

Constraining the properties of a parity doublet model with a quark-hadron-crossover equation of state and the observations for neutron stars

based on our papers:

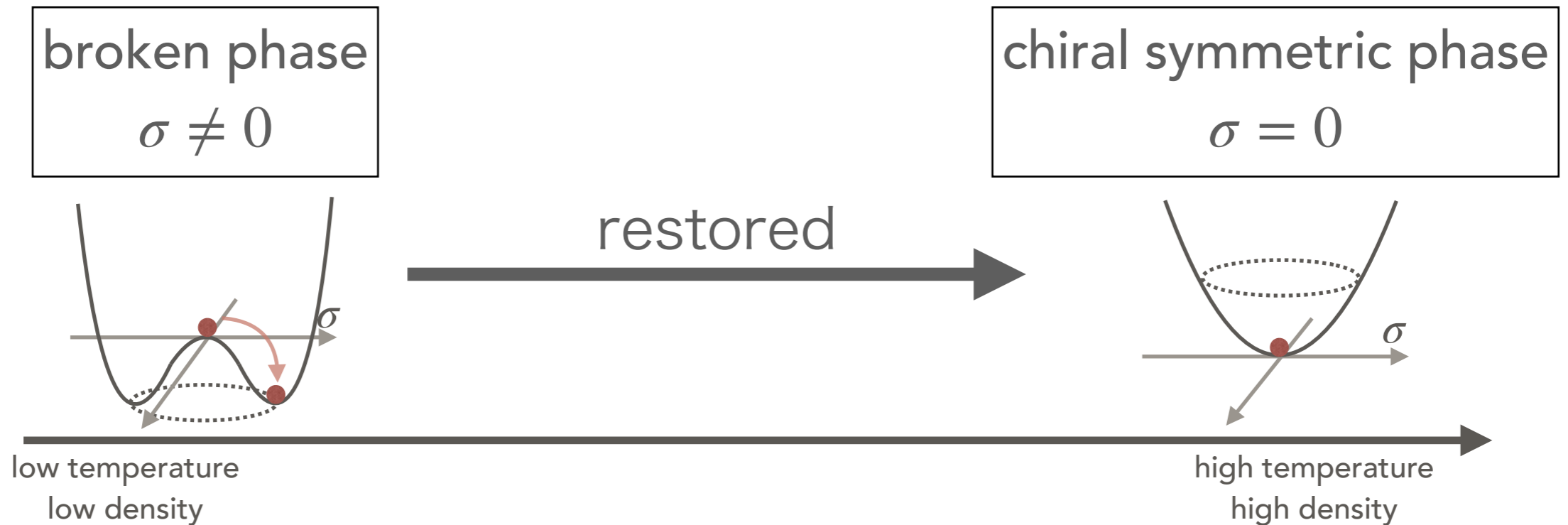
[1] PhysRevC.103.045205 (T.M., M.H., T.K.)

[2] PhysRevC.104.065201 (T.M., M.H., T.K.)

T. Minamikawa (Nagoya Univ.)

collaborators: M. Harada (Nagoya Univ.) and T. Kojo (Tohoku Univ.)

Fate of Nucleon Mass



- In effective models, e.g. the linear-sigma model, $M_N \propto \sigma$

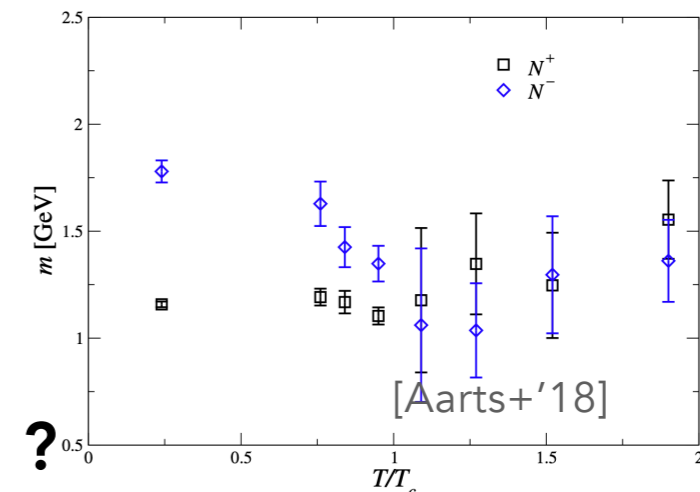
$$M_N \neq 0 \xrightarrow{\sigma \rightarrow 0} M_N \rightarrow 0 ?$$

- On the other hand, lattice QCD at finite T (e.g. e.g. Aarts+'15, '18):

$\sigma \rightarrow 0$ but M_N remains ~ 1 GeV

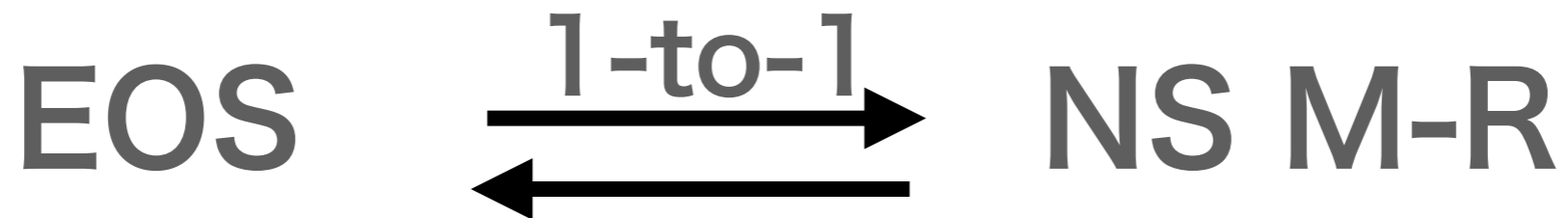
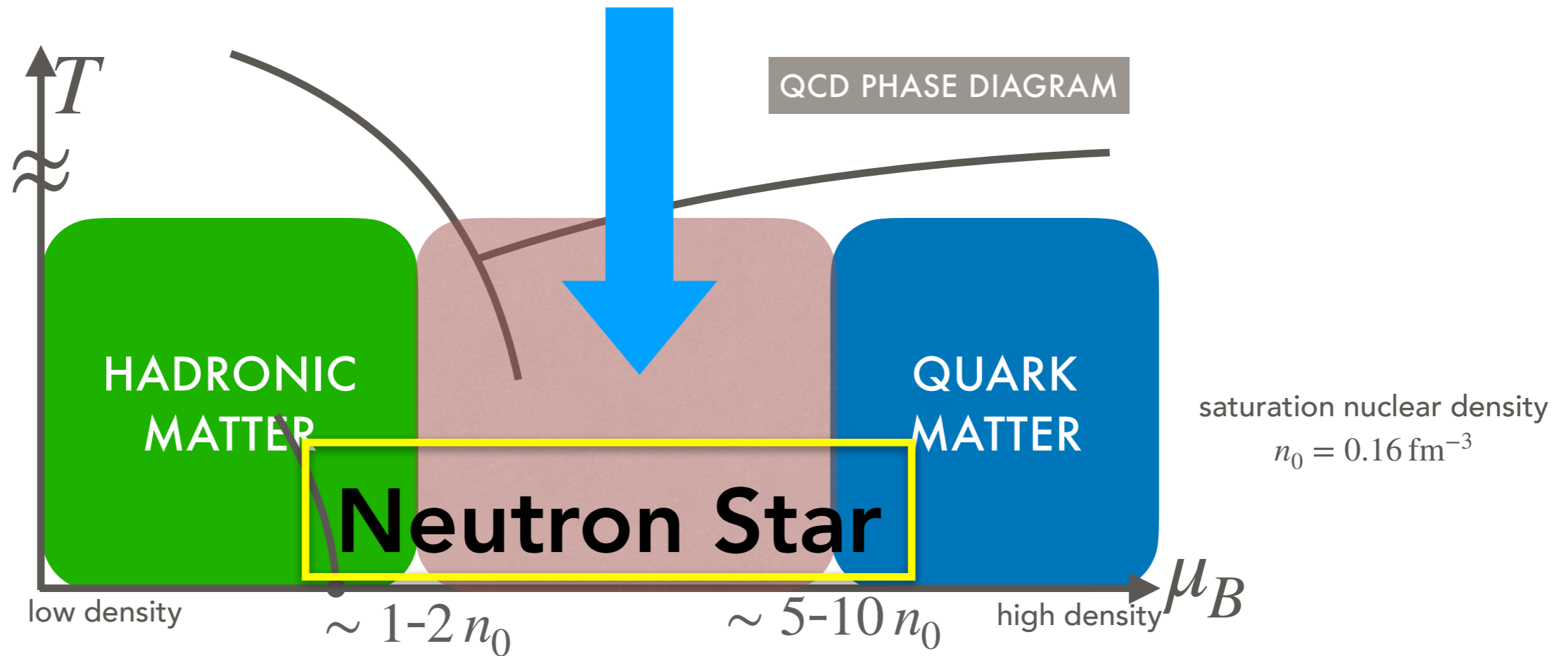
- M_N are not very sensitive to the environment ?

relevant for the physics of heavy ion collisions and Neutron Stars (NSs)



NSs as Cosmic Laboratories^{3 / 13}

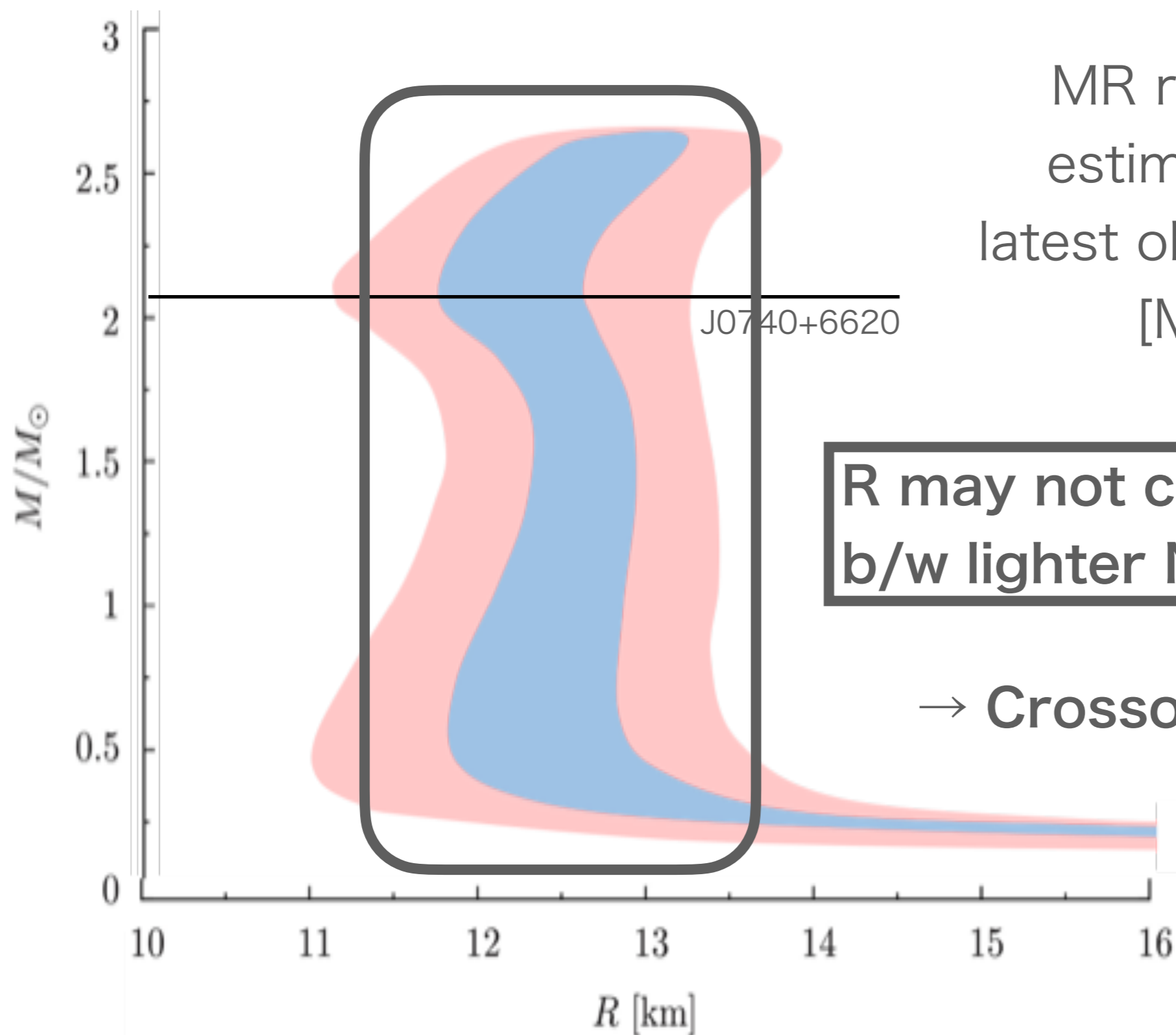
fate of nucleons? chiral?



constraints on the microphysics

Latest Observ. of NSs

[Miller+ (2021)]

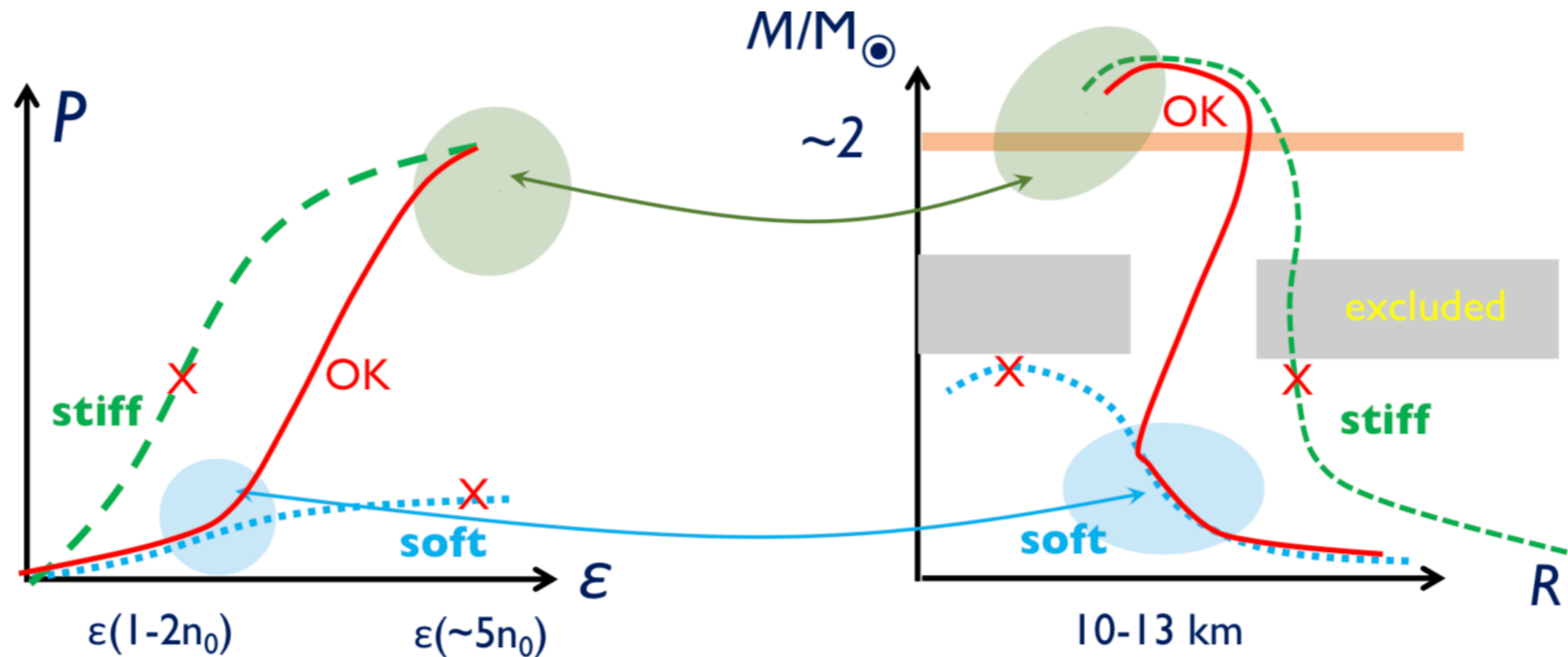


MR relation of NSs
estimated from the
latest observational data
[Miller+, '21]

**R may not change radically
b/w lighter NSs & heavier NSs**

→ **Crossover is preferred**

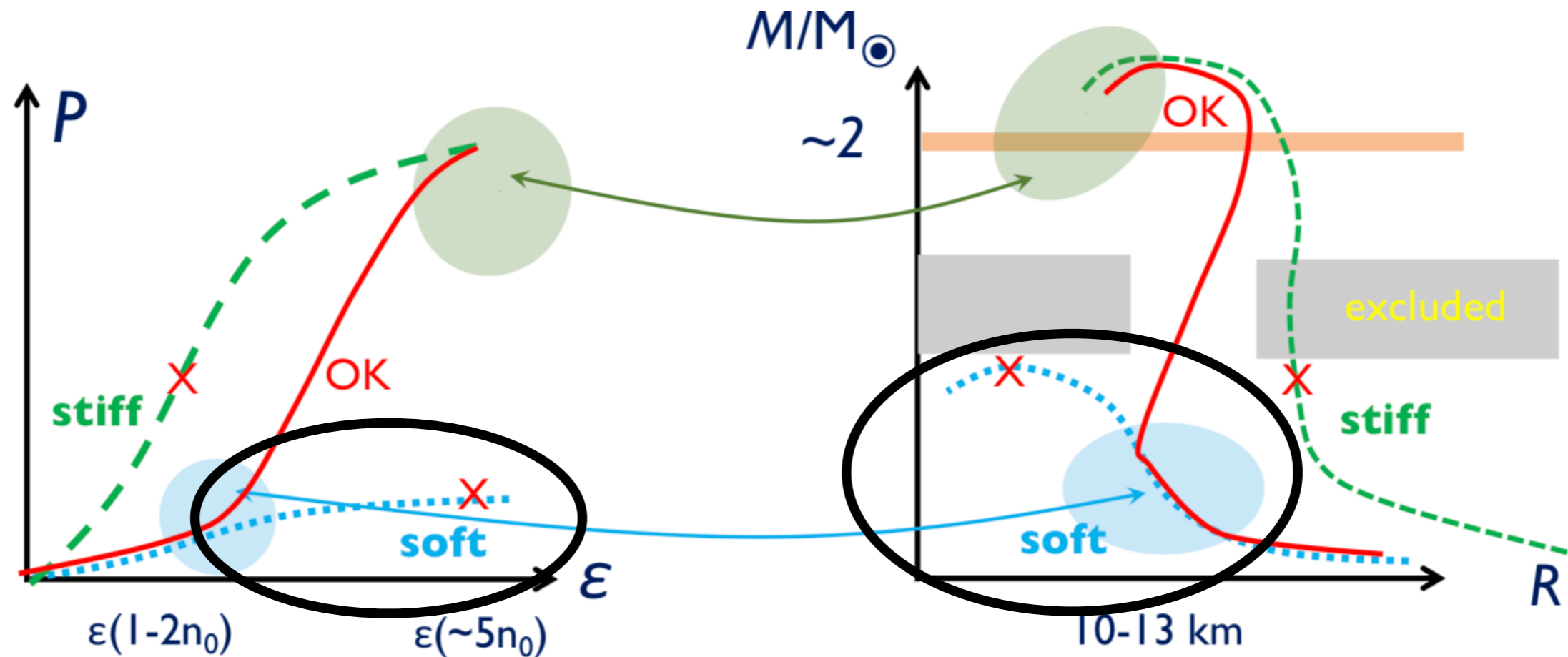
EOS vs M-R relation



considering NSs data, a recent trend of EOS is: from soft to stiff

→ **crossover phase transition from hadrons to quarks**
 (1st transition change a radius of NS radically.)

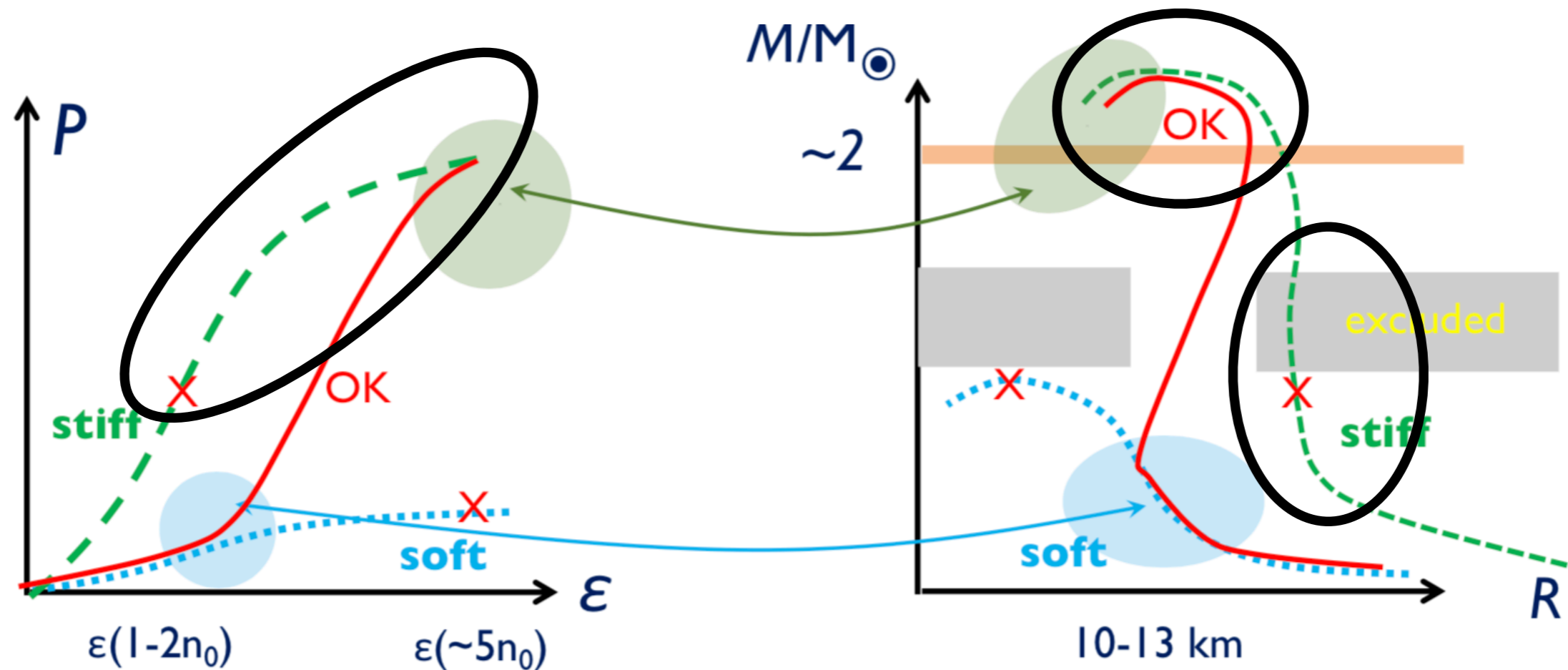
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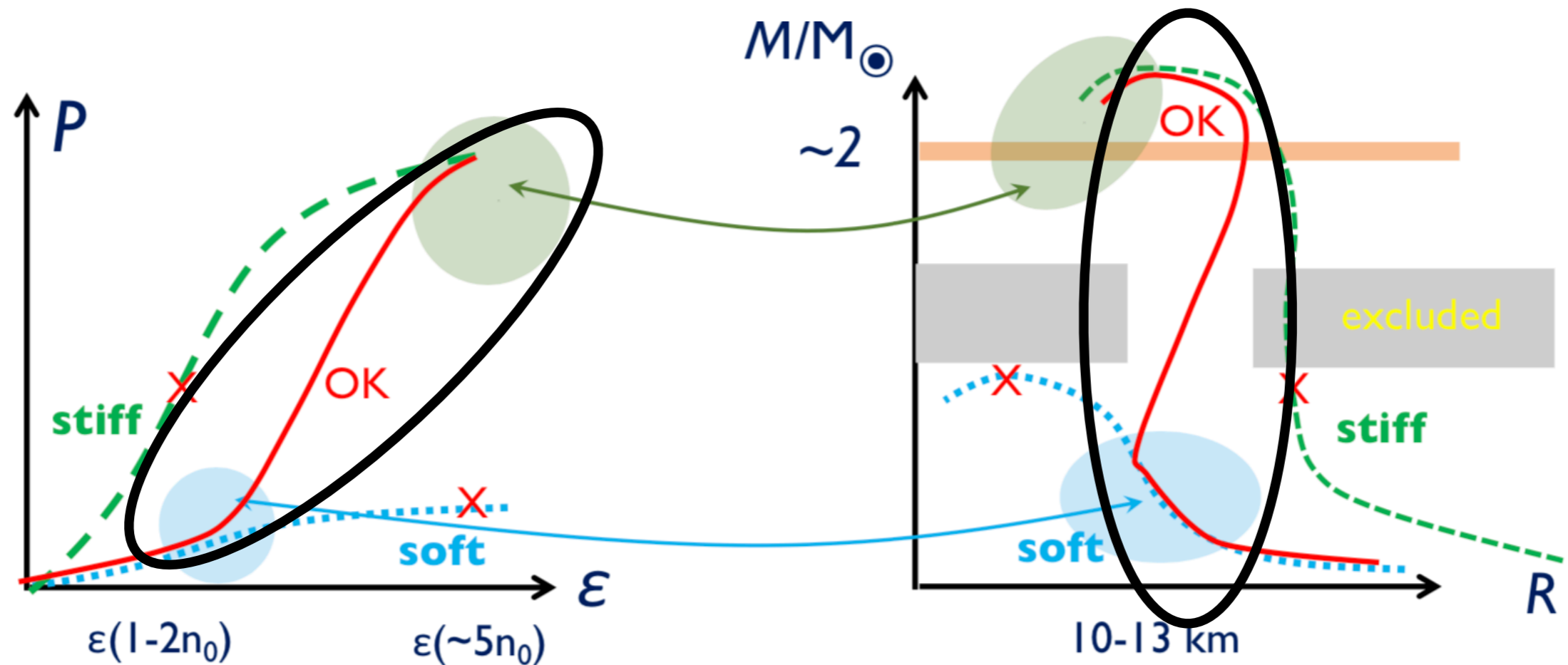
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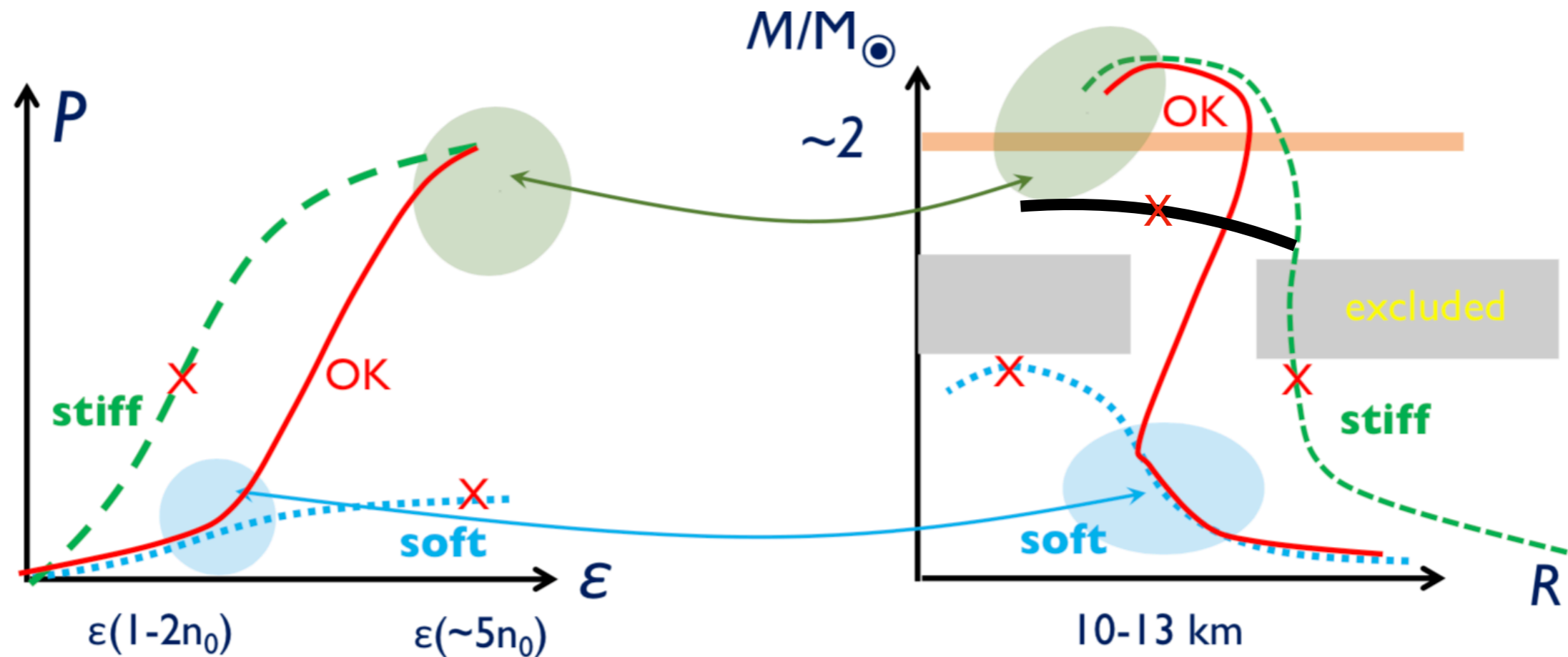
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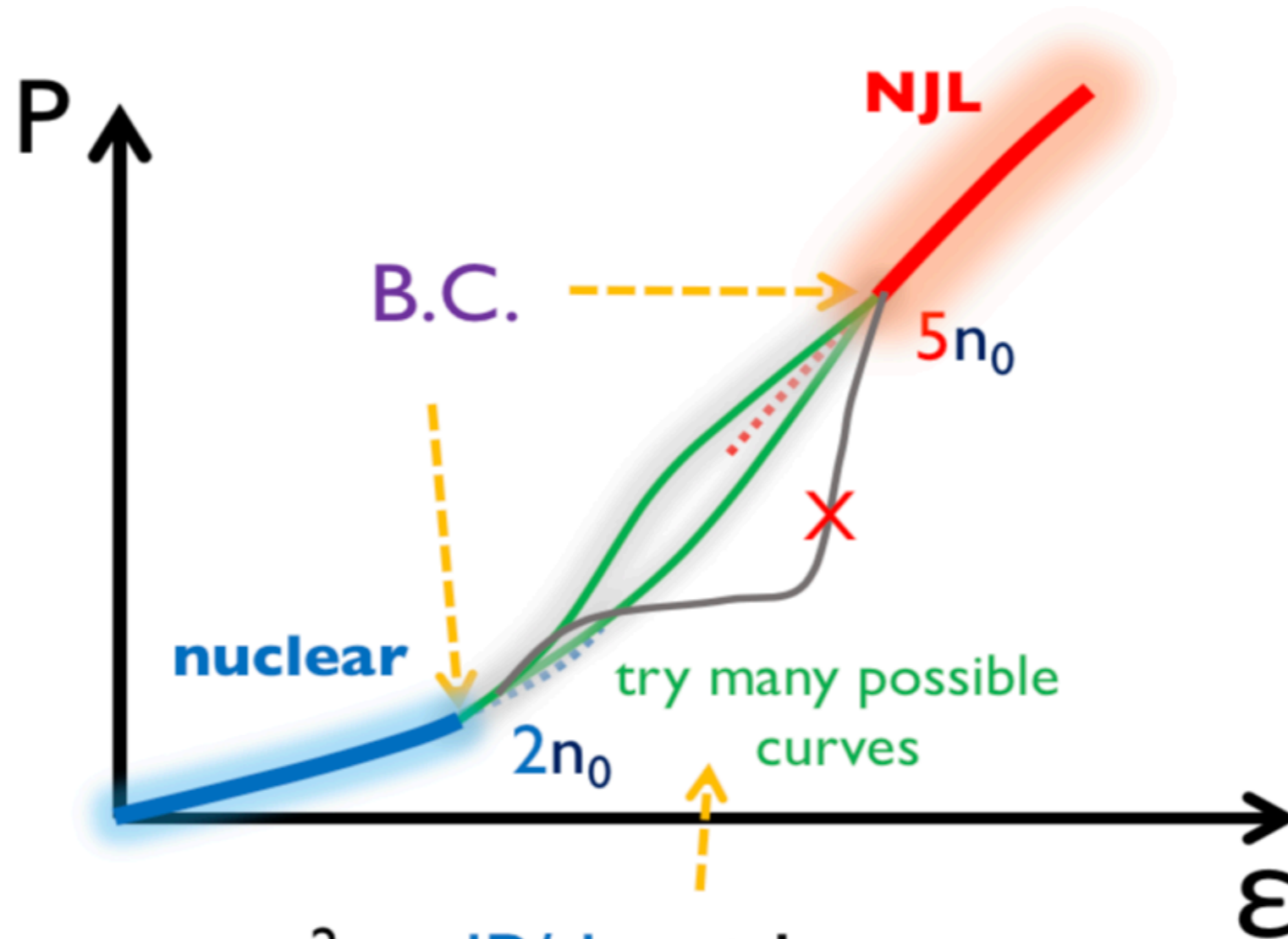


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Crossover Model for Unified EOS

3-window (Masuda+ '11; ...)



$$c_s^2 = dP/d\varepsilon < 1 \quad (\text{causality})$$

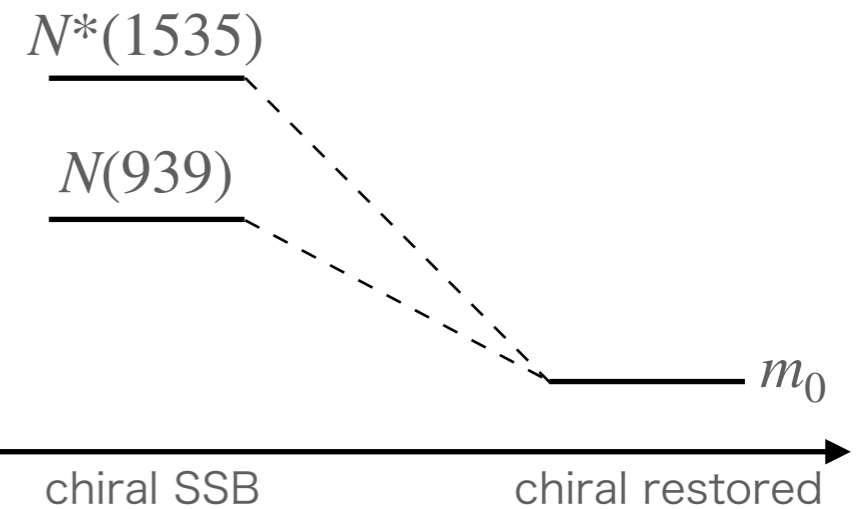
→ removes unphysical curves

quark & nuclear EOS constrain each other

Parity Doublet Model for Nucleons

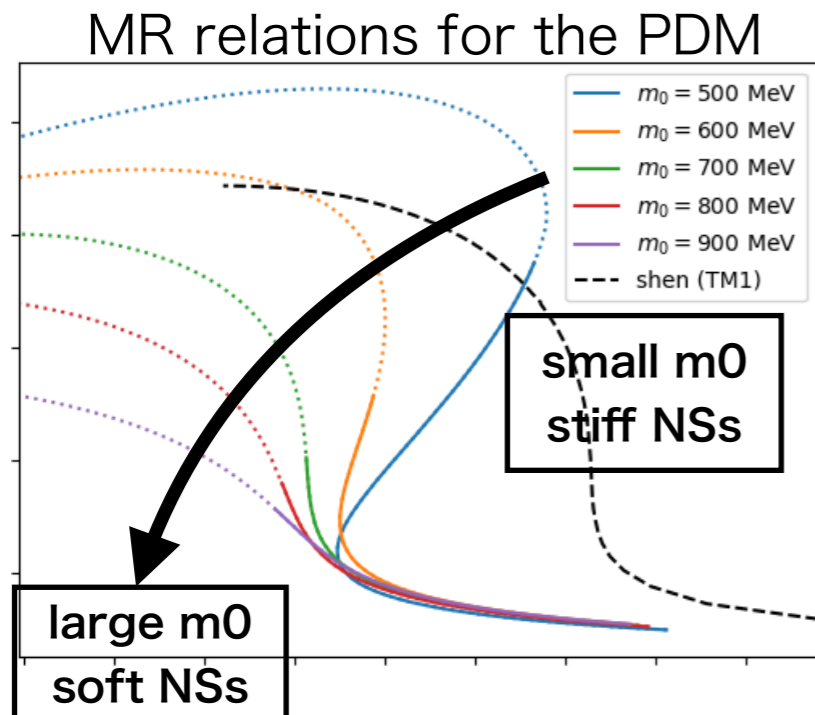
$$\mathcal{L}_{\text{PDM}} = \mathcal{L}_{\text{Nucleon}} + \mathcal{L}_{\text{Meson}}(\sigma, \omega, \rho)$$

$$m_0(\bar{\psi}_1 \gamma_5 \psi_2 - \bar{\psi}_2 \gamma_5 \psi_1) \quad m_0: \text{the chiral invariant mass}$$



nucleons $N(939)[N_+]$ and $N^*(1535)[N_-]$ are degenerate

$$M_{N_{\pm}} = \sqrt{m_0^2 + g_+^2 \sigma^2} \mp g_- \sigma \xrightarrow{\sigma \rightarrow 0} m_0 (\gtrsim 500 \text{ MeV})$$



large $m_0 \longleftrightarrow$ soft EOS

- if m_0 is large \rightarrow small coupling b/w σ & nucleons
- \rightarrow small coupling for omega (balance b/w σ & ω)
- \rightarrow small interaction, soft EOS, light&small NS

NJL Model for Quarks

$$\mathcal{H} = \mathcal{H}_{\text{NJL}} - \underbrace{H(q^T \Gamma_A q)}_{\text{color-mag.}} (\bar{q} \Gamma^A q)^T + \underbrace{g_V}_{\text{nB-nB int.}} (\bar{q} \gamma^0 q)^2$$

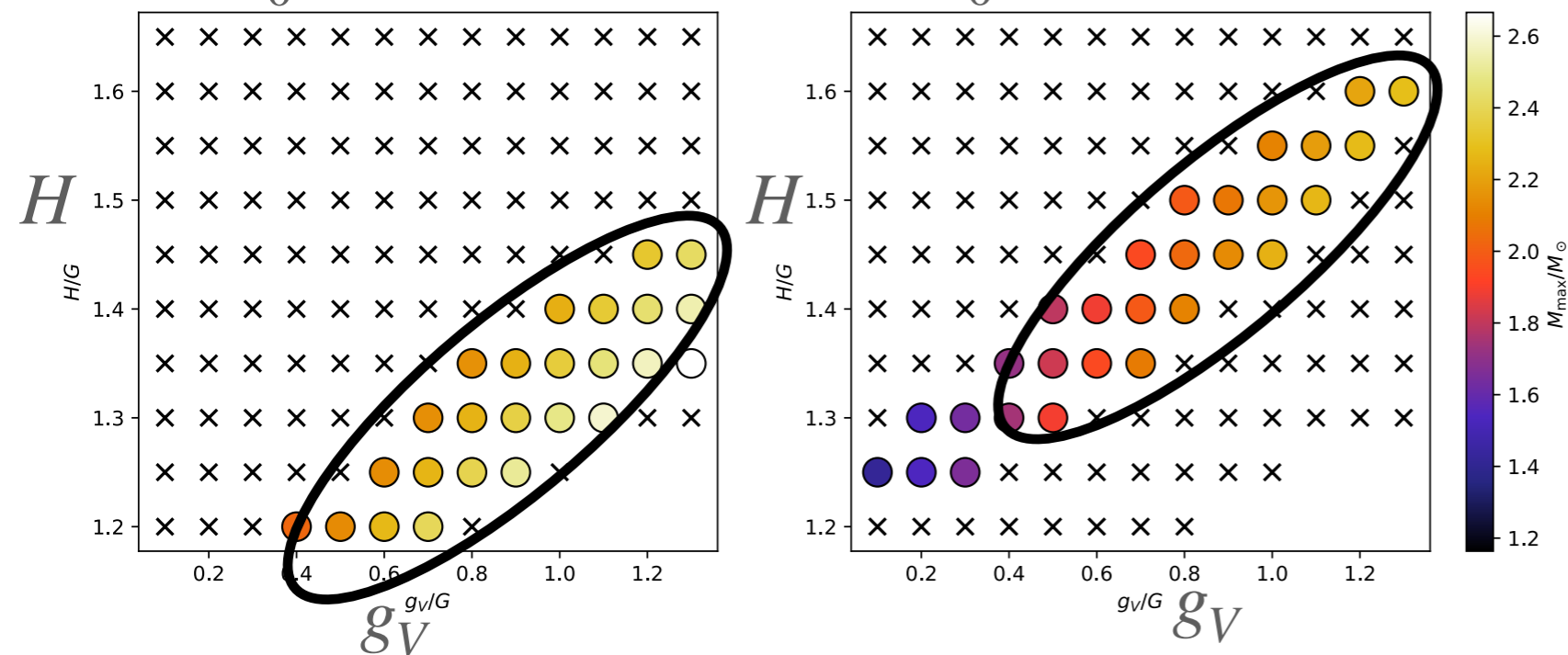
NJL: parameters of Hatsuda-Kunihiro

(H,gV): not well-constrained before

→ survey wide range for given nuclear EOS + NS constraints

stiffer PDM
 $m_0 = 500 \text{ MeV}$

softer PDM
 $m_0 = 800 \text{ MeV}$



x: not
causal

blue:
too light

causality + M_max

**m0 ↔ (H,gV)
constrain each other**

can examine microphysics

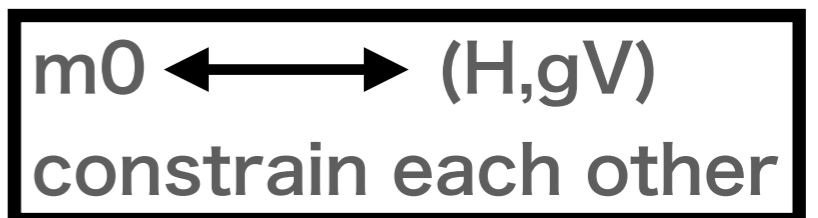
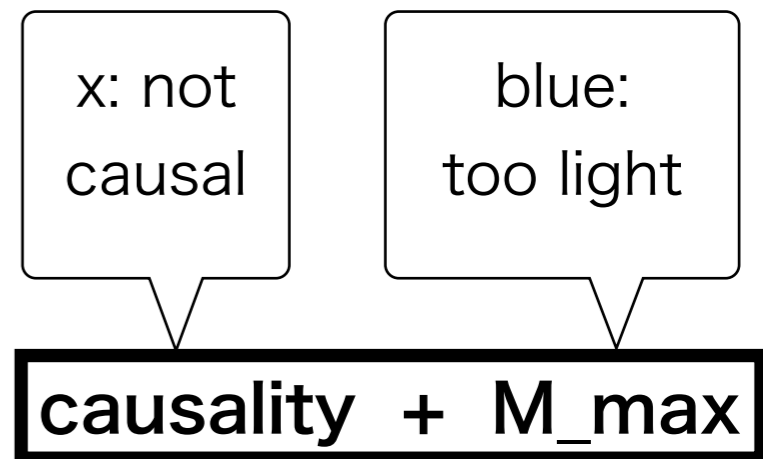
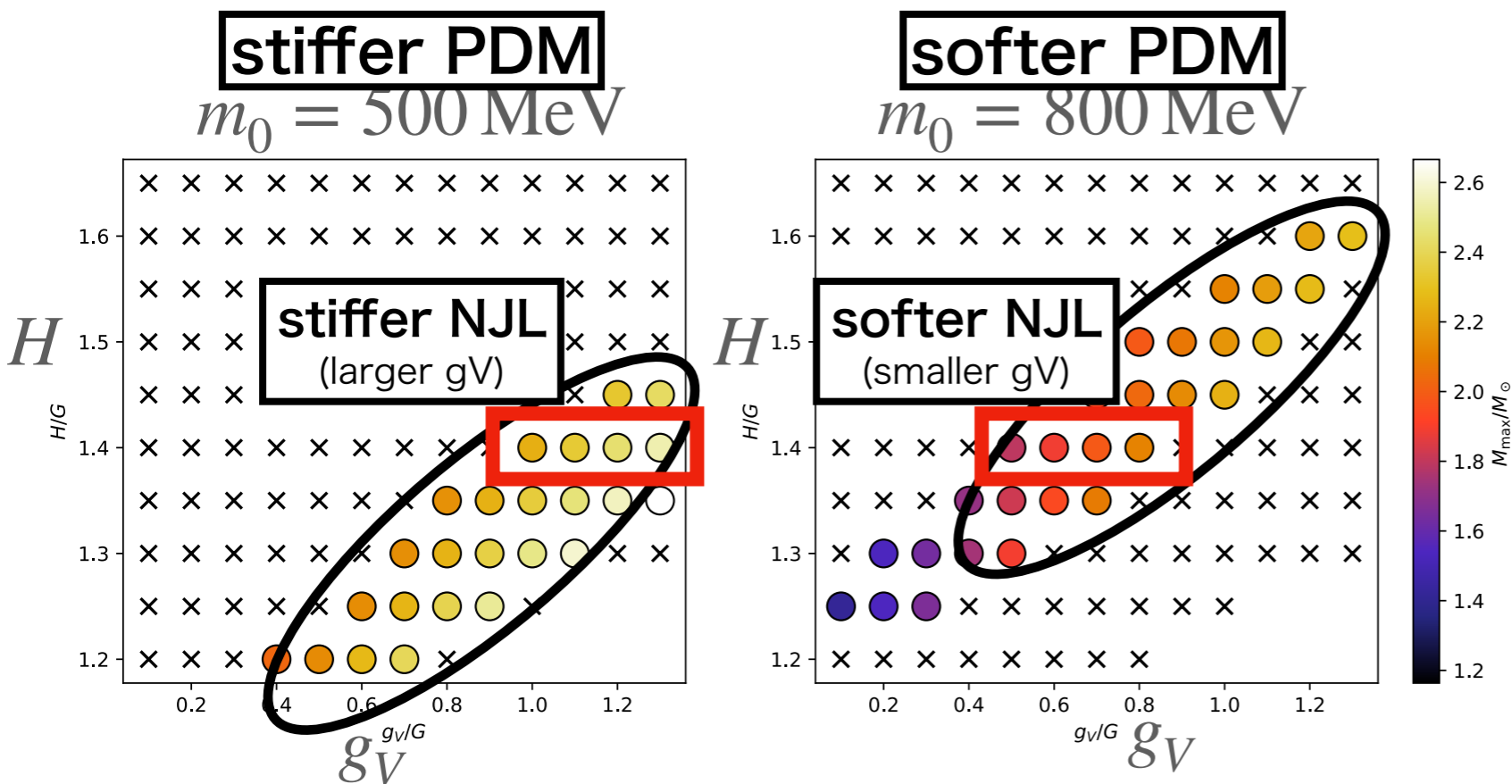
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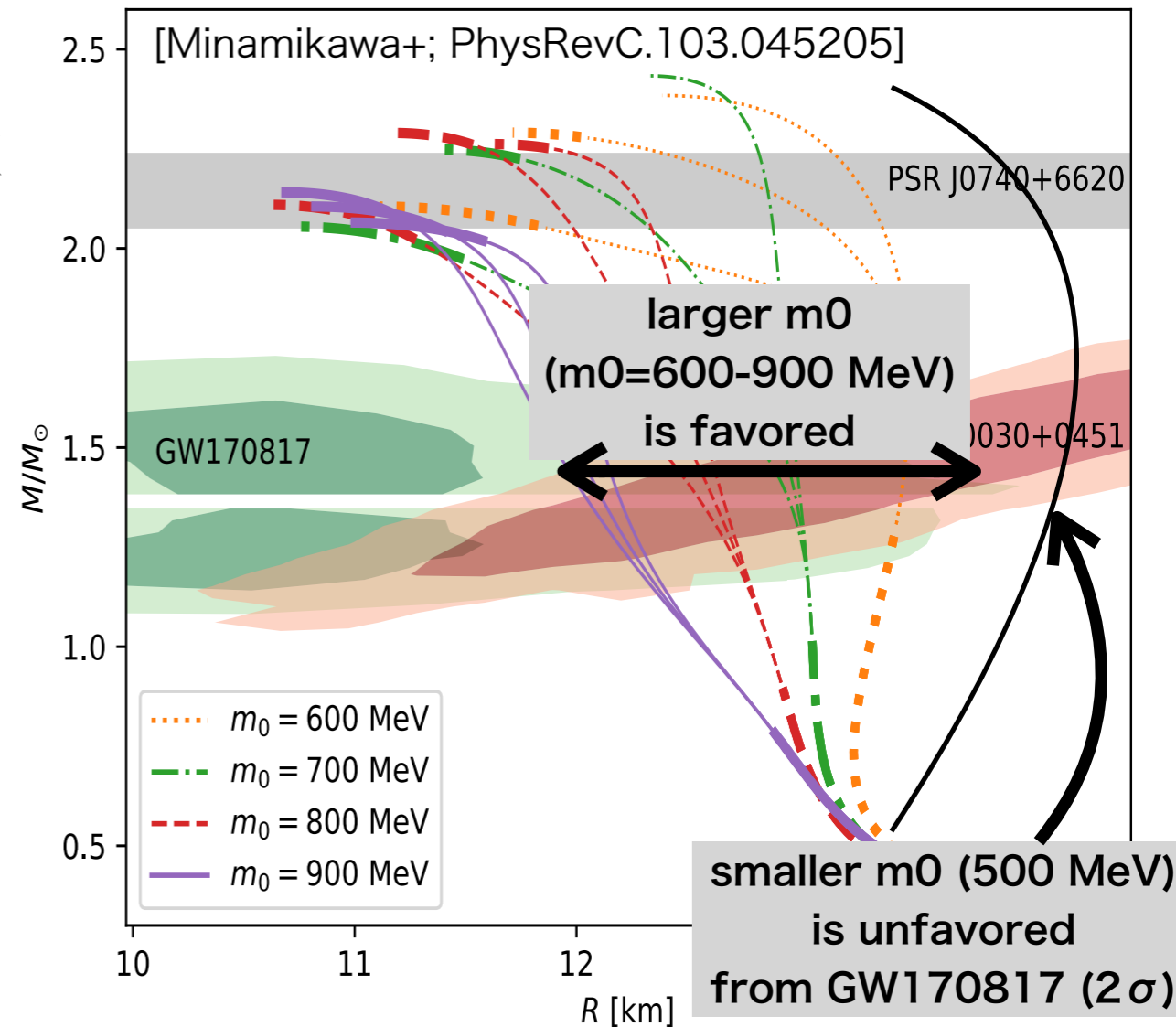
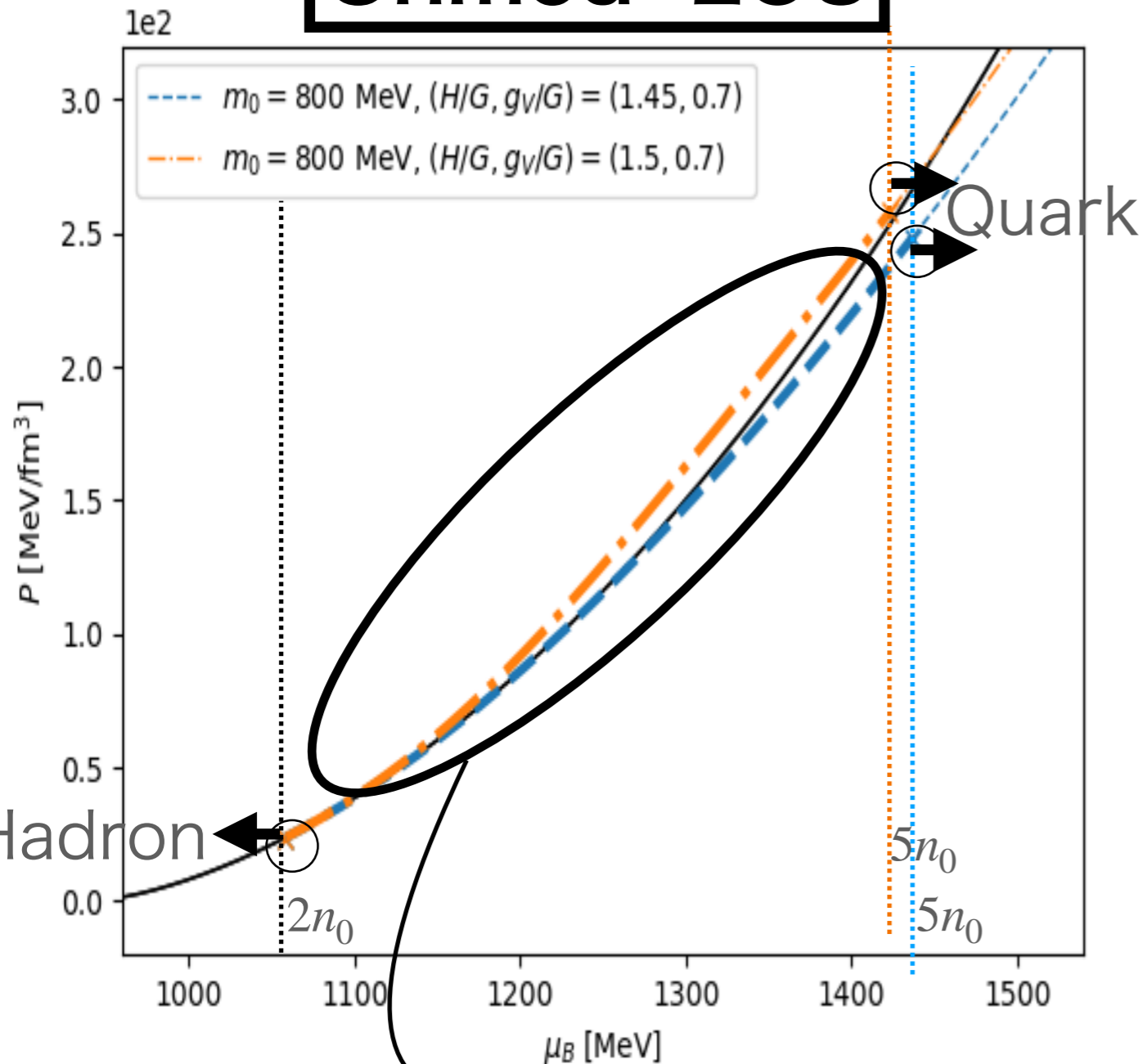
can examine microphysics

Unified EOS and its MR

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Unified EOS

MR relation

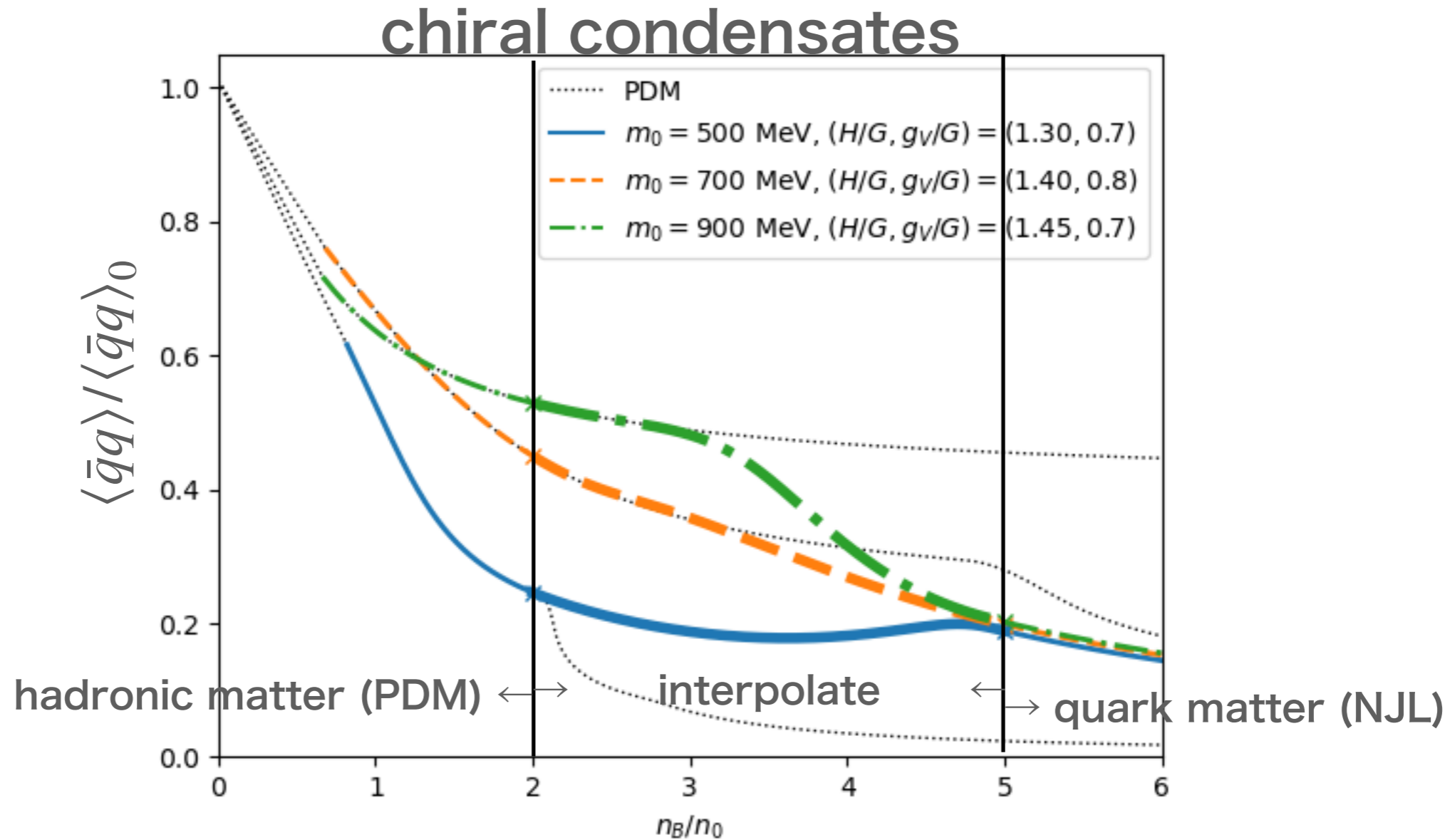


interpolate w/ a polynomial:
$$P(\mu_B) = \sum_{n=0}^5 c_n \mu_B^n$$

(six) Boundary Conditions \rightarrow (six) coefficients c_n

Microphys. in Crossover

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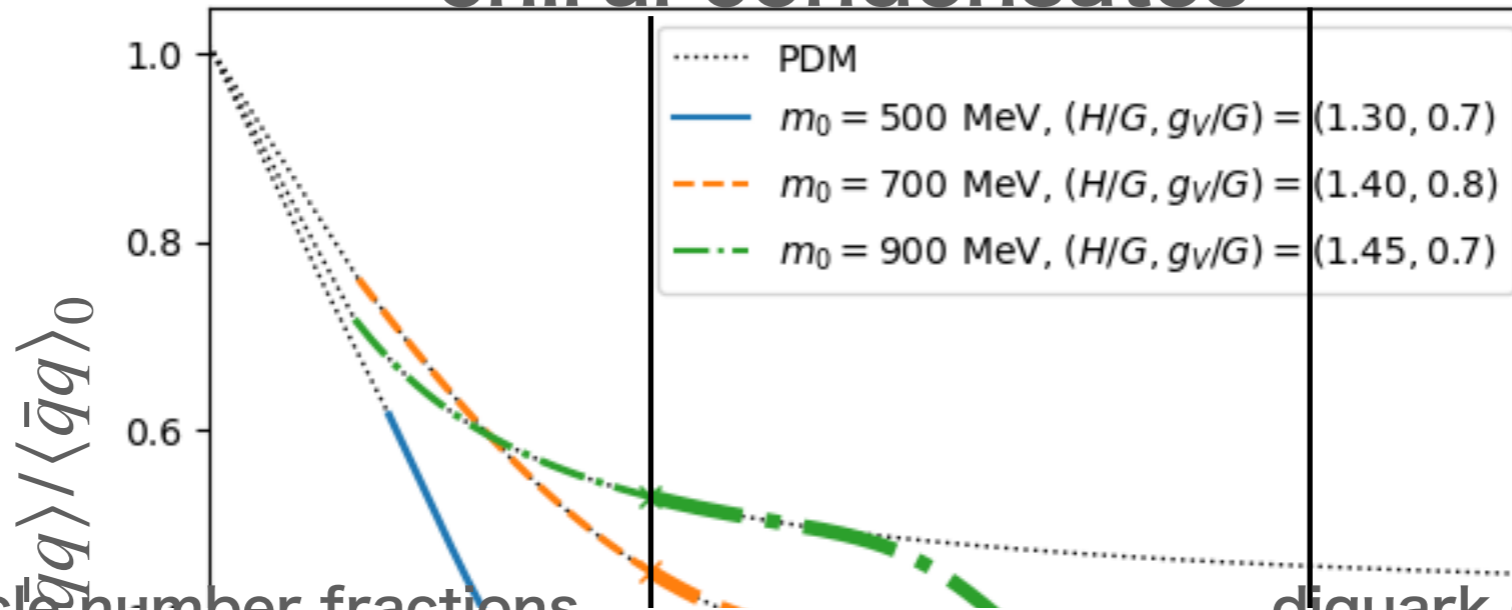
$$\langle \bar{q}q \rangle = \frac{\partial \Omega}{\partial m_q} \quad (\text{Hellmann-Feynman Thm.})$$

$$\text{extend } P(\mu_B) = \sum a_n \mu_B^n \rightarrow P(\mu_B, m_q) = \sum a_n(m_q) \mu_B^n$$

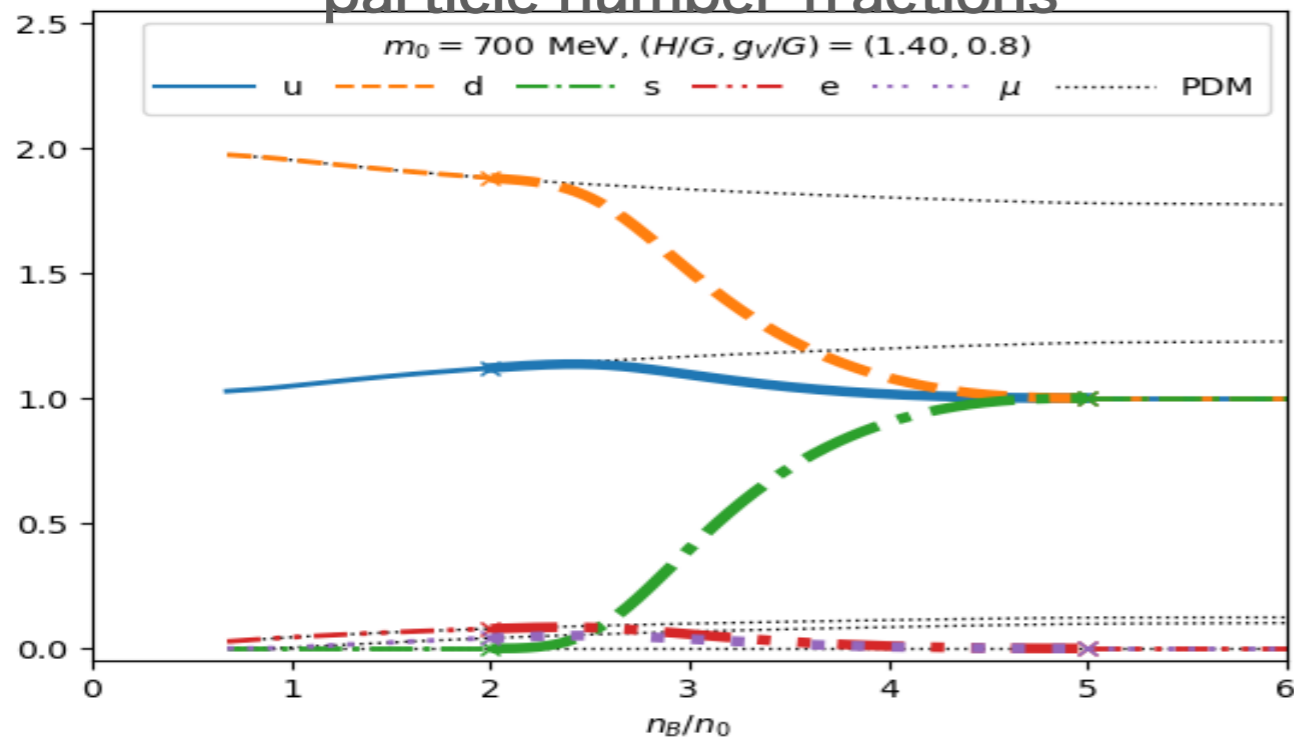
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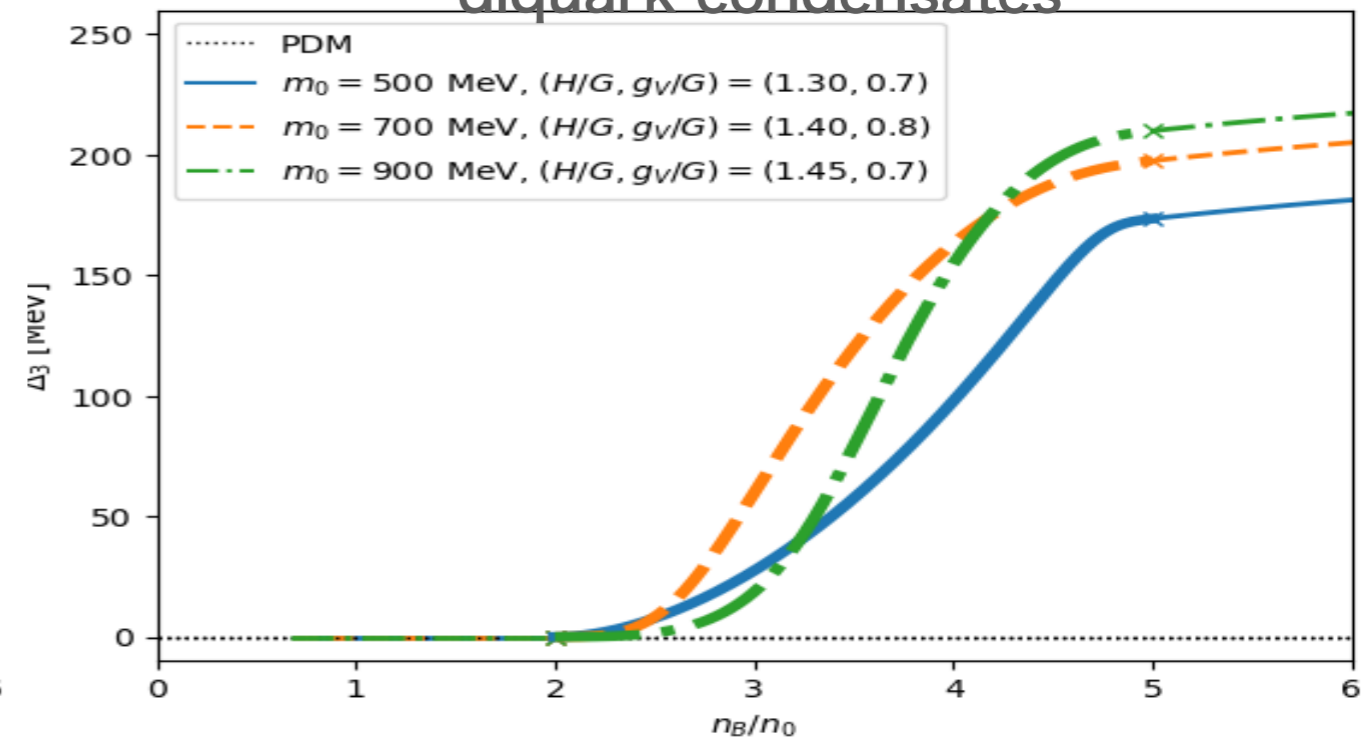
chiral condensates



particle number fractions



diquark condensates

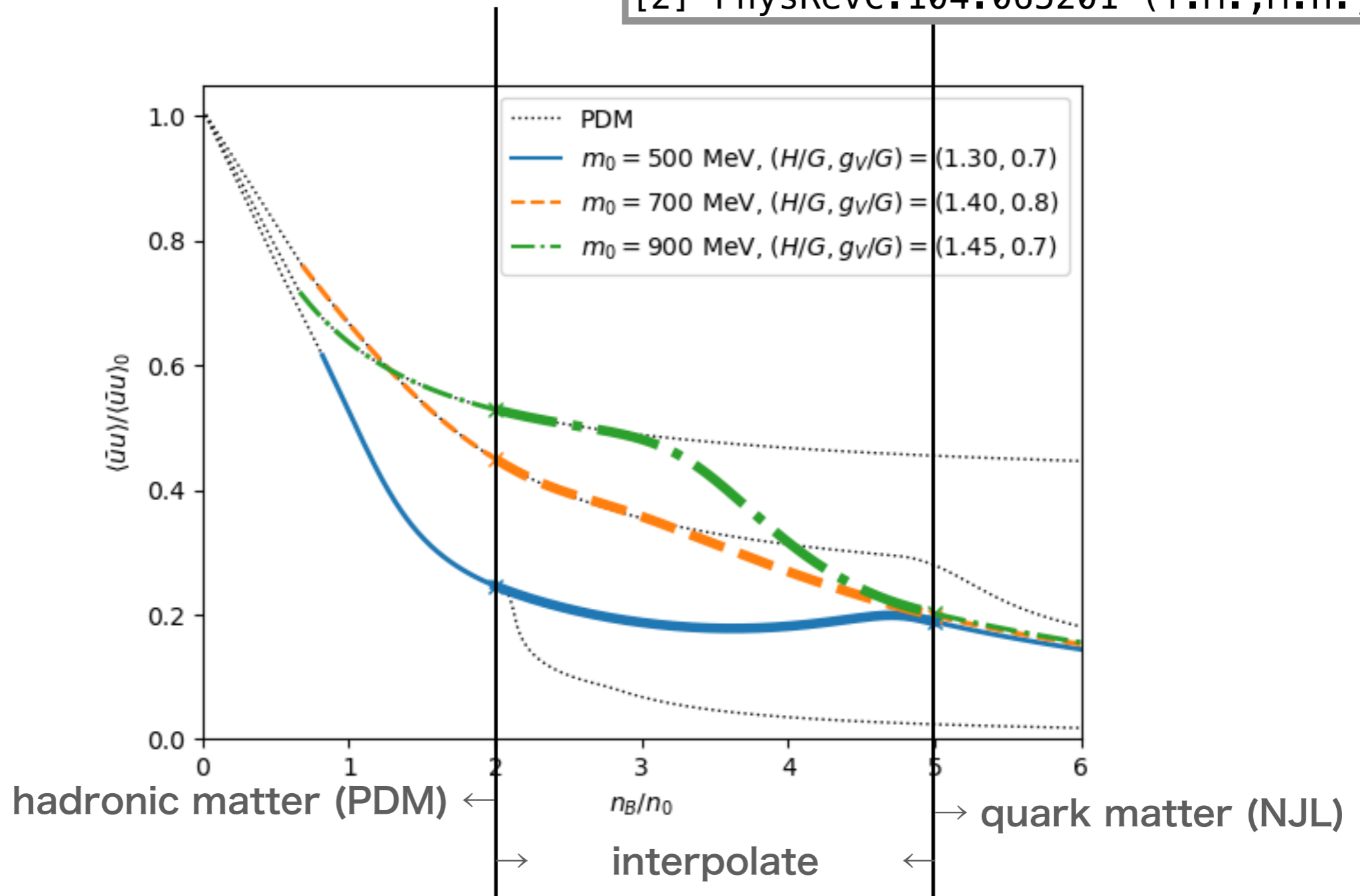


qualitative guideline
to extend models in the future

$$\mu_B^n$$

Unified $\langle \bar{q}q \rangle$

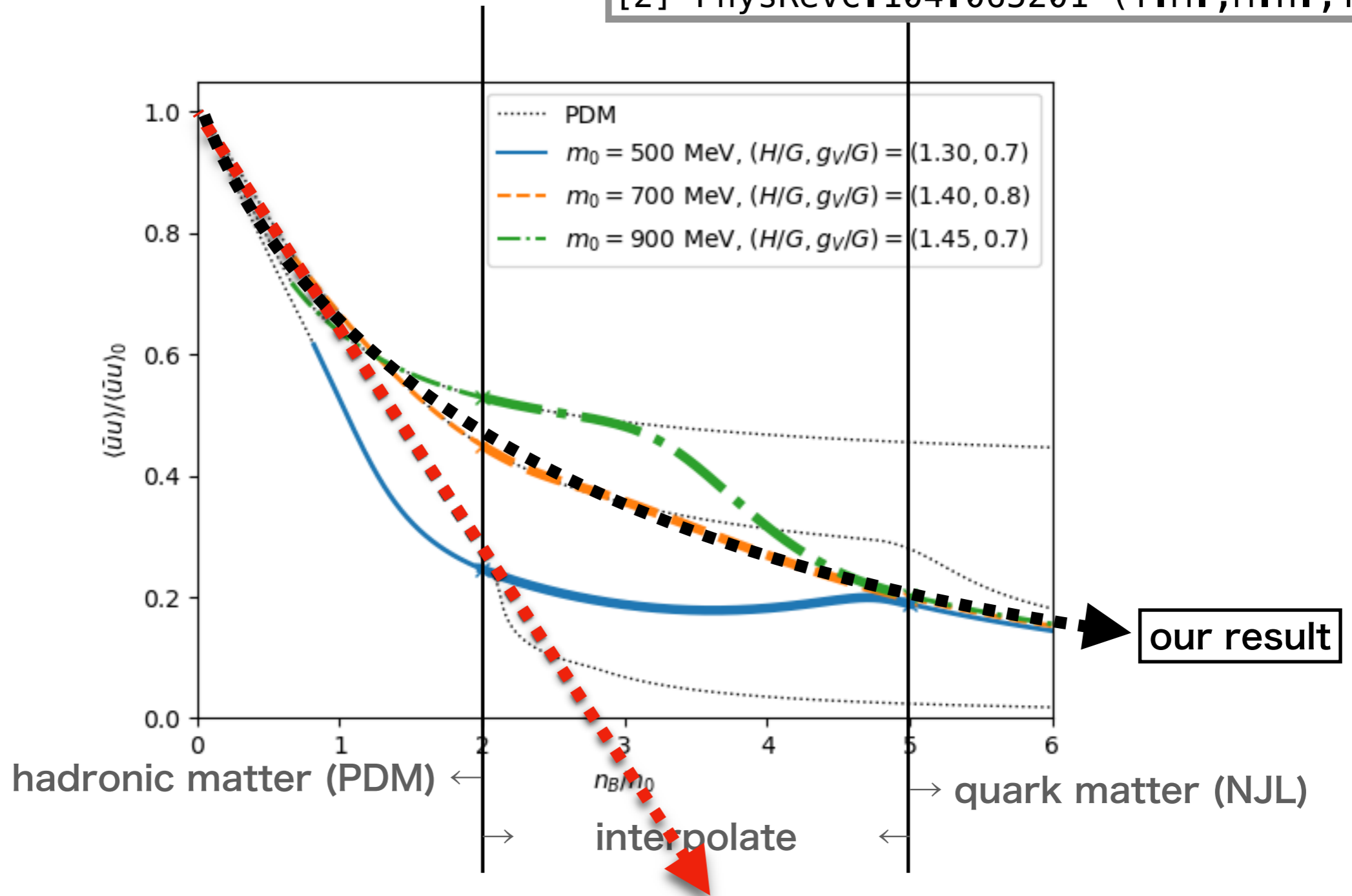
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causality → match w/ both Hadronic & Quark matter

Unified $\langle \bar{q}q \rangle$

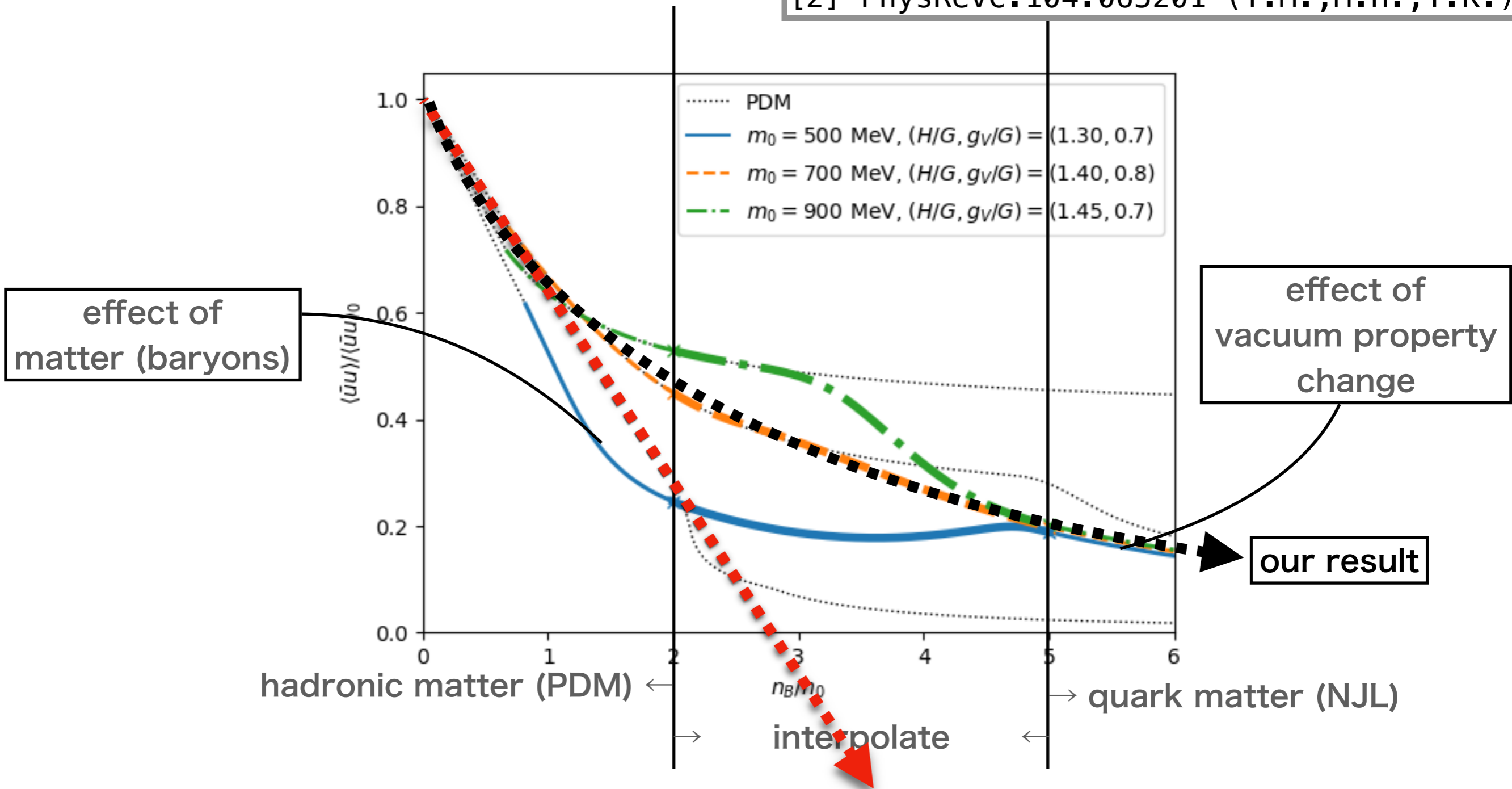
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causality \rightarrow match w/ baryon matter **cf: Linear Density Approx. or (non chiral) QHD** quark matter

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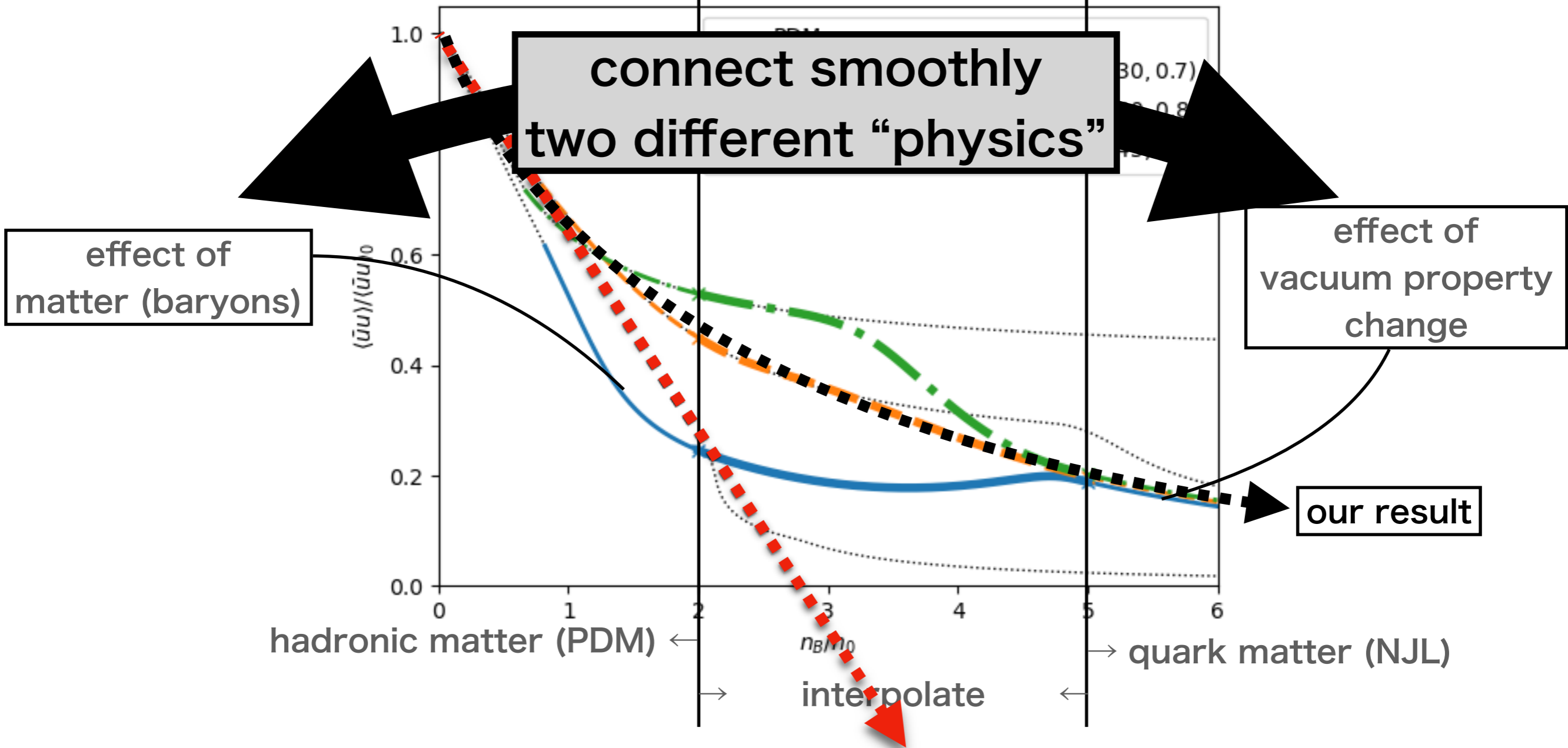
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Inhomogeneous $\langle \bar{q}q \rangle$

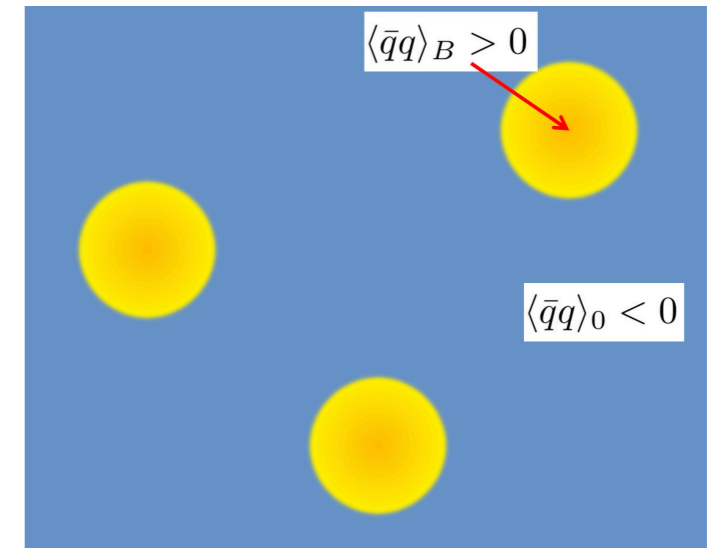
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Linear Density Approx. inside a nucleon

$$\langle \bar{q}q \rangle \approx - (0.25 \text{ GeV})^3 + N_{\text{scalar}} \frac{N_{\text{baryon}}}{V_{\text{baryon}}} > 0$$

↑
opposite signs
& similar magnitudes

≈ (0.3 GeV)³



Inhomogeneous $\langle \bar{q}q \rangle$

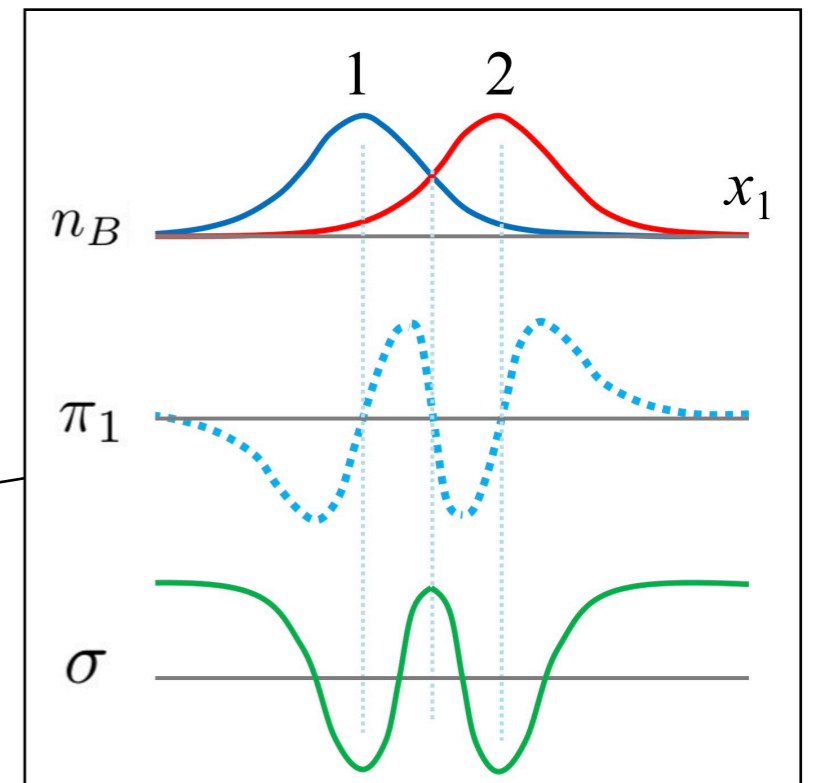
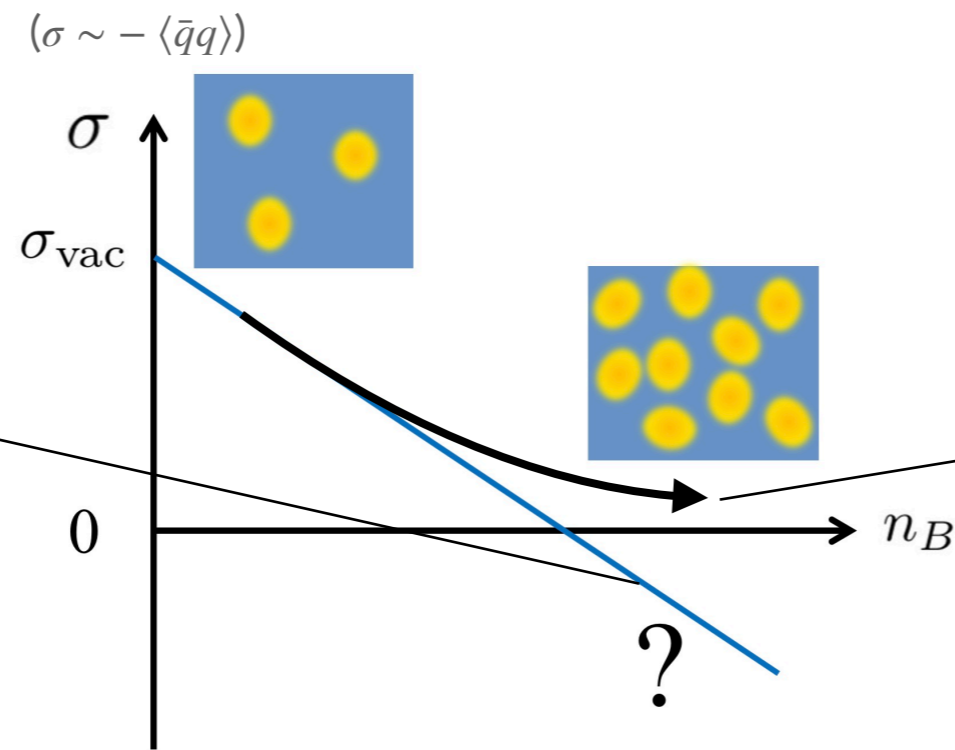
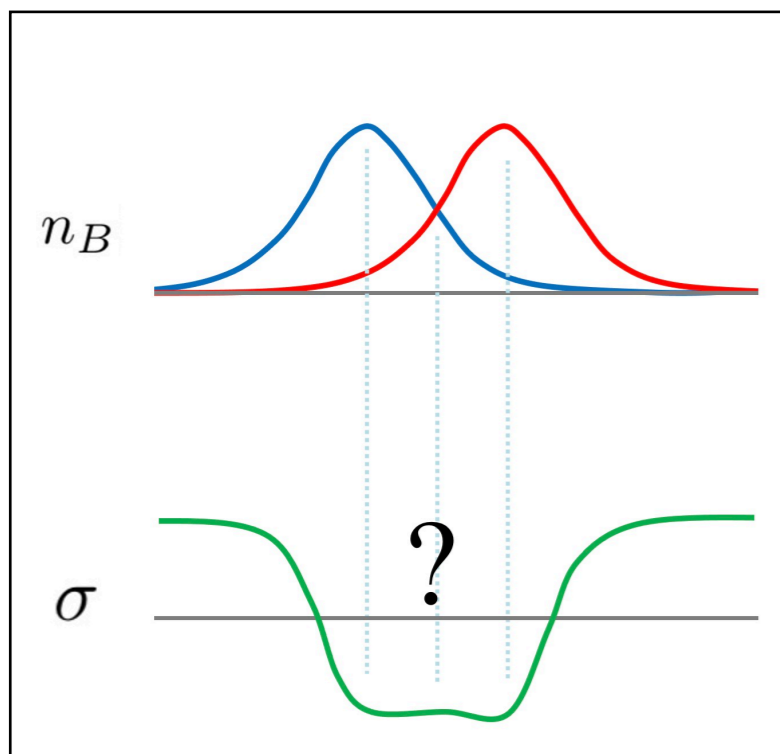
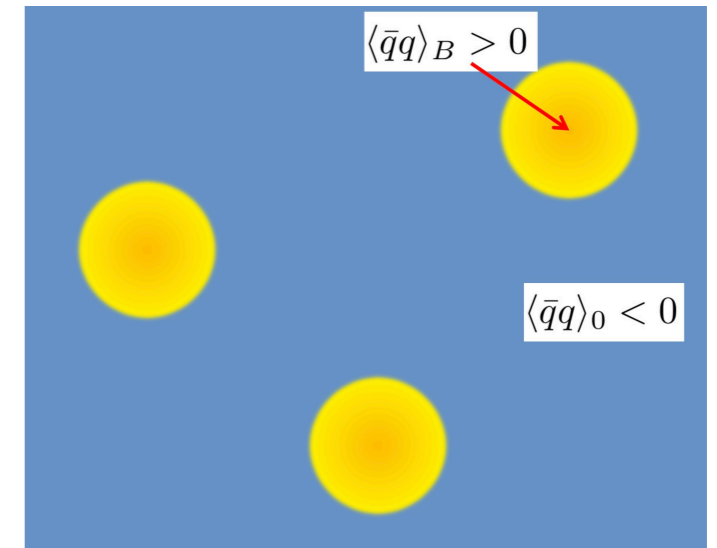
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\uparrow
 opposite signs
 & similar magnitudes

$\approx (0.3 \text{ GeV})^3$



possible configuration w/ our results:
 σ and pions make forms like a wave in nucleons

Summary and Outlook

- unified EOSs for NS constructed by using PDM for nucleons and NJL for quarks
- PDM-EOS & NJL-EOS are constraining each other; the understanding of one EOS improves the other

- observational constraints \rightarrow $600 \text{ MeV} \lesssim m_0 \lesssim 900 \text{ MeV}$

- we calculated the chiral quark condensate $\langle \bar{q}q \rangle$.
the interpolation of $\langle \bar{q}q \rangle_{\text{PDM}}$ and $\langle \bar{q}q \rangle_{\text{NJL}}$ is shallow(smooth).

outlook (work in progress):

- extend the PDM to SU(3) (include hyperons)
- effects of the U(1)_A anomaly