Quark-Hadron Crossover equations of state for neutron stars

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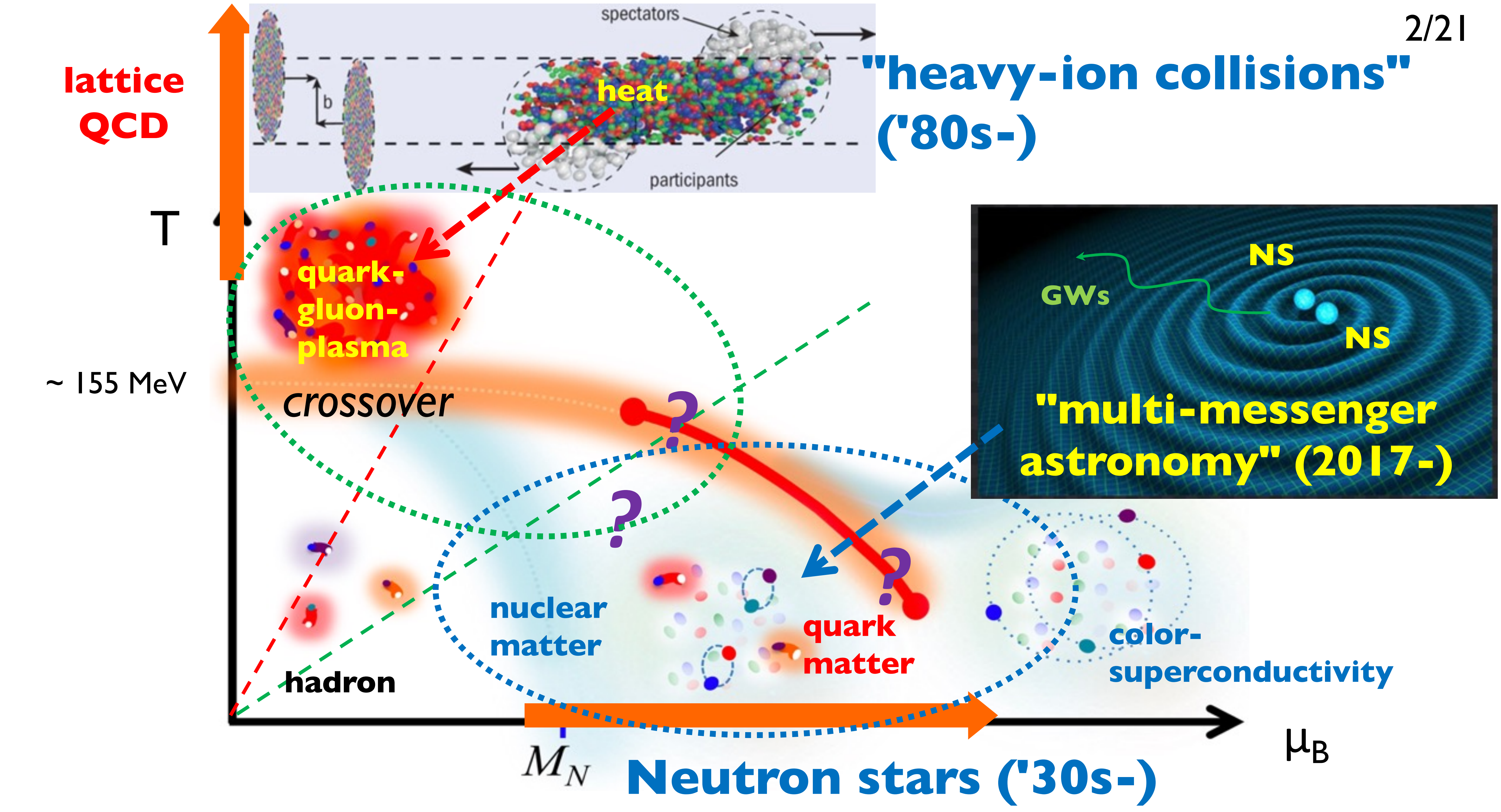
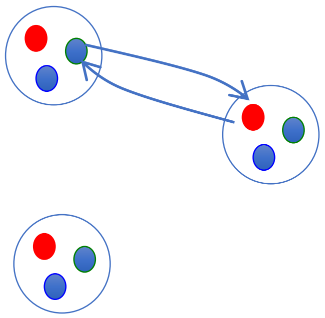
Refs)
- Baym-Hatsuda-TK-Powell-Song-Takatsuka (2018): review for NS
- TK-Baym-Hatsuda (2021): QHC21 EoS
- Fukushima-TK-Weise (2020): Soft- & Hard- Deconfinement
- TK (2021), TK-Suenaga (2021): sound velocity peak, quark saturation effects


lattice
QCD
spectators
b
heat
participants
"heavy-ion collisions"
('80s-)
T
quark-
gluon-
plasma
~ 155 MeV
crossover
?
?
?
GWs
NS
NS
"multi-messenger
astronomy" (2017-)
nuclear
matter
quark
matter
color-
superconductivity
hadron
M_N
μ_B
Neutron stars ('30s-)

State of matter: overview

($n_0 = 0.16\ \mathrm{fm}^{-3}$) [Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

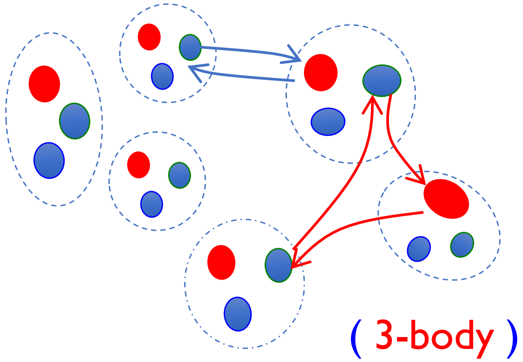


ab-initio nuclear cal.

laboratory experiments

steady progress

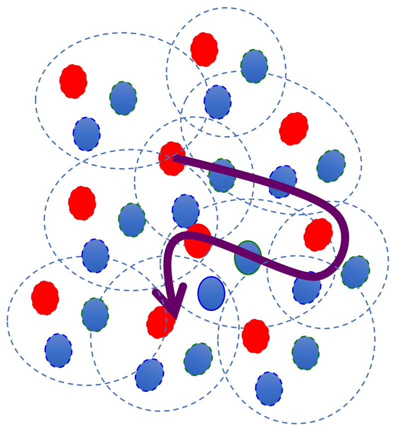
- many-quark exchange
- structural change,...
- hyperons, Δ, ...



most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

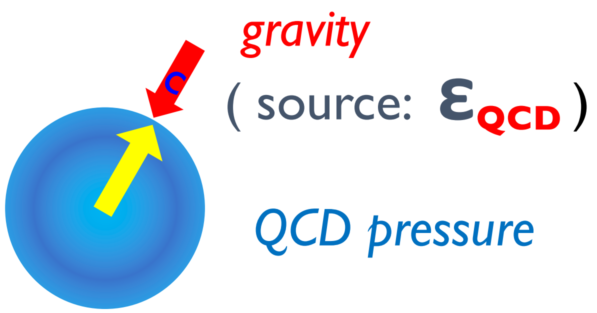
[Freedman-McLerran, Kurkela+, Fujimoto+...]

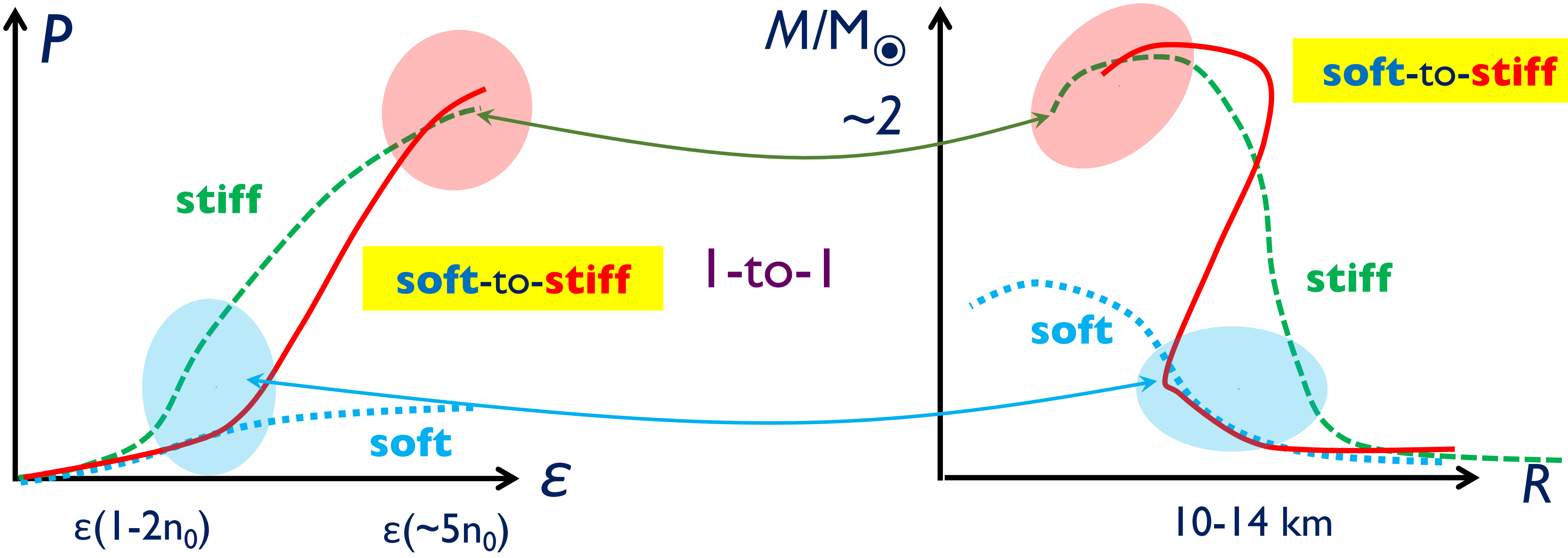
~ 1.4 $M_\odot$ ~ 2 $M_\odot$ n_B

~ $2n_0$ **Hints from NS** ~ $5n_0$ ~ $40n_0$

EoS & Neutron Star M-R relation

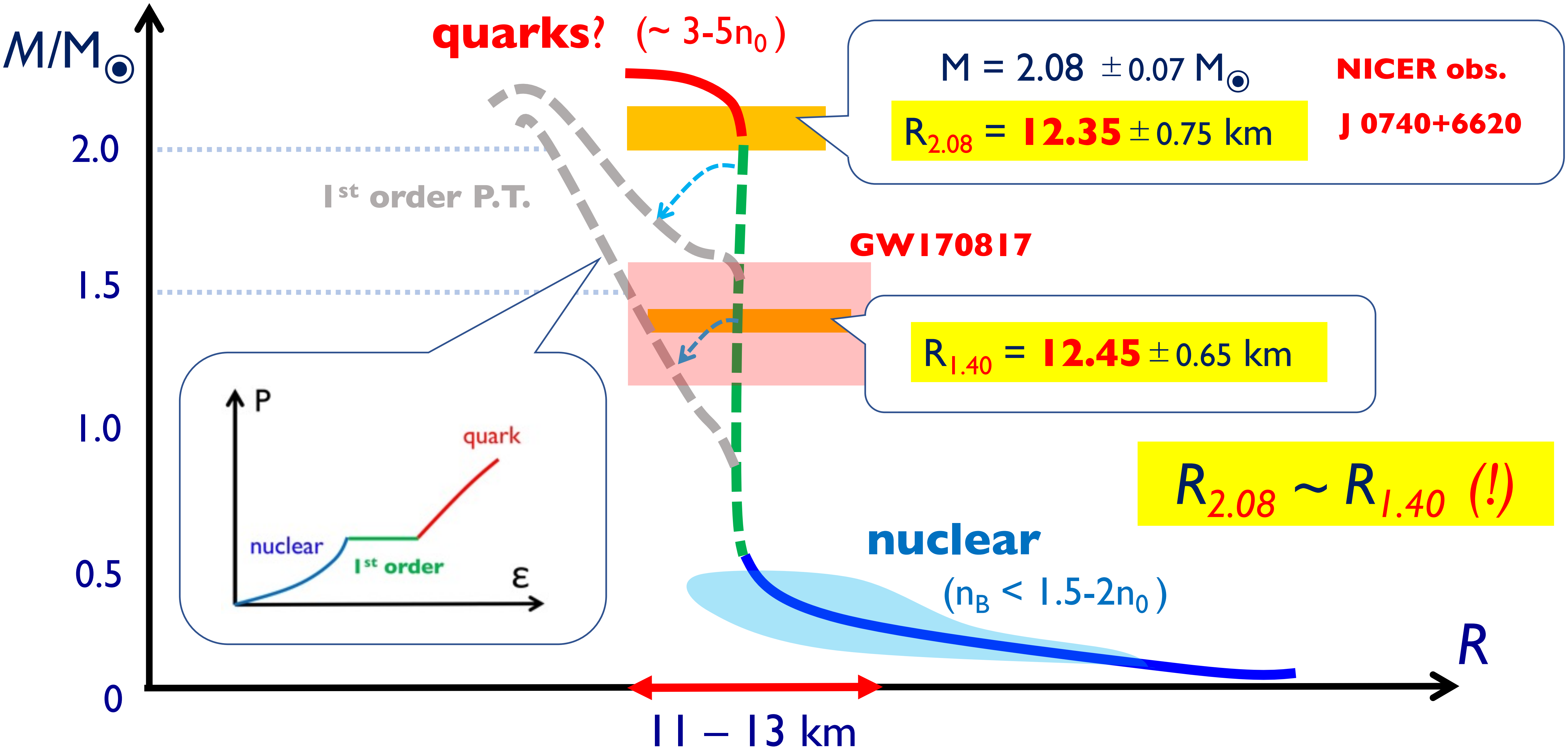
Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ ······· QCD (+EW) EoS





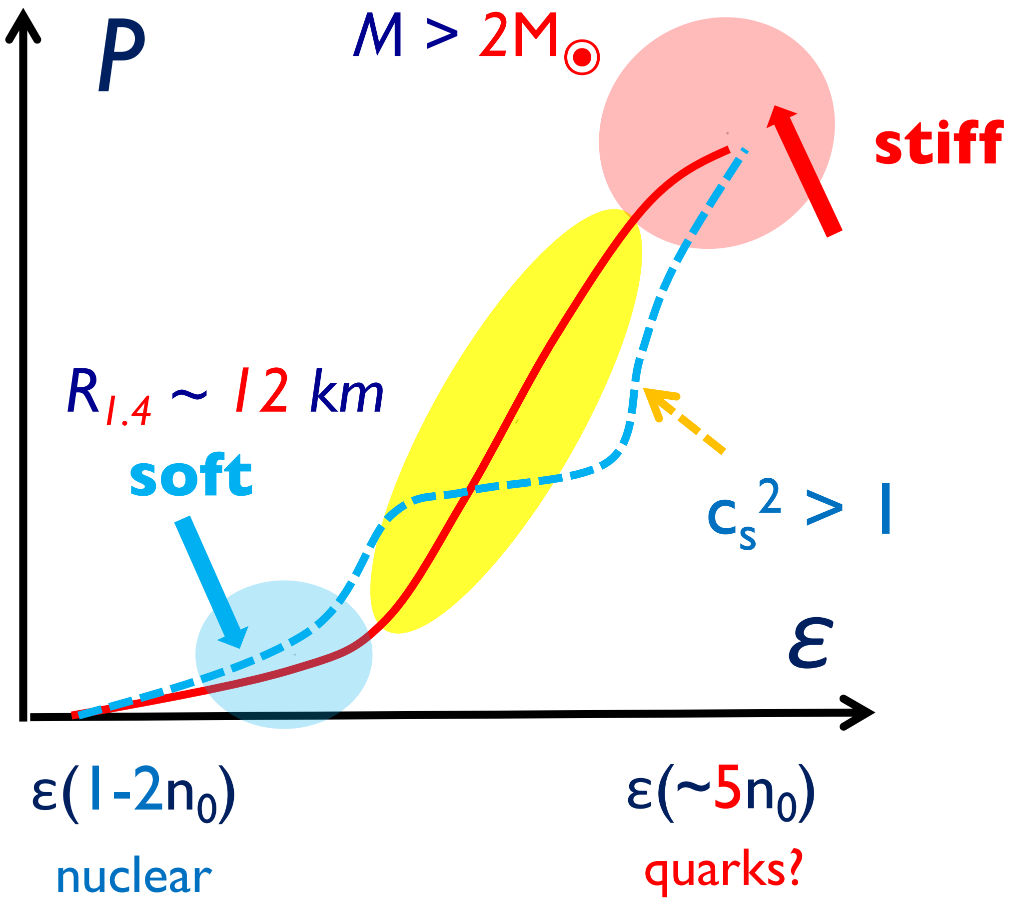
Observations: (NICER, GW170817, nuclear)

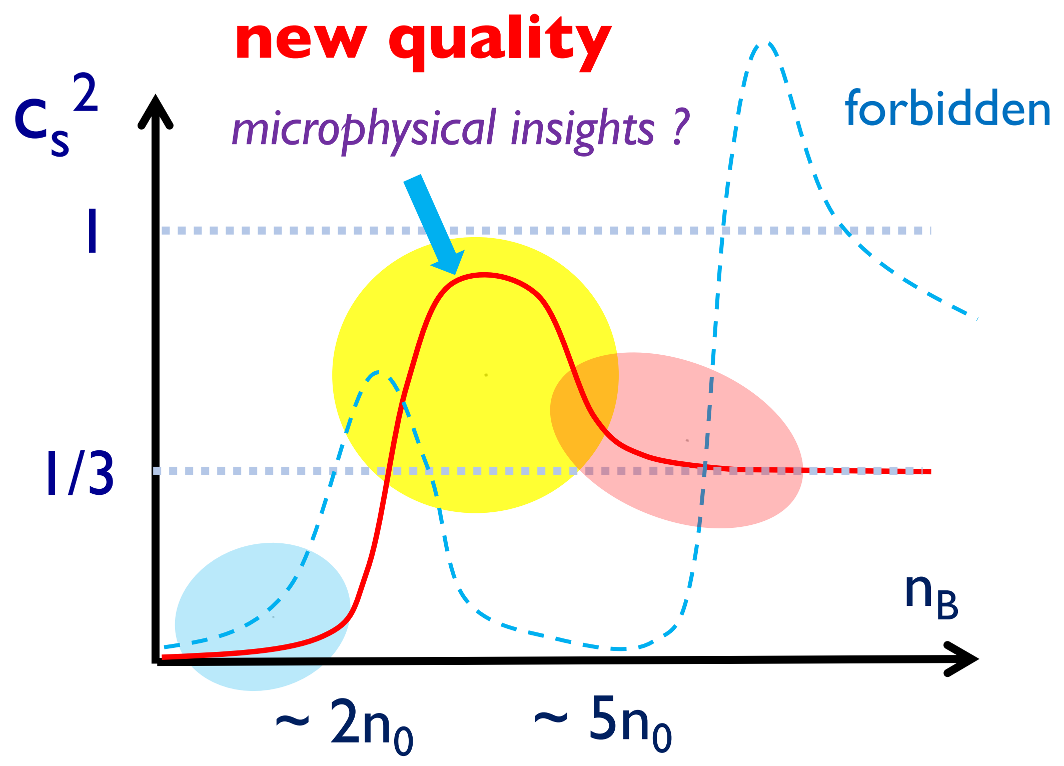
[e.g., Miller+ '21]



Soft to stiff is challenging:

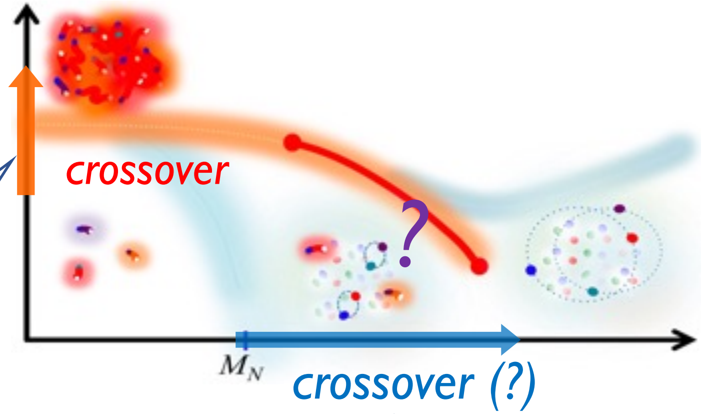
sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*) → *nuclear* & *quark* physics constrain each other

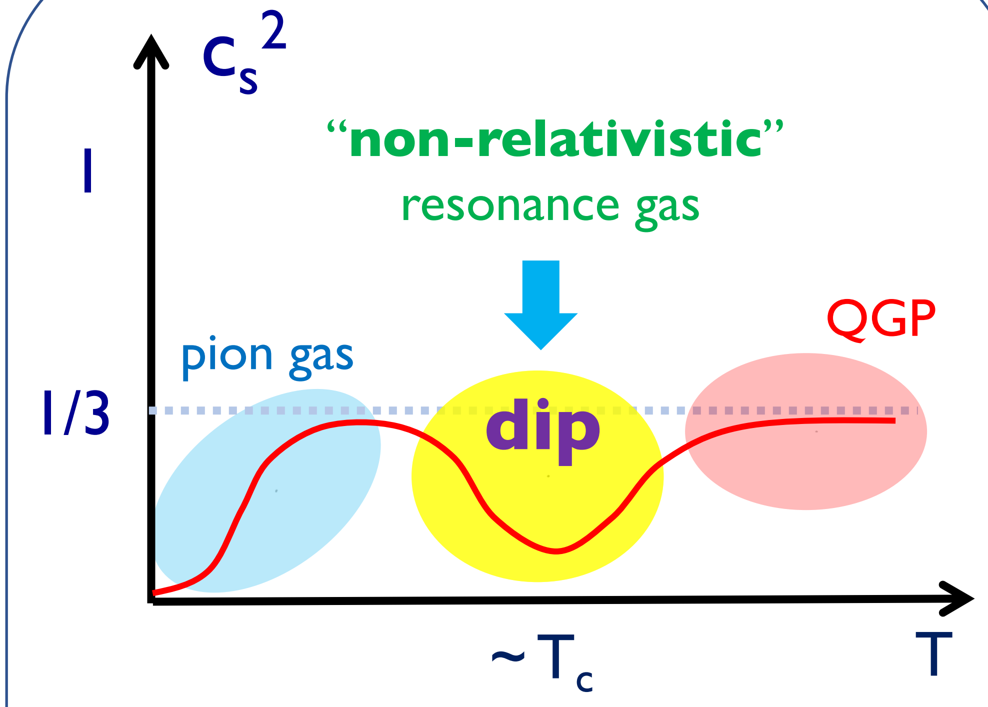




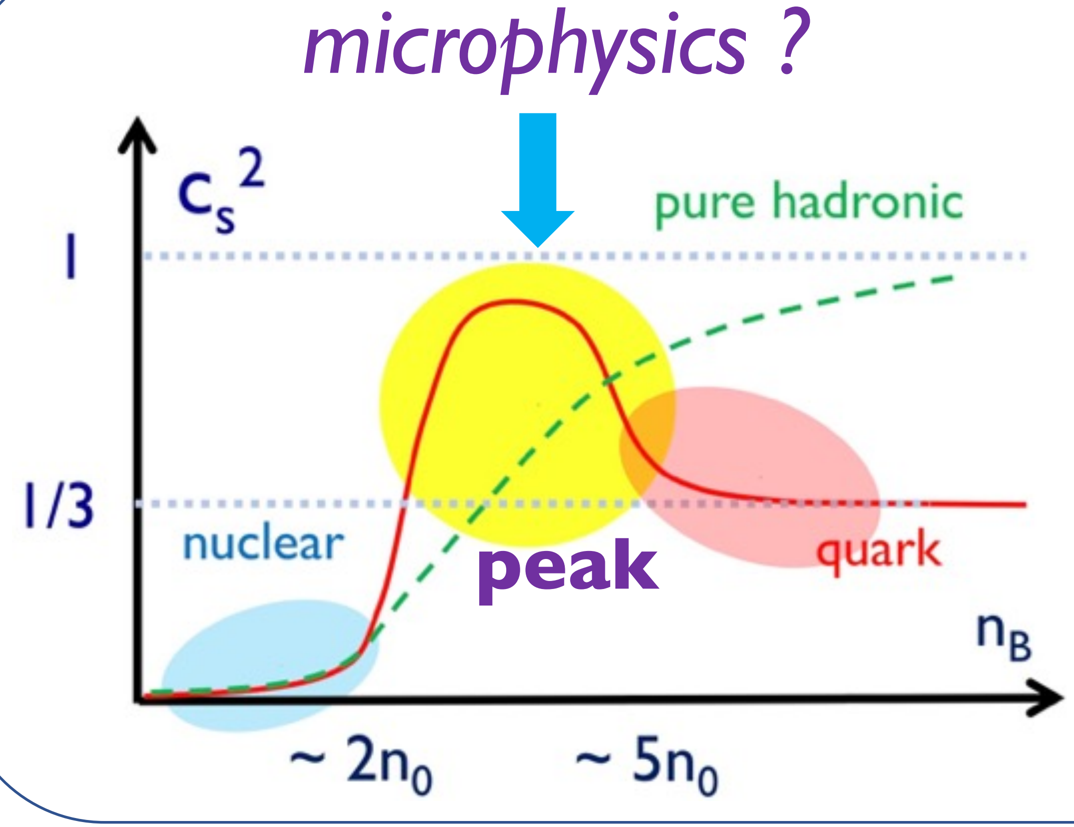
baseline: *quark-hadron continuity* **(QHC)**

Crossovers & $c_s^2 = dP/d\varepsilon$





hadrons with **m >> T** dominate
(**entropic** effects)



2, Stiffing in the crossover domain

- TK (2021): PRD104 (2021)7, 074005
- TK-Suenaga (2021): arXiv 2110.02100

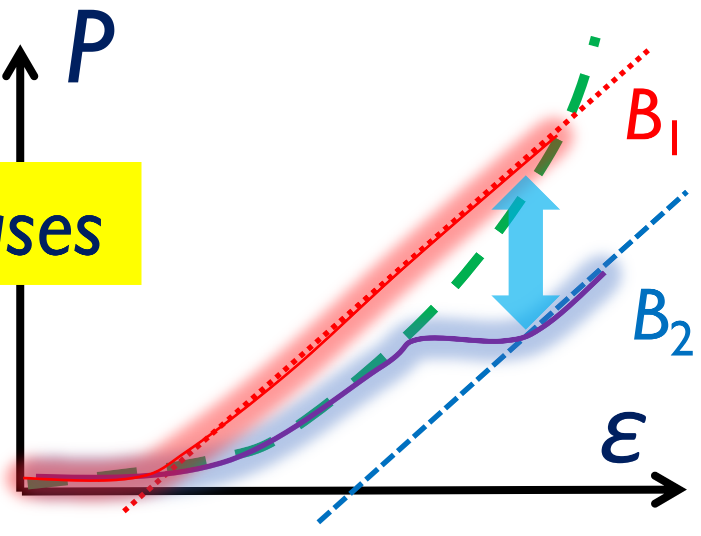
Direct descriptions for 2-5n_0 ?

confusing point:

- *Switching from baryonic to quark bases*

→ *a source of confusions in hybrid models*

(e.g. normalization of energy)



Strategy

Keep track of quark states from nuclear to quark matter

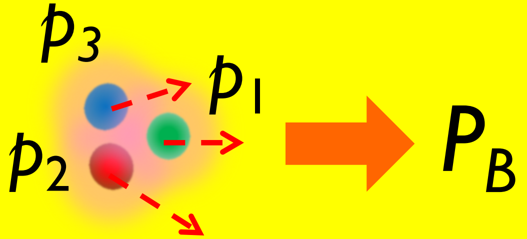
(within a *single* model, e.g., percolation model, Fukushima-TK-Weise '20)

Quarks in a baryon

N_c (=3): number of colors

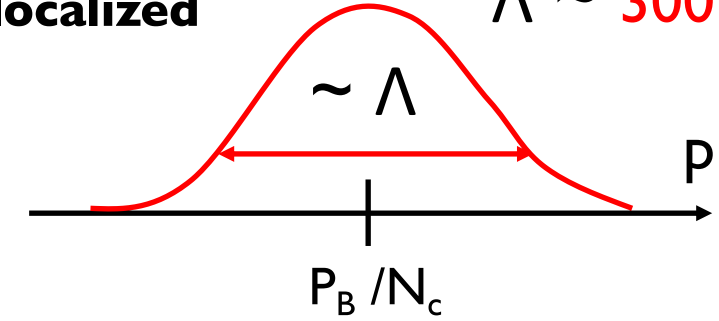
probability density:

$$Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B) = \mathcal{N} \mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_{\rm c}}\right)^2}$$



localized

$\Lambda \sim$ 300 MeV



mean: $\langle \boldsymbol{P}_B \rangle = N_{\rm c} \int_{\boldsymbol{p}} \boldsymbol{p} Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$

variance: $\left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_{\rm c}} \right)^2 \right\rangle \sim \Lambda^2$ *energetic !*

$$\langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B} = \mathcal{N} \int_{\boldsymbol{p}} E_q(\boldsymbol{p}) \mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_{\rm c}}\right)^2} \simeq \langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B = 0} + \frac{1}{6} \left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i} \right\rangle_{\boldsymbol{P}_B = 0} \left(\frac{\boldsymbol{P}_B}{N_{\rm c}} \right)^2 + \cdots$$

average energy (quark)

x N_c x N_c

$\sim N_c (M_q + E_{kin})$ ≫ $\sim P_B^2 / (N_c E_q)$

baryon mass baryon kin. energy

(short range correlations
→ included later)

A new **unified** model for **QHC**

cf) [TK '21, TK-Suenaga '21]

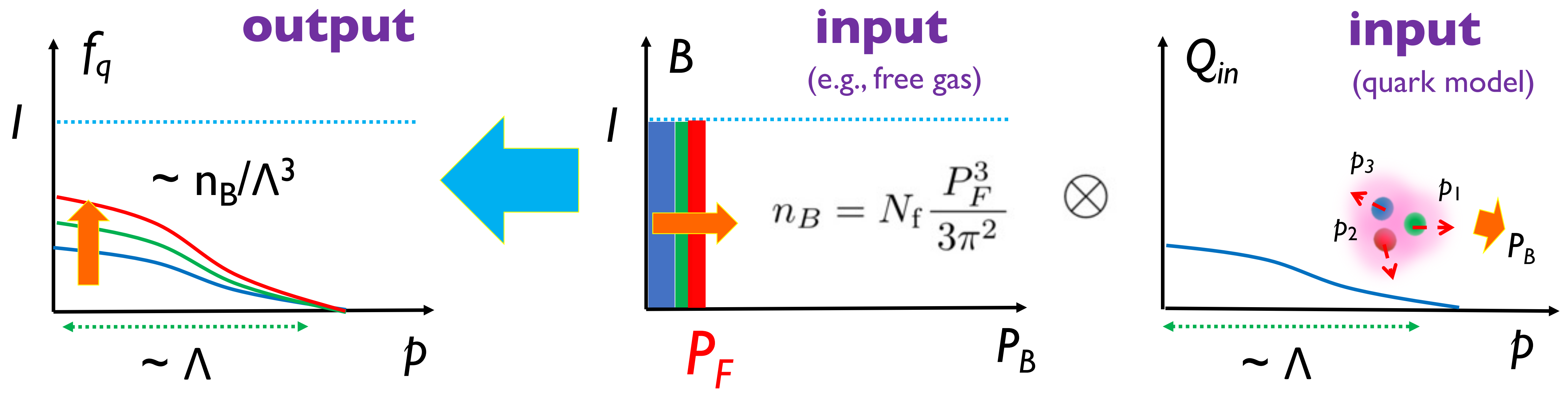
occupation **probability** of **quark** state with p

occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

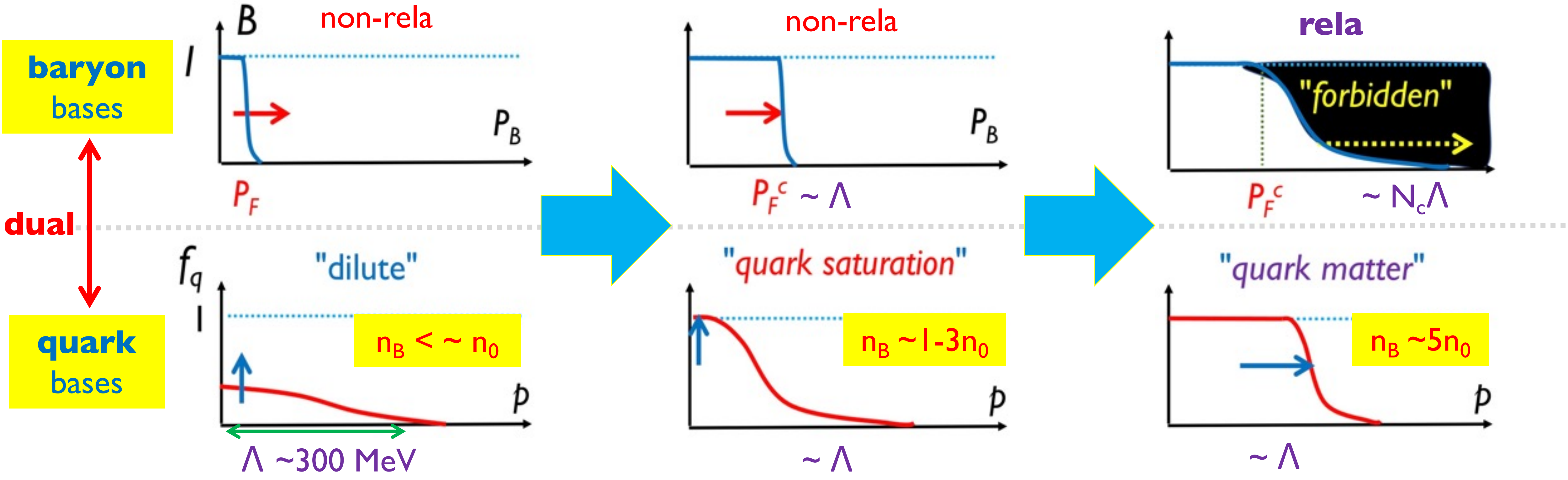
$$f_q(p;n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B;n_B) Q_{\rm in}(\boldsymbol{p},\boldsymbol{P}_B)$$

e.g.) in **ideal** baryonic matter



Evolution of occ. probabilities

$$f_q(p;n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B;n_B) Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$

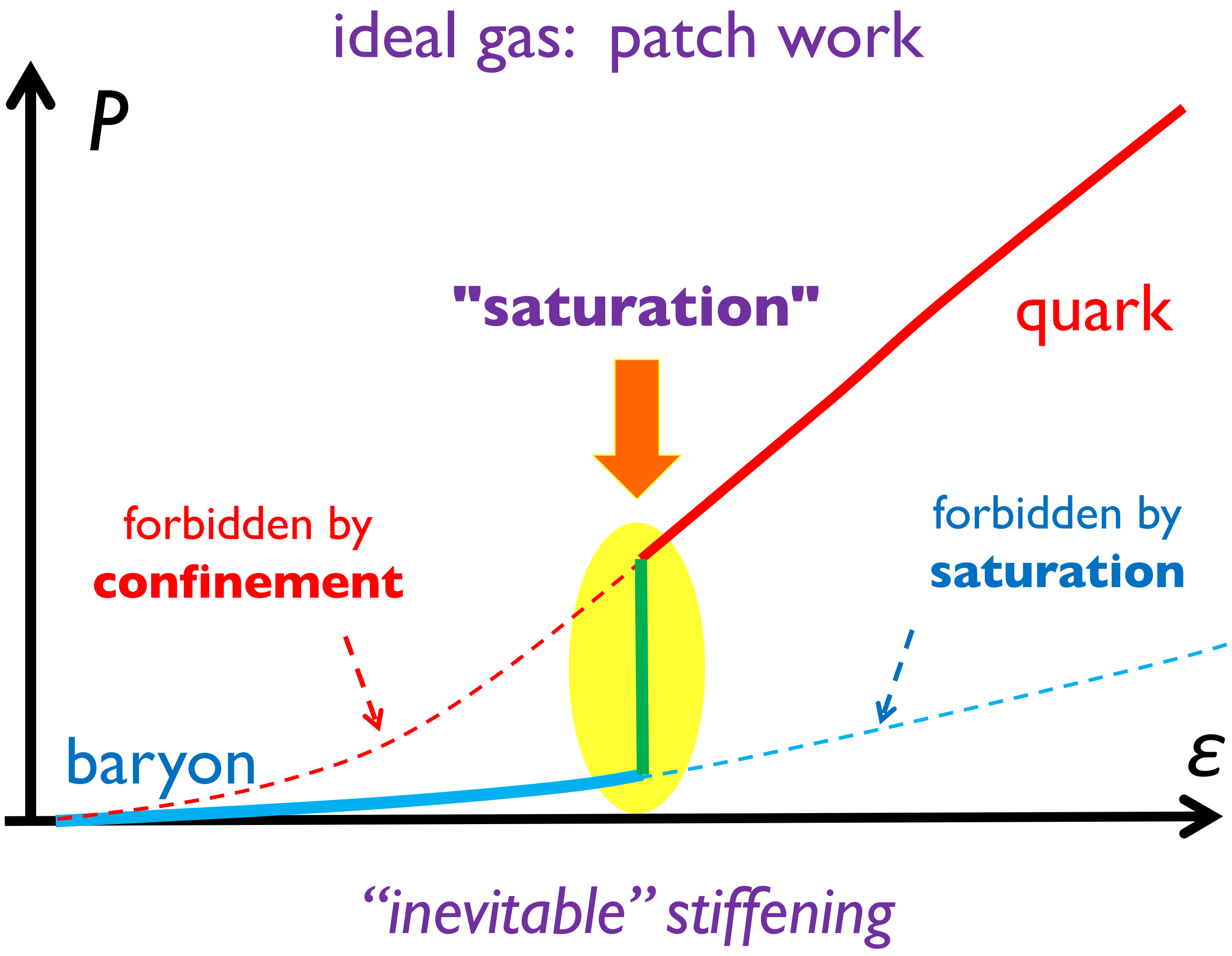


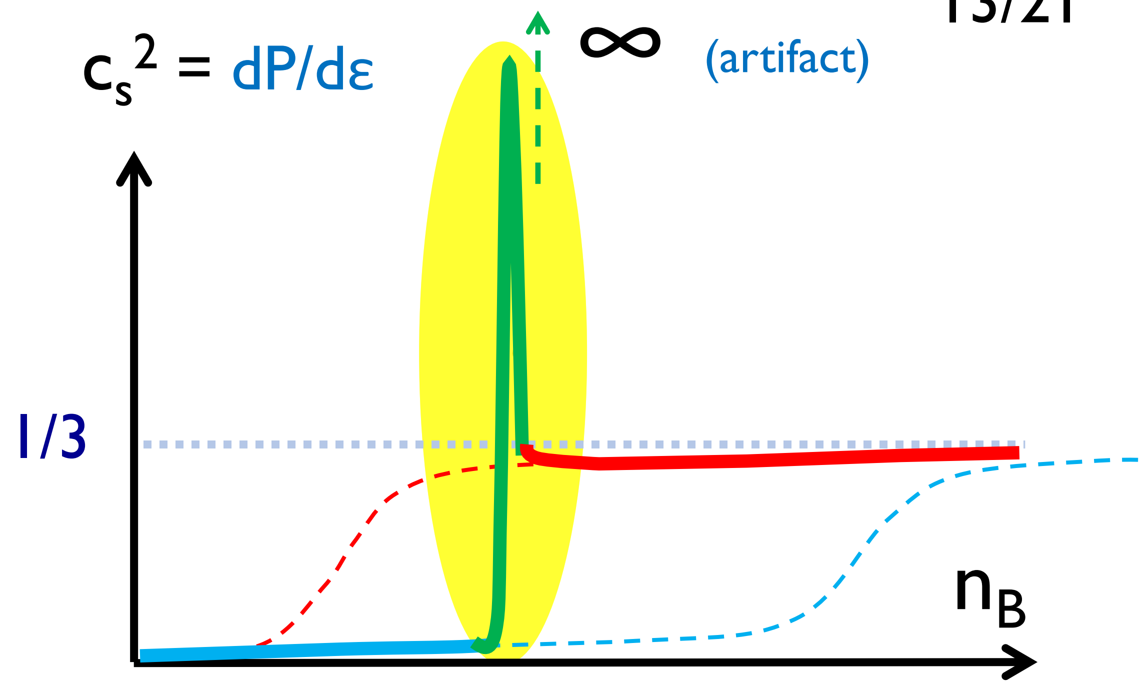
"**quark saturation**" constraint

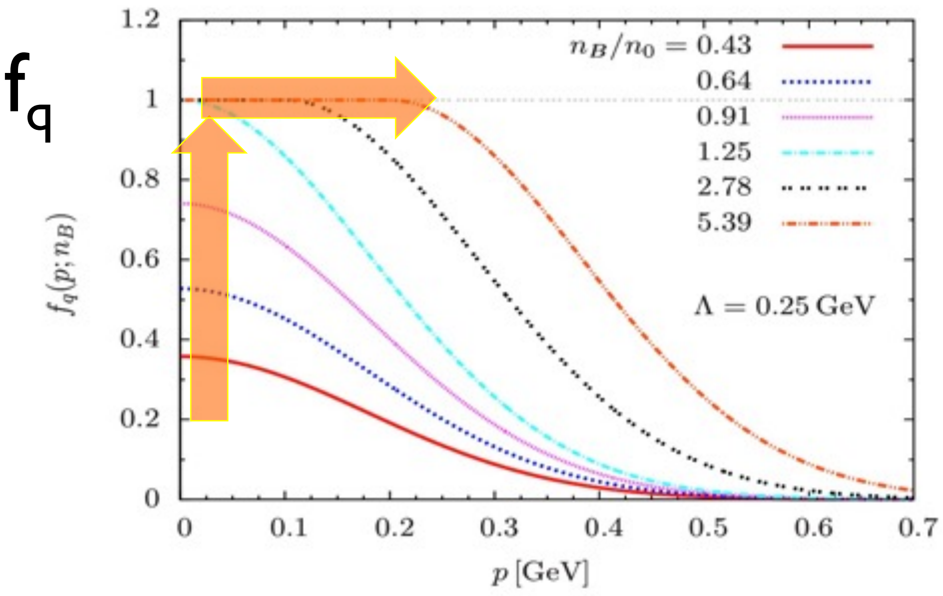
→ **relativistic baryons at low density, n_B ~ 1-3n_0 !**

cf) McLerran-Reddy model (2018) of quarkyonic matter

Peak in sound velocity

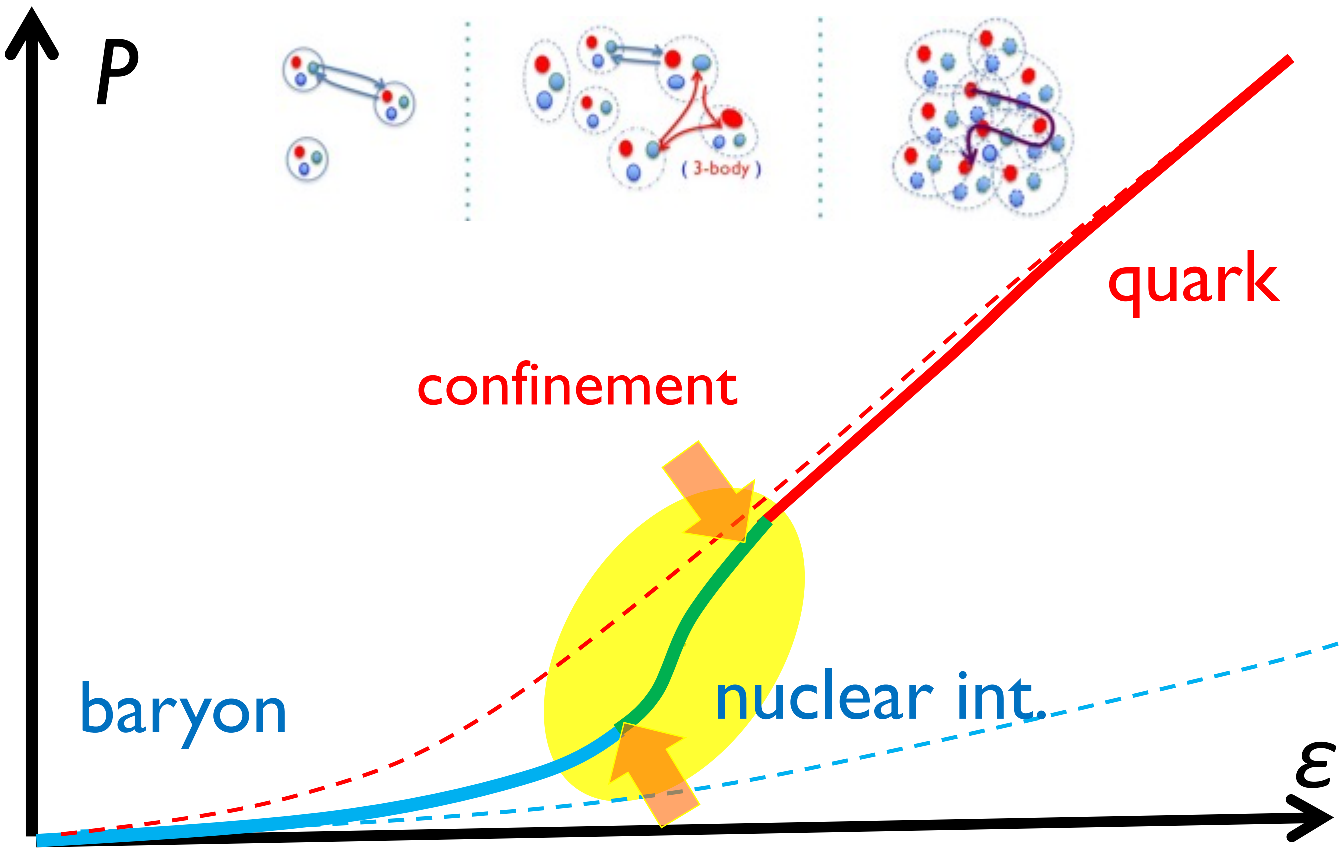




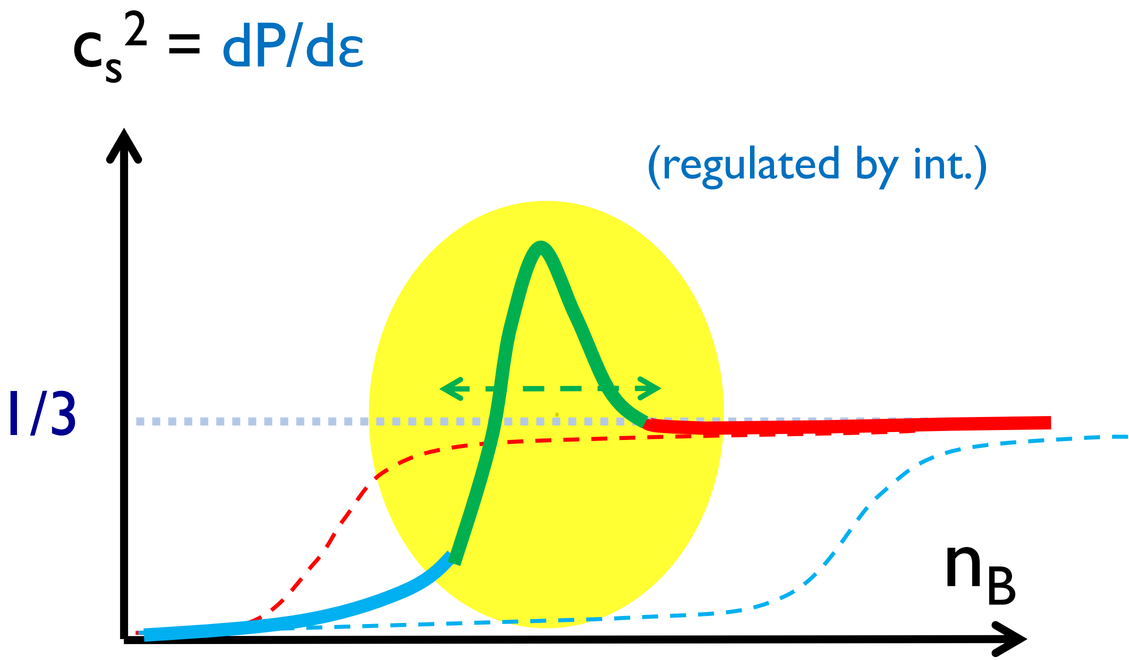


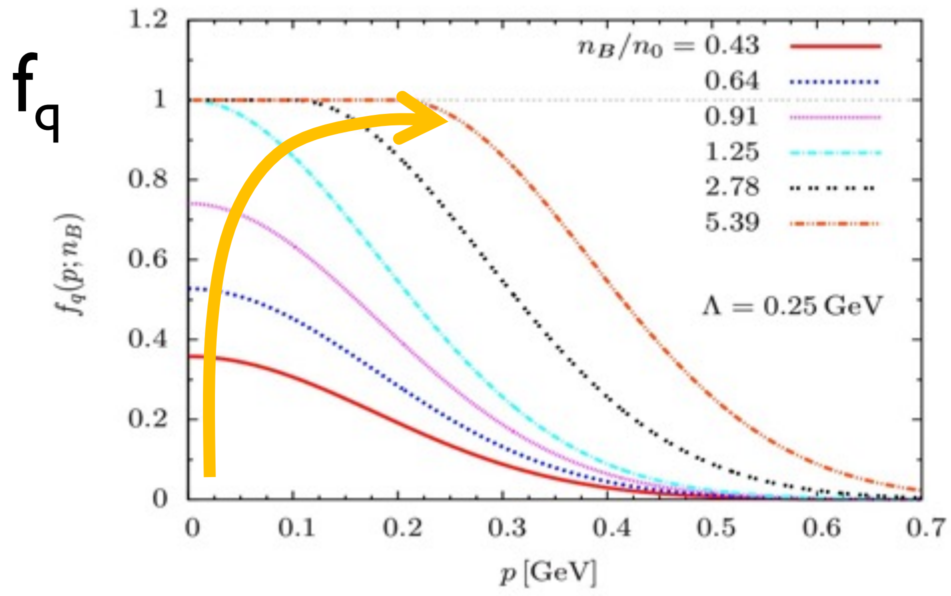
Peak in sound velocity

more realistic picture



nuclear interactions as
"perturbations"

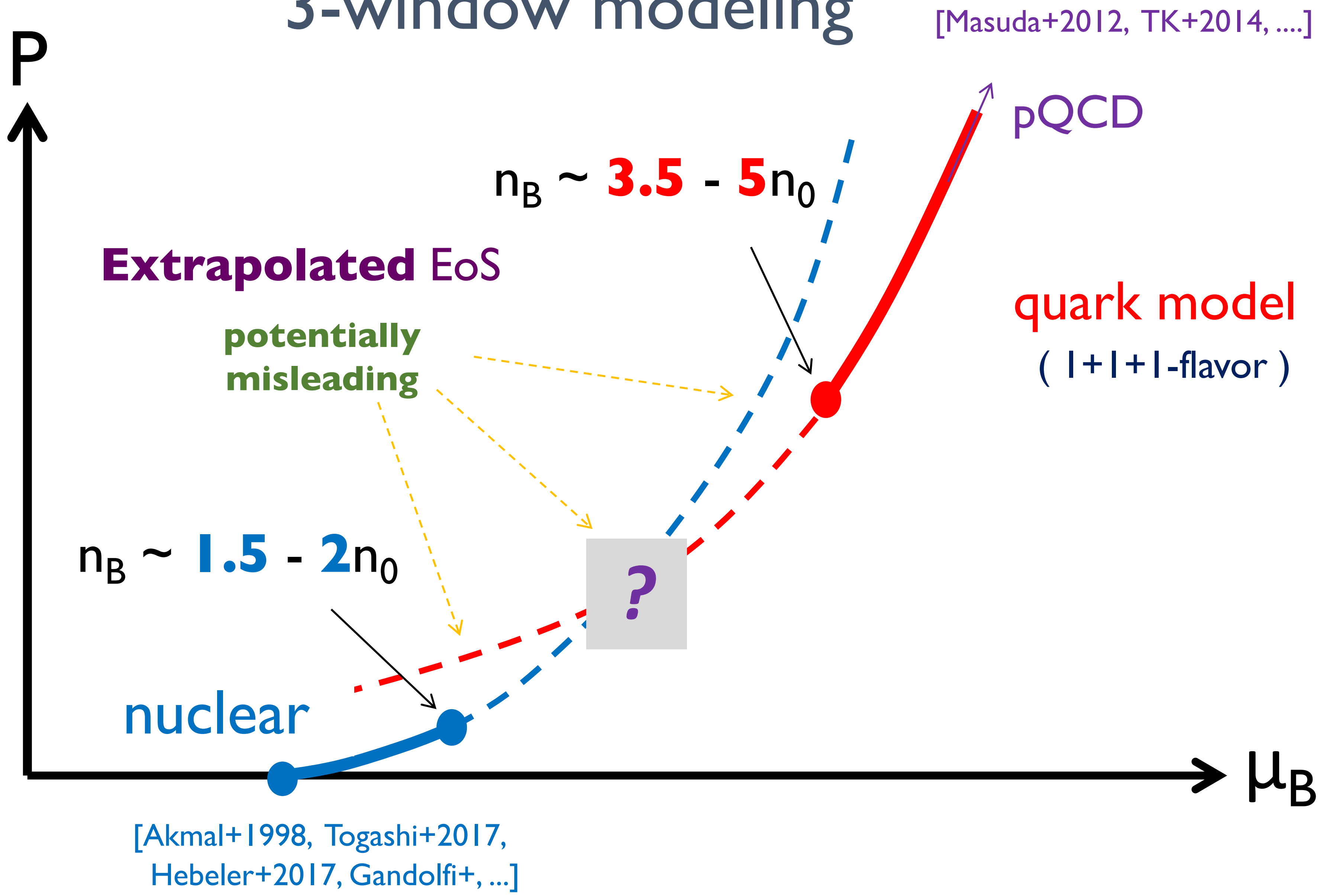




3, QHC21: quark EOS and 3-window modeling

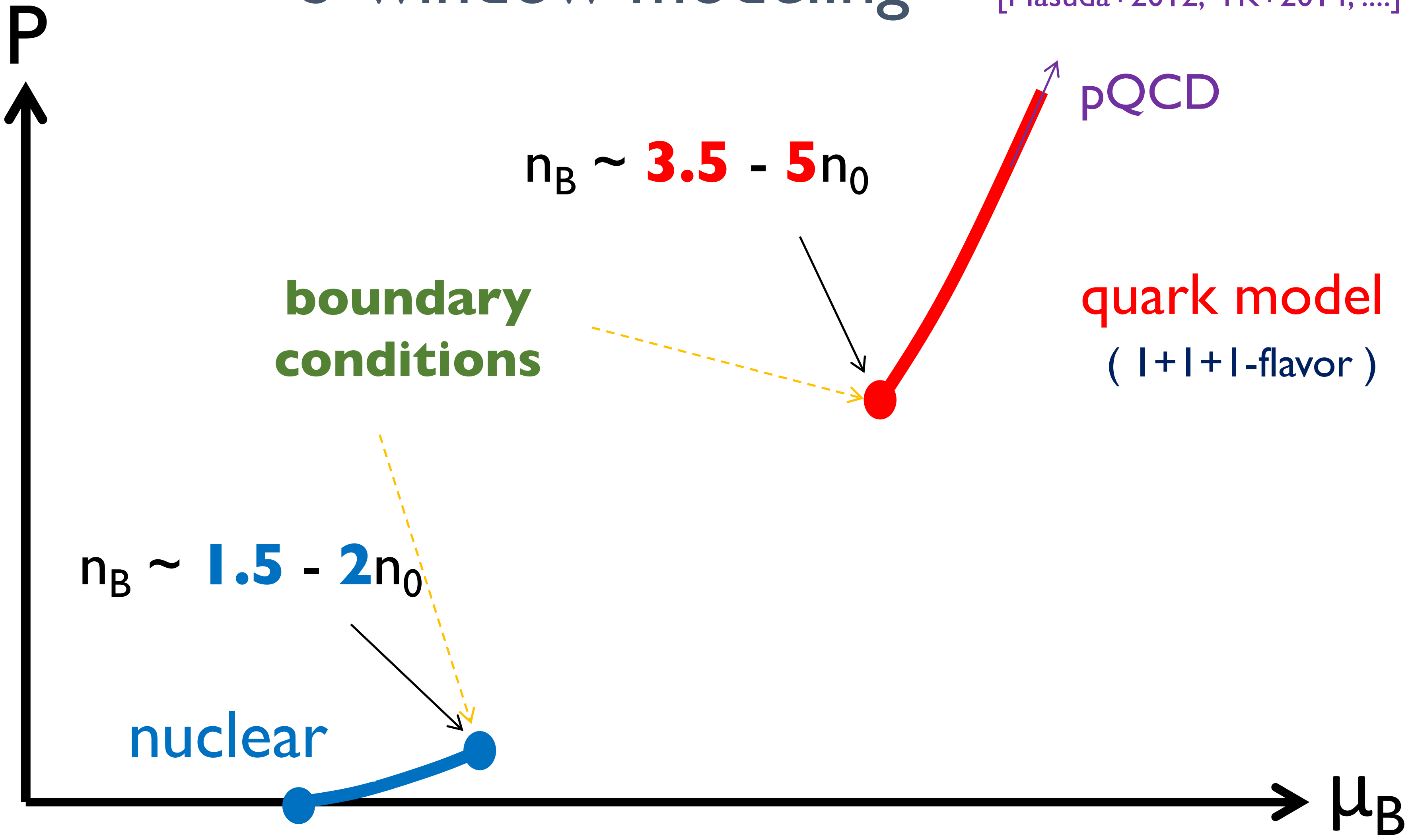
- Baym-Hatsuda-TK-Powell-Song-Takatsuka (2018): QHC18
- Baym-Furusawa-Hatsuda-TK-Togashi (2019): QHC19-Togashi
- TK-Baym-Hatsuda (2021): QHC21-χ & QHC21-T

3-window modeling



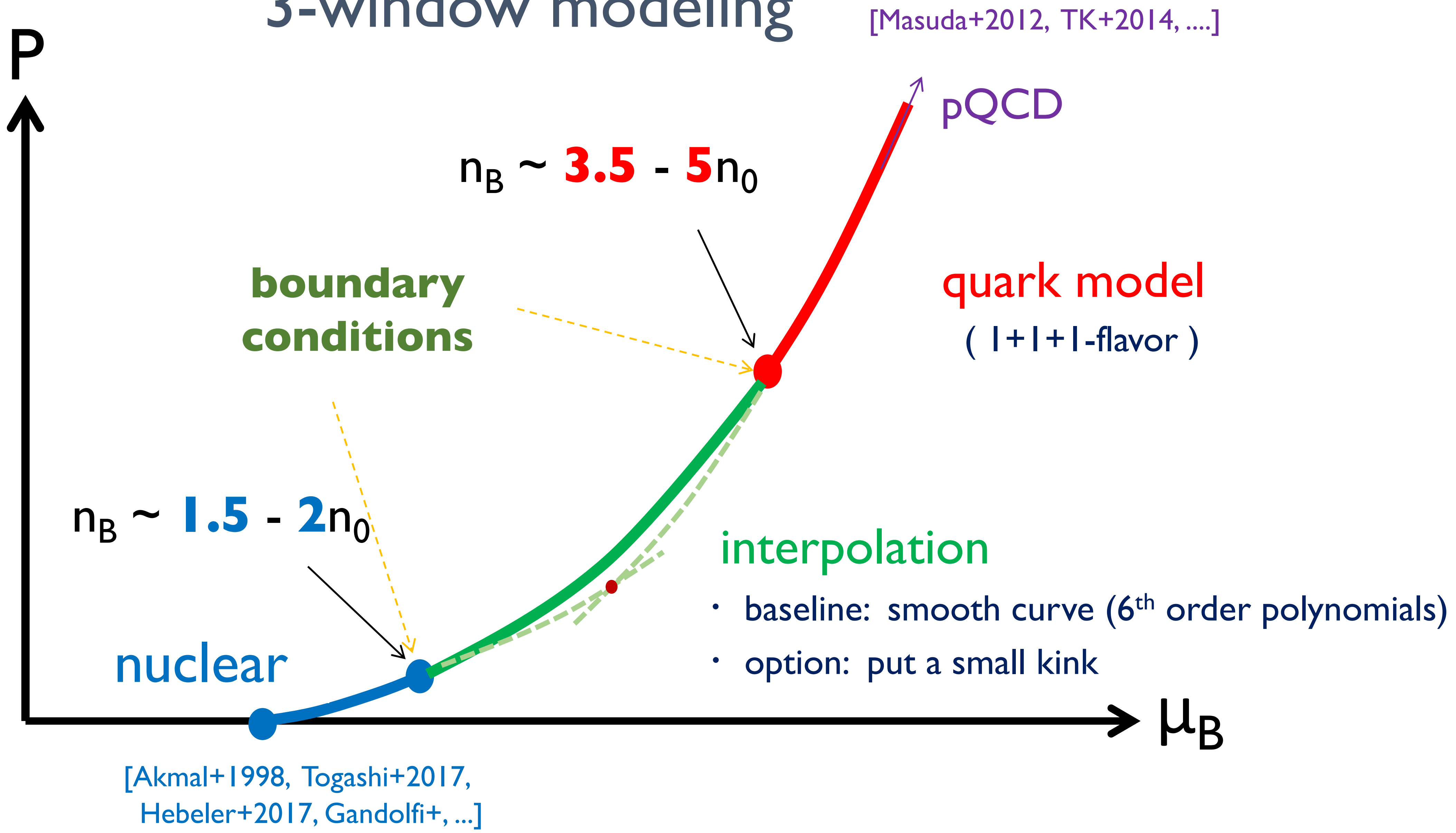
3-window modeling

[Masuda+2012, TK+2014, ...]

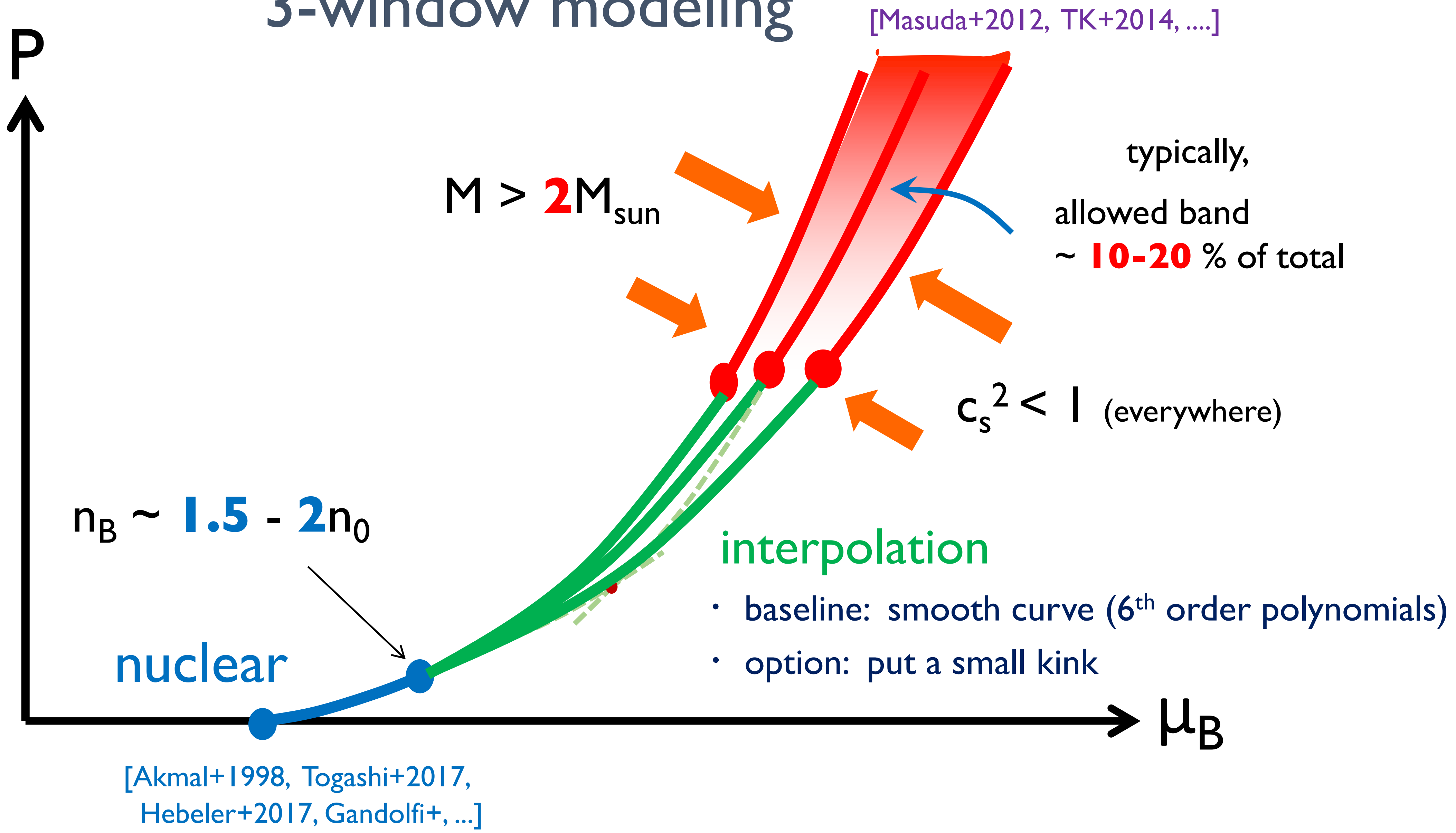


[Akmal+1998, Togashi+2017,
Hebeler+2017, Gandolfi+, ...]

3-window modeling



3-window modeling



Stiff quark EoS ? : *a guide*

cf) [TK-Powell-Song-Baym, '14]

rela. kin. energy   interactions

$$\varepsilon(n) = a n^{4/3} + b n^{\alpha}$$

(n: quark density)

ideal gas   interactions

$$P = \frac{\varepsilon}{3} + b\left(\alpha - \frac{4}{3}\right) n^{\alpha}$$

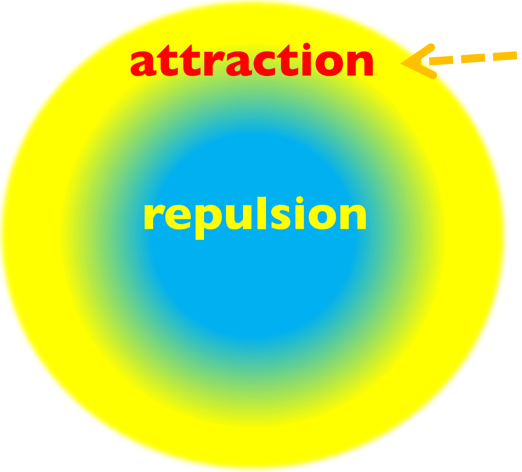
For stiff EoS:

(for large P)

for $\alpha > 4/3$:   b > 0   (e.g. bulk repulsion, ~ $+ n_B^2/\Lambda^2$)

for $\alpha < 4/3$:   b < 0   (e.g. surface pairings, ~ $- \Lambda^2 n_B^{2/3}$)

quark
Fermi sea
(ideal combo)



2- or 3-quark correlations

"Exotic" Fermi surface stiffens EoS !

Reminder: QCD int. are very **channel dependent**

A quark model for $n_B > \sim 5 n_0$ ($\sim 1 fm^{-3}$)

A guide : *Quark-Hadron Continuity* : eff. Hamiltonian continuously evolves from hadron physics

"**3-window**" [Manohar-Georgi 1983, Weinberg 2010,...]

Q < ~0.2 GeV	0.2 GeV < Q < 1-2 GeV	~2 GeV < Q
very long-range (> 1fm)	constituent quarks + OGE (quasi-particles)	short range
confinement	**chiral** SB & color-**mag**. int. & **baryon-baryon**. int.	pQCD

A template) chiral color-mag. nB-nB int.

$$\mathcal{H} = \mathcal{H}_{\rm NJL} - \underline{H} \sum_A (q\Gamma_A q)(\bar{q}\Gamma_A \bar{q}) + \underline{g_V} (\bar{q}\gamma_0 q)^2$$

solve within **MF**
+ color- & charge- neutrality
+ β-equilibrium

[Masuda+2015, TK+2014, Blaschke+....]

(gv, H): both inspired from color-mag. interactions

[Oka-Yazaki '80, Park-Lee-

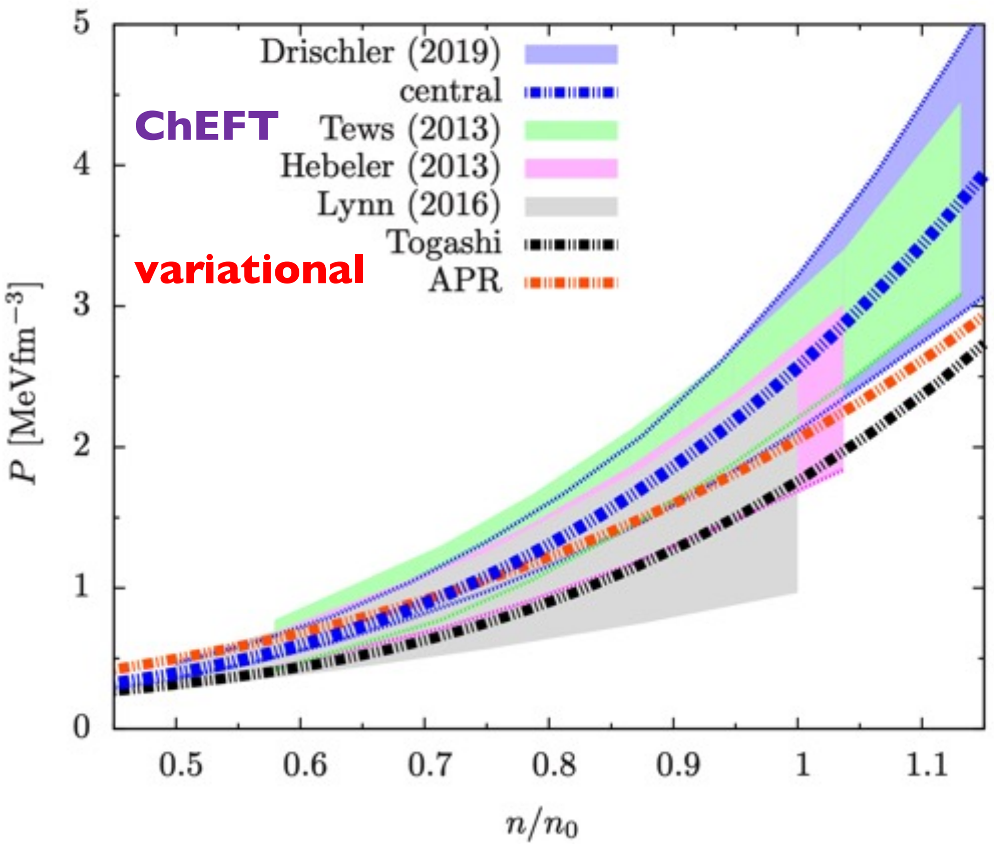
An exercise: survey for (g_V, H)@3.5-5n0

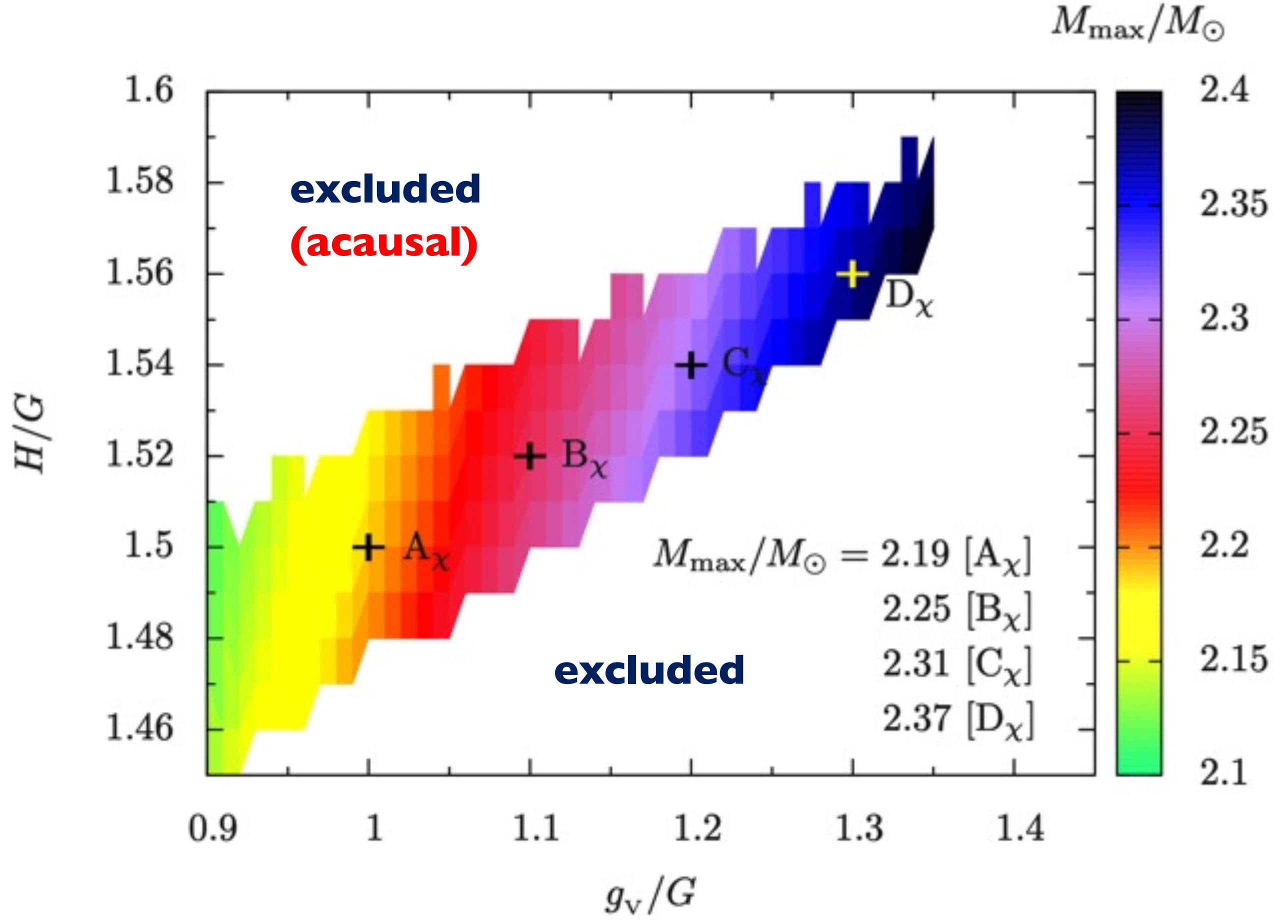
[Baym+ '19, TK '21]

Step1) Prepare **realistic** nuclear EoS up to **1.5-2**n_0

[e.g. Akmal+1998, **Togashi+2017**, **ChEFT**, ...] → 30-40% uncertainties in P @ ~ n_0

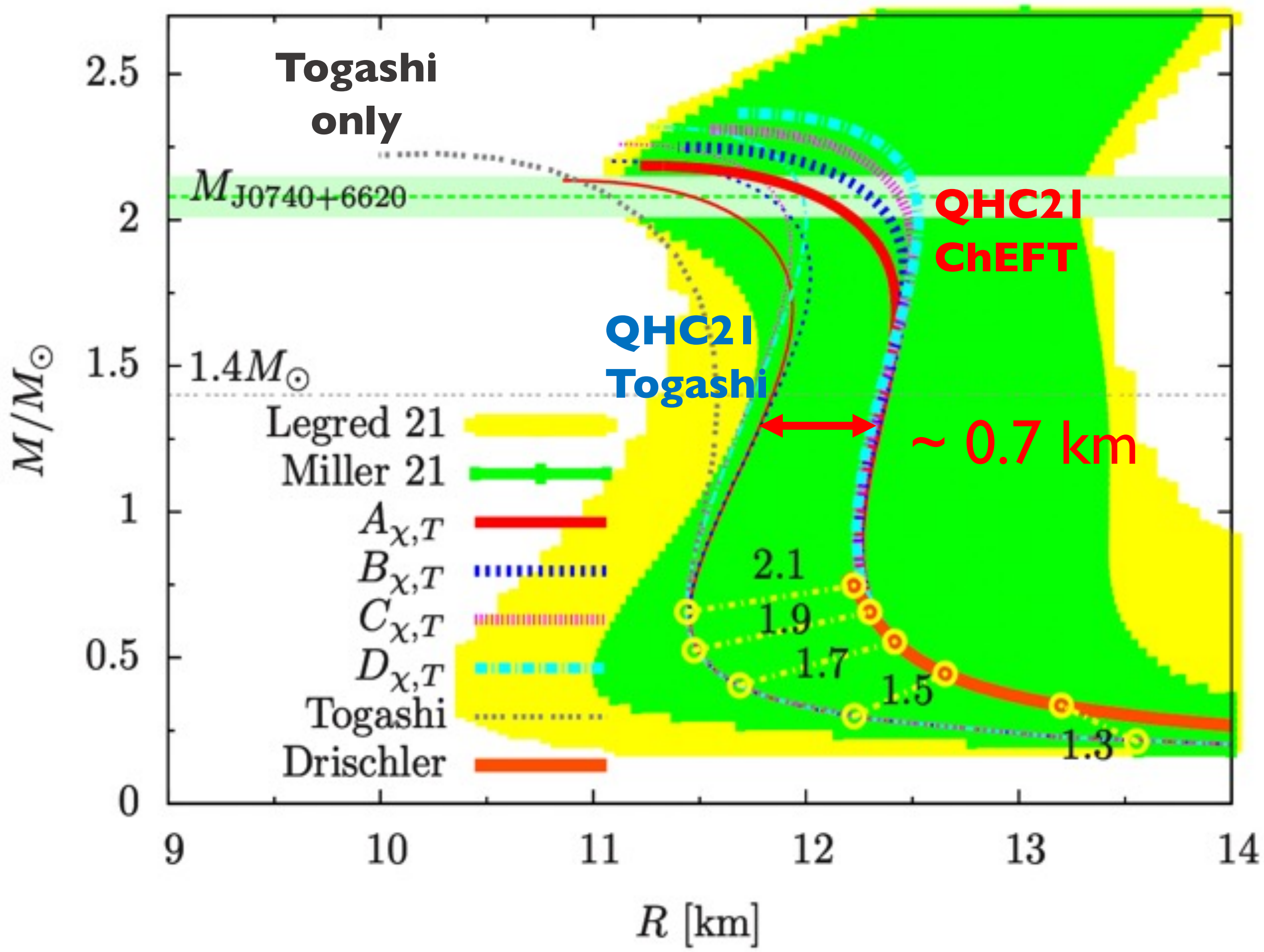
Step2) Survey the range of (g_V, H) consistent with **causality & stability**

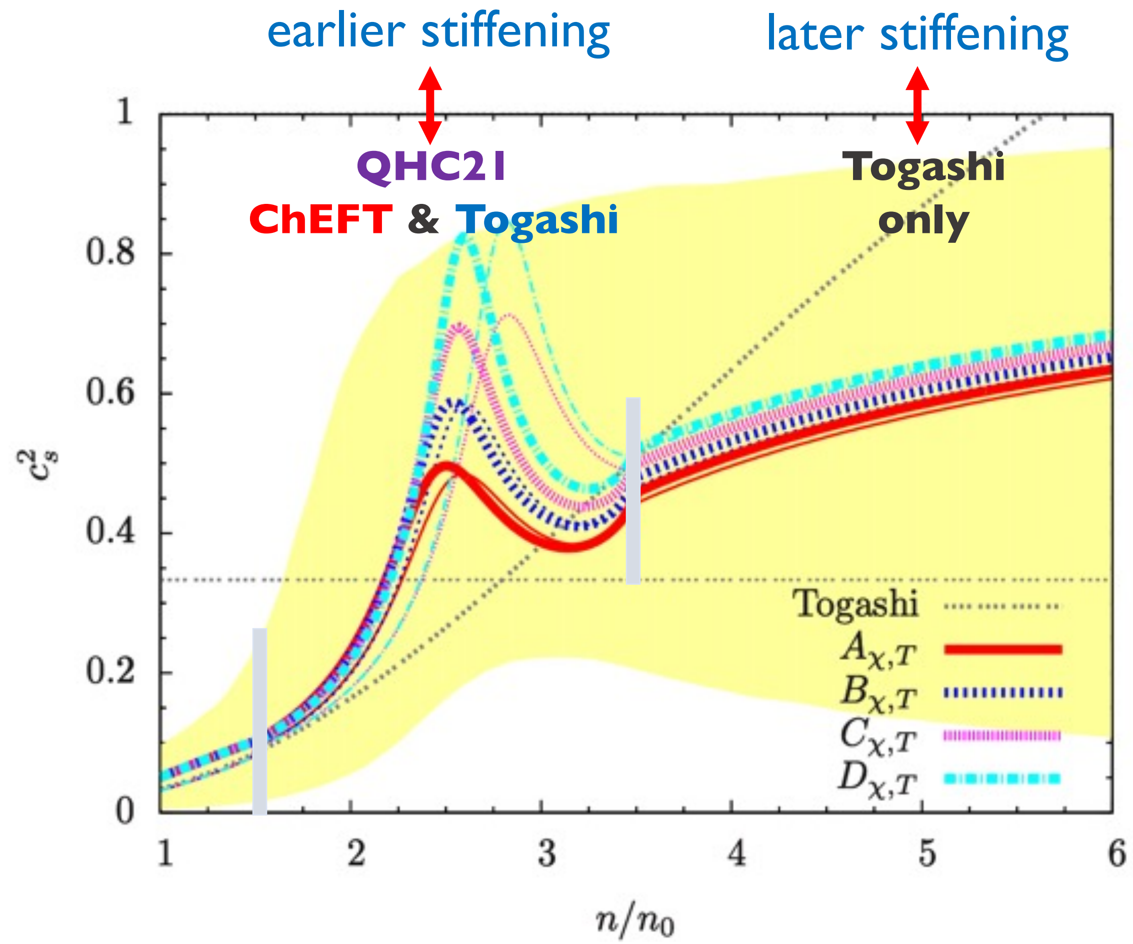




An exercise: survey for (g_V, H) @3.5-5n0

[Baym+ '19, TK '21]





- nuclear uncertainties → $\Delta R_{1.4}$ ~ 0.7 km, but the peak in c_s^2 robust
- QHC type models → earlier stiffening than in pure hadronic models

Summary

$R_{2.08} \sim R_{1.40}$ (!) → **Quark-Hadron-Continuity** : a **baseline**

Peak in sound velocity → signature of quark matter formation

(quark saturation effects)

Stiff quark matter EoS → bulk repulsion & Fermi surface **attraction**

(disparity ← channel dep. of gluon exchanges)

Outlook

- modeling for crossover; 3-window model ⟶ a model of quark saturation (TK '21)
- Structural changes in hadrons & percolation (Fukushima-TK-Weise '20)
- chiral condensates, flavor composition, etc. → Harada-san & Minamikawa-san's talks

Hadron physics for dense QCD