langle B\rangle = -\pi\alpha\left\langle\left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2\right\rangle_B$$ Chiral symmetry order parameter

$$\langle S\rangle = -\frac{22\pi\alpha}{9}\left\langle\left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2\right\rangle_S$$ No relation to chiral symmetry breaking

☞ $\langle B\rangle$ and $\langle S\rangle$ can be determined separately from ρ and a_1 sum rules



Pole	$B\,(\mathrm{GeV}^6)$	$S\,(\mathrm{GeV}^6)$
δ	7.42×10^{-4}	5.65×10^{-4}
B-W^1	6.42×10^{-4}	6.05×10^{-4}
B-W^2	5.75×10^{-4}	7.11×10^{-4}

☞ $\Pi^{VV} = \ldots \frac{1}{Q^6}\left[\frac{14}{9}\langle B\rangle + \langle S\rangle\right]$, $\Pi^{AA} = \ldots \frac{1}{Q^6}\left[-\frac{22}{9}\langle B\rangle + \langle S\rangle\right]$

☞ When only chiral symmetry is restored : $\langle S\rangle \to$ fixed but $\langle B\rangle \to 0$

QCD sum rule analysis gives

$m_\rho = m_{a_1} = m_0 \sim 550 \pm 50$ MeV in the chiral symmetry restored vacuum

☞ QSR: masses of other hadrons when chiral symmetry is restored

Particle	$\bar{m}_{sym}$(MeV)
ρ, a_1	572.5 ± 27.5
ω	655 ± 15
f_1	1060 ± 30
K^*, K_1	545 ∓ 5

Particle	$\bar{m}_{sym}$(MeV)
N	525
Δ	600

Scalar nucleon mass in nuclear matter

$M^* > M + (S)$ where $S = -400$ MeV

☞ Origin of hadron masses in the chiral symmetry restored vacuum

In QCD sum rules

$$\left[\left\langle \left(\bar{q}\gamma_\mu\gamma^5\lambda^a\tau^3 q\right)^2 + \left(\bar{q}\gamma_\mu\lambda^a\tau^3 q\right)^2 \right\rangle\right]_S \propto \left(S(0)+i\gamma^5 S(0) i\gamma^5\right)$$

$$\boxed{\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = 12 \left\langle \sum_\lambda \rho(\lambda) \right\rangle}$$

$$cf.\quad \langle \bar{q}q \rangle = \xrightarrow{m=0} \left\langle \pi\rho(\lambda=0) \right\rangle$$

$\Rightarrow$ Other non perturbative effects in QCD:

scale breaking, confinement etc.

ϕ meson in nuclear medium

Partial restoration of Chiral symmetry in the strangeness sector

+ Other effects in medium

J.Kim, P. Gubler, SHL in preparation

- *QCD sum rules for the* ϕ *meson*

$$\widehat{\Pi}^{\mathrm{OPE}}(M^2) = c_0 + \frac{c_2}{M^2} + \frac{c_4}{M^4} + \frac{c_6}{M^6}$$

$$c_4^A(0) = \frac{1}{12}\left(1 + \frac{7}{6}\frac{\alpha_s}{\pi}\right)\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + 2m_s\left(1 + \frac{1}{3}\frac{\alpha_s}{\pi}\right)\langle \bar{s}s \rangle + A_2^s M_N - \frac{7}{12}\frac{\alpha_s}{\pi} A_2^g M_N$$

$$c_6^A(0) = -\pi\alpha_s\left[\left\langle (\bar{s}\gamma_\mu\gamma_5\lambda^a s)^2 \right\rangle + \frac{2}{9}\left\langle (\bar{s}\gamma_\mu\lambda^a s)\left(\sum_{q=u,d,s}(\bar{q}\gamma_\mu\lambda^a q)\right)\right\rangle\right] + \frac{m_s^2}{6}\left[\frac{1}{3}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle - 8m_s\langle \bar{s}s \rangle\right] - \frac{5}{6} A_4^s M_N^3$$

$$= \frac{7}{9}\langle B_{ss} \rangle_B \frac{\langle \bar{s}s \rangle_\rho^2}{\langle \bar{s}s \rangle^2} + \left\langle (\bar{s}\gamma_\mu\gamma_5\lambda^a s)^2 \right\rangle_{dis,S} + \langle S \rangle_S$$

Dominated by two contribution

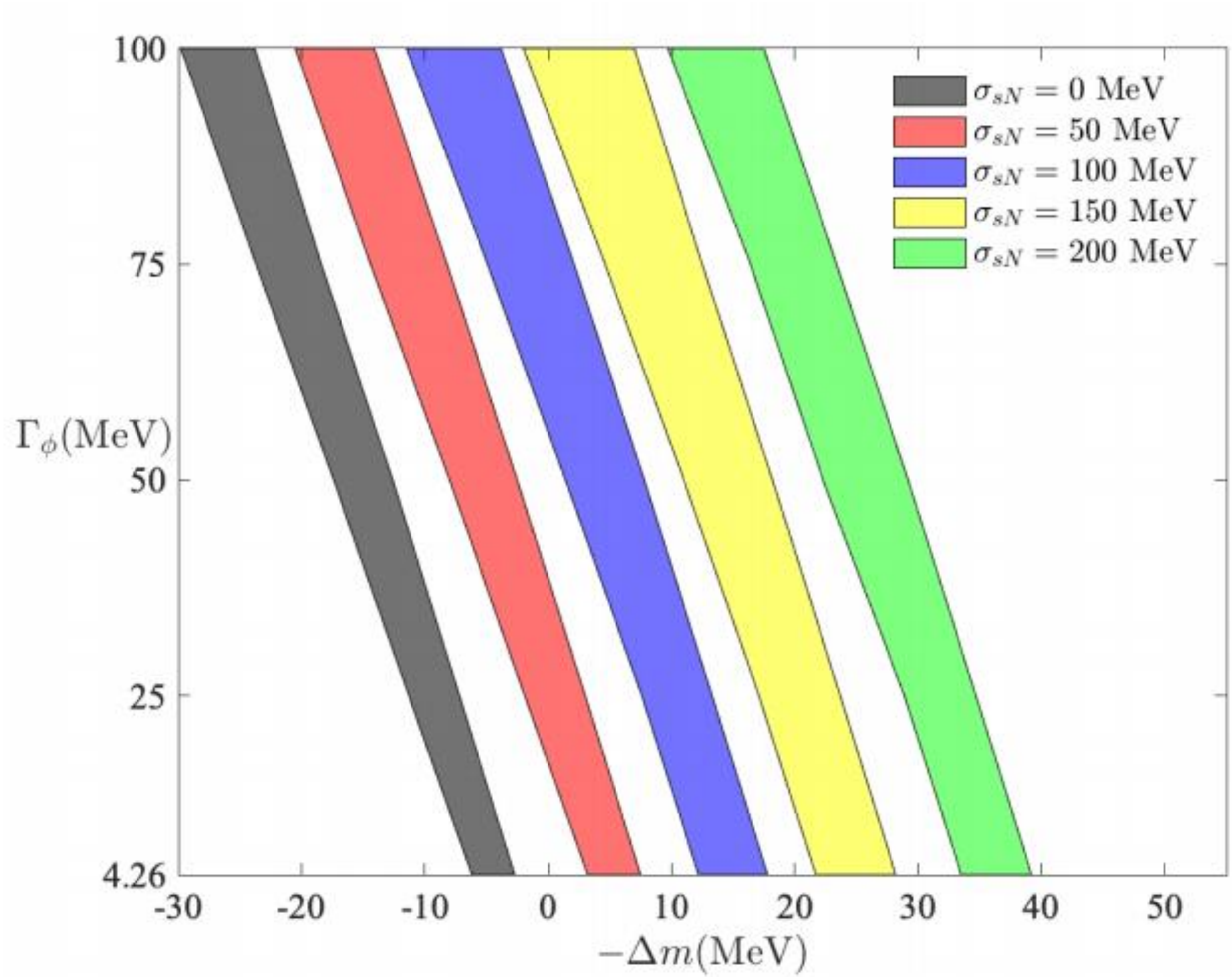
- $$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_\rho \simeq \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle \rho$$
$$= \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle - \frac{8}{9} (M_N - \sigma_{\pi N} - \sigma_{sN}) \rho,$$

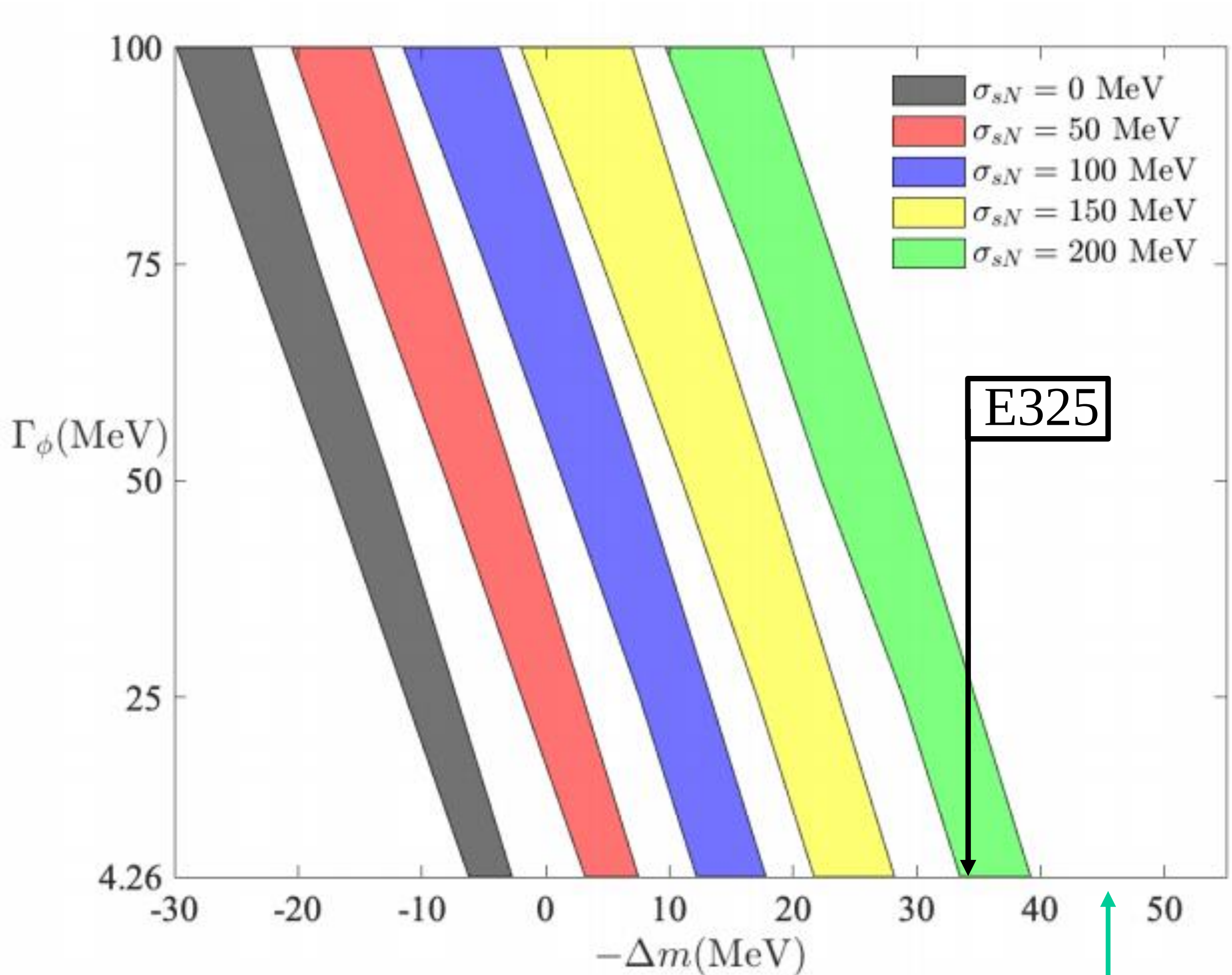
For $\sigma_{\pi N} = 45$ MeV $\sigma_{sN} = 100$ MeV, $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_\rho \approx 0.012 \mathrm{GeV}^4 \left(1 - 0.072 \frac{\rho}{\rho_0} \right)$

- $$\langle \bar{s} s \rangle_\rho \simeq \langle \bar{s} s \rangle + \langle N | \bar{s} s | N \rangle \rho = \langle \bar{s} s \rangle + \frac{\sigma_{sN}}{m_s} \rho,$$

For $\sigma_{sN} = 100$ MeV, $\langle \bar{s} s \rangle_\rho \approx -(0.246)^3 \left(1 - 0.083 \frac{\rho}{\rho_0} \right)$

QCD sum rules constraints for the ϕ mass and width change in nuclear medium





77 MeV

[ALICE], [arXiv:2105.05578 [nucl-ex]].

$$V_{\text{Gaussian}}(r) = -V_{\text{eff}} \exp(-\mu r^2)$$

$$E_{int} = \int d^3x \int d^3x' \rho_\phi(x) V_{\text{Yukawa}}(x - x') \rho_p(x')$$

$$V_{\text{Yukawa}}(r) = -\frac{A}{r} \exp(-\alpha r)$$

Summary

1. *Confinement, chiral symmetry breaking, $U_A(1)$ effects all have different origin and contribute to hadron mass*

2. *Mass difference between chiral partners are directly related to chiral symmetry breaking*

   $\rho - a_1$, $K^* - K_1$ (small width and can be measured at JPARC)

3. *Still, separating the 4-quark operators into chiral symmetric and breaking operators in QCD sum rules, one can identify the mass in the chiral symmetry restored vacuum.*

   $\rho,\ a_1,\ K^*,\ K_1,\ \phi,\ f_1, \ldots\ldots$

4. *Combining theory and Experimental observation of ϕ one can identify contribution of chiral symmetry breaking to hadron mass*

   Looking forward to E16 results