

Study of dense nuclear matter using parity doublet models

Masayasu Harada (Nagoya University)

@ Reimei Workshop “Hadrons in dense matter at J-PARC” (February 23, 2022)

Based on

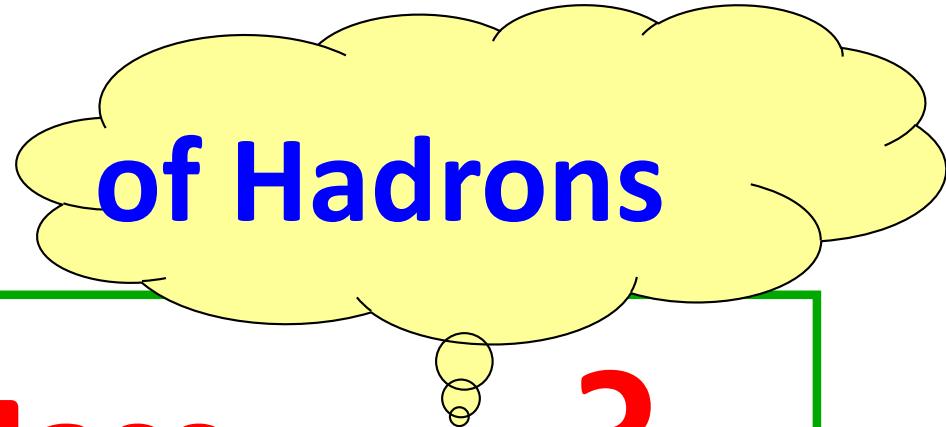
- Y. Motohiro, Y. Kim, M. Harada, Phys. Rev. C 92, 025201 (2015); Erratum: Phys. Rev. C 95, 059903 (2017).
- Y. Takeda, Y. Kim and M. Harada, Phys. Rev. C 97, 065202 (2018).
- Y. Takeda, H. Abuki and M. Harada, Phys. Rev. D 97, 094032 (2018).
- T. Yamazaki and M. Harada, Phys. Rev. D 99, 034012 (2018).
- T. Yamazaki and M. Harada, Phys. Rev. C 100, 025205 (2019).
- T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 103, 045205 (2021).
- T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 104, 065201 (2021).

Introduction

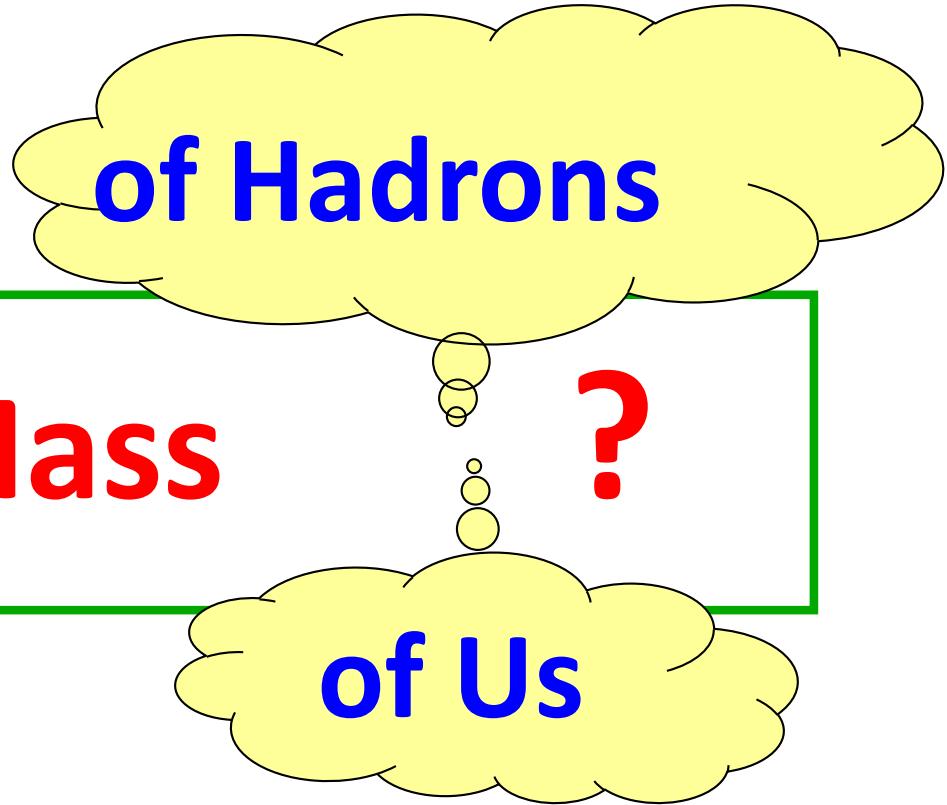
Origin of Mass ?

Origin of Mass

?



Origin of Mass



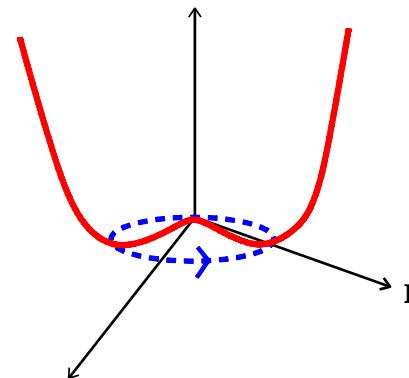
Origin of Mass

of Us

II

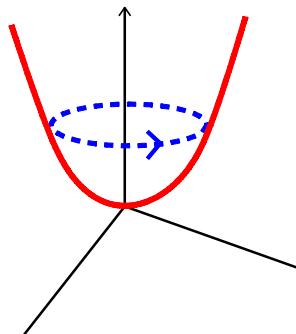
One of the Interesting problems of QCD

spontaneous chiral symmetry breaking



chiral symmetry
broken phase at
vacuum

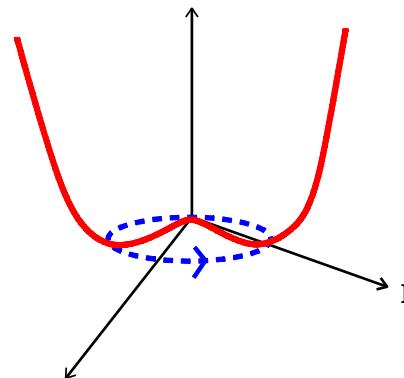
$$\langle \bar{q}q \rangle \neq 0 \text{ (chiral condensate)}$$



chiral symmetric
phase at high T
and/or density

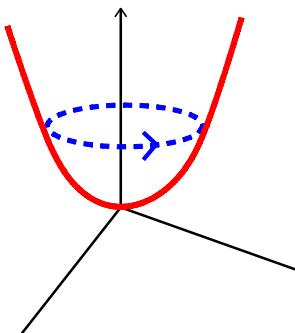
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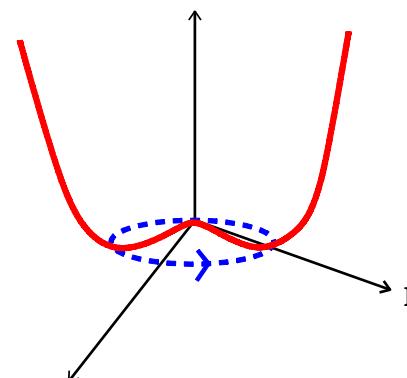


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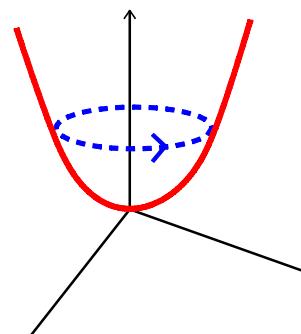
- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

spontaneous chiral symmetry breaking



chiral symmetry
broken phase at
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chiral symmetric
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$$\langle \bar{q}q \rangle = 0$$

- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

- How much mass of nucleon is from the spontaneous chiral symmetry breaking ?
- What is the chiral partner of the nucleon ?

Parity Doublet models for nucleons

- How much mass of nucleon is from the spontaneous chiral symmetry breaking ?
- What is the chiral partner of nucleon ?

- A Parity doublet model for light baryons
 - In [C.DeTar, T.Kunihiro, PRD39, 2805 (1989)], N(1535) is regarded as the chiral partner to the N(939) having the chiral invariant mass.

$$m_N = m_0 + m_{\langle \bar{q}q \rangle}$$

A blue arrow points from the term m_0 to the label "chiral invariant mass". Another blue arrow points from the term $m_{\langle \bar{q}q \rangle}$ to the label "spontaneous chiral symmetry breaking".

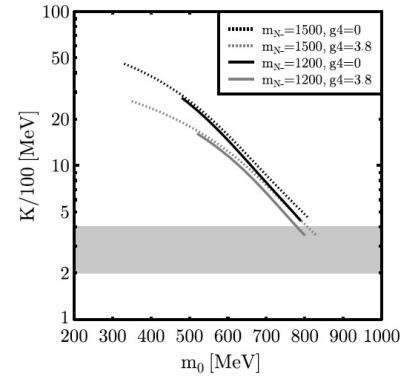
chiral invariant mass spontaneous chiral symmetry breaking

- This model can be extended to include different excited nucleons.

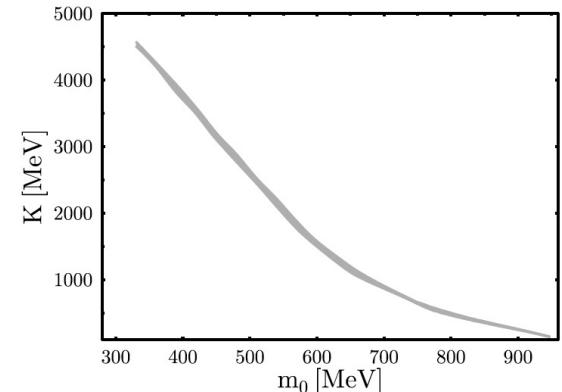
Nuclear matter in parity doublet models

- A parity doublet model including omega meson with 4-point interaction is used in a Walecka-type mean field analysis.
 - Large value of m_0 is needed to reproduce the incompressibility.
- Rho meson is further included with 4-point interaction.
 - $m_0 > 800$ MeV is needed to have $100 < K < 400$ MeV

D.Zschiesche et al., PRC75, 055202 (2007)



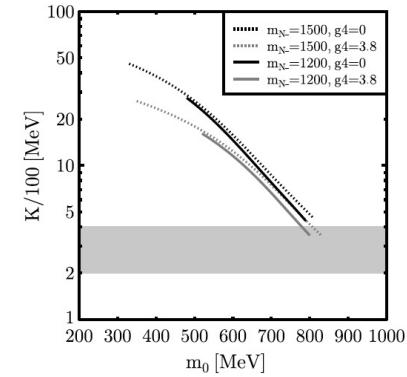
V.Dexheimer et al., PRC77, 025803 (2008)



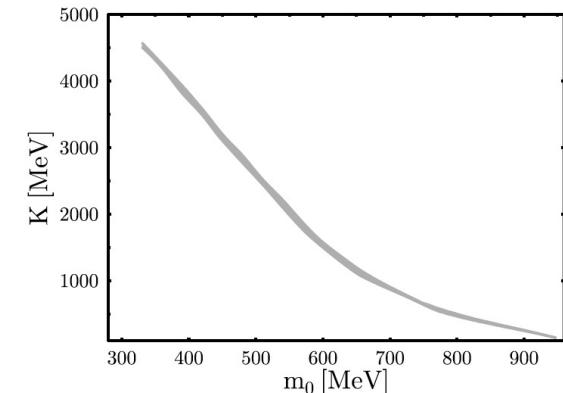
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 - Large value of m_0 is needed to reproduce the incompressibility.
- Rho meson is further included with 4-point interaction.
 - $m_0 > 800$ MeV is needed to have $100 < K < 400$ MeV
- In our analysis [Y.Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)], we constructed a model with **a 6-point interaction of sigma**, but without 4-point interaction for vector mesons.
- Our results show that **K = 240 MeV is reproduced for $m_0 = 500 - 900$ MeV.**

D.Zschiesche et al., PRC75, 055202 (2007)



V.Dexheimer et al., PRC77, 025803 (2008)



Study of dense nuclear matter using parity doublet models (PDMs)

- Construction of nuclear matter from a PDM
 - Y.Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)
- Study of effect of $\Delta(1232)$ to the chiral symmetry breaking in a PDM
 - Y. Takeda, Y. Kim and M. Harada, Phys. Rev. C 97, 065202 (2018).
- Study of a new **dual chiral density wave** (DCDW) in a PDM.
 - Y. Takeda, H. Abuki and M. Harada, Phys. Rev. D 97, 094032 (2018).
- Study of a **constraint** to the chiral invariant mass in a PDM from the **neutron star properties**
 - T. Yamazaki and M. Harada, Phys. Rev. C 100, 025205 (2019).
- Construction of a **unified EOS** connecting a PDM and an NJL-type quark model
 - T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 103, 045205 (2021).
- Study of **density dependence of the chiral condensate** from the unified EOS.
 - T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 104, 065201 (2021).

Outline

1. Introduction
2. Construction of nuclear matter from a PDM
3. Effect of $\Delta(1232)$ to nuclear matter based on parity doublet structure
4. A new Dual Chiral Density Wave (DCDW) in a PDM
5. A constraint to the chiral invariant mass in a PDM from the neutron star properties
6. A unified EOS for NS connecting a PDM and an NJL-type quark model
7. Summary

2. Construction of nuclear matter from a Parity Doublet Model (PDM)

Y.Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)

chiral representation of baryons

C.DeTar, T.Kunihiro, PRD39, 2805 (1989)

D.Jido, M.Oka, A.Hosaka, PTP106, 873 (2001)

- representation of quark under $SU(2)_R \times SU(2)_L$

$$q \sim q_r + q_l \sim (2,1) \oplus (1,2)$$

- representation of baryon under $SU(2)_R \times SU(2)_L$

$$\begin{aligned} \psi &\sim q \otimes q \otimes q \sim [(2,1) \oplus (1,2)]^3 \\ &= 5[(2,1) \oplus (1,2)] \oplus 3[(3,2) \oplus (2,3)] \oplus [(4,1) \oplus (1,4)] \end{aligned}$$

chiral representation of baryons

C.DeTar, T.Kunihiro, PRD39, 2805 (1989)

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- representation of quark under $SU(2)_R \times SU(2)_L$

$$q \sim q_r + q_l \sim (2,1) \oplus (1,2)$$

- representation of baryon under $SU(2)_R \times SU(2)_L$

$$\psi \sim q \otimes q \otimes q \sim [(2,1) \oplus (1,2)]^3$$

$$= \underbrace{5[(2,1) \oplus (1,2)]}_{\text{Chiral symmetry is broken}} \oplus \underbrace{3[(3,2) \oplus (2,3)]}_{\text{isospin symmetry remains}} \oplus \underbrace{[(4,1) \oplus (1,4)]}_{\text{isospin symmetry remains}}$$

Chiral symmetry is broken and the isospin symmetry remains

$I = \frac{1}{2}$ baryons

$I = \frac{3}{2}$ baryons

A Parity Doublet model

C.DeTar, T.Kunihiro, PRD39, 2805 (1989)
 D.Jido, M.Oka, A.Hosaka, PTP106, 873 (2001)
 Y.Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)

- N(1535) = **chiral partner** to N(939)
- chiral transformations:
 - $\psi_{1l} \rightarrow g_L \psi_{1l}, \psi_{1r} \rightarrow g_R \psi_{1r}, \psi_{2l} \rightarrow g_R \psi_{2l}, \psi_{2r} \rightarrow g_L \psi_{2r}; g_L \in \text{SU}(2)_L, g_R \in \text{SU}(2)_R$
- A **chiral invariant mass**:
 - $m_0 [\bar{\psi}_{1l}\psi_{2r} + \bar{\psi}_{2r}\psi_{1l} + \bar{\psi}_{1r}\psi_{2l} + \bar{\psi}_{2l}\psi_{1r}]$
- Yukawa interactions
 - $-g_1 [\bar{\psi}_{1l}M\psi_{1r} + \bar{\psi}_{1r}M^\dagger\psi_{1l}] - g_2 [\bar{\psi}_{2r}M\psi_{2l} + \bar{\psi}_{2l}M^\dagger\psi_{2r}]$



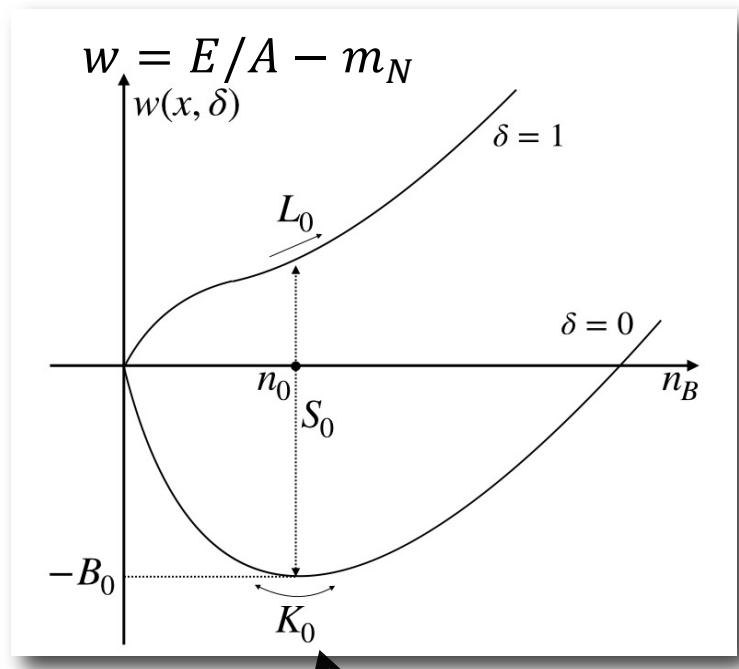
- $M = \sigma + i \vec{\tau} \cdot \vec{\pi}$ transforms $M \rightarrow g_L M g_R^\dagger$
- $\langle M \rangle = \bar{\sigma} \neq 0$ causes the spontaneous chiral symmetry breaking.

$$m_\pm = \frac{1}{2} \left[\sqrt{(g_1 + g_2)^2 \bar{\sigma}^2 + 4m_0^2} \mp (g_2 - g_1) \bar{\sigma} \right]$$

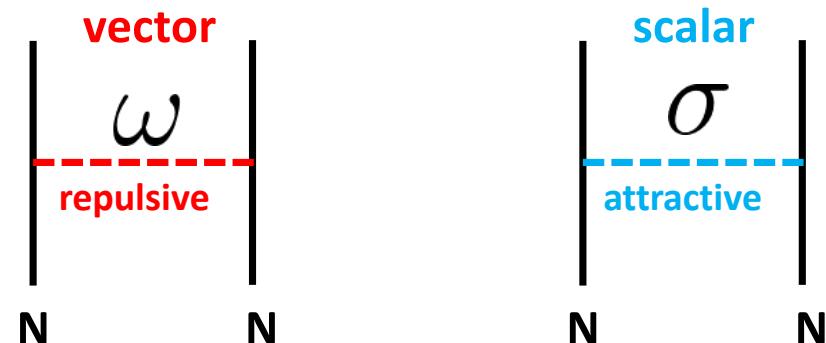
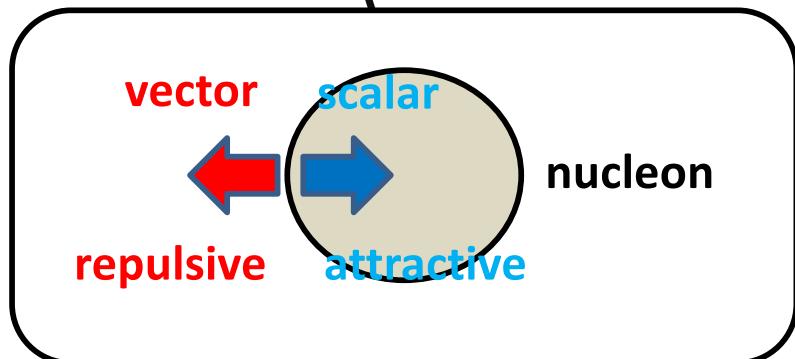
$$\begin{aligned} m_+ &= m(N(939)) \\ m_- &= m(N(1535)) \end{aligned}$$

Nuclear Matter at normal nuclear density

Y. Motohiro, Y. Kim, M. Harada, Phys. Rev. C 92, 025201 (2015); Erratum: Phys. Rev. C 95, 059903 (2017).



- Nuclear saturation density
 - $\rho(\mu_B^* = 923\text{MeV}) = n_0 = 0.16\text{fm}^{-3}$
- Binding energy at normal nuclear density
 - $w = \left[\frac{E}{A} - m(939) \right]_{n_0} = -16\text{MeV}$
- Incompressibility
 - $K_0 = 9\rho_0^2 \frac{\partial^2(E/A)}{\partial\rho^2} \Big|_{n_0} = 240\text{MeV}$
- Symmetry energy
 - $S_0 = 31\text{ MeV}$



3. Effect of $\Delta(1232)$ to nuclear matter based on parity doublet structure

Y. Takeda, Y. Kim and M. Harada, Phys. Rev. C 97, no. 6, 065202 (2018).

Chiral Partner Structure of Delta Baryons

D.Jido, T.Hatsuda, T.Kunihiro, PRL84, 3252 (2000)

D.Jido, M.Oka, A.Hosaka, PTP106, 873 (2001)

- $\Delta(1700)$ = chiral partner to $\Delta(1232)$

$$m_{\Delta^\pm} = \sqrt{(g_{\Delta 1} + g_{\Delta 2})^2 \bar{\sigma}^2 + m_{\Delta 0}^2} \mp (g_{\Delta 1} - g_{\Delta 2}) \bar{\sigma}$$

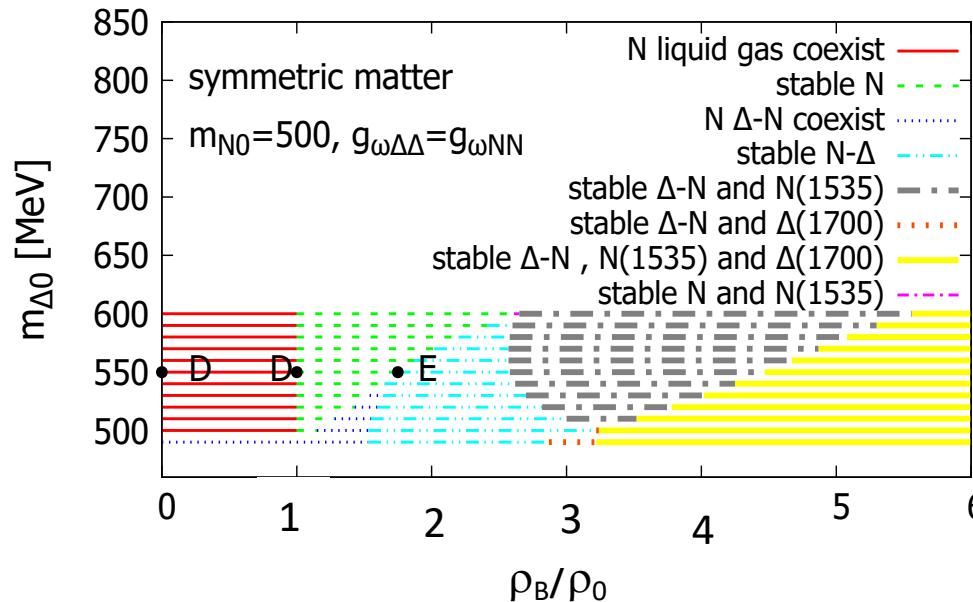
- $m_{\Delta 0}$ = chiral invariant mass for Δ .
- Masses of $\Delta(1232)$ and $\Delta(1700)$ = inputs

Phase structure & Density

$m_{N0} = 500 \text{ MeV}$

$g_{\omega\Delta\Delta} = g_{\omega NN}$

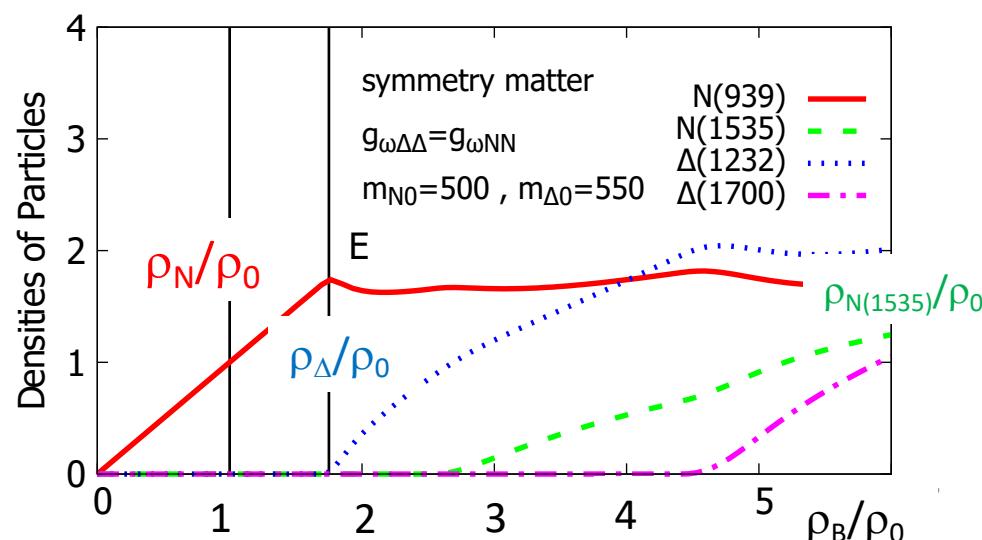
Phase structure



- N liquid-gas coexistence
- stable N phase
- coexistence of Delta-N matter and nuclear matter
- stable N-Delta matter
- stable N-Delta, N(1535) matter
- stable N-Delta, N(1535), Delta(1700) matter
- Stable N-N(1535) matter

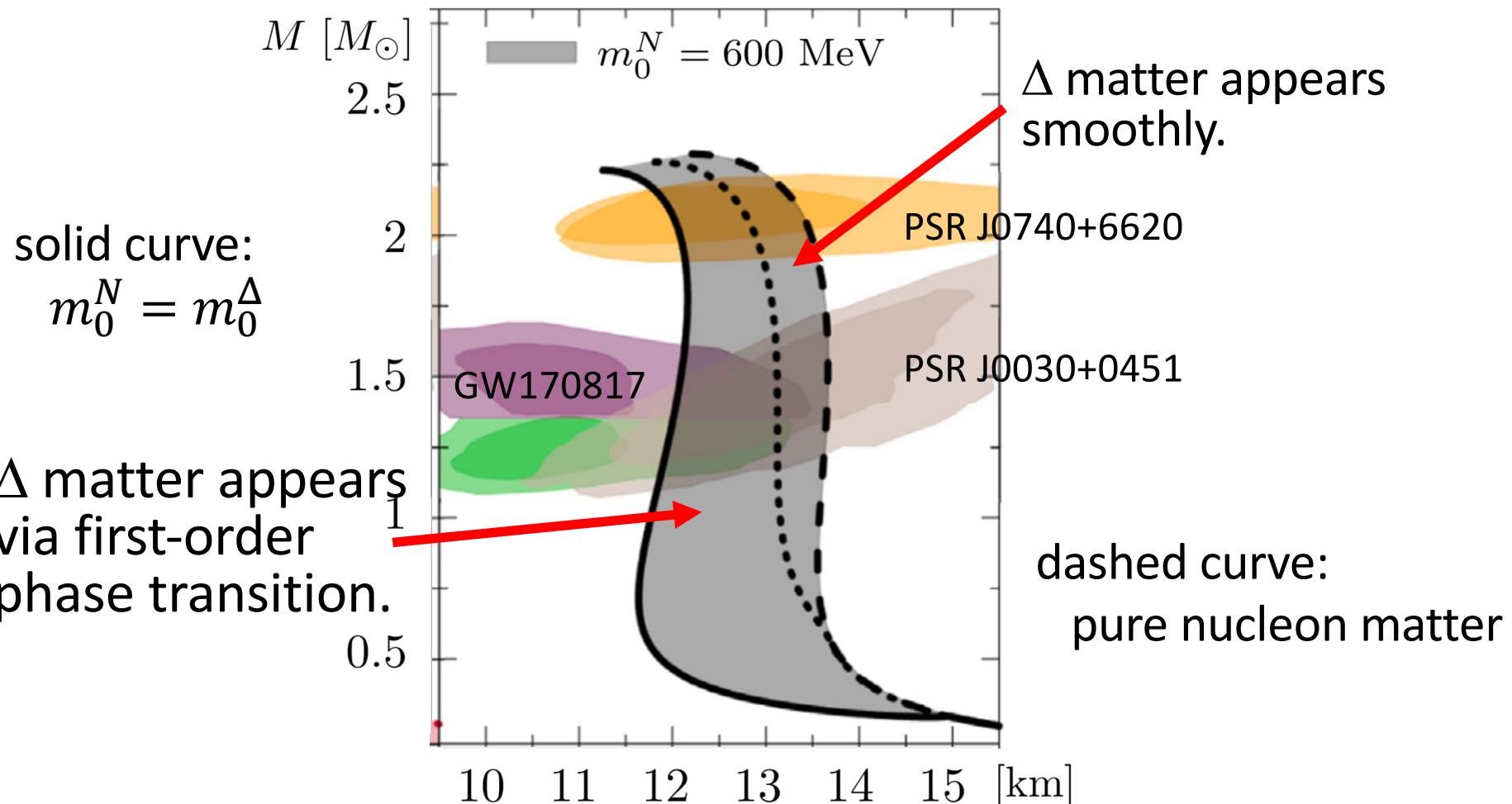
$m_{\Delta 0} = 550 \text{ MeV}$

Baryon Density vs. ρ_B



M-R relation of NS with Δ matter

M. Marczenko, K. Redlich and C. Sasaki, *Astrophys. J. Lett.* 925, L23 (2022).

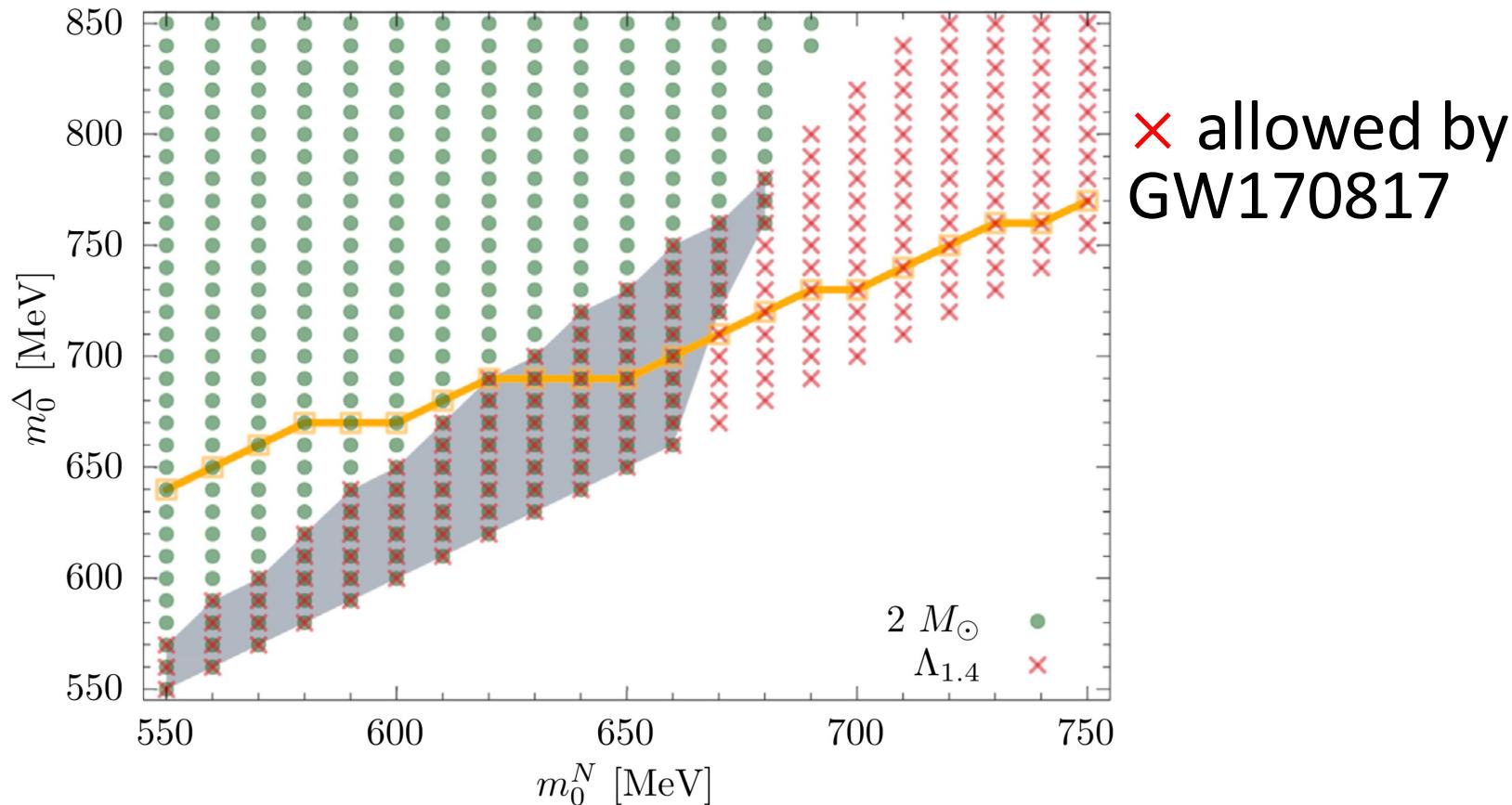


Δ matter softens the EOS, but pure hadronic matter can survive the observational constraints.

Constraint to chiral invariant masses from NS data

M. Marczenko, K. Redlich and C. Sasaki, *Astrophys. J. Lett.* 925, L23 (2022)

- allowed by PSR J0740+6620 $M_{\text{max}} = (2.08 \pm 0.07)M_{\odot}$



Gray area: Δ matter is consistent with NS data.

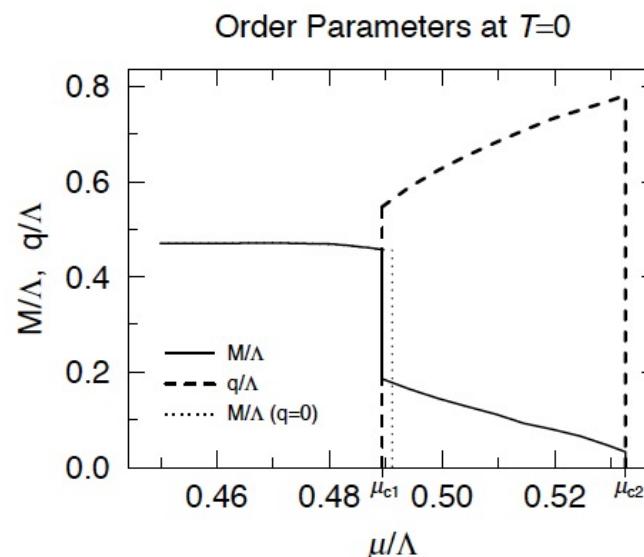
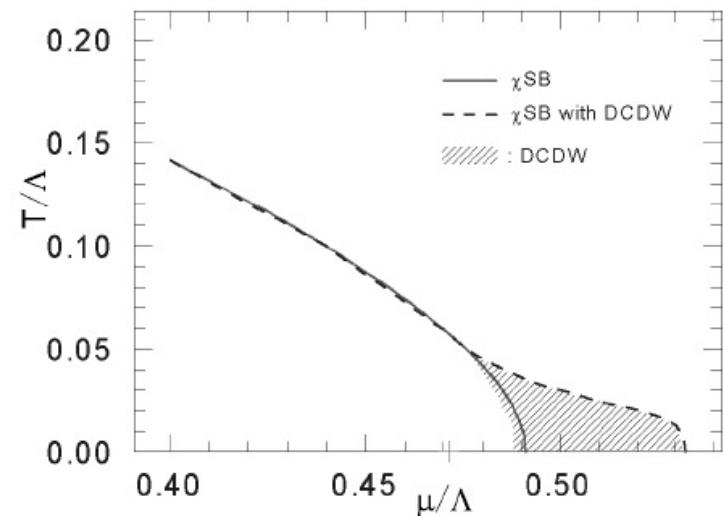
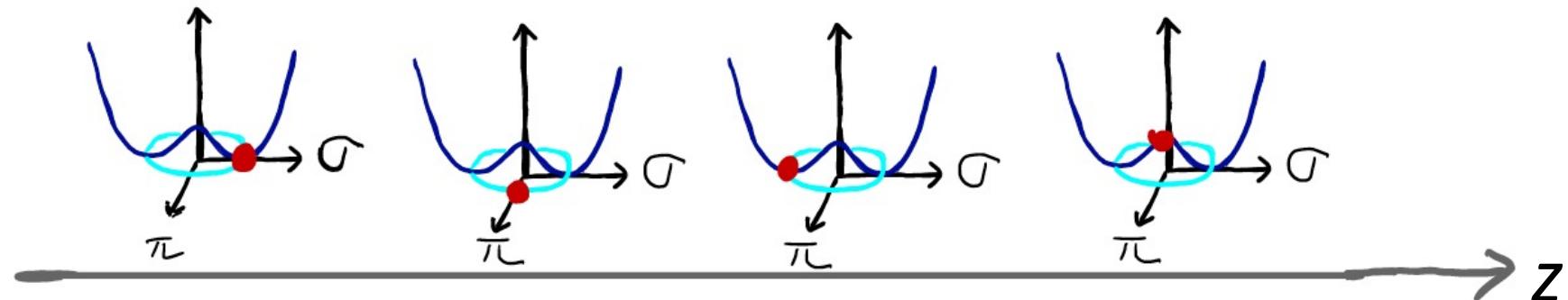
4. A new Dual Chiral Density Wave (DCDW) in a PDM

Y. Takeda, H. Abuki and M. Harada, Phys. Rev. D 97, no. 9, 094032 (2018).

Dual Chiral Density Wave in NJL-type quark model

E.Nakano and T.Tatsumi, PRD71, 114006 (2005)

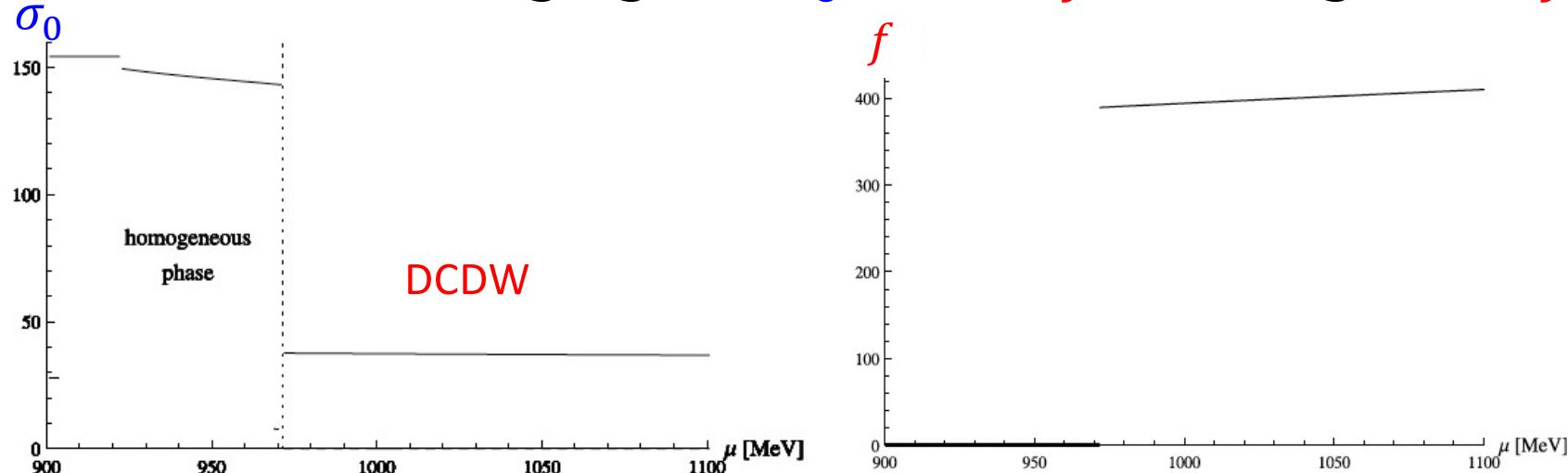
$$\langle M \rangle = \bar{\sigma} + i\tau_3 \bar{\pi}_3 = \sigma_0 (\cos 2fz + i\tau_3 \sin 2fz)$$



DCDW in a Parity Doublet Model

A.Heinz, F.Giacosa, D.H.Rischke, NPA933, 34 (2015)

$$\langle M \rangle = \bar{\sigma} + i\tau_3 \bar{\pi}_3 = \sigma_0 (\cos 2fz + i\tau_3 \sin 2fz)$$



- Phase transition to DCDW at $\mu_B = 973$ MeV .
- Onset density : $\rho_B = 2.4\rho_0$

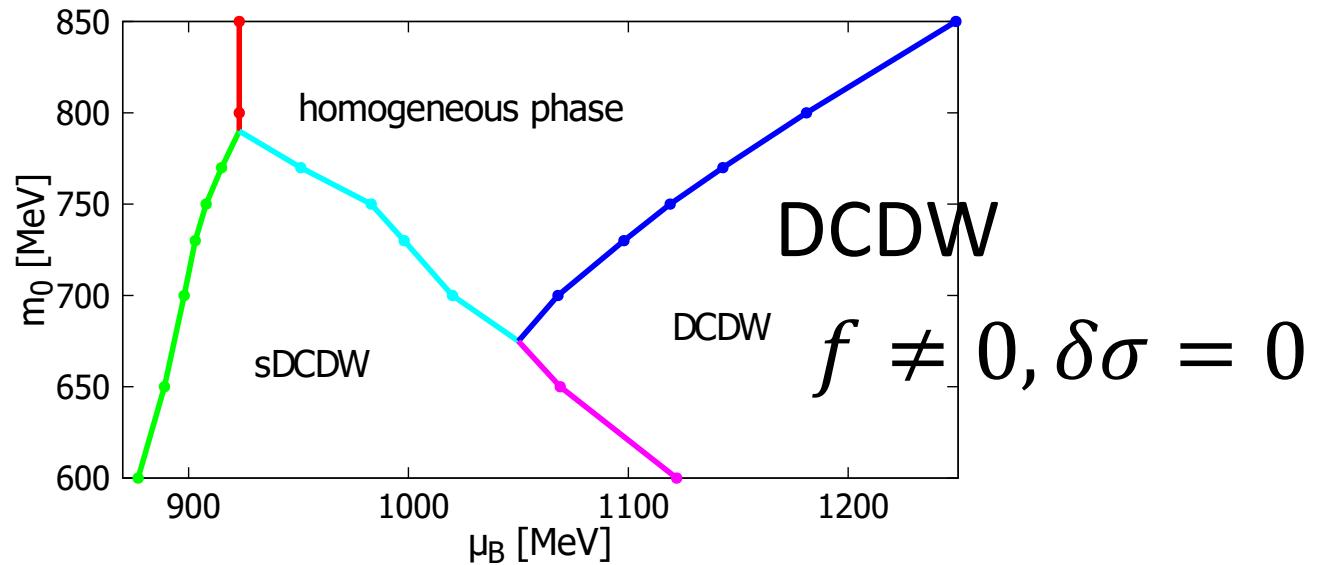
shifted DCDW (sDCDW)

Y.Takeda, H.Abuki, M.Harada, PRD97, 094032 (2018)

$$\langle M \rangle = \delta\sigma + \sigma_0 (\cos 2fz + i\tau_3 \sin 2fz) \quad \text{note: } \int d^3x \langle M \rangle = \delta\sigma$$

Homogeneous phase : $f = 0, \delta\sigma + \sigma_0 \neq 0$

sDCDW
 $f \neq 0, \delta\sigma \neq 0$



stability of normal nuclear matter $\Rightarrow m_0 \geq 780$ MeV

5. A constraint to the chiral invariant mass in a PDM from the neutron star properties

T. Yamazaki and M. Harada, Phys. Rev. D 99, 034012 (2018)

T. Yamazaki and M. Harada, Phys. Rev. C 100, 025205 (2019)

A Parity doublet model with $[(1,2)\oplus(2,1)]$ and $[(2,3)\oplus(3,2)]$ representations

T. Yamazaki and M. Harada, Phys. Rev. D99, 034012 (2018)

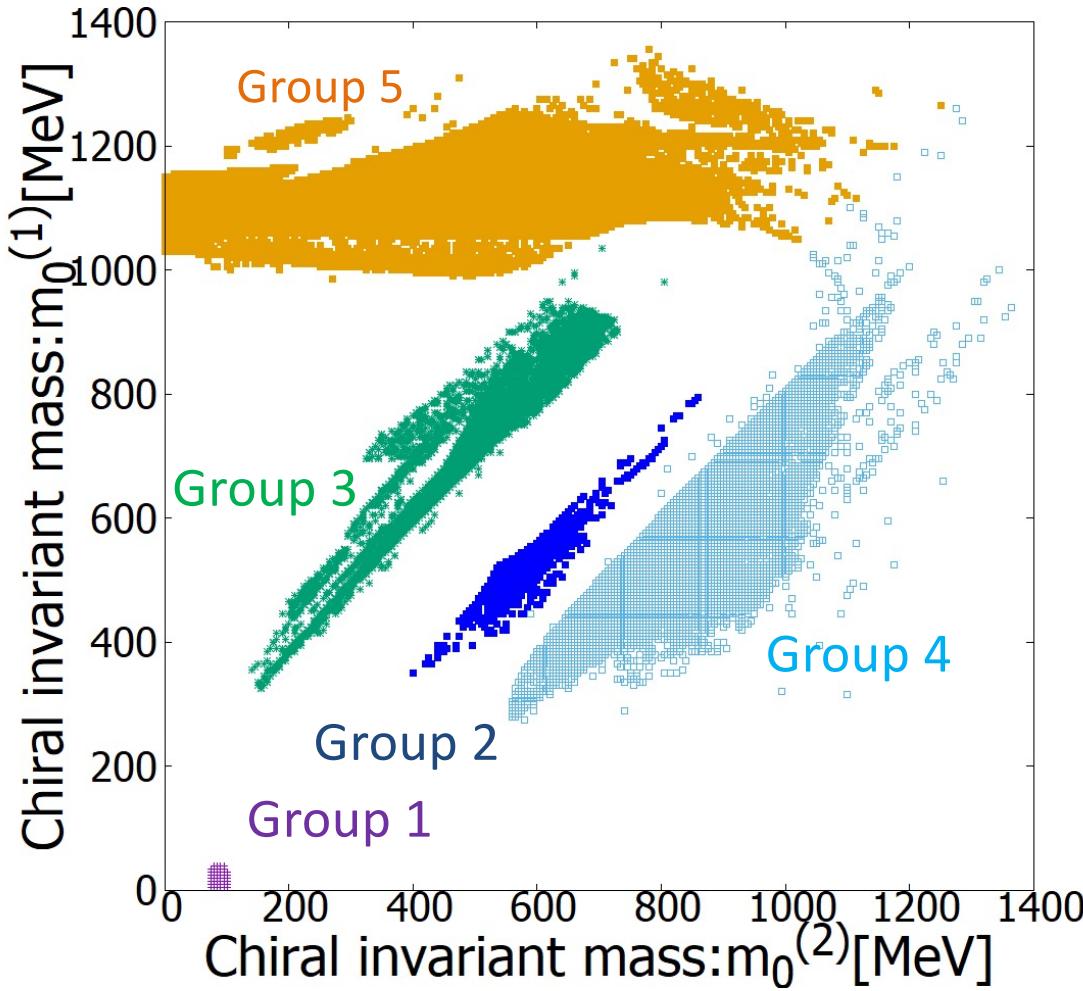
- 2 representations, $[(1,2)\oplus(2,1)]$ and $[(2,3)\oplus(3,2)]$ to study N(939), N(1440), N(1535), N(1650).
- 2 chiral invariant masses.
 - $-m_0^{(1)}[\bar{\psi}_1\gamma_5\psi_2 - \bar{\psi}_2\gamma_5\psi_1] - m_0^{(2)}[\bar{\Psi}_1\gamma_5\Psi_2 - \bar{\Psi}_2\gamma_5\Psi_1]$
- 6 Yukawa Interactions
 - $-g_1[\bar{\psi}_{1l}M\psi_{1r} + \bar{\psi}_{1r}M^\dagger\psi_{1l}] - g_2[\bar{\psi}_{2r}M\psi_{2l} + \bar{\psi}_{2l}M^\dagger\psi_{2r}]$
etc.
- We also have 4 terms with one derivative.
 - $a_1[\bar{\psi}_{1l}\gamma^\mu\partial_\mu M\psi_{2l} - \bar{\psi}_{1r}\gamma^\mu\partial_\mu M^\dagger\psi_{2r}]$ etc

Physical inputs

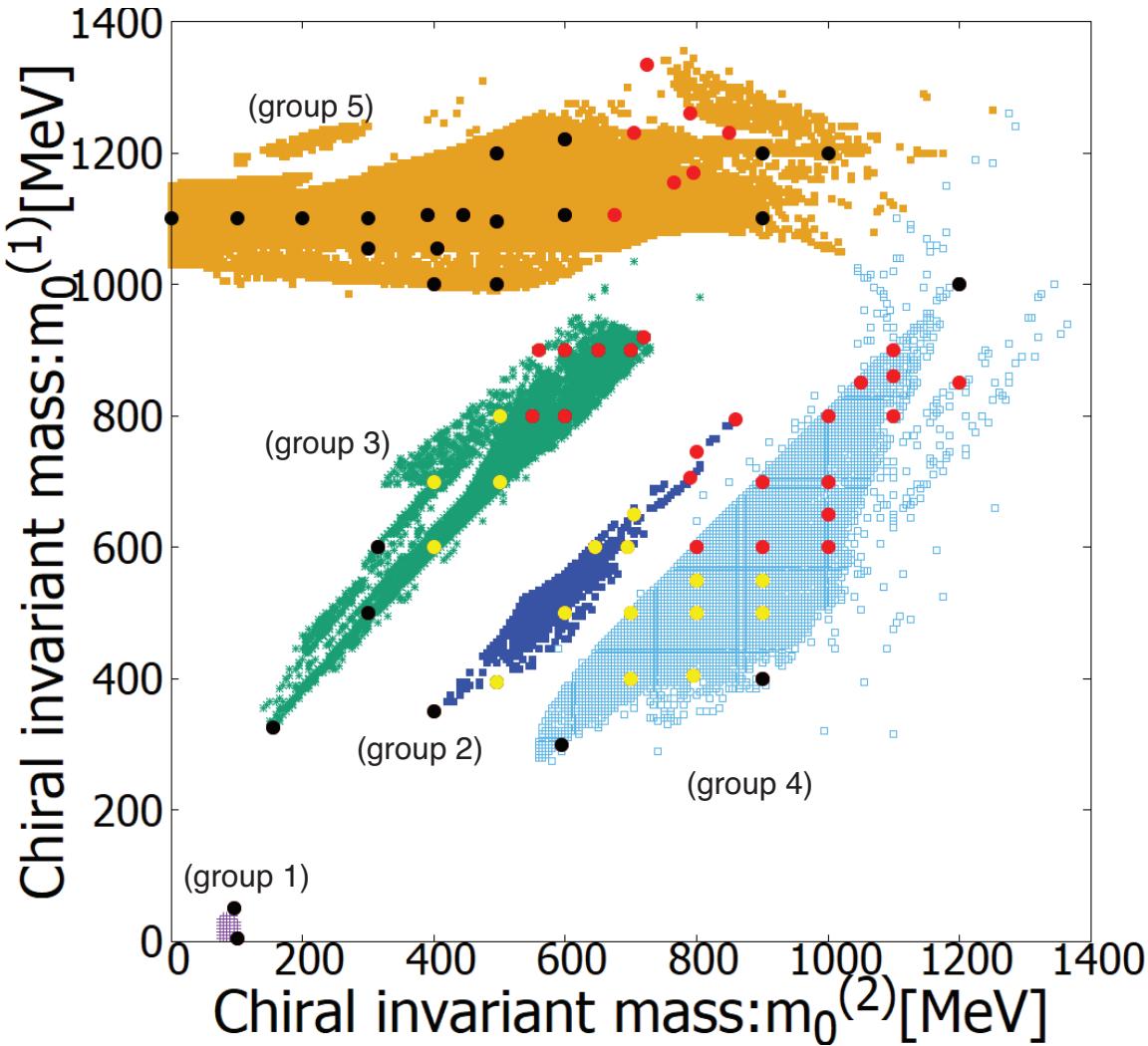
- Fix the values of $m_0^{(1)}$ and $m_0^{(2)}$.
- Use the following 10 physical inputs to determine 10 parameters (10 couplings).

	Mass (MeV)	Width [$\Gamma_{N^* \rightarrow N\pi}$] (MeV)	axial charge
$N(939)$	939	-	1.272
$N(1440)$	1430	228	-
$N(1535)$	1535	68	$-0.25 \leq g_A \leq 0.25$
$N(1650)$	1655	84 [to $N(939)$]	0.55
		22 [to $N(1440)$]	

Constraint from vacuum properties



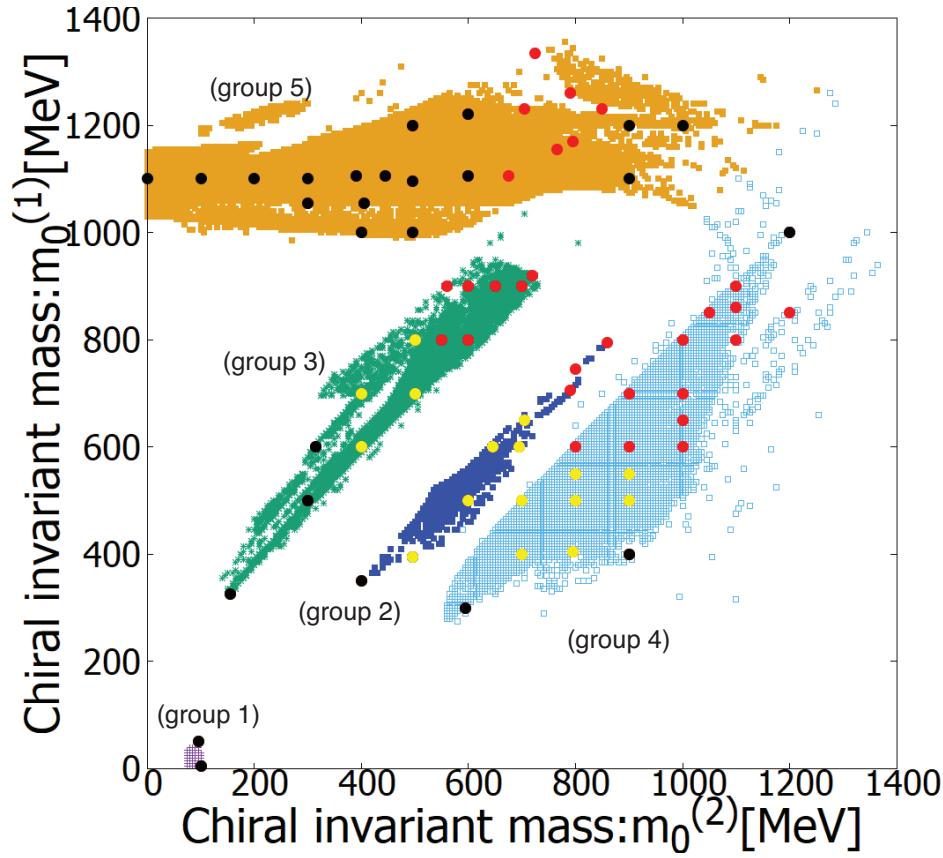
Constraint from saturation properties



“• marks”: the saturation properties are **NOT** satisfied.

Please note that the parameter choices in **Group 1** are all excluded.

Constraint from Neutron Star Properties



Constraint from the tidal deformability:

$$\tilde{\Lambda} \leq 800 \text{ with } M_{\text{chirp}} = 1.188 M_{\odot}$$

GW170817 (LIGO & Virgo)

- (yellow dots) are excluded
- (red dots) are allowed.

$$m_{\pm} = \frac{1}{2} \left[\sqrt{(g_1 + g_2)^2 \bar{\sigma}^2 + 4m_0^2} \mp (g_2 - g_1)\bar{\sigma} \right]$$

m_0	σ -interaction	Attractive force by σ	Repulsive force by ω	EOS
small	large	strong	strong	stiff
large	small	weak	weak	soft

6. A unified EOS for NS connecting a PDM and an NJL-type quark model

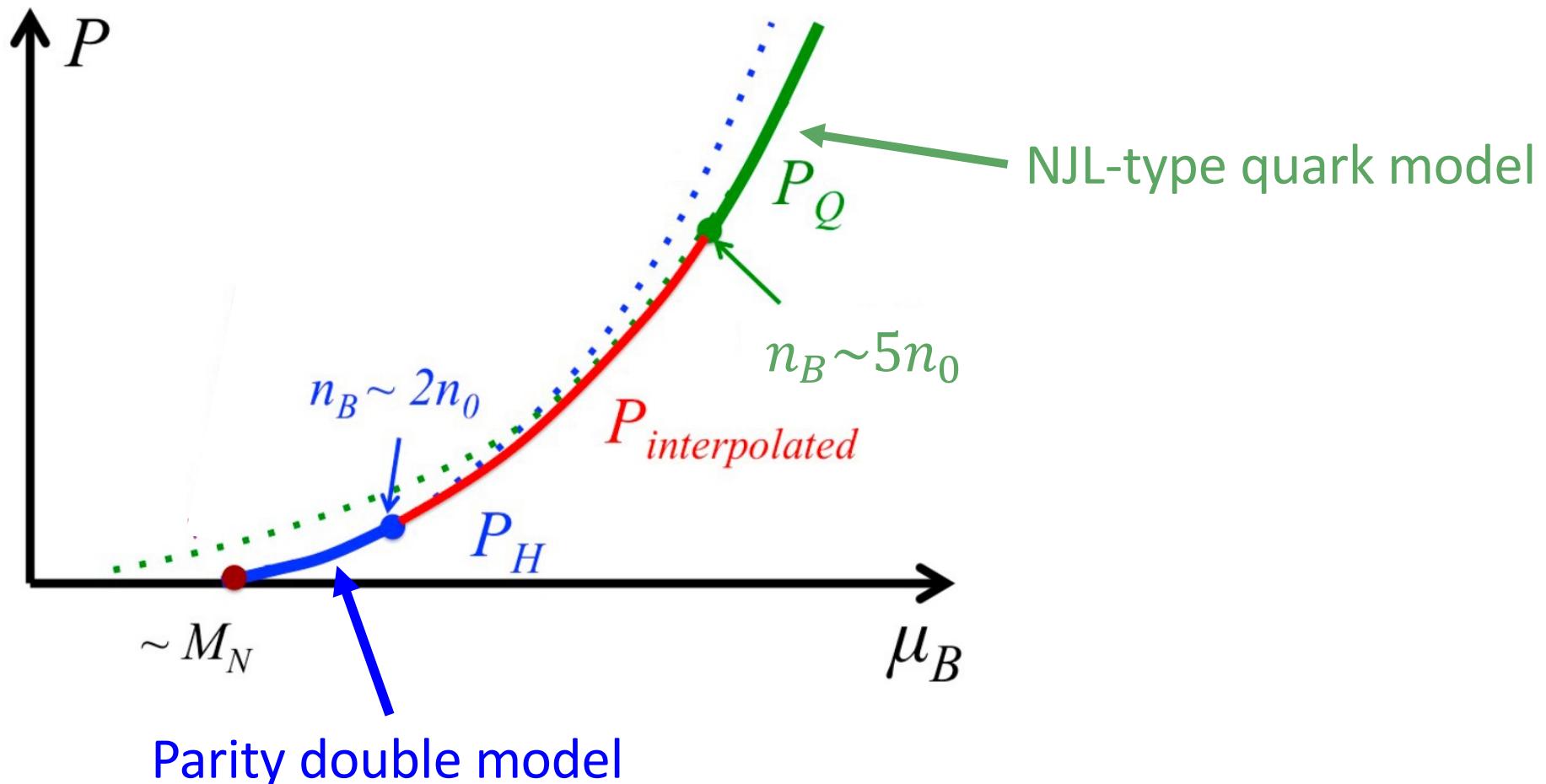
T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 103, 045205 (2021).

T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 104, 065201 (2021).

Unified EOS for NS in 3-window picture

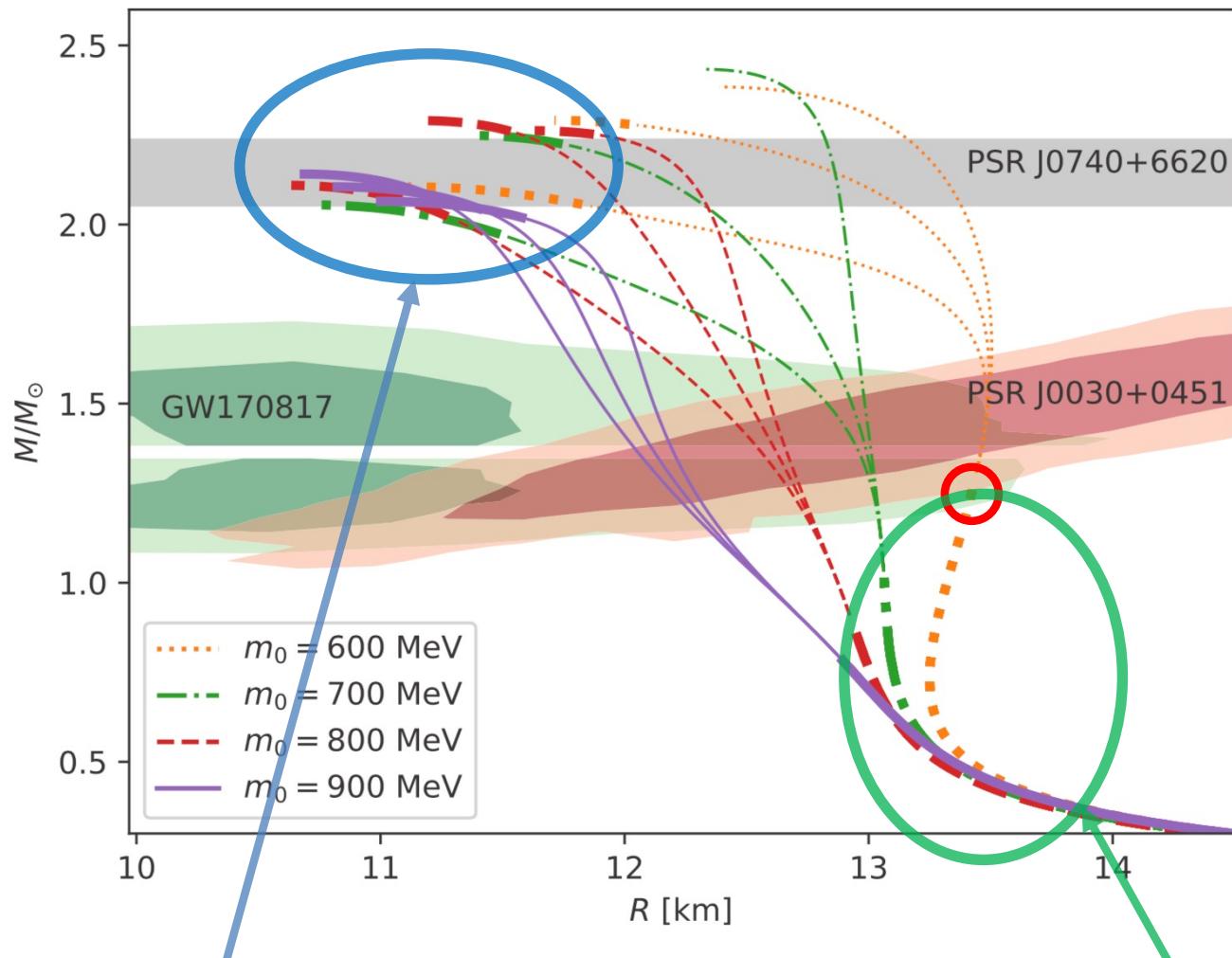
G. Baym et al., Rept. Prog. Phys. 81, 056902 (2018).

T. Minamikawa, T. Kojo and M.H., Phys. Rev. C 103, 045205 (2021).



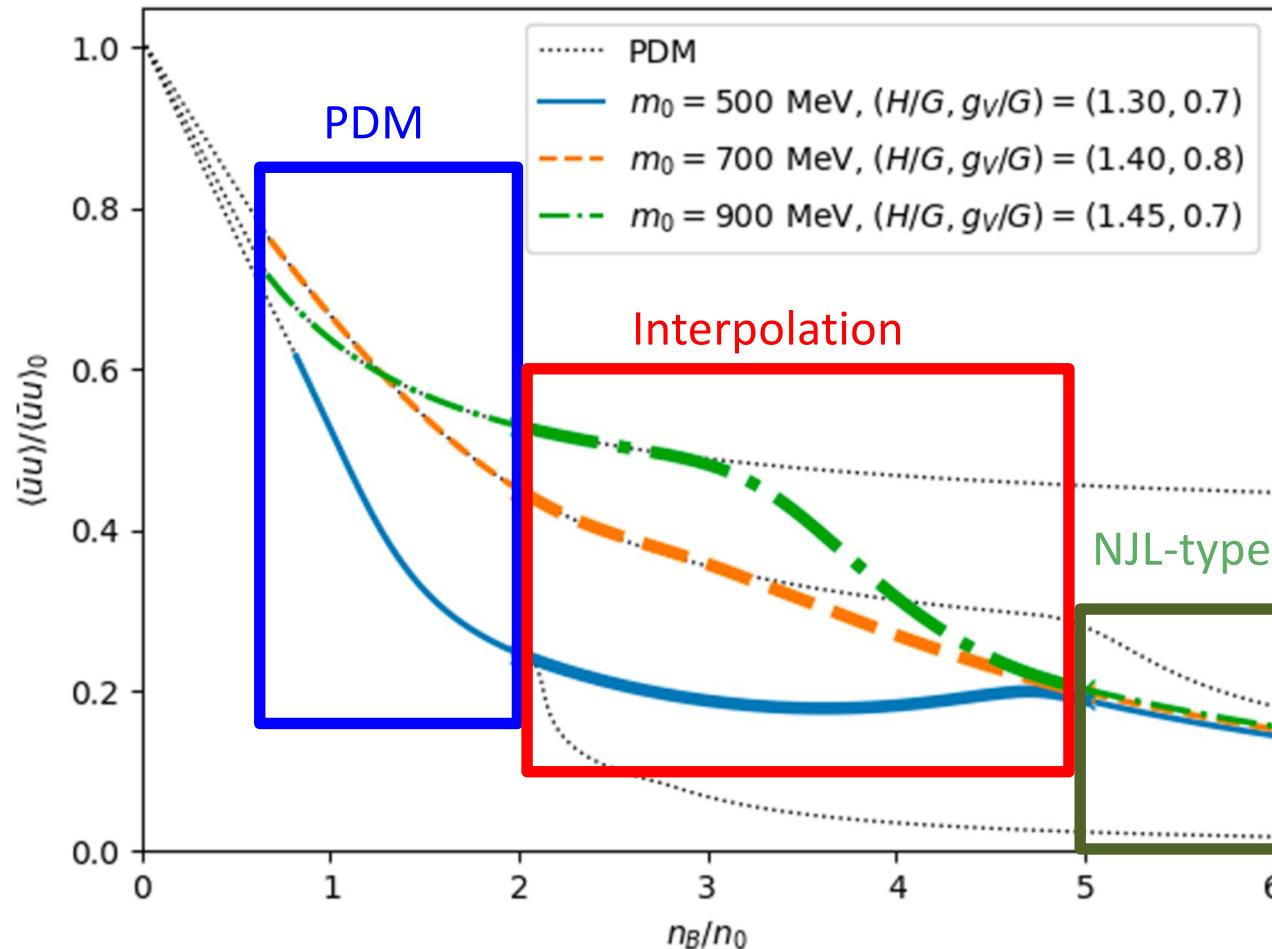
M-R relation

T. Minamikawa, T. Kojo and M.H., Phys. Rev. C 103, 045205 (2021).



Density dependence of chiral condensate

T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 104, 065201 (2021).



See Minamikawa's talk

7. Summary

- I have introduced our study of dense matter based on parity doublet models.
- Although Δ baryon can enter the matter, the EOS still satisfy the constraints from NS.
- There could exist a new type of inhomogeneous condensate: shifted DCDW.
- Comparison of our EOS with constraints from NS implies that **more than half of nucleon mass is the chiral invariant mass.**
 - See Minamikawa's talk.

Origin of chiral invariant mass ?

- Dilaton (Gluon condensate and/or 4-quark condensate)
 - S. Gallas, F. Giacosa, G. Pagliara, Nucl. Phys. A 872, 13 (2011).
 - W. G. Paeng, H. K. Lee, M. Rho and C. Sasaki, Phys. Rev. D 85, 054022 (2012).
 - Y. L. Ma, H. K. Lee, W. G. Paeng and M. Rho, Sci. China Phys. Mech. Astron. 62, 112011 (2019).
- Gluon background in vacuum
 - Y. Kim, I. A. Mazur, M. Harada and H. K. Lee, arXiv:2009.02839 [hep-ph].
- Chiral Scalar density
 - T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 104, 065201 (2021).
 - See Minamikawa's talk



The End

The text "The End" is displayed in a stylized font where each letter has a different color: T is pink, h is red, e is orange, E is green, n is blue, and d is purple. The letters are partially obscured by a series of diagonal grey lines that slope upwards from left to right, creating a sense of depth.